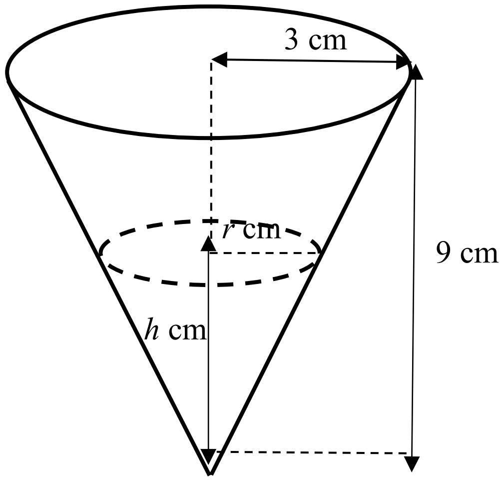
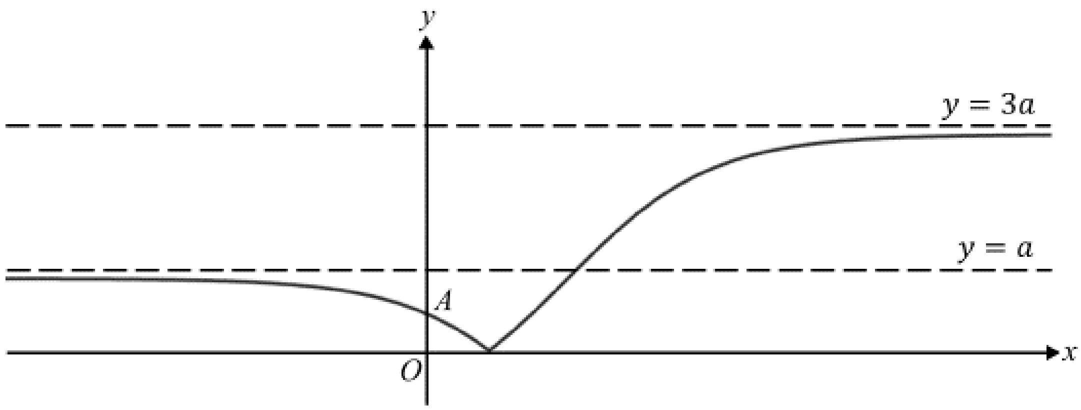
<u>Promos Practice Paper 3 [YIJC 2022] 98marks</u>

1 [It is given that the volume of a circular cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.]



A cone-shaped paper drinking cup with height 9 cm and radius 3 cm is shown above. Water is poured at a constant rate of 5 cm^3 per second into the cup. When the depth of water is h cm, the surface of the water has radius r cm (see diagram). Find the rate of increase of the depth of water when $r = 2$. **[4]**

2 The diagram below shows the graph of $y = |\mathrm{f}(x)|$ with asymptotes $y = a$ and $y = 3a$, where $a > 0$. The curve crosses the y-axis at the point $A\left(0, \frac{a}{2}\right)$.



Given that f is an increasing function, sketch the graphs of

(a) $y = \mathrm{f}(x) + a$, **[2]**

(b) $y = \mathrm{f}(-|x|)$, **[2]**

stating the equations of any asymptotes and the coordinates of the point corresponding to A after the transformation.

3 **MI Promo 9758/2021/PU2/01/Q2**

(i) Given that θ is sufficiently small, show that

$$\sin\left(\frac{\pi}{3}-\theta\right)\approx a+b\theta+c\theta^2,$$

where a, b and c are constants to be determined. **[3]**

(ii) By using the substitution $\theta=\frac{\pi}{10}$, find an approximate value for $\cos\left(\frac{4\pi}{15}\right)$, giving your answer correct to 5 decimal places. **[2]**

(iii) By using a calculator to evaluate $\cos\left(\frac{4\pi}{15}\right)$, correct to 5 decimal places, explain why the approximation in part **(ii)** is not good. **[1]**

4 **MI PU2 Promo 9758/2019/01/Q4**

Do not use a calculator in answering this question.

(i) Given that $z=2+\mathrm{i}$ is a root of the equation $z^3+2z^2-19z+30=0$, find the other roots. [4]

(ii) Hence find in cartesian form the roots of the equation $\mathrm{i}w^3-2w^2+19\mathrm{i}w+30=0$. [2]

5 A curve C has equation $xy^2-3x^2y+144=0$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and the coordinates of the turning point of C. **[5]**

The point P on C has coordinates $(4, k)$ for some constant k.

(ii) Find the equation of the tangent at P. **[3]**

6 An arithmetic series has first term 3 and common difference d, where d is non-zero. A geometric series has first term a and common ratio r. Given that the 37th, 7th and 1st terms of the arithmetic series are consecutive terms of the geometric series, find d. **[3]**

Deduce that the geometric series is convergent. Find, in terms of a, the sum to infinity of the odd-numbered terms (i.e. the 1st, 3rd, 5th, …. terms) of the geometric series. **[3]**

Given further that a = 3072, find the least value of n such that the sum of the first n terms of the arithmetic series exceeds the sum to infinity of the odd-numbered terms of the geometric series. **[2]**

7 A curve C has parametric equations

$$x = 2t, \qquad y = \frac{1}{1-t}, \quad t \neq 1.$$

Show that the normal to C at the point with parameter t has equation

$$(1-t)y + 2(1-t)^3 x = 1 + 4t(1-t)^3. \qquad \textbf{[4]}$$

State the equation of the normal at the point P where $t = 2$. This normal cuts C again at the point Q. Find the exact coordinates of Q. **[4]**

8 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 1 + \frac{2}{x^2 - 4}, \quad x \in \mathbb{R}, \ 0 \leq x < 2,$$

$$\mathrm{g} : x \mapsto \frac{1}{x}, \quad x \in \mathbb{R}, \ x \geq 1.$$

(i) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. **[3]**

(ii) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$, giving the equations of any asymptotes and the exact coordinates of any points where the curves cross the x- and y-axes. **[4]**

(iii) Solve the equation $\mathrm{fg}(x) = \frac{2}{5}$. **[2]**

9 On 1 January 2022, Jerald puts \$500 into a savings account which pays interest at a rate of 0.1% per month on the last day of each month. On the first day of each subsequent month from February 2022, he puts another \$$x$ into the account.

(i) Write down how much Jerald's initial deposit of \$500 will become on 30 November 2022 after adding compound interest earned. **[1]**

(ii) Taking January 2022 as the first month, show that the amount of money in Jerald's account on the last day of the nth month is

$$500(1.001)^n + 1001x\left[(1.001)^{n-1} - 1\right]. \qquad \textbf{[3]}$$

(iii) Find the least integer value of x so that the interest earned for the first six months of the year 2022 exceeds \$30. **[2]**

(iv) Given that $x = 300$, how much will Jerald have in his account on 31 December 2025? Hence state the date on which the amount will first exceed \$15 000. **[3]**

10 The curves C and D have equations $y=\frac{5}{3}-x-\frac{4}{x-3}$ and $\frac{(x-1)^2}{6^2}+\frac{y^2}{k^2}=1$ respectively, where k is a positive constant.

(i) Using an algebraic method, find the exact range of values of y that C can take. **[4]**

(ii) On the same axes, sketch

(a) the graph of C, stating the equations of any asymptotes and the coordinates of the turning points, **[2]**

(b) the graph of D for the case where $k=2$, stating the coordinates of the centre, the turning points and the points of intersection with the x-axis. **[2]**

(iii) State the exact range of values of k such that C and D intersect at more than one point. **[1]**

(iv) State the range of values of m such that the line with equation $y+\frac{4}{3}=m(x-3)$ does not intersect C. **[1]**

11 **(a)** Mr Tan buys 5 bottles of cooking oil, 4 packets of biscuits and 2 packets of rice. Based on the usual retail price in the supermarket, the total amount paid is $73.45.

Currently, there is a “Buy 6 Get 1 Free” promotion for the biscuits. Under this promotion, Mr Suresh pays $53.30 and receives 2 bottles of cooking oil, 14 packets of biscuits and a packet of rice.

Ms Siti receives a 5% membership discount on the total bill and pays $103.93 for 4 bottles of cooking oil, 2 packets of biscuits and 5 packets of rice.

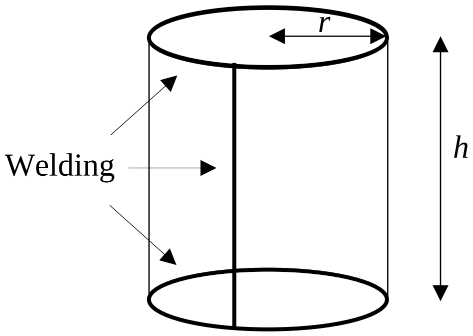
Write down and solve equations to find the usual retail price of a bottle of cooking oil, a packet of biscuits and a packet of rice. **[4]**

(b) Without using a calculator, solve

(i) $\frac{x^2}{6-x}\geq 1$, **[3]**

(ii) $x^2-4\geq (x+2)\left(4x^2-3x-1\right)$. **[3]**

12 A closed cylindrical can with radius r cm and height h cm has fixed volume 20π cm^3. The material for the top and bottom faces costs \$0.50 per cm^2 and the material for the curved surface costs \$0.30 per cm^2. It also costs \$0.80 per cm to weld the top and bottom faces onto the cylinder and \$0.60 per cm to weld the seam up the curved surface of the cylinder (see diagram).



(i) The total cost of the can is \$$C$. Show that $C = \pi r^2 + 3.2\pi r + \dfrac{12\pi}{r} + \dfrac{12}{r^2}$. **[3]**

(ii) Use differentiation to find the values of r and h which give a minimum value of C, proving that C is a minimum. State this value of C. **[7]**

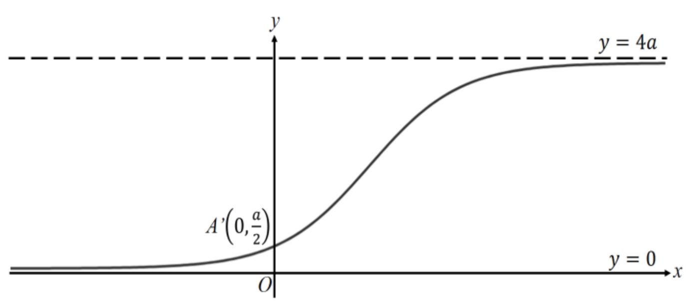
(iii) It is given instead that $0.5 \le r \le 2$. Find the corresponding range of values of C. **[2]**

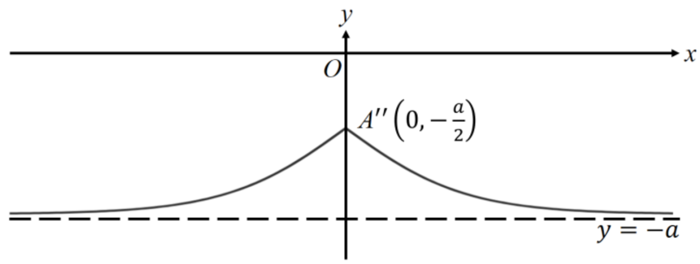
Answers

1	0.398 (or $\frac{5}{4\pi}$) cm/s
2	**Graph**
3	**(i)** $\therefore a=\frac{\sqrt{3}}{2}, b=-\frac{1}{2}, c=-\frac{\sqrt{3}}{4}$ **(ii)** (ii) $0.66621\ (5\text{ d.p})$
4	$z=2-\mathrm{i}$ and $z=-6$
5	(i) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y(6x-y)}{x(2y-3x)}$, $(-2,-12)$ $x=4$
6	$d=2$, Sum to infinity of odd-numbered terms $=\frac{25}{24}a$, least $n=56$
7	$y+2x=7$, $Q\left(\frac{3}{2},4\right)$.
8	(i) $\mathrm{f}^{-1}(x)=\sqrt{4+\frac{2}{x-1}}$, $\mathrm{D}_{\mathrm{f}^{-1}}=\left(-\infty,\frac{1}{2}\right]$ $x\approx 1.22\ (3\text{ s.f.})$
9	(i) \$505.53 (iii) 1798 \$14968.22 , 1st Jan 2026
10	(i) $y\le -\frac{16}{3}$ or $y\ge \frac{8}{3}$ (iii) $k>\frac{8}{3}$ $m\ge -1$
11	(a) Let \$*x*, \$*y* and \$*z* be the usual retail price of a bottle of cooking oil, a packet of biscuits and rice respectively. $x=6.85$, $y=2$ and $z=15.6$ (i) $x\le -3$ or $2\le x<6$, (ii) $x\le -2$ or $x=\frac{1}{2}$
12	$r=1.61$ (3 s.f.) and $h=7.68$ (3 s.f.) . The least cost is \$52.37. $52.37\le C\le 129.21$

Promos Practice Paper 3 [YIJC 2022] Solutions

Qn	Solutions
1	Let V cm^3 represent the volume of water in the cup when the depth of water is h cm. $\frac{r}{h}=\frac{3}{9}$ $r=\frac{h}{3}$ --- (1) Substitute $r=\frac{h}{3}$ into $V=\frac{1}{3}\pi r^2 h$, $V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$ $=\frac{1}{27}\pi h^3$ **Alternatively,** Substitute $h=3r$ into $V=\frac{1}{3}\pi r^2 h$, $V=\pi r^3$ $\frac{dV}{dh}=\frac{1}{9}\pi h^2$ $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$ $=\frac{9}{\pi h^2}\times 5$ $=\frac{45}{\pi h^2}$ **Alternatively,** Differentiate V wrt t, $\frac{dV}{dt}=\frac{3}{27}\pi h^2\frac{dh}{dt}$ $5=\frac{3}{27}\pi h^2\frac{dh}{dt}$ $\frac{dh}{dt}=\frac{45}{\pi h^2}$ **OR** Differentiate V wrt t, $\frac{dV}{dt}=3\pi r^2\frac{dr}{dt}$ $\frac{dh}{dt}=3\frac{dr}{dt}=\frac{5}{\pi r^2}$ When $r=2$, $\frac{dh}{dt}=\frac{5}{4\pi}\approx 0.398$ When $r=2$, $h=2\times 3=6$ When $h=6$, $\frac{dh}{dt}=\frac{45}{\pi(6)^2}=\frac{45}{36\pi}=\frac{5}{4\pi}\approx 0.398$ The rate of increase of the depth of the water is 0.398 (or $\frac{5}{4\pi}$) cm / s

2a	$y=\mathrm{f}(x)+a$ 
2b	$y=\mathrm{f}(-\lvert x\rvert)$


y
x
O
$A''\left(0,-\frac{a}{2}\right)$
$y=-a$

3(i) **[3]**	$\sin\left(\frac{\pi}{3}-\theta\right)=\sin\frac{\pi}{3}\cos\theta-\sin\theta\cos\frac{\pi}{3}$ $=\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta$ $\approx\frac{\sqrt{3}}{2}\left(1-\frac{\theta^2}{2}\right)-\frac{1}{2}\theta$ $=\frac{\sqrt{3}}{2}-\frac{1}{2}\theta-\frac{\sqrt{3}}{4}\theta^2$ $\therefore a=\frac{\sqrt{3}}{2},b=-\frac{1}{2},c=-\frac{\sqrt{3}}{4}$
3(ii) **[3]**	$\sin\left(\frac{\pi}{3}-\frac{\pi}{10}\right)\approx\frac{\sqrt{3}}{2}-\frac{1}{2}\left(\frac{\pi}{10}\right)-\frac{\sqrt{3}}{4}\left(\frac{\pi}{10}\right)^2$ $\sin\left(\frac{7\pi}{30}\right)=0.66621\ (5\text{ d.p})$ using $\cos\theta=\sin\left(\frac{\pi}{2}-\theta\right)$, $\cos\left(\frac{4\pi}{15}\right)=\sin\left(\frac{\pi}{2}-\frac{4\pi}{15}\right)$ $=\sin\left(\frac{7}{30}\pi\right)$ $=0.66621\ (5\text{ d.p})$
3(iii)	Using calculator, $\cos\left(\frac{4\pi}{15}\right)=0.66913\ (5\text{ d.p.})$ The approximation is not good because $\frac{\pi}{10}$ is **not small enough**. ***<u>Alternative explanation:</u>*** The terms with θ^3 and higher powers **are not negligible** / **are significant** enough to affect the accuracy of the approximation.

4(i)	Since all coefficients of the equation $z^3+2z^2-19z+30=0$ are real and $z=2+\mathrm{i}$ is a root, $z=2-\mathrm{i}$ is another root. A quadratic factor is

	$\left[z-(2+\mathrm{i})\right]\left[z-(2-\mathrm{i})\right]$ $=(z-2-\mathrm{i})(z-2+\mathrm{i})$ $=(z-2)^2-\mathrm{i}^2$ $=z^2-2(z)(2)+2^2-(-1)$ $=z^2-4z+4+1$ $=z^2-4z+5$ $\therefore z^3+2z^2-19z+30=(z^2-4z+5)(z+a)$ **To get the 3rd root:** **<u>Method 1</u>** Comparing constant terms $30=5a$ $\Rightarrow a=6$ $z+6=0 \Rightarrow z=-6$ **<u>Method 2</u>** By long division $\begin{array}{r} z+6 \\ z^2-4z+5\overline{\big)\, z^3+2z^2-19z+30} \\ \underline{-(z^3-4z^2+5z)} \\ 6z^2-24z+30 \\ \underline{-(6z^2-24z+30)} \\ 0 \end{array}$ $z+6=0 \Rightarrow z=-6$ $\therefore$ Besides $z=2+\mathrm{i}$, the other roots are $z=2-\mathrm{i}$ and $z=-6$.

5(i)	$xy^2-3x^2y+144=0$ Differentiating w.r.t x: $2xy\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)+y^2-6xy-3x^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}(2xy-3x^2)=6xy-y^2$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6xy-y^2}{2xy-3x^2}$ $=\frac{y(6x-y)}{x(2y-3x)}$

	At the turning point, $\frac{dy}{dx}=0$ $y(6x-y)=0$ $y=0$ or $x=\frac{y}{6}$ For $y=0$, $x(0)^2-3x^2(0)+144=144\neq 0$ Therefore, we reject $y=0$. Substitute $x=\frac{y}{6}$ into the equation for curve C $\left(\frac{y}{6}\right)y^2-3\left(\frac{y}{6}\right)^2 y+144=0$ $\frac{y^3}{12}=-144$ $y^3=-1728$ $y=-12$ $x=-2$ **Alternatively,** Substitute $y=6x$ into the eqn, $x(6x)^2-3x^2(6x)+144=0$ $18x^3=-144$ $x=-2$ $y=-12$ The coordinates of the turning point are $(-2,-12)$
(ii)	At $x=4$, $4k^2-3(4)^2k+144=0$ $4k^2-48k+144=0$ $k^2-12k+36=0$ $k=6$ At $x=4$, $\frac{dy}{dx}=\frac{6(24-6)}{4(12-12)}$ which is undefined. Hence the tangent is parallel to the y-axis. $\therefore$ the equation of the tangent is $x=4$.

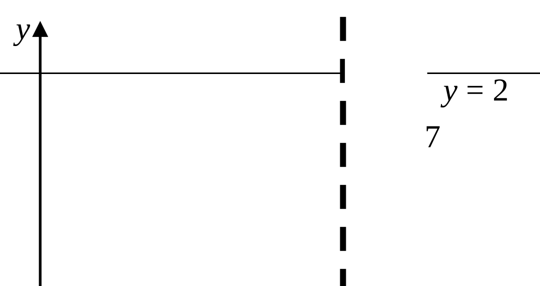
6	$\frac{3+6d}{3+36d}=\frac{3}{3+6d}$ $\frac{1+2d}{1+12d}=\frac{1}{1+2d}$ $(1+2d)^2=1+12d$ $1+4d+4d^2=1+12d$ $4d^2-8d=0$ Since $d\neq 0$, $d-2=0$ $d=2$

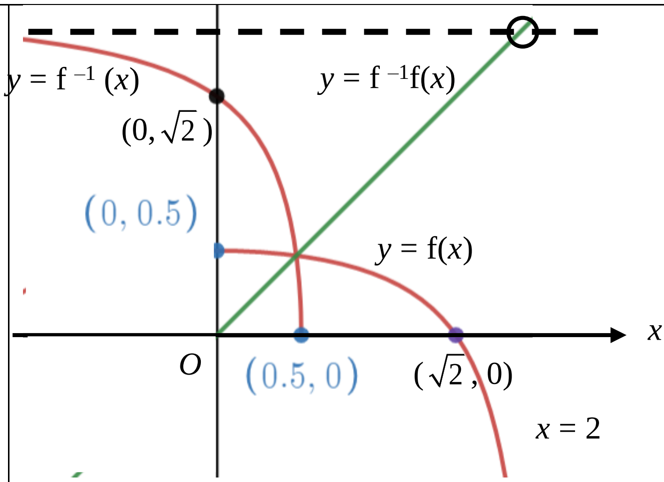
	$r=\frac{3}{3+6d}=\frac{3}{3+6(2)}=\frac{1}{5}$ Since $\lvert r\rvert=\frac{1}{5}<1$, the geometric series is convergent. (Or $-1<r<1$) Sum to infinity of odd-numbered terms $=\frac{a}{1-\left(\frac{1}{5}\right)^2}$ $=\frac{25}{24}a$
	$\frac{n}{2}\left[2(3)+(n-1)(2)\right]>\frac{25}{24}(3072)$ $\frac{n}{2}(4+2n)>3200$ $n^2+2n-3200>0$ $n<-57.577$ or $n>55.577$ $\therefore$ least $n=56$ **<u>Alternative method (using GC)</u>** When n = 55, S_n = 3135 When n = 56, S_n = 3248 When n = 56, S_n = 3363 $\therefore$ least $n=56$

7	$x=2t,\qquad y=\frac{1}{1-t}$ $\frac{\mathrm{d}x}{\mathrm{d}t}=2,\qquad \frac{\mathrm{d}y}{\mathrm{d}t}=-(1-t)^{-2}(-1)=\frac{1}{(1-t)^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{1}{(1-t)^2}}{2}=\frac{1}{2(1-t)^2}$ Gradient of normal = $-2(1-t)^2$ Equation of normal at point with parameter t: $y-\frac{1}{1-t}=-2(1-t)^2(x-2t)$ $(1-t)y-1=-2(1-t)^3(x-2t)$ $(1-t)y+2(1-t)^3x=1+4t(1-t)^3\quad$ (shown)

	Equation of normal at point where $t = 2$: $-y-2x=-7$ $y+2x=7$ Substitute $x=2t$ & $y=\dfrac{1}{1-t}$ into equation of normal: $\dfrac{1}{1-t}+4t=7$ $1+4t(1-t)=7(1-t)$ $1+4t-4t^2=7-7t$ $4t^2-11t+6=0$ $(t-2)(4t-3)=0$ $t=2$ or $t=\dfrac{3}{4}$ At Q, $t=\dfrac{3}{4}$ $x=\dfrac{3}{2}, \quad y=\dfrac{1}{1-\frac{3}{4}}=4$ $\therefore$ coordinates of Q are $\left(\dfrac{3}{2}, 4\right)$.

8(i)	Let $y=1+\dfrac{2}{x^2-4}$. $x^2-4=\dfrac{2}{y-1}$ $x^2=4+\dfrac{2}{y-1}$ $x=\pm\sqrt{4+\dfrac{2}{y-1}}$ (rej. neg since $0\le x<2$) $\therefore \mathrm{f}^{-1}(x)=\sqrt{4+\dfrac{2}{x-1}}$ $\mathrm{D}_{\mathrm{f}^{-1}}=\mathrm{R}_\mathrm{f}=\left(-\infty, \dfrac{1}{2}\right]$
(ii)	





(iii)

$$\mathrm{fg}(x)=\frac{2}{5}$$

$$1+\frac{2}{\left(\frac{1}{x}\right)^2-4}=\frac{2}{5}$$

$$1+\frac{2x^2}{1-4x^2}=\frac{2}{5}$$

$\mathrm{D_{fg}}=\mathrm{D_g}=[1,\infty)$

From GC, $x\approx 1.2247=1.22\ (3\text{ s.f.})$

<u>Alternative method</u>

$$\mathrm{fg}(x)=\frac{2}{5}$$

$$\mathrm{g}(x)=\mathrm{f}^{-1}\left(\frac{2}{5}\right)$$

$$\frac{1}{x}=\sqrt{4+\frac{2}{\frac{2}{5}-1}}$$

$$=\sqrt{\frac{2}{3}}$$

$$x=\sqrt{\frac{3}{2}}$$

9(i)	Required amount $=500(1.001)^{11}$ $\approx \$505.53$

(ii)

Month	Amount of Money in the Account at the Start of the month	Amount of Money in the Account at the End of the month
1	500	500(1.001)
2	$500(1.001) + x$	$500(1.001)^2 + 1.001x$
3	$500(1.001)^2 + 1.001x + x$	$500(1.001)^3 + (1.001)^2 x + 1.001x$

Amount of money in the account on the last day of the nth month

$$= 500(1.001)^n + (1.001)^{n-1} x + (1.001)^{n-2} x \ \dots \ + 1.001x$$

$$= 500(1.001)^n + \left[(1.001)^{n-1} + (1.001)^{n-2} + \dots + 1.001\right] x$$

$$= 500(1.001)^n + \left[\frac{1.001\left((1.001)^{n-1} - 1\right)}{1.001 - 1}\right] x = 500(1.001)^n + 1001\left[(1.001)^{n-1} - 1\right] x \quad \text{(shown)}$$

<u>Alternative method</u>

The first \$$x$ put on 1 Feb 2021 will be in the bank for $(n-1)$ months and will become $1.001^{n-1}x$ by the end of nth month.

The second \$$x$ put on 1 Mar 2021 will be in the bank for $(n-2)$ months and will become $1.001^{n-2}x$ by the end of nth month.

The third \$$x$ put on 1 Apr 2021 will be in the bank for $(n-3)$ months and will become $1.001^{n-3}x$ by the end of nth month.

…

The $(n-1)$ th \$$x$ put on the nth month will be in the bank for 1 month and will become $1.001x$ by the end of nth month.

Also, the initial \$500 put on 1 Jan 2021 will be in the bank for n months and will become $1.001^n(500)$ by the end of nth month.

Hence, total amount at the end of nth month

$$= 500\left(1.001^n\right) + 1.001x + \dots + 1.001^{n-2}x + 1.001^{n-1}x$$

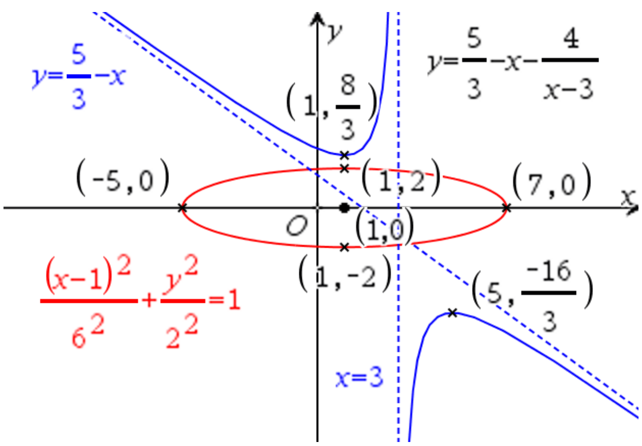
$$= 500\left(1.001^n\right) + x\left(1.001 + \dots + 1.001^{n-2} + 1.001^{n-1}\right)$$

$$= 500\left(1.001^n\right) + x\left[\frac{1.001\left(1.001^{n-1} - 1\right)}{1.001 - 1}\right]$$

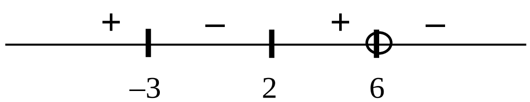
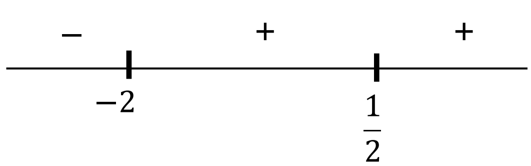
$$= 500\left(1.001^n\right) + 1001\left(1.001^{n-1} - 1\right)x \quad \text{(shown)}$$

(iii)	$500(1.001)^6 + 1001\left[(1.001)^5 - 1\right]x - 500 - 5x > 30$ $\Rightarrow x > 1797.10$ least integer value of x is 1798
(iv)	On 31 Dec 2025, $n = 48$ $500\left(1.001^{48}\right) + 1001(300)\left(1.001^{47} - 1\right) \approx 14968.22$ Jerald has \$14968.22 in his account on 31 Dec 2025. Since $14968.22 + 300 = 15268.22 > 15000$, The amount will first exceed \$15000 on 1st Jan 2026.

10(i)	$y = \frac{5}{3} - x - \frac{4}{x-3}$ $(x-3)y = (x-3)\left(\frac{5}{3} - x\right) - 4$ $xy - 3y = \frac{5}{3}x - x^2 - 5 + 3x - 4$ $3xy - 9y = -3x^2 + 14x - 27$ $3x^2 + (3y - 14)x + (27 - 9y) = 0$ The equation has real roots $\Rightarrow b^2 - 4ac \geq 0$ $(3y - 14)^2 - 4(3)(27 - 9y) \geq 0$ $9y^2 - 84y + 196 - 324 + 108y \geq 0$ $9y^2 + 24y - 128 \geq 0$

	Consider $9y^2+24y-128=0$ $y=\dfrac{-24\pm\sqrt{24^2-4(9)(-128)}}{18}$ $=\dfrac{8}{3}$ *or* $-\dfrac{16}{3}$ For $9y^2+24y-128\geq 0$, $+$ $-\dfrac{16}{3}$ $-$ $\dfrac{8}{3}$ $+$ The range of values that C can take is $y\leq -\dfrac{16}{3}$ or $y\geq\dfrac{8}{3}$
(ii)	
(iii)	For C and D to intersect at more than one point, $k>\dfrac{8}{3}$.
(iv)	The line $y+\dfrac{4}{3}=m(x-3)$ has gradient m and passes through the point $\left(3,-\dfrac{4}{3}\right)$ which is the point of intersection of the vertical and oblique asymptotes. From the graph, the line does not intersect C when $m\geq -1$.

11(a)	Let \$$x$, \$$y$ and \$$z$ be the usual retail price of a bottle of cooking oil, a packet of biscuits and rice respectively. $5x+4y+2z=73.45 \qquad \cdots(1)$

	$2x+12y+z=53.30 \quad \cdots(2)$ $4x+2y+5z=\frac{100}{95}(103.93)=109.4 \quad \cdots(3)$ From GC, $x=6.85$, $y=2$ and $z=15.6$ The usual retail prices of 1 bottle of cooking oil, 1 packet of biscuits and 1 packet of rice are \$6.85, \$2 and \$15.60 respectively.
(b)(i)	$\frac{x^2}{6-x} \geqslant 1$ $\frac{x^2}{6-x}-1\geq 0$ $\frac{x^2-(6-x)}{6-x}\geq 0$ $\frac{(x-2)(x+3)}{6-x}\geq 0$  $\therefore \quad x\leq -3$ or $2\leq x<6$
(b)(ii)	$x^2-4\geq (x+2)(4x^2-3x-1)$ $(x+2)(x-2)-(x+2)(4x^2-3x-1)\geq 0$ $(x+2)\left[x-2-(4x^2-3x-1)\right]\geq 0$ $(x+2)(-4x^2+4x-1)\geq 0$ $(x+2)(4x^2-4x+1)\leq 0$ $(x+2)(2x-1)^2\leq 0$  $\therefore \quad x\leq -2$ or $x=\frac{1}{2}$

12 (i)	$V=\pi r^2 h=20\pi$ $h=\frac{20}{r^2}$

	$C=0.50\times 2\left(\pi r^2\right)+0.30\left(2\pi rh\right)+0.8\times 2\times\left(2\pi r\right)+0.6h$ $=\pi r^2+0.6\pi r\left(\dfrac{20}{r^2}\right)+3.2\pi r+0.6\left(\dfrac{20}{r^2}\right)$ $=\pi r^2+\dfrac{12\pi}{r}+3.2\pi r+\dfrac{12}{r^2}$
12 (ii)	$\dfrac{\mathrm{d}C}{\mathrm{d}r}=2\pi r-\dfrac{12\pi}{r^2}+3.2\pi-\dfrac{24}{r^3}$ Putting $\dfrac{\mathrm{d}C}{\mathrm{d}r}=0$, $2\pi r-\dfrac{12\pi}{r^2}+3.2\pi-\dfrac{24}{r^3}=0$ $2\pi r^4+3.2\pi r^3-12\pi r-24=0$ [*optional*] Using GC, $r\approx 1.613596$ or $r\approx -0.685758$ (Rej since $r>0$) $h=\dfrac{20}{1.613596^2}\approx 7.6814$ $\therefore$ $r=1.61$ (3 s.f.) and $h=7.68$ (3 s.f.) $\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}=2\pi+\dfrac{24\pi}{r^3}+\dfrac{72}{r^4}$ When $r\approx 1.613596$, since $r>0$, $\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}>0$ (or $\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}\approx 34.85>0$) Thus C is minimum. The least cost is \$52.37.
12 (iii)	When $0.5\le r\le 2$, From GC, $52.37\le C\le 129.21$ (2 d.p.)