Class	Register No.

Candidate Name ______________________________

PEIRCE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY 4 EXPRESS / 5 NORMAL(ACADEMIC)

ADDITIONAL MATHEMATICS **4049/01**
Paper 1 **23 August 2022**
2 hours 15 minutes

Additional Materials:
Plain Paper (for rough work)

INSTRUCTIONS TO CANDIDATES
Candidates answer on the Question Paper.

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

PARENT'S SIGNATURE

For Examiner's Use	
Total	

This paper consists of **21** printed pages and **1** blank page.
Paper set by: Mdm Sin Boon Yiah

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

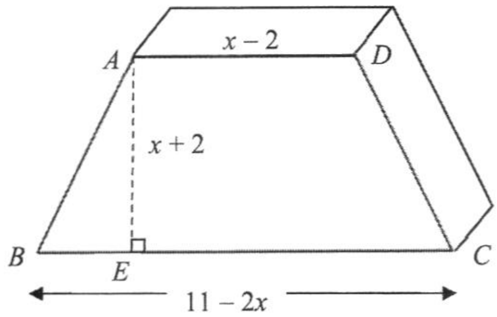
1 The volume of a rectangular block with a square base is $\left(6\sqrt{7}-2\sqrt{3}\right)\text{cm}^3$. The length of each side of the base is $\left(\sqrt{7}-\sqrt{3}\right)\text{cm}$. Find, **without using a calculator**, the height f the block in the form $\left(a\sqrt{7}+b\sqrt{3}\right)\text{cm}$, where a and b are integers. [4]

2 Solve the simultaneous equations

$$2x = y + 3,$$

$$2x^2 + y + 9 = 10x. \qquad [4]$$

3



The diagram shows a design of a paper weight in the shape of a trapezium prism. It is given that $ABCD$ is a trapezium where AD is parallel to BC, $AD = (x-2)$ cm, $BC = (11-2x)$ cm, $AE = (x+2)$ cm and $AB = CD$.

(i) Given that R is the area of the trapezium $ABCD$, show that

$$R = -\frac{1}{2}x^2 + \frac{7}{2}x + 9$$ [2]

(ii) Express R in the form $a(x+b)^2 + c$, where a, b and c are constants. Hence state the maximum value of R and its corresponding value of x. [4]

4 Integrate $\frac{2}{3}(4x+3)^3+\frac{4}{x^2}$ with respect to x. [3]

5 Express $\frac{5x^2-11x-4}{x^2-2x-3}$ in partial fractions. [6]

6 **(a)** Factorise $27x^3 - 8$ and hence prove that $x^3 = \frac{8}{27}$ has only one real solution. [3]

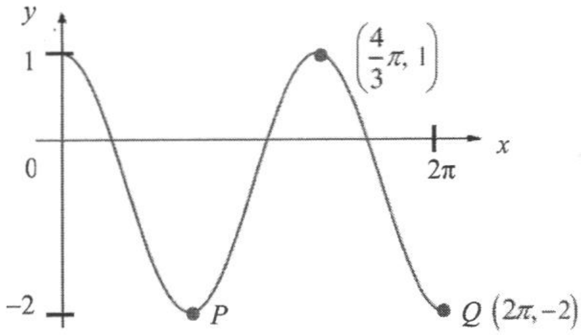
(b) $2x^2 + ax - 3$ is a quadratic factor of $2x^3 - x^2 - 13x - 6$. Find the value of the constant a. [3]

7 **(a)** State the value, in radians, between which each of the following must lie:

(i) the principal value of $\cos^{-1} x$, [1]

(ii) the principal value of $\tan^{-1} x$. [1]

(b)



The diagram shows the graph of the function $y = a\cos bx + c$. The coordinates of Q are $(2\pi, -2)$.

(i) Explain why $c = -\frac{1}{2}$. [1]

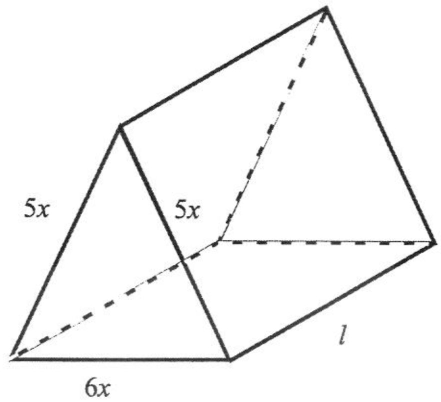
(ii) Explain why $b = \frac{3}{2}$. [2]

(iii) Hence find the equation of the curve. [1]

(iv) Find the coordinates of P. [1]

(v) State the range of values of k for which $a\cos bx + c = k$ has three solutions. [1]

8 The regular cross-section of a triangular prism is an isosceles triangle whose sides are $5x$ cm, $5x$ cm and $6x$ cm. The length of the prism is l cm.



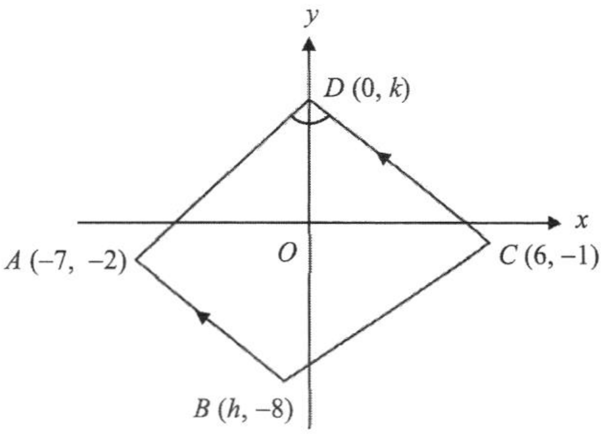
(i) Given that the volume of the prism is 240 cm^3, shown that $l = \frac{20}{x^2}$. [2]

(ii) Given that the total surface area is A cm^2, show that $A = \frac{320}{x} + 24x^2$. [2]

(iii) Find the value of x for which A has a stationary value, giving your answer to 2 decimal places. [4]

(iv) Find this stationary value of A, giving your answer to 3 significant figures. [1]

9



The diagram shows a trapezium *ABCD*, where *A* is (−7, −2), *B* is (*h*, −8) and *C* is (6, −1). Point *D* (0, *k*) lies on the *y*-axis such that *AB* is parallel to *CD* and the *y*-axis bisects angle *ADC*.

(i) Express the gradient of *AD* and *CD* in terms of *k*. [2]

(ii) Hence show that $k = 5$. [2]

(iii) Find the value of h. [2]

(iv) Find the area of trapezium $ABCD$. [2]

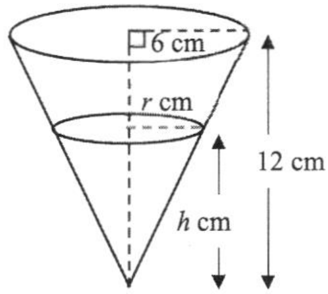
10 **(a)** Prove the identity $\frac{\cos\theta}{1-\sin\theta}+\frac{1-\sin\theta}{\cos\theta}=2\sec\theta$. [4]

(b) Hence solve the equation $\dfrac{\cos 3\beta}{1-\sin 3\beta}+\dfrac{1-\sin 3\beta}{\cos 3\beta}=4$ for $0° \le \beta \le 180°$. [5]

11 **(a)** Solve the equation $7^x\left(5^{2x}\right)=x^{3x}$. [3]

(b) Given that $2\log_a b-\dfrac{5}{\log_b a}=3$, find an expression for b in terms of a. [3]

12



[Volume of a cone of height h and base radius r is $\frac{1}{3}\pi r^2 h$]

The diagram shows a container in the shape of an inverted circular cone of radius 6 cm and of height 12 cm.

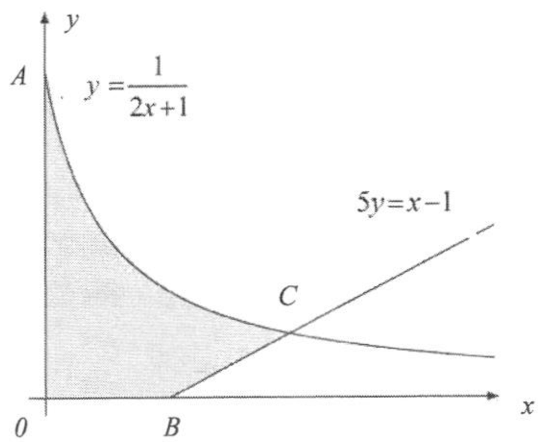
(i) Show that the volume of water, V cm^3, in the container is $V = \frac{1}{12}\pi h^3$. [1]

The water in the container was initially full. When water is poured into this container at a rate of 3 cm^3/s, it leaks at a rate of 5 cm^3/s through a small hole at the vertex.

(ii) Calculate the rate of change of height of the water at the instant when the depth of the water is 3 cm. [3]

(iii) Calculate the rate of change of area of the horizontal surface of the water at the instant when $h = 3$. [4]

13



The diagram shows part of the curve $y=\frac{1}{2x+1}$ and part of the line $5y=x-1$.

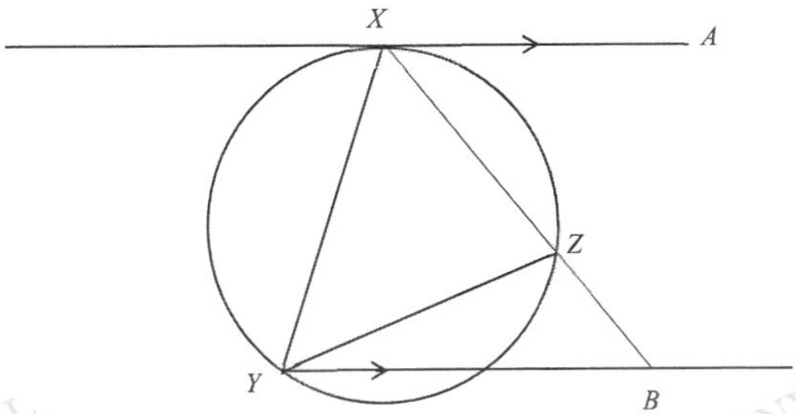
The curve meets the y-axis at point A. The line meets the x-axis at point B. The line and curve intersect at point C.

(a) **(i)** Find the coordinates of A and B. [2]

(ii) Show that x-coordinate of C is 2. [2]

(b) Find the exact area of the shaded region. [4]

14



In the diagram, AX is a tangent to the circle at point X and AX is parallel to BY.
Prove that

(i) $\triangle XYZ$ is similar to $\triangle XBY$, [3]

(ii) $XY^2 = XB \times XZ$. [2]

*** END OF PAPER ***

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PEIRCE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS — **4049/02**
Paper 2 — **25 August 2022**
2 hours 15 minutes

Additional Materials:
Plain Paper (for rough work)

INSTRUCTIONS TO CANDIDATES
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Paper set by: Mdm Sin Boon Yiah

Mathematical Formulae

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For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

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$$\sin^2 A + \cos^2 A = 1$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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$$\Delta = \frac{1}{2}ab \sin C$$

1. Given that $y = 2\left(7^{2x}\right) - 3\left(7^{x+1}\right) + 19$, find the value(s) of x when $y = 30$. [5]

2 The polynomial $p(x) = mx^3 - 29x^2 + 39x + n$, where m and n are constants, has a factor $3x - 1$, and remainder 6 when divided by $x - 1$.

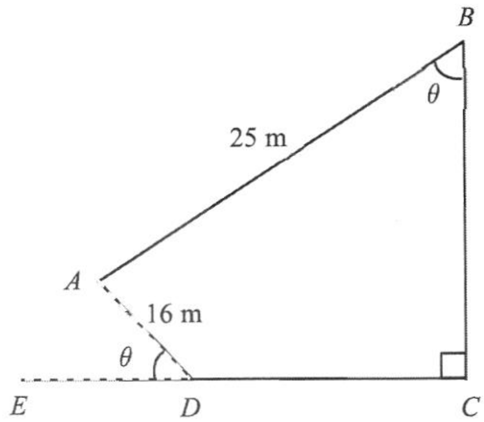
(i) Find the value of m and n. [5]

(ii) Hence solve the equation $p(x) = 0$. [4]

3 **(a)** Differentiate, with respect to x, $y=x^2\ln x$. [2]

(b) Hence find the exact value of $\int_2^4 (x\ln x)\,dx$. [4]

4 Edwin runs along the straight line paths in the park from $ABCD$ as shown in the diagram below. D is a point on the straight line EC. It is given that $AB = 25$ m, $AD = 16$ m, angle $BCD = 90°$ and angle ADE = angle $ABC = \theta$, where θ is an acute angle in degrees.



(i) Show that the total distance L, in metres, of the path $ABCD$ that Edwin runs is $L = 25 + 9\cos\theta + 41\sin\theta$. [3]

(ii) Express L in the form $25 + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Find the value of θ for which $L = 50$ m. [3]

(iv) Is it possible for Edwin to run 70 m using this path? Explain your answer. [2]

5 The equation of a curve is $y = 2x^2 + x - 6$.

(a) Find the set of values of x for which the curve lies above the line $y = 9$ and represent this set on a number line. [4]

The line $y = 3x + k$ is a tangent to the curve at the point H.

(b) Find the coordinates of H. [3]

(c) Find the value of the constant k. [2]

6 **(i)** Write down and simplify the first three terms in the expansion, in ascending powers of x, of $\left(2-\frac{x}{4}\right)^6$. [3]

(ii) In the expansion of $\left(4+kx+x^2\right)\left(2-\frac{x}{4}\right)^6$, the sum of the coefficients of x and x^2 is -4. Find the value of the constant k. [4]

7 **(a)** The variables x and y are known to be connected by the equation $y = ax(x + b)$ where a and b are constants. Values of x for different values of y have been collected. Explain how a straight line graph can be drawn to represent the formula, and state how the values of a and b could be obtained from the line. [3]

(b) The table below shows experimental values of two variables x and y.

x	2	4	6	8
y	8.48	5.99	4.90	4.24

It is known that x and y are connected by the equation $yx^n = k$, where k and n are constants.

(i) Plot $\ln y$ against $\ln x$, using a scale of 4 cm for 1 unit on both axes, for the given data and draw a straight line graph on the grid on next page. [2]

(ii) Use your graph to estimate the value of n and of k. [3]

(iii) Use your graph to **estimate** the value of x when $y=e^2$. [2]

(iv) On the same diagram, draw the straight line representing the equation $y=x^3$ and hence find the value of x for which $x^{3+n}=k$. [3]

8 An object, moving along a straight road, passes a point A and, 10 seconds later, the object passes a point B. The object's speed at B is twice its speed at A. For the journey from A to B, the object's speed, v m/s, t seconds after passing A, is given by

$$v = 15e^{kt} + \frac{3}{4}t \ , \ 0 \le t \le 10,$$

where k is a constant.

(i) Find the value of k. [3]

(ii) Find the distance between from A to B. [4]

Continuation of working space for question 8.

(iii) Find the acceleration of the object when $t = 2$. [3]

9 Points $A(-2, 3)$, $B(3, 0)$ and $C(6, 5)$ lies on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$. [2]

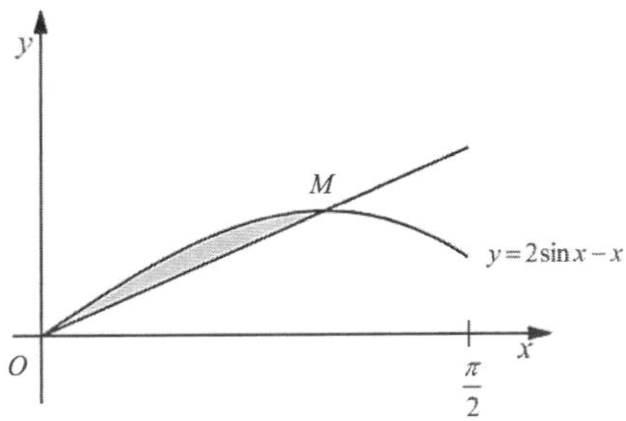
(b) Hence, by giving the reason, state the coordinates of D. [2]

(c) Find the equation of the circle. [2]

The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find the equation of the tangent to the circle at E. [4]

10



The diagram shows the curve $y=2\sin x-x$ for $0\le x\le\frac{\pi}{2}$ radians. The point M is the maximum point of the curve and OM is a straight line.

Show that the area of the shaded region is $1-\frac{\sqrt{3}}{6}\pi$ units^2. [10]

Continuation of working space for question 10.

*** End of Paper ***

Peirce Secondary School
2022 Preliminary Examination 4E/5A Add Math Paper 1 Solutions

Qn	Suggested solution
1	$\text{Height} = \frac{(6\sqrt{7}-2\sqrt{3})}{(\sqrt{7}-\sqrt{3})^2}$ $= \frac{(6\sqrt{7}-2\sqrt{3})}{10-2\sqrt{21}} \times \frac{10+2\sqrt{21}}{10+2\sqrt{21}}$ $= \frac{60\sqrt{7}-20\sqrt{3}+84\sqrt{3}-12\sqrt{7}}{100-4(21)}$ $= \frac{48\sqrt{7}+64\sqrt{3}}{16}$ $= (3\sqrt{7}+4\sqrt{3})$ cm
2	$2x = y+3$ $y = 2x-3$ (1) $2x^2+y+9=10x$ ——— (2) Subst (1) into (2). $2x^2+2x-3+9-10x=0$ $2x^2-8x+6=0$ $x^2-4x+3=0$ $(x-1)(x-3)=0$ When $x=1, y=-1$ When $x=3, y=3$

Qn	Suggested solution
3 (i)	$R=\frac{1}{2}\left[(x-2)+(11-2x)\right](x+2)$ $=\frac{1}{2}(9-x)(x+2)$ $=\frac{1}{2}(18+7x-x^2)$ $R=-\frac{1}{2}x^2+\frac{7}{2}x+9$ (Shown)
3 (ii)	$-\frac{1}{2}x^2+\frac{7}{2}x+9$ $=-\frac{1}{2}(x^2-7x)+9$ $=-\frac{1}{2}\left[\left(x-\frac{7}{2}\right)^2-\left(\frac{7}{2}\right)^2\right]+9$ $=-\frac{1}{2}\left(x-\frac{7}{2}\right)^2+\frac{121}{8}$ $\therefore$ the maximum value of R = $\frac{121}{8}$ when x = $3\frac{1}{2}$ (or $\frac{7}{2}$)
4	$\int\frac{2}{3}(4x+3)^3\,dx+\int\frac{4}{x^2}\,dx$ $=\frac{2}{3}\left[\frac{(4x+3)^4}{16}\right]-\frac{4}{x}+c$ where c is a constant , $=\frac{1}{24}(4x+3)^4-\frac{4}{x}+c$

Qn	Suggested solution
5	$$x^2-2x-3\overline{)5x^2-11x-4}$$ quotient 5; $\underline{-(5x^2-10x-15)}$; $-x+11$ and $x^2-2x-3=(x-3)(x+1)$ $\therefore \dfrac{5x^2-11x-4}{x^2-2x-3}=5+\dfrac{11-x}{x^2-2x-3}$ Let $\dfrac{11-x}{x^2-2x-3}=\dfrac{A}{x-3}+\dfrac{B}{x+1}$ $\Rightarrow$ $11-x=A(x+1)+B(x-3)$ Subst $x=3$, $8=4A$ $A=2$ Subst. $x=-1$, $12=-4B$ $B=-3$ $\therefore \dfrac{5x^2-11x-4}{x^2-2x-3}=5+\dfrac{2}{x-3}-\dfrac{3}{x+1}$
6 (a)	$27x^3-8=(3x-2)(9x^2+6x+4)$ For $27x^3-8=0$ $(3x-2)(9x^2+6x+4)=0$ $3x-2=0,\quad 9x^2+6x+4=0$ $x=\dfrac{2}{3}$, Discriminant for $9x^2+6x+4=36-4(9)(4)$ $=-108$ D < 0 has no real roots. Hence $27x^3-8=0$ has only one real solution.
6 (b)	Let $2x^3-x^2-13x-6=(2x^2+ax-3)(x+2)$ The term in x: $-13x=2ax-3x$ $2a=-10,$ $a=-5$

Qn	Suggested solution
7 (a) (i)	$0 \le \cos^{-1} x \le \pi$
7 (a) (ii)	$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$
7 (b)(i)	$c = \frac{1}{2}$ (max value of y + min value of y) $\therefore c = \frac{1+(-2)}{2}$ $= -\frac{1}{2}$
7 (b)(ii)	period $= \frac{2\pi}{b}$ $\frac{4}{3}\pi = \frac{2\pi}{b}$ $b = \frac{3}{2}$
7 (b)(iii)	$y = \frac{3}{2}\cos\frac{3}{2}x - \frac{1}{2}$
7 (b)(iv)	$P = \left(\frac{2\pi}{3}, -2\right)$
7 (b)(v)	$-2 < k < 1$
8 (i)	Height of the isoceles triangle = $\sqrt{(5x)^2 - (3x)^2}$ $= 4x$ $\text{Volume} = \frac{1}{2}(6x)(4x)l$ $240 = 12x^2 l$ $l = \frac{20}{x^2}$ (shown)
8 (ii)	$A = \frac{1}{2}(6x)(4x)\times 2 + 16x\left(\frac{20}{x^2}\right)$ $A = 24x^2 + \frac{320}{x}$ (shown)
8 (iii)	$A = 24x^2 + \frac{320}{x}$ $\frac{dA}{dx} = 48x - \frac{320}{x^2}$ Let $\frac{dA}{dx} = 0$, $48x - \frac{320}{x^2} = 0$ $x^3 = \frac{320}{48}$ x = 1.88 (correct to 2 d.p.)
8 (iv)	A = 255 (to 3 s.f.)

Qn	Suggested solution
9 (i)	Gradient of $AD = \frac{k+2}{7}$ Gradient of $CD = -\frac{k+2}{7}$ or $\frac{k+1}{-6}$
9 (ii)	$-\frac{k+2}{7} = \frac{k+1}{-6}$ $6(k+2) = 7(k+1)$ $k = 5$ (shown)
9 (iii)	Gradient of DC = Gradient of AB $\frac{5+1}{0-6} = \frac{-2+8}{-7-h}$ $\frac{6}{-6} = \frac{6}{-7-h}$ $h = -1$
9 (iv)	Area of $ABCD = \frac{1}{2}\begin{vmatrix} 0 & -7 & -1 & 6 & 0 \\ 5 & -2 & -8 & -1 & 5 \end{vmatrix}$ $= \frac{1}{2}\left[(0+56+1+30)-(-35+2-48-0)\right]$ $= 84$ sq units.

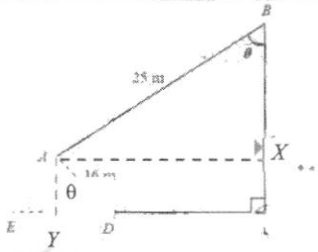
Qn	Suggested solution
10 (a)	From LHS: $=\dfrac{\cos^2\theta+(1-\sin\theta)^2}{\cos\theta(1-\sin\theta)}$ $=\dfrac{\cos^2\theta+1-2\sin\theta+\sin^2\theta}{\cos\theta(1-\sin\theta)}$ $=\dfrac{2-2\sin\theta}{\cos\theta(1-\sin\theta)}$ $=\dfrac{2(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$ $= 2\sec\theta$ (= RHS)
10 (b)	$\dfrac{\cos 3\beta}{1-\sin 3\beta}+\dfrac{1-\sin 3\beta}{\cos 3\beta}=4$ ⇨ $\dfrac{2}{\cos 3\beta}=4$ $\cos 3\beta=\dfrac{1}{2}$ Basic angle = 60° $3\beta = 60°, 360° - 60°, 360° + 60°$ $\beta = 20°, 100°, 140°$.
11 (a)	$7^x(5^{2x})=x^{3x}$ $7^x(25^x)=x^{3x}$ $(7\times25)^x=(x^3)^x$ $175=x^3$ $x = 5.59$ (correct to 3 s.f.)
11 (b)	$2\log_a b-\dfrac{5}{\log_b a}=3$ $2\log_a b-\dfrac{5}{\frac{\log_a a}{\log_a b}}=3$ $2\log_a b-5\log_a b=3$ $-3\log_a b=3$ $\log_a b=-1$ $b=\dfrac{1}{a}$ or $b=a^{-1}$

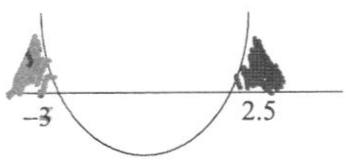
Qn	Suggested solution
12 (i)	$\frac{r}{h}=\frac{6}{12}$ $r=\frac{1}{2}h$ $V=\frac{1}{3}\pi r^2 h$ Where V is the volume of water in the inverted cone $V=\frac{1}{3}\pi\left(\frac{h^3}{4}\right)$ $V=\frac{\pi}{12}h^3$ (Shown)
12 (ii)	$V=\frac{\pi}{12}h^3$ $\frac{dV}{dh}=\frac{\pi}{4}h^2$ $\therefore\ \frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ $-2=\frac{\pi}{4}(3)^2\times\frac{dh}{dt}$ $\frac{dh}{dt}=-\frac{8}{9\pi}$ or -0.283 cm/s
12 (iii)	$A=\pi r^2$ $=\pi\left(\frac{h}{2}\right)^2$ $=\frac{\pi}{4}h^2$ $\frac{dA}{dh}=\frac{\pi}{2}h$ $\therefore\frac{dA}{dt}=\frac{\pi}{2}h\times\frac{dh}{dt}$ $\therefore\frac{dA}{dt}=\frac{\pi}{2}(3)\times\left(\frac{-8}{9\pi}\right)$ $\therefore\frac{dA}{dt}=-\frac{4}{3}\ \text{cm}^2/s$

Qn	Suggested solution	
13 (a) (i)	Subst. $x=0$ into $y=\frac{1}{2x+1}$, => $y=1$ $\therefore A\ (0, 1)$ Subst. $y=0$ into $5y=x-1$, => $x=1$ $\therefore B\ (1, 0)$	
13 (a) (ii)	$y=\frac{1}{2x+1}$ ——— (1) $5y=x-1$ ——— (2) Subst. equation (1) into (2): $5\left(\frac{1}{2x+1}\right)=x-1$ $5=(x-1)(2x+1)$ $2x^2-x-6=0$ $(2x+3)(x-3)=0$ $x=2$ or $x=-\frac{3}{2}$ Hence the x-coordinate of C is 2 (verified)	
13 (b)	Area of the shaded region = $\int_0^2 \frac{1}{2x+1}\,dx-\frac{1}{5}\int_1^2(x-1)\,dx$ $=\frac{1}{2}\left[\ln(2x+1)\right]_0^2-\frac{1}{15}\left[\frac{x^2}{2}-x\right]_0^2$, $=\frac{1}{2}\ln 5-\frac{1}{10}$ units^2	
14 (i)	$\angle YXZ=\angle XBY$ (Common Angle) $\angle XYX=\angle AXZ$ (tangent-chord theorem or alternate segment theorem) and $\angle AXZ=\angle XBY$ (Alternate angle) ⇨ $\angle XYX=\angle XBY$ (3 pairs of corresponding angles are equal) Hence $\triangle XYZ$ is similar to $\triangle XBY$.	
14 (ii)	$\frac{XY}{XB}=\frac{YZ}{BY}=\frac{XZ}{XY}$ (Properties of similar triangles) $\therefore XY^2=XB\times XZ$ (Proven)	

Peirce Secondary School
2022 Preliminary Examination 4E/5A Add Math Paper 2 Solutions

Qn	Suggested solution
1	$y=2(7^{2x})-3(7^{x+1})+19$ $y=2(7^{2x})-3(7)(7^{x})+19$ $y=2(7^{2x})-21(7^{x})+19$ Let $u=7^x$ $y=2u^2-21u+19$ Given $y=30$, $2u^2-21u+19=30$ $2u^2-21u-11=0$ $(2u+1)(u-11)=0$ $u=-\frac{1}{2},\quad u=11$ $7^x=-\frac{1}{2}$ (NA) , $7^x=11$ $x=\frac{\lg 11}{\lg 7}$ $=1.23$ (to 3 s.f.)
2 (i)	Given $p\left(\frac{1}{3}\right)=0$ $p\left(\frac{1}{3}\right)=m\left(\frac{1}{3}\right)^3-29\left(\frac{1}{3}\right)^2+39\left(\frac{1}{3}\right)+n=0$ $\frac{1}{27}m-\frac{29}{9}+13+n=0$ × 27: $m+27n=-264$ ——— (1) Given $p(1)=6$ $m(1)^3-29(1)^2+39(1)+n=6$ $m+n=-4$ ———— (2) Equation (1) – (2): $26n=-260$ $n=-10$ From (2): $m=6$
2 (ii)	$p(x)=6x^3-29x^2+39x-10$ Let $6x^3-29x^2+39x-10=(3x-1)(2x^2+bx+10)$ The term in x: $30x-bx=39x$ $b=-9$ Let $6x^3-29x^2+39x-10=(3x-1)(2x^2+bx+10)$ Let $p(x)=0$, $(3x-1)(2x^2-9x+10)=0$ $(3x-1)=0$, $2x^2-9x+10=0$ $(2x-5)(x-2)=0$

Qn	Suggested solution
	$x=\frac{1}{3}$, $x=2$, $x=\frac{5}{2}$ (accept $2\frac{1}{2}$, 2.5)
3 (a)	$y=x^2\ln x$ $\frac{dy}{dx}=2x(\ln x)+x^2\left(\frac{1}{x}\right)$ $=2x(\ln x)+x$
3 (b)	$\frac{dy}{dx}=2x(\ln x)+x$ $[y]_2^4=\int_2^4 2x(\ln x)\,dx+\int_2^4 x\,dx$ $\left[x^2\ln x\right]_2^4=\int_2^4 2x(\ln x)\,dx+\left[\frac{x^2}{2}\right]_2^4$ $16\ln 4-4\ln 2=\int_2^4 2x(\ln x)\,dx+(8-2)$ $8\ln 4-2\ln 2=\int_2^4 x(\ln x)\,dx+3$ $16\ln 2-2\ln 2-3=\int_2^4 x(\ln x)\,dx$ $\int_2^4 x(\ln x)\,dx=14\ln 2-3$
4 (i)	 Let X be the foot of perpendicular from A to BC. Let Y be the foot of perpendicular from A to DE $BX=25\cos\theta$ and $AY=XC=16\sin\theta$ $AX=25\sin\theta$ and $YD=16\cos\theta$ $\therefore DC=25\sin\theta-16\cos\theta$ and $BC=25\cos\theta+16\sin\theta$ $\therefore L=25+(25\sin\theta-16\cos\theta)+(25\cos\theta+16\sin\theta)$ $L=25+9\cos\theta+41\sin\theta$ (shown)
4 (ii)	Let $R\cos(\theta-\alpha)=9\cos\theta+41\sin\theta$ $R=\sqrt{9^2+41^2}=\sqrt{1762}$

Qn	Suggested solution
	$\alpha=\tan^{-1}\left(\frac{41}{9}\right)=77.61^o$ $\therefore\ L=25+\sqrt{1762}\ \cos\left(\theta-77.6^o\right)$
4 (iii)	$25+\sqrt{1762}\ \cos\left(\theta-77.61^o\right)=50$ $\sqrt{1762}\ \cos\left(\theta-77.61^o\right)=25$ $\cos\left(\theta-77.61^o\right)=\frac{25}{\sqrt{1762}}=0.59557$ $\theta-77.61^o=53.44^o;\ \ 360-53.44^o$ (and include – 53.44° angle in clockwise) $\theta=53.44^o+77.61^o;\ \ 306.55+77.61^o(-53.44^o+77.61^o=24.2^o)$ $\theta=131.05^o;\ \ 384.16$ $=384.16-360$ $=24.2^o$ Answer: $\theta=24.2^o$ given θ is an acute angle.
4 (iv)	$L=25+\sqrt{1762}\ \cos\left(\theta-77.6^o\right)$ Max Length $=25+\sqrt{1762}$ $=66.97$ m Hence it is **not possible** to run the running track of 70 m.
5 (a)	$2x^2+x-6>9$ $2x^2+x-15>0$ $(2x-5)(x+3)>0$ $x<-3,\ x>\frac{5}{2}$  
5 (b)	Gradient of tangent = 3 ⇨ $\frac{dy}{dx}=3$

Qn	Suggested solution
	$4x+1=3$ $x=\frac{1}{2}$ and $y=2\left(\frac{1}{2}\right)^2+\frac{1}{2}-6=-5$ Hence the coordinates of $H=\left(\frac{1}{2},\ -5\right)$
5 (c)	Subst $x=\frac{1}{2}$, $y=-5$ into $y=3x+k$ ⇨ $-5=3(\frac{1}{2})+k$ ⇨ $k=-6\frac{1}{2}$
6 (i)	$\left(2-\frac{x}{4}\right)^6=2^6+\binom{6}{1}(2^5)\left(\frac{-x}{4}\right)+\binom{6}{2}(2^4)\left(\frac{-x}{4}\right)^2+...$ $=64-48x+15x^2-...$
6 (ii)	$(4+kx+x^2)(64-48x+15x^2-...)$ The term in $x=-192x+64kx$ $=(-192+64k)$x The term in x^2 $=60x^2-48kx^2+64x^2$ $=(124-48k)x^2$ Given that $-192+64k+124-48k=-4$ $16k=64$ $k=4$
7 (a)	$y=ax(x+b)$ $\frac{y}{x}=ax+ab$ Plot $\frac{y}{x}$ against x, a straight line graph can be drawn. Gradient of the straight line is a and ab is the vertical intercept ***<u>Alternative solution</u>***: $y=ax(x+b)$ $y=ax^2+abx$ $\frac{y}{x^2}=a+\frac{ab}{x}$ Plot $\frac{y}{x^2}$ against $\frac{1}{x}$, a straight line graph can be drawn. Gradient of the straight line is ab and a is the vertical intercept

Qn	Suggested solution

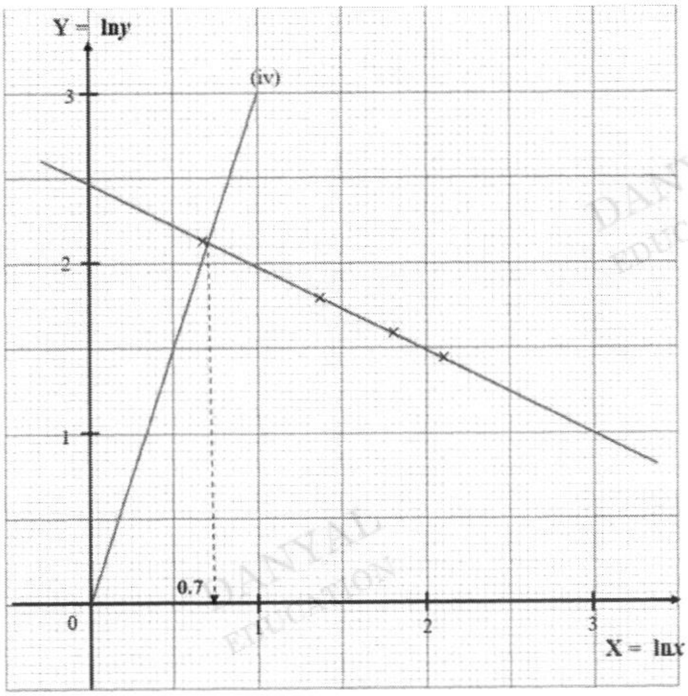
7 (b) (i)

$yx^n = k$

$\ln y + n\ln x = \ln k$

$\ln y = -n\ln x + \ln k$

$X = \ln x$	0.7	1.4	1.8	2.1
$Y = \ln y$	2.2	1.8	1.6	1.4



7 (b)(ii)

From graph, gradient $= \dfrac{2.45-1}{0-3}$

$-n = -0.4833$ (range $\pm$ 0.03)

$\ln k = 2.45$

$k = e^{2.45}$

$= 11.6$ (correct to 3 s.f.) (range $\pm$ 0.5)

7 (b)(iii)

From Graph, $\ln x = 0.925$

$x = e^{0.925} = 2.52$ (to 3 s.f.) ($\pm$ 0.03)

7 (b)(iv)

$\ln y = 3\ln x$

$(3+n)\ln x = \ln k$

$3\ln x = -n\ln x + \ln k$

From graph, $\ln x = 0.7$

$x = e^{0.7} = 2.01$ (to 3 s.f.)

Qn	Suggested solution
8 (i)	$v = 15e^{kt} + \frac{3}{4}t$ When $t = 0$, $v = 15$ m/s When $t = 10$, $30 = 15e^{10k} + \frac{3}{4}(10)$ $15e^{10k} = \frac{45}{2}$ $10k = \ln\frac{3}{2}$ $k = \frac{1}{10}\ln\frac{3}{2}$ or = 0.0405 (to 3 s.f.)
8 (ii)	$s = \int_0^{10}\left(15e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)t} + \frac{3}{4}t\right)dt$ $s = \left[\frac{15e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)t}}{\frac{1}{10}\ln\frac{3}{2}} + \frac{3}{8}t^2\right]_0^{10}$ $s = \left[\frac{15e^{\ln\frac{3}{2}}}{\frac{1}{10}\ln\frac{3}{2}} + \frac{3}{8}(10)^2\right] - \frac{150}{\ln\frac{3}{2}}$ = 222 m (correct to 3 s.f.)
8 (iii)	$v = 15e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)t} + \frac{3}{4}t$ $a = \frac{dv}{dt}$ $\frac{dv}{dx} = 15\left(\frac{1}{10}\ln\frac{3}{2}\right)e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)t} + \frac{3}{4}$ When $t = 2$, $\frac{dv}{dx} = \frac{3}{2}\left(\ln\frac{3}{2}\right)e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)(2)} + \frac{3}{4}$ = 1.41 m/s^2 (correct to 3 s.f.)

Qn	Suggested solution
9 (a)	Gradient of AB, $m_{AB} = \frac{0-3}{3-(-2)} = \frac{-3}{5}$ Gradient of BC, $m_{BC} = \frac{0-5}{3-6} = \frac{5}{3}$ $m_{AB} \times m_{BC} = \frac{-3}{5} \times \frac{5}{3} = -1$ $\therefore \angle ABC = 90^\circ$
9 (b)	$D\ (2, 4)$ AC is the diameter of circle, $\angle$s in semi-circle)
9 (c)	Radius of circle $= \sqrt{(3-2)^2+(0-4)^2} = \sqrt{17}$ units $\therefore$ the equation of circle is $(x-2)^2+(y-4)^2=17$
9(d)	Let $E=(x,\ y)$ $\left(\frac{3+x}{2}, \frac{0+y}{2}\right)=(2,\ 4)$ $\therefore\ E=(1,\ 8)$ Gradient of $DE = \frac{8-4}{1-2} = -4$ ⇨ Gradient of tangent at $E = \frac{1}{4}$. Hence the equation of tangent is $y-8=\frac{1}{4}(x-1)$ $y=\frac{1}{4}x+\frac{31}{4}$ or $y=\frac{1}{4}x+7\frac{3}{4}$ or $4y=x+31$

Qn	Suggested solution
10	$\frac{dy}{dx}=2\cos x-1$ Let $\frac{dy}{dx}=0$, => $2\cos x-1=0$ $\cos x=\frac{1}{2}$ $x=\frac{\pi}{3}$ When $x=\frac{\pi}{3}$, $y=2\sin\frac{\pi}{3}-\frac{\pi}{3}$ $y=2\left(\frac{\sqrt{3}}{2}\right)-\frac{\pi}{3}$ $y=\sqrt{3}-\frac{\pi}{3}$ $\therefore$ Area of the shaded region $=\int_0^{\frac{\pi}{3}}(2\sin x-x)\,dx-\frac{1}{2}\left(\frac{\pi}{3}\right)\left(\sqrt{3}-\frac{\pi}{3}\right)$ $=\left[-2\cos x-\frac{x^2}{2}\right]_0^{\frac{\pi}{3}}-\frac{\pi}{6}\left(\sqrt{3}-\frac{\pi}{3}\right)$ $=\left(-2\cos\frac{\pi}{3}-\frac{\pi^2}{18}\right)-(-2)-\frac{\sqrt{3}}{6}\pi+\frac{\pi^2}{18}$ $=-2\left(\frac{1}{2}\right)+2-\frac{\sqrt{3}}{6}\pi$ $=\left(1-\frac{\sqrt{3}}{6}\pi\right)$ units ^{2}