Name: ______________________________ (   ) Class: ____

WHITLEY SECONDARY SCHOOL

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MID-YEAR EXAMINATION 2018

SUBJECT	:	Additional Mathematics
LEVEL	:	Secondary 3 Express
DATE	:	10 May 2018
DURATION	:	2 hours
SETTER	:	Mr Desmond Kan
VETTER	:	Mr Koh Wei Ping

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions on writing paper provided.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degree to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

1. The term containing the highest power of x in the polynomial $f(x)$ is $3x^3$.

Given that one root of the equation is 2, and that x^2+x+1 is a quadratic factor of $f(x)$, find

(i) an expression for $f(x)$ in *descending* powers of x, [3]

(ii) the remainder when $f(x)$ is divided by $(2x+1)$. [2]

2. The equation of a curve is $y=3x^2-mx-7$ where m is a real constant. The equation of a line is $y-4x=6$.

(i) When $m=6$, find the coordinates of the point(s) of intersection(s) of the line with the curve. [5]

(ii) Show that for all values of m, the line intersects the curve at two distinct points. [2]

3. (i) Express $\dfrac{4x^2-4x+7}{2x^2-3x-2}$ in the form $A+\dfrac{Bx+C}{2x^2-3x-2}$, where A, B and C are constants to be determined. [2]

(ii) Hence, express $\dfrac{4x^2-4x+7}{2x^2-3x-2}$ in partial fractions. [3]

4. The roots of the quadratic equation $2x^2+x-1=0$ are α and β.

(i) Show that the value of $\alpha^2+\beta^2$ is $\dfrac{5}{4}$. [5]

(ii) Find a quadratic equation whose roots are $\dfrac{\alpha}{\beta+2}$ and $\dfrac{\beta}{\alpha+2}$. [5]

5. Find the range of values of k for which the curve $y=kx^2-kx-1$ does not intersect the line $y=3x-5$. [5]

6. (i) Solve $\left|\frac{3x+1}{2}\right|-1=x$. [4]

(ii) Sketch the graph of $y=|3x+1|$. [3]

(iii) The equation in (i) can also be solved by drawing a line $y = ax + a$ on the same sketch in (ii). State the value of a. [1]

Start Question 7 on a fresh sheet of Answer Paper.

Submit Questions 7 to 11 separately from Questions 1 to 6.

7. (a) Given that px^2+qx+4 is always negative, what conditions must apply to the constants p and q? [2]

(b) Solve the inequality $3x^2+13x+9<5$ and represent the solution on a number line. [4]

8. (i) Express $y=2x^2+x-6$ in the form $y=a(x+b)^2+c$, where a, b and c are constants to be determined. [3]

(ii) State the coordinates of the turning point of the graph $y=2x^2+x-6$. [1]

(iii) Sketch the graph of $y=|2x^2+x-6|$, for $-2.5\le x\le 2$. [4]

(iv) State the range of values of h such that the line $y = h$ cuts the graph of $y=|2x^2+x-6|$ at four distinct points. [1]

9. In the expansion $\left(3x^2+\frac{1}{x}\right)^{12}$, find the term independent of x. [5]

10. (i) Write down, and simplify, the first 4 terms in the expansion $(1+q)^8$ in ascending powers of q. [2]

(ii) Using your answer in (i), estimate the value of 1.1^8. [2]

(iii) Determine the coefficient of q^{11} in the expansion $\left[q(1+q)\right]^8$. [3]

11. (a) The expression $2x^3 + ax^2 + bx + 6$, where a and b are constants, has a factor of $(2x-1)$ and a remainder of 20 when divided by $(x+2)$.

(i) Show that $a = 1$ and $b = -13$. [5]

(ii) Hence, solve the equation $2x^3 + ax^2 + bx + 6 = 0$. [5]

(b) (i) Given that α is a root of the equation $x^2 = 4x - 7$, show that $\alpha^4 = 16\alpha^2 - 56\alpha + 49$. [1]

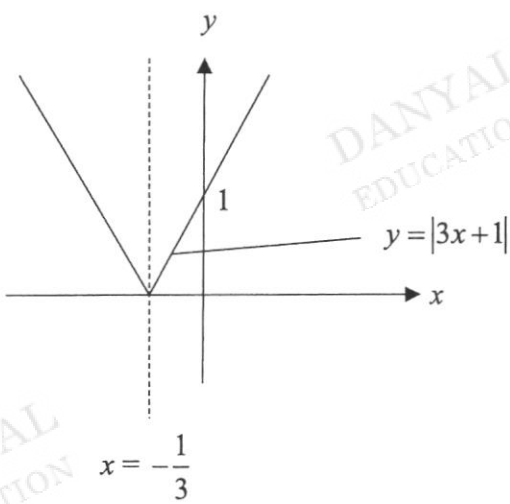
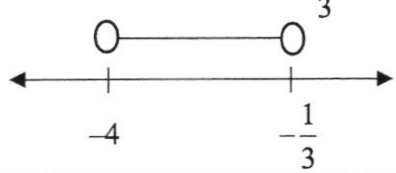
(ii) Hence, find the exact value of

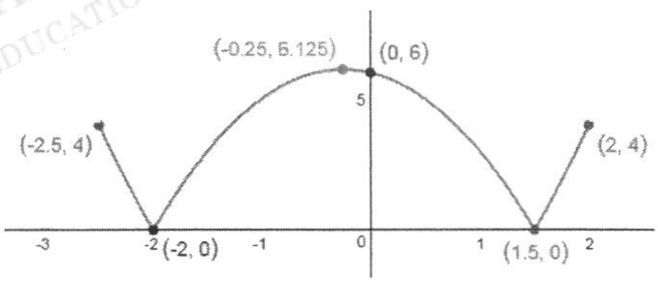
$$\frac{\alpha^4 + 56\alpha + 63}{\alpha^2 + 7}.$$ [2]

-End of Paper-

1(i)	$f(x)=3(x-2)(x^2+x+1)$ $=3(x^3+x^2+x-2x^2-2x-2)$ $=3(x^3-x^2-x-2)$ $=3x^3-3x^2-3x-6$	B1 (o.e.) M1 A1	Multiplying polynomials together
1(ii)	Remainder $=f(-0.5)$ $=3(-0.5)^3-3(-0.5)^2-3(-0.5)-6$ $=-\frac{45}{8}$	M1 A1√	For long division, either 2 marks or 0 Also accept –5.625 (o.e.)
2(i)	$3x^2-6x-7=4x+6$ $3x^2-10x-13=0$ $(3x-13)(x+1)=0$ $x=\frac{13}{3}$ or -1 $y=4\left(\frac{13}{3}\right)+6$ or $y=4(-1)+6$ $y=\frac{70}{3}$ or $y=2$ Coordinates are $\left(\frac{13}{3},\frac{70}{3}\right)$ and $(-1,2)$	M1 DM1 B1 B1√ A1	Equating the line and curve Factorize / Formula Correct values of x Correct values of y c.a.o.
2(ii)	$3x^2-mx-7=4x+6$ $3x^2-(m+4)x-13=0$ Discriminant of equation $=(m+4)^2-4(3)(-13)$ $=(m+4)^2+156$ >0 Since the discriminant is greater than 0 for all values of m, the line will intersect the curve at two distinct points.	M1 A1	Calculating discriminant **without assuming >0** from the very start. Proper concluding statement
3(a)	$\frac{4x^2-4x+7}{2x^2-3x-2}=2+\frac{2x+11}{2x^2-3x-2}$	B2,1,0	$A=2$ $B=2$ $C=11$
3(b)	$\frac{4x^2-4x+7}{2x^2-3x-2}=2+\frac{2x+11}{(2x+1)(x-2)}$ $\frac{2x+11}{(2x+1)(x-2)}=\frac{A}{2x+1}+\frac{B}{x-2}$ $2x+11=A(x-2)+B(2x+1)$ When $x=2$, $2(2)+11=5B$ $B=3$	B1 M1	Correct form of partial fractions Award even if the form of partial fractions above is incorrect (substitution)

	When $x=-\frac{1}{2}$, $2\left(-\frac{1}{2}\right)+11=A\left(-\frac{5}{2}\right)$ $A=-4$ $\frac{4x^2-4x+7}{2x^2-3x-2}=2-\frac{4}{2x+1}+\frac{3}{x-2}$	A1	c.a.o.
4(i)	$\left.\begin{aligned}\alpha+\beta&=-\frac{1}{2}\\ \alpha\beta&=-\frac{1}{2}\end{aligned}\right\}$	B2	B1 for each correct value
	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	B1	Correct use of formula
	$=(-0.5)^2-2(-0.5)$	M1	Substitution
	$=\frac{5}{4}\text{(Shown)}$	A1	
4(ii)	Sum of new roots $=\frac{\alpha}{\beta+2}+\frac{\beta}{\alpha+2}$	M1	Adding the two new roots together
	$=\frac{\alpha^2+2\alpha+\beta^2+2\beta}{\alpha\beta+2(\alpha+\beta)+4}$ $=\frac{\alpha^2+\beta^2+2(\alpha+\beta)}{\alpha\beta+2(\alpha+\beta)+4}$ $=\frac{\frac{5}{4}+2(-0.5)}{(-0.5)+2(-0.5)+4}$		
	$=\frac{1}{10}$	A1√	Condone if value of $\alpha\beta$ from 4(i) is incorrect.
	Product of new roots $=\frac{\alpha}{\beta+2}\times\frac{\beta}{\alpha+2}$	M1	Multiplying the new roots together
	$=\frac{\alpha\beta}{\alpha\beta+2(\alpha+\beta)+4}$ $=\frac{-0.5}{-0.5+2(-0.5)+4}$		
	$=-\frac{1}{5}$	A1√	Condone if value of $\alpha\beta$ from 4(i) is incorrect.
	New equation: $x^2-\frac{1}{10}x-\frac{1}{5}=0$	B1 (o.e.)	Also accept: $10x^2-x-2=0$

5	$kx^2 - kx - 1 = 3x - 5$ $kx^2 - (k+3)x + 4 = 0$ $(k+3)^2 - 4(k)(4) < 0$ $k^2 + 6k + 9 - 16k < 0$ $k^2 - 10k + 9 < 0$ $(k-9)(k-1) < 0$ $1 < k < 9$	M1 M1 B1 M1 A1	Equating the line and curve Using discriminant < 0, with substitution Correct quadratic in k Factorize
6(i)	$\left\|\frac{3x+1}{2}\right\| - 1 = x$ $\left\|\frac{3x+1}{2}\right\| = x + 1$ $\frac{3x+1}{2} = x+1$ or $\frac{3x+1}{2} = -x-1$ $3x+1 = 2x+2$ or $3x+1 = -2x-2$ $x = 1$ or $5x = -3$ $x = 1$ or $x = -\frac{3}{5}$	B1 (o.e.) M1 A2	Correct removal of modulus Algebraic manipulation
6(ii)		B1 B1 B1	Shape x-intercept correctly labelled y-intercept correctly labelled
6(iii)	$a = 2$	B1	
7(i)	$p < 0$ AND $q^2 - 16p < 0$	B2	o.e.
7(ii)	$3x^2 + 13x + 9 < 5$ $3x^2 + 13x + 4 < 0$ $(3x+1)(x+4) < 0$ Hence, $-4 < x < -\frac{1}{3}$ 	B1 M1 A1 B1	Shifting all terms to one side Factorize / Formula

8(i)	$y=2x^2+x-6$ $=2\left(x^2+\frac{1}{2}x-3\right)$ $=2\left[\left(x+\frac{1}{4}\right)^2-3-\left(\frac{1}{4}\right)^2\right]$ $=2\left[\left(x+\frac{1}{4}\right)^2-\frac{49}{16}\right]$ $=2\left(x+\frac{1}{4}\right)^2-\frac{49}{8}$	M1 M1 A1	Factorizing out 2 to form a monic quadratic within brackets Completing the square $a=2$, $b=0.25$, $c=-6.125$
8(ii)	Coordinates of turning point $=\left(-\frac{1}{4},-\frac{49}{8}\right)$	B1√	Award as long as it makes sense from previous answer in 8(i)
8(iii)		B1 B1√ B1 B1	x-intercepts and y-intercepts turning point shape Start point and end point
8(iv)	$0<h\le 4$	B1√	o.e.
9	$(r+1)^{\text{th}}$ term of $\left(3x^2+\frac{1}{x}\right)^{12}$: $\binom{12}{r}\left(3x^2\right)^{12-r}\left(\frac{1}{x}\right)^r$ $=\binom{12}{r}3^{12-r}x^{2(12-r)}x^{-r}$ $=\binom{12}{r}3^{12-r}x^{24-2r}x^{-r}$ $=\binom{12}{r}3^{12-r}x^{24-3r}$ $24-3r=0$ $3r=24$ $r=8$ The term is: $\binom{12}{8}3^4=40095$	M1 A1 M1 A1 B1	Using $(r+1)^{\text{th}}$ term Equating power on x to 0.
10(i)	$(1+q)^8=1+\binom{8}{1}q+\binom{8}{2}q^2+\binom{8}{3}q^3+\ldots$ $=1+8q+28q^2+56q^3+\ldots$	B2,1,0	All terms correct – B2 1 term incorrect – B1 2 or more incorrect – B0

10(ii)	Let $q = 0.1$. $1.1^8 \approx 1 + 8(0.1) + 28(0.1)^2 + 56(0.1)^3$ $= 2.136$	M1 A1√	Using answer from 10(i) Do not accept 2.144
10(iii)	$\left[q(1+q)\right]^8$ $= q^8(1+q)^8$ $= q^8(1 + 8q + 28q^2 + 56q^3 + \ldots)$ $= 56q^{11} + \ldots$ Coefficient of q^{11} is 56. Alternatively, $\left[q(1+q)\right]^8 = (q+q^2)^8$ $\binom{8}{r} q^{8-r} q^{2r} = \binom{8}{r} q^{8+r}$ $8 + r = 11$ $r = 3$ Coefficient $= \binom{8}{3}$ $= 56$	M1 M1 A1√ M1 M1 A1	Simplifying using law of indices Using answer from 10(i) $(r+1)^{th}$ term Equating power to 11
11(a)(i)	Let $f(x) = 2x^3 + ax^2 + bx + 6$. Then, $f(0.5) = 0$ $2(0.5)^3 + a(0.5)^2 + b(0.5) + 6 = 0$ $\frac{1}{4} + \frac{1}{4}a + \frac{1}{2}b + 6 = 0$ $1 + a + 2b + 24 = 0$ $a + 2b = -25 --- (1)$ Then, $f(-2) = 20$ $2(-2)^3 + a(-2)^2 + b(-2) + 6 = 20$ $-16 + 4a - 2b + 6 = 20$ $4a - 2b = 30 --- (2)$ Substitute (1) into (2): $4a + (a + 25) = 30$ $5a + 25 = 30$ $a = 1 --- (3)$ Substitute (3) into (1): $1 + 2b = -25$ $2b = -26$ $b = -13$	B1 B1 M1 A2	Apply Factor Theorem correctly Apply Remainder Theorem correctly Substitution / Elimination For both values of a and b shown correctly

11(a)(ii)	$f(x)=(2x-1)(x^2+qx-6)$ Comparing coefficients of x, $-13=-12-q$ $q=1$ Therefore, $f(x)=(2x-1)(x+3)(x-2)$ Solving $f(x)=0$, we will obtain: $x=0.5$, $x=-3$ or $x=2$.	B1 B1 M1 A2,1,0	Correct value of a or c of the quadratic factor ax^2+bx+c Correct value of b of the quadratic factor ax^2+bx+c Factorizing / Formula All answers correct – A2 1 answer incorrect – A1 2 or more incorrect – A0
11(b)(i)	$\alpha^2=4\alpha-7$ $\alpha^4=(4\alpha-7)^2$ $=16\alpha^2-56\alpha+49$	B1	Squaring both sides
11(b)(ii)	$\dfrac{\alpha^4+56\alpha+63}{\alpha^2+7}=\dfrac{16\alpha^2-56\alpha+49+56\alpha+63}{\alpha^2+7}$ $=\dfrac{16\alpha^2+112}{\alpha^2+7}$ $=\dfrac{16(\alpha^2+7)}{\alpha^2+7}$ $=16$	B1 B1	Substituting answer in (b)(i) c.a.o.

Mistakes made by students

1.
   i. $x^2+x+1\neq(x+1)^2$
   ii. *Ascending* vs *descending*
   iii. $f(x)\neq x^2+x+1$
   iv. $f(x)\neq 3x^3+(x^2+x+1)$
   v. Marks will NOT be awarded for long division unless students obtain the correct remainder from long division.

2. For $m=6$, most students got it correct. Some mistakes include
   i. Solving for x-values and assume that y-coordinates are 0
   ii. Not recognizing that question requires *coordinates*

   For general m, almost all students assumed >0 (or even <0). Only a few calculated the discriminant but none realized that the squared term should not be expanded.

3. Most students are able to obtain A = 2, B = 2 and C = 11. Along the way, a few students wrote down C as 1 instead of 11, leading to careless mistakes.

   Students should note that the form in (i) is NOT partial fractions. The form in (i) is to help students develop answers for (ii).

4. Generally well done. For (ii), equation requires "=0". This distinguishes it from an *expression*.

5. Generally well done, except for students who either
   i. Do not understand that no intersection implies that the discriminant is <0
   ii. Do not know how to solve quadratic inequalities

6. Serious misconception

   $\left|\frac{3x+1}{2}\right| - x = 1$ does not imply that $\frac{3x+1}{2} - x = 1$ or $\frac{3x+1}{2} - x = -1$

   Some students do not even realize that two answers are required.

   For (iii), if students understood the concept that simultaneous equations can be solved graphically, they would be able to nail it. Only a handful were able to state that $a = 2$.

7. Some common answers
   i. Both p and q must be negative
   ii. $b^2 - 4ac$ must be negative (students expect examiner to figure out what does b, a and c stand for)
   iii. Writing down $p < 0$ and $q^2 < 16p$ but having a different concluding statement, e.g. both p and q negative. Marks are awarded based on final answers.

8. Poorly done. Students struggle with
   i. Completing the square
   ii. Stating the start and end points of the graph given its domain
   iii. Comparing the y-coordinates of the start and end points with respect to the y-intercept and turning points to plot a smooth graph

   Note that if graphs do not resemble a quadratic modulus graph, no marks will be awarded even though their intercepts may be calculated correctly.

   For (iv), some students did not include the domain into consideration.

9. There are still a group of students who insist on using the Binomial Theorem to expand $\left(3x^2 + \frac{1}{x}\right)^{12}$ completely. Marks are not awarded unless students obtain the correct answer.

10. Generally well done. Some students came up with a different method which also works fine.

11. For (a)(ii), some students ended off with the factorization but did not state the values of x.

    For (b), only a few students attempted it and managed to get it correct.