Class	Register No.

Candidate Name _____

PEIRCE SECONDARY SCHOOL
MID-YEAR EXAMINATION 2021
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

MATHEMATICS **4048/01**
Paper 1 **7 May 2021**
2 hours

Additional Materials:
Plain Paper (for rough work)

INSTRUCTIONS TO CANDIDATES
Candidates answer on the Question Paper.

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use	
Total	

PARENT'S SIGNATURE

This paper consists of **21** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

For Examiner's Use

Answer **all** the questions.

For Examiner's Use

1 **(a)** The number of people in a shopping mall is given as 8000, correct to the nearest thousand.

Write down the maximum number of people that could be in the shopping mall at this time.

Answer ..... [1]

(b) Write the following numbers in order of size, starting with the **largest**.

$$0.25, \quad \frac{1}{3}, \quad \left(\frac{1}{3}\right)^2, \quad \frac{2}{7}$$

Answer ....., ..., ..., ... [1]

---

2 The stem-and-leaf diagram shows the masses, in kilograms, of 14 infants.

5	0	1	1	2	
6	2	4	4	8	9
7	0	3	7		
8	2	5			

Key : 6 | 4 represents 6.4 kg

For these masses, find

(a) the range,

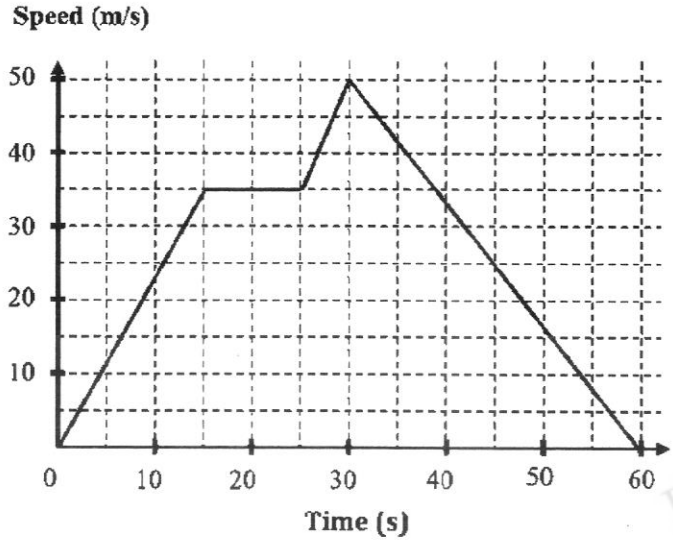
Answer .....kg [1]

(b) the interquartile range.

Answer .....kg [2]

For Examiner's Use

3 The diagram shows a speed-time graph of an object.



Calculate

(a) the total distance travelled by the object in the first 25 seconds,

Answer .....m [1]

(b) the deceleration of the object in the last 30 seconds.

Answer .....m/s^2 [1]

---

4 A regular polygon has p sides. When the number of sides of the polygon is doubled, each exterior angle decreases by 20^o. Find the value of p.

Answer p = ... [2]

For Examiner's Use

5 **(a)** Write as a single fraction in its simplest form $\frac{9}{(x-4)^2}+\frac{2}{4-x}$.

For Examiner's Use

Answer ..... [2]

(b) Hence, solve $\frac{9}{(x-4)^2}=-\frac{2}{4-x}$.

Answer ..... [1]

For Examiner's Use

6 **(a)** $100000 is invested in an account which pays 1.6% per annum compound interest, compounded half yearly.
Find the compound interest earned at the end of one and a half year, giving your answer to the nearest cent.

Answer $..... [2]

(b) The selling price of a Koshiba air-conditioning system is $5000.
The hire purchase price is a deposit of $200 plus 24 equal monthly payments of $275 per month.
Calculate the simple interest rate per annum.

Answer .....% [3]

For Examiner's Use

For Examiner's Use

7 **(a)** Solve the inequalities $-5 < 2x - 4 \leq 8$.

For Examiner's Use

Answer ..... [2]

(b) Write down the largest prime number value of x that satisfies the inequalities in **(a)**.

Answer ..... [1]

---

8 **(a)** Write $2^3 + 2^3 + 2^3 + 2^3$ as a power of 2.

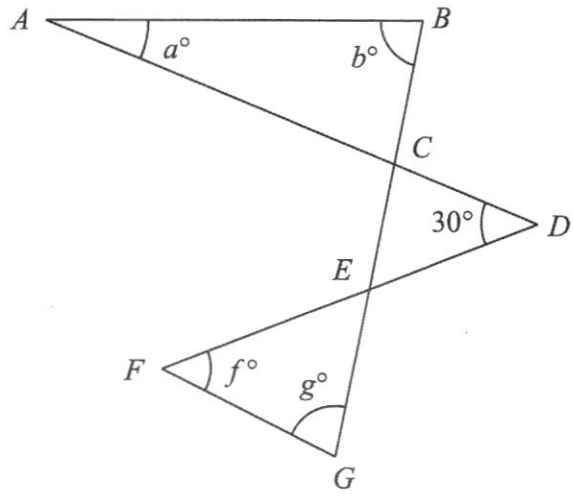
Answer ..... [1]

(b) Simplify $\left(\frac{x^6}{y^8}\right)^{-\frac{3}{2}}$, leaving your answer in positive index.

Answer ..... [2]

For Examiner's Use

9 In the figure, angle $CDE = 30°$.
ACD, FED and $BCEG$ are straight lines.



(a) Find angle ACB in terms of a and b.

Answer ..... [1]

(b) Find angle CED in terms of a and b.

Answer ..... [1]

(c) Hence, express g in terms of a, b and f.

Answer $g =$ ..... [2]

For Examiner's Use

10 Abel cycles at an average speed of x km/h for 30 minutes and then at an average speed of y km/h for 1 hour 20 minutes.
He cycles at total of 50 km.

(a) Write down an equation in x and y to represent this information and show that it simplifies to $3x+8y=300$.

[1]

Cain cycles at an average speed of x km/h for 1 hour 10 minutes and then at an average speed of y km/h for 40 minutes.
He cycles $4\frac{1}{3}$ km further than Abel.

(b) Write down an equation in x and y to represent this information and show that it simplifies to $7x+4y=326$.

[1]

(c) Solve the two equations to find the value of x and the value of y.

Answer x = ...

y = ... [3]

(d) Calculate how much longer it would take for Abel to cycle 50 km at his slower speed as compared to his faster speed.
Give your answer in minutes and seconds, correct to the nearest second.

Answer .....min .....s [2]

For Examiner's Use

11 y is directly proportional to $2x^3$.

(a) When $x = \frac{1}{4}$, $y = 1$.

Find y when $x = \frac{1}{2}$.

Answer $y =$ ..... [2]

(b) A change in x produces a change in y.

When the value of x is multiplied by 2, find the percentage change in y.

Answer ..... [2]

For Examiner's Use

For Examiner's Use

12 **(a)** Find the prime factors of 90, giving your answer in index form.

For Examiner's Use

Answer ..... [1]

(b) The number $\frac{90p}{q}$ is a perfect square.

p and q are prime numbers.

State the values of p and q such that $p < q$.

Answer p = ...

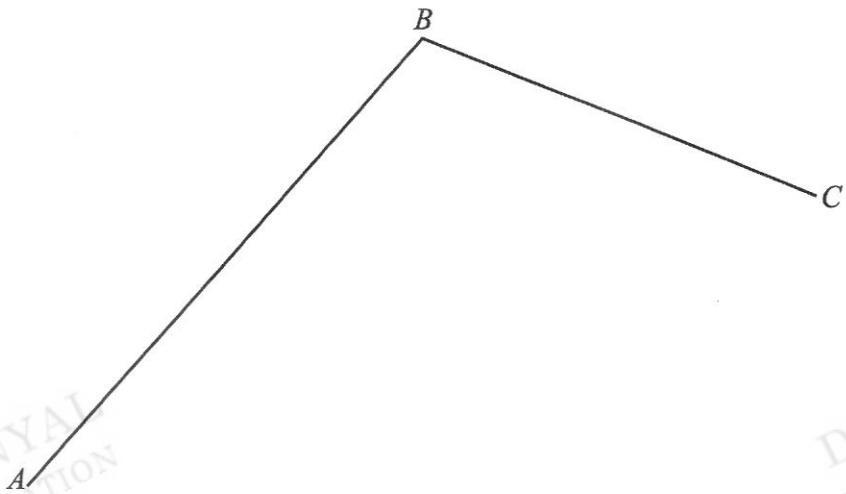
q = ... [1]

(c) Write down the smallest three-digit integer such that the highest common factor of 990 and the integer is 22.

Answer ..... [1]

For Examiner's Use

13 Three points A, B and C are shown below.



(a) Construct the bisector of angle ABC. [1]

(b) Construct the perpendicular bisector of AB. [1]

(c) These two bisectors meet at X.
Complete the statement below.

Answer The point X is equidistant from the points ..... and .....,

and equidistant from the lines ..... and ..... [1]

(d) The point Y is such that angle $BCY = 80°$ and $AY = 8$ cm.
Find the two possible positions of Y, and label them Y_1 and Y_2. [2]

For Examiner's Use

14 **(a)** Box A contains only red and green marbles.

A marble is drawn at random from Box A.
x = Probability that the marble drawn is red
y = Probability that the marble drawn is green

If $x = 5y$, find x.

Answer $x =$ ..... [2]

(b) Box B contains 1 black marble and 1 white marble.
Box C contains 5 black marbles and 4 white marbles.
A marble is drawn at random from Box B and put in Box C.
Then a marble is drawn at random from Box C.

Find the probability that the marble drawn from Box C is white.

Answer ..... [2]

15 A map area of a forest is 400 cm^2.
It is drawn to a scale of 1 : n.

The actual area of the forest is 90 000 m^2.

(a) Write this scale in the form 1 : n.

Answer ..... [2]

(b) The National Park needs to rent a tractor for some work in the forest.
The rental cost of tractor is based on the capacity of the tractor.

Tractor *A*: \$1000 per day. Maximum coverage of 1 hectare per day.
Tractor *B*: \$1400 per day. Maximum coverage of 1.5 hectares per day.
Tractor *C*: \$2000 per day. Maximum coverage of 2.5 hectares per day.

1 hectare = 10 000 m^2

Which choice of tractor will you use for the work?
Show all workings clearly and give your reason for the choice.

Answer Tractor ..... because.....

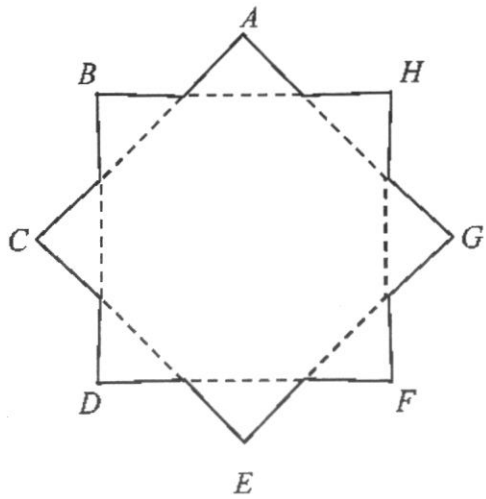
.....

..... [3]

For Examiner's Use

16 The sides of a regular octagon shown in dotted lines are extended to make a 8-sided star.

For Examiner's Use



(a) Prove that angle $CAG = 90°$. [2]

(b) Find the sum of the angles A, B, C, D, E, F, G, and H.

Answer .....° [1]

For Examiner's Use

17 $\varepsilon = \{ x : x \text{ is an integer, } 4 < x \leq 20 \}$

$A = \{x: x \text{ is a factor of } 12\}$

$B = \{x : x \text{ is a multiple of } 2\}$

(a) Find

(i) $A \cap B$,

Answer ..... [1]

(ii) $(A \cup B)'$.

Answer ..... [1]

(b) **(i)** Is A a proper subset of B? Explain.

Answer .....

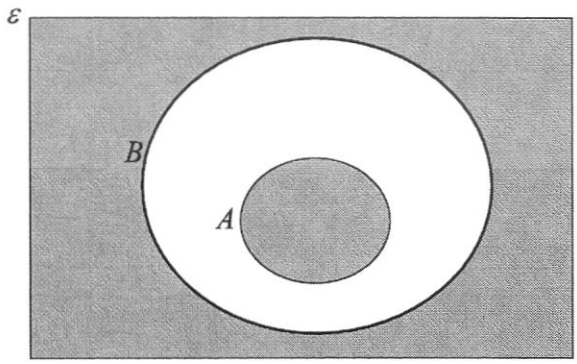
..... [1]

(ii) Write down the set notation to represent your answer in **(b) (i)**.

Answer ..... [1]

(c) Use set notation to describe the shaded region.

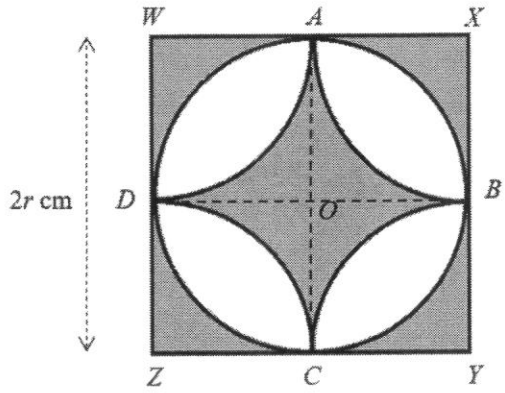
Answer



Answer ..... [1]

For Examiner's Use

18



WXYZ is a square with centre *O* and sides $2r$ cm.
A, *B*, *C* and *D* are the midpoints of the sides of the square.

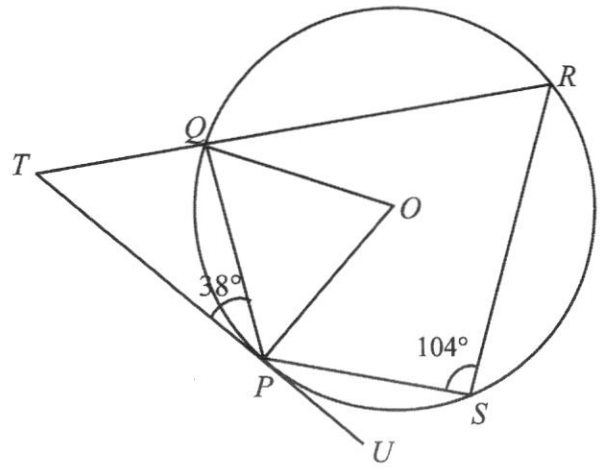
ABCD is a circle, centre *O*.

What fraction of the square *WXYZ* is shaded?
Leave your answer in terms of π.

Answer ..... [3]

For Examiner's Use

19



The diagram shows a circle with center O and four points P, Q, R and S on the circle.
TU is a tangent to the circle at P.
Angle $QPT = 38°$ and angle $PSR = 104°$.

(a) Find angle POQ.

Answer.....° [1]

(b) Find angle OQR.

Answer.....° [1]

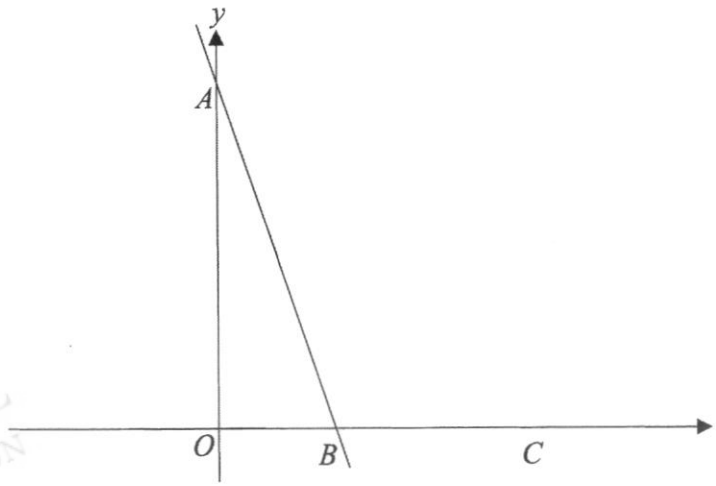
(c) If a circle passes through O, P and U, describe where the centre of this circle should lie. Justify your answer with clear explanation.

Answer

[2]

For Examiner's Use

20 The diagram shows a sketch of the graph of $y = 12 - 3x$.
The line crosses the axes at A and B.
C is a point on x-axis.



(a) Find the coordinates of A and B.

Answer $A($..... , ...$)$

$B($ ..... , ...$)$ [2]

(b) Find the coordinates of the point D, such that it lies on the y-axis, is directly below A and $AB = AD$.

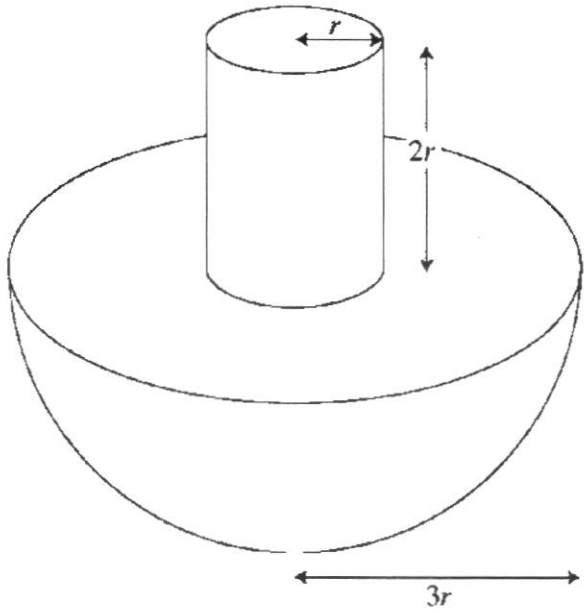
Answer $D($..... , ...$)$ [2]

(c) Write down the value of $\cos \angle ABC$.

Answer .....[1]

For Examiner's Use

21 A container consists of a cylinder and a hemisphere.
The cylinder has radius r and height $2r$.
The hemisphere has radius $3r$.
Water enters the empty container at a constant rate from the top of the cylinder.
It takes 2 minutes to fill the container completely.

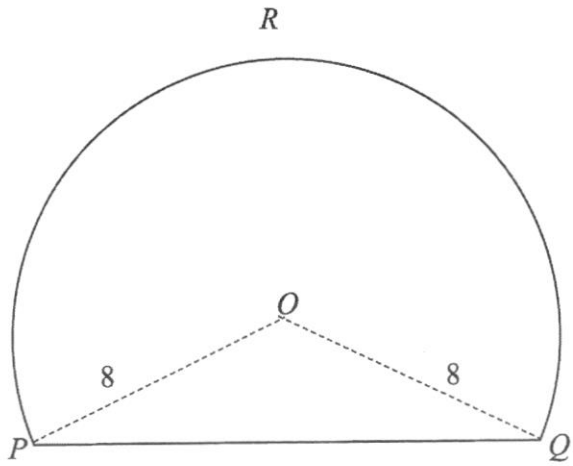


Find the time taken to fill the hemisphere completely. Give your answer in minutes and seconds.

Answer .....min .....s [2]

For Examiner's Use

22 The cross-section of a tunnel is a major segment of a circle, centre O and radius 8 m. The total perimeter of the major sector $POQR$ is 48 m.



Calculate

(a) reflex angle POQ in radians,

Answer .....rad [1]

(b) the total area of cross-section of the tunnel.

Answer .....m^2 [2]

- End of Paper -

Class | Register No.

Candidate Name _____

PEIRCE SECONDARY SCHOOL
MID-YEAR EXAMINATION 2021
SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC

MATHEMATICS **4048/02**
Paper 2 **11 May 2021**
2 hours 30 minutes

Additional Materials:
Plain Paper (for rough work)

INSTRUCTIONS TO CANDIDATES
Candidates answer on the Question Paper.

Write your name, class and register number on all the work you hand in.
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If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use	
Total	

PARENT'S SIGNATURE

This paper consists of **29** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P(1+\frac{r}{100})^n$$

Mensuration

$$\text{Curved Surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** **(i)** Simplify $p^2-(p+q)(p-q)$.

Answer ..... [2]

(ii) Hence evaluate $543896702^2-543896707\times543896697$.

Answer ..... [2]

(b) Factorise completely $12ax-2by+8ay-3bx$.

Answer ..... [2]

(c) $p = \dfrac{4q + 3r}{5 - r}$

(i) Evaluate p when $q = 6$ and $r = -4$.

Answer $p =$ ..... [1]

(ii) Express r in terms of p and q.

Answer $r =$ ..... [2]

2 **(a)** The first four terms in a sequence of numbers, u_1, u_2, u_3, u_4,..., are given below.

$$u_1 = 1 - 0^2 = 1$$
$$u_2 = 4 - 1^2 = 3$$
$$u_3 = 9 - 2^2 = 5$$
$$u_4 = 16 - 3^2 = 7$$

(i) Write down an expression for u_8 and evaluate it.

Answer

[1]

(ii) Find an expression in terms of n for the nth term, u_n, of the sequence.

Answer $u_n =$ ..... [2]

(iii) Evaluate $u_{50} + u_{51}$.

Answer ..... [2]

(iv) Determine if 387 is a term in the sequence.

Answer

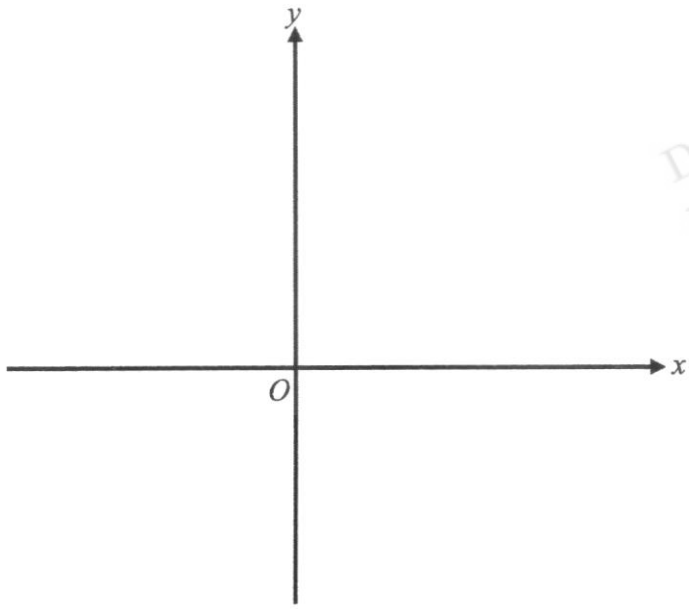
[1]

(b) **(i)** Express $x^2 - 4x + 7$ in the form $(x+a)^2 + b$, where a and b are integers.

Answer ..... [2]

(ii) Hence, sketch the graph of $y = x^2 - 4x + 7$, giving the coordinates of the turning point and of the axial intercept(s).

Answer



[2]

3 **(a)** One day the exchange rate between United States dollar (USD) and Singapore dollar ($) was USD1 = $1.35.

On the same day, the exchange rate between Japanese yen (¥) and United States dollar was ¥1 = USD0.0096.

(i) Cindy changed USD3000 into Singapore dollars.
Calculate how many dollars she received.

Answer $..... [1]

(ii) Eric converted $10000 into Japanese yen.
Calculate how much yen he received, correct to the nearest yen.

Answer ¥..... [2]

(b) The price of a 10-day holiday package to Switzerland is $4288 including an airport tax of $40.50 and 7% Goods and Services Tax (GST).
Note: GST is applied after the inclusion of the airport tax.

Calculate the price of the holiday package before tax, correct to the nearest cent.

Answer $..... [2]

(c) The average distance between Mars and the Sun is approximately 2.27×10^8 km.

(i) 2.27×10^8 can be written as k million. Find k.

Answer $k =$ ..... [1]

(ii) Given that light travels at a speed of 3×10^8 m/s, calculate the time, in minutes, it takes for light to travel from the Sun to Mars.

Answer .....min [2]

4 The table below shows the mean and median monthly household income in a city from 2010 to 2019.

Year	Mean ($)	Median ($)
2010	7214	5000
2011	8039	5624
2012	8637	6000
2013	8692	6257
2014	9176	6500
2015	9495	6819
2016	9359	6804
2017	9633	6913
2018	9679	6984
2019	9763	7000

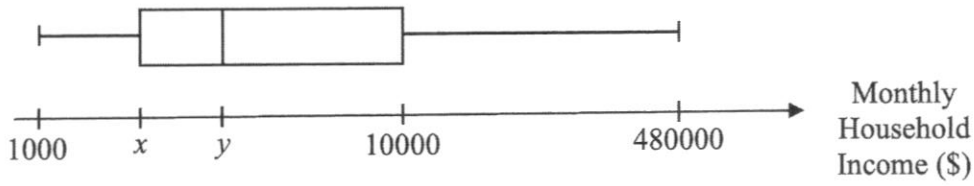
(a) In 2019, the total number of households in the city was 1.37 million.
Calculate the total income received by all households in a month, giving your answer in standard form, correct to 5 significant figures.

Answer $..... [2]

(b) Calculate the percentage increase in the mean monthly household income from 2010 to 2019.

Answer .....% [1]

(c) Suppose a box-and-whisker plot of the monthly household income in 2019 is drawn as shown. The 25th percentile is \$2800.



(i) State the values of x and y.

Answer $x =$ .....

$y =$ ..... [2]

(ii) Find the range of the monthly household income distribution.

Answer \$..... [1]

(iii) Find the probability of a randomly selected household having a monthly income of more than \$10000.

Answer ..... [1]

(d) Explain whether the mean or the median is a better gauge of the average monthly household income in the city.
Give a reason for your answer.

Answer .....

.....

.....

.....

.....

..... [1]

5 The table shows the price of a ticket for each category for a performance.

Child (below 12 years old)	Adult	Senior Citizen (above 60 years old)
\$10	\$40	\$25

(a) Write down a column matrix to represent the information above.

Answer ..... [1]

Mrs Toh bought three tickets for her 65-year-old mother, her 7-year-old son and herself.

(b) Write down two matrices such that the elements of their product under matrix multiplication gives the total amount of money Mrs Toh paid for the tickets. Hence, find this product.

Answer

[1]

The number of tickets sold on one weekend is shown in the table below.

	Child	Adult	Senior Citizen
Saturday	37	a	25
Sunday	b	85	31

(c) Given that $\mathbf{S} = \begin{pmatrix} 37 & a & 25 \\ b & 85 & 31 \end{pmatrix} \begin{pmatrix} 10 \\ 40 \\ 25 \end{pmatrix}$.

Express matrix **S** as a single matrix in terms of a and b.

Answer ..... [1]

(d) Given that $\mathbf{S} = \mathbf{T}$, where $\mathbf{T} = \begin{pmatrix} 3395 \\ 4655 \end{pmatrix}$, find the values of a and b.

Answer $a =$ .....

$b =$ ..... [2]

(e) Explain what the elements in matrix **T** represent.

Answer .....

.....

.....

.....

.....

..... [1]

(f) If the price of a ticket for each category was increased by 30% for Saturday and doubled for Sunday, and **R** is a 1×2 matrix, evaluate **RT** such that the elements of their product under matrix multiplication gives the total amount of money collected from the sale of the tickets on both days.

Answer $\$$..... [1]

6 Kristy owned a fashion pop up stall. She bought x dresses, each at the same price, for a total of \$750.

(a) Write down an expression, in terms of x, for the cost price of each dress.

Answer \$..... [1]

(b) She sold 12 of the dresses for \$630 and the rest at a loss of \$5 per dress.

(i) Write down, in terms of x, the selling price of each dress sold at a loss.

Answer \$..... [1]

(ii) Show that the total amount Kristy received from the sale of all the dresses is $\$\left(1440-\frac{9000}{x}-5x\right)$.

Answer

[2]

(c) Given that Kristy made a profit of \$240 altogether, form an equation in x and show that it reduces to $x^2 - 90x + 1800 = 0$.

Answer

[2]

(d) Solve the equation $x^2 - 90x + 1800 = 0$.

Answer $x =$ ..... or ..... [2]

(e) Given that the cost price of each dress is more than \$15, find the cost price of each dress.

Answer \$..... [1]

7 The variables x and y are connected by the equation

$$y = \frac{1}{10}x(20 - x^2).$$

Some corresponding values of x and y, correct to 1 decimal place, are given in the table below.

x	-2	-1	0	1	2	3	4	5	6
y	-3.2	p	0	1.9	3.2	3.3	1.6	-2.5	-9.6

(a) Find the value of p.

Answer $p =$ ..... [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-2 \leq x \leq 6$.
Using a scale of 2 cm to represent 2 units, draw a vertical y-axis for $-10 \leq y \leq 4$.

On the grid opposite, draw the graph of $y = \frac{1}{10}x(20 - x^2)$. [3]

(c) Use your graph to solve the equation $x(20 - x^2) = 20$.

Answer $x =$ .....or ..... [2]

(d) **(i)** On the same grid, draw the line $y = -x + 2$ for $-2 \leq x \leq 6$. [1]

(ii) Write down the x-coordinates of the points where this line intersects the curve.

Answer $x =$ .....or ..... [1]

(iii) These values of x are solutions of the equation $x^3 + Ax^2 + Bx + 20 = 0$.
Find the values of A and B.

Answer $A =$ .....

$B =$ ..... [2]

8 The table summarises the marks of 50 students in the last Mathematics test.

Marks (x)	**Frequency**
$44 < x \leq 48$	3
$48 < x \leq 52$	13
$52 < x \leq 56$	p
$56 < x \leq 60$	10
$60 < x \leq 64$	q
$64 < x \leq 68$	6

(a) Given that the estimated mean mark is 57.28, show that the values of p and q are 2 and 16 respectively.

Answer

[3]

(b) Find an estimate of the standard deviation.

Answer ..... [2]

(c) The same students also took a Physics test.
Estimates for the mean mark and standard deviation are 62.5 and 5.1 respectively.
Make two comparisons between the marks for the Mathematics test and Physics test.

Answer .....

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..... [2]

(d) Describe how the cumulative frequency curve for the Mathematics test may differ from the curve for the Physics test.

Answer .....

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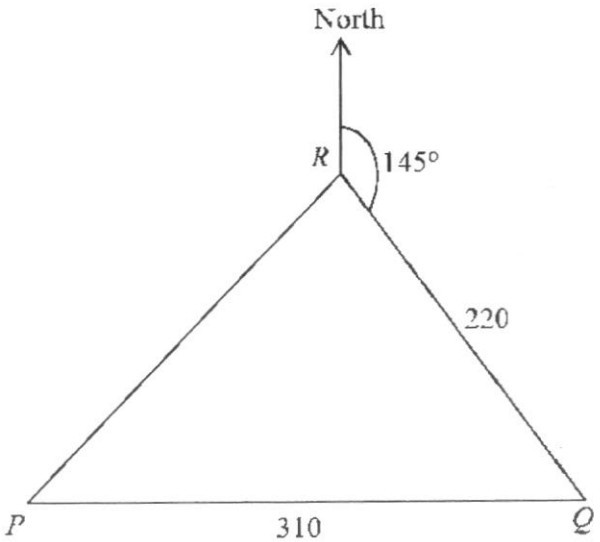
.....[1]

9 P, Q and R are three points on the ground.
P is due west of Q. $PQ = 310$ m and $QR = 220$ m.
The bearing of Q from R is 145°.



(a) Find the bearing of R from Q.

Answer .....° [1]

(b) Find angle PQR.

Answer .....° [1]

(c) Calculate the distance PR.

Answer ..... m [2]

(d) Find the area of triangle PQR.

Answer ..... m^2 [2]

(e) A boy stands at the top of a vertical tower of height 100 m at R, and looks at a bicycle, B, travelling along PQ.

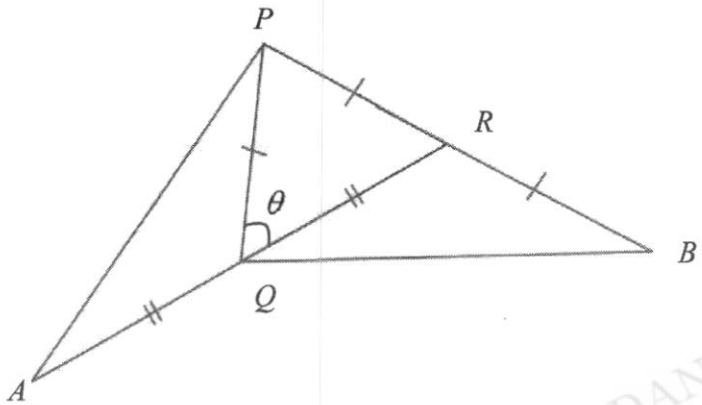
(i) Calculate the shortest distance of B from R during this journey.

Answer ..... m [2]

(ii) Find the greatest angle of depression of B from the top of the tower.

Answer .....° [2]

10 In the diagram, PQR is a triangle in which $PQ = PR$. PR is produced to B so that $PR = RB$. RQ is produced to A so that $RQ = QA$. Angle $PQR = \theta$.

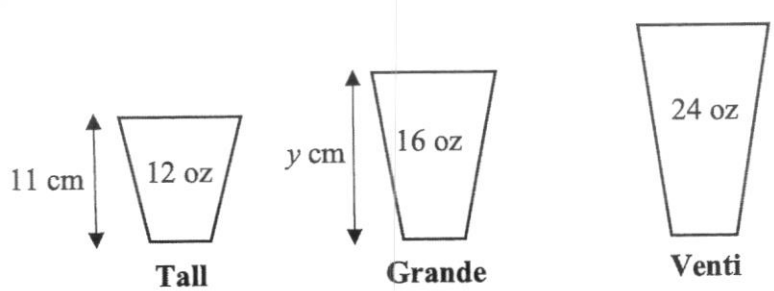


(a) Prove that triangle PQA is congruent to triangle BRQ.

Answer

[3]

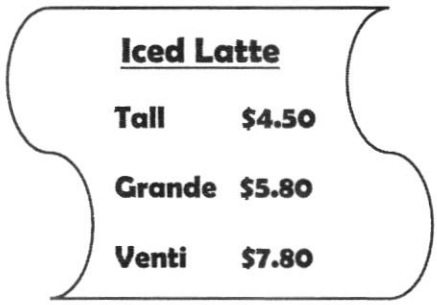
(b) At Starbright café, coffee is sold in three cup sizes: Tall, Grande and Venti. The volumes of coffee in Tall, Grande and Venti sizes are 12 oz, 16 oz and 24 oz respectively. You may assume that they are filled to the brim.



(i) Given that 1 oz = 29.57 m*l*, calculate the volume of Grande in m*l*.

Answer ..... m*l* [1]

(ii) The prices of iced latte for the three cup sizes are as follows.



Which size has the best value for money? Show your calculations clearly.

Answer ..... [2]

Suppose the sizes of Tall, Grande and Venti are geometrically similar.
The heights of Tall and Grande are 11 cm and y cm respectively.

(iii) Calculate the height, y, of Grande.

Answer $y =$ ..... [2]

(iv) Calculate $\frac{\text{base area of Venti}}{\text{base area of Tall}}$, simplifying your answer in the form 2^n, where n is a rational number.

Answer ..... [3]

11 Kenny is considering buying a new car – Hando Civil 1.6 i-VTEC(A).

Fuel Consumption	14.9 km/litre
Engine Capacity (EC)	1597 cc
Open Market Value (OMV)	$19980
Certificate of Entitlement (COE) for month of November 2020	$35990

The purchase price of the car consists of the OMV, COE and miscellaneous costs is $105970 (as of November 2020).

Some additional information regarding purchase of a new car:

Information on Car Loan	
Open Market Value (OMV)	**Maximum Amount of Loan**
Up to $20000	70% of the purchase price
More than $20000	60% of the purchase price

Engine Capacity (EC) in cc	**Annual Road Tax Formulae in $**
Less than 600	400×0.782
600 to 1000	$\left[400 + 0.25 \times (\text{EC} - 600)\right] \times 0.782$
1000 to 1600	$\left[500 + 0.75 \times (\text{EC} - 1000)\right] \times 0.782$
1600 to 3000	$\left[950 + 1.5 \times (\text{EC} - 1600)\right] \times 0.782$
More than 3000	$\left[3050 + 2.0 \times (\text{EC} - 3000)\right] \times 0.782$

Calculate

(a) the amount of miscellaneous costs that is included in the purchase price.

Answer $..... [1]

(b) the amount for the down payment Kenny will have to pay if he wants to take the maximum amount of loan.

Answer $..... [2]

(c) the amount of road tax Kenny will have to pay a year.

Answer $..... [1]

Kenny finds out that DSS Bank offers a car loan with an effective fixed interest rate of 2.55% per annum up to a loan period of 7 years. He will take up a loan period of 7 years if he wants to buy a car. Additional expenditures include:

Types of Expenditure	Cost
Petrol	\$2.13 per litre
Car Servicing	\$400 per half year
Car Insurance	\$1100 per year
Electronic Road Pricing (ERP)	\$12 per week
Parking Fees	\$220 per month

Kenny estimates that he will drive a distance of 650 km per week.
After 3 years, the projected selling price of the car, including COE, is \$74179.

A car rental company offers a lease package of the same type of car at a daily cost of \$55, including road tax, car servicing and car insurance. The minimum contract period is 6 months.

(d) If Kenny intends to use a car for 3 years, should he buy or rent a car?
Justify your decision with calculations.
State an assumption that you make in your calculations.

Answer

[6]

- **End of Paper** -

Math MYE P1 Marking Scheme

1a.	8499	[B1]
1b.	$\frac{1}{3}, \frac{2}{7} 0.25, \left(\frac{1}{3}\right)^2$	[B1]
2a.	$8.5 - 5.0 = 3.5$	[B1]
2b.	Q1 = 5.2 and Q3 = 7.3 IQR = 7.3 – 5.2 = 2.1	[M1] Correct Q1 and Q2 [A1]
3a.	$\frac{1}{2} \times (25 + 10) \times 35 = 612.5$	[B1]
3b.	$\frac{50}{30} = 1\frac{2}{3}$ or 1.67 (3sf)	[B1]
4.	$\frac{360}{p} - \frac{360}{2p} = 20$ $720 - 360 = 40p$ $p = 9$ OR $\frac{(2p-2) \times 180}{2p} - \frac{(p-2) \times 180}{p} = 20$ $360p - 360 - 360p + 720 = 40p$ $40p = 360$ $p = 9$	[M1] Use ext angles [A1] [M1] Use int angles [A1]

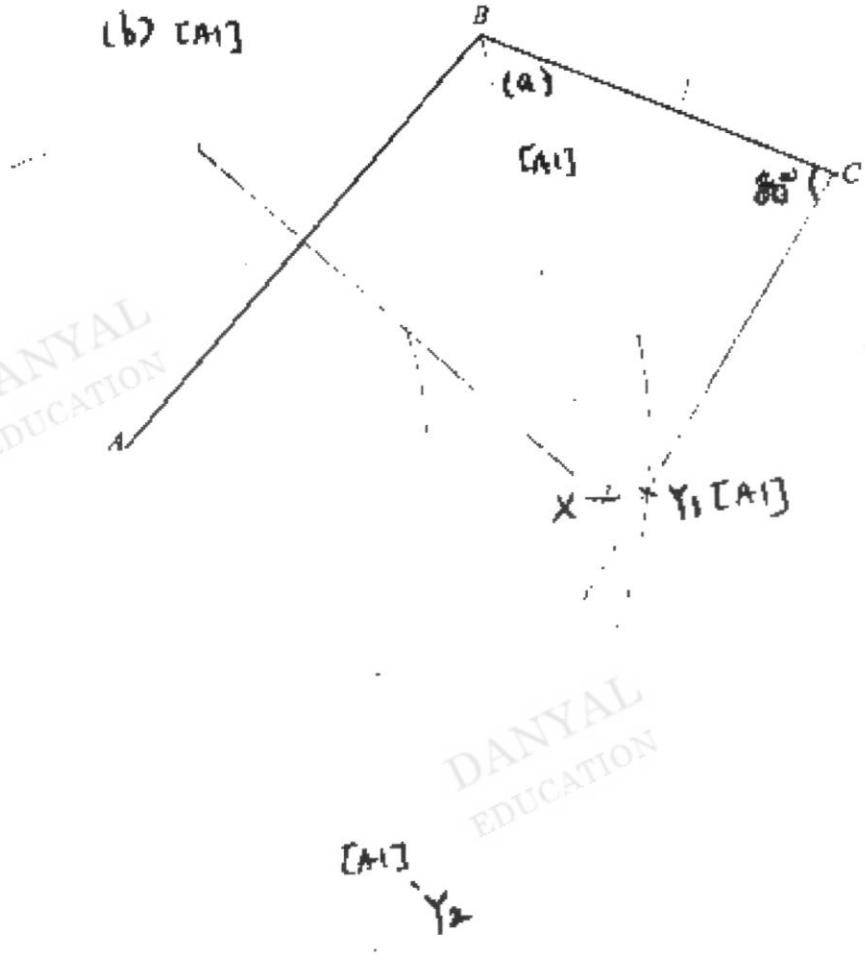
5a.	$= \frac{9}{(x-4)(x-4)} - \frac{2}{x-4}$ $= \frac{9-2(x-4)}{(x-4)(x-4)}$ $= \frac{9-2x+8}{(x-4)(x-4)}$ $= \frac{17-2x}{(x-4)(x-4)}$	[M1] $4-x=-(x-4)$ and/or $(x-4)^2=(x-4)(x-4)$ [A1]
5b.	$\frac{9}{(x-4)(x-4)} + \frac{2}{x-4} = 0$ $\frac{17-2x}{(x-4)(x-4)} = 0$ $17-2x=0$ $x = 8\frac{1}{2}$ or 8.5	[A1]
6a.	Interest $= 100000(1+\frac{1.6}{2}\%)^3 - 100000$ = 102419.25 - 100000 = \$2419.25	[M1] Correct formula for total amount [A1]
6b.	Total $= 200 + 24 \times 275$ = 6800 Interest amount = 6800-5000 = 1800 Principal = 5000-200 = 4800 $\frac{4800 \times r \times 2}{100} = 1800$ $r = 18.75\%$	[M1] Find interest amount [M1] Find principal [A1]

7a.	$-5 < 2x - 4 \quad 2x - 4 \leq 8$ $-1 < 2x \qquad 2x \leq 12$ $-\frac{1}{2} < x \qquad x \leq 6$ $-\frac{1}{2} < x \leq 6$	[M1] [A1]
7b.	5	[B1]
8a.	$= 4(2^3)$ $= 2^2(2^3)$ $= 2^5$	[B1]
8b.	$= \left(\frac{y^8}{x^6}\right)^{\frac{3}{2}}$ or $\frac{x^{-9}}{y^{-12}}$ $= \frac{y^{12}}{x^9}$	[M1] [A1]
9a.	$\angle ACB = 180 - a - b$	[B1] ($\angle$ sum of Δ)
9b.	$\angle DCE = \angle ACB = 180 - a - b$ (vert opp $\angle$s) $\angle CED = 180 - 30 - (180 - a - b)$ ($\angle$sum of Δ) $= a + b - 30$	 [B1]
9c.	$\angle FEG = \angle CED = a + b - 30$ (vert opp $\angle$s) $f + g + a + b - 30 = 180$ ($\angle$ sum of Δ) $g = 210 - a - b - f$	 [M1] [A1]

10a.	$\frac{30}{60}x + 1\frac{20}{60}y = 50$ $\frac{x}{2} + \frac{4y}{3} = 50$ $3x + 8y = 300$	[A1]
10b.	$1\frac{10}{60}x + \frac{40}{60}y = 50 + 4\frac{1}{3}$ $\frac{7x}{6} + \frac{2y}{3} = 54\frac{1}{3}$ $7x + 4y = 326$	[A1]
10c.	$3x + 8y = 300$---- (1) $7x + 4y = 326$----(2) (2) x 2: $14x + 8y = 652$----(3) (3) – (1): $11x = 352$ $x = 32$ $y = 25.5$	[M1] Correct Substitution or Elimination [A1] [A1]
10d.	$\frac{50}{25.5} - \frac{50}{32} = \frac{325}{816}$ = 23 min 54 s	[M1] [A1]

11a.	$y = k(2x^3)$ or $y = 2kx^3$ $1 = 2k(\frac{1}{4})^3$ $k = 32$ $y = 64x^3$ $y = 64(\frac{1}{2})^3$ $y = 8$	[M1] [A1]
11b.	$y = 64x^3$ $y_1 = 64 \times (2x)^3$ $= 512x^3$ Percentage change in y $= \frac{512x^3 - 64x^3}{64x^3} \times 100\%$ or $= \frac{2^3 - 1^3}{1^3} \times 100\%$ =700%	[M1] OR [M1] [A1]
12a.	$990 = 2 \times 3^2 \times 5$	[B1]
12b.	$p = 2 \quad q = 5$	[B1]
12c.	$990 = 2 \times 3^2 \times 5 \times 11$ $22 = 2 \times 11$ Smallest 3-digit integer = $2 \times 7 \times 11 = 154$	[B1]

13 Three points A, B and C are shown below.



(a) Construct the bisector of angle ABC. [1]

(b) Construct the perpendicular bisector of AB. [1]

(c) These two bisectors meet at X.
Complete the statement below.

Answer The point X is equidistant from the points A and B,
and equidistant from the lines AB and BC. [1]

(d) The point Y is such that angle $BCY = 80°$ and $AY = 8$ cm.
Mark the position of Y_1 and Y_2. [2]

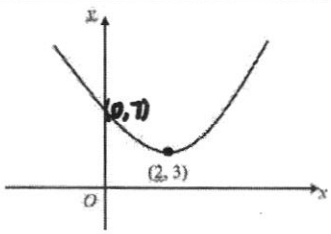
14a.	$x + y = 1$ $5y + y = 1$ $y = \frac{1}{6}$ $x = \frac{5}{6}$	[M1] [A1]
14b.	P(Black from Box B, White from Box C) + P(White from Box B, White from Box C) $= \frac{1}{2} \times \frac{4}{10} + \frac{1}{2} \times \frac{5}{10}$ $= \frac{9}{20}$ or 0.45	 [M1] at least one of the 2 events [A1]
15a.	$\sqrt{400}$cm : $\sqrt{90000}$m = 20 cm : 300 m = 1 cm : 15 m = 1 : 1500	[M1] square root [A1]
15b.	Area of forest = 9 hectares Tractor *A*: 9 days x \$1000 = \$9000 Tractor *B*: 6 days x \$1400 = \$8400 Tractor *C*: 4 days x \$2000 = \$8000 Tractor *C* because the rental cost is the cheapest and it takes only 4 days to complete the work.	 [M1] 1 out of 3 correct [M2] All 3 correct [A1] either or both reasons are acceptable.
16a.	Exterior angle of octagon $= \frac{360}{8} = 45°$ Angle $CAG = 180 - 45 - 45 = 90°$ (proven)	[M1] base angle if isos triangle [A1]
16b.	Sum of angles *A* to *H* $= 8 \times 90 = 720°$	[B1]

17a.	$\{6,12\}$	[B1]
17b.	$\{5,7,9,11,13,15,17,19\}$	[B1]
17c.	Yes since every element in A is in B and set A is not equal to set B.	[B1]
17d.	$A \subset B$	[B1]
17e.	$A \cup B'$	[B1]
18.	Area of square $= 4r^2$ Area of circle $= \pi r^2$ Area of shaded region $= \frac{(4r^2 - \pi r^2)}{4} \times 8$ $= 2r^2(4 - \pi)$ Fraction of the square is shaded $= \frac{2r^2(4-\pi)}{4r^2}$ $= \frac{4-\pi}{2}$	[M1] Area of square and circle [M1] [A1]
19a.	$\angle OPQ = 90 - 38 = 52$ (tangent $\perp$radius) $\angle POQ = 180 - 2(52) = 76$ ($\angle$sum of Δ)	[B1]
19b.	$\angle PQR = 180 - 104 = 76$ ($\angle$s in opp segment) $\angle OQR = 76 - 52 = 24$	[B1]
19c.	$\angle OPU = 90$ (tangent $\perp$radius) OU = diameter of circle (converse of $\angle$in semicircle) Center of the circle is at the midpoint/center of OU.	[M1] diameter [A1] midpoint or center

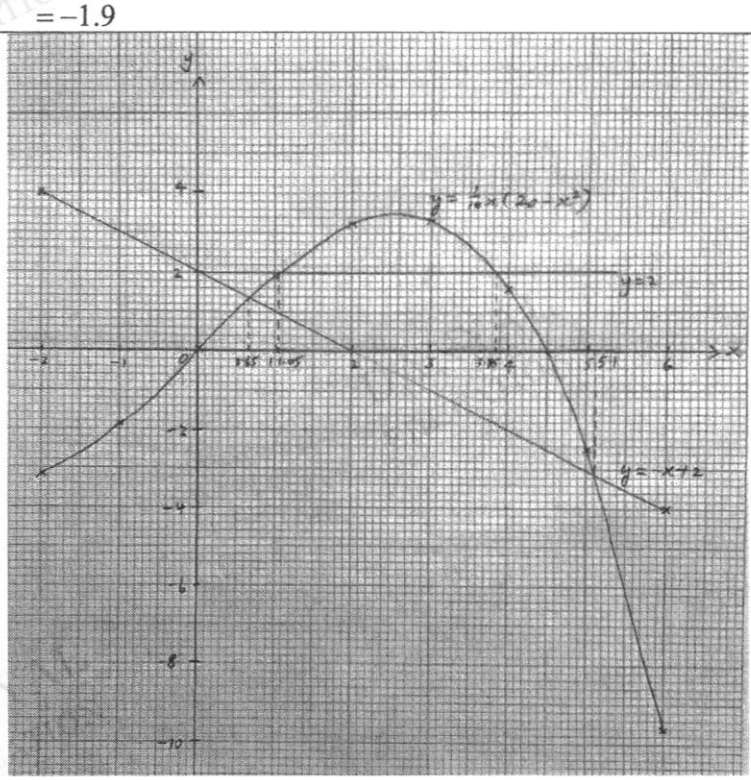
20a.	At A, $x = 0$, $y = 12$ $\quad A(0,12)$ At B, $y = 0$, $x = 4$ $\quad B(4,0)$	[B1] [B1]
20b.	$AB = \sqrt{(0-4)^2 + (12-0)^2}$ $= \sqrt{160}$ $= 12.64911$ Since it lies on the y-axis, directly below A, $D(0,12\text{-}12.64911)$=$D(0,\text{-}0.649)$	[M1] [A1]
20c.	$\cos \angle ABC = -\cos \angle ABO$ $= -\frac{4}{\sqrt{160}}$ $= -0.316$ (3sf)	 [A1]
21.	Volume of hemisphere $= \frac{1}{2} \times \frac{4}{3} \times \pi \times (3r)^3$ $= 18\pi r^3$ Volume of cylinder $= \pi \times r^2 \times 2r$ $= 2\pi r^3$ Volume of container $= 20\pi r^3$ $20\pi r^3 \rightarrow 2min$ $18\pi r^3 \rightarrow \frac{2}{20\pi r^3} \times 18\pi r^3$ $= 1.8$ min $= 1$ min 48 s	 [M1] At least one correct volume [A1]
22a.	Major arc length = 48-8-8 = 32 m $8\theta = 32$ $\theta = \frac{32}{8} = 4$ rad	 [B1]
22b.	$= \frac{1}{2} \times 8^2 \times 4 + \frac{1}{2} \times 8^2 \times \sin(2\pi - 4)$ $= 152.21$ ≈ 152 (3 sf)	[M1] At least one correct area [A1]

Peirce Secondary School
2021 4E5A Math MYE Paper 2 (Marking Scheme)

Qn	Solution	Marking Scheme
1ai	$p^2-(p+q)(p-q)$ $=p^2-(p^2-q^2)$ $=p^2-p^2+q^2$ $=q^2$	 M1 A1
aii	$543896702^2-543896707\times543896697$ $=543896702^2-(543896702+5)(543896702-5)$ $=5^2$ $=25$	M1 A1
b	$12ax-2by+8ay-3bx$ $=4a(3x+2y)-b(2y+3x)$ $=(4a-b)(3x+2y)$	 M1 – Factorisation (condone sign errors) A1
ci	$p=\dfrac{4(6)+3(-4)}{5-(-4)}$ $=1\dfrac{1}{3}$	 B1
cii	$p=\dfrac{4q+3r}{5-r}$ $p(5-r)=4q+3r$ $5p-pr=4q+3r$ $3r+pr=5p-4q$ $r(3+p)=5p-4q$ $r=\dfrac{5p-4q}{3+p}$	 M1 A1
2ai	$u_8=64-7^2=15$	B1
aii	$u_n=n^2-(n-1)^2$ $=n^2-(n^2-2n+1)$ $=2n-1$	M1 A1
aiii	$u_{50}+u_{51}$ $=2(50)-1+[2(51)-1]$ $=100-1+101$ $=200$	M1 A1
aiv	$2n-1=387$ $2n=388$ $n=194$ Since n is a positive integer, 387 is a term in the sequence.	 B1

bi	x^2-4x+7 $=\left(x-\frac{4}{2}\right)^2-2^2+7$ $=(x-2)^2+3$	M1 A1
2bii		B1 Correct Shape B1 Correct turning pt & y-intercept
3ai	$3000 \times 1.35 = \$4050$	B1
aii	$\$10000 = \text{USD}(10000 \div 1.35) = \text{USD}7\ 407.41$ $\text{USD}7407.41 = (7407.41 \div 0.0096)\text{yen}$ $= 771605\text{yen (nearest yen)}$	M1 A1
b	$\frac{4288}{107} \times 100\% = \$4007.4766$ $4007.4766 - 40.50 = \$3966.98$ (nearest cent)	M1 A1
ci	$k = \frac{2.27 \times 10^8}{10^6} = 227$	B1
cii	Time taken $= \frac{2.27 \times 10^8 \times 1000}{3 \times 10^8}$ $= 756.667\text{s}$ $= 12.6\text{ min (3sf)}$	M1 A1
4a	Total income in 2019 $= 1.37 \times 10^6 \times 9763$ $= 1.337531 \times 10^{10}$ $= 1.3375 \times 10^{10}$ (5sf)	M1 A1
b	% increase $= \frac{9763 - 7214}{7214} \times 100\%$ $= 35.3\%$	 B1
ci	$x = 2800$ $y = 7000$	B1 B1
cii	Range = 480000-1000 =\$479000	 B1
ciii	Since \$10000 is the 75th percentile, 25% of households earned more than \$10000. Probability $= \frac{1}{4}$	 B1
d	Median is a better gauge because it is not affected by extreme values, e.g. extremely high income will inflate the mean income disproportionately.	B1

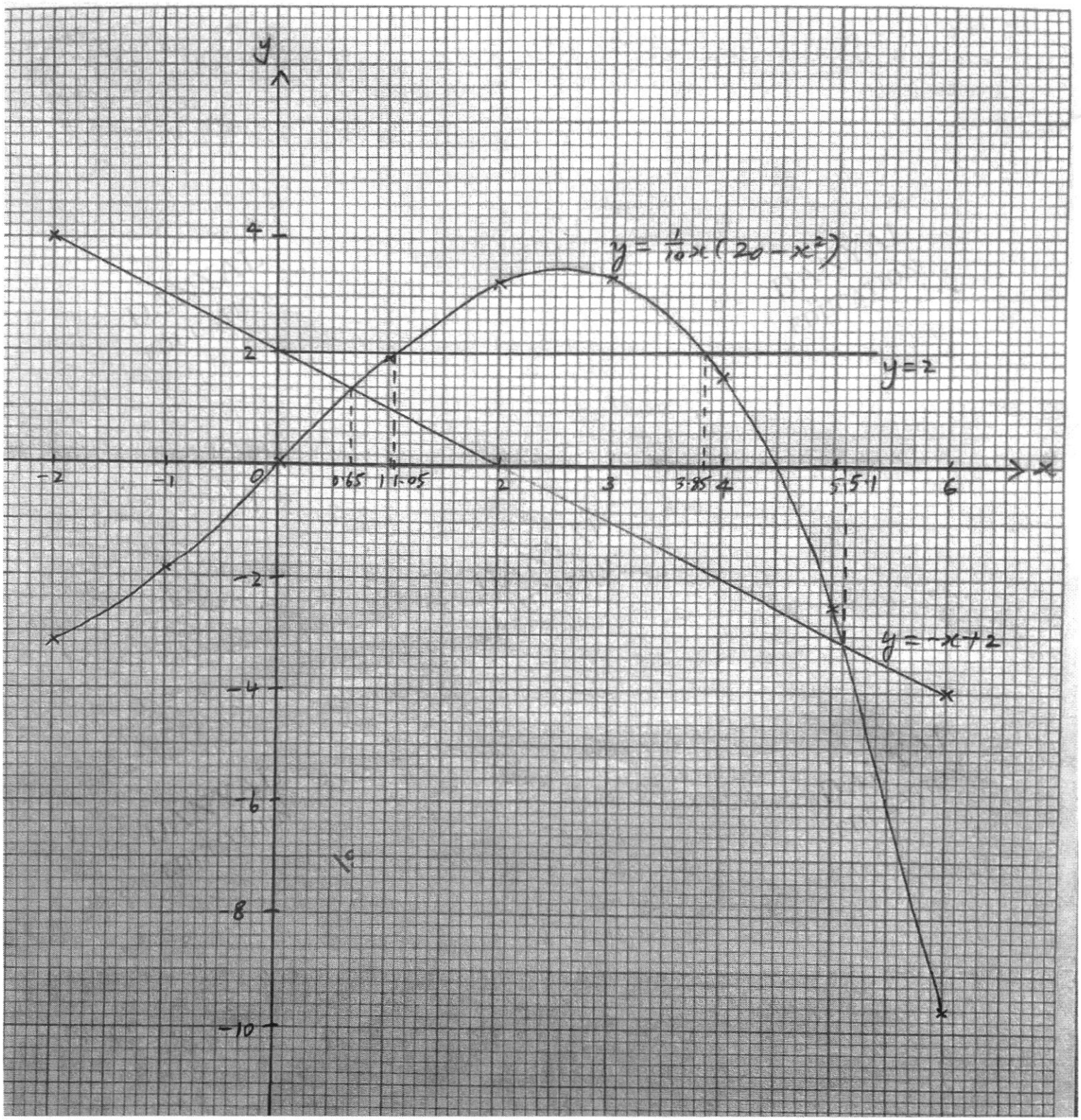
5a	$\begin{pmatrix}10\\40\\25\end{pmatrix}$	B1
b	$\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}10\\40\\25\end{pmatrix}=(75)$	B1
5c	$S=\begin{pmatrix}37 & a & 25\\ b & 85 & 31\end{pmatrix}\begin{pmatrix}10\\40\\25\end{pmatrix}$ $=\begin{pmatrix}370+40a+625\\10b+3400+775\end{pmatrix}$ $=\begin{pmatrix}995+40a\\4175+10b\end{pmatrix}$	B1
d	$\begin{pmatrix}995+40a\\4175+10b\end{pmatrix}=\begin{pmatrix}3395\\4655\end{pmatrix}$ $995+40a=3395$ $a=60$ $4175+10b=4655$ $b=48$	M1 Form linear equation A1 Both answers correct
e	Elements in **T** represent the amount of money collected from ticket sales on each day - \$3395 and \$4655 was collected on Saturday and Sunday respectively.	B1
f	$RT=(1.3 \quad 2)\begin{pmatrix}3395\\4655\end{pmatrix}$ $=(13723.5)$ Total amount collected on both days = \$13723.50	B1
6a	$\$\frac{750}{x}$	B1
bi	$\$\left(\frac{750}{x}-5\right)$	B1
bii	No. of dresses sold at a loss = $x-12$ Total amount received from sale of all dresses $=\$\left[630+(x-12)\left(\frac{750}{x}-5\right)\right]$ $=\$\left(630+750-5x-\frac{9000}{x}+60\right)$ $=\$\left(1440-\frac{9000}{x}-5x\right)$ (shown)	M1 A1

6c	$1440-\frac{9000}{x}-5x=750+240$ $1440x-9000-5x^2=990x$ $5x^2-450x+9000=0$ $x^2-90x+1800=0$ (shown)	M1 A1
d	$x^2-90x+1800=0$ $(x-60)(x-30)=0$ $x=60$ or $x=30$	M1 A1
e	Cost price $=\frac{750}{60}$ $=\$12.50$ (rej, >15) Cost price $=\frac{750}{30}$ $=\$25$	B1
7a	$p=\frac{1}{10}(-1)\left[20-(-1)^2\right]$ $=-1.9$	B1
b		B1 Correct scales B1 Correct pts plotted B1 Correct smooth curve
c	$x(20-x^2)=20$ $\frac{1}{10}x(20-x^2)=\frac{20}{10}$ $\frac{1}{10}x(20-x^2)=2$ Insert the line $y=2$ From the graph, $x=1.05\ (\pm0.1)$ or $x=3.85\ (\pm0.1)$	M1 A1
di	$x=-2,\ y=4;\quad x=6,\ y=-4$ Draw line $y=-x+2$ (See Graph)	B1

dii	From the graph, $x = 0.65\ (\pm 0.1)$ or $x = 5.1\ (\pm 0.1)$	B1
diii	$\frac{1}{10}x(20 - x^2) = -x + 2$ $20x - x^3 = -10x + 20$ $x^3 - 30x + 20 = 0$ $A = 0, \quad B = -30$	M1 A1
8a	$3 + 13 + p + 10 + q + 6 = 50$ $p + q = 18$ --- (1) $\frac{46 \times 3 + 50 \times 13 + 54p + 58 \times 10 + 62q + 66 \times 6}{50} = 57.28$ $1764 + 54p + 62q = 2864$ $54p + 62q = 1100$ $27p + 31q = 550$ --- (2) $(1) \times 27$: $27p + 27q = 486$ --- (3) $(2) - (3)$: $27p + 31q - 27p - 27q = 550 - 486$ $4q = 64$ $q = 16$ $p = 18 - 16 = 2$	M1 M1 A1
b	$\sum fx^2 = 3 \times 46^2 + 13 \times 50^2 + 2 \times 54^2 + 10 \times 58^2 + 16 \times 62^2 + 6 \times 66^2$ $= 165960$ $\text{SD} = \sqrt{\frac{165960}{50} - 57.28^2}$ $= 6.18$ (3sf)	 M1 A1
c	Since mean marks for Physics test > mean marks for Math test, on average, the students performed better for the Physics test. Since the standard deviation for Physics test < standard deviation for Math test, Physics test marks are less spread out than Math test marks.	B1 B1
d	Cumulative frequency curve for Physics test is steeper than the curve for Math test.	B1
9a	$\angle RQN_1 = 180 - 145$ $= 35°$ (interior $\angle$s, parallel lines) Bearing of R from $Q = 360 - 35$ $= 325°$ ($\angle$s at a point) 	 B1
b	$\angle PQR = 90 - 35$ $= 55°$	 B1
c	$PR = \sqrt{220^2 + 310^2 - 2(220)(310)\cos 55°}$ $PR = 257.42$ $PR = 257$ m (3sf)	M1 A1

9d	Area of triangle $PQR = \frac{1}{2}(310)(220)\sin 55°$ $= 27933.08$ $= 27900\ \text{m}^2$ (3sf)	M1 A1
ei	Let the shortest distance be d Area of triangle $= \frac{1}{2}\times 310\times d$ $27933.08 = \frac{1}{2}\times 310\times d$ $d = 180.21$ $= 180$ m (3sf)	 M1 A1
eii	Let the angle of depression be θ $\theta = \tan^{-1}\left(\frac{100}{180.21}\right)$ $\theta = 29.026°$ $= 29.0°$ (1dp)	 M1 A1
10a	$PQ = BR$ (given) $AQ = QR$ (given) $\angle PQA = 180° - \theta$ $\angle PRQ = \theta$ (base angles of isosceles triangle) $\angle BRQ = 180° - \theta$ (angles on a straight line) $\therefore \angle PQA = \angle BRQ$ $\therefore \Delta PQA \equiv \Delta BRQ$ (SAS)	B1 B1 B1
bi	Volume of Grande $= 16\times 29.57$ $= 473.12$ m*l*	 B1
bii	Cost per unit oz for 'Tall' $= 4.50 \div 12$ $= \$0.375 / \text{oz}$ Cost per unit oz for 'Grande' $= 5.80 \div 16$ $= \$0.3625 / \text{oz}$ Cost per unit oz for 'Venti' $= 7.80 \div 24$ $= \$0.325 / \text{oz}$ By comparison, Venti has the best value for money.	M1 Find unit price A1
biii	$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$ $\frac{16}{12} = \left(\frac{y}{11}\right)^3$ $y^3 = \frac{16\times 1331}{12}$ $y = \sqrt[3]{\frac{5324}{3}}$ $= 12.1$ (3sf)	 M1 A1

10biv	$\frac{24}{12} = \left(\frac{\text{Height of Venti}}{\text{Height of Tall}}\right)^3$ $\frac{\text{Height of Venti}}{\text{Height of Tall}} = 2^{\frac{1}{3}}$ $\frac{\text{Base Area of Venti}}{\text{Base Area of Tall}} = \left(\frac{\text{Height of Venti}}{\text{Height of Tall}}\right)^2$ $= \left(2^{\frac{1}{3}}\right)^2$ $= 2^{\frac{2}{3}}$	M1 M1 A1
11a	105970 – 19980 – 35990 = \$50000	B1
b	(100 – 70)% × 105970 = \$31791	M1 A1
c	[500 + 0.75 × (1597 – 1000)]× 0.782 = \$741.1405 = \$741.14 (2dp)	B1
d	<u>To buy a car</u> Amount to loan from bank = 105970 – 31791 = \$74179 Total interest (over 7 years) = 74179×2.55% × 7 = 13240.9515 Total loan amount = 74179 + 13240.9515 = 87419.9515 Total cost of owning car for 3 years = 31791+ 87419.9515 + 741.1405×3 + 400×2×3 + 1100×3 – 74179 = \$52955.373 *[Note: Expenditure for petrol, ERP and parking fees are not included in the calculation for comparison, as they are the same whether to buy or rent a car.]* Assume there are 365 days in a year, Cost per day = $\frac{52955.373}{365\times3}$ = \$48.36 (2dp) **<u>or</u>** [Cost of owning car per year = $\frac{52955.373}{3}$ = \$17651.79, Cost of renting car per year = 55 ×365 = \$20075] Since 48.36 < 55 (**<u>or</u>** 17651.79 < 20075), Kenny should buy a car.	 M1 Down payment (b) + total loan + road tax + servicing + insurance – 74179 M1 – any 2 correct M2 – all correct B1 M1 (Compare cost per day or year or total cost) } A1


y
x
4
2
-2
-4
-6
-8
-10
-2
-1
0
0·65
1
1·05
2
3
3·85
4
5
5·1
6
$y = \frac{1}{10}x(20 - x^2)$
$y = 2$
$y = -x + 2$