Name:	Index Number:	Class:

YIO CHU KANG SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2018
SECONDARY THREE EXPRESS

ADDITIONAL MATHEMATICS
Paper 1

4047/01
2 hours

Additional materials: Answer Paper

4 October 2018 (Thursday)

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

Setter: Mr Tan Thiam Boon

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \ \cos A$$

$$\Delta = \frac{1}{2}ab \ \sin C$$

1 **(i)** On the same axes, sketch the graphs of $y^2 = 36x$ and $y = \dfrac{6}{\sqrt{x^3}}$ for $x > 0$. [2]

(ii) Calculate the coordinates of the point of intersection of your graphs. [3]

2 Solve the simultaneous equations

$$2x + y + 2 = 0,$$
$$\frac{1}{x} + \frac{2}{y} = \frac{1}{2}.$$
[5]

3 **(i)** Find the set of values of *k* for which the curve $y = (k-2)x^2 + 2kx + (k+3)$ lies entirely above or below the *x*-axis. [3]

(ii) Justify whether the curve lies entirely above or below the *x*-axis. [2]

4 The graph of $y = \log_a x$ passes through the points with coordinates (27, 3), (1, *b*) and (*c*, –1).

(i) Determine the value of each of the constants *a*, *b* and *c*. [3]

(ii) Sketch the graph of $y = \log_a x$. [2]

5 Given that $\dfrac{3x^3 + 11x - 4}{(x^2+4)(x-1)}$ can be expressed in the form $A + \dfrac{Bx + C}{x^2 + 4} + \dfrac{D}{x-1}$ for all real values of *x*, find the values of *A*, *B*, *C* and *D*. [6]

6 Given that $\dfrac{9^{n+2} - 3^{2n+2}}{2^5} = 2^a 3^b$, where *a* and *b* are integers,

(i) find the value of *a* and express *b* in terms of *n*, [5]

(ii) hence, or otherwise, solve the equation $\dfrac{9^{n+2} - 3^{2n+2}}{2^5} = \dfrac{1}{4}$. [2]

7 **(a)** Express $2\log_5 x - \log_5(x-6) = 1$ as a quadratic equation in *x* and explain why there are no real solutions. [5]

(b) Given that $\log_{16} x^2 = \log_8 u$, express *u* in terms of *x*. [3]

8 A curve has the equation $y=-(3x-2)^2+9$.

(i) Explain why the highest point on the curve has coordinates $(\frac{2}{3}, 9)$. [1]

(ii) Find the coordinates of the points at which the curve intersects the x-axis. [2]

(iii) Sketch the graph of $y=\left|-(3x-2)^2+9\right|$ indicating clearly the coordinates of the turning point and the points where the curve meets the x and y axes. [3]

(iv) Using your graph, state the number of solutions to each of the following equations.

(a) $\left|-(3x-2)^2+9\right|=10$, [1]

(b) $\left|-(3x-2)^2+9\right|+3=0$, [1]

(c) $\left|-(3x-2)^2+9\right|=-x+\frac{5}{3}$. [1]

9 Solve the following equations.

(a) $2(3^x)-3^{2-x}=3$, [5]

(b) $7^x=e^{3x+5}$. [3]

10 **(a)** **(i)** Factorise completely the polynomial $2x^3+15x^2+6x-7$. [3]

(ii) Hence solve the equation $2(x+1)^3+15(x+1)^2+6x-1=0$. [2]

(b) If x^2+1 is a factor of $2x^4+3x^3-8x^2+px+q$, find the value of p and of q [5]

11 A circle C_1, centre $C(3,-1)$, has a diameter AB where A is the point (6, 3).

(i) Find the radius of the circle C_1 and the coordinates of B. [3]

(ii) Find the equation of the circle C_1. [1]

(iii) Show that the equation of the tangent to the circle at A is $4y+3x-30=0$. [3]

The circle C_2 is the reflection of the circle C_1 along the y-axis.

(iv) Find the equation of the circle C_2. [2]

(v) Find the coordinates of the points of intersection of the two circles. [3]

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YIO CHU KANG SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2018
SECONDARY THREE EXPRESS

ADDITIONAL MATHEMATICS
PAPER 2

4047/02
2 hours

Additional materials: Answer Paper

9 October 2018 (Tuesday)

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For Examiner's Use
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Setter: Mdm Ng Hui Yin

This document consists of **5** printed pages and **3** blank pages.

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Answer all the questions

1 The equation of a curve is $y = 3x^2 - kx - 5$, where k is a constant, and the equation of a line is $y - 6x = 10$.

(i) In the case where $k = 6$, find the coordinates of the points of intersection of the line with the curve. [4]

(ii) Show that, for all values of k, the line intersects the curve at two distinct points. [2]

2 A cylinder has a radius of $\left(\sqrt{10} - \sqrt{2}\right)$ cm and a height of h cm. The volume of the cylinder is $\left(3 + 2\sqrt{5}\right)\pi$ cm^3. **Without using a calculator**, show that h can be expressed as $a + b\sqrt{5}$, where a and b are rational numbers. [5]

3 It is given that $2^{3-x} \times 7^{2x-1} = 7^{3x+2}$.

(i) Show that $14^x = \dfrac{8}{343}$. [3]

(ii) Hence find the value of x, correct to 2 decimal places. [2]

4 Express $\dfrac{4-x}{x^3 + 4x^2 + 4x}$ in partial fractions. [5]

5 **(a)** Without using a calculator, express $\left(\dfrac{\sqrt{48}}{6} + \dfrac{2}{\sqrt{12}} + \dfrac{36}{\sqrt{75}}\right) \times \dfrac{6}{\sqrt{2}}$ in the form of $p\sqrt{6}$. [3]

(b) Prove that $2^x + \dfrac{1}{2}\left(2^{x+4}\right) - 2^{x+2}$, where x is a positive integer, is exactly divisible by 5. [3]

6 **(a)** Find the range of values of x for which $(2x-3)^2 > x$. [3]

(b) The expression $6x^3 + px^2 + qx + 10$, where p and q are constants, has a factor of $2x - 1$ and leaves a remainder of -20 when divided by $x + 2$. Find the value of p and of q. [4]

7 **(a)** Solve the equation $\log_3 x^2 - 1 = 3\log_x 3$. [4]

(b) Given that $u = \log_3 z$, find, in terms of u,

(i) $\log_3 9z$, [1]

(ii) $\log_3\left(\frac{z}{27}\right)$, [1]

(iii) $\log_z 27$. [2]

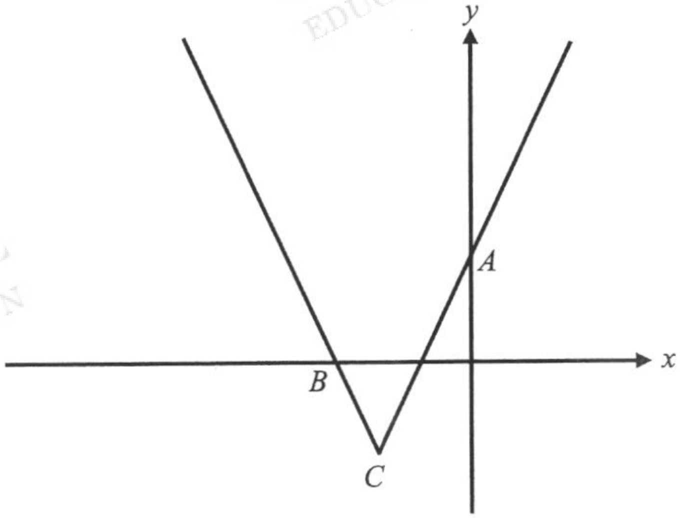
8 The roots of the quadratic equation $4x^2 - 9x + 16 = 0$ are α^2 and β^2 where both α and β are positive.

(i) Show that $\alpha + \beta = \frac{5}{2}$. [3]

(ii) Find the value of $\alpha^3 + \beta^3$. [2]

(iii) Find a quadratic equation with roots $\alpha^2 + \beta$ and $\beta^2 + \alpha$. [4]

9 The diagram shows part of the graph of $y = |3x+5| - 2$.



(i) Find the coordinates of the points A, B and C. [3]

(ii) Solve the equation $|3x+5| - 2 = x + 4$. [3]

(iii) Determine the number of solutions of the equation $|3x+5| - 2 = mx + 4$, justifying your answer, when

(a) $m = -1$, [2]

(b) $m = 3$. [2]

10 A radioactive substance of mass 500 grams was left in a laboratory to decay. The mass, M grams, after t seconds, of the radioactive substance is given by the formula $M = Ae^{-kt}$, where A and k are constants.

(i) Explain why $A = 500$. [1]

The time taken for the substance to be half of its mass is 55 seconds.

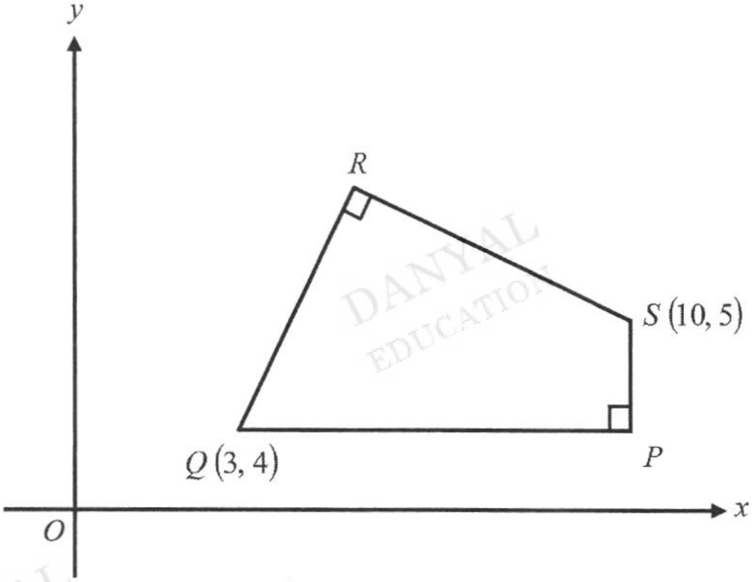
(ii) Find the time taken for the substance to be one quarter of its initial mass. [5]

(iii) Another formula used to calculate the mass of the substance is given by

$M = A\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where A is the same constant as the first equation and h is a constant.

Express h in terms of k. [3]

11



The diagram shows a quadrilateral $PQRS$ in which SR is perpendicular to RQ and QP is perpendicular to PS. The point Q is $(3, 4)$ and the point S is $(10, 5)$.

Given that QR is parallel to the line $6x - 2y = 13$, find

(i) the equation of QR, [2]

(ii) the coordinates of R, [4]

(iii) the area of the quadrilateral $PQRS$. [2]

T is a point on the line SR such that the area of triangle QTR : area of triangle $QTS = 3 : 2$.

(iv) Find the coordinates of the point T. [2]

END OF PAPER

1(i)	$3x^2-6x-5-6x=10$ $3x^2-12x-15=0$	**M1**	Quad Eqn
	$x^2-4x-5=0$ $(x-5)(x+1)=0$ $x=5, x=-1$	**M1**	Show factors
	$y=40, y=4$ (5, 40) and (−1,4)	**A2**	
(ii)	$3x^2-kx-6x-15$ b^2-4ac $=(-k-6)^2-4(3)(-15)$ $=(-k-6)^2+180$	**M1**	
	Since $(-k-6)^2 \geq 0$, therefore $D \geq 180$ Hence for all values of *k*, the line will intersect the curve at 2 distinct points.	**A1**	
2	$h=\frac{(3+2\sqrt{5})\pi}{\pi(\sqrt{10}-\sqrt{2})^2}$	**M1**	M1 for making *h* the subject
	$=\frac{(3+2\sqrt{5})}{10-2\sqrt{20}+2}$	**M1**	M1 for correct expansion of denominator.
	$=\frac{3+2\sqrt{5}}{12-2\sqrt{20}}$		
	$=\frac{3+2\sqrt{5}}{12-4\sqrt{5}}\times\frac{12+4\sqrt{5}}{12+4\sqrt{5}}$	**M1**	M1 for rationalising their denominator
	$=\frac{36+12\sqrt{5}+24\sqrt{5}+40}{(12)^2-(4\sqrt{5})^2}$	**M1**	
	$=\frac{76+36\sqrt{5}}{64}$		
	$=\frac{19}{16}+\frac{9}{16}\sqrt{5}$	**A1**	Accept equivalent.

3(i)	$2^{3-x}\times 7^{2x-1}=7^{3x+2}$		
	$2^3\times 2^{-x}\times 7^{2x}\times 7^{-1}=7^{3x}\times 7^2$	**M1**	
	$\dfrac{2^3\times 7^{-1}}{7^2}=\dfrac{7^{3x}}{7^{2x}\times 2^{-x}}$		
	$\dfrac{2^3}{7\times 7^2}=\dfrac{7^{3x}\times 2^x}{7^{2x}}$	**M1**	
	$7^x\times 2^x=\dfrac{2^3}{7^3}$		
	$14^x=\dfrac{8}{343}$ (shown)	**A1**	
	OR		
	$2^{3-x}\times 7^{2x-1}=7^{3x+2}$ $2^{3-x}=7^{3x+2-(2x-1)}$		
	$2^{3-x}=7^{x+3}$	**M1**	
	$2^3\left(2^{-x}\right)=7^x\left(7^3\right)$		
	$\dfrac{2^3}{7^3}=7^x\left(2^x\right)$	**M1**	
	$14^x=\dfrac{8}{343}$ (shown)	**A1**	
(ii)	$\lg 14^x=\lg\dfrac{8}{343}$	**M1**	
	$x=-1.42$	**A1**	

4	$\frac{4-x}{x^3+4x^2+4x}=\frac{4-x}{x(x+2)^2}$ $=\frac{A}{x}+\frac{B}{(x+2)}+\frac{C}{(x+2)^2}$	**M1**	
	$4-x=A(x+2)^2+B(x)(x+2)+C(x)$	**M1**	
	When $x=0$, $4=A(2)^2$ $A=1$ When $x=-2$ $6=-2C$ $C=-3$ When $x=1$, $3=9A+B(1)(3)+C(1)$ $3=9+3B-3$ $3B=-3$ $B=-1$	**B2**	
	$\frac{4-x}{x^3+4x^2+4x}=\frac{1}{x}-\frac{1}{(x+2)}-\frac{3}{(x+2)^2}$	**A1**	

5(a)	$\left(\frac{\sqrt{48}}{6}+\frac{2}{\sqrt{12}}+\frac{36}{\sqrt{75}}\right)\times\frac{6}{\sqrt{2}}$		
	$=\left(\frac{4\sqrt{3}}{6}+\frac{2}{2\sqrt{3}}+\frac{36}{5\sqrt{3}}\right)\times\frac{6}{\sqrt{2}}$	**M1**	simplify
	$=\left(\frac{2}{3}\sqrt{3}+\frac{\sqrt{3}}{3}+\frac{36}{15}\sqrt{3}\right)\times\frac{6}{\sqrt{2}}$	**M1**	rationalize correctly
	$=\left(\frac{17}{5}\sqrt{3}\right)\times\frac{6\sqrt{2}}{2}$		
	$=\frac{51}{5}\sqrt{6}$	**A1**	
(b)	$2^x+\frac{1}{2}(2^{x+4})-2^{x+2}=2^x+2^{-1}(2^x\times 2^4)-(2^x\times 2^2)$	**M1**	**OR**
	$=2^x+2^{-1}(2^x)(2^4)-(2^x)(2^2)$		**Let** $y=2^x$
	$=2^x(1)+(2^x)(2^{4-1})-(2^x)(2^2)$ $=(2^x)(1+2^3-2^2)$	**M1**	$=y+8y-4y$---M1 $=5y$
	$=(2^x)(1+8-4)$		
	$=(2^x)(5)$ $(2^x)(5)$ is multiple of 5, divisible by 5. (Proven)	**A1**	Show $5y$ and conclude ----A1

6(a)	$(2x-3)^2 > x$ $4x^2-12x+9>x$	**M1**	Simplification
	$4x^2-13x+9>0$ $(4x-9)(x-1)>0$	**M1**	Factorisation
	$x>\frac{9}{4}=2\frac{1}{4}$ or $x<1$	**A1**	
(b)	Let $f(x)=6x^3+px^2+qx+10$ Since $f\left(\frac{1}{2}\right)=0$, $6\left(\frac{1}{2}\right)^3+p\left(\frac{1}{2}\right)^2+q\left(\frac{1}{2}\right)+10=0$	**M1**	Use Factor Theorem
	$\frac{3}{4}+\frac{p}{4}+\frac{q}{2}+10=0$ $\frac{p}{4}+\frac{q}{2}+10\frac{3}{4}=0$ $p+2q=-43$ - Eq (1)		
	Since $f(-2)=-20$, $6(-2)^3+p(-2)^2+q(-2)+10=-20$	**M1**	Use Remainder Theorem
	$-48+4p-2q+10=-20$ $4p-2q=18$ $2p-q=9$ - Eq (2) From (1), $p=-43-2q$ - Eq (3) Sub (3) into (2), $2(-43-2q)-q=9$ $-86-4q-q=9$ $-5q=95$		
	$q=-19$	**A1**	
	$p=-5$	**A1**	

7(a)	$\log_3 x^2 - 1 = 3\log_x 3$ $2\log_3 x - 1 = \frac{3}{\log_3 x}$	**M1**	Convert base
	Let $u = \log_3 x$. $2u - 1 = \frac{3}{u}$ $2u^2 - u - 3 = 0$ $(2u-3)(u+1) = 0$ $u = \frac{3}{2}$ or $u = -1$	**M1**	
	$\log_3 x = \frac{3}{2}$ or $\log_3 x = -1$ $x = 3^{\frac{3}{2}}$ or $x = 3^{-1}$ $x = \sqrt{3^3}$ $x = \sqrt{27}$ or $x = \frac{1}{3}$ $x = 3\sqrt{3} = 5.20$ (3sf)	**A2**	Both correct
(b)(i)	$\log_3 9z = \log_3 9 + \log_3 z$ $= \log_3 3^2 + \log_3 z$ $= 2 + u$	**B1**	
(ii)	$\log_3\left(\frac{z}{27}\right) = \log_3 z - \log_3 27$ $= \log_3 z - \log_3 3^3$ $= u - 3$	**B1**	
(iii)	$\log_z 27 = \frac{\log_3 27}{\log_3 z}$	**M1**	
	$= \frac{3}{u}$	**A1**	

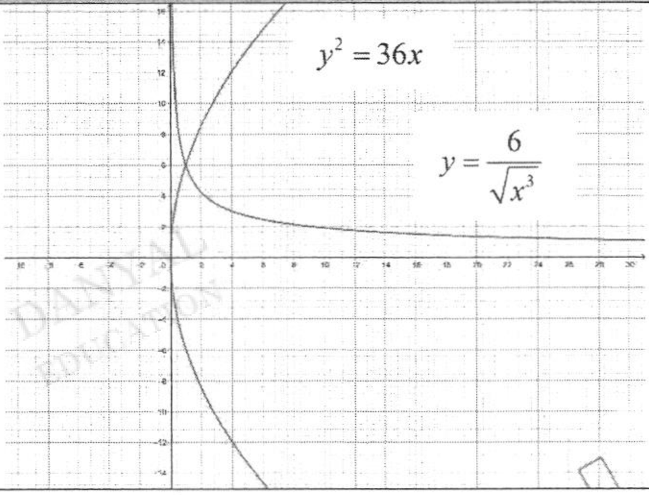
8(i)	$\alpha^2\beta^2 = 4 \quad \Rightarrow \quad \alpha\beta = 2\ (rej - ve)$ $\alpha^2 + \beta^2 = -\left(\frac{-9}{4}\right)$ $(\alpha+\beta)^2 - 2\alpha\beta = \frac{9}{4}$ $(\alpha+\beta)^2 = \frac{9}{4} + 2(2)$ $\alpha + \beta = \frac{5}{2}\ (rej - ve) \quad (shown)$	**M1** **M1** **A1**	
(ii)	$\alpha^3 + \beta^3$ $= (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ $= \frac{5}{2}\left(\frac{9}{4} - 2\right)$ $= \frac{5}{8}$	**M1** **A1**	
(iii)	$\alpha^2 + \beta + \beta^2 + \alpha$ $= \frac{9}{4} + \frac{5}{2}$ $= \frac{19}{4}$ $(\alpha^2 + \beta)(\beta^2 + \alpha)$ $= \alpha^2\beta^2 + \alpha\beta + \alpha^3 + \beta^3$ $= 2^2 + 2 + \frac{5}{8}$ $= \frac{53}{8}$ Quadratic equation is $x^2 - \frac{19}{4}x + \frac{53}{8} = 0 \quad or \quad 8x^2 - 38x + 53 = 0$	**M1** **M1** **M1** **A1**	

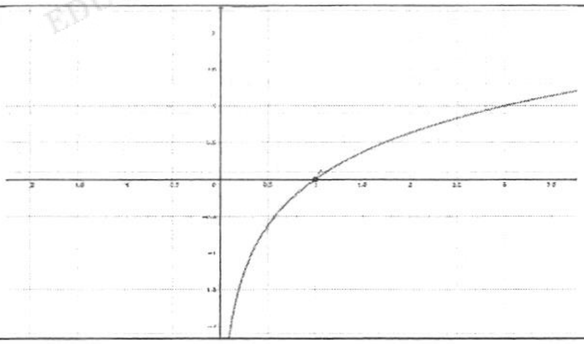
9(i)	$y=\lvert 3x+5\rvert-2$ At $x=0$, $y=\lvert 3(0)+5\rvert-2$ $y=3$ At $y=0$, $0=\lvert 3x+5\rvert-2$ $3x+5=2$ or $-(3x+5)=2$ $x=-1$ or $x=-2\frac{1}{3}$ Coordinates are $A(0,3)$, $C\left(-\frac{5}{3},-2\right)$ and $B\left(-2\frac{1}{3},0\right)$.	**B1** **B1** **B1**	
(ii)	$\lvert 3x+5\rvert-2=x+4$ $\lvert 3x+5\rvert=x+6$ $3x+5=x+6$ or $-(3x+5)=x+6$ $2x=1$ or $4x=-11$ $x=\frac{1}{2}$ or $x=-2\frac{3}{4}$	**M1** **A2**	
(iii)(a)	Gradient of line is < than gradient of left arm. Line cuts both left and right arms at two points respectively. 2 solutions	**B1** **B1**	Award M1 only if the solution worked out is correct for both values
(b)	Gradient of line is parallel to right arm. Line cuts left arm at only 1 point. 1 solution	**B1** **B1**	Award M1 only if the solution worked out is correct for the one answer (and not rejected)

10(i)	when $t = 0$, $M = 500$		
	$500 = Ae^{-k(0)}$	**M1**	
	$A = 500$		
(ii)	when $t = 55$, $M = 250$, $A = 500$		
	$250 = 500e^{-k(55)}$	**M1**	
	$\frac{1}{2} = e^{-55k}$		
	$\ln\left(\frac{1}{2}\right) = \ln e^{-55k}$	**M1**	or M1 for splitting into $\ln 250 = \ln 500 + \ln e^{-k(55)}$
	$-55k = \ln\left(\frac{1}{2}\right)$		
	$k = 0.012602$ or $\frac{\ln 2}{55}$	**M1**	
	when $M = 125$, $A = 500$, $k = 0.012602$,	**M1**	ecf for their k
	$125 = 500e^{-0.012602t}$		
	$e^{-0.012602t} = \frac{1}{4}$		
	$-0.012602t \ln e = \ln\left(\frac{1}{4}\right)$		
	$t = 110s$	**A1**	
	Time taken = 110 s		
(iii)	$M = Ae^{-kt}$ --------(1)		
	$M = A\left(\frac{1}{2}\right)^{\frac{t}{h}}$ -------(2)		
	Sub. (2) into (1)		
	$A\left(\frac{1}{2}\right)^{\frac{t}{h}} = Ae^{-kt}$		
	$\left(\frac{1}{2}\right)^{\frac{t}{h}} = e^{-kt}$	**M1**	
	$\ln\left(\frac{1}{2}\right)^{\frac{t}{h}} = \ln e^{-kt}$		
	$\frac{t}{h}\ln\left(\frac{1}{2}\right) = -kt$	**M1**	
	$h = \frac{-\ln\frac{1}{2}}{k}$ or $= -\frac{1}{k}\ln\frac{1}{2}$	**A1**	
	Or $h = \frac{\ln 2}{k}$		

11(i)	Let the equation of QR be $y=mx+c$. $m=\frac{6}{2}=3$ Sub $(3, 4)$, $4=3(3)+c$ or $y-4=3(x-3)$ $c=-5$ $y=3x-5$	**M1** **A1**	
(ii)	Let equation of RS be $y=mx+c$. $m=-\frac{1}{3}$ Sub $(10, 5)$, $5=\left(-\frac{1}{3}\right)(10)+c$ $c=8\frac{1}{3}$ $y=-\frac{1}{3}x+8\frac{1}{3}$ To find R, $y=3x-5$ - Eq (1) $y=-\frac{1}{3}x+8\frac{1}{3}$ - Eq (2) Sub (1) into (2), $3x-5=-\frac{1}{3}x+8\frac{1}{3}$ $9x-15=-x+25$ $10x=40$ $x=4$ $y=3(4)-5=7$ $R\,(4, 7)$	**M1** **M1** **M1** **A1**	Equating 2 eqns correctly and attempt to solve
(iii)	$P\,(10, 4)$ Area of $PQRS$ $=\frac{1}{2}\times\begin{vmatrix}10 & 10 & 4 & 3 & 10\\ 4 & 5 & 7 & 4 & 4\end{vmatrix}$ $=13.5$ units^2	**M1** **A1**	Show criss-cross mthd
(iv)	$T=\left(4+\left(\frac{3}{5}\times 6\right),\ 7-\left(\frac{3}{5}\times 2\right)\right)$ $T=\left(7\frac{3}{5},\ 5\frac{4}{5}\right)$	**M1** **A1** **Or B2**	

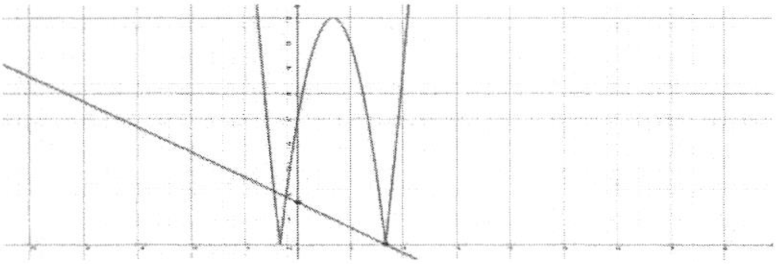
Yio Chu Kang Secondary School
EOY 2018 Sec 3E Add Mathematics Paper 1
Marking Scheme

Qn	Part	Solutions	Marks	Marker's Comments
1	i		B2	Poor. Clearly, many did not study the shapes of graphs. No mark for graphs with sharp point. Many did not sketch for $y < 0$. Did not penalize for graph drawn for $x < 0$. Some plotted points. Those with enough pts, the shape is ok.
	ii	$36x = \left(\frac{6}{\sqrt{x^3}}\right)^2$ $36x = \frac{36}{x^3}$ $x^4 = 1$ $x = 1 \quad (rej \quad -1)$ $y = 6$ Pt of intersection $= (1, 6)$	M1 M1 A1	Ave. No marks given if no calculation is shown. Many did not reject $x = -1$, did not penalize.
2	i	$2x + y + 2 = 0$ $y = -2x - 2 \quad ---(1)$ $\frac{1}{x} + \frac{2}{y} = \frac{1}{2}$ $2y + 4x = xy \quad ---(2)$ sub (1) into (2), $2(-2x-2) + 4x = x(-2x-2)$ $-4x - 4 + 4x = -2x^2 - 2x$ $2x^2 + 2x - 4 = 0$ $2(x+2)(x-1) = 0$ $x = -2 \quad or \quad 1$ $y = 2 \quad or \quad -4$ $Ans: \quad x = -2, y = 2 \quad \& \quad x = 1, y = -4$	M1 M1 M1 A1 A1	Well done. –1 mark if working for solving quadratic eqn is not shown.

3	a	For curve entirely above or below *x*-axis ➔no real roots $b^2-4ac<0$ $(2k)^2-4(k-2)(k+3)<0$ $4k^2-4k^2-4k+24<0$ $-4k<-24$ $k>6$	M1 M1 A1	Ave Many made mistake simplifying after expansion. Others did not change inequality sign after dividing by –4.
	ii	For $k>6$, coeff of x^2: $k-2>0$, Hence curve lies entirely above the *x*-axis.	M1 A1	Poorly answered 1 m only if - Only state coeff of $x^2>0$ without ref to k - Sub approp k value to explain $x^2>0$ - Only state $k>6$ without ref to $k-2$ 0 m if - no justification - Sub approp k value but did not explain $x^2>0$
4	i	$y=\log_a x$, $For \quad (27,3)$, $3=\log_a 27$ $a^3=27$ $a=3$ $For \quad (1,b)$, $b=\log_3 1$ $b=0$ $For \quad (c,-1)$, $-1=\log_3 c$ $c=3^{-1}=\frac{1}{3}$	B1 B1 B1	Well done
	ii		S1 P1	P1 – *x*-int = 1 must be clearly shown –1 m if curve touch the *x*-axis

5		$\frac{3x^3+11x-4}{(x^2+4)(x-1)}=A+\frac{Bx+C}{x^2+4}+\frac{D}{x-1}$ $\therefore 3x^3+11x-4=A(x^2+4)(x-1)+(Bx+C)(x-1)+D(x^2+4)$ *Compare coeff of* x^3 : $A=3$ $sub\ x=1,\quad 10=5D \quad ==> D=2$ $sub\ x=0,\quad -4=3(-4)-C+2(4) \quad ==> C=0$ $sub\ x=2,\quad 3(2)^3+11(2)-4=3(2^2+4)+2B+2(2^2+4)$ $42=24+2B+16 \quad ==> B=1$ Alternative Solution $(x^2+4)(x-1)=x^3-x^2+4x-4$ $\begin{array}{r} 3 \\ x^3-x^2+4x-4\,/\,\overline{3x^3+0x^2+11x-4} \\ -(3x^3-3x^2+12x-12) \\ \hline 3x^2-x\quad+8 \end{array}$ $\therefore A=3$ $\frac{3x^2-x+8}{(x^2+4)(x-1)}=\frac{Bx+C}{x^2+4}+\frac{D}{x-1}$ $3x^2-x+8=(Bx+C)(x-1)+D(x^2+4)$ $sub\quad x=1,\quad 3-1+8=5D \quad ==> D=2$ $sub\quad x=0,\quad 8=-C+4(2) \quad ==> C=0$ $sub\quad x=2,\quad 3(2)^2-2+8=2B+2(2^2+4) ==> B=1$	M1 A1 A1 A1 M1 A1 M1 A1 M1 A1 A1 A1	Good. Most students used the Long Division meth. Some used a different set of letters *A,B & C* –1 if final answer did not state the answers for original set of letters.
6	i	$\frac{9^{n+2}-3^{2n+2}}{2^5}$ $=\frac{3^{2(n+2)}-3^{2n+2}}{2^5}$ $=\frac{3^{2n}(3^4-3^2)}{2^5}$ $=3^{2n}\left(\frac{9}{4}\right)$ $=3^{2n+2}\times 2^{-2}$ $\therefore a=-2 \quad \& \quad b=2n+2$	M1 M1 M1 A2	Poorly done M1 for $3^{2(n+2)}$ *o.e.* Many did not evaluate 3^4-3^2 and just state $a=-5$

	ii	$3^{2n}\left(\frac{9}{4}\right)=\frac{1}{4}$ $3^{2n}=\frac{1}{9}$ $=3^{-2}$ $\therefore 2n=-2$ $n=-1$	M1 A1	Those who were able to do part (i) will get the answer for *n*.
7	a	$2\log_5 x-\log_5(x-6)=1$ $\log_5\left(\frac{x^2}{x-6}\right)=1$ $\frac{x^2}{x-6}=5$ $x^2=5(x-6)$ $x^2-5x+30=0$ $b^2-4ac=(-5)^2-4(1)(30)$ $=-95<0$ $\therefore$ no real solutions.	M1 M1 M1 M1 A1	Ave Many gave $\frac{x^2}{x-6}=1$ and lost 2 marks 1m for correct use of *D* for incorrect quad eqn formed. A handful used formula to solve quad eqn and conclude no solution. They need to explain $\sqrt{neg}$ has no answer. Did not penalize this time
	b	$\log_{16}x^2=\log_8 u$ $\frac{\log_2 x^2}{\log_2 16}=\frac{\log_2 u}{\log_2 8}$ $\frac{2\log_2 x}{\log_2 2^4}=\frac{\log_2 u}{\log_2 2^3}$ $\frac{2\log_2 x}{4}=\frac{\log_2 u}{3}$ $\frac{3}{2}\log_2 x=\log_2 u$ $u=x^{\frac{3}{2}}$	M1 M1 A1	Poorly done. 1m for correct application of change of base. Many start to make mistakes after change of base. Those with even power of *u* should state "rej –ve"
8	i	Coeff of $x^2=-9<0$, curve will have a max pt when $3x-2=0$ $x=2/3$ & $y=9$ Hence max pt = (2/3, 9)	B1	Poorly done - as many did not explain why it is a highest(max) pt.
	ii	$(3x-2)^2=9$ $3x-2=\pm 3$ $x=\frac{5}{3}$ or $-\frac{1}{3}$ $x-$int are $\left(\frac{5}{3},0\right)$ & $\left(-\frac{1}{3},0\right)$.	M1 A1	Good though quite a number of students lost 1 mark for not leaving their answers in coordinates form

	iii		S1 I1 P1	Shape – 1m x- & y- Intercepts – 1m Turning Pt – 1m Ave – many lost the I1 mark for not stating the y-intercept Many also did indicate the coord of max pt – did not penalize if the max pt is drawn at correct place.
	iva	2 solutions	A1	Ecf – if shape of curve is correct
	b	0 solution	A1	Ecf – if shape of curve is correct
	c	3 solutions	A1	Poor – some drew the line but fail to pass through (5/3, 0)
9	a	$2\left(3^x\right)-3^{2-x}=3$ $2\left(3^x\right)-\frac{9}{3^x}-3=0$ $let\quad u=3^x,$ $2u-\frac{9}{u}-3=0$ $2u^2-3u-9=0$ $(2u+3)(u-3)=0$ $u=3\quad or\quad u=-\frac{3}{2}(rej)$ $3^x=3 ==> x=1$	M1 M1 M1 A1 A1	Ave Some were not able to simplify to $\frac{9}{3^x}$ Some did not wrote "let …." – did not penalize this time Those weak in algebra were unable to get the correct quad eqn
	b	$7^x=e^{3x+5}$ $x\ln 7=3x+5$ $x(\ln 7-3)=5$ $x=\frac{5}{\ln 7-3}$ $=-4.74\,(3sf)$	M1 M1 A1	Ave Many wrote $x=\frac{5-\ln 7}{2}$, not able to see the terms in x

10	ai	$let\ f(x) = 2x^3 + 15x^2 + 6x - 7$ $f(-1) = 2(-1)^3 + 15(-1)^2 + 6(-1) - 7 = 0$ $\therefore x+1$ is a factor. $2x^3 + 15x^2 + 6x - 7 = (x+1)(2x^2 + bx - 7)$ or using Long Division $Compare\ coeff\ of\ x:\quad 6 = -7 + b$ $b = 13$ $2x^3 + 15x^2 + 6x - 7 = (x+1)(2x^2 + 13x - 7)$ $= (x+1)(2x-1)(x+7)$	M1 M1 A1	Ave No working – 0 mark Many use Long Division to show a factor. Must state "…. is a factor" Some went to solve when question clearly says "Factorise" – did not penalize this time
	ii	$2(x+1)^3 + 15(x+1)^2 + 6x - 1 = 0$ $2(x+1)^3 + 15(x+1)^2 + 6(x+1) - 7 = 0$ $(x+1+1)(2(x+1)-1)(x+1-7) = 0$ --- from part (i) $x = -2, -\frac{1}{2}, -8$	M1 A1	
	b	$let\ 2x^4 + 3x^3 - 8x^2 + px + q = (x^2+1)(2x^2 + bx + q)$ $= 2x^4 + bx^3 + (q+2)x^2 + bx + q$ Compare coeffs of: (or Long Division) $x^3:\quad b = 3$ $x^2:\quad -8 = q + 2 \Longrightarrow q = -10$ $x:\quad p = b = 3$ Long Division: $2x^2 + 3x - 10$ $x^2 + 1\ /\ 2x^4 + 3x^3 - 8x^2 + px + q$ $-(2x^4 \quad + 2x^2)$ $3x^3 - 10x^2 + px + q$ $-(3x^3 \quad + 3x)$ $-10x^2 + (p-3)x + q$ $-(-10x^2 \quad -10)$ 0	M1 M1 M1 A1 A1 M1 M1 (use of LD) M1	Poorly done. Many sub $x = 1$ or -1 and will not be awarded any mark. Mostly use Long Division but make careless mistakes.

11	i	$r=\sqrt{(6-3)^2+(3+1)^2}=5$ units Let $B=(p,q)$ Midpt of $AB=C$, $\left(\frac{p+6}{2},\frac{q+3}{2}\right)=(3,-1)$ $\therefore p+6=6$ & $q+3=-2$ $p=0$ $q=-5$ $B=(0,-5)$	B1 M1 A1	Good Many did not show working to find *B* – did not penalize
	ii	Eqn C_1: $(x-3)^2+(y+1)^2=5^2$	B1	Good
	iii	Grad $AC=\frac{3+1}{6-3}=\frac{4}{3}$ Grad tangent $=-\frac{3}{4}$ Eqn of tangent : $y-3=-\frac{3}{4}(x-6)$ $y=-\frac{3}{4}x+\frac{15}{2}$ $4y+3x-30=0$ (*shown*)	M1 M1 A1	Ave Incorrect meth used was to sub coord of *A* into the eqn of tangent – 0 mark
	iv	Centre of $C_2=(-3,-1)$ Eqn C_2: $(x+3)^2+(y+1)^2=5^2$	M1 A1	Ave Not able to visualize the reflection to find centre of C_2
	v	Pts of intersection are the *y*-ints ➔sub $x=0$, $3^2+(y+1)^2=5^2$ $(y+1)^2=16$ $y+1=\pm4$ $y=3$ *or* -5 Pts of intersection are (0, 3) & (0, –5).	M1 M1 A1	Ave Many were not able to see that pts of intersection are on *y*-axis and went to solve Simultaneous Eqns