Class | Register Number

Candidate Name ______________________________

PEIRCE SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2021
SECONDARY 3 EXPRESS

ADDITIONAL MATHEMATICS

4049/01
28 Sep 2021
2 hour 30 minutes

Additional Materials:
Plain Paper (for rough work)

INSTRUCTIONS TO CANDIDATES
Candidates answer on the Question Paper.

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

PARENT'S SIGNATURE

For Examiner's Use	
Total	

~~This paper consists of **23** printed pages and **1** blank page.~~

Setter: Mr Goh

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ⊗ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1 Solve the inequality $(1-x)^2 \geq 17-2x$. [2]

2 Factorise $81x^3 - 24z^6$ completely. [2]

3 **(i)** Show that $3x-1$ is a factor of $g(x) = 6x^3 - 5x^2 + 10x - 3$. [1]

(ii) Hence, show that $g(x)$ has only 1 real root. Show all workings clearly. [3]

4 Given that $\dfrac{3^{2x+4} \times 5^{x-3}}{25^x} = 27^x$, without using a calculator, find the value of 15^x. [3]

5 **(i)** Solve the equation $e^{2x} = 10$. [1]

(ii) Solve the equation $25^x - 6(5^x) + 5 = 0$. [3]

6 Express $\dfrac{7x^2-9x+29}{(x-3)(x^2+4)}$ as a sum of partial fractions. [5]

7 Find the range of values of n such that $y = x^2 + x + 4n$ is above the line $y = 11 - nx$.

[4]

8 (i) Given that $\tan A = -\frac{1}{2}$ and $90^\circ < A < 180^\circ$, state the exact value of $\cos A$ and $\sin A$ without the use of calculator. [2]

(ii) Find the exact value of $\cos(A - \frac{\pi}{3})$ and show that $\cos(A - \frac{\pi}{3}) = \frac{\sqrt{3}-2}{2\sqrt{5}}$. [2]

(iii) Hence, find the exact value of $\sec(A - \frac{\pi}{3})$ and leave it in the form $a\sqrt{15} + b\sqrt{5}$, where a and b are integers. [2]

9 The vertical height, y m, of a rider above the ground in a section of a roller coaster ride is given by $y=\frac{3}{4}x^2-4x+10$, where x m is the rider's distance from the start of the ride.

(i) Express $y=\frac{3}{4}x^2-4x+10$ in the form $y=a(x-h)^2+k$, where a, h and k are constants.

[2]

(ii) State the minimum height of the rider above the ground. [1]

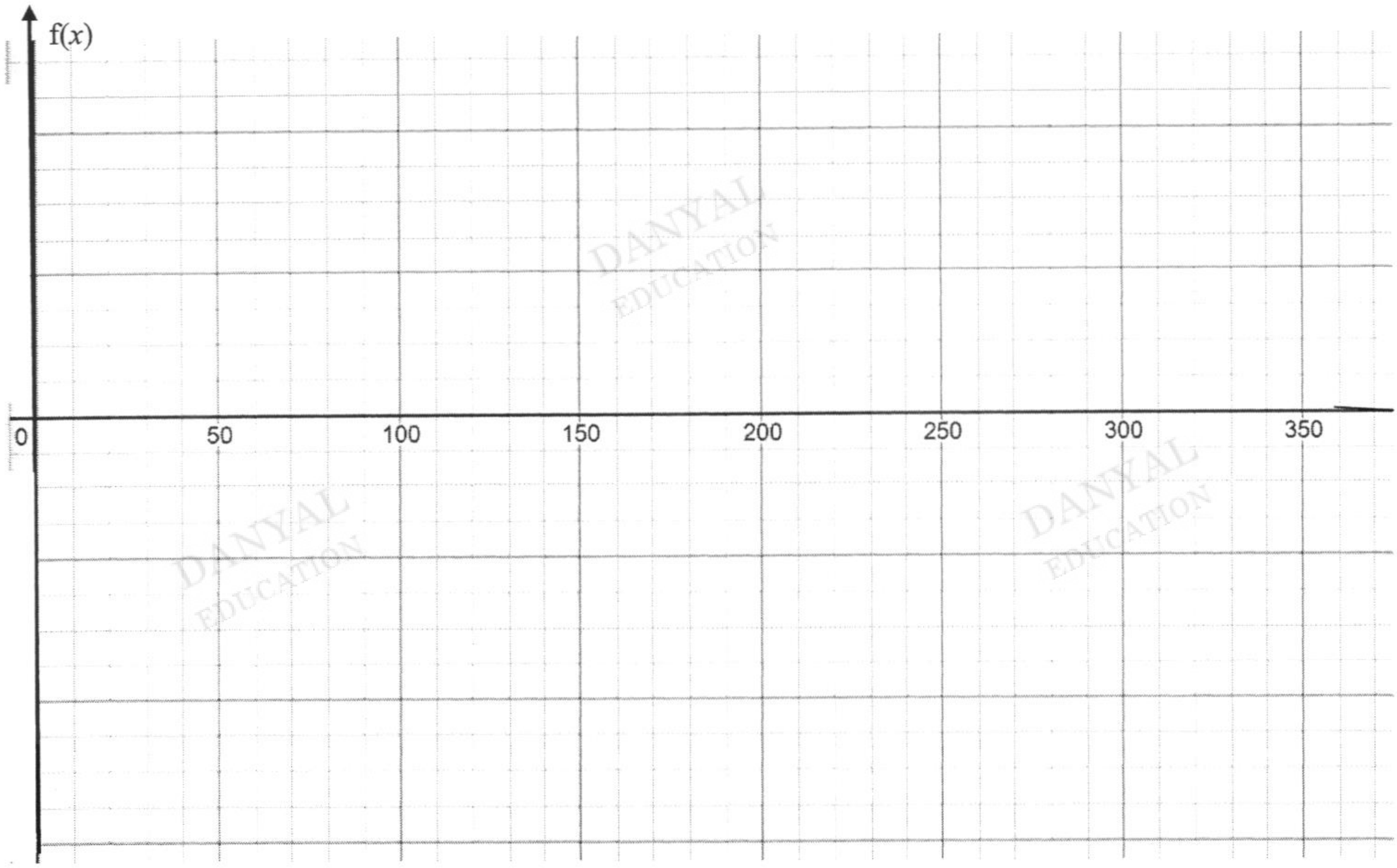
(iii) If the rider is 70 m above the ground after the ride starts, find the rider's horizontal distance from the beginning of the ride. [2]

10 **(a)** The equation of the curve $f(x) = q\sin 3x + p$ where q is a positive integer.

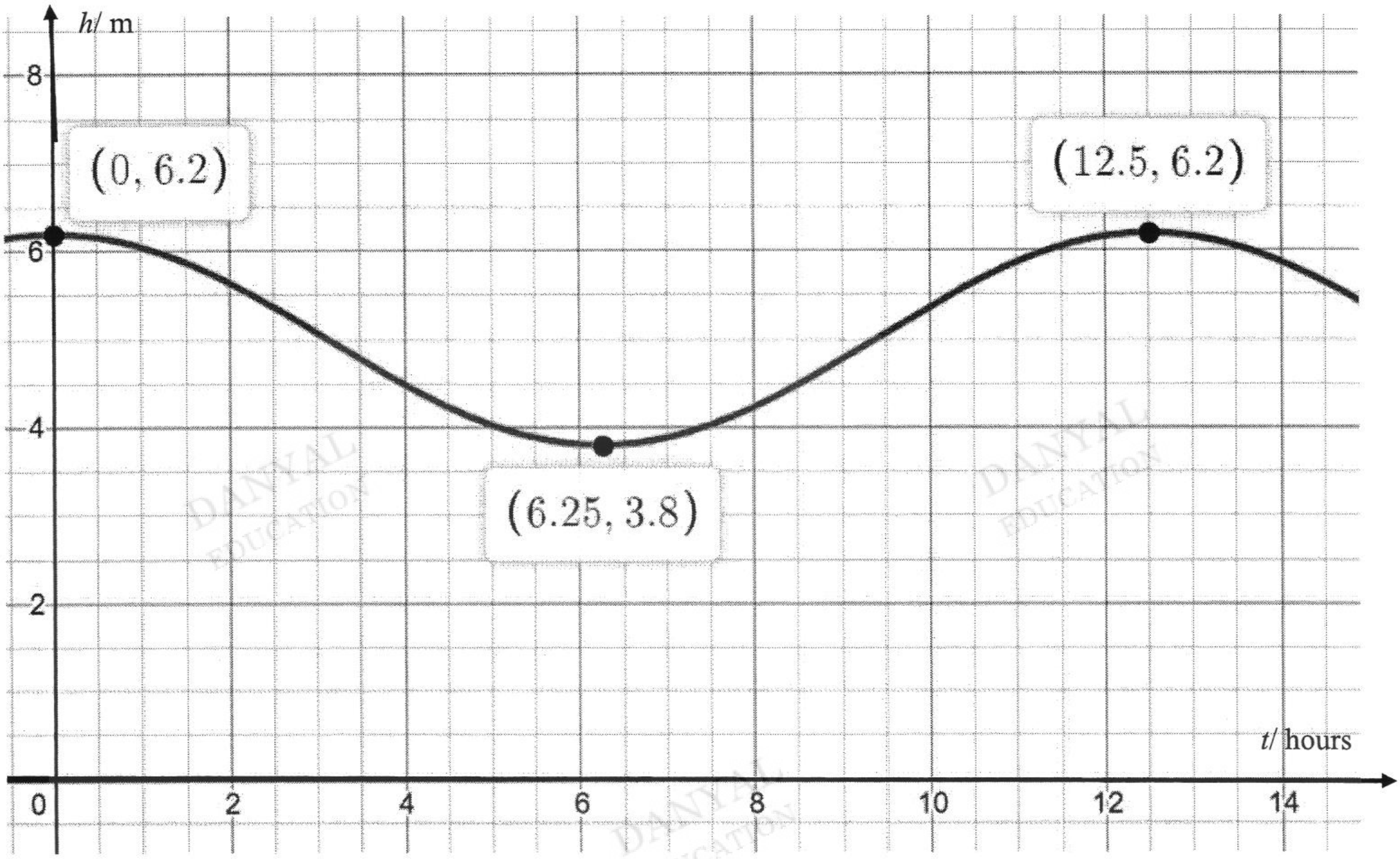
(i) State the period of f(x). [1]

(ii) It is given that that the minimum and maximum values of f(x) are -6 and 2 respectively. Find the value of p and the value of q. [2]

(iii) From the values of p and q found in **(ii)**, sketch the graph of $f(x) = q\sin 3x + p$ on the axis below for $0^\circ \leq x \leq 360^\circ$. [2]



(b) The depth, h m, of the water in a port t hours after noon is given by $h = 1.2\cos(kt) + 5$, where k is a constant.



(i) By showing all workings clearly, show that $k = \dfrac{4\pi}{25}$. [1]

(ii) The draft of a boat is defined as the vertical distance between the waterline and the bottom of the boat and it determines the minimum depth of water a boat can safely navigate.
A fishing boat with a draft of 4.2 m arrived at the port at 12 noon. Explain if the boat can leave the port safely at 7 pm sharp. [2]

11 Solve, for x and y in the simultaneous equations. [6]

$$\left(\frac{1}{9}\right)\left(9^{y}\right)=81^{x}$$
$$\left(2^{2x^{2}+xy}\right)=16\left(2^{-y^{2}}\right).$$

12 **(i)** Prove the following identity .

$$\frac{\cos x}{1+\cos 2x}+\frac{\sin x}{1-\cos 2x}=\frac{1}{2}(\operatorname{cosec} x+\sec x)$$

[3]

(ii) Hence, find all the angles A in the equation $\frac{\cos 2A}{1+\cos 4A}+\frac{\sin 2A}{1-\cos 4A}=0$ for $0<A<2\pi$.

Leave your answers in terms of π. [3]

13 (i) Without using a calculator, find the value of $(\log_2 8)^2 \times \frac{1}{\log_5 3} \times \log_{25} 3$. [3]

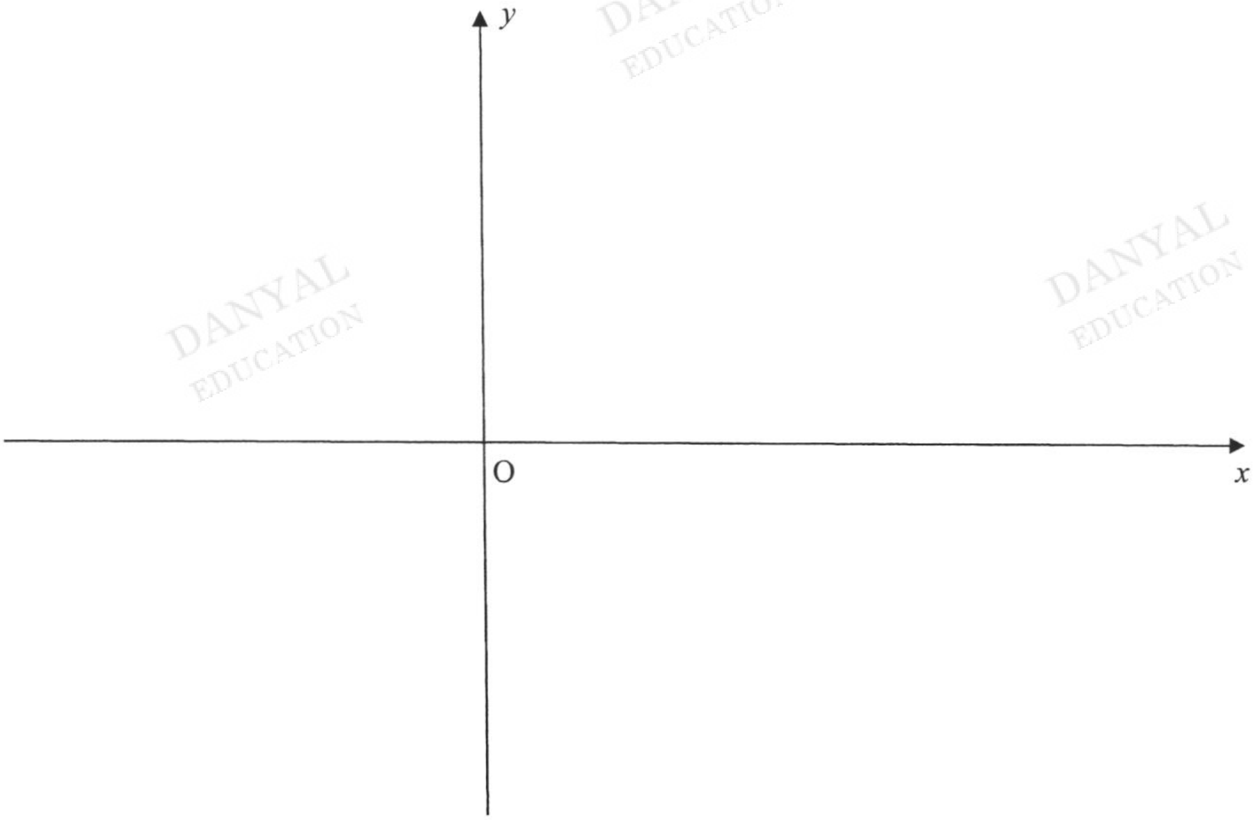
(ii) Solve the equation $\log_3 x - \log_9 (x+6) = 0$ and show that the equation has only 1 real solution. [4]

14 **(a)** Some bacteria are grown in a gel medium in a petri dish.
The number, N, of bacteria in the culture, after t hours may be modelled by
$N = 120(1.05^{0.847t})$.

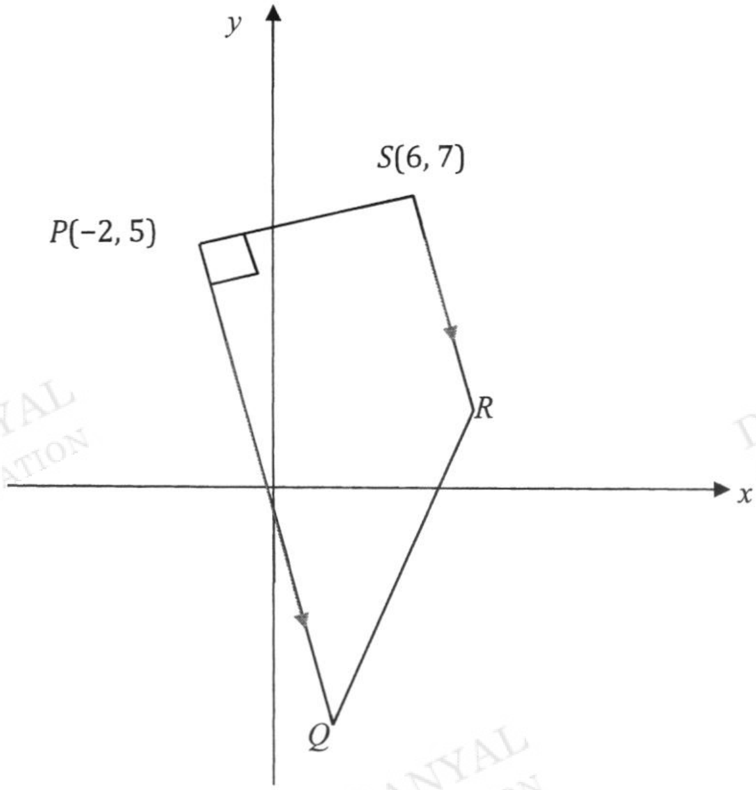
(i) Find the initial number of bacteria in the petri dish. [1]

(ii) Find the number of bacteria in the petri dish after 8 hours. [2]

(b) Sketch the graph of $y = (\frac{1}{5})^x$. Show all intercept(s) clearly. [1]



15 **Solutions to this question by accurate drawing will not be accepted.**



The diagram above (***not drawn to scale***) shows a trapezium *PQRS*.

PQ is parallel to *SR*, *PS* is perpendicular to *PQ*,

P is $(-2, 5)$ and *S* is $(6,7)$. The equation of *QR* is $3y = 4x - 29$.

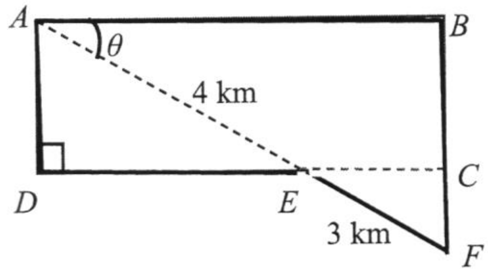
(i) Find the equation of *PQ*. [2]

(ii) Hence, show that the coordinates of *Q* is $(1.25, -8)$. [2]

(iii) Find the coordinates of *R*. [3]

(iv) Find the area of the trapezium *PQRS*. [2]

16



The diagram shows a park *FBADE* that is formed by a rectangle *CBAD* and a triangle *FCE*. *BCF*, *DEC* and *AEF* are straight lines. The park is often used by joggers.

Tom started from point *F*, ran along the straight path *F-B-A-D-E-F*.

It is given that angle $BAE = \theta$, $AE = 4$ km and $EF = 3$ km.

(i) Show, clearly with all workings, that the distance *L*, covered by Tom , can be expressed as $L = 11\cos\theta + 11\sin\theta + 3$. [3]

(ii) Express *L* in the form $R\cos(\theta - \alpha) + 3$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [2]

(iii) State the maximum value of L and the corresponding value of θ. [2]

(iv) Given that the distance covered by Tom is 15 km,

calculate the corresponding value of θ. [2]

17 By rewriting $3\tan A = (\cot A)(1 - 4\sec A)$ as a quadratic equation in sec A, or otherwise, find A for $0^\circ < A < 360^\circ$. [5]

18 **(i)** The equation of a circle, C_1, is $x^2+y^2-30x+8y+232=0$.

Find the radius and coordinates of centre of C_1. [3]

(ii) For a second circle C_2, it passes through points $A(2,4)$ and $B(-4,-6)$.
The centre of C_2 passes through the line $y=-3x+8$.
Find the equation of the perpendicular bisector of AB. [3]

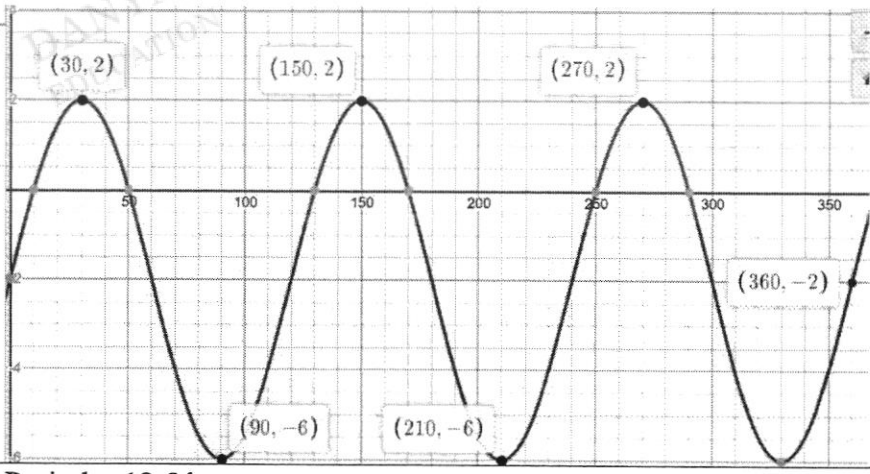
(iii) Hence, find the coordinates of the centre and radius of C_2. [3]

(iv) Caleb states the circles C_1 and C_2 will not touch each other. Do you agree with Caleb ? Explain your answer with clear working. [2]

End of Paper

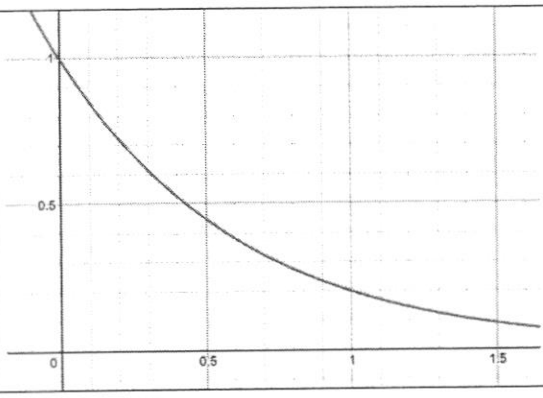
Qn	Working
1	$(1-x)^2 \geq 17-2x$ $1+x^2-2x \geq 17-2x$ $x^2-16 \geq 0$ $(x+4)(x-4) \geq 0$ $x \geq 4$ or $x \leq -4$
2	$81x^3-24z^6 = 3(27x^3-8z^6)$ $=3[(3x)^3-(2z^2)^3]$ $=3[(3x-2z^2)(9x^2+6xz^2+4z^4)]$
3(i)	Substitute $x=\frac{1}{3}$ into g(x) $g(\frac{1}{3})=6(\frac{1}{3})^3-5(\frac{1}{3})^2+10(\frac{1}{3})-3$ $=0$ $3x-1$ is a factor
3(ii)	$2x^2-x+3$ $3x-1 \overline{)6x^3-5x^2+10x-3}$ $6x^2-2x^2$ $-3x^2+10x$ $-3x^2+x$ $9x-3$ $9x-3$
	$b^2-4ac=(-1)^2-4(2)(13)$ $=-23\ (<0)$
	Since 2.. has no real roots, g(x) has only 1 solution.
4	$\frac{3^{2x}\times 3^4\times 5^x}{5^{2x}\times 5^3}=3^{3x}$ $\frac{3^{2x}\times 5^x}{3^{3x}\times 5^{2x}}=\frac{5^3}{3^4}$ $\frac{1}{3^x\times 5^x}=\frac{125}{81}$ $15^x=\frac{81}{125}$
5(a)	$e^{2x}=10$ $\ln e^{2x}=\ln 10$ $2x=\ln 10$ $x=1.15$

(b)	$25^x - 6(5^x) + 5 = 0$ $u = 5^x$ $u^2 - 6u + 5 = 0$ $u = 5$ or $u = 1$ $5^x = 5$ or $5^x = 1$ $x = 1$ or 0
6	$\dfrac{7x^2 - 9x + 29}{(x-3)(x^2+4)} = \dfrac{A}{x-3} + \dfrac{Bx+C}{x^2+4}$ A, B, C are constants $7x^2 - 9x + 29 = A(x^2+4) + (Bx+C)(x-3)$ $x = 3 : 7(3)^2 - 9(3) + 29 = A(3^2+4)$ $65 = 13A$ $A = 5$ $x = 0$ $7(0)^2 - 9(0) + 29 = 5(4) + (C)(0-3)$ $29 = 20 - 3C$ $C = -3$ $x = 1 : 7(1)^2 - 9(1) + 29 = 5(1^2+4) + (B-3)(1-3)$ $27 = 25 - 2B + 6$ $2B = 4$ $B = 2$ $\dfrac{7x^2 - 9x + 29}{(x-3)(x^2+4)} = \dfrac{5}{x-3} + \dfrac{2x-3}{x^2+4}$
7	$x^2 + x + 4n$ $x^2 + x + 4n + n$ $x^2 + (n+1)x + 4n - 1 \geq 0$ $(n+1)^2 - 4(1)(4n-11) < 0$ $n^2 + 1 + 2n - 16n + 44 < 0$
	$(n-9)(n-5) < 0$ $5 < n < 9$
8(i)	$\sin A = \dfrac{1}{\sqrt{5}}$ $\cos A = \dfrac{-2}{\sqrt{5}}$

(ii)	$\cos(A-\frac{\pi}{3})=\cos A\cos\frac{\pi}{3}+\sin A\sin\frac{\pi}{3}$ $=-\frac{2}{\sqrt{5}}\times\frac{1}{2}+(\frac{1}{\sqrt{5}}\times\frac{\sqrt{3}}{2})$ $=\frac{\sqrt{3}-2}{2\sqrt{5}}$
(iii)	$\sec A=1\div\frac{\sqrt{3}-2}{2\sqrt{5}}$ $=\frac{2\sqrt{5}}{\sqrt{3}-2}\times\frac{\sqrt{3}+2}{\sqrt{3}+2}$ $=\frac{2\sqrt{15}+4\sqrt{5}}{3-4}=-2\sqrt{15}-4\sqrt{5}$
9	$y=\frac{3}{4}(x^2-\frac{16}{3}x)+10$ $y=\frac{3}{4}(x^2-\frac{16}{3}x+(-\frac{8}{3})^2-(-\frac{8}{3})^2)+10$ $y=\frac{3}{4}(x-\frac{8}{3})^2+\frac{14}{3}$
(ii)	$4\frac{2}{3}$ m
(iii)	$70=\frac{3}{4}(x-\frac{8}{3})^2+\frac{14}{3}$ $(x-\frac{8}{3})^2=87\frac{1}{9}$ $x=12$m
10(a)(i)	Period = 120°
(ii)	$p=-2$, $q=4$
(iii)	
b(i)	Period = 12.5 hours

	$k=\frac{2\pi}{12.5}=\frac{4}{25}$
b(ii)	$h=1.2\cos(\frac{4\pi}{25}\times 7)+5$ t = 7, $h=$ $h=3.88$ Since the draft of the boat is more than the height of water at t = 7 hours, it is not safe for the boat to leave at 7 pm.
11	$3^{-2}\times 3^{2y}=3^{4x}$ $3^{2y-2}=3^{4x}$ $2y-2=4x$ $y=2x+1$.......(1)
	$2^{2x^2+xy}=2^{4-y^2}$ $2x^2+xy=4-y^2$ From (1):$y=2x+1$ $2x^2+x(2x+1)-4+(2x+1)^2=0$ $2x^2+2x^2+x-4+4x^2+1+4x=0$ $8x^2+5x-3=0$ $(8x-3)(x+1)=0$ $x=\frac{3}{8}$ or $x=-1$ $y=1\frac{3}{4}$ or $x=-1$
12(i)	$\frac{\cos x}{1+2\cos^2 x-1}+\frac{\sin x}{1-(1-2\sin^2 x)}$ $=\frac{\cos x}{2\cos^2 x}+\frac{\sin x}{2\sin^2 x}$ $=\frac{1}{2\cos x}+\frac{1}{2\sin x}$ $=\frac{1}{2}(\operatorname{cosec} x+\sec x)$

(ii)	$\frac{1}{2}(\operatorname{cosec} 2A+\sec 2A)=0$ $\frac{1}{\sin 2A}+\frac{1}{\cos 2A}=0$ $\frac{1}{\sin 2A}=\frac{-1}{\cos 2A}$ $\frac{\sin 2A}{\cos 2A}=-1$ $\tan 2A=-1$ Reference angle $\alpha=\frac{\pi}{4}$ $2A=\pi-\frac{\pi}{4}, 2\pi-\frac{\pi}{4}, 2\pi+\pi-\frac{\pi}{4}, 2\pi+2\pi-\frac{\pi}{4}$ $A=\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$
13(i)	$(\log_2 2^3)^2\times\frac{1}{(\log_5 3)}\times\frac{\log_5 3}{\log_5 25}$ $=3^2\times\frac{1}{\log_5 5^2}$ $=\frac{9}{2}$
13(ii)	$\log_3 x-\frac{\log_3(x+6)}{\log_3 9}=0$ $\log_3 x-\frac{\log_3(x+6)}{\log_3 9}=0$ $\log_3 x-\frac{\log_3(x+6)}{\log_3 3^2}=0$ $\log_3 x-\frac{\log_3(x+6)}{2}=0$ $2\log_3 x-\log_3(x+6)=0$ $\log_3 x^2=\log_3(x+6)$ $x^2=x+6$ $x^2-x-6=0$ $(x-3)(x+2)=0$ $x=3$ or $x=-2$ As $x>0$, x = -2 is rejected.
14a(i)	$t=0, N=120$

a(ii)	$N = 120(1.05^{0.847t})$ $t = 8$ $N = 120(1.05^{0.847\times 8})$ $N = 167$
14(b)	
15 (i)	gradient of $PS = \frac{7-5}{6-(-2)} = \frac{1}{4}$ gradient of $PQ = -4$ Equation of $PQ: y-5 = -4(x+2)$ $y = -4x-3$
(ii)	$3(-4x-3) = 4x-29$ $-12x-9 = 4x-29$ $16x = 20$ $x = 1\frac{1}{4}$ $y = -4(1\frac{1}{4})-3 = -8$ Q is (1.25, −8) (proven)
(iii)	Eqn of $SR: y-7 = -4(x-6)$ $y = -4x+31$ Solve simultaneously $x = 7\frac{5}{8}$ $y = 0.5$ $R = (7\frac{5}{8}, \frac{1}{2})$

(iv)	$\frac{1}{2}\begin{vmatrix} -2 & 1.25 & 7\frac{5}{8} & 6 & -2 \\ 5 & -8 & \frac{1}{2} & 7 & 5 \end{vmatrix}$ $=\frac{1}{2}\lvert 100-(-65.75)\rvert = 82.875\text{ unit}^2$
16(i)	$\angle CEF=\theta$ (corrs angles) Triangle CEF: $\cos\theta=\frac{CE}{4}, CE=4\cos\theta$ $\sin\theta=\frac{CF}{4}, CF=4\sin\theta$ Triangle ABF: $\cos\theta=\frac{AB}{7}, AB=7\cos\theta$ $\sin\theta=\frac{BF}{7}, BF=7\sin\theta$ $L=7\sin\theta+7\cos\theta+4\sin\theta+4\cos\theta+3$ $=11\cos\theta+11\cos\theta+3$ (proven)
(ii)	$R=\sqrt{11^2+11^2}=\sqrt{242}$ $\tan\alpha=\frac{11}{11}, \alpha=45^\circ$ $L=\sqrt{242}\cos(\theta-45^\circ)+$
(iii)	Maximum $L=\sqrt{242}+3=18.6$ when $\theta=45^\circ$
(iv)	$\sqrt{242}\cos(\theta-45^\circ)+3=15$ $\cos(\theta-45^\circ)=\frac{12}{\sqrt{242}}$ reference angle $=39.52^\circ$ $\theta=45^\circ+39.52^\circ=84.5^\circ$

17	$3\tan A=\frac{1-4\sec A}{\tan A}$ $3\tan^2 A=1-4\sec A$ $3(\sec^2 A-1)=1-4\sec A$ $3\sec^2 A-3-1+4\sec A=0$ $3\sec^2 A+4\sec A-4=0$ $\sec A=\frac{2}{3}$ or $\sec A=-2$ $\cos A=\frac{3}{2}$ (rejected) or $\cos A=-\frac{1}{2}$ $\cos A=-\frac{1}{2}$ Reference angle $=60^\circ$ $A=120^\circ$ or 240°
18(i)	$2g=-30, 2f=8, c=232$ centre = (15,–4) radius $=\sqrt{(-15)^2+4^2-232}=3$
(ii)	$mAB=\frac{-6-4}{-4-2}=\frac{5}{3}$ m_2(perpendicular line) $=-\frac{3}{5}$ Mid point of $AB=(\frac{2-4}{2},\frac{4-6}{2})=(-1,-1)$ Equation of perpendicular bisector $y-(-1)=-\frac{3}{5}(x-(-1))$ $y=-\frac{3}{5}x-\frac{8}{5}$
(iii)	$y=-\frac{3}{5}x-\frac{8}{5}$ ...(1) $y=-3x+8$...(2) Solving simultaneously $x=4, y=-4$ Centre of C_2 = (4,-4)
	Radius of $C_2=\sqrt{(4-2)^2+(-4-4)^2}=\sqrt{68}$

(iv)	Distance between end point of C_1 and C_2 $(15+3)-(4-\sqrt{68})=22.246$ = Sum of diameters = $6+2\sqrt{68}=22.49$ Since the distance between the 2 end points is shorter than the sum of the diameters, the 2 circles C_1 and C_2 will intersect each other. Hence, I do not agree with Caleb.
	Alternative Method C1 : (15,-4), radius = 3 C2 : (4,-4), radius = $\sqrt{68}$ Distance between centres = 15 – 4 =11 Sum of radii = $\sqrt{68}+3=11.25$ Since distance between centres is less than sum of radii, the circles will intersect. Hence, I do not agree with Caleb.