TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2020
Secondary 4

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS
Paper 1

4048/01
Wednesday 5 August 2020
2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN THE MARGINS.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r \ell$

Curved surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

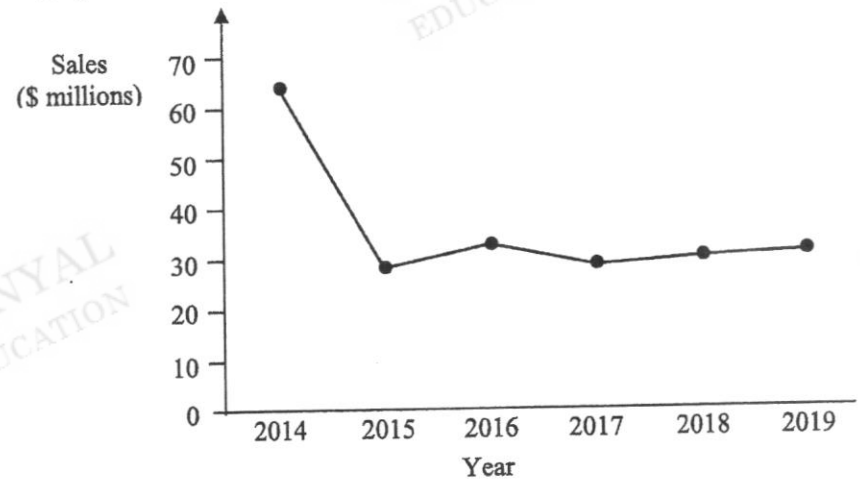
$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 Given that x is an integer where $1 \leq x \leq 4$, find the minimum value of $\frac{x}{2} + \frac{2}{x}$.

Answer _____ [1]

2 The graph shows the sales at a particular mall.



Explain why the mean sales is not a good indication as a central measure.

Answer

_____

_____

_____ [1]

3 (a) Simplify $\left(16^{12x^2}\right)^{\frac{5}{48x}}$.

Answer _____ [2]

(b) Express 5^{-2} as a percentage.

Answer _____ [1]

---

4 (i) Express 84 as a product of prime factors.

Answer _____ [1]

(ii) The sequence $84p$, $84q$, $84r$, are perfect cubes arranged in ascending order. Find the smallest possible integer value of q.

Answer $q =$ _____ [2]

---

5 The amount of plastic waste in Singapore was 900 000 tonnes in 2019.
This is an increase of 10% from 2018.
Find the amount of plastic waste in 2018.

Answer _____ tonnes [2]

---

6 Express $\frac{2}{3x-1} - \frac{3}{1+2x}$ as a single term, in its simplest form.

Answer _____ [2]

7 A city has people with Blood Types as follows:

Type O	45%
Type A	40%
Type B	11%
Type AB	4%

Two persons are selected at random from the city.
Find the probability that at least one person is of Blood Type O.

Answer _____ [2]

8 Factorize completely,

(i) $x^2y^4 - z^6$,

Answer _____ [2]

(ii) $15a^2 - 6a + 20ab - 8b$.

Answer _____ [2]

9 **(i)** Choose the most appropriate symbol from the list shown to make a correct statement.

$<$ $=$ $>$ $\approx$

Answer $0.\dot{5}$ _____ $\frac{5}{9}$ [1]

(ii) The distance between two drones is 12.5 metres, correct to the nearest 10 centimetres. What is the least possible distance between them?

Answer _____ m [1]

---

10 The force, F units, between two objects is inversely proportional to the square of the distance, d units, between them. If the distance decreases by 10%, find the percentage change in the force.

Answer _____ [3]

---

11 Solve $4(2x-5)^2-105=0$.

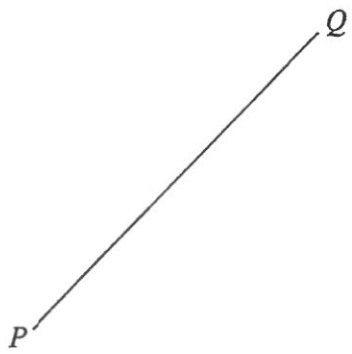
Answer $x =$ _____ or _____ [2]

---

12 The diagram shows a line segment PQ.

(i) Construct the perpendicular bisector of PQ, showing your constructions clearly.

Answer



[1]

(ii) Explain clearly why your construction method gives the perpendicular bisector of PQ.

Answer

_____

_____

_____

_____ [2]

13 The points A, B and C have coordinates (1, 0), (3, 7) and (3, 5) respectively. Find angle BAC.

Answer _____ [4]

14 **(a)** Jane claims that the first three terms of the sequence 1, 2, 4, ... can also be represented by an expression that is different from $T_n = \frac{1}{2}n^2 - \frac{1}{2}n + 1$. Is she correct? Justify your answer.

Answer

[2]

(b) ξ = {Students in a college}
M = {Students who take Mathematics}
E = {Students who take Economics}
P = {Students who take Psychology}
Represent the following using set language.

(i) There are students who take Mathematics and Economics.

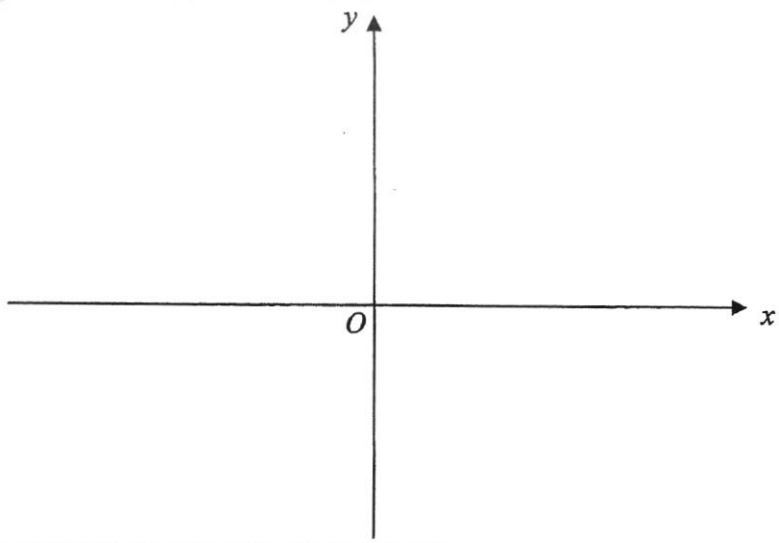
Answer _____ [1]

(ii) All students who take Psychology also take Mathematics.

Answer _____ [1]

---

15 Sketch the graph of $y = 16 - (x + 3)^2$ on the axes below, indicating clearly the coordinates of the intercepts and the turning point on the graph.



[3]

---

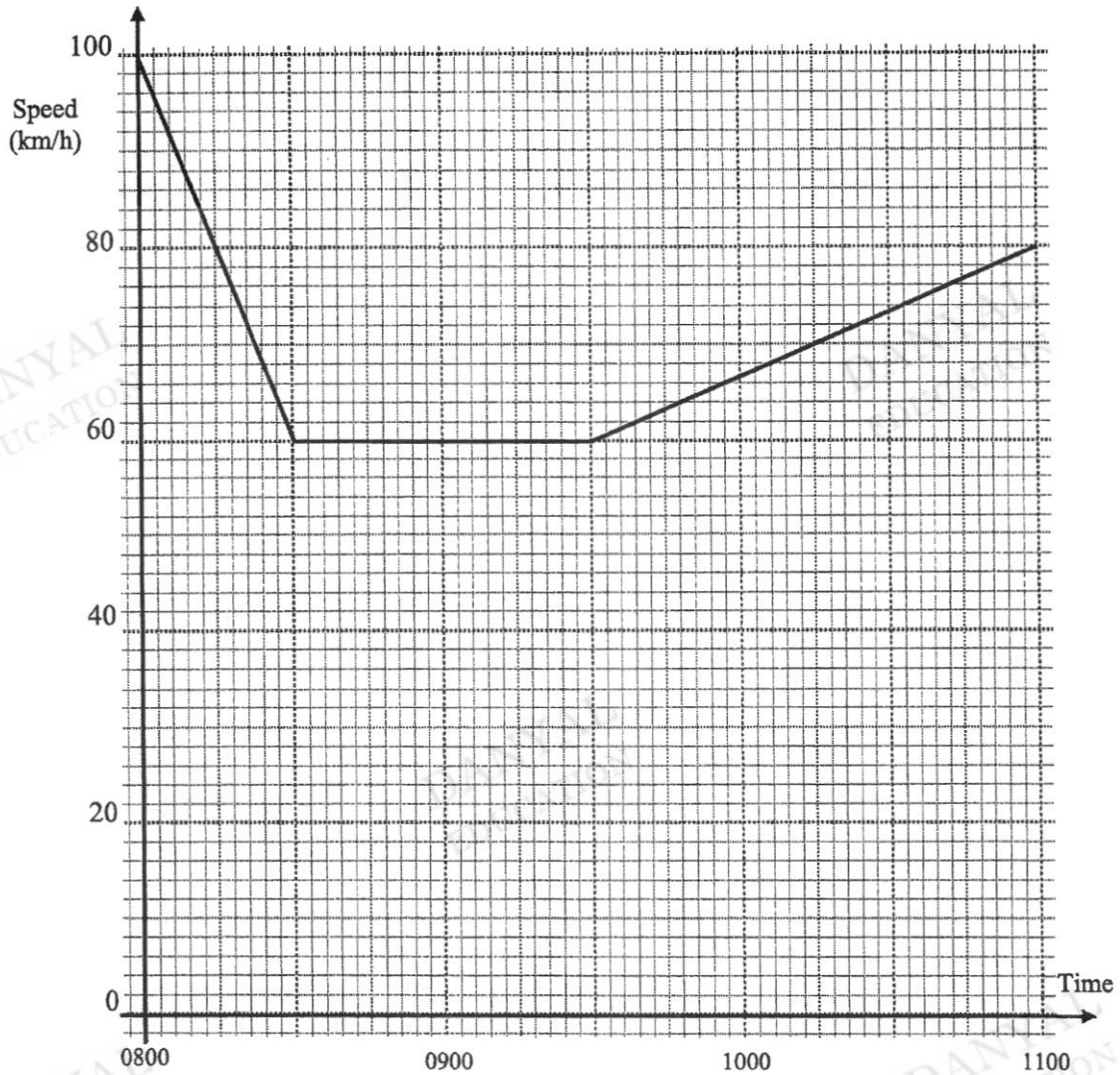
16 **(a)** The sizes of four of the exterior angles of a decagon are in the ratio 1 : 2 : 2 : 3. The remaining exterior angles are each of size 36°. Find the size of the largest interior angle of the decagon.

Answer _____ [2]

(b) The area of a triangle ABC is 35 cm^2 and D is a point such that CD is parallel to AB. The ratio of $CD : AB$ is 1 : 4. Calculate the area of the triangle ACD.

Answer _____ cm^2 [2]

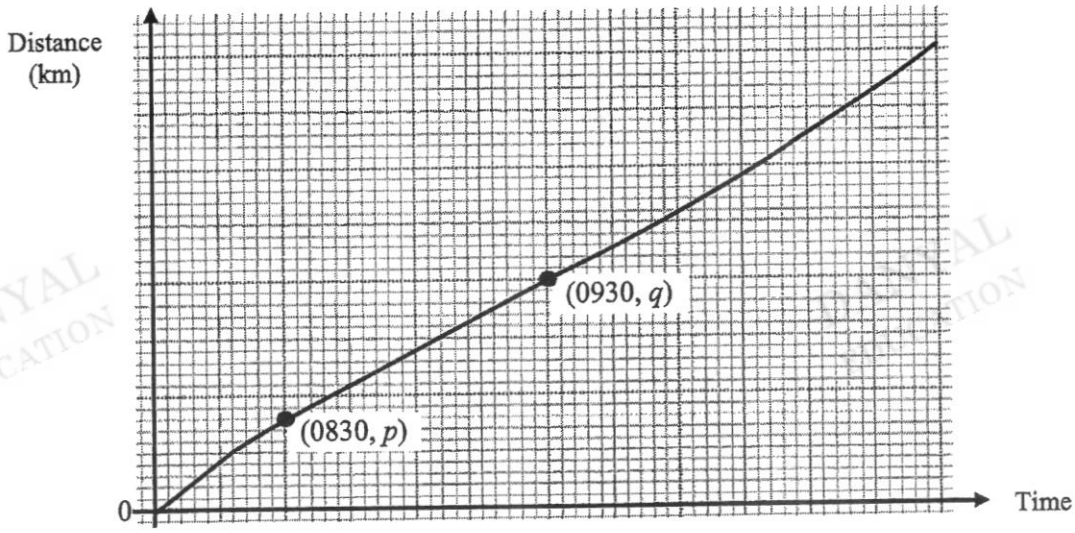
17 The speed-time graph shows the journey that Lim makes during a three-hour journey.



(a) Find the acceleration at 0815.

Answer _____ km/h^2 [1]

The corresponding distance-time graph for Lim's journey is shown below.



(b) Find the value of p and of q.

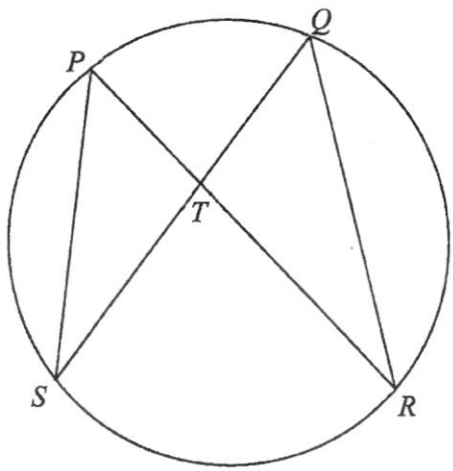
Answer $p =$ _____ $q =$ _____ [3]

---

18 Mr Smith plans to invest his money in unit trust with a bank. His target is to earn an interest of $10,000 after 5 years. The bank pays 3% compound interest per annum compounded yearly. Calculate, to the nearest hundred dollars, the minimum amount of money Mr Smith has to invest.

Answer $ _____ [3]

19 In the diagram, P, Q, R and S are points on a circle. PR and QS meet at T.



(a) Show that triangle PTS and triangle QTR are similar, giving a reason for each statement you make.

Answer

[3]

(b) Show that $QT \times ST = PT \times RT$.

Answer

[1]

20 **(i)** A rectangular fish tank has length 100 cm, width 30 cm and height 40 cm.
It is filled with water to 90% of its capacity.
Find the volume of water in the fish tank, in litres.
1 litre = 1000 cm^3.

Answer _____ *l* [2]

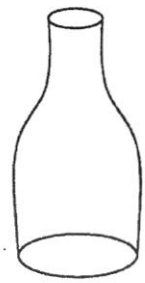
(ii) A particular species of fish need 1600 cm^3 of water per fish.
What is the maximum number of such fish that could be kept in this fish tank?

Answer _____ [2]

(iii) Given that this species of fish grow at a rate such that in each week the need for space increases by 200 cm^3 per fish.
How much space, in cm^3, does each fish need after one month?

Answer _____ cm^3 [1]

21 A manufacturer sells drinks in bottles of two sizes that are geometrically similar.
Some specifications for the bottles are shown below.



Regular Size Bottle	**Large Size Bottle**
• Capacity of bottle = 330 ml • Amount of material needed to manufacture bottle = 24 cm^3	• Capacity of bottle = 1500 ml • Amount of material needed to manufacture bottle = x cm^3

(i) Find the ratio of height of the regular size bottle to the height of the large size bottle.
Give your answer in the form $1 : n$.

Answer _____ : _____ [2]

(ii) Given that the thickness of the bottles are the same, find the value of x.

Answer $x =$ _____ [2]

22 The prices of noodle set and rice set at two shops are shown below.

	Noodle set	Rice set
Shop R	\$2.50	\$3.50
Shop S	\$2.80	\$3.20

(a) Find $\begin{pmatrix} 2.5 & 3.5 \\ 2.8 & 3.2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

Answer _____ [2]

(b) Describe what your answer in **(a)** represents.

_____

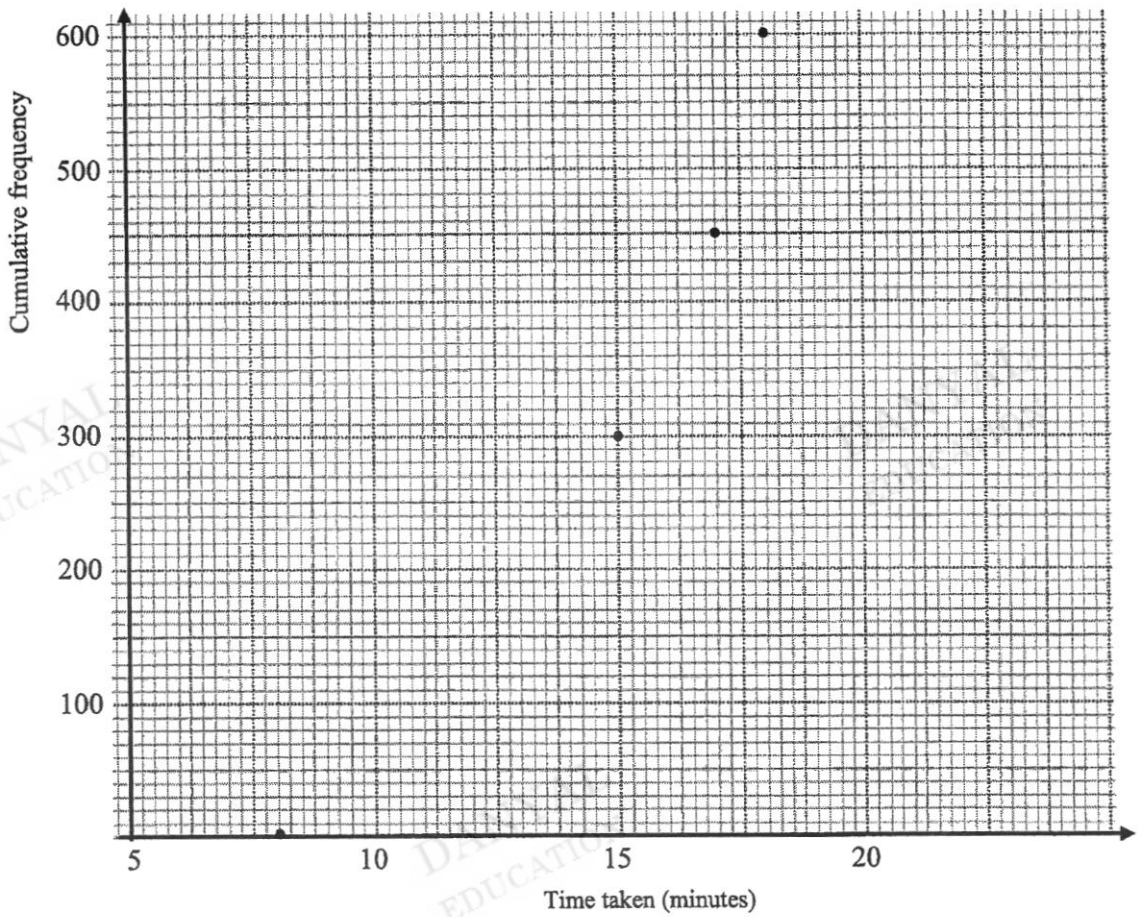
_____

_____ [2]

(c) Sally has to buy a total of 6 meal sets from one particular shop.
How many noodle sets and rice sets does she have to buy such that the total cost is the same regardless of whether she buys from Shop R or Shop S?

Answer _____ noodle set(s), _____ rice set(s) [2]

23



Six hundred runners took part in a race.
The fastest runner took 8 minutes to complete the race while the slowest took 18 minutes.
The points (8, 1), (15, 300), (17, 450) and (18, 600) are points on the cumulative frequency curve for the runners as shown on the grids above.

It is given that the interquartile time for the race is 4 minutes.

(i) State the coordinates of the point that represents the lower quartile for the cumulative frequency curve.

Answer (_____, _____) [1]

(ii) Draw the cumulative frequency curve on the grids above. [2]

(iii) "The bottom 10% runners have more consistent timings than the top 10% runners."
Do you agree with the statement above? Give your reasons clearly.

Answer

_____

_____

_____ [2]

24 **(i)** By completing the square or otherwise, express x in terms of y given that $y = x^2 - 4x + 6$.

Answer $x =$ _____ or _____ [3]

(ii) Solve the equation $38 = a^6 - 4a^3 + 6$.

Answer $a =$ _____ or _____ [3]

End of Paper

TANJONG KATONG SECONDARY SCHOOL

Preliminary Examination 2020
Secondary 4

CANDIDATE NAME

CLASS | | | INDEX NUMBER | | |

MATHEMATICS **4048/02**

Paper 2 **Thursday 13 August 2020**

2 hours and 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Curved surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all questions.

1 **(a)** Express $\frac{12xy^2}{15} \div \frac{2xy^3}{(3x)^2}$ as a single fraction in its simplest form.

Answer ..... [2]

(b) Given that $\frac{v}{3} = \sqrt{\frac{2s+v}{s-3}}$, express s in terms of v.

Answer ..... [3]

(c) **(i)** Solve the equation $\frac{3}{2x-1}=\frac{5}{x-2}$.

Answer ..... [2]

(ii) Simplify $(x^{\frac{1}{2}}+y^{\frac{1}{2}})(x^{\frac{1}{2}}-y^{\frac{1}{2}})$.

Answer ..... [1]

(d) Show that $2(9^{n+1})+3^{2n+3}-9^n$ is a multiple of 11 for all positive integer values of n.

[4]

Answer

2 20 female shoppers went to buy their groceries.
The amount of time (in minutes) each female shopper spent is shown in the stem-and-leaf diagram.

1	*a*	0	1	4	7	8	8	8	9
2	*b*	3	*c*	4	4	4	*d*		
3	2	2	5						
4									
5	*e*								

Key: 3 | 2 means 32 minutes

The modal time taken is 24 minutes while the median is 22 minutes.
Two shoppers spent 23 minutes and the range is 46 minutes.

(i) State the values of *a*, *b*, *c*, *d* and *e*.

Answer $a =$ ....., $b =$ ....., $c =$ ....., $d =$ ....., $e =$ ..... [3]

(ii) 20 male shoppers went to buy their groceries. The interquartile range of the time spent by the males shoppers was 4.5 minutes.

Use this information to make one comment comparing the time spent by the male and female shoppers.

Answer .....

.....

.....[2]

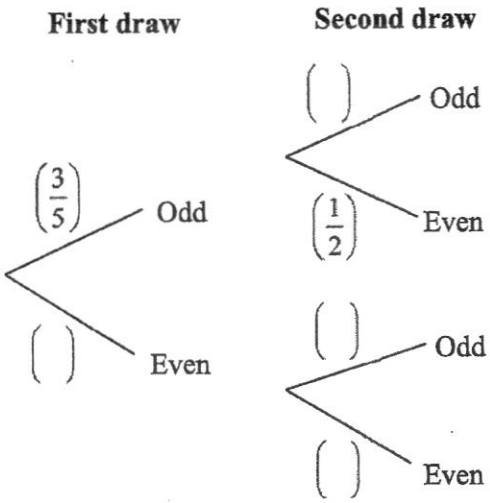
(iii) The standard deviation of the time taken by the male shoppers was 4.53 minutes. Given that the sum of the squared timings of the 20 male shoppers was 9050, find the mean time of the male shoppers for their grocery trip.

Answer .....minutes [2]

3 Five discs numbered 1, 3, 4, 6 and 7 are placed in a bag.
Two disc are taken out of the bag at random **without replacement**.

(a) Complete the tree diagram to show the probabilities of the possible outcome.

Answer



[2]

(b) Find the probability that one disc is odd and the other is even.

Answer ..... [1]

(c) Peter thinks that there is more than 10% chance that both numbers drawn is less than 4.
Is he correct or wrong? Explain your answer.

Answer .....
.....
.....
..... [1]

(d) Calculate the probability where the sum of both numbers drawn is a prime number.

Answer ..... [2]

4 **(a)** The employees of a company are offered an increase in salary.
Employees have the option to choose from either Scheme *A* or Scheme *B*.

Scheme *A* offers an increase of 8% of the current monthly salary.
Scheme *B* offers an increase of 5% of the current monthly salary plus an additional \$87 per month.

(i) Mr Lim claims that his new monthly salary would be the same under either scheme. Calculate his current monthly salary.

Answer \$ ..... [2]

(ii) The company has a total of 1.19×10^5 employees.
The ratio of employees who opted for Scheme *A* to Scheme *B* is 7 : 3.
How many more employees choose Scheme *A* over Scheme *B*?
Leave your answer in standard form.

Answer ..... [2]

(b) Mdm Ang worked for a company in Germany from 2018 to 2019.
In 2018, her monthly salary was €3500 and the exchange rate between Singapore Dollars ($) and Euro (€) then was $1 = €0.67.

(i) Calculate her monthly salary in Singapore Dollars ($) for 2018.

Answer $..... [1]

Mdm Ang received a pay cut in 2019, but she still earns the same amount of salary in Singapore dollars. The exchange rate between Singapore Dollars ($) and Euro (€) in 2019 was $1 = €0.61.

(ii) What is the percentage change in her salary for 2019?

Answer .....% [3]

5 The diagram shows the first three of a sequence of figures that are formed by squares of the same size.

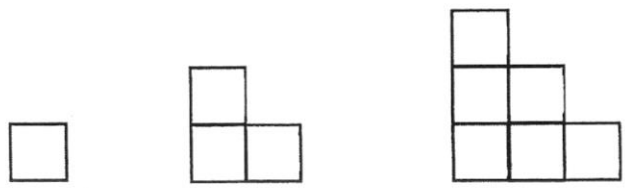


Figure 1 Figure 2 Figure 3

The number of vertical sides V and the total number of sides S are recorded in the table.

Figure Number (n)	**Number of vertical sides (V)**	**Total number of sides (S)**
1	2	4
2	5	10
3	9	18
. . .	. . .	. . .
6	p	q

(a) Find the value of p and of q.

Answer $p =$ ....., $q =$ ..... [2]

(b) The relationship between S and n is given by $S = n^2 + 3n$.
Find the value of n when the total number of sides is 270.

Answer .....[2]

(c) Find an expression, in terms of n, for V.

Answer ..... [1]

(d) Explain why the difference between the total number of sides and the number of vertical sides for any figure cannot be 100.

Answer .....

.....

.....

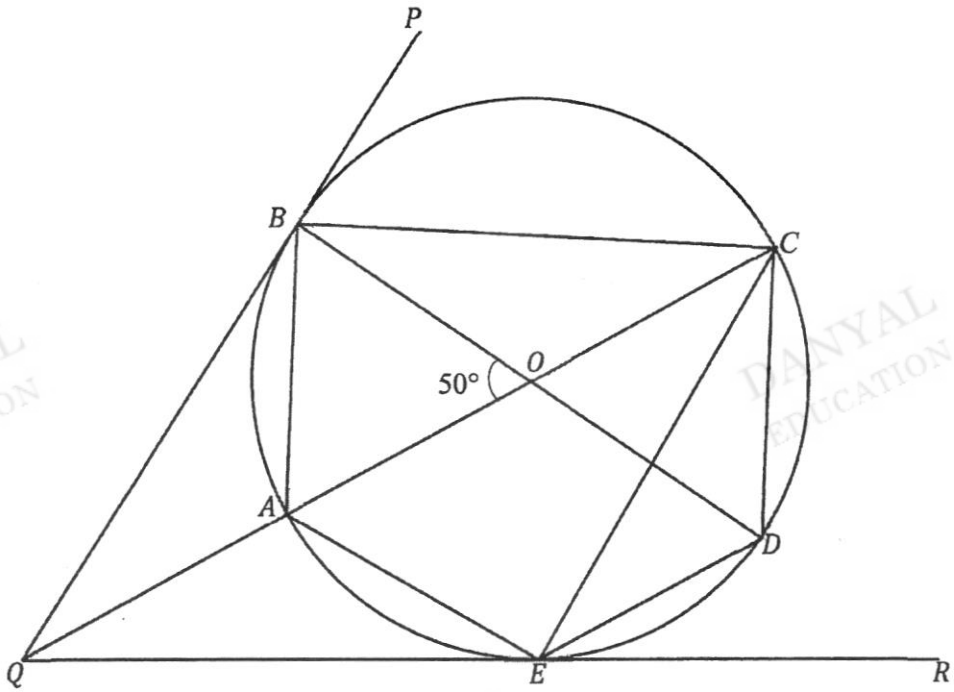
.....

.....

.....

..... [3]

6



A, B, C, D and *E* are points on the circle, centre *O*.
QP and *QR* are tangents to the circle at *B* and *E* respectively.
AC intersects *BD* at *O* and *QAOC* is a straight line.

Angle *BOA* = 50°. Find, giving reasons for each answer,

(a) angle *CED*,

Answer .....° [2]

(b) angle BQE.

Answer .....° [3]

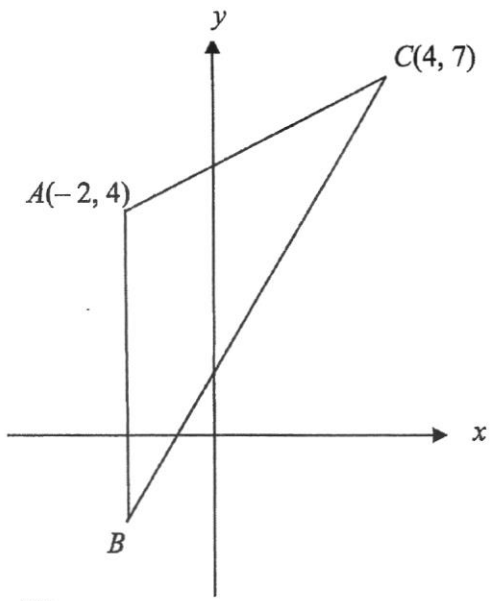
(c) Determine if BD is parallel to AE.

Answer .....

.....

..... [2]

7 In the diagram, A is the point $(-2, 4)$, and C is the point $(4, 7)$.
AB is a vertical line. The line BC has gradient $\frac{3}{2}$.



(a) Find the coordinates of B.

Answer ..... [3]

(b) Find the shortest distance from A to BC.

Answer .....units [3]

(c) The equation of line L is $\frac{4}{3}y - \frac{2}{3}x = 9$

Show how you can tell that the line does not intersect the line AC.

Answer .....

.....

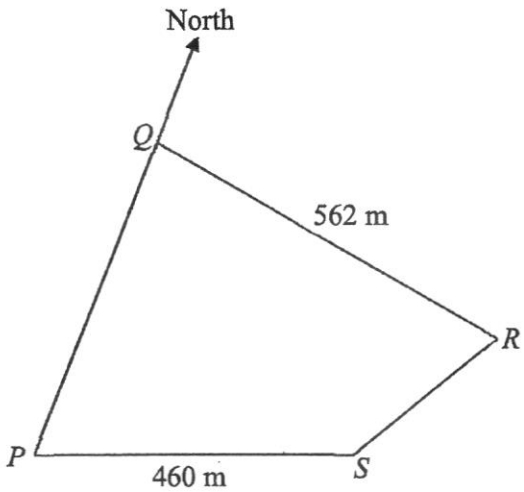
.....

..... [2]

(d) Another triangle PQR, is formed by reflecting triangle ABC about the y-axis.
Calculate the overlapping area between triangle ABC and triangle PQR.

Answer .....units^2 [3]

8 The diagram shows four points P, Q, R and S which lie on level ground in a garden.
P is due south of Q. The bearing of S from P and Q are 068° and 126° respectively.
$PS = 460$ metres and $QR = 562$ metres.



(a) Find QS,

Answer .....metres [2]

It is given that the bearing of R from Q is $098°$.

(b) Find RS.

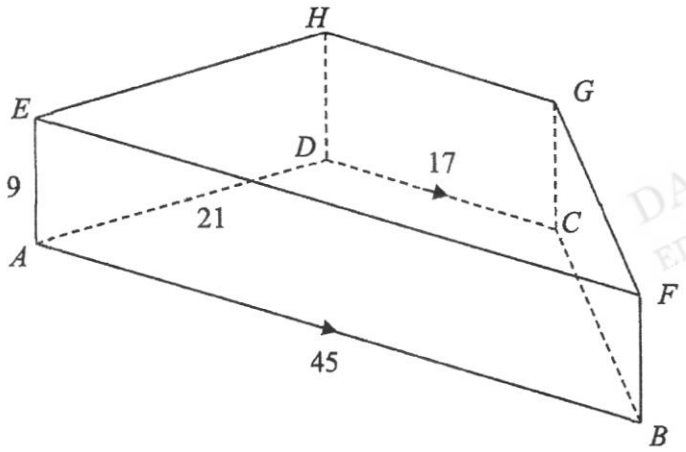
Answer .....metres [3]

(c) Joseph walks from P to Q, how far is he away from Q when he is west of R?

Answer .....metres [2]

9 The diagram shows a lecture theatre in the shape of a prism with height 9 metres.
The floor of the lecture theatre, $ABCD$, is an isosceles trapezium.
$AB = 45$ metres, $CD = 17$ metres, $AD = 21$ metres, and AB is parallel to DC.
J is a point on AB such that DJ and AB are perpendicular.

The lecture theatre is positioned on horizontal ground and the walls are vertical.



(a) Find the floor area of the lecture theatre.

Answer .....m^2 [3]

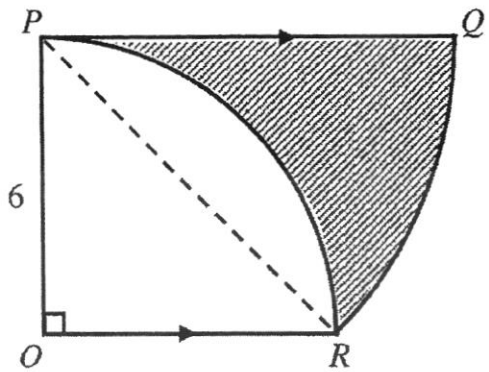
(b) Find the volume of the lecture theatre.

Answer .....m^3 [2]

(c) Find the angle of elevation of G from J.

Answer .....° [3]

10 In the diagram, POR is a quadrant of a circle with radius 6 cm. OR and PQ are parallel. QR is an arc of a circle with centre P.



(a) Calculate angle QPR in radians.

Answer ..... [1]

(b) Calculate the area of the shaded region.

Answer .....cm^2 [4]

11 A solid cylinder of height h cm and volume V cm^3 is cut from a solid sphere of radius 7 cm. The rim of each base of the cylinder touches the surface of the sphere.

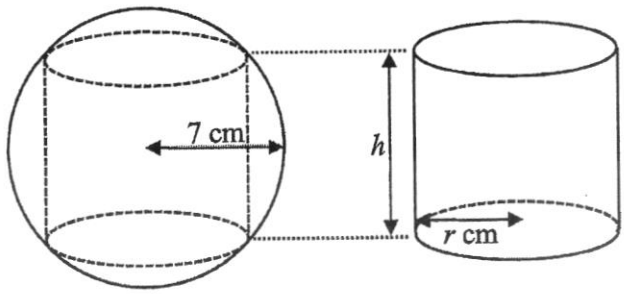


Diagram 1 **Diagram 2**

(a) The radius of the cylinder is r cm.

Show that $r^2 = 49 - \frac{h^2}{4}$.

Answer

[1]

(b) The volume of the cylinder is given by the equation $V = \pi h\left(49 - \frac{h^2}{4}\right)$.

Some corresponding values of h and V, correct to the nearest whole number, are given in the table below.

h	0	2	4	7	10	12
V	0	302	p	808	754	490

(i) Find the value of p.

Answer ..... [1]

(ii) On the grid opposite, draw the graph of $V = \pi h\left(49 - \frac{h^2}{4}\right)$ for $0 \leq h \leq 12$.

[3]

(c) Use your graph to find

(i) the maximum volume of the cylinder,

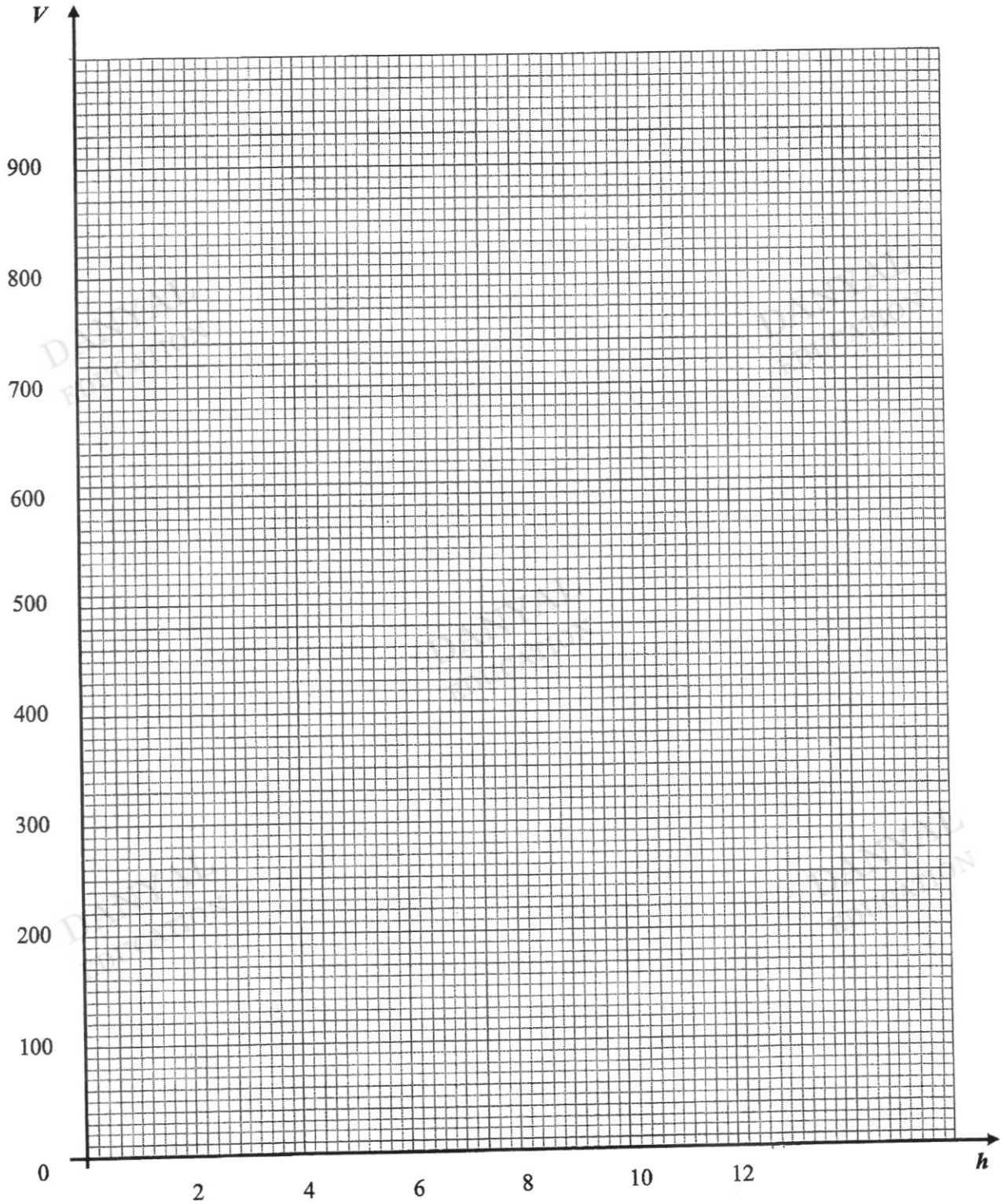
Answer ..... cm^3 [1]

(ii) the solutions to the equation $2400 = \pi h\left(196 - h^2\right)$.

Answer h = ... or ..... [2]

(d) The line $V = kh + 500$, where k is a constant, is a tangent to the curve. By drawing a suitable straight line on the graph, find the value of k.

Answer ..... [2]


V
900
800
700
600
500
400
300
200
100
0
2
4
6
8
10
12
h

12 The cash price of a new motorcycle is $25 120.
Jack paid a down payment of 20% of the price of the motorcycle and the balance with a fixed simple interest rate of 4% per annum for a period of 3 years.

(a) Calculate his monthly instalment.

Answer $..... [3]

The technical specification of his new motorbike, as stated by the manufacturer, is shown in **Table 1**.

Fuel capacity :	3.7 gallons
Fuel efficiency:	6 litres/100 km
Dimension front tyre :	120 / 70 R17
Dimension rear tyre :	180 / 55 R17
Recommended replacement:	Front tyre, every 6000km
	Rear tyre, every 2900km

Conversions:

1 gallon = 3.785 litres *1 inch = 2.54 cm*

(b) **(i)** Find the fuel capacity of his motorbike, correct to the nearest litre.

Answer .....litres [1]

Jack starts riding with a full tank of petrol. After riding for a total distance of 190 km, he filled up the petrol tank to the brim again, by pumping 13 litres of petrol.

(ii) Determine if the fuel efficiency stated by the manufacturer in **Table 1** is true.

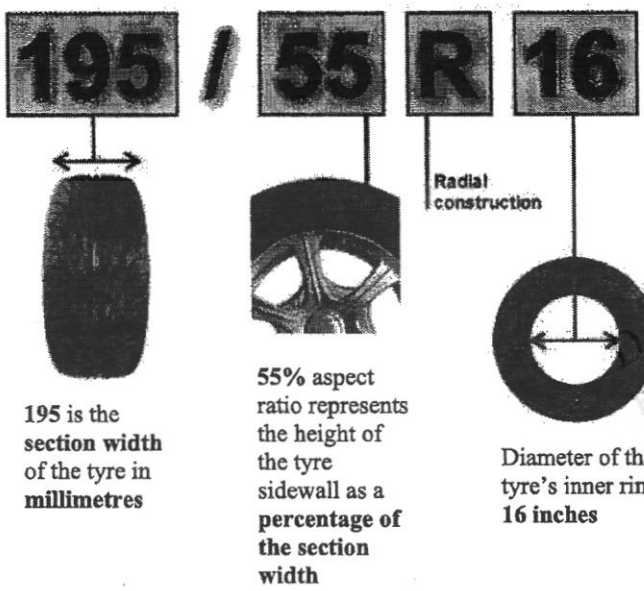
Answer .....

.....

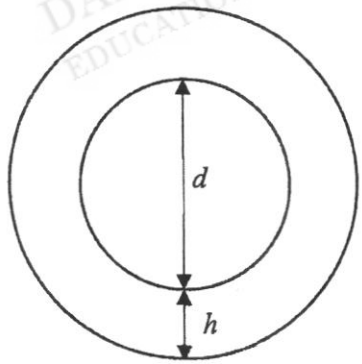
.....

..... [2]

An **example** of the dimensions for a motorbike tyre **195 / 55 R16** is explained below.



The cross-section of a wheel can be presented in **Diagram II**, where d is the diameter of inner rim, and h the height of the sidewall.



(c) Jack replaces his tyres at the recommended distance travelled.

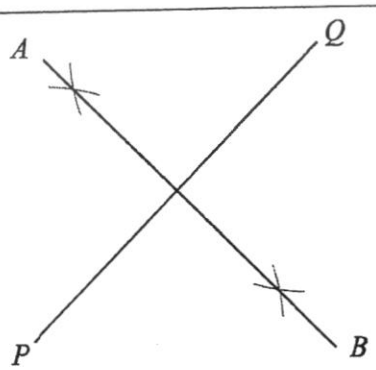
What would be the number of complete revolutions made, when Jack has to **first** replace a **front tyre**?

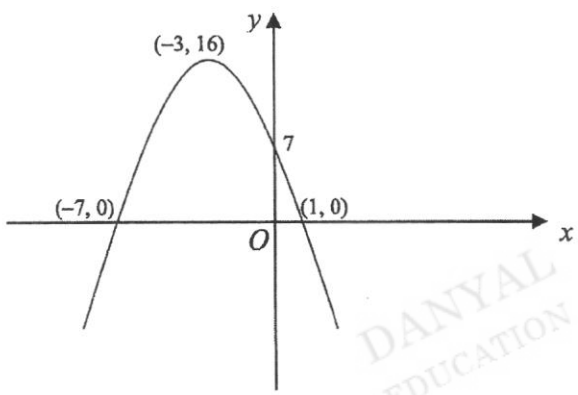
Use the information in **Table 1** and show your calculations clearly.

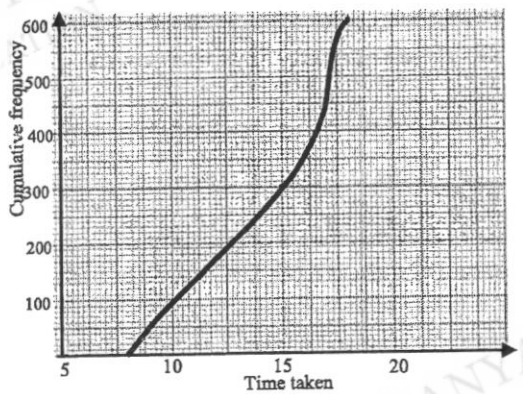
Answer ..... [5]

END OF PAPER

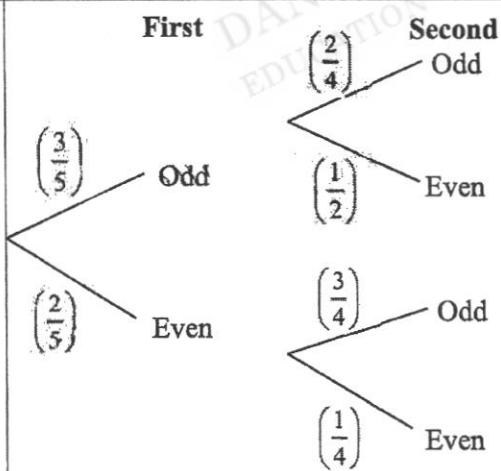
Answer Key:

No.	Answer
1	2
2	Mean is not a good indicator as there is an outlier.
3a	2^{5x} or 32^{x}
3b	4%
4i	$2^2 \times 3 \times 7$
4ii	$q = 7056$
5	818 000
6	$\frac{5(1-x)}{(3x-1)(1+2x)}$ or $\frac{5-5x}{(3x-1)(1+2x)}$
7	0.6975 or $\frac{279}{400}$
8i	$(xy^2 - z^3)(xy^2 + z^3)$
8ii	$(3a + 4b)(5a - 2)$
9i	$0.\dot{5} = \frac{5}{9}$
9ii	12.45
10	23.5%
11	5.06 or –0.0617
12i	

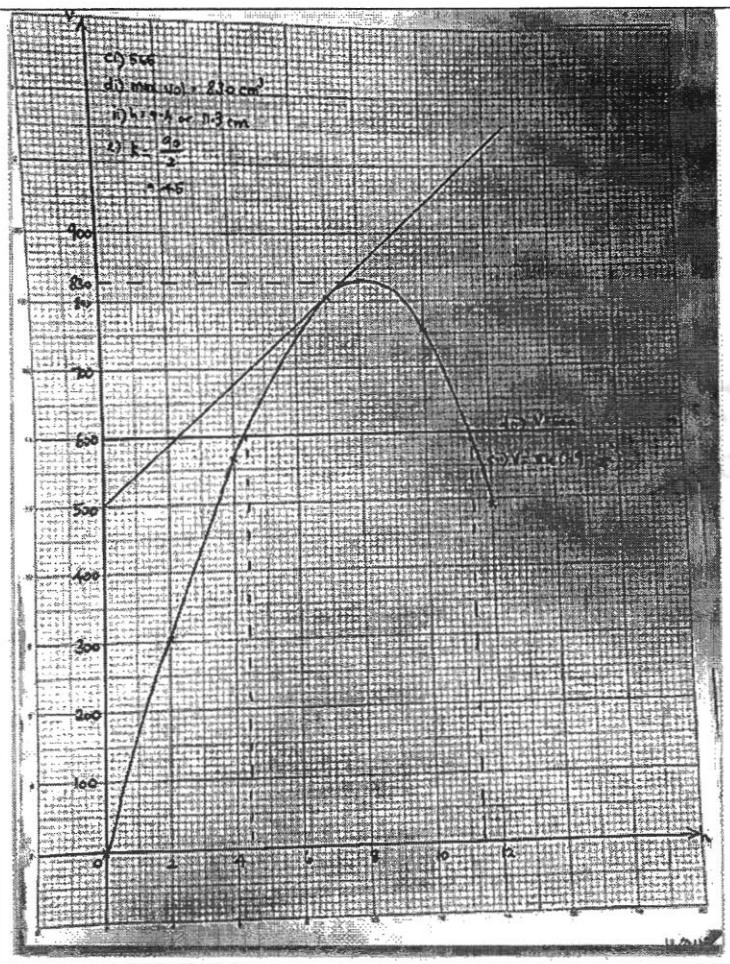
12ii	$AP = AQ = BP = BQ$ $\therefore AB$ and PQ are diagonals of a rhombus. Diagonals of rhombus bisect each other at 90°.
13	$\angle BAC = 74.05° - 68.20°$ $= 5.9°$
14a	Jane is correct, because $T_n = 2^{n-1}$ could also be a general term.
bi	$M \cap E \neq \phi$
bii	$P \subset M$
15	
16a	162°
16b	$8\frac{3}{4}$ or 8.75cm^2
17a	–80 km/h^2
17b	$p = 40, q = 100$
18	\$62800
19a	$\angle PST = \angle QRT$ ($\angle$, same seg) $\angle SPT = \angle RQT$ ($\angle$, same seg) $\angle PTS = \angle QTR$ (vert opp $\angle$s) $\therefore \Delta PTS$ and ΔQTR are similar.
19b	From 1, $\frac{PT}{QT} = \frac{TS}{TR}$ Hence $QT \times ST = PT \times RT$
20i	108 litres

20ii	67 fish
20iii	2 400 cm^3
21i	1 : 1.657
21ii	65.9 cm^3
22a	$\begin{pmatrix} 8.5 \\ 8.8 \end{pmatrix}$
22b	The cost for two rice sets and one noodle set from Shop R and Shop S respectively.
22c	3 noodles, 3 rice
23i	(13, 150)
23ii	
23iii	Agree. The spread of timing for the bottom 10% is 0.5 min whereas the spread in timing for the top 10% is 3 min.
24i	$x = 2 \pm \sqrt{y-2}$
24ii	$a = 2, -1.59$

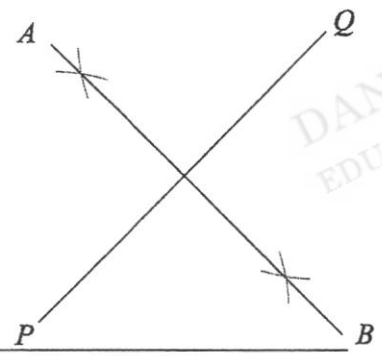
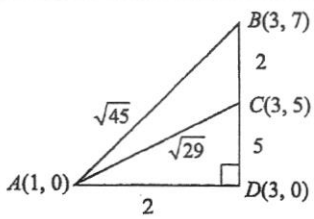
Answers

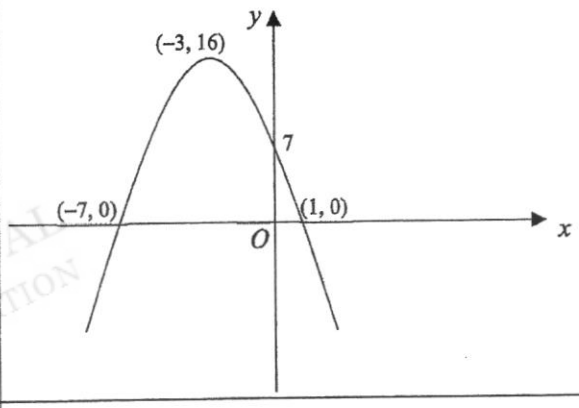
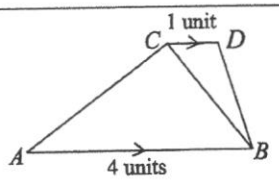
1	**(a)**	$\frac{18x^2}{5y}$
	(b)	$s = \frac{3v^2 + 9v}{v^2 - 18}$
	(ci)	$x = -\frac{1}{7}$
	(cii)	$x - y$
	(di)	$2(9^{n+1}) + 3^{2n+3} - 9^n = (3^{2n})(4)(11)$ Since 11 is a factor of $2(9^{n+1}) + 3^{2n+3} - 9^n$, and 2^{3n} is a whole number, $2(9^{n+1}) + 3^{2n+3} - 9^n$ is a multiple of 11 for all positive integer values of n.
2	**(ai)**	b = 1, c = 3, d = 4, a = 0, e = 6
	(aii)	Interquartile range for female = 24 – 17.5 = 6.5mins The time taken by the male shoppers are more consistent than the female shopper because IQR for male shoppers is smaller than female shoppers.
	(aiii)	20.78
3	**(a)**	
	(b)	$\frac{3}{5}$
	(c)	Disagree. The chance is exactly10%, not more.
	(d)	0.5
4	**(ai)**	a = $2900
	(aii)	4.76×10^4
	(bi)	$5223.88
	(bii)	-8.96%

5	**(a)**	p =27, q = 54
	(b)	n = 15
	(c)	$V = \frac{1}{2}n^2 + \frac{3}{2}n$
	(d)	$n = \frac{-3 \pm \sqrt{809}}{2}$ Since n is not an integer, it is not possible.
6	**(a)**	$\angle COD$ =50°(vertically opp angles) $\angle CED$ = 0.5 (50) ($\angle$ at centre = 2 $\angle$ *at circumference)* = 25°
	6(b)	$\angle BQE = 80°$
	6(c)	$\angle BAE = 130°$ $\angle DBA + \angle BAE = 65 + 130$ = 195° Since $\angle DBA + \angle BAE > 180°$, they are not interior angles and BD is not parallel to AE.
7	**(a)**	B: (–2, –2)
	(b)	Shortest distance = 3.33 units
	(c)	Since Gradient of AC = Gradient of line L, the lines are parallel and will not intersect because their y-intercepts are different,
	(d)	8 units^2
8	**(a)**	527.189
	(b)	265.7
	(c)	78.2
9	**(a)**	485 m^2
	(b)	4370m^3
	(c)	21.3°
10	**(a)**	0.7854 *or* $\frac{\pi}{4}$
	(b)	18
11	**(a)**	$(\frac{h}{2})^2 + r^2 = 7^2$ $r^2 = 49 - \frac{h^2}{4}$
	(bi)	565

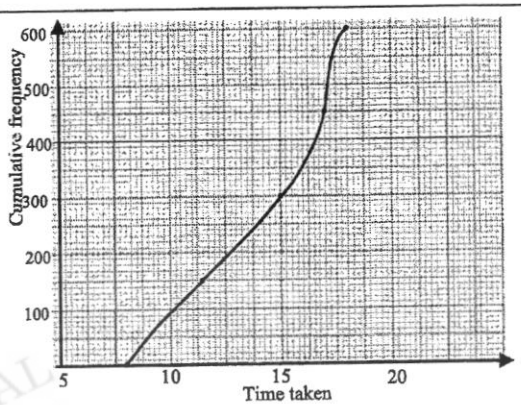
	(bii)	
	(ci)	V max = 840
	(cii)	Draw line $y = 600$ $h = 4.3$ or 11.1
	(d)	44.6
12	**(a)**	\$625.21
	(bi)	$3.785 \times 3.7 = 14l$
	(bii)	It is not true because he travelled less than 216.67km. or It is not true because it consumes more than 6 litres.
	(c)	3184514

No.	Solution/Key Steps	Remarks	
1	By trial, $x=1;\quad \frac{x}{2}+\frac{2}{x}=2.5$ $x=2;\quad \frac{x}{2}+\frac{2}{x}=2$ $x=3\quad \frac{x}{2}+\frac{2}{x}=2\frac{1}{6}$ $x=4\quad \frac{x}{2}+\frac{2}{x}=2.5$ Min value = 2	B1	**1**
2	Mean is not a good indicator as there is an outlier.	B1 o.e.	**1**
3(a)	$\left(16^{12x^2}\right)^{\frac{5}{48x}}=\left(2^4\right)^{12x^2\times\frac{5}{48x}}$ Ans: 2^{5x} or 32^x	M1 2^4 or $(16)^{\frac{5x}{4}}$ seen A1	
(b)	Ans: 4%	A1	**3**
4(i)	Ans: $2^2\times3\times7$	A1	
(ii)	$(2^2\times3\times7)\times2^4\times3^2\times7^2$ or $(2^2\times3\times7)\times2\div(3\times7)$ $q=2^2\times3^2\times2^3$ or $2\div(3\times7)$ Ans: q = 7056 or 2/21	B1 $2^4\times3^2\times7^2$ or $2\div(3\times7)$ soi B1	**3**
5	$1.1\,x=900\,000$ $x=818\,181.8$ Ans: 818 000	B1 oe B1 3 sf or better	**2**
6	$\frac{2}{3x-1}-\frac{3}{1+2x}$ $=\frac{2(1+2x)-3(3x-1)}{(3x-1)(1+2x)}$ $=\frac{5-5x}{(3x-1)(1+2x)}$ or $\frac{5(1-x)}{(3x-1)(1+2x)}$	B1 single fraction B1 accept deno. in expanded form	**2**
7	$1-0.55^2$ $=0.6975$ Ans: 0.6975 or $\frac{279}{400}$	M1 alternative method A1	**2**
8(i)	$x^2y^4-z^6$ $=(xy^2)^2-(z^3)^2$ $=(xy^2-z^3)(xy^2+z^3)$	B1 Diff of 2 sq seen B1	
(ii)	$15a^2-6a+20ab-8b$ $=3a(5a-2)+4b(5a-2)$ $=(3a+4b)(5a-2)$	B1 Use of grouping B1	**4**

No.	Solution/Key Steps	Remarks	
9(i)	Ans: $0.\dot{5}=\frac{5}{9}$	B1	
(ii)	$12.45 \leqslant 12.5 < 12.55$ Ans: 12.45	B1	**2**
10	$F=\frac{k}{d^2}$ $F_2=\frac{k}{(0.9d)^2}$ $=1.23456\ \frac{k}{d^2}$ Ans: % change = 23.5%	B1 variation relation seen B1 1.1^2 seen B1	**3**
11	$4(2x-5)^2-105=0$ $(2x-5)^2=26.25$ $(2x-5)=\pm\sqrt{26.25}$ $x=5.06$ or -0.0617	M1 $(2x-5)^2$ as subject or solve by formula A1 both	**2**
12(i)		B1 correct line with constructions seen	
(ii)	$AP=AQ=BP=BQ$ $\therefore AB$ and PQ are diagonals of a rhombus. Diagonals of rhombus bisect each other at 90°.	B1 "rhombus" seen B1 "⊥ bisector" seen	**3**
13	 $\tan BAD=\frac{7}{2}$, $\angle BAD=74.05°$ $\tan CAD=\frac{5}{2}$ $\angle CAD=68.20°$ $\angle BAC=74.05°-68.20°$ $=5.9°$	M1 Using tangent ratio M1 Finding angles B1 Clear presentation A1 1 dp or better	**4**

No.	Solution/Key Steps	Remarks	
14(a)	Jane is correct, because $T_n = 2^{n-1}$ could also be a general term.	B1 with justification seen B1 formula seen	
b (i)	$M \cap E \neq \phi$	B1 oe	
b (ii)	$P \subset M$	B1 oe	**4**
15		B1 max pt B1 All intercepts C1 Symm curve	**3**
16(a)	Let one of the ext. angle be $x°$ $x+2x+2x+3x+6(36)=360$ $x=18$ Largest int. angle $=180°-18°=162°$ Ans: 162°	M1 sum of exterior angle, oe. A1	
(b)	 $\frac{\text{Area of triangle } ACD}{\text{Area of triangle } ABC} = \frac{1}{4}$ Area of triangle $ACD = \frac{1}{4}(35) = 8\frac{3}{4}$ Ans: $8\frac{3}{4}$ or 8.75cm^2	B1 Diagram soi B1	**4**
17(a)	$\text{accn} = -\frac{40}{0.5}$ Ans: $-80\ \text{km/h}^2$	B1	
(b)	0800 to 0830 Dist travelled $= \left(\frac{100+60}{2}\right) \times 0.5 = 40$ km 0830 to 0930 Dist travelled $= 60 \times 1 = 60$ Ans: $p = 40$, $q = 100$	M1 A1, A1	**4**

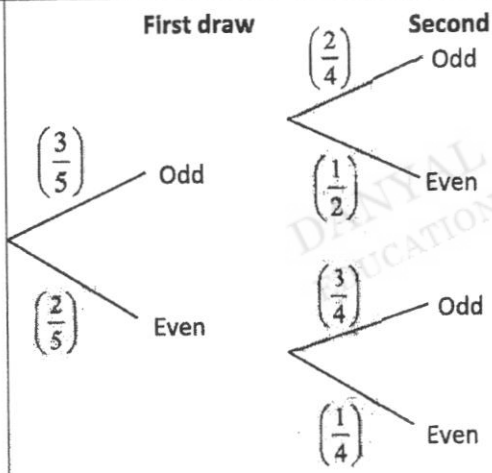
No.	Solution/Key Steps	Remarks	
18	$P\left(1+\frac{3}{100}\right)^5 - P > 10000$ $P\left(1.03^5 - 1\right) > 10000$ $P > 62784.85713$ Ans: \$62800	B1 $P\left(1+\frac{3}{100\times 4}\right)^{5\times 4}$ seen Accept equation form B1 B1	**3**
19(a)	$\angle PST = \angle QRT$ ($\angle$, same seg) $\angle SPT = \angle RQT$ ($\angle$, same seg) $\angle PTS = \angle QTR$ (vert opp $\angle$s) $\therefore \Delta PTS$ and ΔQTR are similar.	B2 for any 2 reasons seen B1 Statement seen. Only if 1st B2 awarded.	
(b)	From 1, $\frac{PT}{QT} = \frac{TS}{TR}$ Hence $QT \times ST = PT \times RT$	B1 award only if connection to (a) seen	**4**
20(i)	Vol of water $= 0.1 \times 0.03 \times 0.04 \times 90\%$ Ans: 108 litres	B1 conversion or 90% seen B1	
(ii)	$108 \div 1.6$ Ans: 67 fish	B1 $\frac{\text{their water vol}}{\text{vol per fish}}$ B1	
(iii)	$1600 + 200 \times 4$ Ans: 2 400 cm^3	B1	**5**
21(i)	Height R : Height L $= \sqrt[3]{330} : \sqrt[3]{1500}$ Ans: 1 : 1.657	B1 cube root seen B1 3 sf or better	
(ii)	Area R : Area L $= 1^2 : 1.657^2$ $= 1 : 2.744$ $\therefore \frac{x}{24} = \frac{2.744}{1}$ Ans: $x = 65.9$ cm^3	B1 Squaring seen B1 cao	**4**
22(a)	$\begin{pmatrix} 2.5 & 3.5 \\ 2.8 & 3.2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ Ans: $\begin{pmatrix} 8.5 \\ 8.8 \end{pmatrix}$	B1, B1	
(b)	The cost for two rice sets and one noodle set from Shop R and Shop S respectively.	B1 2 rice, 1 noodle seen B1 Shop R, Shop S seen	
(c)	Ans: 3 noodles, 3 rice	B1, B1	**6**

No.	Solution/Key Steps	Remarks	
23(i)	Ans: (13, 150)	B1	
(ii)		B1 (13, 150) seen B1 Smooth curve	
(iii)	Agree. The spread of timing for the bottom 10% is 0.5 min whereas the spread in timing for the top 10% is 3 min.	B1 (their value of t at f=60) – 8 & 18 – (their value of t at f=540) seen B1 Spread/variance/consistency of timing must be seen. Reject terms of description: "Difference" / "Range"	**5**
24(i)	$x^2 - 4x = y - 6$ $x^2 - 4x + 2^2 = y - 6 + 2^2$ $(x-2)^2 = y - 2$ $x = 2 \pm \sqrt{y-2}$	B1 $+2^2$ seen B1, B1	
(ii)	$a^6 - 4a^3 - 32 = 0$ $(a^3)^2 - 4(a^3) - 32 = 0$ $(a^3 - 8)(a^3 + 4) = 0$ $a^3 = 8$ or $a^3 = -4$ Ans: $a = 2, -1.59$	M1 A1, A1	**6**

Secondary 4 Mathematics Prelim 2020

Paper 2 Marking Scheme

Question No		Solutions	Marks	Remarks
1	(a)	$\frac{12xy^2}{15}\times\frac{9x^2}{2xy^3}$ $=\frac{18x^2}{5y}$	M1 A1	Expand $(3x)^2$
	(b)	$\left(\frac{v}{3}\right)^2=\frac{2s+v}{s-3}$ $sv^2-3v^2=18s+9v$ $s(v^2-18)=3v^2+9v$ $s=\frac{3v^2+9v}{v^2-18}$	M1 M1 A1	Square both sides Isolate 's'
	(ci)	$\frac{3}{2x-1}=\frac{5}{x-2}$ $3x-6=10x-5$ $7x=-1$ $x=-\frac{1}{7}$	 M1 A1	 Remove fraction by cross multiplying
	(cii)	$(x^{\frac{1}{2}}+y^{\frac{1}{2}})(x^{\frac{1}{2}}-y^{\frac{1}{2}})$ $=(x^{(\frac{1}{2})2}-y^{(\frac{1}{2})2})$ $=(x-y)$	 B1	
	(di)	$2(9^{n+1})+3^{2n+3}-9^n = 2(3^{2n+2})+3^{2n+3}-3^{2n}$ $=2(3^{2n})(3^2)+(3^{2n})(3^3)-3^{2n}$ $=(3^{2n})(18)+(3^{2n})(27)-3^{2n}$ $=(3^{2n})(18+27-1)$ $=(3^{2n})(44)$ $=(3^{2n})(4)(11)$ Since 11 is a factor of $2(9^{n+1})+3^{2n+3}-9^n$, and 2^{3n} is a whole number, $2(9^{n+1})+3^{2n+3}-9^n$ is a multiple of 11 for all positive integer values of n.	B1 B1 B1 B1	Base 3 seen throughout Law of addition Factorise Conclude
			12 marks	

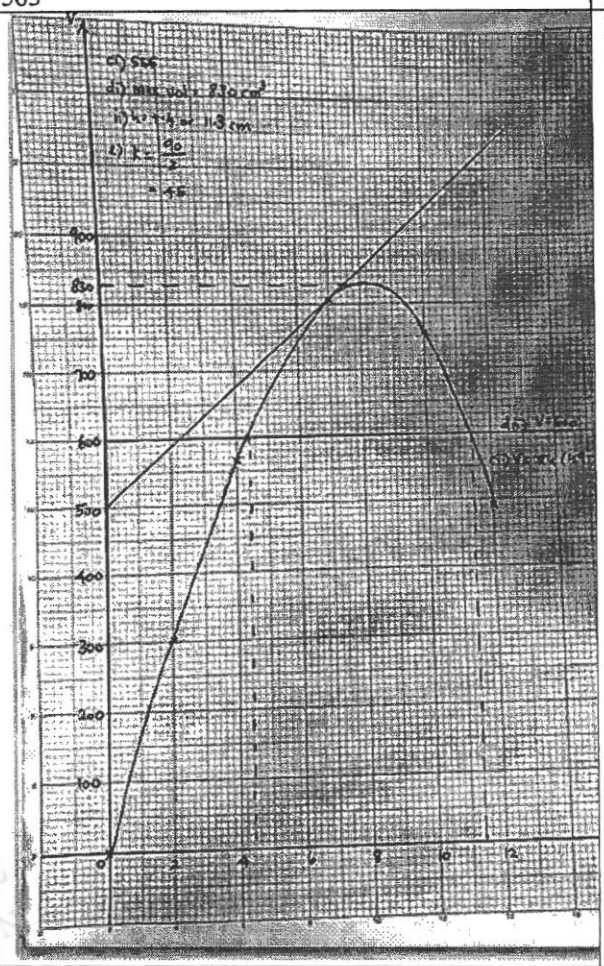
Question No		Solutions	Marks	Remarks
2	(ai)	b = 1, c = 3, d = 4, a = 0, e = 6	B1 B1 B1	Use median Use mode, both correct Use range, both correct
	(aii)	Interquartile range for female = 24 – 17.5 = 6.5mins The time taken by the male shoppers are more consistent than the female shopper because IQR for male shoppers is smaller than female shoppers.	B1 B1	Supported by comparison of IQR
	(aiii)	$4.53 = \sqrt{\frac{9050}{20} - mean^2}$ $mean^2 = 431.979$ $mean = 20.78$	M1 A1	
			7 marks	
3	(a)		A2	o.e Deduct 1 mark for each error A1, A0
	(b)	$\left(\frac{3}{5}\times\frac{2}{4}\right)+\left(\frac{2}{5}\times\frac{3}{4}\right)=\frac{3}{5}$	A1	
	(c)	P(select 1 or 3) = $\left(\frac{2}{5}\times\frac{1}{4}\right)=\frac{1}{10}$ Disagree. The chance is exactly10%, not more.	B1	
	(d)	<table><tr><td></td><td>1</td><td>3</td><td>4</td><td>6</td><td>7</td></tr><tr><td>1</td><td>×</td><td>4</td><td>5</td><td>7</td><td>8</td></tr><tr><td>3</td><td>4</td><td>×</td><td>7</td><td>9</td><td>10</td></tr><tr><td>4</td><td>5</td><td>7</td><td>×</td><td>10</td><td>11</td></tr><tr><td>6</td><td>7</td><td>9</td><td>10</td><td>×</td><td>13</td></tr><tr><td>7</td><td>8</td><td>10</td><td>11</td><td>13</td><td>×</td></tr></table> P(Sum of two numbers drawn is a prime no.) = 0.5	M1 A1	Possibility diagram drawn
			6 marks	

Question No		Solutions	Marks	Remarks
4	(ai)	Let his salary be a $1.08\,a = 1.05\,a + 87$ $0.03a = 87$ $a = \$2900$	M1 A1	Form eqn
	(aii)	$(1.19\times10^5)\times\frac{4}{10}$ $=4.76\times10^4$	M1 A1	
	(bi)	$3500\div0.67$ $=\$5223.88$	B1	
	(bii)	Pay in 2019 = Their ci) ×0.61 = €3186.5668 Percentage change = $\frac{3500-Their\_2019\_pay}{3500}\times100$ $=-8.96\%$	M1 M1 A1	2019 pay with their *ci)*
			8 marks	
5	(a)	p =27, q = 54	B1 B1	
	(b)	$270=n^2+3n$ $n^2+3n-270=0$ $(n-15)(n+18)=0$ $n=15$	M1 A1	Solve
	(c)	$V=\frac{1}{2}n^2+\frac{3}{2}n$	B1	
	(d)	$100=S-V$ $100=n^2-3n-(\frac{1}{2}n^2+\frac{3}{2})$ $100=\frac{1}{2}n^2+\frac{3}{2},$ $n^2+3n-200=0$ $n=\frac{-3\pm\sqrt{3^2-4(1)(-200)}}{2(1)}$ $=\frac{-3\pm\sqrt{809}}{2}$ Since n is not an integer, it is not possible.	M1 M1 A1	Form eqn in n Solve with quadratic formula Conclude with reason
			8 marks	

Question No		Solutions	Marks	Remarks
6	(a)	$OB = OC$ $\angle OBC = 50° \div 2 (ext \angle \Delta)$ $= 25°$ $\angle CED = 25° (\angle same\ seg)$ *or* $\angle ABC = 90° (\angle in\ semicircle)$ $\angle OBA = (180° - 50°) \div 2$ $= 65°$ $\angle OBC = 90° - 65°$ $= 25°$ $\angle CED = 25° (\angle same\ segment)$ Or $\angle ABO = (180 - 50) \div 2$ (Base angles of isos triangle OBA) $= 65°$ $\angle CED = 180 - 90 - 65$ (angles in the opp. Seg) $= 25$ or $\angle COD = 50°$(vertically opp angles) $\angle CED = 0.5\ (50)$ ($\angle$ at centre $= 2 \angle$ at circumference) $= 25°$	B1 B1 OR B1 B1 B1 OR B1 B1 OR B1 B1	With reason With reason
	6(b)	$\angle OBQ = 90° (\tan \perp rad)$ $\angle BQO = 90° - 50°$ $= 40°$ $since\ BQ = QE (\tan ext\ point)$ $\angle BQE = 40° \times 2$ $= 80°$	B1 B1 B1	
	6(c)	$\angle BAE = 130°$ $\angle DBA + \angle BAE = 65 + 130$ $= 195°$ Since $\angle DBA + \angle BAE > 180°$, they are not interior angles and BD is not parallel to AE. OR $\angle OEQ = 90°$(Tgt perpendicular to radius) $\angle AOF = 180° - 90 - 40$ (Angle sum of triangle) $= 50°$ $\angle OAE = 0.5(180 - 50°)$(Base angles of isos triangle) $= 65°$	M1 A1 OR M1	Find 1 more relevant angle to explain With reason related to parallel lines

Question No		Solutions	Marks	Remarks
		If $BD \parallel AE$, $\angle COD = \angle CAE$ (corresponding angles). Since the angles are not equal, BD and AE are not parallel.	A1	
			7 marks	
7	(a)	$\frac{y-7}{x-4}=\frac{3}{2}$ $2y-14=3x-12$ $2y=3x+2$ or $y=\frac{3}{2}x+1$	M1	Find eqn of BC
		$2y=3(-2)+2$ $y=-2$	M1	Sub $x=-2$ into their eqn
		B: $(-2,-2)$	A1	
	(b)	$BC=\sqrt{[7-(-2)]^2+[4-(-2)]^2}$ $=\sqrt{117}$	M1	Find length of BC
		$0.5(6)(6) = 0.5$(shortest distance) $\sqrt{117}$ Shortest distance = 3.33 units	M1 A1	Use Area or trigo
	(c)	Gradient of AC is $\frac{7-4}{4-(-2)}=\frac{1}{2}$.	B1	
		Since Gradient of AC = Gradient of line L, the lines are parallel and will not intersect because their y-intercepts are different,	B1	
	(d)	$\frac{y-7}{x-4}=\frac{1}{2}$ Equation of AC: $y=\frac{1}{2}x+5$	M1	Find y-intercept of AC.
		y-intercept: 5		
		Overlapping area: $2\times\frac{1}{2}\times4\times2=8 units^2$	M1 A1	o.e
			11 marks	
8	(a)	$\angle PQS=180°-126°=54°$ $\frac{QS}{\sin 68}=\frac{460}{\sin 54}$	M1	Apply sine rule
		$QS=527.189$	A1	
	(b)	$\angle RQS=126-98=28$ $RS^2=562^2+Their\_QS^2-2(562)(their\_QS)\cos 28$	M2 A1	Apply cos rule
		$RS=265.7$		
	(c)	Let the distance be x $\angle XQR=180-98=82$		
		$\cos 82=\frac{x}{562}$	M1	Use cosine
		$x=78.2$	A1	
			7 marks	

Question No		Solutions	Marks	Remarks
9	(a)	Height of trapezium $= \sqrt{21^2 - 14^2}$ $= \sqrt{245}$ $= 15.652$ units	M1	Find height of trapezium
		Floor area $= \frac{1}{2}(17+45)\times\sqrt{245}$ $= 485.226$ $= 485\ m^2$	M1 A1	
	(b)	Volume $= Their(a)\times 9$ $= 4367.04$ $= 4370m^3$	M1 A1	$\times 9$
	(c)	$JC^2 = 245+17^2$ $JC = 23.1084$ $\tan\angle GJC = \frac{9}{23.1084}$ $\angle GJC = 21.3°$	M1 M1 A1	Find JC
			8 marks	
10	(a)	$\angle QPR = \angle PRO$ $\tan\angle PRO = 1$ $\angle PRO = 0.7854$ *or* $\frac{\pi}{4}$	B1	
	(b)	$PR = \sqrt{72}$ Area of sector QPR $= \frac{1}{2}(\sqrt{72})^2(\frac{\pi}{4})$ $= 9\pi\ cm^2$ Shaded area $= 9\pi - [\frac{1}{2}(6)^2(\frac{\pi}{2}) - \frac{1}{2}(6)^2\sin(\frac{\pi}{2})]$ $= 18$	B1 M1 M1 A1	Area of sector Area of segment
			5 marks	

Question No		Solutions	Marks	Remarks
11	(a)	$(\frac{h}{2})^2 + r^2 = 7^2$ $r^2 = 49 - \frac{h^2}{4}$	B1 AG	Pyt theorem seen
	(bi)	565	B1	
	(bii)		P2 C1	2 points wrong P1
	(ci)	V max = Accept 820 to 840	B1	
	(cii)	$2400 = \pi h(196 - h^2)$ $600 = \frac{\pi h(196 - h^2)}{4}$ $600 = \pi h\left(49 - \frac{h^2}{4}\right)$ Draw line $y = 600$ $h = 4.3$ or 11.3 (+/- 0.1)	B2	
	(d)	Tangent drawn, y –int =500 $k = \frac{870 - 500}{8.3}$ $k = 44.6$	L1 B1	Accept 43 to 45
			10 marks	

Question No		Solutions	Marks	Remarks
12	(a)	Principal = 0.8×25120 $= \$20096$ Monthly instalment $= \frac{(Their\_P + Their\_P \times 0.04 \times 3)}{36}$ $= \$625.21$	B1 M1 A1	Correct P Find I = PRT
	(bi)	$3.785 \times 3.7 = 14l$	B1	
	(bii)	$6l \rightarrow 100km$ $13l \rightarrow 216.67km$ It is not true because he travelled less than 216.67km. Fuel efficiency is lower than stated. *or* $190km \rightarrow 13l$ $100km \rightarrow 6.84l$ It is not true because it consumes more than 6 litres. Fuel efficiency is lower than stated. Or 100 km used 6 litres 190 km used 11.4 litres It is not true because he should have just used 11.4 litres, not 13 litres. Since he used up more than 11.4 litres, fuel efficiency is lower than stated. Or 13 litres travel 190km 6 litres travel 87.7km It is not true as he only travelled 87.7km instead of 100km with 6 litres. Fuel efficiency is lower than stated. Or comparison for how much distance can be travelled per 1 litre: 13 litres travel 190km (actual) 1 litre travel 14.61km (actual) 6 litres travel 100km (as stated) 1 litre travel 16.67km (as stated) Since he can only travel 14.61km per litre of petrol instead of 16.67 km per litre of petrol as stated, fuel efficiency is actually lower.	B1 B1	190km < 216.67km $6.84l > 6l$

Question No		Solutions	Marks	Remarks
	(c)	Front tyre: $h = 0.7 \times 120$ $= 84$ mm or 8.4 cm	B1	Find height using % of width
		$d=17(2.54)+2$(their height) $= 59.98$ cm OR $r = 17\ (2.54)(0.5)$ + their height $= 29.99$ cm	M1	Find diameter or radius in cm using their height
		Circumference (front tyre) $= 2\pi$ (their radius) $= 188.43$ cm $= 1.884$ m $= 0.001884$ km	M1	Find circumference
		Number of complete revolutions $= 6000 \div$ (their circumference) $= 3184514$	M1 A1	Find no. of revolutions using circumference
			11 marks	