YISHUN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023
SECONDARY 4 EXPRESS

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS   **4052/01**
Paper 1   **21 August 2023**
                                                     2 hours 15 minutes
Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For examiner's use

90

[Turn over

2

This document consists of **21** printed pages and **1** blank page.

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

3

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** questions

1   Calculate $121 - \sqrt{\dfrac{98}{0.61}} - 80$.

Write your answer correct to 4 significant figures.

$121 - \sqrt{\dfrac{98}{0.61}} - 80$

$= 112.019$

$= 112.0 (4sf)$ ------ B1

Answer .... [1]

---

2   The estimated population of Taiwan is 23.6 million.

The estimated total land area is 3.62×10^4 km².

Calculate the average number of people per square kilometre of land area in Taiwan, giving your answers in standard form.

$\dfrac{23.6 \times 10^6}{3.62 \times 10^4}$ ------- B1 convert million

$= 651.933$

$= 6.52 \times 10^2$ ------- A1

Answer .... [2]

---

3   When 540 is divided by m, a perfect square is formed.
Find the smallest value of m, where m is an integer.

[Turn over

4

Answer m=............................ [2]

4   The scale of a map is 4 cm : 28 km.
     (a)   Find the length of a canal that is represented by 10 cm on the map.

Answer .... [1]

(b)   A lake is represented by an area of 400 cm² on the map.
           Calculate the actual area of the lake in square kilometres.

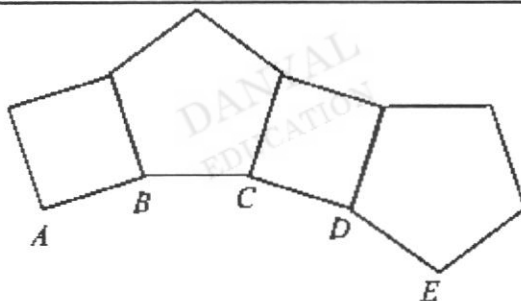*Answer* .... km² [2]

5   **(a)**   Express $22+10x+x^2$ in the form of $p+(q+x)^2$.

5

Answer .... [2]

(b)   Write down the equation of line of symmetry for $y = 22 + 10x + x^2$.

Answer .... [1]

6

A regular pentagon is joined to a square repeatedly until it forms a regular polygon.
AB, *BC*, *CD* and *DE* are edges of the regular polygon.

Find the number of sides in this regular polygon.

[Turn over

6

Answer .... [3]

___

7   r is directly proportional to t^3 .
     When t is decreased by 25%, find the percentage change in r.

Answer .... [2]

8   Sketch the graph of $y = -(x+2)(x-3)$ on the axes below.
     Indicate clearly the values where the graph crosses the x- and y- axes and its turning point.

[2]

___

7

9   The solution of the inequality $-2 \le \dfrac{4-2x}{3} \le 2k+5$, where k is a constant, is represented on the number line below. Find the value of k.

Answer $k =$ .... [4]

10 Zac has written down five numbers.
    The mean of these numbers is 5.2, the median is 4 and the mode is 2.
    The largest number is five times the smallest number.

    Find the five numbers.

Answer ...., ..., ..., ..., ... [2]

[Turn over

8

11　(a)　Simplify $\left(\dfrac{2x^3}{\sqrt{x^4}}\right)^{-3}$.

Answer .... [2]

(b)　Solve $\dfrac{3^{x+1}}{27^x}=1$.

Answer $x=$ .... [2]

12　Object A and object B are geometrically similar.

The ratio of the base area of object A to the base area of object B is $4:9$.
Find the ratio of the volume of object A to volume of object B.

9

Answer .... : .... [2]

13   *n* is a positive integer.

Show that, for all *n*, $(2n+3)^2 - (2n-3)^2$ is not a prime number.

Answer
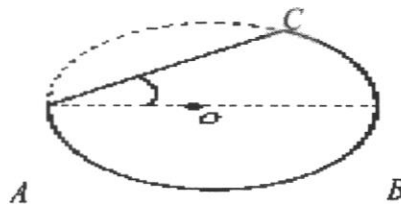
$$(2n+3)^2 - (2n-3)^2 = 4n^2 + 12n + 9 - (4n^2 - 12n + 9)$$
$$= 24n \qquad \text{---- M1 quadratic expansion}$$

A prime number can only be divided by 1 or itself.

Since 24 is not prime, $(2n+3)^2 - (2n-3)^2$ can be divided by all factors of 24, making it not prime.

A1 – Explanation based on definition of prime numbers.

[2]

14

The diagram shows a major segment *ABC* of a circle with centre *O* and radius 5 cm.

Angle $OAC = \dfrac{\pi}{6}$.

[Turn over

10

Find the area of the major segment *ABC*.

Answer ....cm^2 [3]

---

15 **(a)** Factorise completely $12pq - 3p + q - 4q^2$.

Answer .... [2]

(b) Simplify $\dfrac{3x^2 + 15x - 42}{x^2 - 4}$.

Answer .... [3]

---

16 The volume of a cone with radius x and height y is twice the volume of a hemisphere with radius x.

$\dfrac{1}{27}$ of the cone is filled with water.

This water is poured into the hemisphere.

Find the fraction of the hemisphere that is filled with water.

[Turn over

12

Answer .... [4]

17   The graph shows information about prices of the Certificate of Entitlement (COE) for buses for each
      month in year 2021 and 2022.

COE is a license for owning a vehicle in Singapore.

PartnerInLearning377

13
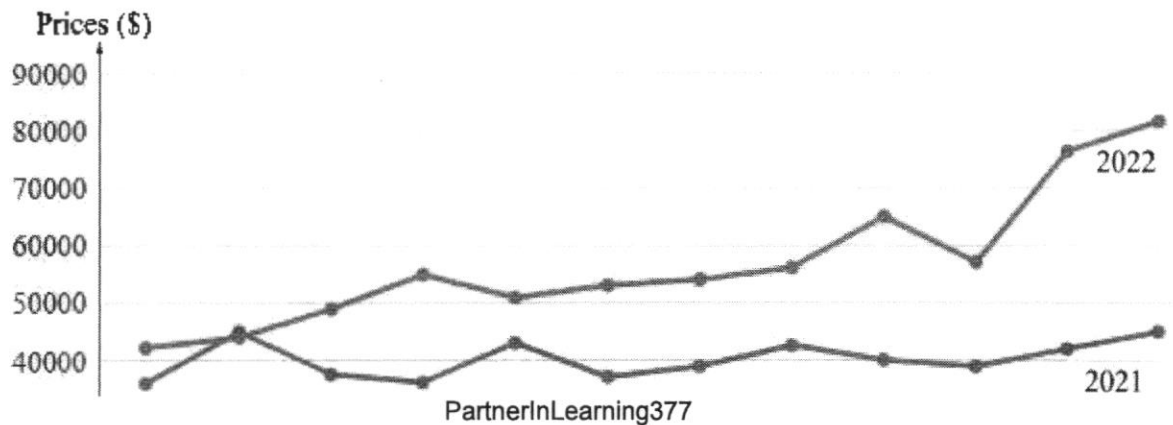
(a) Estimate the price of COE in July 2022.

Answer $............................. [1]

(b) A school bus company claims that the cost of buying new buses has increased and intends to increase the prices of the services they provide for the year 2023.

What feature of the graph can the company use to justify the company's claim?

Answer

[1]

(c) Make one comparison between the mean COE prices in 2021 and 2022. Justify your answer with references to the graph.

Answer

. [2]

---

18 **(a)** The cash price of a TV is $2100.
Ben buys this TV on hire purchase.
He pays a deposit of one fifth of the cash price.
He then makes 12 monthly payments of $160.

Find the total amount that Ben pays for the TV.

Answer $............................. [2]

14

(b) Alicia invested a sum of money in an account paying compound interest at 3% per annum. After 5 years, she earned a total interest of $3981.

Calculate the sum of money Alicia invested in the account.
Give your answer to the nearest dollar.

Answer $............................... [3]

---

19 In a sequence, the same number is subtracted each time to obtain the next term.
The first five terms of the sequence are

$$72 \quad p \quad q \quad 57 \quad r$$

(a) Find the values of p, q and r.

Answer $p =$ ....

$q =$ ....

$r =$ ....[2]

15

(b) Write down an expression for the *n*th term of this sequence.

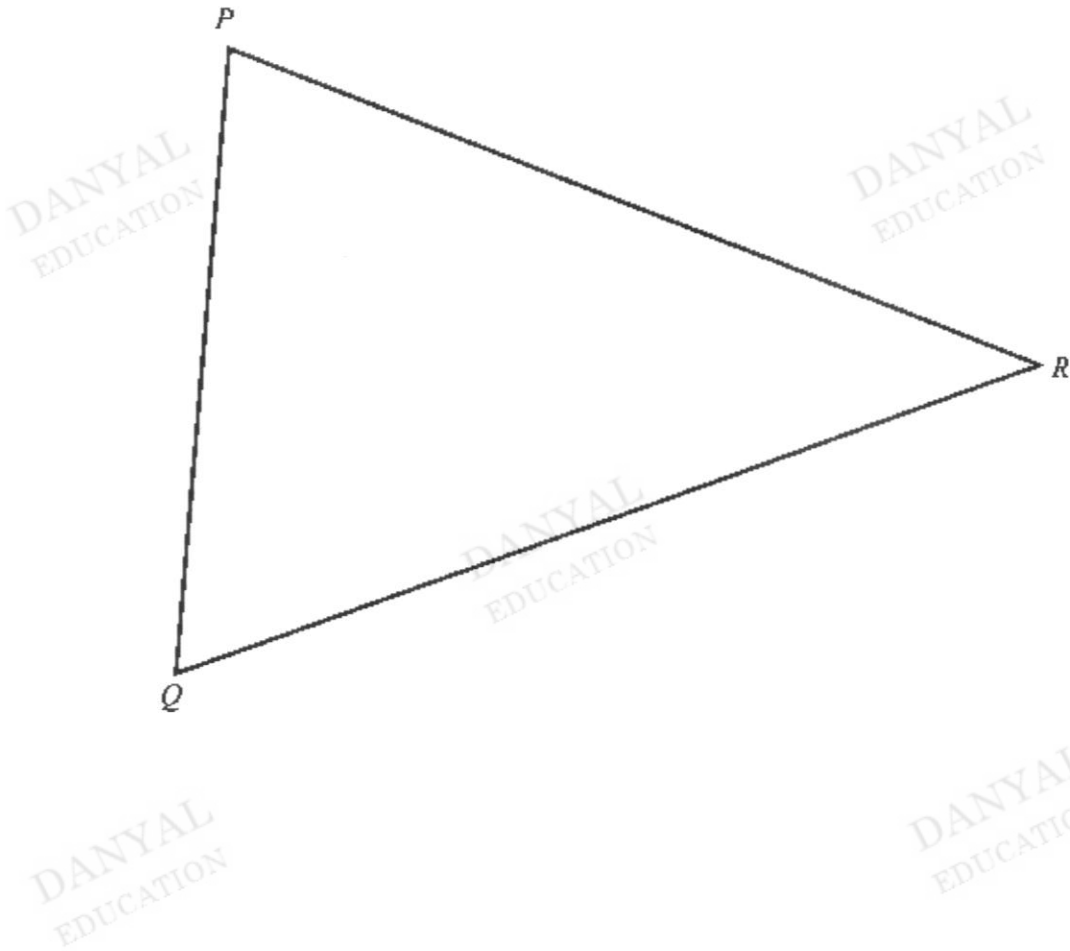*Answer* .... [2]

(c) Explain why -100 is not a term of this sequence.

[2]

[Turn over

20   The diagram shows a triangle *PQR*.

(a)   Construct the perpendicular bisector of *QR*.   [1]

(b)   Construct the bisector of angle *PRQ*.   [1]

(c)   Point *B* is equidistant from *Q* and *R* and equidistant from *PR* and *RQ*.
           Mark the point *B* on the diagram and measure the length *BP*.

Answer BP = ... cm [1]

17

21    $\xi = \{$integers $x : 2 \le x < 16\}$

$A = \{$multiples of 3$\}$

$B = \{$factors of 20$\}$

(a)   (i)   List all the elements in A'.

Answer .... [1]

(ii)   List all the elements in $A \cap B$.

Answer .... [1]

(iii)   Find the value of $n(A \cup B)'$.

Answer .... [1]

(b)   On the Venn diagram, shade the region which represents $(P' \cup Q) \cap Q'$.

[Turn over

18

[1]

22   The diagram show triangle QRM with coordinates $Q(-12, 0)$, $R(4, 0)$ and $M(8, 10)$.
      P is a point on the y-axis.
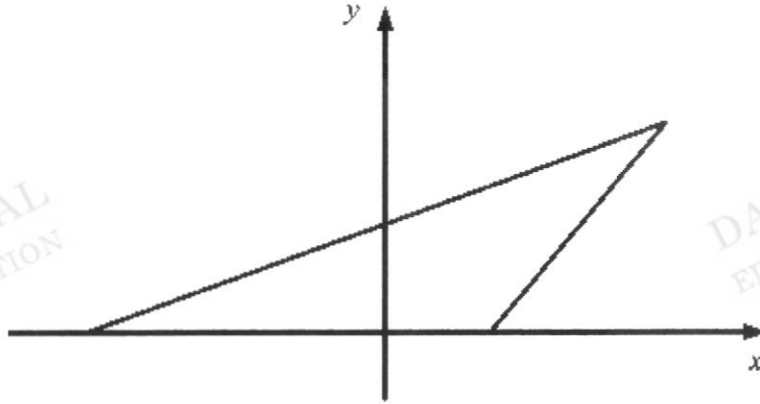
(a) Find length QM.

Answer ....units [2]

(b) Find the coordinates of P.

Answer (............ , ...) [2]

(c) S is a point such that it forms a parallelogram $QRMS$.
Find the area of $QRMS$.

19

Answer ....units^2 [1]

---

23   A shop sells two different flavours of waffles, coconut (C) and peanut (P).

In a morning, an average of 10 coconut and 8 peanut waffles were sold.
In an afternoon, an average of 21 coconut and 15 peanut waffles were sold.
In an evening, an average of 7 coconut and 14 peanut waffles were sold.

(a)   Represent this information in a 3×2 matrix, **S**.

$$\text{\textit{Answer} } \mathbf{S} = \begin{array}{cc} \text{C} & \text{P} \\ \left(\begin{array}{cc} & \\ & \\ & \end{array} \right) & \begin{array}{l} \text{morning} \\ \text{afternoon} \\ \text{evening} \end{array} \end{array} \quad [1]$$

The price of one coconut waffle and one peanut waffle is \$2.10 and \$2.70 respectively.

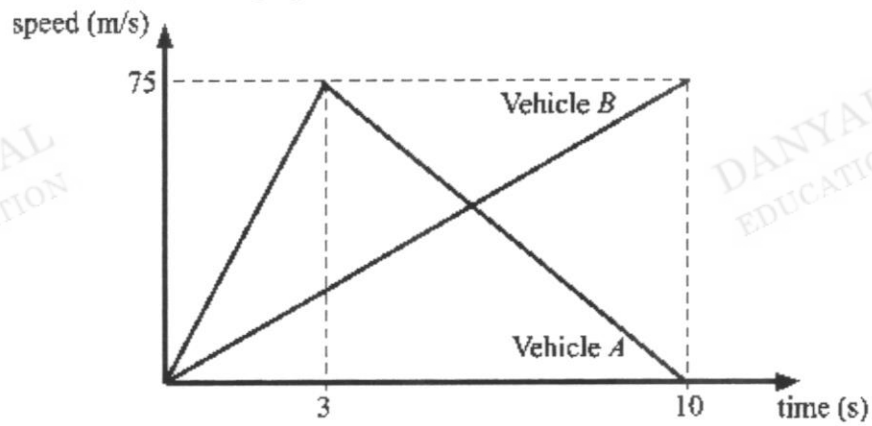(b)   Find the matrix $\mathbf{T} = 7\mathbf{S} \begin{pmatrix} 2.10 \\ 2.70 \end{pmatrix}$.

Answer **T** =   [2]

(c)   State what the elements of **T** represents.

[Turn over

20

[1]

___

24   The diagram shows the speed-time graph of vehicle A and vehicle B.

(a)   Find the deceleration of vehicle A.

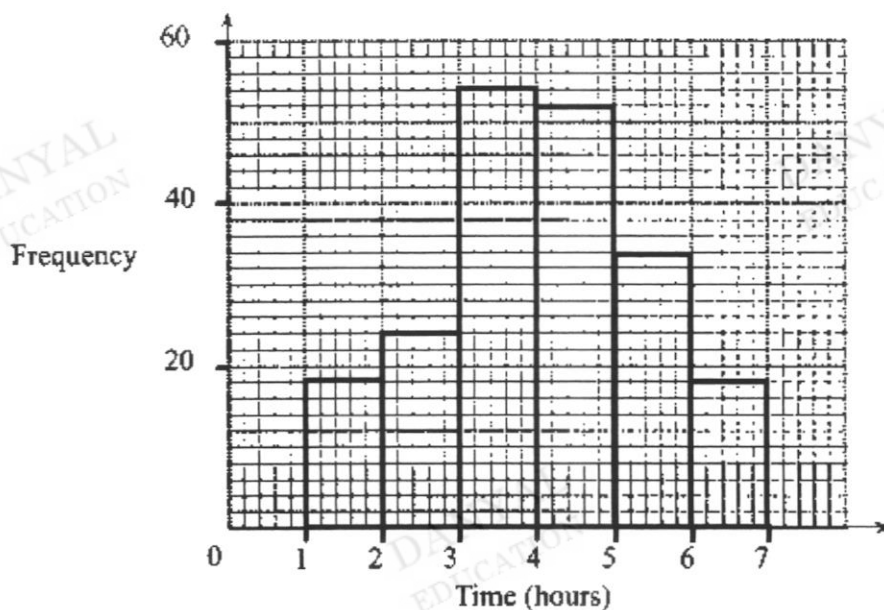*Answer* ....m/s^2 [1]

(b)   Both vehicles travel the same distance at the end of 10 seconds.
Determine if this statement is true.

[2]

(c)   Draw the distance-time graph for vehicle A from 0 to 10 seconds.

Distance (m)

[2]

---

25 (a) The histogram shows the distribution of the time spent exercising by 200 students from Jurong.

(i) Which interval contains the median?

Answer .... hours [1]

(ii) Calculate an estimate of the mean.

Answer .... hours [1]

(iii) Calculate an estimate of the standard deviation.

Answer .... hours [1]

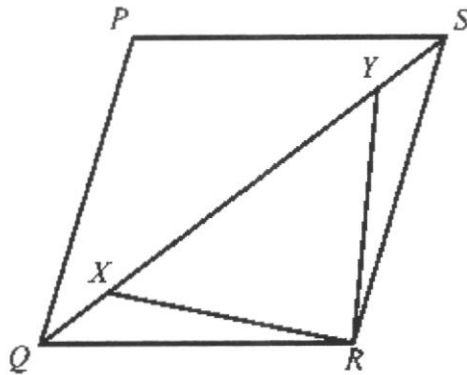(iv) Explain why the mean and the standard deviation is just an estimate.

[1]

[Turn over

22

(b) The standard deviation of the time spent exercising by a group of 200 students from Yishun is 2.21. Make a comparison between the distribution of the two groups of students.

[1]

26

PQRS is a rhombus.
X and Y are points on QS such that $QX = SY$.
Show that triangle RXY is an isosceles triangle.

[3]

(b) Explain why it is not possible to draw a circle that passes through P, Q, R and S.

If a circle passes through all the points P, Q, R and S, by **angles in the opposite segment**, angle QPS + angle QRS = 180.

23

[2]

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[Turn over

YISHUN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023
SECONDARY 4 EXPRESS

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS
Paper 2

Candidates answer on the Question Paper.

4052/02
23 August 2023
2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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Answer **all** the questions.
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The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For examiner's use

90

[Turn over

2

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) It is given that $b = \dfrac{c+b}{3b-c}$,

    (i) find the value of b when $c = 2b$,

Answer .... [3]

(b) Solve $\dfrac{x+1}{(3-x)^2} - \dfrac{4}{x-3} = 1$.

[Turn over

4

Answer .... [3]

(c) A teacher wanted to give her 40 students a burger each. The cost of a chicken burger is $2 and the cost of a fish burger is $2.50. She spent a total of $95 for x chicken burgers and y fish burgers. Use algebraic method to find the number of fish burgers that she ordered.

Answer .... [3]

2 (a) John spends 1.5 hours at a sports centre. The ratio of the times he spends changing, warming up and running is 2 : 3 : 7. Calculate

(i) the time he spends to change.

Answer ....min [1]

(ii) the percentage of time he spends running at the sports centre.

5

Answer ....% [1]

(iii)

(b) In 2021, John's fastest time for a 10 km run was 54 minutes 30 seconds.
In 2022, he went to Thailand to take part in a 10 km competition.
His time taken was 6% less than his fastest time in 2021.
Calculate, in minutes and seconds, his best time in 2022.

Answer …………..…minutes……………….seconds [2]

(c) John saw a pair of branded running shoe which cost 17 430 Baht.
The same pair costs 500 US Dollars (USD) in the United States.
To buy these shoes, John will pay using his Singapore credit card and will be
charged a fee of 1.7% fee for currency conversion.

The exchange rate between Singapore dollars ($) and USD is $1 = USD 0.74 and
between Thai Baht and Singapore Dollar is 1 Baht = 0.039 Singapore Dollars.

Show, with clear working, if John should buy the pair of running shoes in Thailand
or the United States.

[3]

3
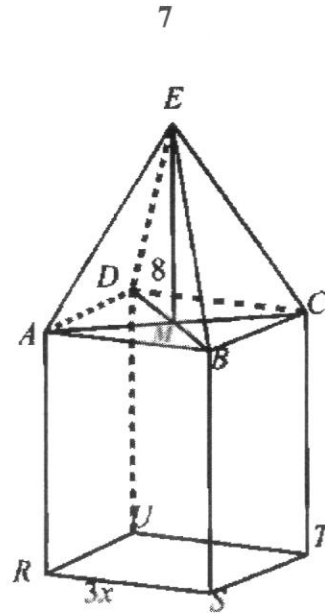
The diagram shows a solid consisting of a cuboid attached to a pyramid.

The pyramid has a square base of length $3x$ cm and a vertical height of 8 cm.

The height of the cuboid is twice the length of one side of its square base.

(a) Form an equation, in terms of x, to represent this information and show that it simplifies to

$$9x^2 + 4x - 100 = 0 .$$

[3]

(b) Solve the equation $9x^2 + 4x - 100 = 0$, giving your answers to 3 decimal places.

Answer x =……….……… or ……….……… [4]

(c) Hence, calculate the slant length of the pyramid.

Answer ……….……………….cm [3]

4
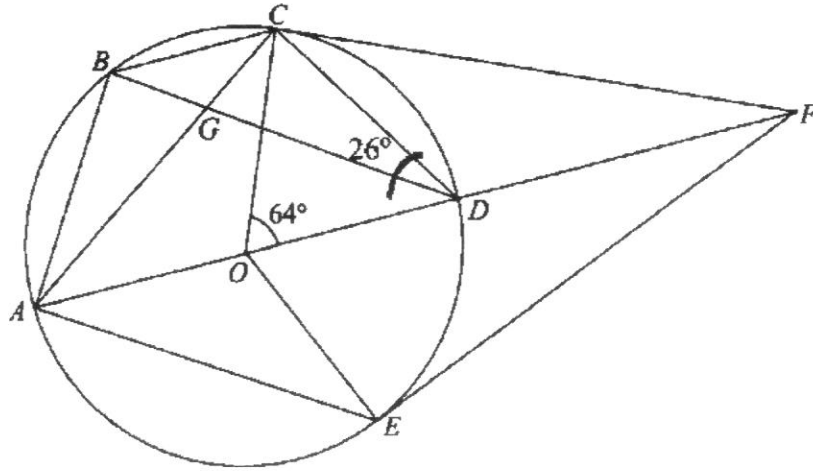
The diagram shows a circle *ABCDE* with centre at *O*.
CF and *EF* are tangents to the circle at points *C* and *E* respectively.
Angle *COD* = 64° and *BDC* = 26°.[3]

(a) Find

(i) angle *CAD*,

(ii) angle *OFE*,

Answer ....° [2]

(iii) angle *DCF*.

[**Turn over**

10

Answer ....° [2]

(b) Show that triangle *BGC* is similar to triangle *AGD*.
Give a reason for each statement you make.

(c) Show with clear working and reasons, that *ABCD* is a trapezium.

[3]

5 *A*, *B*, *C* and *D* are four points on level ground.
   A is due west of *D* and the bearing of *C* from *A* is 050°.
   AD = 25 m, *DC* = 45 m, *DB* = 70 m and angle *BDC* =100°.
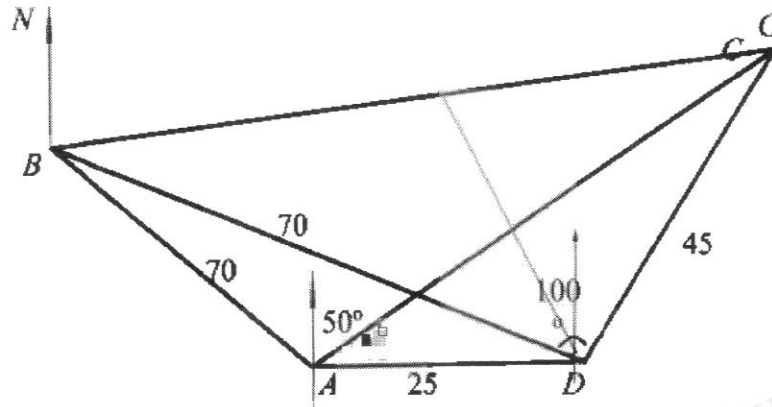
(a) Show that *BC* = 89.5 m, correct to 3 significant figures.

Answer .... [2]

(b)   Calculate angle *ACD*.

Answer .... °[2]

(c) Calculate the bearing of *D* from *C*.
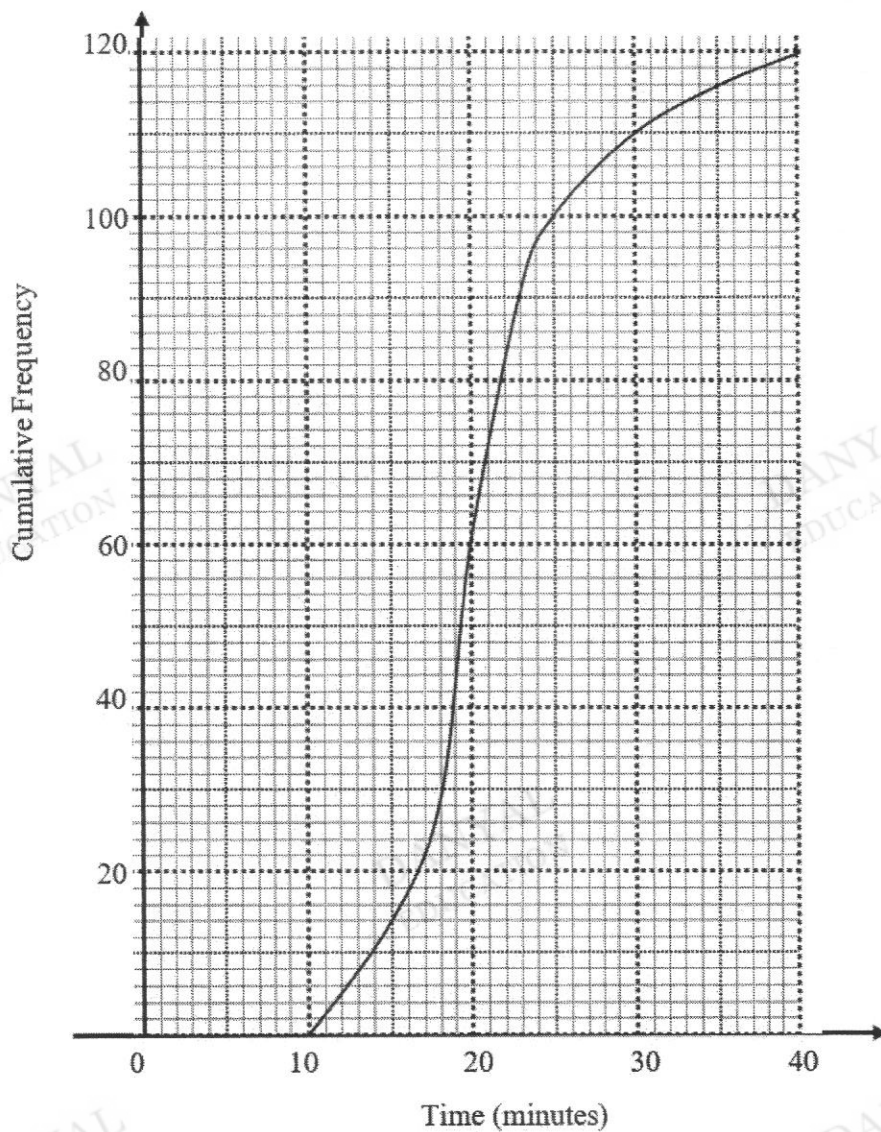
Answer .... °[2]

(d) *T* is a point 53 m vertically above *D*.
A car travels along a straight path from *B* to *C* until it reaches a point *E* where the angle of elevation of *T* from *E* is the greatest.
Calculate this greatest angle of elevation.

| 5c | $\frac{1}{2}(BC)(DE) = \frac{1}{2}(DC)(BD)\sin CDB$

 $\frac{1}{2}(89.5487)(DE) = \frac{1}{2}(45)(70)\sin 100$

 $DE = [(45)(70)\sin 100] \div 89.5487$

 $DE = 34.64196$

 $\tan x = \dfrac{53}{34.64196}$

 $x = \tan^{-1}(1.52994)$

 $= 56.830°$

 $= 56.8°$ | M1 find DE using area of triangle BDC or ½ x 89.5 x ht seen

 A1

 M1 **correct angle** of elevation identified

 A1 | |

Answer .... °[4]

[Turn over

14

6 **(a)** The cumulative frequency graph shows the distribution of the time taken for 120 cars to travel a stretch of road from a research company.

(i) Use the curve to estimate
   (a) the median time,

Answer .... minutes [1]

   (b) the interquartile range of the time.

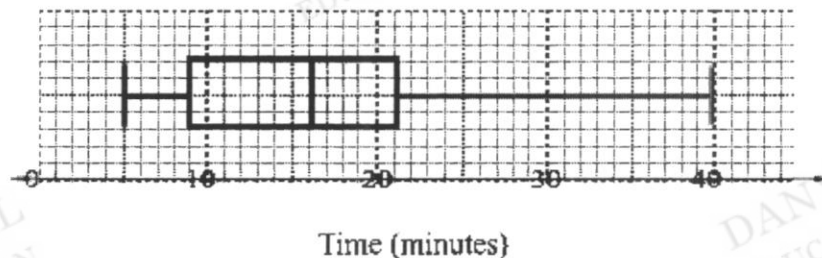*Answer* .... minutes [2]

(ii) A survey states that 10% of the cars took more than 31 minutes to travel this stretch of road. Comment whether the data from the research centre supports this statement.

Answer [2]

(v) Some candidates demonstrated their understanding of what was required by stating a relevant numerical value or values from which they could draw a conclusion. Most attempted to use the curve to find the cumulative frequency for babies weighing 2.8 kg, leading to a value that could be compared with the 90% quoted. The most common method was to read 9 babies from the graph and equate this to 82% of babies, leading to a correct conclusion. Some quoted 9 babies less than 2.8 kg or 41 more than 2.8 kg but did not compare their value with 9 or 45 resulting from the 90% given in the question. A common error was to misread the horizontal scale, for example reading at 2.65 or 2.9 instead of 2.8. Some candidates misinterpreted the curve and stated that 90% of babies had a mass of 3.9 kg or more.

(iii) The box-and-whisker plot below represents the distribution of the time taken for 120 motorcycles to travel the same stretch of road.

Time (minutes)

Make two comparisons between these two modes of transport.
Give a reason for your answer.
Answer

[2]

[Turn over

6 **(b)** This table shows information about a group of students in a playgroup.

	Boys	Girls
Korea	4	3
Japan	2	7

(i) A student of the group is selected at random.
Find, as a fraction in its lowest terms, the probability that the student is a boy from Korea.

(ii) Two students are selected at random.
Find, as a fraction in its lowest terms, the probability that
(a) they are both boys,

(b) they are both girls but only one is from Japan. [2]

7   **(a)** The point P has coordinates $(-3, 8)$ and $\overrightarrow{PQ} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$.

    (i) Find the equation of the line PQ.

Answer .... [2]
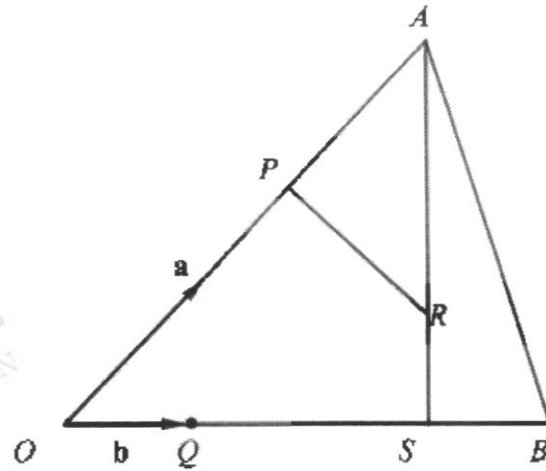
    (ii) The equation of the line ST is $3y + x - 10 = 0$.

    Explain, with clear working, if line PQ intersects line ST.

[2]

(b)

OAB is a triangle.

Q and *S* are points on *OB* such that $SB = OQ$ and $OQ = \frac{1}{3}OS$.

P is a point on *OA* such that $OP = \frac{2}{3}OA$.

R is a point on *AS* such that *SR* : *RA* = 1 : 2.

$\overrightarrow{OP} = \boldsymbol{p}$, $\overrightarrow{OQ} = \boldsymbol{q}$.

(i) Express, as simply as possible, in terms of *p* and/or *q*,

Answer .... [2]

(b) $\overline{PB}$

Answer .... [1]

19

(ii) Show that *P*, *R* and *B* lie on a straight line.

[3]

(iii) Find the numerical value of $\dfrac{\text{area of triangle } ASB}{\text{area of triangle } AOB}$.

Answer .... [1]

8   The variables x and y are connected by the equation $y = -\dfrac{1}{5}x^3 + 2x + 1$.

(a) Complete the table of values for $y = -\dfrac{1}{5}x^3 + 2x + 1$.

x	-4	-3	-2	-1	0	1	2	3	4
y	5.8	0.4		-0.8	1	2.8	3.4	1.6	-3.8

[1]

(b) On the grid opposite, draw the graph of $y = -\dfrac{1}{5}x^3 + 2x + 1$ for $-4 \leq x \leq 4$.
   [3]

(c) By drawing a suitable tangent, find the gradient of the curve at $x = -3$.

Answer .... [2]

(d) (i) On the same grid, draw the graph of $2y + x = 1$ for $-4 \leq x \leq 4$.

[1]

(ii) Show that the points of intersection of the line and the curve gives the solution of $-2x^3 + 25x + 5 = 0$.

[2]

(iii) Use your graph to solve the equation $-2x^3 + 25x + 5 = 0$.

Answer $x =$ .... or .... or .... [2]

[**Turn over**

22

9 To prevent vehicles from speeding, speed humps are built on the roads to regulate the speed of cars.

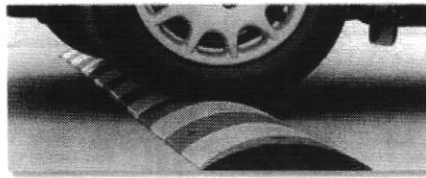Diagram 1 shows the length, width and height of one speed hump. Each speed hump along a road has 4 identical speed humps joined together.
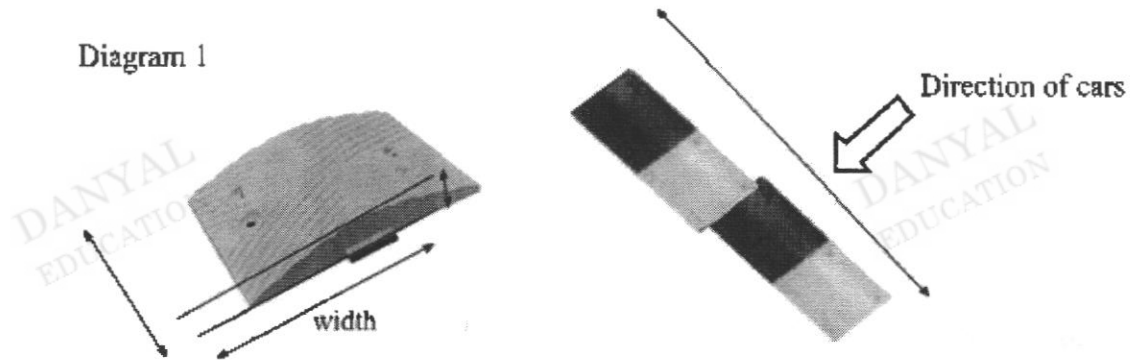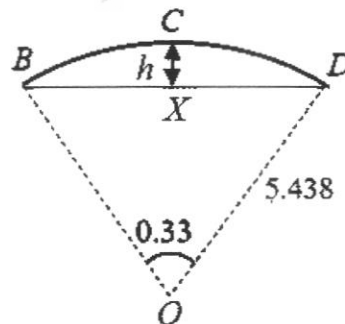
Diagram 1

width

Direction of cars

Diagram 2 shows the cross-section of one speed hump.
BCD is part of sector *OBCD* with centre *O*, radius 5.438 m and angle *BOD* of 0.33 radians.

Diagram 2 : Cross-section of one speed hump

(a) Find the length of the arc *BCD* of the speed hump as shown in Diagram 2.

Answer ....m [1]

(b) Diagram 3 shows the top view of 4 identical speed humps joined together. The thickness of the rubber to cover the top of each speed hump is 0.5 cm and the total cost to cover four such speed humps is $240.

Using the information given in the table, calculate the length of one speed hump.

Diagram 3 : Top view of four speed humps
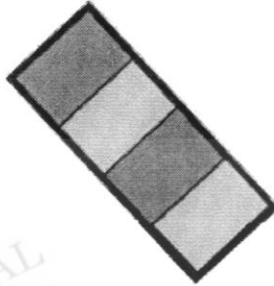
Information for material used and cost	
Density of rubber	1060 kg/m³
Cost of rubber per kg	$20
Cost to cover the **top** part of 4 speed humps	$240

Answer ....m [3]

[Turn over

(c) The engineers are to ensure that the height (h), width (w) and length (l) of **each** speed hump meet the safety criteria as shown in the table.

Height of the hump (h)	$0.0732 \text{ m} \leq h < 0.1016 \text{ m}$
Width of the hump (w)	$0.9144 \text{ m} \leq w < 1.8288 \text{ m}$
Gradient of the hump (g), where g is given as the ratio of the height of the hump : horizontal length of BX or XD	$g < 0.1$
Length of one speed hump (l)	$0.3048 \text{m} \leq l \leq 0.9144 \text{ m}$

Show with clear working if the engineers should approve this speed hump.

YISHUN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023
SECONDARY 4 EXPRESS

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS
Paper 1

Candidates answer on the Question Paper.

4052/01
21 August 2023
2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For examiner's use

90

[Turn over

This document consists of 21 printed pages and 1 blank page.

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1 + \frac{r}{100}\right)^Y$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

3

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all questions

1   Calculate $121 - \sqrt{\dfrac{98}{0.61} - 80}$

Write your answer correct to 4 significant figures.

$121 - \sqrt{\dfrac{98}{0.61} - 80}$

$= 112.019$

$= 112.0 (4sf)$ ------ B1

Answer .... [1]

---

2   The estimated population of Taiwan is 23.6 million.

The estimated total land area is 3.62×10^4 km².

Calculate the average number of people per square kilometre of land area in Taiwan, giving your answers in standard form.

$\dfrac{23.6 \times 10^6}{3.62 \times 10^4}$ ------ B1 convert million

$= 651.933$

$= 6.52 \times 10^2$ ------ A1

Answer .... [2]

---

3   When 540 is divided by m, a perfect square is formed.
Find the smallest value of m, where m is an integer.

[Turn over

4

$540 = 2^2 \times 3^3 \times 5$ ----- M1 prime factorise 540

$m = 3 \times 5 = 15$ ----- A1

Answer m −..................................... [2]

4 The scale of a map is 4 cm : 28 km.

(a) Find the length of a canal that is represented by 10 cm on the map.

4 cm : 28 km
1 cm : 7 km
10 cm : 70 km ----- B1

Answer .... [1]

(b) A lake is represented by an area of 400 cm² on the map.
Calculate the actual area of the lake in square kilometres.

1 cm : 7 km
1 cm² : 49km² ----- M1 find area scale
400 cm² : 49 x 400 = 19600 km² ----- A1

Answer .... km² [2]

5 (a) Express $22 + 10x + x^2$ in the form of $p + (q + x)^2$.

5

$$22 + 10x + x^2 = (x+5)^2 - 52 + 22$$
$$= (x+5)^2 - 3$$
$$= -3 + (5+x)^2 \text{ ----- B1,B1 \quad or M1 complete square seen,A1}$$

Answer .... [2]

(b) Write down the equation of line of symmetry for $-22 + 10x + x^2$.

$x = -5$ B1

Answer .... [1]

6
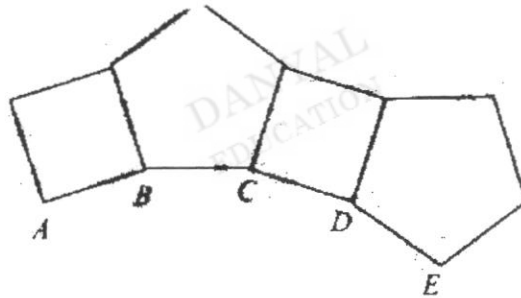
A regular pentagon is joined to a square repeatedly until it forms a regular polygon AB, BC, CD and DE are edges of the regular polygon

Find the number of sides in this regular polygon

Interior angle of square = 90

Interior angle of pentagon $\dfrac{(5-2) \times 180}{5} = 108$

Int. angle of reg. polygon = 360 − 108 − 90 = 162 ----- M1 new interior angle

Method 1
Ext. angle of reg. polygon = 180 − 162 = 18 ----- M1 find one exterior angle

No. of sides of reg. polygon = $\dfrac{360}{18}$ = 20 ----- A1

[Turn over

6

Method 2

$(n-2) \times 180 = 162n$ ----- M1 use interior angle method to find n

$180n - 360 = 162n$

$18n = 360$

$n = 20$ ----- A1

Answer .... [3]

---

7   r is directly proportional to t^3 .

When t is decreased by 25%, find the percentage change in r.

$r = kt^3$

new $r = k(0.15t)^3$ ----- M1 cube 0.75   iP

% change = $\dfrac{k\left(\dfrac{27}{64}t^3\right) - kt^3}{kt3} \times 100$

$= \left(\dfrac{21}{64} - 1\right) \times 100$

$= -57.8125\%$ or $-\dfrac{925}{16}\%$ or $-57\dfrac{13}{16}\%$ -----A1

Answer .... [2]
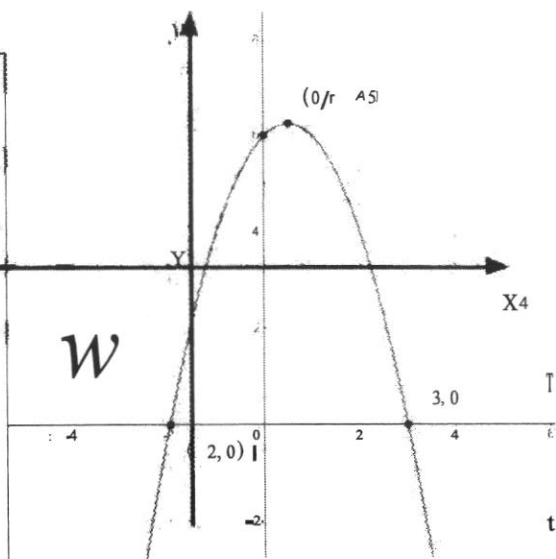
8   Sketch the graph of $y = -(x-2)(x-3)$ on the axes below.
     Indicate clearly the lines where the graph crosses the x- and y- axes and its turning point.

Four marking points: (1)
Shape. (∧tfenteMepts.
(3) y-iniereefts, (4)
turning point
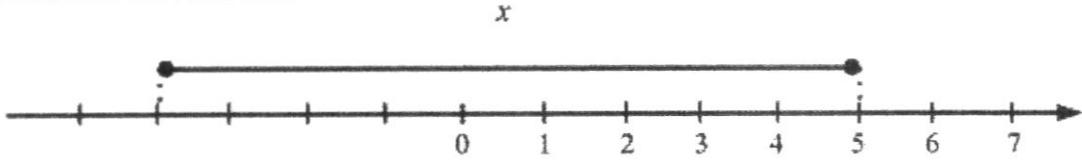
B2: all 4 marking points
correct

B1: 2 ─ 3 marking points.

[2]

7

9   The solution of the inequality $-2 \leq \dfrac{4-2x}{3} \leq 2k+5$, where k is a constant, is represented on the number line below. Find the value of k.

x

$-2 \leq \dfrac{4-2x}{3} \leq 2k+5$

$-2 \leq \dfrac{4-2x}{3}$ and $\dfrac{4-2x}{3} \leq 2k+5$ ----- M1 Split

$-6 \leq 4-2x$ $4-2x \leq 6k+15$

$-10 \leq -2x$ $-2x \leq 6k+11$ ----- M1 divide by negative and flip sign

$x \leq 5$ $x \geq \dfrac{6k+11}{-2}$

$\dfrac{6k+11}{-2} = -4$ ----- M1 equate

$6k+11 = 8$

$6k = -3$

$k = -\dfrac{1}{2}$ ----- A1

Answer $k =$ .... [4]

---

10   Zac has written down **five numbers**.

The mean of these numbers is **5.2**, the median is 4 and the mode is 2.

The largest number is five times the smallest number.

Find the five numbers.

2, 2, 4, 8, 10

B2 – all 5 correct
B1 – 3 or 4 correct

Answer ...., ..., ..., ..., ... [2]

11 **(a)** Simplify $\left(\dfrac{2x^3}{\sqrt{x^4}}\right)^{-3}$.

$$\left(\dfrac{2x^3}{\sqrt{x^4}}\right)^{-3} = \left(\dfrac{\sqrt{x^4}}{2x^3}\right)^{3}$$ ----- M1 negative or distributive law applied correctly

$$= \dfrac{x^6}{8x^9}$$

$$= \dfrac{1}{8x^3}$$ ----- A1

Answer .... [2]

(b) Solve $\dfrac{3^{x+1}}{27^x} = 1$.

$$\dfrac{3^{x+1}}{27^x} = 1$$

$$3^{x+1} = 27^x$$

$$3^{x+1} = 3^{3x}$$ ----- M1 make same base for all terms

$$x + 1 = 3x$$

$$x = \dfrac{1}{2}$$ ----- A1

Answer $x =$ .... [2]

---

12 Object A and object B are geometrically similar.

The ratio of the base area of object A to the base area of object B is $4:9$.

Find the ratio of the volume of object A to volume of object B.

$$\dfrac{A_A}{A_B} = \dfrac{4}{9}$$

$$\dfrac{l_A}{l_B} = \sqrt{\dfrac{4}{9}} = \dfrac{2}{3}$$ ----- M1 sq rt area ratio

$$\dfrac{V_A}{V_B} = \left(\dfrac{2}{3}\right)^3 = \dfrac{8}{27}$$

Hence, ratio 8:27 ----- A1

Answer .... : .... [2]

---

13   *n* is a positive integer.

Show that, for all *n*, $(2n+3)^2 - (2n-3)^2$ is not a prime number.

Answer

$$(2n+3)^2 - (2n-3)^2 = 4n^2 + 12n + 9 - (4n^2 - 12n + 9)$$
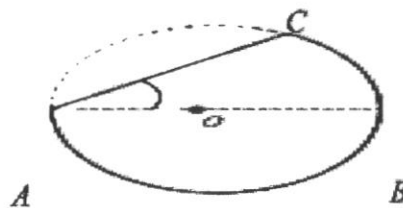$$= 24n \qquad \text{---- M1 quadratic expansion}$$

A prime number can only be divided by 1 or **itself**

Since 24 is not prime, $(2n+3)^2 - (2n-3)^2$ can be **divided** by all factors of 24, making it not prime.

A1 – Explanation based on definition of prime numbers

[2]

14

The diagram shows a major segment *ABC* of a circle with centre *O* and radius 5 cm.

Angle $OAC = \dfrac{\pi}{6}$.

[Turn over

10

Find the area of the major segment *ABC*.

Reflex angle $AOC = 2\pi - \left(\pi - \dfrac{\pi}{6} - \dfrac{\pi}{6}\right) = \dfrac{4}{3}\pi$ ----- B1 reflex angle

Area of major segment = Area of major sector + Area of triangle

$$= \frac{1}{2}(5^2)\left(\frac{4\pi}{3}\right) + \frac{1}{2}(5^2)\sin\left(\pi - \frac{\pi}{6} - \frac{\pi}{6}\right) \quad \text{---- M1}$$

$$= 52.3598 + 10.8253$$

$$= 63.1851$$

$$= 63.2 \quad \text{-----A1}$$

Answer .... cm^2 [3]

---

15 **(a)** Factorise completely $12pq - 3p - q - 4q^2$

$$12pq - 3p + q - 4q^2 = 3p(4q-1) + q(1-4q)$$
$$= 3p(4q-1) - q(4q-1)$$
$$= (3p-q)(4q-1) \quad \text{------ B2}$$

11

(b) Simplify $\dfrac{3x^2+15x-42}{x^2-4}$.

$$\frac{3x^2+15x-42}{x^2-4} = \frac{3(x^2+5x-14)}{(x-2)(x+2)} \quad \text{-----M1 factorise denominator}$$

$$= \frac{3(x-2)(x+7)}{(x-2)(x+2)} \quad \text{----- M1 quadratic factorisation}$$

$$= \frac{3(x+7)}{x+2} \quad \text{-----A1}$$

Answer .... [3]

16   The volume of a cone with radius x and height y is twice the volume of a hemisphere with radius x.
$\dfrac{1}{27}$ of the cone is filled with water.
This water is poured into the hemisphere.

Find the fraction of the hemisphere that is filled with water.

Volume of cone = 2 volume of hemisphere

[Turn over

12

$$\frac{1}{3}\pi x^2 y = (2)\frac{1}{2}\left(\frac{4}{3}\pi x^3\right) \quad \text{----- B1}$$

$$\frac{1}{3}\pi x^2 y = \frac{4}{3}\pi x^3$$

$$y = 4x \qquad \qquad \text{---------- M1 make y the subject}$$

$$\begin{aligned}
\text{Amt of water} &= \frac{1}{27}\left(\frac{1}{3}\pi x^2 y\right)\\
&= \frac{1}{81}\pi x^2(4x)\\
&= \frac{4}{81}\pi x^3 \qquad \text{------- M1 find volume of water in terms of x}\\
&= \frac{2}{27}\left(\frac{2}{3}\pi x^3\right)\\
&= \frac{2}{27}(\text{vol. of hemisphere}) \quad \text{— A1}
\end{aligned}$$

Answer .... [4]

17   The graph shows information about prices of the Certificate of Entitlement (COE) for buses for each month in year 2021 and 2022

COE is a license for owning a vehicle in Singapore.

13
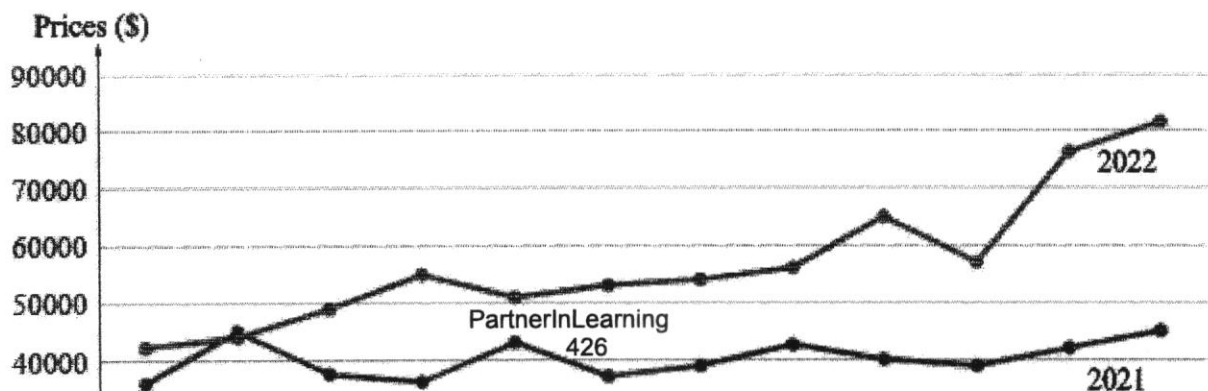
(a) Estimate the price of COE in July 2022.

$54000/555000 — B1

Answer $ .... [1]

(b) A school bus company claims that the cost of buying new buses has increased and intends to increase the prices of the services they provide for the year 2023!

What feature of the graph can the company use to justify the company's claim

Answer **The line graph of 2022 shows an increasing trend for the COE prices from January 2022 to December 2022. Following the read, there is a likelihood that it will continue to increase tor 2023, hence, justifying the company's claim.** [1]

(c) Make one comparison between the mean COBE prices In 202f and 2022. Justify your answer with references to the graph.

Answer **The mean COE price for 2022 is higher than 2021 [Bl] as**

the COE prices of every month expect February in 2022 is higher than in

2021; fBlj This canbe seen from Ore data points of 2022 lying above that of 2021.

[2]

---

18 **(a)** The cash price of a TV: $2100.
Ben buys this TV on hire purchase.
He pays a deposit of one fifth of the cash price
He then makes 12 monthly paypments of $160

fisFind »total amount that Ben navs for the TVj

$\frac{1}{5} \times 2100 = 420$ **1**

M1 one-fifth

Total = $420 + 12 \times 160 = 2340$ ----- A1

Answer $ .... [2]

[Turn over

14

(b) Alicia invested a sum of money in an account paying compound interest at 3% per annum. After 5 years, she earned a total interest of $3981.

Calculate the sum of money Alicia invested in the account.
Give your answer to the nearest dollar.

Total amount $= P\left(1+\dfrac{r}{100}\right)^{n}$

$P+3981=P\left(1+\dfrac{3}{100}\right)^{5}$ ———— M1   sub r = 3, n=5 with P remaining as unknown

$P\left(1+\dfrac{3}{100}\right)^{5}-P=3981$ ———— M1 bring P to one side

$0.15927P=3981$

$P=\dfrac{3981}{0.15927}=24994.65=\mathbf{24995}$ ———— A1

Answer $.............................. [3]

---

19   In a sequence, the same number is subtracted each time to obtain the next term.
The first five terms of the sequence are

$$72 \quad p \quad q \quad 57 \quad r$$

(a)   Find the values of p, q and r.

Common difference $= \dfrac{72-57}{3}=5$ ———— B1

$p=67$

$q=62$

$r=52$ ———— B1 All 3 correct

Answer $p=$ ....

$q=$ ....

$r=$ ....[2]

15

(b) Write down an expression for the nth term of this sequence.

$$77 - 5n$$

Answer .... [2]

(c) Explain why -100 is not a term of this sequence

Answer

$$77 - 5n = -100$$

$$n = \frac{-100 - 77}{-5} = \frac{177}{5} \quad \text{----- M1 find } n$$

Since n is **not a positive integer**, 100 cannot be a term in the sequence ----- A1 conclude

....

. .. . [2]
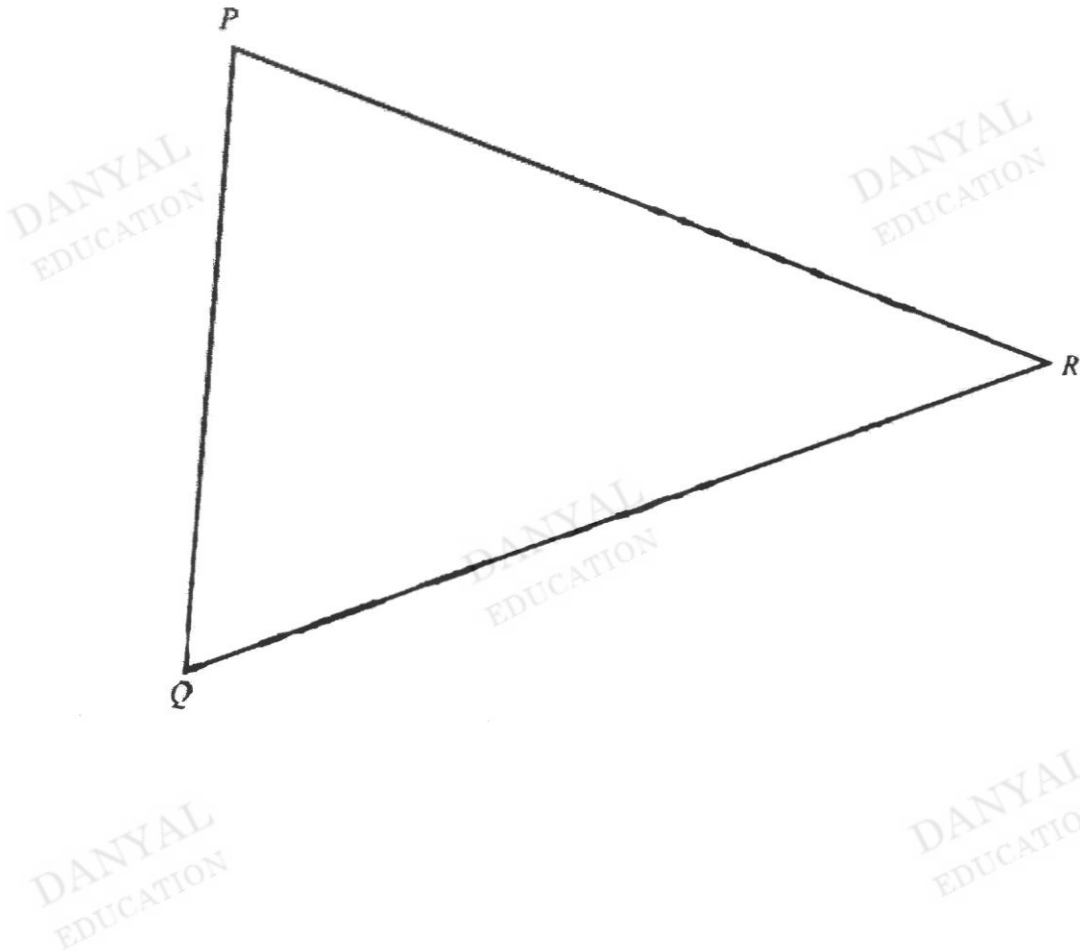
[Turn over

16

20 The diagram shows a triangle PQR.

(a) Construct the perpendicular bisector of QR. [1]

(b) Construct the bisector of angle PRQ. [1]

(c) Point B is equidistant from Q and R and equidistant from PR and RQ.
Mark the point B on the diagram and measure the length BP.

Answer BP = ... cm [1]

17

21   ξ = {integers $x : 2 \le x < 16$}

  A = {multiples of 3}

  B = {factors of 20}

(a)   (i)   List all the elements in A' .

      $A' = \{2, 4, 5, 7, 8, 10, 11, 13, 14\}$ ----- B1

      Answer .... [1]

   (ii)   List all the elements in $A \cap B$.

      $A = \{3, 6, 9, 12, 15\}$
      $B = \{2, 4, 5, 10\}$
      $A \cap B = \phi$ or { }   ----- B1

      Answer .... [1]

   (iii)   Find the value of $n(A \cup B)'$

      $A = \{3, 6, 9, 12, 15\}$
      $B = \{2, 4, 5, 10\}$
      $A \cup B$ {2, 3, 4, 5, 6, 9, 10, 12, 15}
      $(A \cup B)'$ {7, 8, 11, 13, 14}
      $n(A \cup B)'$ 5   ----------- B1

      Answer .... [1]

(b)   **On the Venn diagram, shade the region which represents** $(P' \cup Q) \cap Q'$.
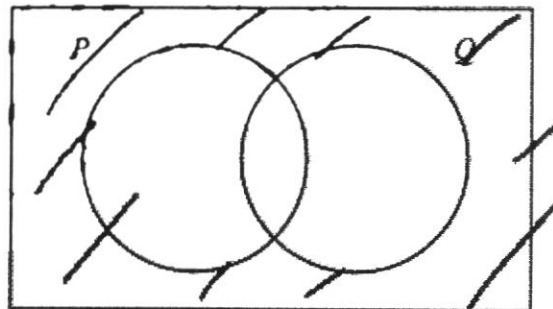
[Turn over

22   The diagram show triangle QRM with coordinates $Q(-12,0)$, $R(4,0)$ and $M(8,10)$.
      P is a point on the y-axis.
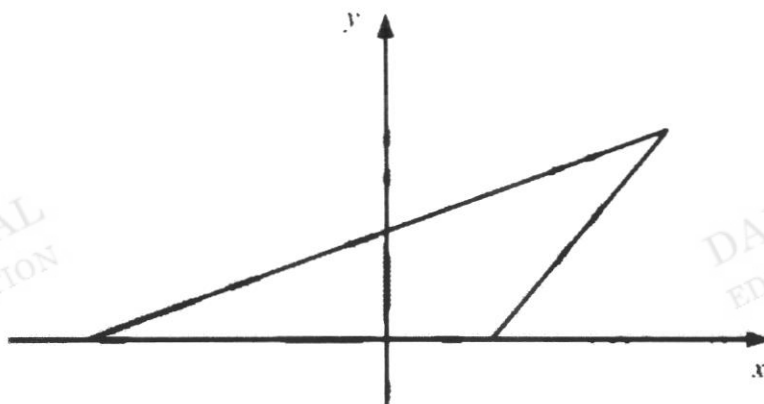
(a)   Find length QM.

$$QM = \sqrt{(-12-8)^2 + (0-10)^2} \quad \text{------ M1 sub coords into formula}$$

$$= \sqrt{500}$$

$$= 22.3606$$

$$= 22.4 \text{ units (3sf)} \quad \text{----- A1}$$

Answer ....units [2]

(b)   Find the coordinates of P.

Method 1 – By similar triangles	Method 2 – By equation of straight line
$\dfrac{y}{10} = \dfrac{12}{20}$ ----- M1 form eqn using ratio	$gradient = \dfrac{10}{8-(-12)} = \dfrac{1}{2}$ ----- M1 find gradient
$20y = 120$	$10 = \dfrac{1}{2}(8) + c$
$y = 6$	$c = 6$
$P(0,6)$ ----- A1	$P(0,6)$ ----- A1

Answer (............ , ...) [2]

(c) S is a point such that it forms a parallelogram $QRMS$.
       Find the area of $QRMS$.

19

Area of parallelogram = 2 Area of triangleQRM

$$= \frac{1}{2}(16)(10) \times 2 = 160 \quad \text{----- B1}$$

Answer ....units^2 [1]

---

23   A shop sells two different flavours of waffles, coconut (C) and peanut (P).

In a morning, an average of 10 coconut and 8 peanut waffles were sold.
In an afternoon, an average of 21 coconut and 15 peanut waffles were sold.
In an evening, an average of 7 coconut and 14 peanut waffles were sold.

(a)   Represent this information in a 3×2 matrix, S.

----- B1

$$\begin{array}{cc} \text{C} & \text{P} \end{array}$$

Answer **S** = $\begin{pmatrix} & \\ & \\ & \end{pmatrix}$ morning
afternoon
evening   [1]

The price of one coconut waffle and one peanut waffle is $2.10 and $2.70 respectively.

(b)   Find the matrix **T** = 7S $\begin{pmatrix} 2.10 \\ 2.70 \end{pmatrix}$.

$$7 \begin{pmatrix} 10 & 8 \\ 21 & 15 \\ 7 & 14 \end{pmatrix} \begin{pmatrix} 2.1 \\ 2.7 \end{pmatrix} = \begin{pmatrix} 70 & 56 \\ 147 & 105 \\ 49 & 98 \end{pmatrix} \begin{pmatrix} 2.1 \\ 2.7 \end{pmatrix}$$

$$= \begin{pmatrix} 298.2 \\ 592.2 \\ 367.5 \end{pmatrix} \quad \text{----- B2 all three terms correct, B1 two terms correct}$$

Answer **T** =   [2]

(c)   State what the elements of **T** represents.

[Turn over

20

Answer

It represents the **total amount collected from the sales of** of the coconut and peanut waffles in **morning, afternoon and evening respectively** in **7 days**.

.... [1]

---

24   The diagram shows the speed-time graph of vehicle A and vehicle B.
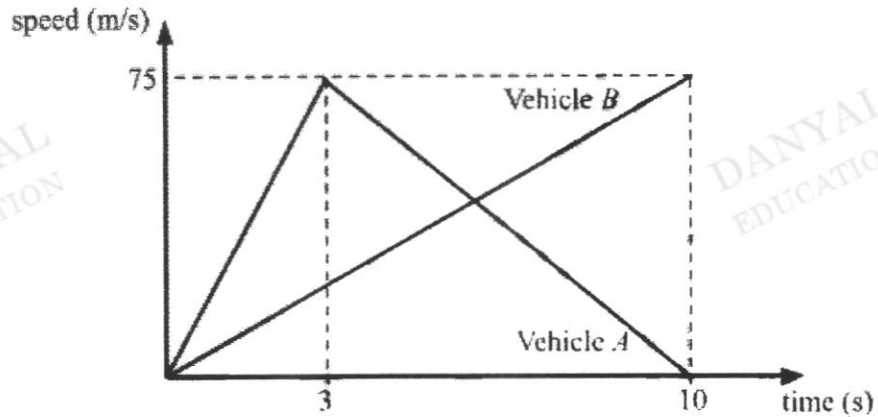
(a)   Find the deceleration of vehicle A.

Deceleration = $\dfrac{75}{7}$ or $10\dfrac{5}{7}$

Answer ....m/s² [1]

(b)   Both vehicles travel the same distance at the end of 10 seconds.
Determine if this statement is true.

Answer

Distance travelled by vehicle B = $\dfrac{1}{2}(10)(75) = 375$

Distance travelled by vehicle A = $\dfrac{1}{2}(3)(75) + \dfrac{1}{2}(7)(75) = 375$

The statement is true. Both vehicle travels 375m.

M1 – find distance of either vehicle A or B.
A1 – conclude.

[2]

(c)   Draw the distance-time graph for vehicle A from 0 to 10 seconds.
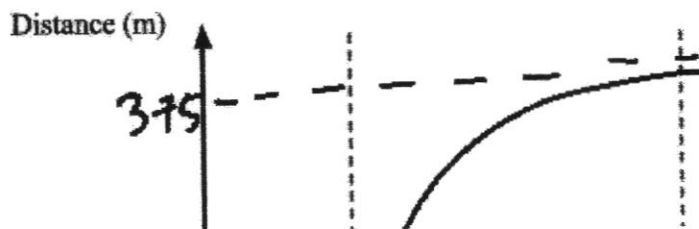
B1 – correct shape, B1 – correct axis labels *or*

B1 – first 3 seconds correct shape + distance labelled correctly, B1 for the next part of the journey (graph +

112.5 ----- ⌐

[2]

---

25   **(a)**   The histogram shows the distribution of the time spent exercising by 200 students from Jurong.
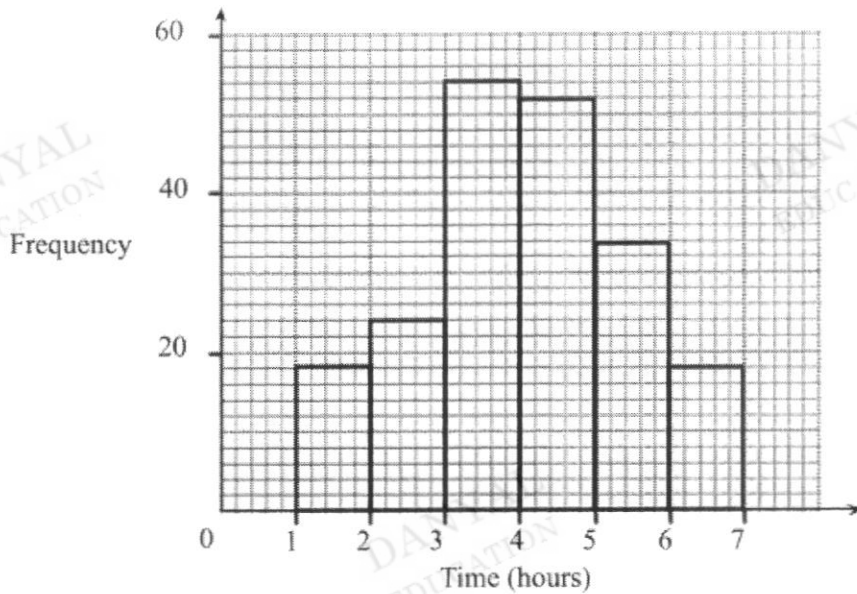
(i)   Which interval contains the median?
Median position = 100.5 (between 100 and 101)
4 to 5 hours ----- B1 or equivalent

Answer .... hours [1]

(ii)   Calculate an estimate of the mean.
4.07   B1

Answer .... hours [1]

(iii)   Calculate an estimate of the standard deviation.
1.3755 = 1.38   B1

Answer .... hours [1]

(iv)   Explain why the mean and the standard deviation is just an estimate.

Answer
The exact values of the number of hours are **not given./Mid-value** of the interval
are used in the caculation. .... [1]

22

(b) The standard deviation of the time spent exercising by a group of 200 students from Yishun is 2.21. Make a comparison between the distribution of the two groups of students.

Answer

The number of hours spent exercising is more consistent among the students in
Jurong than Yishun as the standard deviation of students in Jurong is smaller than
that of students in Yishun, 1.38 < 2.21 .... [1]

26
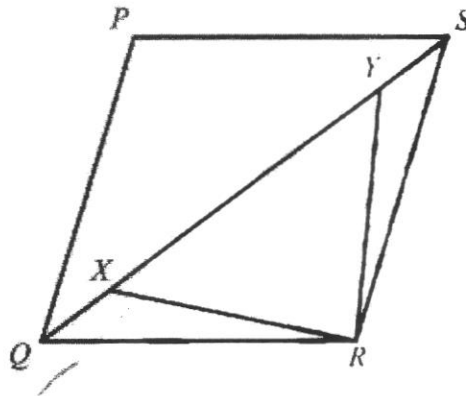
PQRS is a rhombus.
X and *Y* are points on *QS* such that $QX = SY$.
Show that triangle *RXY* is an isosceles triangle.

Answer

$QX = SY$ (given)

$QR = SR$ (sides of rhombus are equal in length)

$\angle XQR = \angle YSR$ (triangle *QRS* is an isosceles triangle since $QR = RS$)

M1 – two statements that are not given.

By SAS congruency test, triangle *QXR* is congruent to triangle *SYR*. ----- M1 congruency test.

Therefore, $RX = RY$, making *RXY* an isosceles triangle. ----- A1

[3]

(b) Explain why it is not possible to draw a circle that passes through *P*, *Q*, *R* and *S*.

If a circle passes through all the points *P*, *Q*, *R* and *S*, by **angles in the opposite segment**, angle *QPS* + angle *QRS* = 180.

23

....

....

....

....

[2]

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YISHUN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023
SECONDARY 4 EXPRESS

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS   **4052/02**
Paper 2   **23 August 2023**
                                                                     2 hours 30 minutes
Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For examiner's use

90

[Turn over

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) It is given that $b = \dfrac{c - b}{3b - c}$.

(i) find the value of b when $c = 2b$,

1(a)(i)	$b = \dfrac{c - b}{3b - c}$ $b = \dfrac{2b + b}{3b - 2b}$ $b = \dfrac{3b}{b}$ $b = 3$	A1	

Answer .... [1]

(ii) express c in terms of b.

1(a)(ii)	$b = \dfrac{c - b}{3b - c}$ $b(3b - c) = c + b$ $3b^2 - bc = c + b$ $3b^2 - b = c + bc$ $3b^2 - b = c(1 - b)$ $c = \dfrac{3b^2 - b}{1 + b} = \dfrac{b(3b - 1)}{1 + b} = \dfrac{b - 3b^2}{-1 - b}$	M1 (cross multiply) M1 (Take out c as factor) A1	

Answer .... [3]

(b) Solve $\dfrac{x - 1}{(3 - x)^2} - \dfrac{4}{x - 3} = 1$.

[Turn over

4

| 1b | $\dfrac{x+1}{(3-x)^2} - \dfrac{4}{x-3} = 1$

$\dfrac{x+1}{(x-3)^2} - \dfrac{4}{x-3} = 1$

$\dfrac{(x+1)-4(x-3)}{(x-3)^2} = 1$

$x+1-4x+12 = x^2 - 6x + 9$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4 \ or \ x = -1$ | M1
denominator/
single fraction

M1
simplified
quadratic
equation

A1 | |

Answer .... [3]

(c) A teacher wanted to give her 40 students a burger each. The cost of a chicken burger is $2 and the cost of a fish burger is $2.50. She spent a total of $95 for x chicken burgers and y fish burgers. Use algebraic method to find the number of fish burgers that she ordered.

| 1c | $x - y = 40 \ ---(1)$
$2x - 2.5y = 95 \ ---(2)$

From 1 : $x = 40 - y$
Sub into eqn 2 :
$2(40 - y) + 2.5y = 95$
$80 - 2y + 2.5y = 95$
$0.5y = 15$
$y = 30$

number of fish burger $= 30$ | B1 eqn of 1
or 2.

M1 : solve
sim equation
by
elimination or
substitution

A1 | |

Answer .... [3]

2 **(a)** John spends 1.5 hours at a sports centre. The ratio of the times he spends changing, warming up and running is 2 : 3 : 7. Calculate
(i) the time he spends to change.

| 2a(i) | $12\,units\,rep\,1.5h$

$warm-up: \dfrac{1.5}{12} \times 2 = \dfrac{1}{4}$

$=15$ min | | 1u -7.5 min

A1 |

Answer .... min [1]

(ii) the percentage of time he spends running at the sports centre.

| 2a(ii) | $\frac{7}{12} \times 100\%$ $= 58\frac{1}{3}\%$ or 58.3% | A1 | |

Answer ....% [1]

(iii)

(b) In 2021, John's fastest time for a 10 km run was 54 minutes 30 seconds.
In 2022, he went to Thailand to take part in a 10 km competition.
His time taken was 6% less than his fastest time in 2021.
Calculate, in minutes and seconds, his best time in 2022.

2b	54 min 30 sec = 3270 sec 100% --- 3270 sec 94% --- $\dfrac{3270}{100} \times 94$ = 3073.8 sec = 51 min 13.8 sec	M1 for 94% A1	

Answer ....minutes................seconds [2]

(c) John saw a pair of branded running shoe which cost 17 430 Baht.
The same pair costs 500 US Dollars (USD) in the United States.
To buy these shoes, John will pay using his Singapore credit card and will be
charged a fee of 1.7% fee for currency conversion.

The exchange rate between Singapore dollars ($) and USD is $1 = USD 0.74 and
between Thai Baht and Singapore Dollar is 1 Baht = 0.039 Singapore Dollars.

Show, with clear working, if John should buy the pair of running shoes in Thailand
or the United States.

Answer   [3]

2(c)	In Singapore, convert USD to SGD: $\dfrac{500}{0.74} \times 1.017$ = S687.16 In Thailand, convert Baht to SGD: 17 430 x 0.039 x 1.017 = 691.33 John should buy the shoe from the US since it is cheaper (687.16 < 691.16).	M1 for $\dfrac{500}{0.74}$ conversion from USD to SGD or conversion of Baht to SGD M1 for their SGD x 1.017 A1 conclude	

3
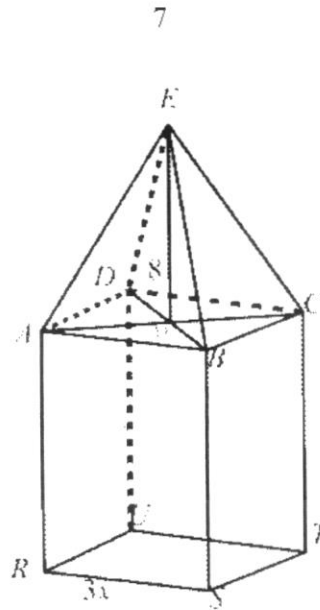
The diagram shows a solid consisting of a cuboid attached to a pyramid.

The pyramid has a square base of length $3x$ cm and a vertical height of 8 cm.

The height of the cuboid is twice the length of one side of its square base.

(a)   Form an equation, in terms of x, to represent this information and show that it simplifies to

$$9x^2 + 4x - 100 = 0 .$$

Answer [3]

3(a)	$6x(3x)(3x) + \dfrac{1}{3}(3x)(3x)8 = 600x$ $54x^3 - 24x^2 - 600x = 0$ $(\div 6x)$ $9x^2 + 4x - 100 = 0$	M2 for vol of cuboid and pyramid **and equate** to 600x M1 for 1 correct volume seen A1	

(b) Solve the equation $9x^2 + 4x - 100 = 0$, giving your answers to 3 decimal places.

| 3(b)	$9x^2 - 4x - 100 = 0$ $x = \dfrac{-4 \pm \sqrt{(4)^2 - 4(9)(-100)}}{2(9)}$ $x = \dfrac{-4 \pm \sqrt{3616}}{18}$ $x - 3.11850 \ or \ -3.56295$ $x = 3.119 \ or \ -3.563$	B1 or $\sqrt{3616}$ B1 for $\dfrac{-3 \pm \sqrt{their\ 3616}}{18}$ B2 (ans in 3 dp) B1 for either 3.119 or -3.563 or both roots seen to more decimal places.	

[Turn over

8

Answer x .... or .... [4]

(c)   Hence, calculate the slant length of the pyramid.

3(c)	$AC^2 = AB^2 - BC^2$		
	$AC^2 = (3x)^2 + (3x)^2$ $AC^2 = 18x^2$ $AM = \dfrac{\sqrt{18x^2}}{2}$	M1 to find AC^2 using their positive x	
	$AE^2 = \left(\dfrac{\sqrt{18x^2}}{2}\right)^2 + 8^2$ $AE^2 = \dfrac{18(3.11850)^2}{4} + 8^2$ $AE = \sqrt{107.763}$ $= 10.3809$ $= 10.4$	M1 to find AE^2 If $x = 3.119$ is used, $AE = \sqrt{107.7767} = 10.3816$ A1 (ans is the same if they use $x = 3.119$)	

Answer ....cm [3]

4
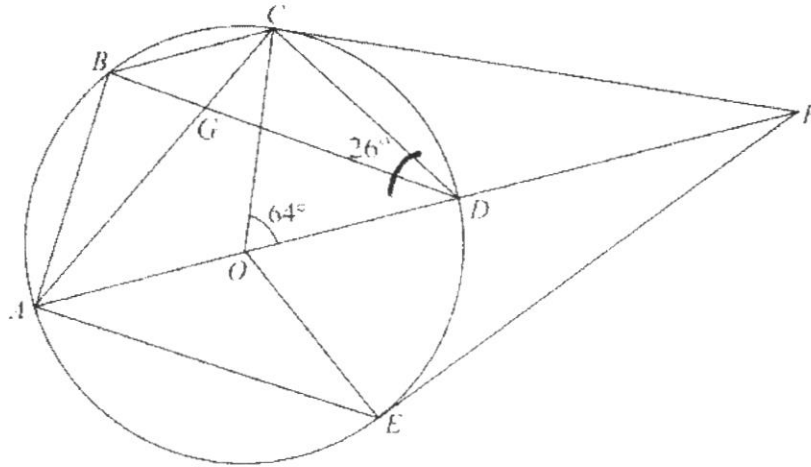
The diagram shows a circle ABCDE with centre at O.
CF and EF are tangents to the circle at points C and E respectively.
Angle COD = 64° and BDC = 26°. [3]

(a) Find

    (i) angle CAD.

4a(i)	$\angle CAD = \dfrac{64}{2}$ ($\angle$ at centre = twice $\angle$ at circumference) $= 32$	A1	

    (ii) angle OFE,

4a(ii)	$\angle OFE = \angle OFC$ (tangents from external point) $\angle OCF = 90$ (tangent perpendicular to radius) $\angle OFC = 180 - 90 - 64$ ($\angle$ sum of triangle) $= 26$	M1 for either OFE = OFC or OCF =90 A1	

                                                        Answer ....° [2]

    (iii) angle DCF.

4a(iii)	$\angle OCD = \dfrac{180-64}{2}$ (OC = OD radius of circle) $= 58$ $\angle OCF = 90$ (tangent perpendicular to radius) $\angle DCF = 90 - 58 = 32$ OR	B1 to find angle OCD A1 B1 find angle CAD A1	

	$\angle CAD = 64 \div 2$		
	($\angle$ at centre is twice angle at circumference)		
	$=32$		
	$\angle DCF = 32 (\angle$ in alternate segment)		

Answer .... " [2]

(b) Show that triangle BGC is similar to triangle AGD.
Give a reason for each statement you make.

4b)	$\angle BGC = \angle AGD$ (vertically opposite angles)	B2, B1, B0 for 3,	
	$\angle GBC = \angle GAD$ (angle in the same segment are equal)	2, 1 correct reasons with	
	or $\angle BCG = \angle ADG$ (angle in the same segment are equal)	angle/ AA ppty	
	By AA, triangle BGC is similar to triangle AGD		

(c) Show with clear working and reasons, that $ABCD$ is a trapezium.
Answer

[3]

4c	$\angle ACD = 90$ ($\angle$ in a semi-circle is 90)	B1 or $\angle ABD = 90$ with reason	
	$\angle BDA = 58 - 26 = 32$ (OCD is an isosceles triangle)		
	$\angle BDA = \angle BCA = 32 (\angle$ in the same segment)		
	$\angle NCD + \angle CDA$	M1 to show $\angle BCD + \angle CDA = 180$ or $\angle ABC + \angle BAD = 180$	
	$= 32 + 90 + 58$		
	$= 180$		
	By interior angles of parallel sides,		
	line BC is parallel to AD,		
	hence ABCD is a trapezium.	B1 to state int angles of parallel sides to conclude	
	OR		
	$\angle ACD = 90$ ($\angle$ in a semi-circle is 90)	B1 or $\angle ABD = 90$ with reason	
	$\angle BDA = 58 - 26 = 32$ (OCD is an isosceles triangle)		
	$\angle BDA = \angle BCA = 32 (\angle s$ in the same segment)		
	$\angle CAD = 64 \div 2 = 32$	M1 to show angle CAD with reason and =angle BDA using a(i) and part b	
	(from a(i) or $\angle$ at centre is twice angle at circumference)		
	$\angle CBD = \angle CAD = 32$ ($\angle s$ in the same segment) OR		
	$\angle CBG = 32$ (corresponding $\angle s$ in similar triangles, shown in (b))		
	Hence, $\angle CBD = \angle BDA = 32$	B1 to state alternate angles used for parallel sides to conclude	
	By alternate angles, line BC is parallel to AD,		
	hence ABCD is a trapezium.		

5 A, B, C and D are four points on level ground.
A is due west of D and the bearing of C from A is 050°.
AD = 25 m, DC = 45 m, DB = 70 m and angle BDC = 100°.

(a) Show that BC = 89.5 m, correct to 3 significant figures.

5a)	$BC^2 = 70^2 + 45^2 - 2(70)(45)\cos100$	M1	
	$BC = \sqrt{8018.9835}$		
	$= 89.549$		
	$= 89.5$	A1	

Answer .... [2]

(b) Calculate angle ACD.

5b)	$\dfrac{45}{\sin 40} = \dfrac{25}{\sin \angle ACD}$	M1 use sin rule to find angle DCA with angle CAD =40	
	$\sin \angle ACD = \dfrac{25\sin 40}{45}$		
	$\angle ACD = 20.922$		
	$= 20.9 \ (1 \ dp)$	A1	

Answer .... °[2]

(c) Calculate the bearing of D from C.

| 5c)	Bearing of D from C	M1 to identifying correct bearing	
	$= 180° + (50° - \angle ACD)$		
	$= 180° + 50° - 20.922$		
	$= 180° + 29.078°$		
	$- 209.1°$	A1	

Answer .... °[2]

(d) T is a point 53 m vertically above D.
A car travels along a straight path from B to C until it reaches a point E where the angle of elevation of T from E is the greatest.
Calculate this greatest angle of elevation.

| 5c | $\frac{1}{2}(BC)(DE) = \frac{1}{2}(DC)(BD)\sin CDB$ $\frac{1}{2}(89.5487)(DE) = \frac{1}{2}(45)(70)\sin 100$ $DE = [(45)(70)\sin 100] \div 89.5487$ $DE = 34.64196$ $\tan x = \dfrac{53}{34.64196}$ $x = \tan^{-1}(1.52994)$ $= 56.830°$ $= 56.8°$ | M1 find DE using area of triangle BDC or ½ x 89.5 x ht seen A1 M1 **correct angle** of elevation identified A1 | |

Answer .... °[4]

6 **(a)** The cumulative frequency graph shows the distribution of the time taken for 120
cars to travel a stretch of road from a research company.
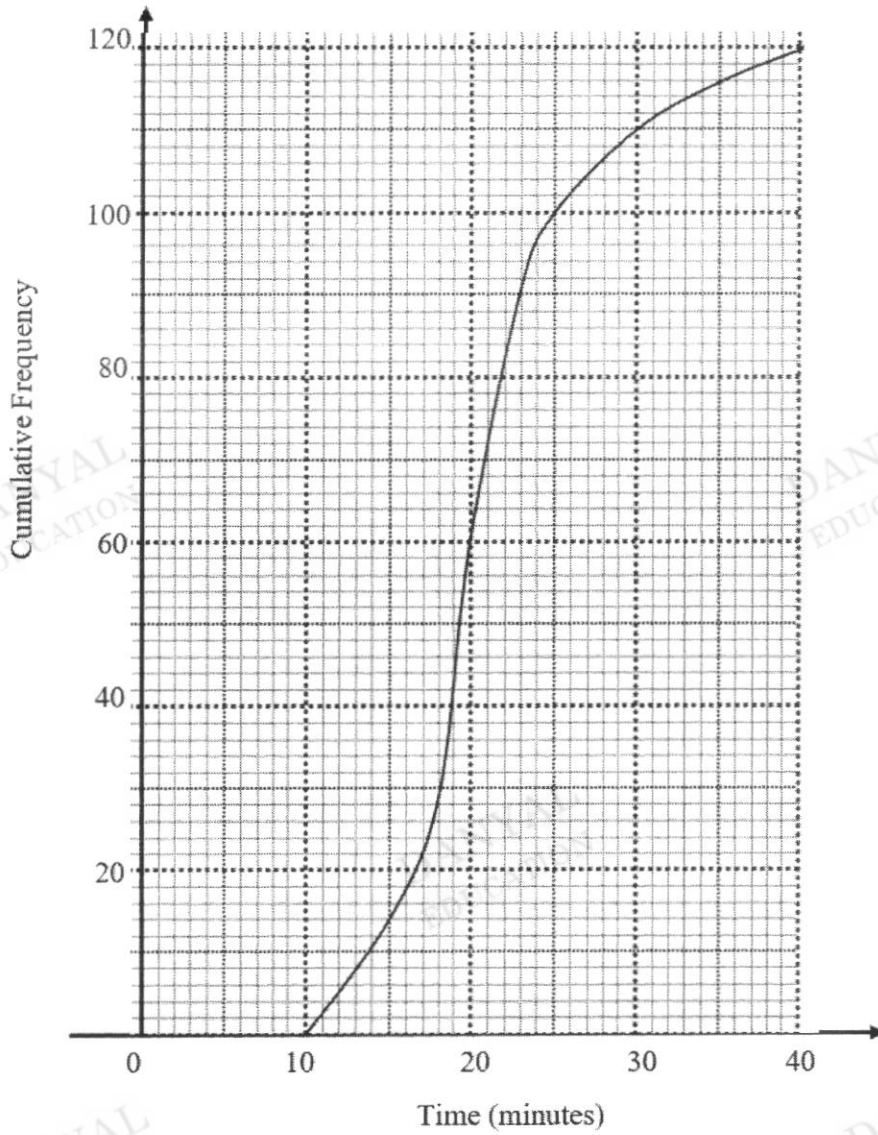
(i) Use the curve to estimate
   (a) the median time.

6(a)(i)(a)	20 min	A1	

Answer .... minutes [1]

   (b) the interquartile range of the time.

6(a)(i)(b)	23 − 18 = 5 min	M1 UQ − LQ A1	

Answer .... minutes [2]

(ii) A survey states that 10% of the cars took more than 31 minutes to travel this stretch of road. Comment whether the data from the research centre supports this statement.
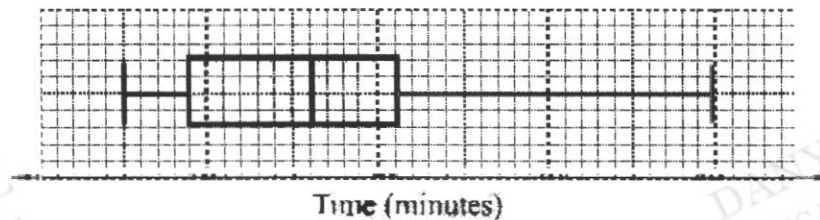
Answer [2]

6(ii)	$120 - 112 = 8$ $8/120 \times 100\% = 6.67\%$ Data does not support this statement 6.67% < 10% Accepted 10% of the car took more than 29min 8 cars took more than 31min 112 cars took less than 31 min	B1 A1	State the number of 8 or 112 cars Compare to number 12 cars or 108 cars resulting from 10% given in the qn)

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(v) Some candidates demonstrated their understanding of what was required by stating a relevant numerical value or values from which they could draw a conclusion. Most attempted to use the curve to find the cumulative frequency for babies weighing 2.8 kg, leading to a value that could be compared with the 90% quoted. The most common method was to read 9 babies from the graph and equate this to 82% of babies, leading to a correct conclusion. Some quoted 9 babies less than 2.8 kg or 41 more than 2.8 kg but did not compare their value with 9 or 41 resulting from the 90% given in the question. A common error was to misread the horizontal scale, for example reading at 2.65 or 2.9 instead of 2.8. Some candidates misinterpreted the curve and stated that 90% of babies had a mass of 3.9 kg or more.

(iii) The box-and-whisker plot below represents the distribution of the time taken for 120 motorcycles to travel the same stretch of road.

Time (minutes)

Make two comparisons between these two modes of transport.
Give a reason for your answer.
Answer

[2]

6(iii)	Median time for motorcycle (16min) is lower than the cars (20min), motorcycles is a faster mode of transport.	B1	
	IQR for cars (5 min) is lower than IQR of motorcycles (21- 9 = 12 min), hence the time taken by cars is more consistent.	B1	

[Turn over

16

6   **(b)**   This table shows information about a group of students in a playgroup.

	Boys	Girls
Korea	4	3
Japan	2	7

(i)   A student of the group is selected at random.
Find, as a fraction in its lowest terms, the probability that the student is a boy from Korea

6b(i)	$\dfrac{4}{16} = \dfrac{1}{4}$	B1	

(ii)   Two students are selected at random.
Find, as a fraction in its lowest terms, the probability that
(a) they are both boys,

(b) they are both girls but only one is from Japan. [2]

6b(ii) (a)	$\dfrac{6}{16} \times \dfrac{5}{15}$		
6b(ii) (b)	$\dfrac{7}{16} \times \dfrac{3}{15} + \dfrac{3}{16} \times \dfrac{7}{15}$	M1 of either	B1
	$= \dfrac{7}{40}$	B1	

7 **(a)** The point P has coordinates $(-3, 8)$ and $PQ = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

   (i) Find the equation of the line PQ.

| 7a(i) | $\text{grad} = \dfrac{-\frac{2}{6}}{}$ $y = -\dfrac{1}{3}x - c$ $8 = -\dfrac{1}{3}(-3) + c$ $c = 7$ $y = -\dfrac{1}{3}x - 7$ | M1: grad of -1/3 seen and/ or attempt to find c with their gradient found A1 | |

Answer .... [2]

   (ii) The equation of the line ST is $3y + x - 10 = 0$.
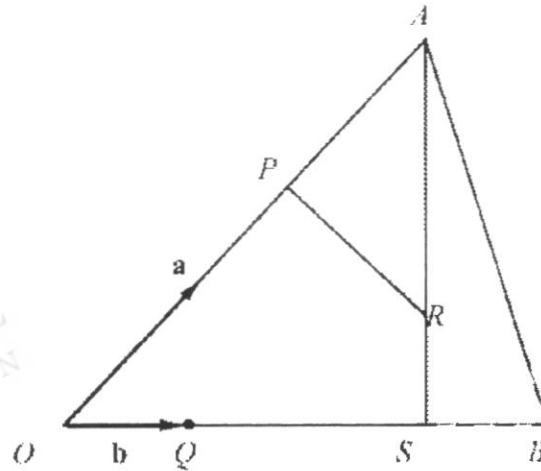
   Explain, with clear working, if line PQ intersects line ST.

   Answer [2]

| 7a(ii) | $3y + x - 10 = 0$ $y = -\dfrac{1}{3}x + \dfrac{10}{3}$ Gradient of ST = gradient of PQ = $-\dfrac{1}{3}$ Lines are parallel, hence line PQ does not intersects line ST. OR $3y + x - 10 = 0 \ ---(1)$ $y = -\dfrac{1}{3}x + 7 \ ---(2)$ Sub (2) into (1) $3(-\dfrac{1}{3}x - 7) - x - 10 = 0$ no solution Hence, line PQ does not intersect line ST | M1: grad of $-\dfrac{1}{3}$ and compare with their grad found in (i) B1: explain -parallel lines /same grad with different y intercepts   M1 to solve sim eqn   B1: explain -no soln | |

(b)

OAB is a triangle.

Q and S are points on OB such that $SB = OQ$ and $OQ = \dfrac{1}{3} OS$.

P is a point on OA such that $OP = \dfrac{2}{3} OA$

R is a point on AS such that $SR : RA = 1 : 2$.

$\vec{OP} = p$, $\vec{OQ} = q$.

(i) Express, as simply as possible, in terms of p and/or q,

(a) $\vec{SA}$

7(b) (i) (a)	$\vec{OS} = 3b$ $\vec{SA} = \vec{SO} + \vec{OA}$ $= -3b + \dfrac{3}{2}a$	B1 : correct $\vec{OS}$ or $\vec{OA}$ A1	

Answer .... [2]

(b) $\vec{PB}$

7(b) (i) (a)	$\vec{PB} = \vec{PO} + \vec{OB}$ $= -a + 4b$	A1	

Answer .... [1]

(ii) Show that P, R and B lie on a straight line.

Answer [3]

| 7b(ii) | $PR = PA - AR$

$= -\dfrac{1}{2}a + \dfrac{2}{3}AS$

$= \dfrac{1}{2}a + \dfrac{2}{3}(3b - \dfrac{3}{2}a)$

$= -\dfrac{1}{2}a + 2b$

$= \dfrac{1}{2}(-a + 4b)$

$= \dfrac{1}{2}PB$

Since $\dfrac{PR}{PB} = \dfrac{1}{2}$ or PR= ½ PB and P is a common point P,R,B lie on a straight line. | M1:find AR or PA correctly

M1 : make PR as factor with PB seen

A1 conclude with correct ratio and <u>P as common point</u> | |

(iii) Find the numerical value of $\dfrac{\text{area of triangle } ASB}{\text{area of triangle } AOB}$.

| 7b (iii) | $\dfrac{\text{area of triangle } ASB}{\text{area of triangle } AOB}$

$= \dfrac{\dfrac{1}{2}(SB)(h)}{\dfrac{1}{2}(OB)(h)}$

$= \dfrac{1}{4}$ | | A1 | |

Answer .... [1]

8 The variables x and y are connected by the equation $y = -\frac{1}{5}x^3 + 2x - 1$

(a) Complete the table of values for $y = -\frac{1}{5}x^3 + 2x - 1$

x	4	3	2	1	0	1	2	3	4
y	5.8	0.4		0.8	1	2.8	3.4	1.6	3.8

[1]

8a	1.4	A1	

(b) On the grid opposite, draw the graph of $y = -\frac{1}{5}x^3 + 2x + 1$ for $-4 \leq x \leq 4$
[3]

Marker's report:

Curve should be drawn with smooth curve using SHARP pencil

note curve has point (-3.5, 2.6) and (-2.5, 0.9)

8b	**Smooth curve** with correct 9 points	B3 Or B2FT for 9 points plotted correctly Or B1FT for 7 or 8 points plotted correctly Tolerance ±1mm for plotting points and drawing curve through points	

(c) By drawing a suitable tangent, find the gradient of the curve at $x=-3$.

8c	Draw tangent at $x=-3$	M1	
	$grad = -3.4$	A1 (accept -2.1 to -4.1)	

Answer .... [2]

(d) (i) On the same grid, draw the graph of $2y+x=1$ for $-4 \le x \le 4$.

8d	$2y+x=1$	L1 cuts at 0 5 at y-axis	

[1]

(ii) Show that the points of intersection of the line and the curve gives the solution of $-2x^3+25x+5=0$.

Answer

8d(ii)	$-\dfrac{2}{5}x^3 - 5x+1 = \dfrac{1}{2}x - \dfrac{1}{2}$ $-2x^3 +20x+10 = -5x+5$ $2x^3 +25x+5 = 0$ (shown)	B1 Equate line and curve **B1 (x 10)**	

[2]

(iii) Use your graph to solve the equation $-2x^3+25x+5=0$.

8d(iii)	x -3.4, -0.2 or 3.7	A2 FT intersections of their line and their curve Or A1FT for two correct solutions Tolerance ±0.1 for readings	

Answer $x=$ .... or .... or .... [2]

[Turn over

9   To prevent vehicles from speeding, speed humps are built on the roads to regulate the speed of cars.
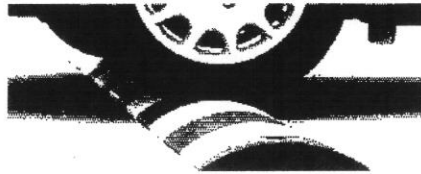

Diagram 1 shows the length, width and height of one speed hump. Each speed hump along a road has 4 identical speed humps joined together.
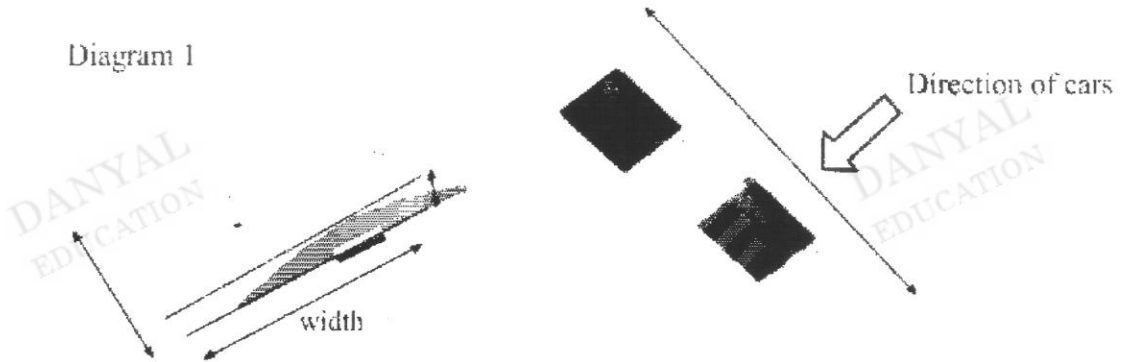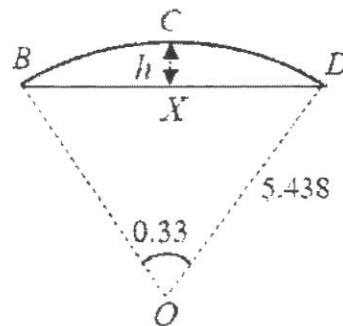
Diagram 1

Direction of cars

width

Diagram 2 shows the cross-section of one speed hump.
BCD is part of sector $OBCD$ with centre O, radius 5.438 m and angle BOD of 0.33 radians.

Diagram 2 : Cross-section of one speed hump

(a)   Find the length of the arc BCD of the speed hump as shown in Diagram 2.
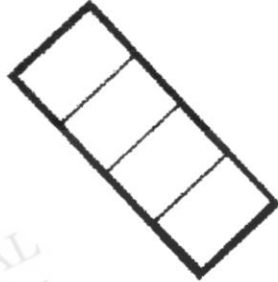
9a)	$s = 5.438 (0.33)$ $= 1.79454$ $= 1.79$ m	A1	

Answer .... m [1]

(b) Diagram 3 shows the top view of 4 identical speed humps joined together. The thickness of the rubber to cover the top of each speed hump is 0.5 cm and the total cost to cover four such speed humps is \$240.

Using the information given in the table, calculate the length of one speed hump.

Diagram 3 : Top view of four speed humps

Information for material used and cost	
Density of rubber	1060 kg/m³
Cost of rubber per kg	\$20
Cost to cover the **top** part of 4 speed humps	\$240

| 9b | Let the length of 1 bump be x m

 Using volume of rubber
 0.005 x **1.79 454 x length**
 = 0.0089 727 x m³

 OR area of sector x length
 (½ (5.438)²(0.33) – ½ (5.438 – 0.005)² (0.33)
 =½ (5.438)²(0.33) – ½ (5.433)² (0.33)
 =4.879 354 – 4.870 386
 =0.0089 68315 m³

 Mass of rubber
 = 1060 x **0.0089 727** x *(mass = density x vol)*
 = 9.511 062 x kg

 Cost of rubber for 1 bump : 240 ÷ 4 = \$60

 20 (9.511 062 x) = 60
 $x = \dfrac{60}{20(9.511062)}$ OR $x = \dfrac{240}{4 \times 20(9.511062)}$

 $x = 0.315 422$ m

 length of bump of 0.315 m. | M1 (find volume of 1 bump using **0.005** (in metre) x their arc length BCD x length or difference in area of sector x length

 M1 find Mass of rubber 1060 x their vol
 OR
 M1 (find length using \$20 x mass of 1 rubber = 60) or \$20 x their mass x 4 =240
 (M1 if find mass of 1 bump :
 (240 ÷ 4) ÷ 20 = 3kg

 A1 (use of diff in sector, ans for 1 length will be 0.315568) | |

	Alternative Mass of 1 hump - $\dfrac{240}{4 \times 20} = 3\,kg$ Vol = $3 \div 1060 = 0.0028301\ m^{3}$ $x\ (1.79454 \times 0.005 = 0.00\ 28301$ $x = 0.315\ m$		

Answer ....m [3]

(c) The engineers are to ensure that the height (h), width (w) and length (l) of **each** speed hump meet the safety criteria as shown in the table.

Height of the hump (h)	$0.0732\ m \le h < 0.1016\ m$
Width of the hump (w)	$0.9144\ m \le w < 1.8288\ m$
Gradient of the hump (g), where g is given as the ratio of the height of the hump : horizontal length of BX or XD	$g < 0.1$
Length of one speed hump (l)	$0.3048m \le l \le 0.9144\ m$

Show with clear working if the engineers should approve this speed hump.

Answer

9c	Height of the hump: $\cos 9.4538° = \dfrac{OX}{5.438}$ or $\cos 0.165\,rad = \dfrac{OX}{5.438}$ $OX = 5.438 \cos 9.538$ $OX = 5.36414$ $h = 5.438 - 5.36414$ $= 0.0738569$ $= 0.0739\ m$ Chord BD $\sin 9.4538 = \dfrac{BX}{5.438}$ or $\sin 0.165\,rad = \dfrac{BX}{5.438}$ $BX = 5.438 \sin 9.5288$ $BX = 0.893204$ $BD = 2 \times 0.893204$ $= 1.786408$ $= 1.79\ m$	M1 finding OX using trigo ratio or sine rule Pythagoras thm if they have found BD first A1 for correct h or BD or BX M1 finding BX using trigo ratio / sine rule or A1	

	OR	M1	
	Gradient of hump using ratio $$grad = \frac{0.073857}{0.893204}$$ $$= 0.0827$$	(their h ÷ their BX or their BD)	
	Gradient < 0.1	B1 for comparison of length from part b (to award only if length < 1m) AND conclusion based for all safety criteria	
	From 9b) length of bump of 0.316 m (0.3048m $\leq$ l ≤ 0.9144 m		
	All criteria are met, the engineers can approve the speed bump.		

ORIGINAL. [6]

| 9c | Height of the hump:

$\cos 9.4538° = \dfrac{OX}{5.438}$ $\quad or \quad \cos 0.165\,rad = \dfrac{OX}{5.438}$

$OX = 5.438\cos 9.538$

$OX = 5.36414$

$h = 5.438 - 5.36414$

$= 0.0738569$

$= 0.0739$ m

<u>Chord BD</u>

$\sin 9.4538 = \dfrac{BX}{5.438}$ $\quad or \quad \sin 0.165\,rad = \dfrac{BX}{5.438}$

$BX = 5.438\sin 9.5288$

$BX = 0.893204$

$BD = 2 \times 0.893204$

$= 1.786408$

$= 1.79$ m

<u>Gradient of hump using ratio</u>

$grad = \dfrac{0.073857}{0.893204}$

$= 0.0827$

Gradient < 0.1

From 9b) length of bump of 0.316 m
$(0.3048m \leq l \leq 0.9144$ m$)$

All criteria are met, the engineers can approve the speed bump. | M1 finding OX using trigo ratio or sine rule

A1

M1 finding BX using trigo ratio / sine rule

A1

M1
(their h / their BX or their BD)

B1 for comparison of length from part b (to award only if length < 1m) AND conclusion based for all safety criteria | |