PRESBYTERIAN HIGH SCHOOL
2021 END-OF-YEAR EXAMINATION
SECONDARY THREE EXPRESS
ADDITIONAL MATHEMATICS (4049)

Name: ______________________________ (   )   Class: 3 __________

Duration: 2 hours 15 minutes   Date: 7 October 2021

INSTRUCTIONS TO CANDIDATES

Write your name, index number and class on the spaces above.
Write in dark blue or black ink.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the spaces provided below the questions.
Give non exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Omission of essential working will result in loss of marks.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

For Examiner's Use															
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	14	*Marks Deducted*
Marks															

Category	Accuracy	Units	Symbols	Others
Question No.				

Total Marks
90

Setter: Mr Gregory Quek
Vetter: Mrs Yim Meng Choo

This question paper consists of **20** printed pages (including this cover page) and **0** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots\ldots\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

1 The value of the shares in a stock market is given by the function $y = 2x^2 - 8x + 15$, where y is the value of the shares in thousand dollars and x is the time in years after it was first listed.

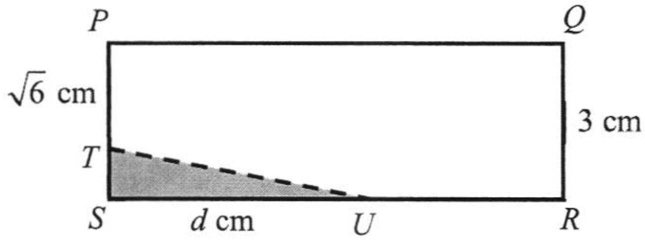
(i) Express the function in the form $y = a(x-h)^2 + k$. [2]

(ii) Hence or otherwise, show that the value of the shares is always positive. [1]

2 Find the range of values of the constant p for which the line $y = p(x-1)$ intersects the curve $y = x^2 + 6x + p$ at two distinct points. [4]

3 The diagram shows a triangular corner, STU, of rectangle $PQRS$ being cut off.

QR is 3 cm, PT is $\sqrt{6}$ cm, SU is d cm, and the area of triangle STU is $\sqrt{24}$ cm^2.



Express d in the form of $a+b\sqrt{6}$, where a and b are integers. [4]

4 Food from a particular restaurant is served at a temperature of 75°C. Subsequently the food cools down in such a way that t minutes after being served, its temperature, T°C, is given by $T = 25 + Ae^{-wt}$, where A and w are constants.

(i) Show that the value of A is 50. [1]

(ii) After 15 minutes, the temperature of the food is 40°C.
Find the value of w. [2]

(iii) State, with explanations, the value that T approaches when t becomes very large. [2]

5 **(a)** Evaluate $\dfrac{\log_3 18 - \log_3 2}{\log_a a}$. [3]

(b) Solve the equation $3\log_2 x - \log_4 x = 15$. [3]

6 **(a)** Solve $\sqrt{x-5}=x-3$. [3]

(b) Using the substitution $u=3^x$, solve $3^{2x+1}-10\left(3^x\right)+3=0$ [3]

7 **(i)** Prove that $\dfrac{2-2\cos^2\theta}{\cos^2\theta} = 2\tan^2\theta$. [2]

(ii) Hence solve the equation $\dfrac{2-2\cos^2\theta}{\cos^2\theta} = 10 - \tan\theta$ for $0° \le \theta \le 360°$. [5]

8 It is given that $f(x)=4x^3+6x^2-9x+2k,$ where k is a constant.

(i) $f(x)$ has a remainder of –125 when divided by $x+2$. Find the value of k. [2]

(ii) Show that $x-3$ is a factor of $f(x)$. [2]

(iii) Explain why $f(x)=0$ has only one real root. Show all workings clearly. [3]

9 **(a)** Factorise $x^3 - 27y^3$ completely. [2]

(b) Express $\dfrac{x^2 + 2x + 7}{(2x+3)(x-1)^2}$ in partial fraction. [5]

10 **(a)(i)** Write down and simplify the first four terms in the expansion of $(2-3x)^5$ in ascending powers of x. [2]

(a)(ii) Hence obtain the coefficient of x^3 in the expansion of $(5x+2x^2)(2-3x)^5$. [2]

10 **(b)** Find the term independent of x in the expansion of $\left(x^2+\dfrac{3}{x}\right)^6$. [3]

11 The variables x and y are known to be related by the equation of the form $y = \frac{x}{ax+b}$.

In an experiment, the values of y are found for certain values of x. A student recorded these values in the following table.

x	1	2	3	4	5
y	−1.25	2.10	1.11	0.89	0.80

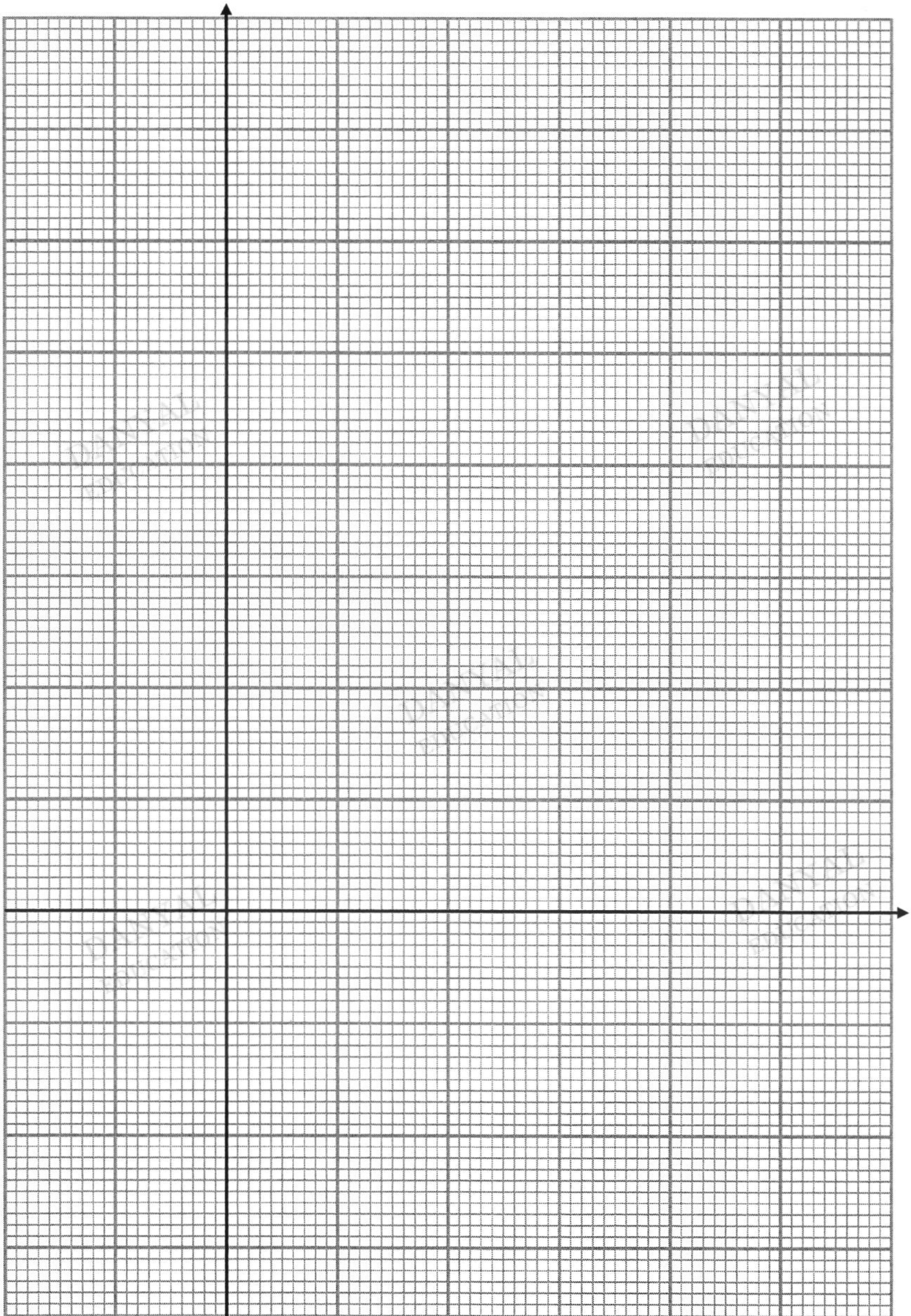
(i) Complete the table below. [1]

x	1	2	3	4	5
$\frac{x}{y}$					

(ii) Using a scale of 2 cm to 1 unit, draw a straight line graph of $\frac{x}{y}$ against x. [2]

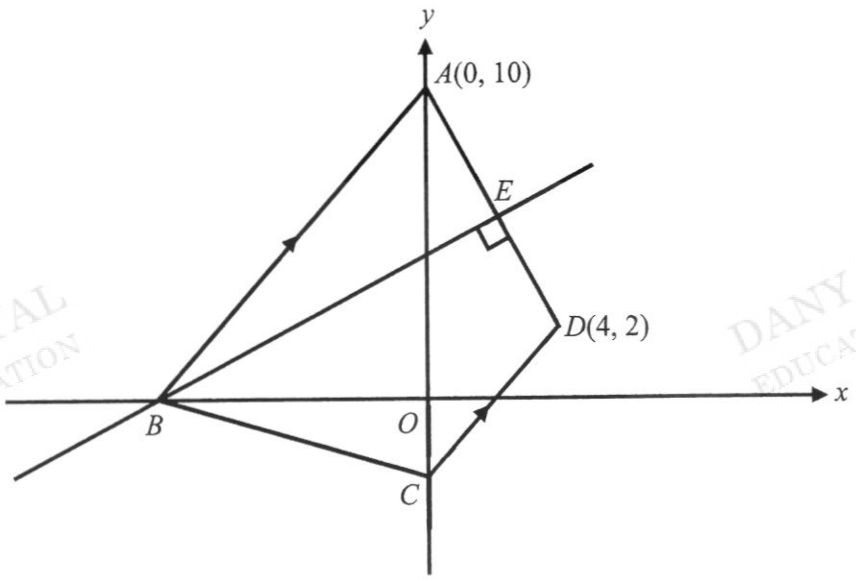
(iii) Use your graph to estimate the value of a and of b. [3]

(iv) Another student claims that by plotting $\frac{1}{y}$ against $\frac{1}{x}$, a straight line graph is obtained. Is he correct? Explain your answer. [2]



12 **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows a quadrilateral $ABCD$, where A is (0, 10) and D is (4, 2). Line BE is the perpendicular bisector of the line AD. B lies on the x-axis and C lies on the y-axis. The lines AB and CD are parallel.



(a) Show that the coordinates of B is (–10, 0). [3]

(b) Find angle EBO. [1]

(c) Find the equation of CD. [2]

(d) Find the area of $ABCD$. [2]

13 **(a)** If $\cos\theta = \frac{5}{13}$ and $180° < \theta < 360°$, evaluate without using a calculator,

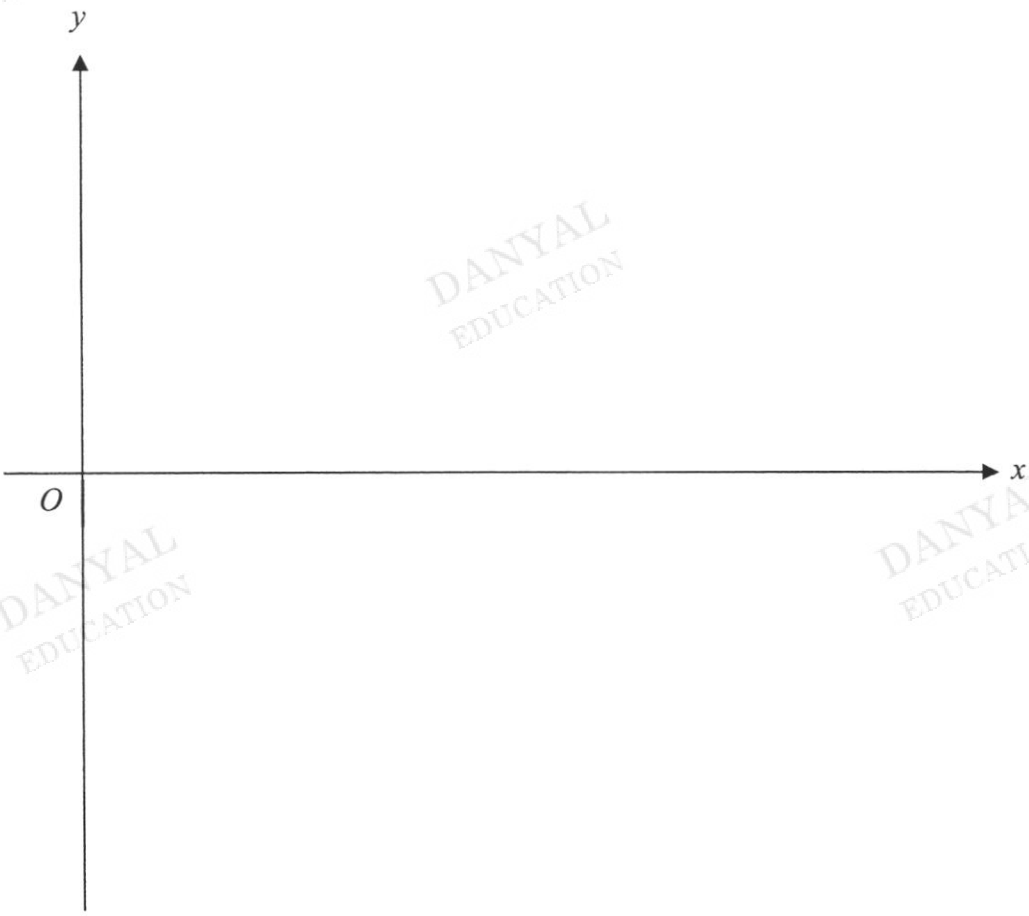
(i) $\sin\theta$, [1]

(ii) $\tan(-\theta)$, [1]

(iii) $\sec\theta$. [2]

13 **(b)(i)** State the amplitude and the period of $y = 3\cos 2x - 1$. [2]

(ii) Sketch the graph of $y = 3\cos 2x - 1$ for $0 \le x \le 2\pi$. [3]



14 The equation of a circle is $x^2 - 12x + y^2 + 6y - 5 = 0$.

The line $y = x - 9$ intersects the circle at P and Q.

(a) Find the radius of the circle and the coordinates of its centre. [3]

(b) Find the coordinates of P and Q. [4]

(c) Determine whether PQ is the diameter of the circle. Justify your answer. [2]

END OF PAPER

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Name: ______________________________ (   )   Class: 3 __________

Duration: 2 hours 15 minutes   Date: 7 October 2021

MARK SCHEME

Mr Mohan – Questions 1 to 6 [28 marks]
Mrs Yim – Questions 7 to 9 [21 marks]
Mr Quek – Questions 10 to 14 [41 marks]

Setter: Mr Gregory Quek
Vetter: Mrs Yim Meng Choo

1 The value of the shares in a stock market is given by the function $y = 2x^2 - 8x + 15$, where y is the value of the shares in thousand dollars and x is the time in years after it was first listed.

(i) Express the function in the form $y = a(x-h)^2 + k$. [2]

$y = 2x^2 - 8x + 15$

$y = 2\left(x^2 - 4x + 7.5\right)$

$y = 2\left[\left(x-2\right)^2 - 2^2 + 7.5\right]$ **M1** (attempt to complete the square)

$y = 2\left(x-2\right)^2 + 7$ **A1**

(ii) Hence or otherwise, show that the value of the shares is always positive. [1]

Since $\left(x-2\right)^2 \geq 0,\ 2\left(x-2\right)^2 + 7 \geq 7$ for all real values of x, **B1**
hence the value of the shares is always positive.

OR

Since $a = 2 > 0$ and discriminant $= \left(-8\right)^2 - 4\left(2\right)\left(15\right) = -56 < 0$, **B1**
thus the value of the shares is always positive.

2 Find the range of values of the constant p for which the line $y = p(x-1)$ intersects the curve $y = x^2 + 6x + p$ at two distinct points. [4]

Let $x^2 + 6x + p = p(x-1)$

$x^2 + 6x + p = px - p$

$x^2 + (6-p)x + 2p = 0$ **M1** (equate and attempt to form quadratic)

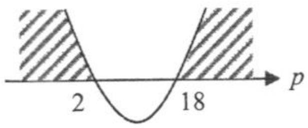
Let $b^2 - 4ac > 0$

$(6-p)^2 - 4(1)(2p) > 0$ **M1** (attempt to apply discriminant > 0)

$p^2 - 12p + 36 - 8p > 0$

$p^2 - 20p + 36 > 0$

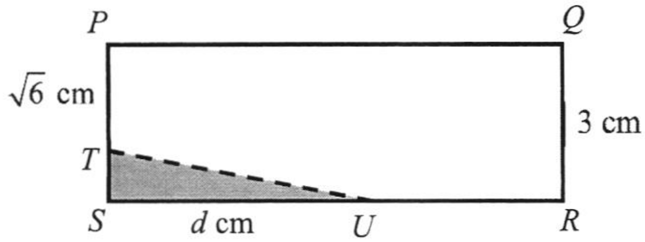
$(p-2)(p-18) > 0$ **M1** (factorisation)



$p < 2$ *or* $p > 18$ **A1**

3 The diagram shows a triangular corner, STU, of rectangle $PQRS$ being cut off.

QR is 3 cm, PT is $\sqrt{6}$ cm, SU is d cm, and the area of triangle STU is $\sqrt{24}$ cm^2.



Express d in the form of $a+b\sqrt{6}$, where a and b are integers. [4]

$\frac{1}{2}d\left(3-\sqrt{6}\right)=\sqrt{24}$ **M1** (seen $A = ½bh$)

$d=\frac{2\sqrt{24}}{3-\sqrt{6}}\times\frac{3+\sqrt{6}}{3+\sqrt{6}}$ **M1** (seen conjugate surds)

$d=\frac{6\sqrt{24}+2\sqrt{144}}{3^2-\left(\sqrt{6}\right)^2}$ **M1** (either numerator or denominator correct)

$d=\frac{12\sqrt{6}+24}{3}$

$d=8+4\sqrt{6}$ **A1**

4 Food from a particular restaurant is served at a temperature of 75°C. Subsequently the food cools down in such a way that t minutes after being served, its temperature, T°C, is given by $T = 25 + Ae^{-wt}$, where A and w are constants.

(i) Show that the value of A is 50. [1]

Let $75 = 25 + Ae^{-w(0)}$
$75 = 25 + A$
$A = 75 - 25 = 50$ (shown)
AG1

(ii) After 15 minutes, the temperature of the food is 40°C.

Find the value of w. [2]

Let $40 = 25 + 50e^{-w(15)}$

$e^{-15w} = \dfrac{40-25}{50} = 0.3$

$\ln e^{-15w} = \ln 0.3$ **M1** (attempt to take ln on both sides)

$-15w = \ln 0.3$

$w = \dfrac{\ln 0.3}{-15} = 0.08026$

$w \approx 0.0803$ (3sf) **A1**

(iii) State, with explanations, the value that T approaches when t becomes very large. [2]

As t becomes very large, e^{-wt} **approaches zero**. **M1**

So the value of T approaches 25°C. **A1**

5 **(a)** Evaluate $\dfrac{\log_3 18 - \log_3 2}{\log_a a}$. [3]

$$\frac{\log_3 18 - \log_3 2}{\log_a a}$$

$$= \frac{\log_3\left(\frac{18}{2}\right)}{1}$$ **M1, M1** (simplify numerator & denominator)

$$= \log_3 3^2$$

$$= 2$$ **A1**

(b) Solve the equation $3\log_2 x - \log_4 x = 15$. [3]

$$3\log_2 x - \log_4 x = 15$$

$$3\log_2 x - \frac{\log_2 x}{\log_2 4} = 15$$ **M1** (change of base law)

$$3\log_2 x - \frac{\log_2 x}{2} = 15$$

$$\frac{5}{2}\log_2 x = 15$$

$$\log_2 x = 6$$ **M1** (make log the subject)

$$x = 2^6 = 64$$ **A1**

6 **(a)** Solve $\sqrt{3x-5}=x-3$. [3]

$\sqrt{3x-5}=x-3$

$3x-5=(x-3)^2$ **M1** (take square on both sides)

$3x-5=x^2-6x+9$

$x^2-9x+14=0$

$(x-2)(x-7)=0$

$x=2$ (rejected) *or* $x=7$ **A1** (seen rejected), **A1**

(b) Using the substitution $u=3^x$, solve $3^{2x+1}-10\left(3^x\right)+3=0$ [3]

$3\left(3^{2x}\right)-10\left(3^x\right)+3=0$

$3u^2-10u+3=0$ **M1** (attempt to substitute $u=3^x$)

$(3u-1)(u-3)=0$

$u=\frac{1}{3}$ *or* $u=3$ **M1** (attempt to solve quadratic equation)

$3^x=\frac{1}{3}$ *or* $3^x=3$

$x=-1$ *or* $x=1$ **A1** (correct pair of answers)

7 **(i)** Prove that $\dfrac{2-2\cos^2\theta}{\cos^2\theta}=2\tan^2\theta$. [2]

$$LHS=\frac{2-2\cos^2\theta}{\cos^2\theta}$$
$$=2\sec^2\theta-2 \qquad \textbf{M1}$$
$$=2\left(\sec^2\theta-1\right)$$
$$=2\tan^2\theta \qquad \textbf{AG1}$$
$$=RHS$$

OR

$$LHS=\frac{2-2\cos^2\theta}{\cos^2\theta}$$
$$=\frac{2\left(1-\cos^2\theta\right)}{\cos^2\theta} \qquad \textbf{M1}$$
$$=\frac{2\sin^2\theta}{\cos^2\theta}$$
$$=2\tan^2\theta \qquad \textbf{AG1}$$
$$=RHS$$

(ii) Hence solve the equation $\dfrac{2-2\cos^2\theta}{\cos^2\theta}=10-\tan\theta$ for $0°\le\theta\le 360°$. [5]

$$\frac{2-2\cos^2\theta}{\cos^2\theta}=10-\tan\theta$$

$2\tan^2\theta=10-\tan\theta$ **M1** (substitute $2\tan^2\theta$ from (i))

$2\tan^2\theta+\tan\theta-10=0$

$(2\tan\theta+5)(\tan\theta-2)=0$

$\tan\theta=-\dfrac{5}{2}$ *or* $\tan\theta=2$ **M1** (attempt to solve quadratic equation)

$\alpha=\tan^{-1}\left(\dfrac{5}{2}\right)=68.19°$ *or* $\alpha=\tan^{-1}(2)=63.43°$ **M1** (find reference angle)

$\theta=180°-68.19°, 360°-68.19°$ *or* $\theta=63.43°, 180°+63.43°$

$\theta=63.43°, 111.81°, 243.43°, 291.81°$

$\theta=63.4°, 111.8°, 243.4°, 291.8°$ **A1, A1** (each correct pair of angles)

8 It is given that $f(x)=4x^3+6x^2-9x+2k,$ where k is a constant.

(i) $f(x)$ has a remainder of -125 when divided by $x+2$. Find the value of k. [2]

Let $f(-2)=-125$

$4(-2)^3+6(-2)^2-9(-2)+2k=-125$ **M1** (apply remainder theorem)

$10+2k=-125$

$k=\dfrac{-125-10}{2}=-67.5$ **A1**

(ii) Show that $x-3$ is a factor of $f(x)$. [2]

$f(3)=4(3)^3+6(3)^2-9(3)+2(-67.5)=0$ **M1** (apply factor theorem)

By factor theorem, $x-3$ is a factor of $f(x)$. **A1** (seen conclusion)

(iii) Explain why $f(x)=0$ has only one real root. Show all workings clearly. [3]

$f(x)=4x^3+6x^2-9x-135=0$

$(x-3)(4x^2+18x+45)=0$ **M1** (attempt to obtain quadratic factor)

$x=3$ or $x=\dfrac{-18\pm\sqrt{18^2-4(4)(45)}}{2(4)}$ **M1** (seen no real roots)

$x=\dfrac{-18\pm\sqrt{-396}}{8}$ [**no real roots**]

Therefore $f(x)=0$ has only one real root $x=3$. **A1** (seen conclusion)

9 **(a)** Factorise $x^3 - 27y^3$ completely. [2]

$$x^3 - 27y^3 = (x)^3 - (3y)^3$$
$$= (x-3y)\left[(x)^2 + (x)(3y) + (3y)^2\right] \quad \textbf{M1} \text{ (difference of 2 cubes)}$$
$$= (x-3y)\left[x^2 + 3xy + 9y^2\right] \quad \textbf{A1 or B2}$$

(b) Express $\dfrac{x^2+2x+7}{(2x+3)(x-1)^2}$ in partial fraction. [5]

Let $\dfrac{x^2+2x+7}{(2x+3)(x-1)^2} = \dfrac{A}{2x+3} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$ **M1** (correct partial fractions)

$x^2 + 2x + 7 = A(x-1)^2 + B(2x+3)(x-1) + C(2x+3)$

Let $x = 1$, $10 = 5C \Rightarrow C = 2$

Let $x = -\dfrac{3}{2}$, $6.25 = 6.25A \Rightarrow A = 1$

Let $x = 0$, $7 = 1 - 3B + 6 \Rightarrow B = 0$

M1 (at least 1 correct method)
A2 (2 correct) or **A1** (1 correct)

$$\frac{x^2+2x+7}{(2x+3)(x-1)^2} = \frac{1}{2x+3} + \frac{2}{(x-1)^2}$$ **A1**

10 **(a)(i)** Write down and simplify the first four terms in the expansion of $(2-3x)^5$ in ascending powers of x. [2]

$$(2-3x)^5 = 2^5 + \binom{5}{1}2^4(-3x) + \binom{5}{2}2^3(-3x)^2 + \binom{5}{3}2^2(-3x)^3 + \ldots$$ **M1**

$$(2-3x)^5 = 32 - 240x + 720x^2 - 1080x^3 + \ldots$$ **A1**

(a)(ii) Hence obtain the coefficient of x^3 in the expansion of $(5x+2x^2)(2-3x)^5$. [2]

$$(5x+2x^2)(2-3x)^5 = (5x+2x^2)\left[32-240x+720x^2-1080x^3+\ldots\right]$$

Coefficient of $x^3 = (5)(720)+(2)(-240)$ **M1** (attempt to compare coefficients)

$= 3120$ **A1**

(b) Find the term independent of x in the expansion of $\left(x^2 + \dfrac{3}{x}\right)^6$. [3]

General term $= \dbinom{6}{r}\left(x^2\right)^{6-r}\left(\dfrac{3}{x}\right)^r$ **M1** (find general term)

$= \dbinom{6}{r}(3)^r x^{12-3r}$

Let $12 - 3r = 0 \Rightarrow r = 4$ **M1** (equate power of x to zero and find r)

Term independent of x is $\dbinom{6}{4}(3)^4 x^0 = 1215$ **A1**

11 The variables x and y are known to be related by the equation of the form $y = \dfrac{x}{ax+b}$.

In an experiment, the values of y are found for certain values of x. A student recorded these values in the following table.

x	1	2	3	4	5
y	−1.25	2.10	1.11	0.89	0.80

(i) Complete the table below. [1]

x	1	2	3	4	5
$\dfrac{x}{y}$	−0.8	0.95	2.70	4.49	6.25

B1 (all 5 values correct)

(ii) Using a scale of 2 cm to 1 unit, draw a straight line graph of $\dfrac{x}{y}$ against x. [2]

P1 (any 3 correct points), **C1** (straight line)
[Deduct 1 mark if axes are not labelled.]

(iii) Use your graph to estimate the value of a and of b. [3]

$gradient = \dfrac{6.25-(-0.8)}{5-1} = 1.7625$ **M1** (finding gradient of line)

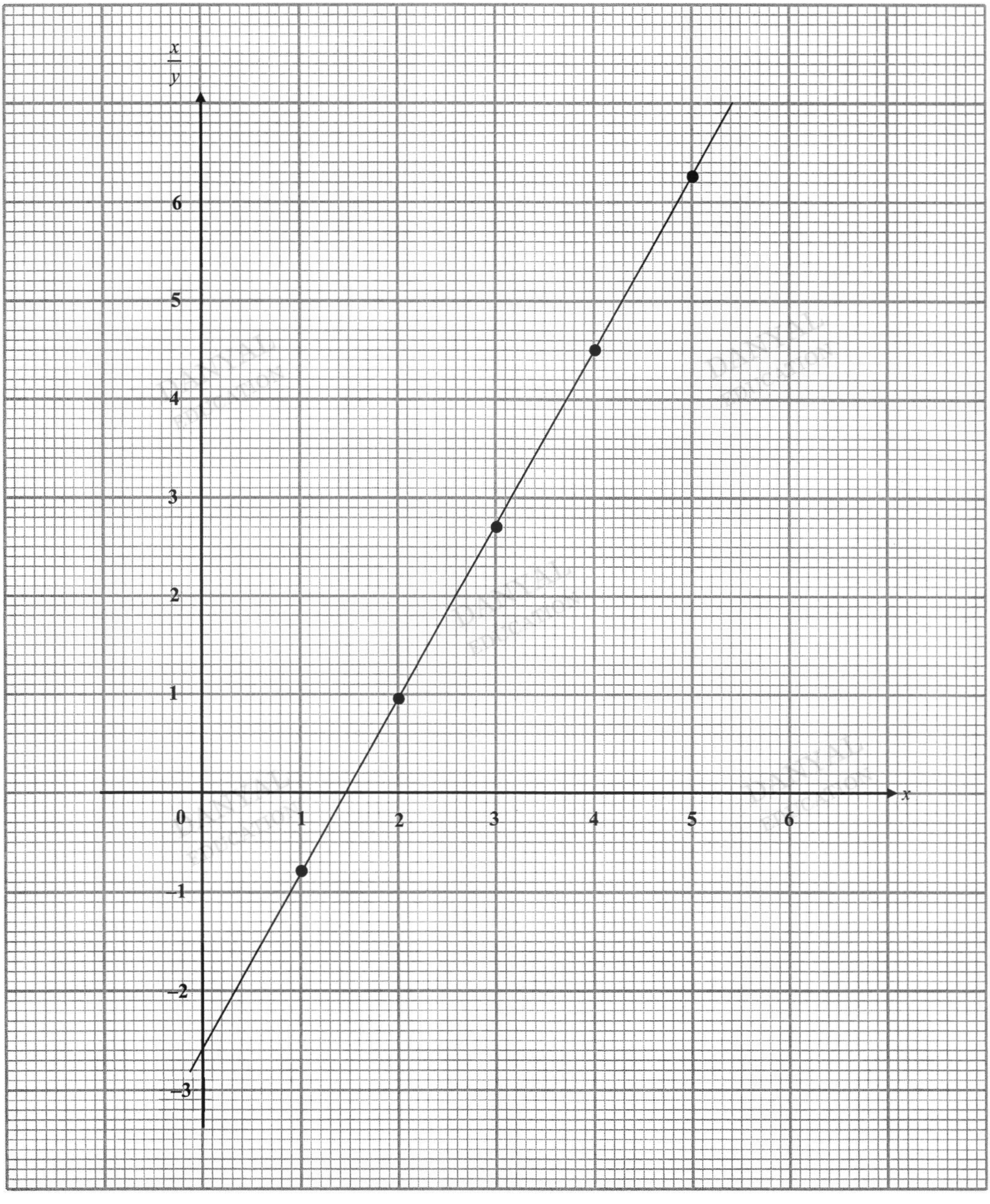
$a = gradient = 1.76\ (3sf)$ **A1**

$b = \dfrac{x}{y}\text{-intercept} = -2.6$ **B1**

(iv) Another student claims that by plotting $\dfrac{1}{y}$ against $\dfrac{1}{x}$, a straight line graph is obtained. Is he correct? Explain your answer. [2]

$$y = \frac{x}{ax+b}$$
$$\Rightarrow \frac{1}{y} = \frac{ax+b}{x}$$
$$\therefore \frac{1}{y} = b\left(\frac{1}{x}\right) + a$$
M1

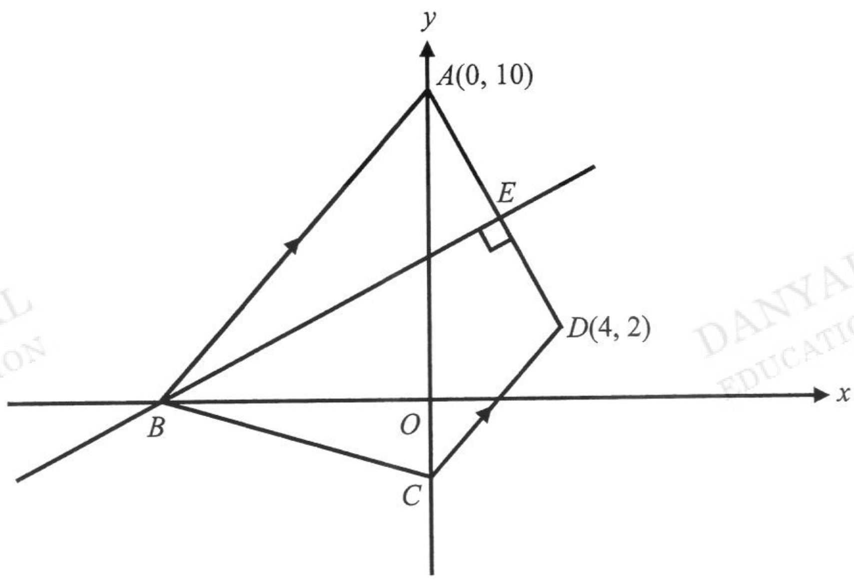
Yes, he is <u>correct</u>. **A1**



$\frac{x}{y}$
6
5
4
3
2
1
0
−1
−2
−3
1
2
3
4
5
6
x

12 **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows a quadrilateral $ABCD$, where A is (0, 10) and D is (4, 2). Line BE is the perpendicular bisector of the line AD. B lies on the x-axis and C lies on the y-axis. The lines AB and CD are parallel.



(a) Show that the coordinates of B is (–10, 0). [3]

$E = \left(\frac{0+4}{2}, \frac{10+2}{2}\right) = (2,6)$ **M1** (midpoint formula)

$m_{AD} = \frac{10-2}{0-4} = -2$

$m_{BE} = \frac{-1}{-2} = \frac{1}{2}$ **M1** (gradient of perpendicular lines)

$y - 6 = \frac{1}{2}(x-2)$

$\Rightarrow y = \frac{1}{2}x + 5$

Let $y = 0$, $x = -10$

$\therefore B = (-10, 0)$ (shown)

AG1 (finding B using equation of line BE)

OR

Let $B = (b, 0)$,

$m_{BE} = \frac{0-6}{b-2} = \frac{1}{2}$

$\Rightarrow b = -10$

$\therefore B = (-10, 0)$ (shown)

AG1 (finding B using gradient)

(b) Find angle EBO. [1]

angle $EBO = \tan^{-1}\left(\frac{1}{2}\right) = 26.56° \approx 26.6°$ (1dp) **B1**

(c) Find the equation of CD. [2]

$m_{CD} = m_{AB} = \frac{10-0}{0-(-10)} = 1$ **M1** (gradient of CD)

$y - 2 = 1(x - 4)$

$y = x - 2$ **A1**

(d) Find the area of $ABCD$. [2]

From (c), $C = (0, -2)$

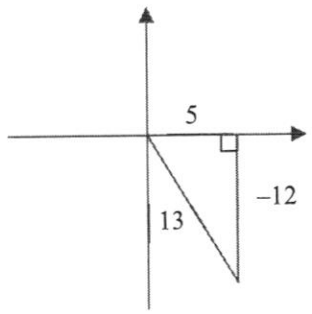
area of $ABCD = \frac{1}{2}\begin{vmatrix} 0 & -10 & 0 & 4 & 0 \\ 10 & 0 & -2 & 2 & 10 \end{vmatrix}$

$= \frac{1}{2}\left|(0)(0)+(-10)(-2)+(0)(2)+(4)(10)-(10)(-10)-(0)(0)-(-2)(4)-(2)(0)\right|$ **M1**

$= \frac{1}{2}\left|168\right|$

$= 84 \text{ units}^2$ **A1**

13 **(a)** If $\cos\theta = \frac{5}{13}$ and $180° < \theta < 360°$, evaluate without using a calculator,



(i) $\sin\theta$, [1]

$\sin\theta = -\frac{12}{13}$ **B1**

(ii) $\tan(-\theta)$, [1]

$\tan(-\theta) = -\tan\theta = -\left(-\frac{12}{5}\right) = \frac{12}{5}$ **B1**

(iii) $\sec\theta$. [2]

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{5}{13}}$ **M1** (seen either one)

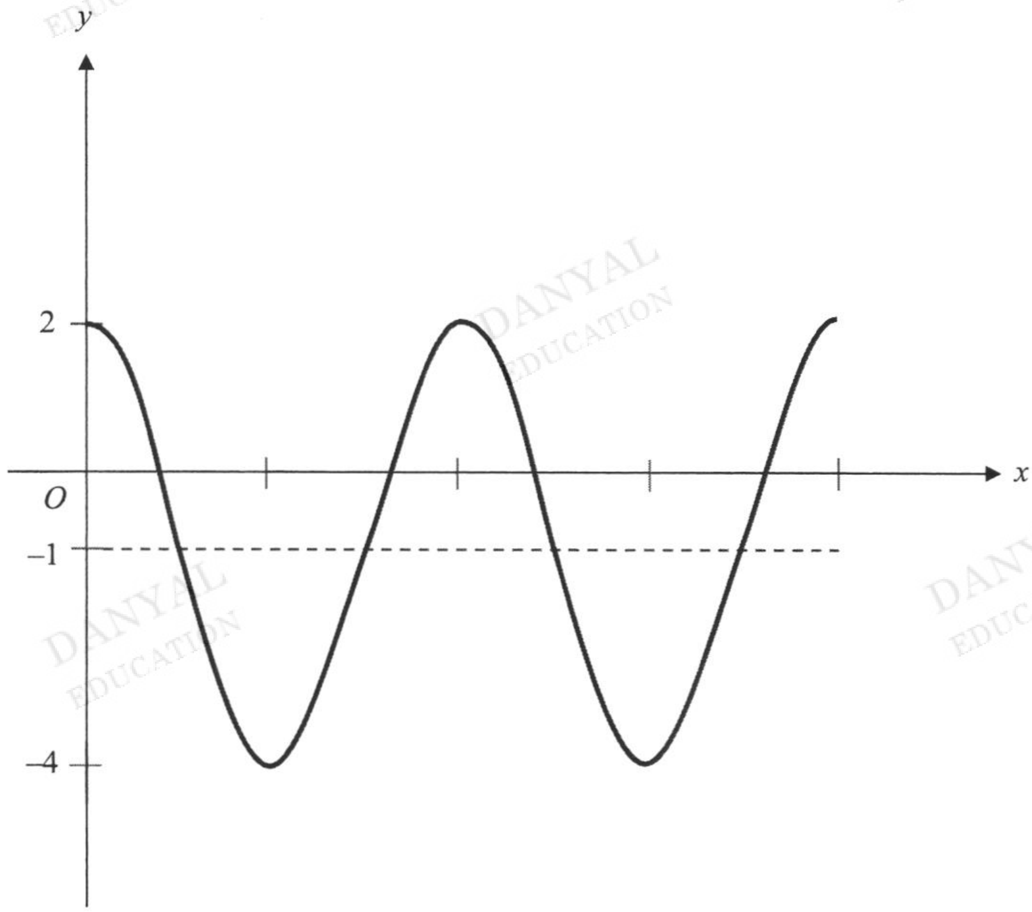
$\sec\theta = \frac{13}{5}$ **A1 or B2**

13 **(b)(i)** State the amplitude and the period of $y=3\cos 2x-1$. [2]

Amplitude = 3 **B1**

Period = $180°$ **or** π **B1**

(ii) Sketch the graph of $y=3\cos 2x-1$ for $0 \le x \le 2\pi$. [3]



C1 (correct shape)
P1 (correct period in radians)
P1 (correct maximum & minimum values)

14 The equation of a circle is $x^2-12x+y^2+6y-5=0$. The line $y=x-9$ intersects the circle at P and Q.

(a) Find the radius of the circle and the coordinates of its centre. [3]

$x^2-12x+y^2+6y-5=0$

$(x-6)^2-6^2+(y+3)^2-3^2-5=0$

$(x-6)^2+(y+3)^2=50$ **M1** (completing the square, o.e)

radius $=\sqrt{50}=5\sqrt{2}$ units **A1**

centre $=(6,-3)$ **A1**

(b) Find the coordinates of P and Q. [4]

$x^2-12x+(x-9)^2+6(x-9)-5=0.$ **M1** (substitution)

$x^2-12x+x^2-18x+81+6x-54-5=0$

$2x^2-24x+22=0$

$x^2-12x+11=0$

$(x-1)(x-11)=0$

$x=1$ *or* $x=11$ **M1** (solving for x)

$y=-8$ *or* $y=2$

The coordinates are $(1,-8)$ and $(11,2)$ **A1, A1**

(c) Determine whether PQ is the diameter of the circle. Justify your answer. [2]

$PQ=\sqrt{(11-1)^2+(2-(-8))^2}=\sqrt{200}=10\sqrt{2}$ units **M1** (find length of PQ)

Since $PQ=2r$, PQ is the diameter of the circle. **A1**

OR

Midpoint of $PQ=\left(\frac{1+11}{2},\frac{-8+2}{2}\right)=(6,-3)$ **M1** (find midpoint of PQ)

Since midpoint = centre, PQ is the diameter of the circle. **A1**