BP~313

Name: .... Register no: .... Class: ....

NGEE ANN SECONDARY SCHOOL

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PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **28 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

Instructions to Candidates

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Write in dark blue or black pen.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Total	**/90**

Checked by student: ____________________ Date: ____________

This document consists of 23 printed pages and 1 blank page.

DANYAL EDUCATION

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots\dots\dots + \binom{n}{r} a^{n-r} b^r + \dots\dots\dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots\dots\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
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$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Express $-2x^2-12x-14$ in the form $a(x-h)^2+k$. [2]

(b) Using your answer to part **(a)**, explain why the maximum value of $-2x^2-12x-14$ is k. [2]

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2 Solve the equation $x = 3\sqrt{3} + \frac{12}{x}$, where $x \neq 0$, giving your answer in its simplest surd form. [4]

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3 The spread of bird flu in a certain duck farm is given by $B(t) = \frac{300}{1+e^{5-t}}$, where t is the number of days since the flu first appeared, and $B(t)$ is the total number of ducks which have caught the flu to date.

(a) Estimate the initial number of ducks infected with the flu.
Give your answer correct to the nearest integer. [1]

(b) If the day that the flu first appeared is Day 0, find out on which day would the number of infected ducks first reach 100. [2]

DANYAL EDUCATION

(c) Explain clearly why the number of infected ducks will never exceed 300. [2]

4 **A calculator must not be used in this question.**

(a) Show that $\tan 15° = 2 - \sqrt{3}$. [3]

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(b) Using your answer in part **(a)**, find an expression for $\sec^2 15°$ in the form $a + b\sqrt{3}$ where a and b are integers. [2]

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5 **(a)** Express $\frac{48x^3+86x^2-2x-32}{6x^2+13x+6}$ as the sum of a polynomial and a proper fraction. [2]

(b) Hence, express $\frac{48x^3+86x^2-2x-32}{6x^2+13x+6}$ as the sum of a polynomial and partial fractions. [4]

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6 **(a)** Find the range of values of h for which the equation $x^2-(h+1)x+h+9=0$ has no real roots. [4]

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(b) Explain why the equation $5x^2-kx+2k^2+8=0$ has no real roots for all real values of k. [2]

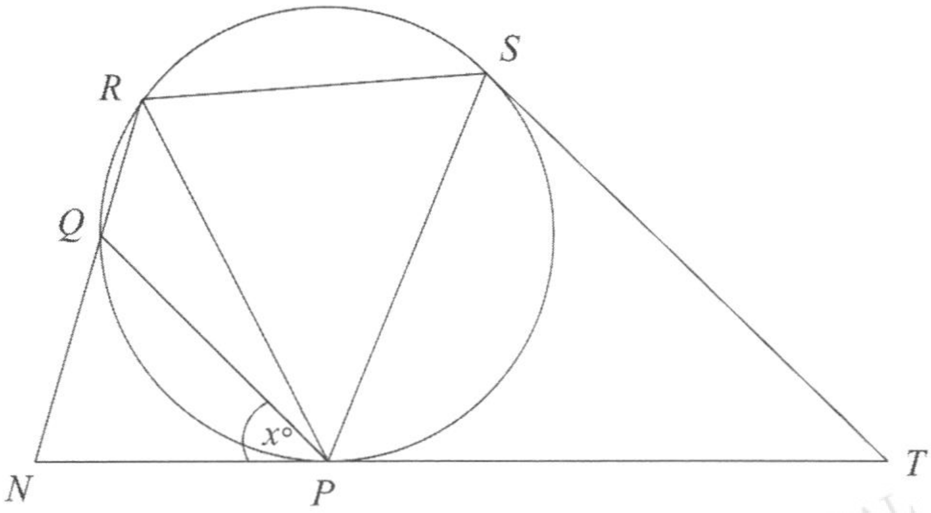
7 The equation of a curve is $y = 3\cos 2x - 1$.

(a) State the maximum and minimum values of y. [1]

(b) Sketch the graph of $y = 3\cos 2x - 1$ for $0 \le x \le 2\pi$. [3]

(c) By drawing a straight line on the same diagram in **(b)**, determine the number of solutions for the equation $3\cos 2x = -\frac{2}{\pi}x + 2$ for $0 \le x \le 2\pi$. [2]

8



In the diagram, P, Q, R and S lie on a circle such that QP is parallel to ST.

Lines NT and ST are tangents to the circle at P and S respectively. NQR is a straight line and angle $NPQ = x°$.

(a) Prove that angle $QRS = 90° + \frac{x°}{2}$. [4]

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(b) It is given further that $QR // PS$.
Show that triangle PRS is isosceles. [2]

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9 Given that $\int_0^4 f(x)dx = \int_4^9 f(x)dx = 12$, find

(a) $\int_0^9 f(x)dx$, [1]

(b) $\int_0^4 f(x)dx + \int_9^4 f(x)dx$, [2]

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(c) find the value of the constant m for which $\int_4^9 [f(x) - mx]dx = 0$. [3]

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10 A point P (2, 4) lies on the curve $y = \dfrac{4}{(x-3)^2}$.

(a) Find the equation of the normal at point P. [4]

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(b) The tangent at another point Q on the curve is parallel to the normal at point P.

Show that the coordinates of Q is $(7, \frac{1}{4})$. [3]

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11 (a) A thin circular metal disk changes size (but not shape) when cooled. The disk is decreasing at a rate of 12π mm^2/s. When the disk has a radius of 200 mm, find the rate of change of the radius of the disk at this instant. [3]

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(b) A 3-dimensional printer prints a chocolate in the shape of a cone. During the printing process, the volume of the cone is increasing at a constant rate of 3π cm^3/s. The height of the cone is always twice of the radius. Find the rate of increase of the radius when the volume of the chocolate cone is 9π cm^3. [4]

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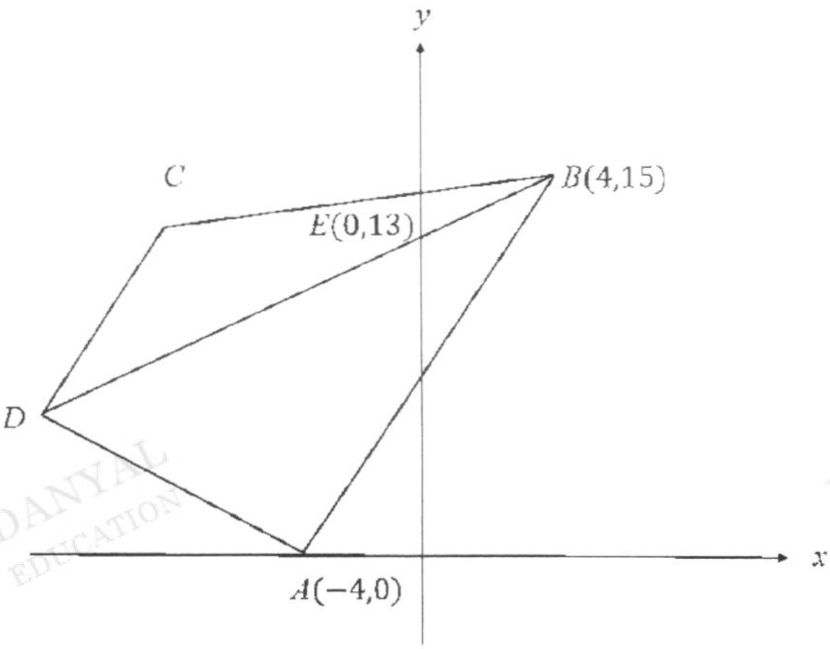

The diagram shows a trapezium with vertices $A(-4,0)$, $B(4,15)$, C and D.
The diagonal BD of the trapezium intersects the y-axis at $E(0,13)$.
Given that $AB = 2DC$ and $AD = 13$ units, find the perimeter of $ABCD$. [8]

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Continuation of working space for Question 12

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13 A small forest has been declining in its tree population since 1980. To assess this decline, a census of the tree population of the forest was conducted on January 1st at intervals of ten years from 1980 to 2020. The table below shows the data from the census.

Year	1980	1990	2000	2010	2020
x (decade)	0	1	2	3	4
y (number of trees)	186624	139968	104976	78732	59049

It is believed this decline follows a $y = e^p q^x$ curve, where p and q are constants.

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(a) Draw a suitable straight line graph to show that the model is valid for the years 1980 to 2020. [3]

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(b) Estimate the number of trees in the forest on 1st January 2040. [2]

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(c) Use your graph to estimate the value of q. [2]

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(d) Give a reason why this model might not be accurate in later decades. [1]

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14 The velocity, v m/s, of a particle travelling in a straight line at time t seconds after leaving a fixed point O, is given by $v = 2t^2 + (2-3k)t + 4k - 5$, where k is a constant.

(a) Given that the minimum velocity occurs at $t = \frac{13}{4}$, show that the value of $k = 5$. [3]

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(b) Find the time(s) the particle comes to instantaneous rest. [2]

(c) Find the distance travelled in the first 7 seconds after passing through O. [4]

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(d) Given that R is the point when the particle has zero acceleration, and P is the point when the particle first comes to rest, determine, with full working whether R is nearer to O or to P. [3]

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Name: .... Register no: .... Class: ....

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PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **29 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

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DANYAL EDUCATION

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n\,(n-1)\ldots(n-r+1)}{r!}$

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2. TRIGONOMETRY

Identities

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$$a^2 = b^2 + c^2 - 2bc \cos A$$
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1 **(a)** Express, in terms of π, the principal value of $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$. [1]

(b) State the values between within each of the following must lie:

(i) the principal value of $\sin^{-1} x$, [1]

(ii) the principal value of $\cos^{-1} x$. [1]

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2 **(a)** In the binomial expansion of $\left(\sqrt{x}+\dfrac{h}{\sqrt{x}}\right)^8$, find in terms of h, the term independent of x and the coefficient of $\dfrac{1}{x}$. [4]

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(b) Write down the first three terms in the expansion of $(1+kx)^n$ in ascending powers of x, where n is a positive integer greater than 2.
Hence, find the possible values of k if the coefficient of x^2 in the expansion of $(1+6x+4x^2)(1+kx)^6$ is 31. [4]

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3 **(a)** Prove the identity $\tan 2\theta(\operatorname{cosec}\theta - 2\sin\theta) = 2\cos\theta$. [4]

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(b) Hence, solve the equation $\tan 2\theta(\operatorname{cosec}\theta - 2\sin\theta) = \frac{2}{9}\sec\theta$ for $0° \leq \theta \leq 360°$. [4]

4 **(a)** It is given that $f(x) = x^3 + ax^2 - 3x + b$, where a and b are constants, has a factor of $(x+2)$ and leaves a remainder of 30 when divided by $(x-3)$. Find the value of a and of b. [5]

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(b) Using these values of a and b, factorise $f(x)$ completely into three linear factors, using surds where necessary and thus solve $f(x) = 0$. [4]

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5 **(a)** Show that $f(x)=\frac{x^2-4}{x}$ is an increasing function for $x>0$. [2]

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(b) Find the coordinates of the stationary points of the curve $y=2x^2(3-x)^4$ and determine the nature of each stationary point. [7]

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Continuation of working space for Question 5(b)

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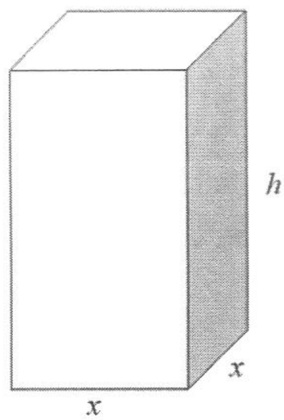
6 Samuel designs an open rectangular tank with the capacity of $216\ m^3$. The tank has a square base with length of x metres and a height of h metres. Material cost for the four walls of the tank costs $\$36/m^2$ and the material cost of the base is $\$90/m^2$.



(i) If C is the total material cost for making the tank in dollars, show that

$$C = 90x^2 + \frac{31104}{x}.$$

[3]

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(ii) Given that x can vary, find the value of x for which C is stationary. [3]

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(iii) Determine whether this value of x gives a maximum or minimum value for C. Hence state the value of C. [3]

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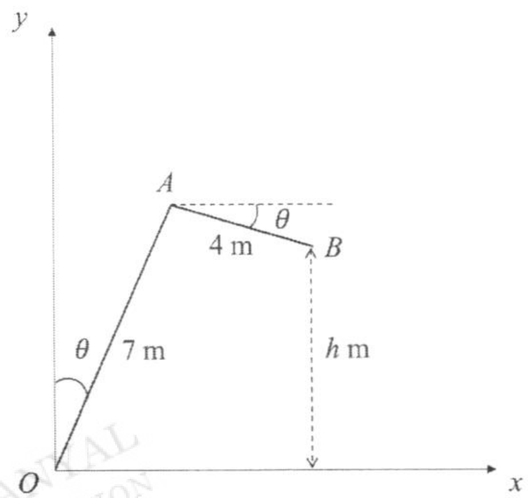

The diagram shows a metal bar OA, which is hinged at O, and another metal bar AB, which is hinged at A. The bars can move in the xy-plane with origin O where x and y axes are horizontal and vertical respectively. The bar OA can turn about O and is inclined at an angle θ to the y-axis, where $0° \le \theta \le \omega$ and ω is an acute angle. The bar AB can turn about A in such a way that the inclination to the horizontal is also θ, and that B is above the x-axis. The lengths of OA and AB are 7 m and 4 m respectively.

It is given that the perpendicular distance of B from the x-axis is h m.

(a) Express h in the form $a\cos\theta - b\sin\theta$, where a and b are integers. [2]

(b) Show that ω is approximately $60.255°$ corrected to 3 decimal places. [2]

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(c) Using your answer in **(a)**, express h in the form $R\cos(\theta+\alpha)$, where $R>0$ and $0°<\alpha<90°$. [3]

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(d) **Without doing any further calculation**, explain why there is a solution for θ at $h = 6$. [1]

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(e) Find the value of θ such that $h = 6$. [2]

8 A circle has equation $x^2 + y^2 - 4x + 8y - 5 = 0$.

(a) Find the radius and the coordinates of the centre of the circle C. [3]

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The line $3y = -x - 5$ cuts the circle at two points A and B.

(b) Find the shortest distance of the centre C from the line AB, expressing your answer in the form $\frac{1}{2}\sqrt{n}$, where n is an integer. [7]

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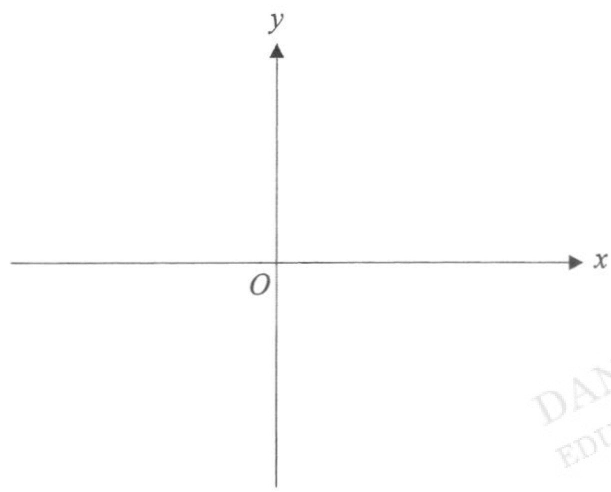
9 (a) (i) Sketch the graph of $y = 6\left(3^{x-2}\right)$ in the space provided below. [1]



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(ii) Given that $m = 3^x$, express $3^{2x-2} + k = 6\left(3^{x-2}\right)$ as a quadratic equation in m. [2]

(iii) If $k > 1$, explain why there are no real solutions for $3^{2x-2} + k = 6\left(3^{x-2}\right)$. [3]

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(b) **(i)** Prove that $7^{k+1} + 28\left(7^{k-1}\right)$ is divisible by 11 for all positive integers of *k*. [2]

(ii) Solve the equation $\log_2 \sqrt{5x+1} + 2\log_{25} 5 = \log_4 (x-2) + \log_3 27$. [4]

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10 **(a)** Show that $\frac{d}{dx}\left[(x-1)e^{x}\right] = xe^{x}$. [2]

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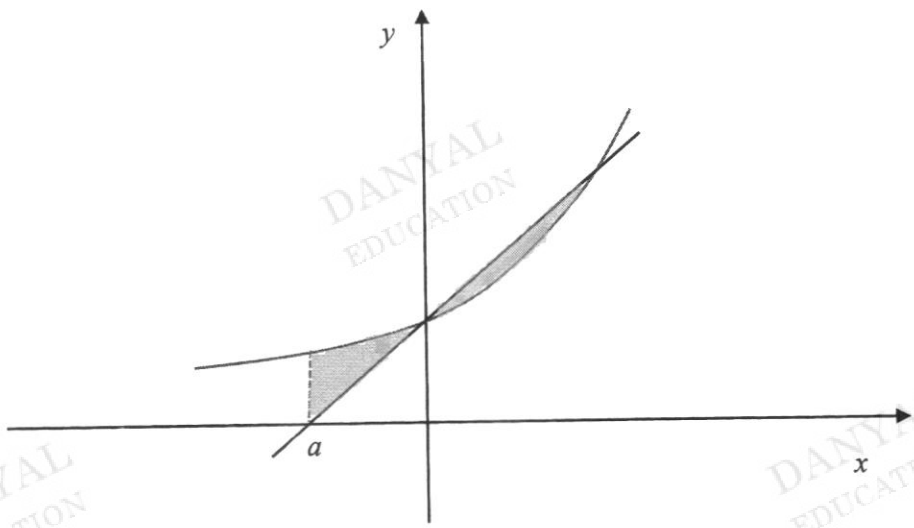
(b) The diagram shows part of the curve $y = 2xe^{x} + 3$. Find the shaded area bounded by the curve, the line $y = 10x + 3$ and the line $x = a$. [10]



Continuation of working space for Question 10(b)

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~End of Paper~

BP~361

Name: Suggested Solution Register no: .... Class: ....

NGEE ANN SECONDARY SCHOOL

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PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS 4049/01

Paper 1 **28 August 2023**

2 hours 15 minutes

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$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Express $-2x^2-12x-14$ in the form $a(x-h)^2+k$. [2]

		Solutions
1	(a)	$-2x^2-12x-14$ $=-2\left[x^2+6x\right]-14$ $=-2\left[(x+3)^2-3^2\right]-14$ $=-2(x+3)^2+18-14$ $=-2(x+3)^2+4$

(b) Using your answer to part **(a)**, explain why the maximum value of $-2x^2-12x-14$ is k. [2]

		Solutions
1	(b)	Since $(x-3)^2\geq 0$ $-2(x-3)^2\leq 0$ $-2(x-3)^2+4\leq 4$ Therefore, the **maximum** value of $-2x^2-12x-14$ is 4. [Alternative Solution] Note that the coefficient of x^2 is negative (or $a<0$). Since the maximum point of $y=-2x^2-12x-14$ is $(-3,4)$, its maximum value is the y-coordinate.

DANYAL EDUCATION

2 Solve the equation $x = 3\sqrt{3} + \frac{12}{x}$, where $x \neq 0$, giving your answer in its simplest surd form. [4]

	Solutions
2	$x^2 - 3\sqrt{3}x - 12 = 0$ (accept $x^2 - \sqrt{27}x - 12 = 0$) $x = \frac{3\sqrt{3} \pm \sqrt{27-4(-12)}}{2}$ $x = \frac{3\sqrt{3} \pm \sqrt{75}}{2}$ Since $\sqrt{75} = 5\sqrt{3}$, $x = \frac{3\sqrt{3} \pm 5\sqrt{3}}{2}$ $x = 4\sqrt{3}$ or $x = -\sqrt{3}$

DANYAL EDUCATION

3 The spread of bird flu in a certain duck farm is given by $B(t) = \frac{300}{1+e^{5-t}}$, where t is the number of days since the flu first appeared, and $B(t)$ is the total number of ducks which have caught the flu to date.

(a) Estimate the initial number of ducks infected with the flu.
Give your answer correct to the nearest integer. [1]

		Solutions
3	(a)	When $t = 0$, $B(0) = \frac{300}{1+e^{5-0}}$ $= 2.01$ (3 s.f.) Initial number of infected ducks is 2.

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(b) If the day that the flu first appeared is Day 0, find out on which day would the number of infected ducks first reach 100. [2]

		Solutions
3	(b)	$\frac{300}{1+e^{5-t}} = 100$ $300 = 100 + 100e^{5-t}$ $100e^{5-t} = 200$ $e^{5-t} = 2$ $5-t = \ln 2$ $t = 4.31$ (to 3 s.f.) The number of infected ducks first reached 100 on Day 4.

(c) Explain clearly why the number of infected ducks will never exceed 300. [2]

		Solutions
3	(c)	For all values of t, $e^{5-t} > 0$. $1 + e^{5-t} > 1$ $1 > \frac{1}{1+e^{5-t}}$ $\frac{1}{1+e^{5-t}} < 1$ $B(t) = \frac{300}{1+e^{5-t}} < 300$ Therefore the number of infected ducks will never exceed 300.

DANYAL EDUCATION

4 **A calculator must not be used in this question.**

(a) Show that $\tan 15° = 2 - \sqrt{3}$. [3]

		Solutions
4	(a)	$\tan 15°$ $= \tan(45° - 30°)$ $= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$ $= \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$ $= \frac{3-\sqrt{3}}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}}$ $= \frac{9-6\sqrt{3}+3}{6}$ $= 2 - \sqrt{3}$
		Alternative solution $\tan 15°$ $= \tan(60° - 45°)$ $= \frac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°}$ $= \frac{\sqrt{3}-1}{1+(\sqrt{3})(1)}$ $= \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$ $= \frac{\sqrt{3}-3-1+\sqrt{3}}{1-3}$ $= 2 - \sqrt{3}$ (shown)

DANYAL EDUCATION

(b) Using your answer in part **(a)**, find an expression for $\sec^2 15°$ in the form $a + b\sqrt{3}$ where a and b are integers. [2]

		Solutions
4	(b)	$\tan 15° = 2 - \sqrt{3}$ $\sec^2 15° = 1 + \tan^2 15°$ $\sec^2 15°$ $= 1 + \left(2 - \sqrt{3}\right)^2$ $= 1 + (4 - 4\sqrt{3} + 3)$ $= 8 - 4\sqrt{3}$

DANYAL EDUCATION

5 **(a)** Express $\dfrac{48x^3+86x^2-2x-32}{6x^2+13x+6}$ as the sum of a polynomial and a proper fraction. [2]

		Solutions
5	(a)	$$\begin{array}{r} 8x - 3 \\ 6x^2+13x+6\,\big)\,\overline{48x^3+86x^2-2x-32} \\ -(48x^3+104x^2+48x) \\ \hline -18x^2-50x-32 \\ -(-18x^2-39x-18) \\ \hline -11x-14 \end{array}$$ $\therefore \dfrac{48x^3+86x^2-2x-32}{6x^2+13x+6} = 8x-3+\dfrac{-11x-14}{6x^2+13x+6}$ $= 8x-3-\dfrac{11x+14}{(3x+2)(2x+3)}$

	(b)	Hence, express $\dfrac{48x^3+86x^2-2x-32}{6x^2+13x+6}$ as the sum of a polynomial and partial fractions.	[4]

		Solutions
5	(b)	Let $\dfrac{11x+14}{(3x+2)(2x+3)} = \dfrac{A}{3x+2}+\dfrac{B}{2x+3}$. $11x+14 = A(2x+3)+B(3x+2)$ Let $x=-\dfrac{2}{3}$: $11\left(-\dfrac{2}{3}\right)+14 = A\left(\dfrac{5}{3}\right)$ $\dfrac{20}{3} = \dfrac{5}{3}A$ $A=4$ Equating coefficients of x, $11 = 8+3B$ $3=3B$ $B=1$ $\dfrac{11x+14}{(3x+2)(2x+3)} = \dfrac{4}{3x+2}+\dfrac{1}{2x+3}$ $\therefore \dfrac{48x^3+86x^2-2x-32}{6x^2+13x+6} = 8x-3-\dfrac{4}{3x+2}-\dfrac{1}{2x+3}$

DANYAL EDUCATION

6	(a)	Find the range of values of h for which the equation $x^2-(h+1)x+h+9=0$ has no real roots.	[4]

		Solutions
6	(a)	$x^2-(h+1)x+h+9=0$ For the equation to have no real roots, Discriminant <0 $[-(h+1)]^2-4(1)(h+9)<0$ $h^2+2h+1-4h-36<0$ $h^2-2h-35<0$ $(h-7)(h+5)<0$ $\therefore$ **$-5<h<7$**

DANYAL EDUCATION

(b) Explain why the equation $5x^2-kx+2k^2+8=0$ has no real roots for all real values of k. [2]

		Solutions
6	(b)	$5x^2-kx+2k^2+8=0$ Discriminant $=(-k)^2-4(5)(2k^2+8)$ $=k^2-40k^2-160$ $=-39k^2-160$ Since $k^2\geq 0$ for all real values of k, then $-39k^2\leq 0$, i.e. $-39k^2-160\leq -160$. Since discriminant <0, the equation has no real roots for all real values of k. (shown)

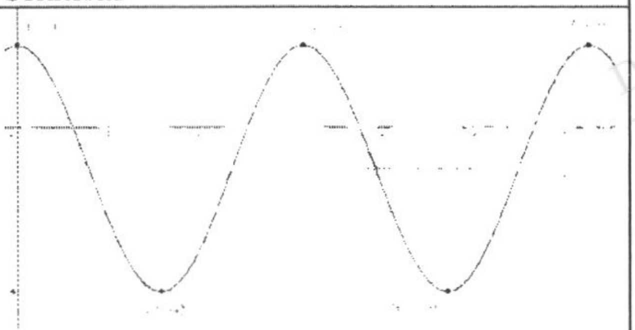
DANYAL EDUCATION

7 The equation of a curve is $y = 3\cos 2x - 1$.

(a) State the maximum and minimum values of y. [1]

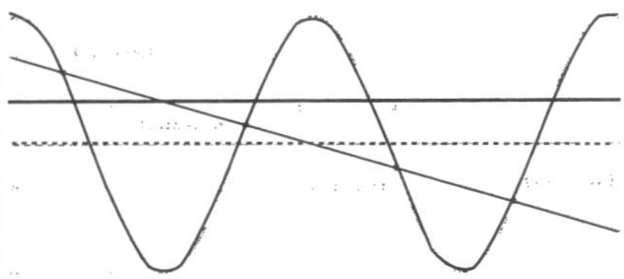
		Solutions
7	(a)	Min = −4 Max = 2

(b) Sketch the graph of $y = 3\cos 2x - 1$ for $0 < x < 2\pi$. [3]

		Solutions
7	(b)	

DANYAL EDUCATION

(c) By drawing a straight line on the same diagram in **(b)**, determine the number of solutions for the equation $3\cos 2x = \frac{2}{\pi}x + 2$ for $0 \le x \le 2\pi$. [2]

		Solutions
7	**(c)**	Draw $y = -\frac{2}{\pi}x + 1$  4 solutions

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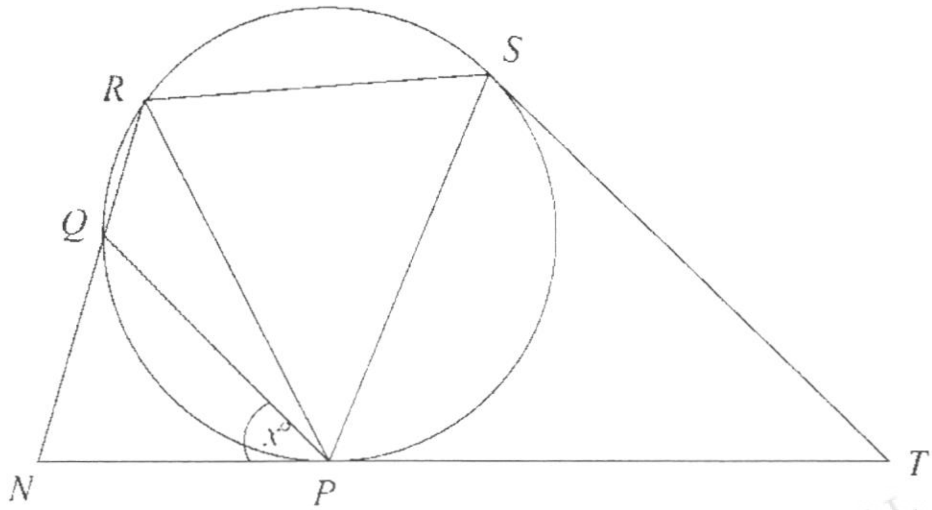
8



In the diagram, P, Q, R and S lie on a circle such that QP is parallel to ST.

Lines NT and ST are tangents to the circle at P and S respectively. NQR is a straight line and angle $NPQ = x°$.

(a) Prove that angle $QRS = 90° + \frac{x°}{2}$. [4]

		Solutions
8	**(a)**	By tangent-chord theorem, $\angle PRQ = x°$ (tangent-chord theorem) $\angle PTS = x°$ (corresponding angles, QP// ST) Note that $PT = ST$ (tangents from an external point), so $\angle TPS = \angle TSP$. $\angle TPS = \frac{180° - x°}{2}$ (base angles of isosceles triangle) $\angle TPS = 90° - \frac{x°}{2}$ $\angle PRS = 90° - \frac{x°}{2}$ (tangent chord theorem) $\angle QRS = x + 90° - \frac{x°}{2}$ $\angle QRS = 90° + \frac{x°}{2}$ (proven)

		Alternative solution $\angle PTS = x°$ (corresponding angles, QP//ST) Note that $PT = ST$ (tangents from an external point), so $\angle TPS = \angle TSP$. $\angle SPT = \frac{180° - x°}{2}$ (base angles of isosceles triangle) $\angle SPT = 90° - \frac{x°}{2}$ $\angle QPS = 180° - x° - \left(90° - \frac{x°}{2}\right) = 90° - \frac{x°}{2}$ (adjacent angles on a straight line) $\angle QRS = 180° - \left(90° - \frac{x°}{2}\right) = 90° + \frac{x°}{2}$ (angles in opposite segment)

(b) It is given further that *QR*// *PS*.
Show that triangle *PRS* is isosceles. [2]

8	**(b)**	Since QR//PS, $\angle RPS = x°$ (alternate angles) From (a), $\angle PRS = 90° - \frac{x°}{2}$ $\angle RSP = 180° - \left(90° - \frac{x°}{2}\right) - x°$ $= 90° - \frac{x°}{2}$ Therefore PRS is an isosceles triangle.

DANYAL EDUCATION

9 Given that $\int_0^4 f(x)dx = \int_4^9 f(x)dx = 12$, find

(a) $\int_0^9 f(x)dx$, [1]

		Solutions
9	**(a)**	$\int_0^9 f(x)dx = \int_0^4 f(x)dx + \int_4^9 f(x)dx$ $= 12 + 12$ $= 24$

(b) $\int_0^4 f(x)dx + \int_9^4 f(x)dx$, [2]

		Solutions
9	**(b)**	$\int_0^4 f(x)dx + \int_9^4 f(x)dx$ $= \int_0^4 f(x)dx - \int_4^9 f(x)dx$ $=12-12$ $= 0$

(c) find the value of the constant m for which $\int_4^9 [f(x) - mx]dx = 0$. [3]

		Solutions
9	**(c)**	$\int_4^9 [f(x) - mx]dx = 0$ $\int_4^9 f(x)dx - \int_4^9 mxdx = 0$ $\int_4^9 mxdx = 12$ $\left[\frac{mx^2}{2}\right]_4^9 = 12$ $\frac{81m}{2} - \frac{16m}{2} = 12$ $\frac{65m}{2} = 12$ $m = \frac{24}{65}$

DANYAL EDUCATION

10 A point P (2, 4) lies on the curve $y = \frac{4}{(x-3)^2}$.

(a) Find the equation of the normal at point P. [4]

		Solutions
10	(a)	$\frac{dy}{dx} = 4(-2)(x-3)^{-3}$ $\frac{dy}{dx} = \frac{-8}{(x-3)^3}$ Gradient of tangent: 8 when $x = 2$ Gradient of normal: $-\frac{1}{8}$ $(y-4) = -\frac{1}{8}(x-2)$ $y = -\frac{1}{8}x + \frac{1}{4} + 4$ $y = -\frac{1}{8}x + 4\frac{1}{4}$

(b) The tangent at another point Q on the curve is parallel to the normal at point P.

Show that the coordinates of Q is $(7, \frac{1}{4})$. [3]

		Solutions
10	(b)	$\frac{-8}{(x-3)^3} = -\frac{1}{8}$ $64 = (x-3)^3$ $x = 7$ Substitute $x = 7$ into $y = \frac{4}{(x-3)^2}$ $y = \frac{4}{(7-3)^2} = \frac{1}{4}$ Coordinates of Q: $(7, \frac{1}{4})$

DANYAL EDUCATION

11 (a) A thin circular metal disk changes size (but not shape) when cooled. The disk is decreasing at a rate of 12π mm^2/s. When the disk has a radius of 200 mm, find the rate of change of the radius of the disk at this instant. [3]

		Solutions
11	**(a)**	Let the area of the disk be A $A = \pi r^2$ $\frac{dA}{dr} = 2\pi r$ $\frac{dA}{dt} = -12\pi$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $-12\pi = 2\pi r \times \frac{dr}{dt}$ When $r = 200$, $-12\pi = 2\pi(200) \times \frac{dr}{dt}$ $\frac{dr}{dt} = -0.03$ mm/s

DANYAL EDUCATION

(b) A 3-dimensional printer prints a chocolate in the shape of a cone. During the printing process, the volume of the cone is increasing at a constant rate of 3π cm^3/s. The height of the cone is always twice of the radius. Find the rate of increase of the radius when the volume of the chocolate cone is 9π cm^3. [4]

		Solutions
11	(b)	Let r be the radius of the chocolate cone after t seconds. $V=\frac{1}{3}\pi r^2(2r)$ $=\frac{2}{3}\pi r^3$ $9\pi=\frac{2}{3}\pi r^3$ $\frac{27}{2}=r^3$ $r=\sqrt[3]{\frac{27}{2}}$ $\frac{dV}{dr}=2\pi r^2$ $\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$ $3\pi=2\pi r^2\times\frac{dr}{dt}$ $\frac{dr}{dt}=\frac{3}{2\left(\sqrt[3]{\frac{27}{2}}\right)^2}$ 0.264566842 cm/s = 0.265 cm/s (3 sf)

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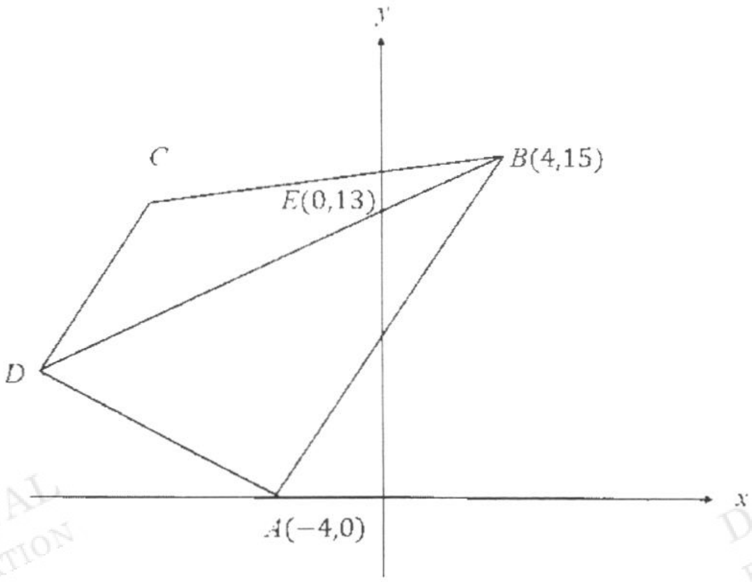
12



DANYAL EDUCATION

The diagram shows a trapezium with vertices $A(-4,0)$, $B(4,15)$, C and D.
The diagonal BD of the trapezium intersects the y-axis at $E(0,13)$.
Given that $AB = 2DC$ and $AD = 13$ units, find the perimeter of $ABCD$. [8]

	Solutions
12	Find equation of BE: $m_{BE} = \frac{15-13}{4-0} = \frac{1}{2}$ Since point E corresponds to the y-intercept of line BE, $y = \frac{1}{2}x + 13$ Let $D(x, y)$ Then $y = \frac{1}{2}x + 13$, that is $D\left(x, \frac{1}{2}x + 13\right)$ Since $AD = 13$ units (given) $\sqrt{(x+4)^2 + \left(\frac{1}{2}x + 13\right)^2} = 13$ $(x+4)^2 + \left(\frac{1}{2}x + 13\right)^2 = 169$ $x^2 + 8x + 16 + \frac{1}{4}x^2 + 13x + 169 = 169$

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	$\frac{5}{4}x^2 + 21x^2 + 16 = 0$ $5x^2 + 84x + 64 = 0$ $x = -\frac{4}{5}$ (reject) or $x = -16$ $y = \frac{1}{2}(-16) + 13 = 5$ Therefore $D(-16,5)$ x-coordinates of $C = -16 + 4$ y-coordinates of $C = 5 + 7.5$ $C(-12,12.5)$ Perimeter $AD = \sqrt{(-4-(-16))^2 + (0-5)^2} = 13$ $AB = \sqrt{(-4-4)^2 + (0-15)^2} = 17$ $CD = \frac{17}{2} = 8.5$ $BC = \sqrt{(4-(-12))^2 + (15-12.5)^2}$ $= 16.194$ Perimeter $= 54.7$ units

DANYAL EDUCATION

13 A small forest has been declining in its tree population since 1980. To assess this decline, a census of the tree population of the forest was conducted on January 1st at intervals of ten years from 1980 to 2020. The table below shows the data from the census.

Year	1980	1990	2000	2010	2020
x (decade)	0	1	2	3	4
y (number of trees)	186624	139968	104976	78732	59049

It is believed this decline follows a $y = e^p q^x$ curve, where p and q are constants.

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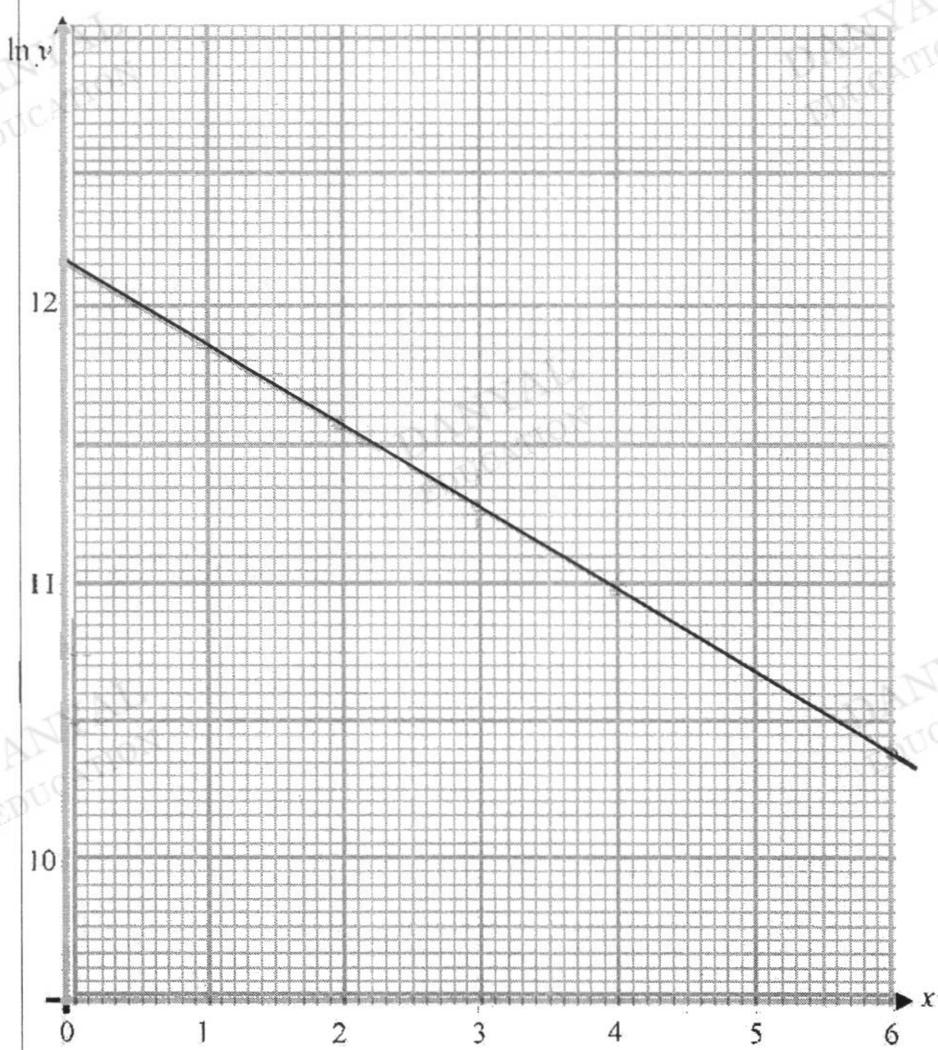
		Solutions
13	(a)	$y = e^p q^x$ $\ln y = \ln\left(e^p q^x\right)$ $\ln y = \ln e^p + \ln q^x$ $\ln y = p + x\ln q$ Draw a graph of $\ln y$ against x.

x	0	1	2	3	4
$\ln y$	12.14	11.85	11.56	11.27	10.99



(b) Estimate the number of trees in the forest on 1st January 2040. [2]

		Solutions
13	**(b)**	From the graph, when $x = 6$, $\ln y = 10.4$ $y = e^{10.4} = 32859$

(c) Use your graph to estimate the value of q. [2]

		Solutions
13	**(c)**	$\ln q = \dfrac{12.14 - 10.4}{0 - 6}$ $\ln q \quad -0.29$ $q \quad e^{0.29} \quad 0.748$ (to 3 s.f.)

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(d) Give a reason why this model might not be accurate in later decades. [1]

		Solutions
13	**(d)**	Possible acceptable reasons: conservation, change in attitude regarding protection of forest trees, climate change, less deforestation, etc. Unacceptable reasons: That the rate might change or that y (or number of trees) would become 0 or negative

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14 The velocity, v m/s, of a particle travelling in a straight line at time t seconds after leaving a fixed point O, is given by $v = 2t^2 + (2-3k)t + 4k - 5$, where k is a constant.

(a) Given that the minimum velocity occurs at $t = \frac{13}{4}$, show that the value of $k = 5$. [3]

		Solutions
14	**(a)**	$v = 2t^2 + (2-3k)t + 4k - 5$ minimum velocity happens when acc = 0, t = $\frac{13}{4}$ $a = \frac{dv}{dt} = 4t + 2 - 3k = 0$ $4\left(\frac{13}{4}\right) + 2 - 3k = 0$ $k = 5$

14 **(b)** Find the time(s) the particle comes to instantaneous rest. [2]

		Solutions
14	**(b)**	particle comes to instantaneous rest, v = 0 $v = 2t^2 - 13t + 15 = 0$ $(2t-3)(t-5) = 0$ $t = 1.5$ or 5

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14 **(c)** Find the distance travelled in the first 7 seconds after passing through O. [4]

		Solutions
14	(c)	Distance $= \int \left(2t^2 - 13t + 15\right) dt$ $= \frac{2t^3}{3} - \frac{13}{2}t^2 + 15t + c$ when t = 0 , s = 0, c = 0 Distance $= \frac{2t^3}{3} - \frac{13}{2}t^2 + 15t$ when $t = 1.5$, $s = \frac{2(1.5)^3}{3} - \frac{13}{2}(1.5)^2 + 15(1.5) = \frac{81}{8}$ when $t = 5$, $s = \frac{2(5)^3}{3} - \frac{13}{2}(5)^2 + 15(5) = -\frac{25}{6}$ when $t = 7$, $s = \frac{2(7)^3}{3} - \frac{13}{2}(7)^2 + 15(7) = \frac{91}{6}$ Total distance travelled $= \frac{81}{8} + \left(\frac{81}{8} + \frac{25}{6}\right) + \left(\frac{25}{6} + \frac{91}{6}\right) = 43.75\ m$

14 **(d)** Given that R is the point when the particle has zero acceleration, and P is the point when the particle first comes to rest, determine, with full working whether R is nearer to O or to P. [3]

		Solutions
14	(d)	At $t = \frac{13}{4}$ (R), Distance $= \frac{2}{3}\left(\frac{13}{4}\right)^3 - \frac{13}{2}\left(\frac{13}{4}\right)^2 + 15\left(\frac{13}{4}\right) = \frac{143}{48}$ $OR = \frac{143}{48} = 2.98$ $PR = \frac{81}{8} - \frac{143}{48} = 7.15$ Particle is nearer to point O.

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-- End of Paper --

Name: Suggested Solution Register no: .... Class: ....

NGEE ANN SECONDARY SCHOOL

O

PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS

4049/02

Paper 2

29 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

Instructions to Candidates

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Total	**/ 90**

Checked by student: ____________________ Date: ______________

This document consists of 21 printed pages and 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

DANYAL EDUCATION

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Express, in terms of π, the principal value of $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$. [1]

		Solutions
1	(a)	$\frac{\pi}{4}$

(b) State the values between within each of the following must lie:

(i) the principal value of $\sin^{-1} x$, [1]

		Solutions
1	b (i)	$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$ OR $-90° \leq \sin^{-1} x \leq 90°$

(ii) the principal value of $\cos^{-1} x$. [1]

		Solutions
1	b(ii)	$0 \leq \cos^{-1} x \leq \pi$ OR $0° \leq \cos^{-1} x \leq 180°$

DANYAL EDUCATION

2 **(a)** In the binomial expansion of $\left(\sqrt{x}+\frac{h}{\sqrt{x}}\right)^8$, find in terms of h, the term independent of x and the coefficient of $\frac{1}{x}$. [4]

		Solutions
2	**(a)**	The general $(r+1)^{th}$ term can be represented by: $\binom{8}{r}(\sqrt{x})^{8-r}(\frac{h}{\sqrt{x}})^r$ with the power of x as $4-r$. $4-r=0$ $r=4$ Term independent of x, $T_5=\binom{8}{4}(h)^4=70h^4$ $4-r=-1$ $r=5$ Coefficient of $\frac{1}{x}$, $T_6=\binom{8}{5}(h)^5=56h^5$

(b) Write down the first three terms in the expansion of $(1+kx)^n$ in ascending powers of x, where n is a positive integer greater than 2.
Hence, find the possible values of k if the coefficient of x^2 in the expansion of $(1+6x+4x^2)(1+kx)^6$ is 31. [4]

		Solutions
2	**(b)**	$(1+kx)^n=1+n(kx)+\frac{n(n-1)}{2}(kx)^2+\ldots$ $=1+nkx+\frac{n(n-1)k^2}{2}x^2+\ldots$ $(1+6x+4x^2)(1+kx)^6=(1+6x+4x^2)(1+6kx+15k^2x^2+\ldots)$ Coefficient of $x^2=15k^2+6(6k)+4$ $=31$ $5k^2+12k-9=0$ $(5k-3)(k+3)=0$ $k=-3$ or $\frac{3}{5}$

3 **(a)** Prove the identity $\tan 2\theta(\operatorname{cosec}\theta \quad 2\sin\theta) = 2\cos\theta$. [4]

		Solutions
3	**(a)**	LHS $= \tan 2\theta\ (\operatorname{cosec}\theta - 2\sin\theta)$ $= \frac{\sin 2\theta}{\cos 2\theta}\left(\frac{1}{\sin\theta} - 2\sin\theta\right)$ $= \frac{\sin 2\theta}{\cos 2\theta}\left(\frac{1-2\sin^2\theta}{\sin\theta}\right)$ $= \frac{2\sin\theta\cos\theta}{1-2\sin^2\theta}\left(\frac{1-2\sin^2\theta}{\sin\theta}\right)$ $= 2\cos\theta$ $=$ RHS (proven)

(b) Hence, solve the equation $\tan 2\theta(\operatorname{cosec}\theta - 2\sin\theta) \quad \frac{2}{9}\sec\theta$ for $0° < \theta < 360°$. [4]

		Solutions
3	**(b)**	$\tan 2\theta\ (\operatorname{cosec}\theta - 2\sin\theta) = \frac{2}{9}\sec\theta$ From part (a), $2\cos\theta = \frac{2}{9}\sec\theta$ $\cos^2\theta = \frac{1}{9}$ $\cos\theta = \frac{1}{3}$ or $\cos\theta = -\frac{1}{3}$ Basic angle $= 70.529°$ $\theta = 70.529°, 180° - 70.529°,$ $180° + 70.529°, 360° - 70.529°$ $\theta = 70.5°, 109.5°, 250.5°, 289.5°$

DANYAL EDUCATION

4 **(a)** It is given that $f(x) = x^3 + ax^2 - 3x + b$, where a and b are constants, has a factor of $(x + 2)$ and leaves a remainder of 30 when divided by $(x - 3)$. Find the value of a and of b. [5]

		Solutions
4	(a)	$f(x) = x^3 + ax^2 - 3x + b$ Given $x - 2$ is a factor, by Factor Theorem, $f(-2) = 0$ $(-2)^3 + a(-2)^2 - 3(-2) + b = 0$ $-8 + 4a + 6 + b = 0$ $4a + b = 2$..........(1) When divided by $x - 3$, by, Remainder Theorem, $f(3) = 30$ $(3)^3 + a(3)^2 - 3(3) + b = 30$ $27 + 9a - 9 + b = 30$ $9a + b = 12$..........(2) (2) − (1): $5a = 10$ $a = 2$ subst into (1) $4(2) + b = 2$ $b = -6$ $\therefore a = 2$ and $b = -6$

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(b) Using these values of a and b, factorise $f(x)$ completely into three linear factors, using surds where necessary and thus solve $f(x) = 0$. [4]

		Solutions
4	(b)	$f(x) = x^3 - 2x^2 - 3x - 6$ Using the given factor $(x - 2)$ $x^3 - 2x^2 - 3x - 6 = (x+2)(px^2 + qx + r)$ By observation, $p = 1$ and $r = -3$ $x^3 - 2x^2 - 3x - 6 = (x+2)(x^2 - qx - 3)$ comparing coefficients of x^2 : $2 = q - 2$ $q = 0$ $x^3 - 2x^2 - 3x - 6 = (x+2)(x^2 - 3)$ $= (x-2)(x+\sqrt{3})(x-\sqrt{3})$ $f(x) = 0$ $(x + 2)(x + \sqrt{3})(x - \sqrt{3}) = 0$ $\therefore x = -2, -\sqrt{3}$ *and* $\sqrt{3}$

DANYAL EDUCATION

5 **(a)** Show that $f(x) = \frac{x^2-4}{x}$ is an increasing function for $x > 0$. [2]

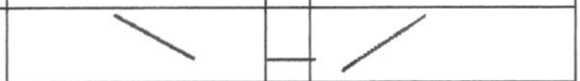
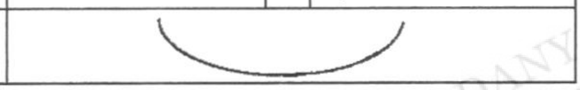
		Solutions
5	**(a)**	Let $f(x) = \frac{x^2-4}{x}$ $f'(x) = \frac{(x)(2x)-(x^2-4)}{x^2}$ $f'(x) = \frac{x^2-4}{x^2} = 1 + \frac{4}{x^2}$ Since $x^2 > 0$ and $x \neq 0$, $f'(x) > 0$. Since $f'(x) > 0$, $\frac{x^2-4}{x}$ is an increasing function for $x > 0$.

(b) Find the coordinates of the stationary points of the curve $y = 2x^2(3-x)^4$ and determine the nature of each stationary point. [7]

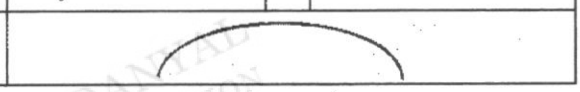
		Solutions
	(b)	$\frac{dy}{dx} = 4x(3-x)^4 + 2x^2(4)(3-x)^3(-1)$ $\frac{dy}{dx} = (3-x)^3[(-8x^2)+12x-4x^2]$ $\frac{dy}{dx} = (3-x)^3[-12x^2+12x]$ $(3-x)^3[-12x^2+12x] = 0$ $3-x = 0$ or $12x(1-x) = 0$ x-coordinates for all stationary points are 0, 1 and 3

DANYAL EDUCATION

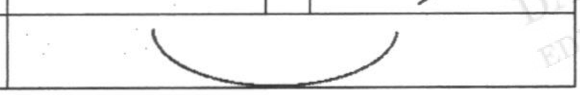
		Solutions
	(b)	$\dfrac{dy}{dx} = (3-x)^3[-12x^2+12x]$

x	-0.1	0	0.1
$\dfrac{dy}{dx}$	$(3+0.1)^3[-12(-0.1)^2+12(-0.1)]<0$	0	$(3-0.1)^3[-12(0.1)^2+12(0.1)]>0$
Sketch of tangent			
Outline			

(0, 0) is a minimum point.

x	0.9	1	1.1
$\dfrac{dy}{dx}$	$(3-0.9)^3[-12(0.9)^2+12(0.9)]>0$	0	$(3-1.1)^3[-12(1.1)^2+12(1.1)]<0$
Sketch of tangent			
Outline			

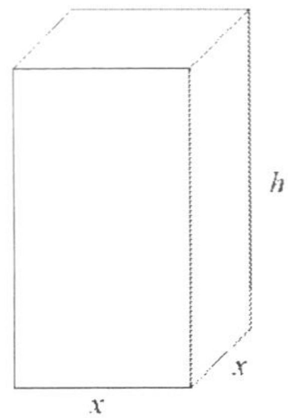
(1, 32) is a maximum point.

x	2.9	3	3.1
$\dfrac{dy}{dx}$	$(3-0.9)^3[-12(0.9)^2+12(0.9)]>0$	0	$(3-3.1)^3[-12(3.1)^2+12(3.1)]>0$
Sketch of tangent			
Outline			

(3, 0) is a minimum point.

DANYAL EDUCATION

6 Samuel designs an open rectangular tank with the capacity of 216 m^3. The tank has a square base with length of x metres and a height of h metres. Material cost for the four walls of the tank costs \$36/m^2 and the material cost of the base is \$90/m^2.



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(i) If C is the total material cost for making the tank in dollars, show that

$$C = 90x^2 + \frac{31104}{x}.$$

[3]

		Solutions
6	**(i)**	$x^2h = 216$ $h = \frac{216}{x^2}$ $C = 36(4xh) + 90x^2$ $C = 36(4x)\left(\frac{216}{x^2}\right) + 90x^2$ $C = 90x^2 + \frac{31104}{x}$

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(ii) Given that x can vary, find the value of x for which C is stationary. [3]

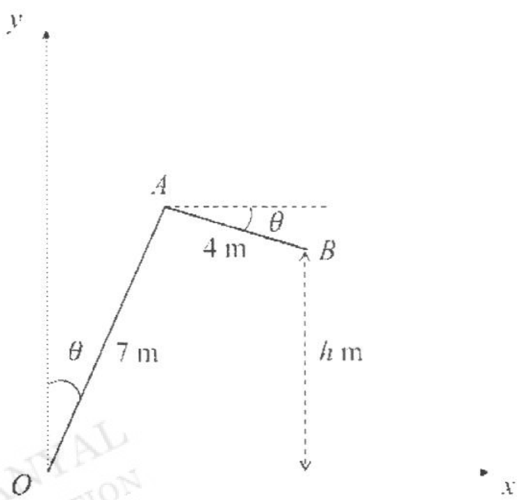
		Solutions
6	**(ii)**	$\frac{dC}{dx} = 180x + (-1)\left(\frac{31104}{x^2}\right)$ $180x + (-1)\left(\frac{31104}{x^2}\right) = 0$ $180x^3 - 31104 = 0$ $x = \sqrt[3]{\frac{864}{5}}$ $= 5.5699066$ $= 5.57$ m (3 significant figures)

(iii) Determine whether this value of x gives a maximum or minimum value for C. Hence state the value of C. [3]

		Solutions
6	**(iii)**	When $x = 5.5699066$, $\frac{d^2C}{dx^2} = 180 + (2)\left(\frac{31104}{5.699066^3}\right) > 0$ Since $\frac{d^2C}{dx^2} > 0$, C is a minimum value. Minimum cost: $90(5.56991)^2 - \frac{31104}{(5.56991)}$ $= \$8376.44$

DANYAL EDUCATION

7



The diagram shows a metal bar OA, which is hinged at O, and another metal bar AB, which is hinged at A. The bars can move in the xy-plane with origin O where x and y axes are horizontal and vertical respectively. The bar OA can turn about O and is inclined at an angle θ to the y-axis, where $0° < \theta \le \omega$ and ω is an acute angle. The bar AB can turn about A in such a way that the inclination to the horizontal is also θ, and that B is above the x-axis. The lengths of OA and AB are 7 m and 4 m respectively.

It is given that the perpendicular distance of B from the x-axis is h m.

(a) Express h in the form $a\cos\theta - b\sin\theta$, where a and b are integers. [2]

		Solutions
7	**(a)**	Let h_1 and h_2 be the vertical component of OA and AB respectively. $h_1 = 7\cos\theta$ $h_2 = 4\sin\theta$ Therefore $h = 7\cos\theta - 4\sin\theta$

(b) Show that ω is approximately $60.255°$ corrected to 3 decimal places. [2]

		Solutions
7	(b)	For B to be above the x-axis, $h \geq 0$ $7\cos\theta - 4\sin\theta \geq 0$ $\tan\theta \leq \frac{7}{4}$ $\omega = \tan^{-1}\frac{7}{4}$ $= 60.25511°$ $= 60.255°$ (shown)

(c) Using your answer in **(a)**, express h in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

		Solutions
7	(c)	$h = 7\cos\theta - 4\sin\theta$ $R = \sqrt{7^2 + 4^2} = \sqrt{65}$ $\alpha = \tan^{-1}\frac{4}{7} = 29.745°$ $h = \sqrt{65}\cos(\theta + 29.7°)$

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(d) **Without doing any further calculation**, explain why there is a solution for θ at $h = 6$. [1]

		Solutions
7	**(d)**	$h = 6$ is equivalent to $h = \sqrt{65}\cos(\theta + 29.745°)$ Since the maximum of h is $\sqrt{65}$, which is greater than 6, there is a θ which satisfy the equation.

DANYAL EDUCATION

(e) Find the value of θ such that $h = 6$. [2]

		Solutions
7	**(e)**	$\sqrt{65}\cos(\theta + 29.745°) = 6$ $\cos(\theta + 29.745°) = \frac{6}{\sqrt{65}}$ Basic angle $= 41.909°$ $\theta + 29.745° = 41.909°$ $\theta = 12.2°$ $\theta = 12.2°$

DANYAL EDUCATION

8 A circle has equation $x^2 + y^2 - 4x - 8y - 5 = 0$.

(a) Find the radius and the coordinates of the centre of the circle C. [3]

		Solutions
8	**(a)**	Let $2g = -4, g = -2$ Let $2f = 8, f = 4$ Centre $= (2, -4)$ Radius $= \sqrt{(-2)^2 + (4)^2 - 5}$ $= 5$ units

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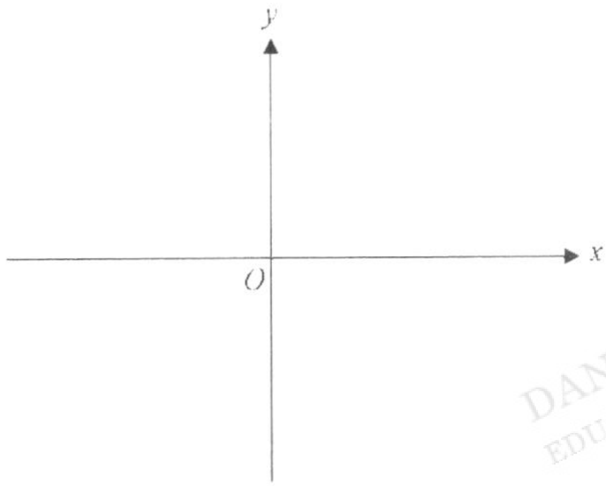
The line $3y = -x - 5$ cuts the circle at two points A and B.

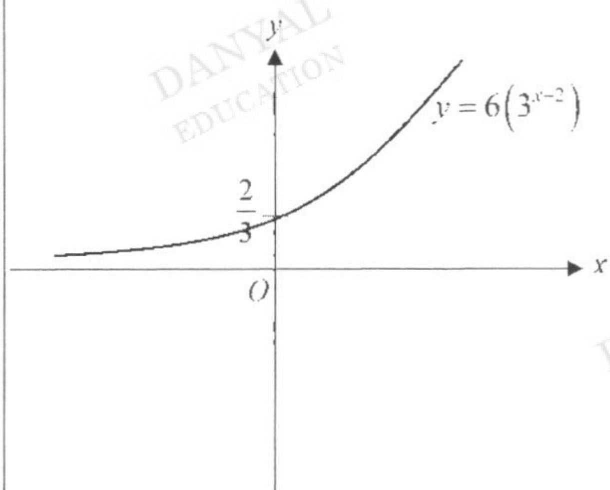
(b) Find the shortest distance of the centre C from the line AB, expressing your answer in the form $\frac{1}{2}\sqrt{n}$, where n is an integer. [7]

		Solutions
8	(b)	$3y = -x - 5$ $x = -3y - 5 \ldots (1)$ $x^2 + y^2 - 4x + 8y - 5 = 0$ $(-3y-5)^2 + y^2 - 4(-3y-5) + 8y - 5 = 0$ $9y^2 + 30y + 25 + y^2 + 12y + 20 + 8y - 5 = 0$ $10y^2 + 50y + 40 = 0$ $y^2 + 5y + 4 = 0$ $(y+1)(y+4) = 0$ $y = -1$ or $y = -4$ When $y = -1$, $x = -2 \rightarrow (-2, -1)$ When $y = -4$, $x = 7 \rightarrow (7, -4)$ **The shortest distance from centre C to AB can be found by calculating the distance from C to the midpoint of AB.** $M\left(\frac{-2+7}{2}, \frac{-1+(-4)}{2}\right)$ $M(2.5, -2.5)$ $C(2, -4)$ $CM = \sqrt{(2.5-2)^2 + (-2.5-(-4))^2}$ $= \frac{1}{2}\sqrt{10}$

DANYAL EDUCATION

9 (a) (i) Sketch the graph of $y = 6\left(3^{x-2}\right)$ in the space provided below. [1]



		Solutions
9	(a) (i)	

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(ii) Given that $m = 3^x$, express $3^{2x-2} + k = 6(3^{x-2})$ as a quadratic equation in m. [2]

		Solutions
9	(a) (ii)	$3^{2x-2} + k = 6(3^{x-2})$ $\frac{(3^x)^2}{3^2} + k = 6\left(\frac{3^x}{3^2}\right)$ Since $m = 3^x$, $\frac{m^2}{9} + k = \frac{6m}{9}$ $m^2 - 6m + 9k = 0$

(iii) If $k > 1$, explain why there are no real solutions for $3^{2x-2} + k = 6(3^{x-2})$. [3]

		Solutions
9	(a) (iii)	Discriminant $= (-6)^2 - 4(1)(-9k)$ $= 36 - 36k$ If $k > 1$, $-36k < -36$ $36 - 36k < 36 - 36$ Discriminant < 0 Therefore $3^{2x-2} + k = 6(3^{x-2})$ has no real solutions.

DANYAL EDUCATION

(b) **(i)** Prove that $7^{k+1} + 28(7^{k-1})$ is divisible by 11 for all positive integers of k. [2]

		Solutions
9	**(bi)**	$7^{k+1} + 28(7^{k-1})$
		$= 7(7^k) + \frac{28(7^k)}{7}$
		$= (7^k)(7+4)$
		$= 11(7^k)$
		Since $7^{k+1} + 28(7^{k-1}) = 11(7^k)$, $7^{k+1} + 28(7^{k-1})$ is divisible by 11 for all positive integers of k.

(ii) Solve the equation $\log_2 \sqrt{5x+1} + 2\log_{25} 5 = \log_4 (x-2) + \log_9 27$. [4]

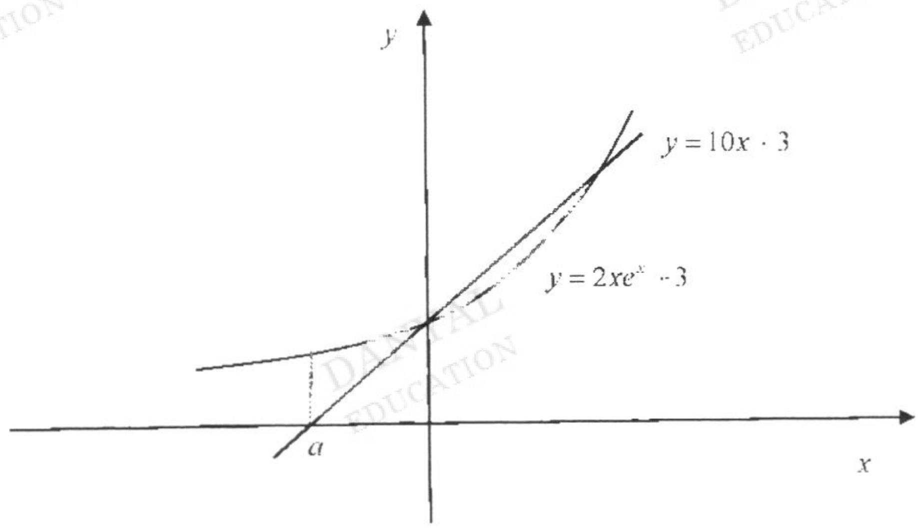
		Solutions
9	**(bii)**	$\log_2 \sqrt{5x+1} + 2\log_{25} 5 = \log_4 (x-2) + \log_9 27$
		$\log_2 \sqrt{5x+1} + \log_{25} 5^2 = \frac{\log_2 (x-2)}{\log_2 4} + \log_3 3^3$
		$\log_2 \sqrt{5x+1} + 1 = \frac{1}{2}\log_2 (x-2) + 3$
		$\log_2 \sqrt{5x+1} - \log_2 \sqrt{x-2} = 3 - 1$
		$\log_2 \sqrt{\frac{5x+1}{x-2}} = 2$
		$\sqrt{\frac{5x+1}{x-2}} = 2^2$
		$\frac{5x-1}{x-2} = 16$
		$5x-1 = 16x-32$
		$x = 3$

DANYAL EDUCATION

10 **(a)** Show that $\frac{d}{dx}\left[(x\ \ 1)e^{x}\right] = xe^{x}$. [2]

		Solutions
10	(a)	$\frac{d}{dx}\left[(x-1)e^{x}\right]\ \ (x-1)e^{x} - e^{x}$ $= xe^{x}$

(b) The diagram shows part of the curve $y\ \ 2xe^{x}+3$. Find the shaded area bounded by the curve, the line $y\ \ 10x-3$ and the line $x\ \ a$. [10]



		Solutions
10	(b)	Point of intersection between curve and line, $y = 2xe^{x} - 3\ \ 10x+3$ $2xe^{x}\ \ 10x = 0$ $x\left(e^{x} \cdot 5\right) = 0$ $x\ \ 0$ or $e^{x}\ \ 5$ $x\ \ \ln 5$

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$$Area\ \text{A} = \int_0^{\ln 5}(10x+3)\,dx - \int_0^{\ln 5}\left(2xe^x+3\right)dx$$

$$= \int_0^{\ln 5}\left(10x - 2xe^x\right)dx$$

$$= \left[5x^2\right]_0^{\ln 5} - 2\left[(x-1)e^x\right]_0^{\ln 5}$$

$$= 5(\ln 5)^2 - 2\left[(\ln 5 - 1)e^{\ln 5} - (-1)e^{0}\right]$$

$$= 4.8571$$

when $y = 0, x = -0.3 = a$

$$Area\ \text{B} = \int_{-0.3}^{0}\left(2xe^x - 10x\right)dx$$

$$= 2\left[(x-1)e^x\right]_{-0.3}^{0} - \left[5x^2\right]_{-0.3}^{0}$$

$$= 2\left[(-1)e^0 - (-1.3)e^{-0.3}\right] - \left[-5(-0.3)^2\right]$$

$$= 0.3761$$

Total area = Area A + Area B

= 4.8571+ 0.3761

= 5.2332

= 5.23

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