SPRINGFIELD SECONDARY SCHOOL
End-Of-Year Examination 2022
Sec 3 Express

STUDENT NAME	

CLASS			REGISTER NUMBER		

ADDITIONAL MATHEMATICS **4049**

4 October 2022
2 hours 15 minutes

Candidates answer on the question paper

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Total	**/90**

Do not turn over this question paper until you are told to do so.

This question paper consists of 19 printed pages.

1 **(a)** Factorise x^3+8. [1]

(b) Does the equation $x^3+8=0$ has three real roots? Justify your answer. [2]

2 A triangle has a base of $(2+\sqrt{5})$ cm and a perpendicular height of h cm. The area of the triangle is $(-8+5\sqrt{5})\,\text{cm}^2$. **Without using a calculator**, show that h can be expressed, in cm, in the form of $(a+b\sqrt{5})$, where a and b are integers. [4]

3 The curve $x^2 = xy + 12$ and the line $x + 2y = 6$ intersect at the points A and B.

Find the coordinates of A and of B. [4]

4 **(a)** Express $y=-2x^2-4x+1$ in the form $y=a(x+h)^2+k$. [2]

(b) State the coordinates of the turning point of $y=-2x^2-4x+1$. [1]

(c) A curve has equation $y=-2x^2-4x+1+p$, where p is an integer. The x-axis is a tangent to the curve. Find the value of p. [1]

5 Find the range of values of c for which $3x^2 + cx + 7 > 4$ for all values of x. [4]

6 Solve the equation $3^{2x+1} - 28(3^x) + 9 = 0$. [4]

7 The population of a type of insect was observed in an experimental environment. After t days, the number of insects was given by the equation $N = 500 + 3000e^{kt}$, where k is a constant.

(a) Find the initial population of the insects. [1]

(b) Find the value of k given that the population of the insects decreased to 3000 after 5 days. [2]

(c) The population of the insects approaches a particular value A after a long period of time.

(i) Find the value of A. [1]

(ii) Explain why the population of the insects can never reach A. [1]

8 Express $\dfrac{4-x}{x(x+2)^2}$ in partial fractions. [6]

9 **(a)** Given that $\sin A = \frac{4}{5}$, where A is acute, find the exact value of

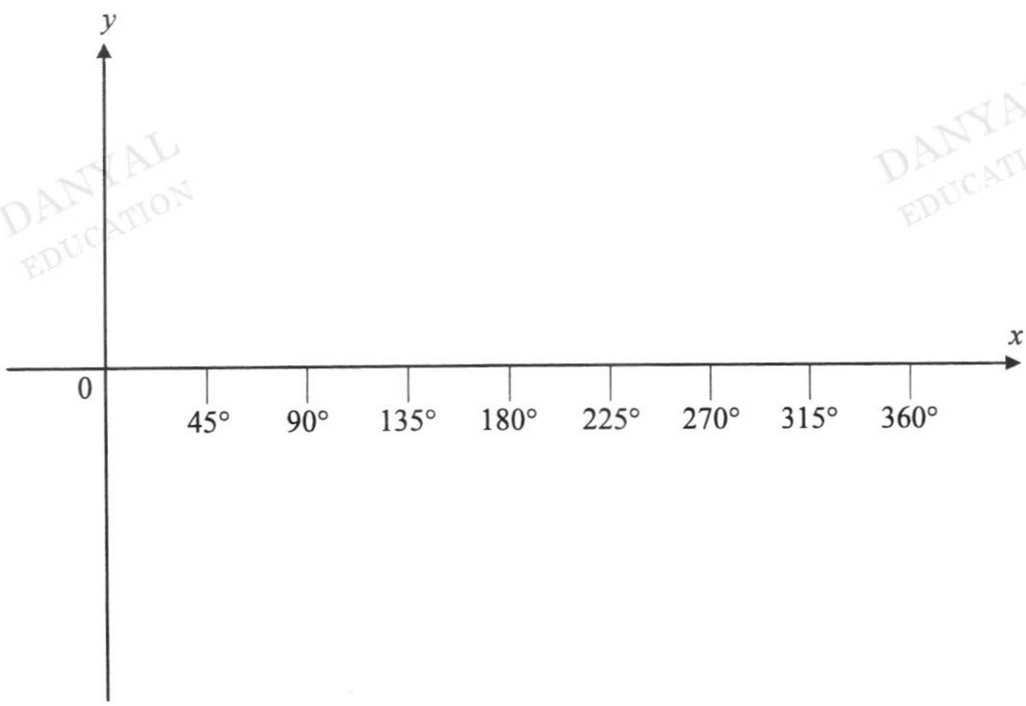
(i) $\sec A$, [2]

(ii) $\tan(-A)$. [1]

(b) It is given that $f(x) = 2\cos 2x + 1$.

(i) State the period of $f(x)$. [1]

(ii) Sketch the graph of $y = f(x)$ for $0° \le x \le 360°$. [2]



10 The equation of a circle is $x^2 + y^2 - 10x + 4y + 25 = 0$.

(a) Find the radius of the circle and the coordinates of its centre. [4]

The point $P(3, -2)$ lies on the circle.

(b) Explain why the tangent to the circle at P is parallel to the y-axis. [2]

11 **(a)** It is given that $1 - \ln y = \ln(x + y)$. Express x in terms of y. [3]

(b) Solve the equation $6\log_y 2 = 5 - \log_2 y$. [5]

12 The polynomial $f(x)$ where $f(x) = 3x^3 - 4x^2 + qx + 6$ where q is a constant, leaves a remainder of -12 when divided by $x-1$.

(a) Show that $q = -17$. [2]

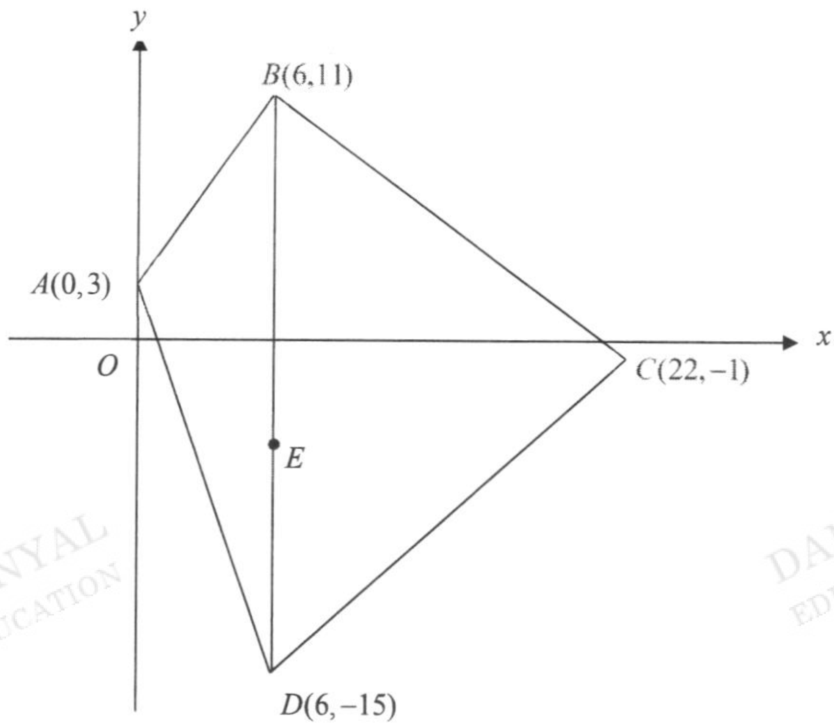
(b) Solve the equation $f(x) = 0$. [4]

(c) Use your answer to part **(b)** to solve the equation

$$3(4^x)^3 - 4(4^x)^2 - 17(4^x) + 6 = 0 .$$ [2]

(d) Explain why there are only two solutions for x. [1]

13



The diagram shows a quadrilateral $ABCD$ with vertices $A(0,3)$, $B(6,11)$, $C(22,-1)$ and $D(6,-15)$. The point E lies on BD and the perpendicular bisector of BC.

(a) Find the coordinates of E. [5]

(b) Find the area of the quadrilateral *ABCD*. [2]

(c) Is quadrilateral *ABCD* a trapezium? [2]

14 A rectangle has an area of y m^2, width x m and length $(Ax + B)$ m, where A and B are constants. Corresponding values of x and y are shown in the table below.

x	1	3	5	7
y	3	12	25	42

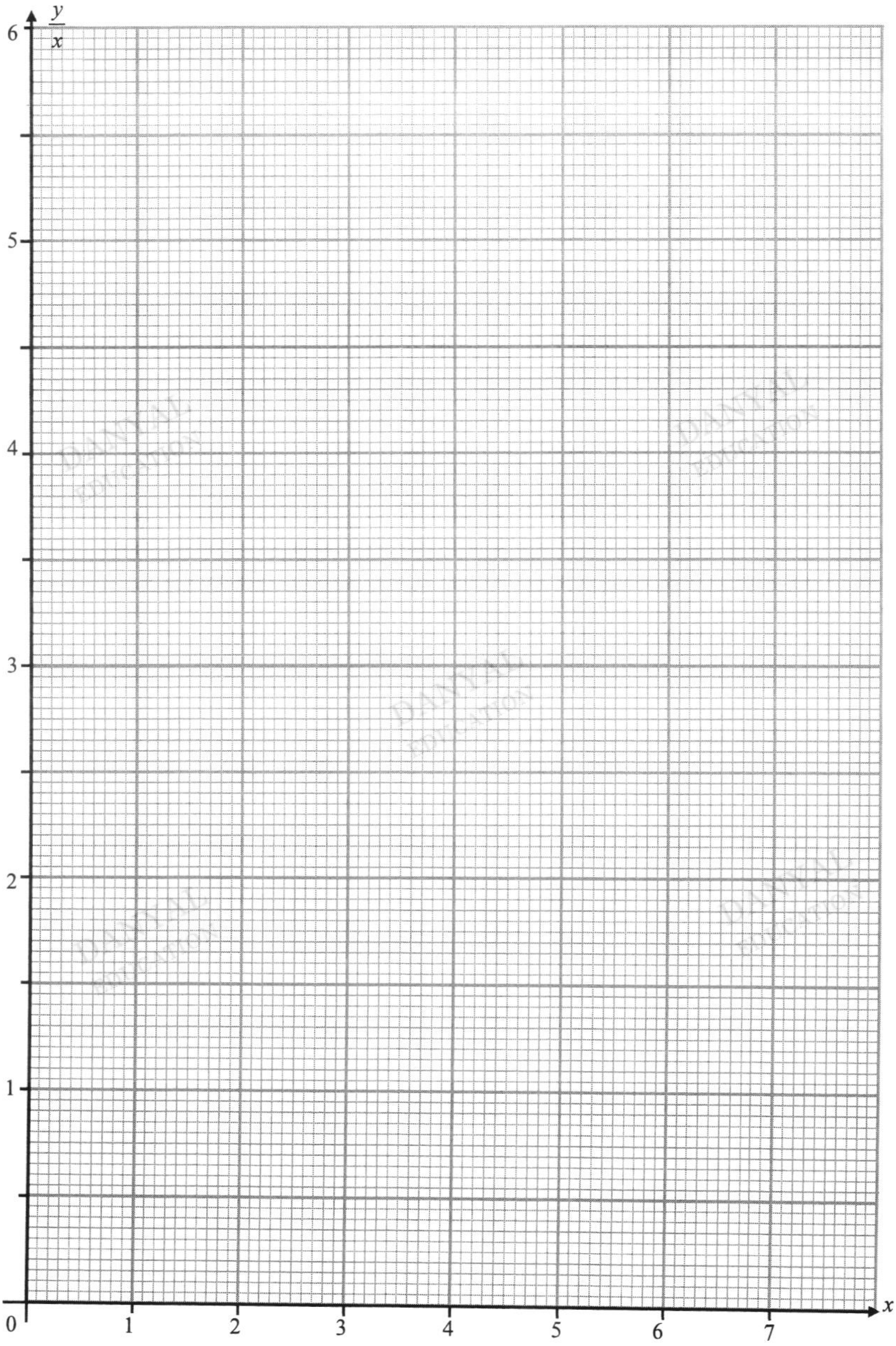
(a) On the grid on page 17, plot $\frac{y}{x}$ against x and draw a straight line. [2]

(b) Use your graph to estimate the value of A and of B. [3]

(c) Using your values of A and B, write down an expression, in terms of x, for the area of the rectangle. [1]

(d) **(i)** Explain how another straight line drawn on your graph, can lead to an estimate of the value of x for which the rectangle is a square. [1]

(ii) Draw this line and find this value of x. [2]



$\frac{y}{x}$
6
5
4
3
2
1
0
1
2
3
4
5
6
7
x

15 **(a)** Find the term independent of x in the binomial expansion of $\left(x-\frac{1}{x^2}\right)^6$. [3]

(b) **(i)** Write down, and simplify, the first 3 terms in the expansion of $(2-3x)^5$ in ascending powers of x. [2]

(ii) Given that the first three terms in the expansion of $(a+bx)(2-3x)^5$ are $32+cx+660x^2$ where a, b and c are constants, find the value of a, of b and of c. [4]

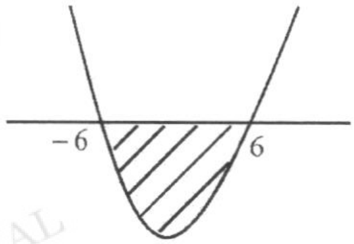
THE END

Springfield Secondary School
Mathematics Department
Marking Scheme

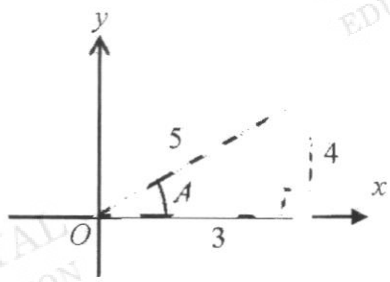
Year	2022
Examination:	End of Year Examination
Format of Paper:	Secondary 3 Express Additional Mathematics
Paper:	1
Setter:	Leung Yong Kang

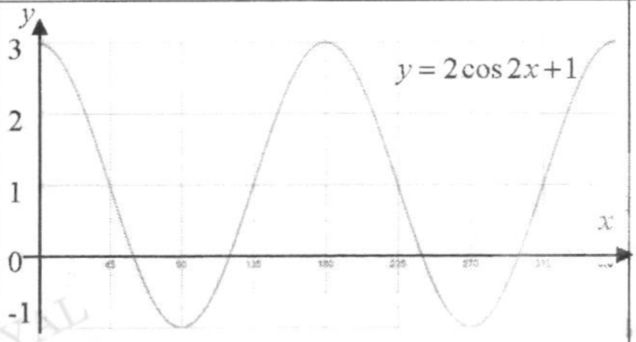
S/N	Solutions	Marks	Remarks	AOs
1(a)	$x^3+8=(x+2)\left(x^2-2x+4\right)$	B1		AO1
1(b)	$x^3+8=0$ $(x+2)\left(x^2-2x+4\right)=0$ $x+2=0$ or $x^2-2x+4=0$ $x=-2$ Discriminant $=(-2)^2-4(1)(4)$ $=-12<0$	M1		AO3
	Therefore, x^2-2x+4 has no real roots. Hence, the equation $x^3+8=0$, **does not** have three real roots. $x=-2$ is the only real root of the equation $x^3+8=0$.	A1		
3 MARKS				
2	Area of triangle $=\frac{1}{2}\times(2+\sqrt{5})\times h$ $-8+5\sqrt{5}=\frac{1}{2}\times(2+\sqrt{5})\times h$	M1		AO2
	$h=\frac{2(-8+5\sqrt{5})}{2+\sqrt{5}}$ $=\frac{(-16+10\sqrt{5})(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$	M1	$2-\sqrt{5}$ is multiplied to both the numerator and denominator.	
	$=\frac{(-16)(2)+16\sqrt{5}+20\sqrt{5}-10(5)}{4-5}$	M1	Correct expansion.	
	$=82-36\sqrt{5}$ The height of the triangle is $82-36\sqrt{5}$.	A1		
4 MARKS				

3	$x^2 = xy + 12 - - - (1)$ $x = 6 - 2y - - - (2)$ Sub (2) into (1): $(6-2y)^2 = (6-2y)y + 12$ $36 - 24y + 4y^2 = 6y - 2y^2 + 12$ $6y^2 - 30y + 24 = 0$ $y^2 - 5y + 4 = 0$ $(y-4)(y-1) = 0$ $y - 4 = 0$ or $y - 1 = 0$ $y = 4$ or $y = 1$ $x = -2$ or $x = 4$ Hence the coordinates of A and B are $(-2, 4)$ and $(4,1)$ respectively. OR Hence the coordinates of A and B are $(4,1)$ and $(-2, 4)$ respectively.	M1 M1 M1 A1	For correct substitutio n For both correct values of y.	AO2
4 MARKS				
4(a)	$y = -2x^2 - 4x + 1$ $= -2(x^2 + 2x) + 1$ $= -2\left[(x+1)^2 - (1)^2\right] + 1$ $= -2(x+1)^2 + 2 + 1$ $= -2(x+1)^2 + 3$	M1 A1 or B2		AO1
4(b)	$(-1,3)$	B1		AO1
4(c)	$p = -3$	B1		AO2
4 MARKS				

5	$3x^2+cx+7>4$ $3x^2+cx+3>0$ $b^2-4ac<0$ $c^2-4(3)(3)<0$ $c^2-6^2<0$ $(c+6)(c-6)<0$  $\therefore -6<c<6$	M1, M1 M1 A1	1M for correct discriminant 1M for correct inequality sign, < Give M1 if student is able to factorise even if inequality sign from above is wrong.	AO2
	4 MARKS			
6	$3^{2x+1}-28(3^x)+9=0$ $3(3^x)^2-28(3^x)+9=0$ Let $u=3^x$, $3u^2-28u+9=0$ $(3u-1)(u-9)=0$ $3u-1=0$ or $u-9=0$ $u=\frac{1}{3}$ or $u=9$ $3^x=\frac{1}{3}$ or $3^x=3^2$ $x=-1$ or $x=2$	M1 M1 M1 A1	For expressing 3^{2x+1} as $3(3^x)^2$.	AO2
	4 MARKS			

7(a)	When $t = 0$ $N = 500 + 3000e^0$ $= 3500$	B1		AO2
7(b)	When $N = 3000$, $t = 5$ $3000 = 500 + 3000e^{5k}$ $2500 = 3000e^{5k}$ $\frac{5}{6} = e^{5k}$ $\ln(\frac{5}{6}) = 5k$ $k = -0.036464$ ≈ -0.0365 (3s.f.)	M1 A1		AO2
7(c)(i)	$A = 500$	B1		AO1
7(c) (ii)	As $t \to \infty$, $3000e^{-0.36464t} \to 0$ $N \to 500$ N will only approaches 500 and hence can never reach 500. OR $N = 500$ is the asymptote for the exponential graph of $N = 500 + 3000e^{-0.36464t}$. Hence, population of the insects can never reach 500.	B1 [B1]		AO3
5 MARKS				

8	$\dfrac{4-x}{x(x+2)^2} = \dfrac{A}{x} + \dfrac{B}{(x+2)} + \dfrac{C}{(x+2)^2}$	M1		AO1
	$4 - x = A(x+2)^2 + B(x)(x+2) + C(x)$	M1		
	When $x = 0$, $4 = A(2)^2$ $A = 1$	M1		
	When $x = -2$ $6 = -2C$ $C = -3$	M1		
	When $x = 1$, $3 = 9A + B(1)(3) + C(1)$ $3 = 9 + 3B - 3$ $3B = -3$ $B = -1$	M1		
	$\dfrac{4-x}{x^3+4x^2+4x} = \dfrac{1}{x} - \dfrac{1}{(x+2)} - \dfrac{3}{(x+2)^2}$	A1		
	6 MARKS			
9(a)(i)				AO1
	$\sec A = \dfrac{1}{\cos A}$ $= \dfrac{1}{\left(\dfrac{3}{5}\right)}$	M1	For $\dfrac{3}{5}$	
	$= \dfrac{5}{3}$	A1	o.e.	
9(a)(ii)	$\tan(-A) = -\tan A$ $= -\dfrac{4}{3}$	B1	o.e.	AO1

9(b)(i)	Period $= \dfrac{360^\circ}{b}$ or $\dfrac{2\pi}{b}$ $= \dfrac{360^\circ}{2}$ or $\dfrac{2\pi}{2}$ $= 180^\circ$ or π	B1		AO1
9(b)(ii)		B1 B1	Correct Shape with 2 cycles Maximum points, minimum points and points on the axis of the curves are correct.	AO1
	6 MARKS			
10(a)	$x^2 + y^2 - 10x + 4y + 25 = 0$ $2g = -10$, $2f = 4$, $c = 25$ $g = -5$, $f = 2$, $c = 25$ Centre C of circle $= (-g, -f)$ $= (5, -2)$ Radius of circle $= \sqrt{g^2 + f^2 - c}$ $= \sqrt{(-5)^2 + (2)^2 - (25)}$ $= 2$ units	M1 A1 M1 A1	For 2g and 2f	AO1
10(b)	$C(5, -2)$, $P(3, -2)$ The radius CP is a horizontal line. Since the **radius is perpendicular to the tangent**, then the tangent to the circle at P is a **vertical line**. Therefore it is parallel to the y-axis.	M1 A1		AO3
	6 MARKS			

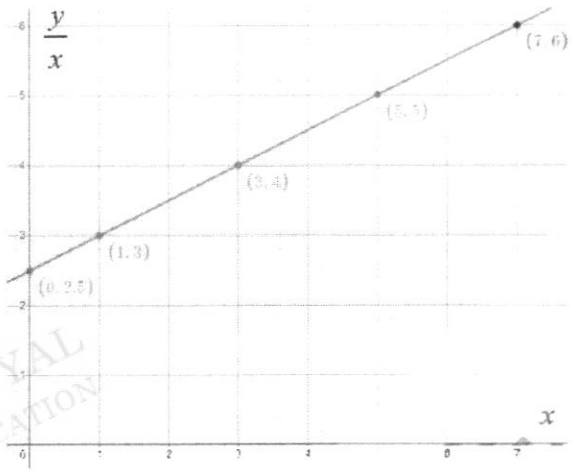
11(a)	$1-\ln y=\ln(x+y)$ $\ln(e)-\ln y=\ln(x+y)$ $\ln\left(\frac{e}{y}\right)=\ln(x+y)$ $\frac{e}{y}=x+y$ $x=\frac{e}{y}-y$	M1 M1 A1	For $1=\ln(e)$	AO2
11(b)	$6\log_y 2=5-\log_2 y$ $\frac{6\log_2 2}{\log_2 y}=5-\log_2 y$ Let $u=\log_2 y$ $\frac{6}{u}=5-u$ $6=5u-u^2$ $u^2-5u+6=0$ $(u-2)(u-3)=0$ $u=2$ or $u=3$ $\log_2 y=2$ or $\log_2 y=3$ $y=4$ or $y=8$	M1 M1 M1 M1 A1	For both correct values of y.	AO2
	8 MARKS			
12(a)	$\mathrm{f}(x)=3x^3-4x^2+qx+6$ Using remainder theorem, $f(1)=3-4+q+6$ $3-4+q+6=-12$ $q=-17$ (Shown)	M1 A1		AO2
12(b)	$\mathrm{f}(x)=3x^3-4x^2-17x+6$ $\mathrm{f}(-2)=3(-2)^3-4(-2)^2-17(-2)+6$ $=0$ Therefore by the factor theorem, $x+2$ is a factor of f(x). $3x^3-4x^2-17x+6=(x+2)(ax^2+bx+c)$ By observation, $a=3$ and $c=3$ $3x^3-4x^2-17x+6=(x+2)(3x^2+bx+3)$ Equating the coefficient of x^2:	M1		AO1

	$-4=6+b$ $b=-10$ $3x^3-4x^2-17x+6=(x+2)(3x^2-10x+3)$ $=(x+2)(3x-1)(x-3)$ Therefore, $3x^3-4x^2-17x+6=0$ $(x+2)(3x-1)(x-3)=0$ $x=-2$ or $x=\frac{1}{3}$ or $x=3$ OR (alternative solution) $f(x)=3x^3-4x^2-17x+6$ $f(-2)=3(-2)^3-4(-2)^2-17(-2)+6$ $=0$ Therefore by the factor theorem, $x+2$ is a factor of $f(x)$. By Long Division, $x+2\overline{)3x^3-4x^2-17x+6}$ quotient $3x^2-10x+3$ $\underline{-3x^3-6x^2}$ $-10x^2-17x$ $\underline{+10x^2+20x}$ $+3x+6$ $\underline{-3x-6}$ 0 $3x^3-4x^2-17x+6=(x+2)(3x^2-10x+3)$ $=(x+2)(3x-1)(x-3)$ Therefore, $3x^3-4x^2-17x+6=0$ $(x+2)(3x-1)(x-3)=0$ $x=-2$ or $x=\frac{1}{3}$ or $x=3$	M1 M1 A1 [M1] [M1] [M1] [A1]	For $3x^2-10x+3$ For correct long division and $3x^2-10x+3$	

12(c)	$4^x = -2$ or $4^x = \frac{1}{3}$ or $4^x = 3$ $4^x = -2$ (no soln) or $4^x = \frac{1}{3}$ $x = \frac{\lg\left(\frac{1}{3}\right)}{\lg 4}$ $= -0.79248$ ≈ -0.792 or $4^x = 3$ $x = \frac{\lg 3}{\lg 4}$ $= 0.79248$ ≈ 0.792	M1 A1	For both −0.792 and 0.792 .	AO2
12(d)	$4^x = -2$ has no solution because $4^x > 0$ for all values of x. Hence, there are only two solutions for x.	B1		AO3
	9 MARKS			

13(a)	Gradient of line $BC = \frac{11-(-1)}{6-22}$ $= \frac{12}{-16}$ $= -\frac{3}{4}$ Gradient of perpendicular bisector $= -\frac{1}{\left(-\frac{3}{4}\right)}$ $= \frac{4}{3}$ M is the midpoint of line BC, then $M = \left(\frac{6+22}{2}, \frac{11+(-1)}{2}\right)$ $= (14,5)$ Equation of perpendicular bisector of BC: $5 = \frac{4}{3}(14) + C$ $C = -\frac{41}{3}$ Hence, $y = 1\frac{1}{3}x - \frac{41}{3}$. Since E lies on BD, then the x-coordinate of E is 6. When $x = 6$, $y = 1\frac{1}{3}(6) - 13\frac{2}{3}$ $= -\frac{17}{3}$ Hence the coordinates of E is $\left(6, -\frac{17}{3}\right)$.	M1 M1 M1 M1 A1	o.e. For substituting $x = 6$ into the equation of perpendicular bisector. o.e.	AO2

13(b)	Area $= \frac{1}{2}\begin{vmatrix} 0 & 6 & 22 & 6 & 0 \\ 3 & 11 & -1 & -15 & 3 \end{vmatrix}$ $= \frac{1}{2}[(18)+(242)+(-6)+(0)]$ $-\frac{1}{2}[(0)+(-6)+(-330)+(18)]$ $= \frac{1}{2}(254) - \frac{1}{2}(-318)$ $= 286 \text{ units}^2$	M1 A1		AO1
13(c)	Gradient of line $AD = \frac{3-(-15)}{0-6}$ $= \frac{18}{-3}$ $= -6$ Gradient of line $BC = -\frac{3}{4}$ (obtained from 13(a)) Therefore, AD is not parallel to BC. Gradient of line $AB = \frac{11-3}{6-0}$ $= \frac{8}{6}$ $= \frac{4}{3}$ Gradient of line $CD = \frac{-1-(-15)}{22-6}$ $= \frac{14}{16}$ $= \frac{7}{8}$ Therefore, AB is not parallel to CD. Since the quadrilateral $ABCD$ does not have a pair of parallel sides, then $ABCD$ is **not** a trapezium.	M1 A1	1M for finding gradient of line AB, line CD and line AD.	AO3
	9 MARKS			

<table>
<tr><td>14(a)</td><td>
<table>
<tr><td>x</td><td>1</td><td>3</td><td>5</td><td>7</td></tr>
<tr><td>$\frac{y}{x}$</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
</table>


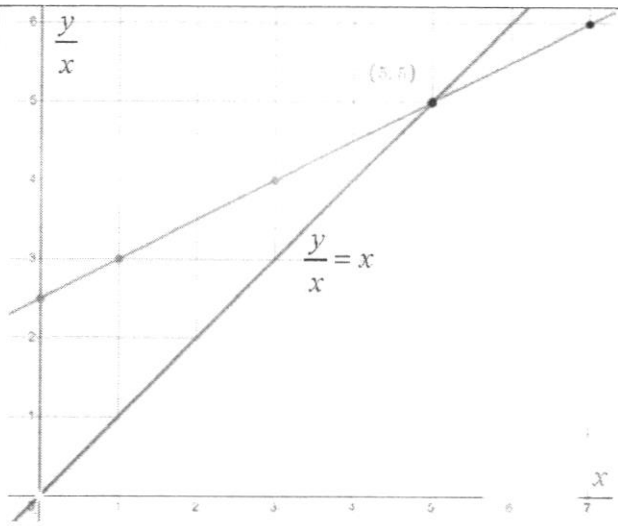
</td><td>B1

B1</td><td>B1 for at least 3 out of 4 correctly plotted points.

B1 for correctly plotted line that connects at least 2 points.</td><td>AO1</td></tr>
<tr><td>14(b)</td><td>$\frac{y}{x} = Ax + B$
Gradient = A
$A = \frac{6-3}{7-1}$
$= 0.5 \pm 0.02$

$\frac{y}{x}$ − intercept = B
$B = 2.5 \pm 0.025$</td><td>M1
A1

B1</td><td></td><td>AO1</td></tr>
<tr><td>14(c)</td><td>$y = 0.5x^2 + 2.5x$</td><td>B1</td><td></td><td>AO2</td></tr>
<tr><td>14(d)(i)</td><td>When the rectangle is a square, the length=breadth. Hence, the line to draw on the graph is $\frac{y}{x} = x$.</td><td>B1</td><td></td><td>AO3</td></tr>
</table>

14(d)(ii)	 Hence, $x = 5$.	M1 A1	1M for the line $\frac{y}{x} = x$.	AO1
	9 MARKS			
15(a)	General Term of $\left(x - \frac{1}{x^2}\right)^6$ $= \binom{6}{r} x^{6-r} \left(-\frac{1}{x^2}\right)^r$ $= \binom{6}{r} (-1)^r x^{6-3r}$ $= \binom{6}{r} (-1)^r x^{6-3r}$ For term independent of x, $r = 2$ Term independent of $x = \binom{6}{2} (-1)^2 x^{6-6}$ $= 15$	M1 M1 A1		AO2
15(b)(i)	$(2-3x)^5$ $= \binom{5}{0}(2)^5 + \binom{5}{1}(2)^4(-3x)^1 + \binom{5}{2}(2)^3(-3x)^2 + \ldots$ $= 32 - 240x + 720x^2 + \ldots$	M1 A1		AO1
15(b)(ii)	$(a+bx)(2-3x)^5$ $= (a+bx)\left(32 - 240x + 720x^2 + \ldots\right)$ $= 32a - 240ax + 32bx + 720ax^2 - 240bx^2 + \ldots$ Since $(a+bx)(2-3x)^5 = 32 + cx + 660x^2 + \ldots$, then	M1		AO2

	Comparing the constants: $32a = 32$ $a = 1$	A1		
	Comparing the coefficient of x^2: $720a - 240b = 660$ $-240b = 660 - 720$ $b = \frac{1}{4}$	A1		
	Comparing the coefficient of x: $c = -240a + 32b$ $= -240(1) + 32\left(\frac{1}{4}\right)$ $= -232$	A1		
9 MARKS				
THE END				