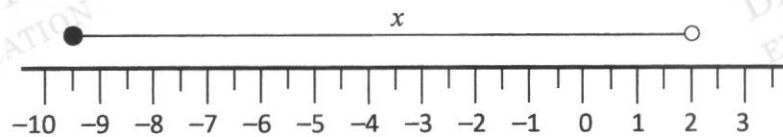


Answer **all** questions.

- 1 Given that $2^{-w} + 2^{-w} + 2^{-w} + 2^{-w} = 8^w$, find w .

Answer $w = \dots\dots\dots$ [2]

- 2 A range of values for x is represented on the number line below.



Write down inequalities that represent this range of values of x .

Answer $\dots\dots\dots$ [1]

- 3 A survey was done to find the number of hours each student spent on social media per day. The results are shown in the table below.

Number of hours (hrs)	5	6	7	8	9
Number of students	2	8	6	x	5

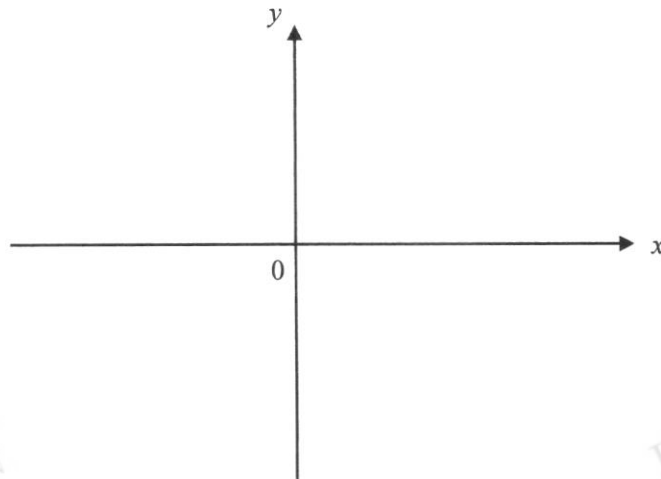
- (a) Find the range.

Answer $\dots\dots\dots$ hrs [1]

- (b) Calculate the smallest possible value of x when the median is 8.

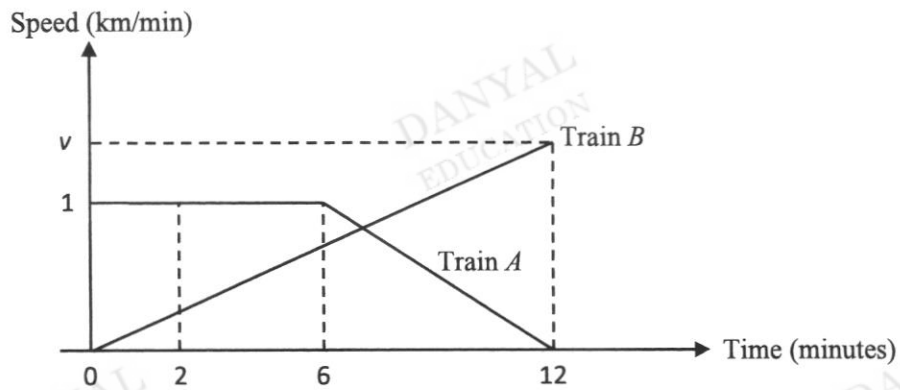
Answer $\dots\dots\dots$ [1]

- 4 **Sketch** the graph of $y = (3 - x)(x + 10)$ on the axes below.
 Indicate clearly the values where the graph crosses the x - and y - axes.



[2]

- 5 The diagram shows the speed-time graphs of both trains during a period of 12 minutes.
 Train A and B started from the same point at the same time and travel in the same direction.



- (a) Calculate the retardation of Train A at 11 minutes.

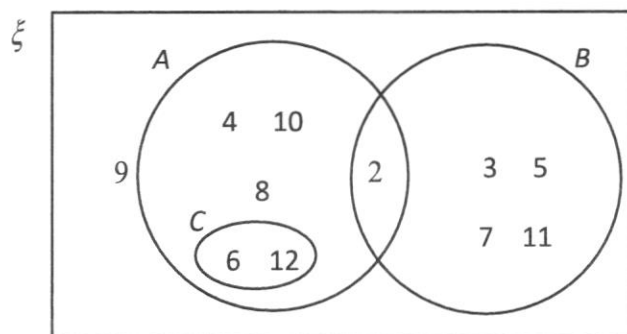
Answer km/min^2 [1]

- (b) Calculate the value of v , the speed of Train B at the end of 12 minutes, given that the two trains travelled the same distance during the period of 12 minutes.

Answer km/min [1]

6 $\xi = \{\text{integers } x : 2 \leq x \leq 12\}$

The Venn diagram shows the elements of ξ and three sets A , B and C .



Use one of the notations below to complete each statement.

$\not\subset$ \subset \in ϕ \notin

(a) $9 \dots\dots\dots (A \cup B)'$

[1]

(b) $\{3, 5\} \dots\dots\dots A$

[1]

(c) $B \cap C = \dots\dots\dots$

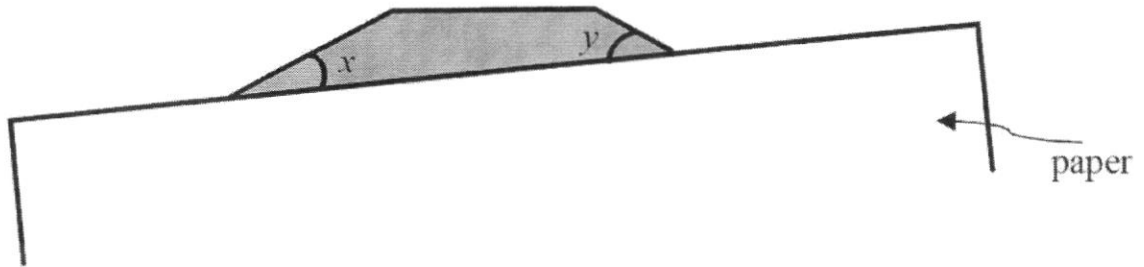
[1]

- 7 A swimming pool is 60% full.
16% of the water in the swimming pool is removed.
There are 1260 litres of water in the pool.

Calculate the capacity of the swimming pool when full.

Answer litres [2]

- 8 In the figure below, a regular shaded polygon is partially covered with a sheet of blank paper. Given that $x + y = 80^\circ$, calculate the number of sides this polygon has.

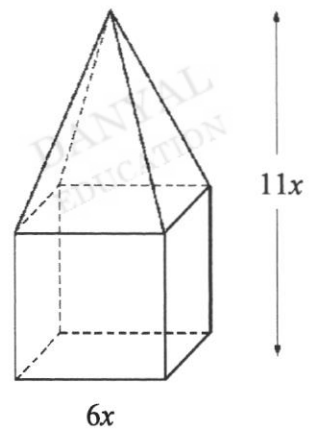


Answersides [3]

- 9 A solid shape consists of a cube with a pyramid on top has a total height of $11x$ cm. The pyramid sits perfectly on one surface of the cube.

Each side of the cube is $6x$ cm.

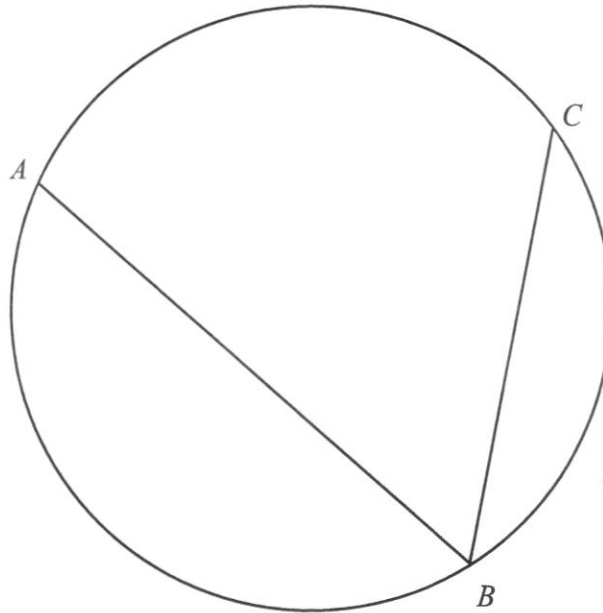
Find an expression, in terms of x , for the surface area of the solid.



Answercm² [3]

- 10 The diagram shows a circle with two chords AB and BC .

Answer

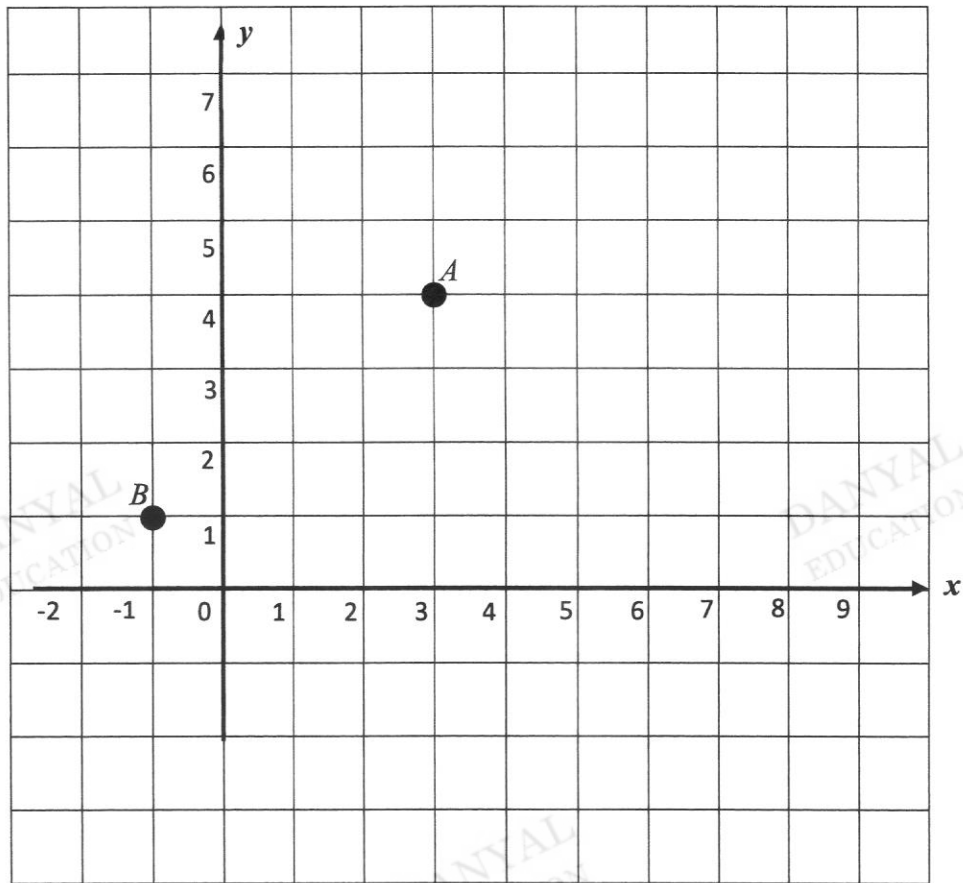


- (a) Construct the perpendicular bisector of AB . [1]
- (b) Construct the bisector of angle ABC . [1]
- (c) Shade the region inside the circle that is closer to AB than to BC and closer to B than to A . [1]

11 Solve $\frac{x-3}{4} - \frac{x+2}{3} = \frac{1}{2}$.

Answer $x = \dots\dots\dots$ [3]

12 On the grid below, the point A is $(3, 4)$ and the point B is $(-1, 1)$.



(a) Mark out and label point C such that $\overrightarrow{BC} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$. [1]

(b) Find $|\overrightarrow{BC}|$.

Answer $|\overrightarrow{BC}| = \dots\dots\dots$

[1]

- 13** John deposits a sum of money in a bank that pays a compound interest of 3.8% per year. After 5 years, the money is expected to earn a total interest of \$1619.50

Calculate the sum of money John deposits.

Give your answer correct to the nearest dollar.

Answer \$ [2]

- 14** (a) Factorise $2x^2 - 7x - 15$.

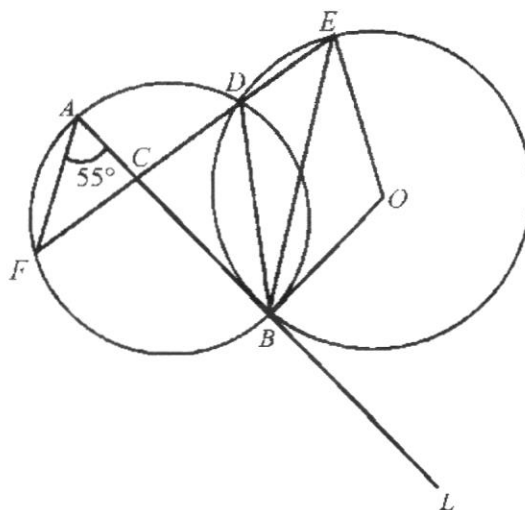
Answer [2]

- (b) Hence, factorise $2(3y-1)^2 - 7(3y-1) - 15$.

Write your answer as simply as possible.

Answer [2]

- 15 In the diagram, $ACBL$ is a tangent to the circle DEB with centre O , at B .
 $\angle CAF = 55^\circ$ and $FCDE$ is a straight line.



Find, stating the reasons clearly,

(a) $\angle BDE$,

Answer $\angle BDE = \dots\dots\dots^\circ$ [2]

(b) $\angle ABE$,

Answer $\angle ABE = \dots\dots\dots^\circ$ [3]

- 16 Ethan observed that the queue at Stall A in his school’s canteen on a particular day. He decided to do a survey to improve the current situation.

Queueing Time (t seconds)	$0 \leq t < 40$	$40 \leq t < 80$	$80 \leq t < 120$	$120 \leq t < 160$	$160 \leq t < 200$	$200 \leq t < 240$
Number of students	6	20	24	30	32	8

- (a) Calculate an estimate of the mean queuing time.

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Answerseconds [1]

- (b) Calculate an estimate of the standard deviation of these times.

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Answerseconds [1]

- (c) Eddie claims that 80% of students queuing at Stall A had to wait at most 180 seconds. Is Eddie’s claim true? Explain your answer.

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Answer

.....

.....

..... [2]

17 The first four terms of a sequence are

278, 269, 260, 251,

(a) Write down the 8th term of the sequence.

Answer [1]

(b) Write down an expression, in terms of n , for the n^{th} term of the sequence.

Answer [1]

(c) Find the first negative term of the sequence.

Answer [2]

18 An area of 5 cm^2 on a map represents an actual area of $32\,000 \text{ m}^2$.
Find the linear scale of the map, giving your answer in the form $1 : n$.

Answer $1 : \dots\dots\dots$ [2]

- 19** The cash price of a laptop is \$ x .
 John bought the laptop on hire purchase.
 He paid a deposit of one-third of the cash price followed by 18 monthly instalments of \$120.
 Given that the total amount he paid for the laptop is \$3300, find the value of x .

Answer $x = \dots\dots\dots$ [2]

- 20** At a sale, all prices are reduced by 30%.
 The price of a watch during the sale is \$693.

(a) Find its original price.

Answer \$..... [2]

(b) The sale price of the watch is exclusive of 8% Goods and Services Tax (GST).
 Find the amount of GST payable.

Answer \$..... [1]

21 $a = \frac{b^2 + 44}{b^2 - c}$

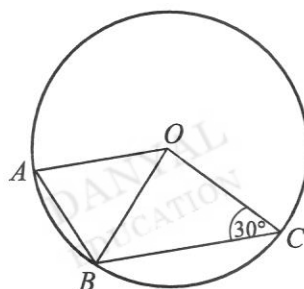
- (a) Find a when $b = -8$ and $c = -11$.

Answer $a = \dots\dots\dots$ [1]

- (b) Rearrange the formula to make b the subject.

Answer $b = \dots\dots\dots$ [3]

22



In the diagram, A , B and C are points on a circle, centre O .
Angle $OCB = 30^\circ$ and angle OAB is 2.5 times of angle OCB .

- (a) Find reflex angle of AOC .

Answer $\dots\dots\dots^\circ$ [3]

(b) Explain why AO is parallel to BC .

.....
.....
..... [2]

23 (a) Express 6300 as a product of its prime factors.

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Answer 6300 = [1]

(b) Given that $6300 \times 15p = q^3$, where p and q are integers.
Find the smallest values of p and q .

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Answer $p =$

$q =$ [2]

(c) The lowest common multiple of the two numbers is 6300.
The highest common factor of the two numbers is 6.
Both numbers are greater than 100.

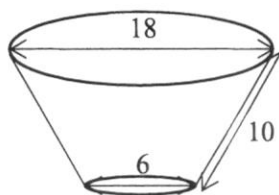
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Find the two numbers.

Answer and [2]

24



The figure shows a solid in the form of a frustum.
 Its circular top and base have diameters 18 cm and 6 cm respectively.
 The slant height is 10 cm long.

- (a) Find the height of the frustum.

Answercm [2]

- (b) Find, in its simplest form, the ratio of the volume of the original cone to that of the frustum.

Answer : [1]

- (c) Calculate the total surface area of the frustum, leaving your answer in terms of π .

Answercm² [4]

25 (a) p is directly proportional to cube root of q .
 Given that $q = 125$ and $p = 3$,

(i) find an expression for p in terms of q

Answer $p = \dots\dots\dots$ [2]

(ii) find the value of q when $p = 0.2$.

Answer $q = \dots\dots\dots$ [1]

(b) y is inversely proportional to x .

(i) When x has a certain value, $y = a$.
 Find an expression of y , in terms of a , when x is halved.

Answer $y = \dots\dots\dots$ [2]

(ii) Sketch the graph of y against x .



[1]

- 26 Tickets to a carnival cost \$10 for adults (A), \$8 for senior citizens (S) and \$5 for children (C). This information can be represented by the matrix **Q** below.

$$\mathbf{Q} = \begin{pmatrix} 10 \\ 8 \\ 5 \end{pmatrix}$$

- (a) 68 adults, 15 senior citizens and 70 children bought tickets through ticket counter. x adults and 88 children bought tickets through online. Represent this information in a 2×3 matrix **P**.

$$\text{Answer } \mathbf{P} = \begin{pmatrix} & \text{A} & \text{S} & \text{C} \\ & & & \\ \text{counter} & & & \\ \text{online} & & & \end{pmatrix} \quad [1]$$

- (b) Find the matrix **R**, in terms of x such that $\mathbf{R} = \mathbf{PQ}$.

$$\text{Answer } \mathbf{R} = \begin{pmatrix} & & & \\ & & & \end{pmatrix} \quad [2]$$

- (c) Explain what each elements in matrix **R** represents.

Answer

..... [1]

- (d) The total amount of money collected from ticket counter is less than online sales. Work out the least value of x .

$$\text{Answer } x = \dots\dots\dots [2]$$

- (e) During a promotion, there is a 15% discount for adults, 25% discount for senior citizens and 20% discount for children.

Write down matrix **D** such that the elements in matrix multiplication of \mathbf{PDQ} gives the amount of money collected from the sales of tickets through ticket counter and online respectively after discount.

$$\text{Answer } \mathbf{D} = \begin{pmatrix} & & & \\ & & & \end{pmatrix} \quad [1]$$

Answer **all** the questions.

1 (a) Simplify $\left(\frac{27x^6}{125y^{-3}}\right)^{-\frac{1}{3}}$.

Answer [2]

(b) Simplify $\frac{8p^5q^0}{r} \div \frac{32r^4q^3}{p^2}$.

Answer [2]

(c) Solve the inequality $\frac{6-x}{3} - \frac{1-x}{4} \geq \frac{5}{6}$.

Answer [2]

- (d) Express as a single fraction in its simplest form $\frac{x}{(x-5)^2} - \frac{9x}{10-2x}$.

Answer [2]

- (e) Solve the equation $\frac{8}{x+1} + \frac{2}{x-3} = 3$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [3]

2 (a) A is the point $(3, -7)$ and B is the point $(5, 3)$.

(i) Find the length of AB .

Answerunits [2]

(ii) Find the equation of line AB .

Answer [2]

(iii) Find the equation of the line l which passes through B and is parallel to the x -axis.

Answer [1]

- (b) The area bounded by a line and the axes has an area of 10 square units.
The line passes through the point $P(0, 4)$.
Max claims that there are 2 possible equations of the lines, and their gradients are the negative of each other.

Do you agree with Max's claim?
Justify your answer showing your calculations.

Answer

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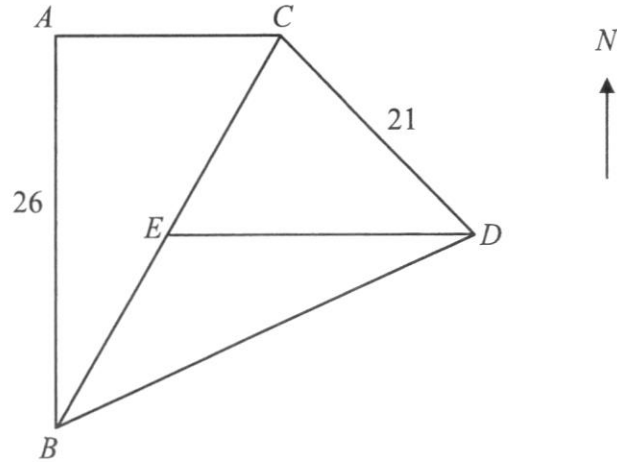
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[4]

3



In the diagram, A , B , C and D are four points on level ground and B , C and D are three corners of a playground.

Two trees are planted at E and D .

E is due west of D , A is due north of B and C is due east of A .

$AB = 26$ m, $BC = 30$ m, $CD = 21$ m and the bearing of D from C is 128° .

Calculate

- (a) the bearing of C from B ,

Answer $^\circ$ [2]

- (b) the angle BCD ,

Answer $^\circ$ [1]

(c) the length BD ,

Answer m [3]

(d) area of the playground,

Answerm² [2]

(e) the distance between the two trees.

Answer m [2]

A boy whose eye level above the ground is 130 cm, stopped at B and saw a bird at the top of the tree at D .

- (f) Calculate the height of the tree at D if the angle of elevation of the bird as seen by the boy was 16° .

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Answer m [2]

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- 4 The value of Mr Wong's luxurious car, currently estimated at \$525 000, depreciates 20% each year.

- (a) Explain why the nett value of the luxurious car, $\$V$ thousands after t years from now is given by $V = 525(0.8)^t$.

Answer

.....

.....

.....

[1]

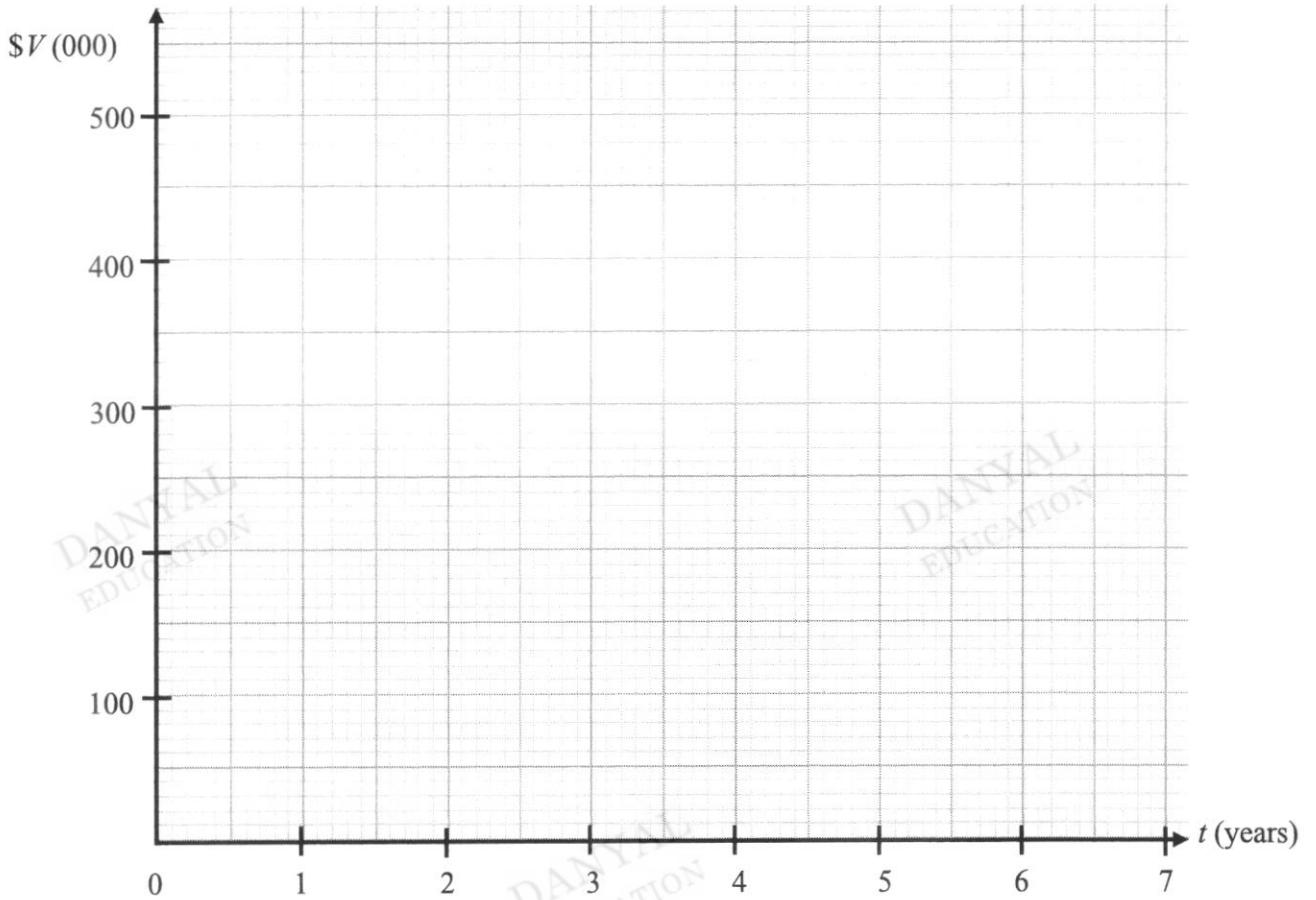
The table shows some corresponding values of t and V thousands where the values of V thousands are corrected to 3 significant figures.

t years	0	1	2	3	4	5	6	7
$\$V$ (thousands)	525	420	336	269	a	172	138	110

- (b) Calculate the value of a .

Answer $a =$ [1]

(c) On the grid, draw the graph of $V = 525(0.8)^t$ for $0 \leq t \leq 7$. [2]



(d) Use your graph to estimate when Mr Wong must sell the car if the loss in the car value is not more than 40% of the original value.

Answer after..... years [1]

(e) (i) By drawing a tangent, find the gradient of the curve at the point where $t = 3$.

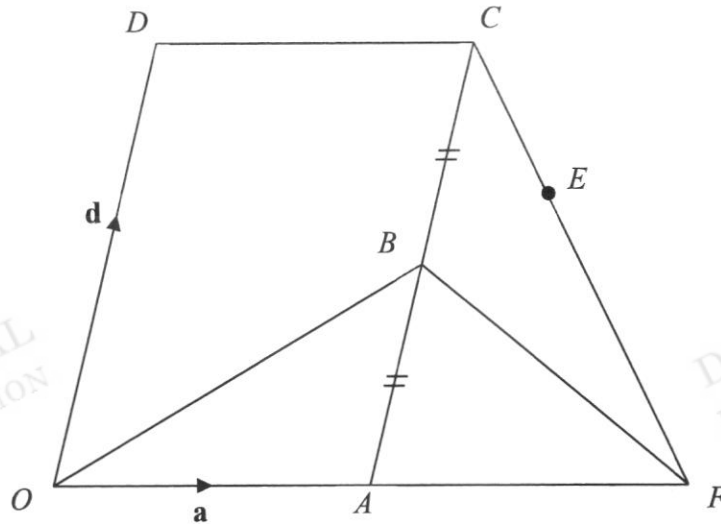
Answer [2]

(ii) Explain the significance of this gradient.

Answer

..... [1]

- 5 In the diagram below, $OACD$ is a parallelogram and B is the midpoint of AC .
 CE and OA produced intersect at the point F .
 $CE : CF = 1 : 3$ and $OA : OF = 1 : 2$.
 Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OD} = \mathbf{d}$.



- (a) Express and simplify the following vectors in terms of \mathbf{a} and \mathbf{d} ,

(i) \overrightarrow{OB} ,

Answer [1]

(ii) \overrightarrow{CF} ,

Answer [1]

(iii) \overrightarrow{OE} .

Answer [2]

- (b) Show that points O , B and E lie on a straight line. [2]

Answer

- (c) P is a point on OA and T is a point on AC such that $\overline{OB} = 2\overline{PT}$.

Show that triangles APT and AOB are similar.
Give a reason for each statement you make.

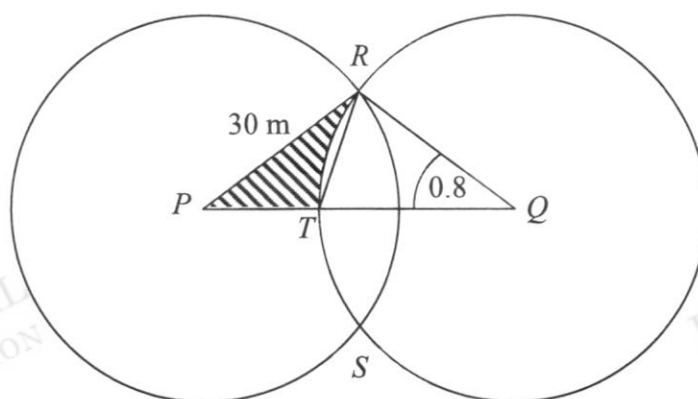
[3]

Answer

- (d) Find the ratio of area triangle OAB : area parallelogram $OACD$.

Answer : [1]

- 6 A landscaping company was tasked to decorate a garden. The diagram shows the top view of the actual garden. There are two circles, centres P and Q , with equal radii of 30 m. The circles meet at R and S . PQ meets the circle with centre Q at T . Angle $TQR = 0.8$ radians.



- (a) (i) Calculate the length of PT .

Answer m [3]

- (ii) Find the perimeter of the shaded region.

Answer m [2]

(iii) Find the area of the shaded region.

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Answer m² [3]

(b) The shaded region is the base of a pillar of height 210 cm for a sculpture.

Calculate the volume of the cement, in m³, used to make the pillar.

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Answer m³ [2]

- 7 A swimming pool is connected to two pumps, A and B .
 Pump A can fill up the swimming pool in $2x$ hours.
 Pump B can fill up the same swimming pool in $(3x-4)$ hours.

(a) Write an expression, in terms of x , for the fraction of the swimming pool that is filled up by

(i) pump A only in 1 hour,

Answer [1]

(ii) pump B only in 1 hour.

Answer [1]

(b) When both taps are used together, the same swimming pool can be filled up in 5 hours.

Write an equation in x to represent this information and show that it reduces to

$$6x^2 - 33x + 20 = 0. \quad [3]$$

- (c) Solve the equation $6x^2 - 33x + 20 = 0$.
(Leave your answer correct to 3 decimal places.)

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [3]

- (d) If only pump *B* is used, calculate how much more time it will take to fill up the swimming pool compared to when 2 pumps are used together.
Leave your answer in hours and minutes (correct to the nearest minute).

Answer hours mins [2]

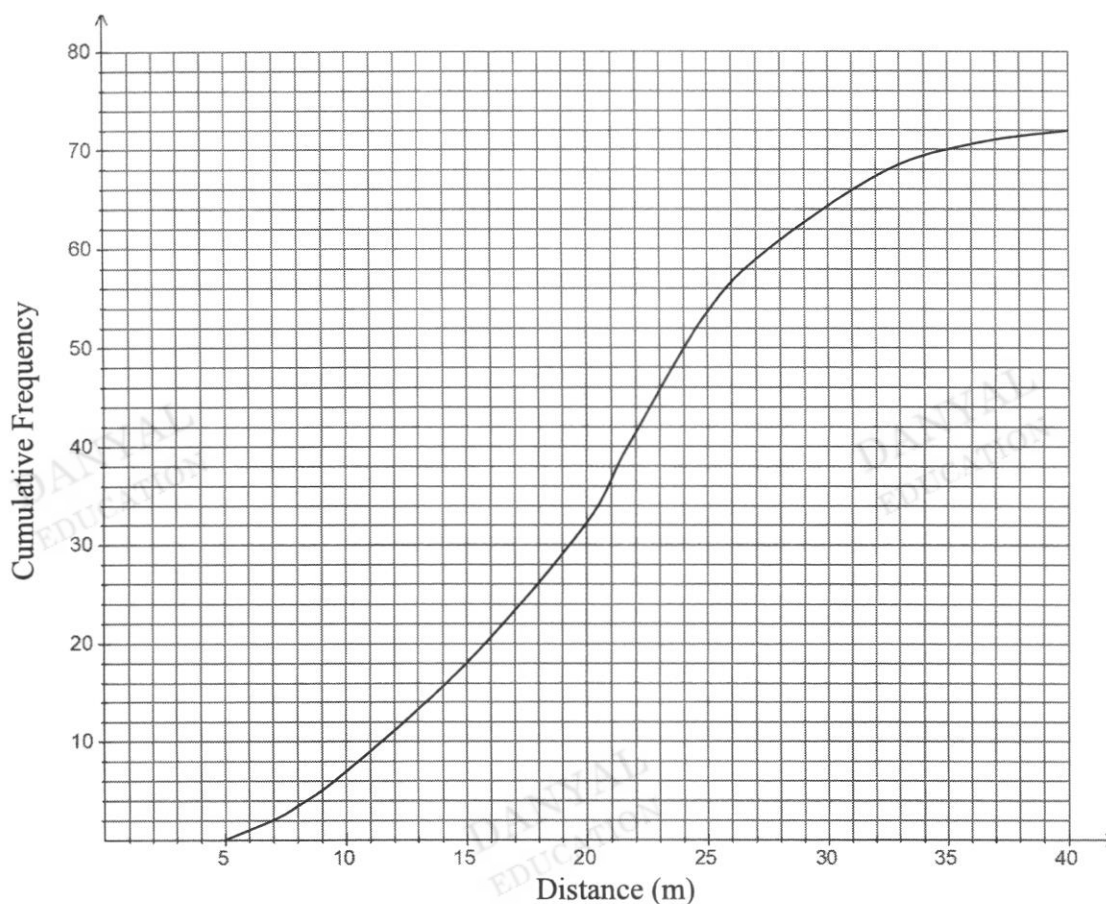
- (e) List an assumption that has been made while forming the equation in (b).

Answer

.....

..... [1]

- 8 (a) A group of 72 children took a swimming test to find the distance they could swim. The results of the test are shown in the cumulative frequency curve below.



(i) Use the curve to estimate

(a) the median distance,

Answer m [1]

(b) the interquartile range of the distances.

Answer m [2]

(ii) The qualifying distance for Intermediate Class is 31 metres. Estimate the percentage of the children that can qualify for the Intermediate Class.

Answer% [1]

- (iii) The distance swam by another group of 72 children had the same interquartile range as the first group's distance but a lower median.

Describe how the cumulative frequency curve for the second group of children may differ from the curve of the first group.

Answer

..... [1]

- (b) The table shows the ages of the 144 children who took the swimming test.

Age (a years)		$2 \leq a < 7$	$7 \leq a < 10$	$10 \leq a < 12$
Frequency	Girls	23	57	14
	Boys	12	31	7

- (i) One of the children is selected at random.

Find, as a fraction in its lowest terms, the probability that the child is

- (a) a boy under 7,

Answer [1]

- (b) aged 10 or more.

Answer [1]

- (ii) Two of the children are selected at random.

Find the probability that **both** of them are girls aged under 10.
Give your answer as a fraction, in its simplest form.

Answer [2]

- 9 Mrs Lee wants to take a cab to Gardens by the Sea.
The distance from her house to Gardens by the Sea is 16.1 km.
She plans to go on a weekday.

Cab Company Transport Rate		
Company	Description	Unit cost
G Cab	Base fare	\$9.00
	Per kilometre	\$0.48
	Per minute	\$0.22
K Cab	Base fare	\$9.00
	Per kilometre	\$0.90
T Cab	First km or less	\$4.10
	Every 400 m thereafter or less up to 10 km	\$0.24
	Every 350 m thereafter or less after 10 km	\$0.24
	Peak Period Surcharge Monday to Friday (Except Public Holidays): 6.00 am – 9.39 am Monday to Sunday & Public Holidays: 6.00 pm – 11.59 pm	25% of metered fare
	Midnight – before 5.59 am	50% of metered fare
	Location Charges (pickups from the area) City Area (within CBD ERP cordon): 5 pm – 11.59 pm Chalet World Centosa: whole day Gardens by the Sea: whole day Tanah Sandra Ferry Terminal: whole day	\$3.00

Cab Company Promotion	
Company	Promotion Details
G Cab	<ul style="list-style-type: none"> • 10% off your ride from 4 pm to 7.59 pm • \$5 off cab fare from 8 pm onwards
K Cab	<ul style="list-style-type: none"> • \$3 off cab fare from 6 am to 7.59 am • \$4 off cab fare from 8 pm to 11 pm
T Cab	<ul style="list-style-type: none"> • \$3 off cab fare from 10 am to 12 pm • \$3.50 off cab fare from 8 pm onwards

Mrs Lee intends to leave her house at 3 pm and wants to arrive at 3.19 pm.

Calculate her cab fare if she takes the cab from

(a) K cab,

Answer \$ [1]

(b) T cab.

Answer \$ [2]

Mrs Lee estimates that she will be spending **at least** 1 hour in the Flora Dome, 1 hour in the Sky Dome and another 1 hour for dinner at the M Restaurant.

She plans to use the cab promotions offered by the various companies to return home by midnight.

Her return journey is estimated to take 30 minutes.

- (c) Suggest a suitable **time** for Mrs Lee to leave the garden and the **cab company** that she should take for her return journey.

Justify the decision you make and show your calculations clearly.

[7]

Answer

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End of paper

Answer Key

1	$w = 0.5$
2	$-9.5 \leq x < 2$
3a	4
3b	12
4	
5a	$1/6$
5b	$v = 1.5$
6a	\in
6b	\notin
6c	\emptyset
7	2500 litres
8	9 sides
9	$250x^2$ or $(180 + 12\sqrt{34})x^2$
10	<p>The diagram shows a circle with two chords intersecting at point P. The central angle formed by the radii to the endpoints of the chords is labeled (c). The angle formed by the two radii and the minor arc between the endpoints of the chords is labeled (a). The angle formed by the two radii and the major arc between the endpoints of the chords is labeled (b).</p>
11	$x = -23$
12a	
12b	7.28
13	\$7900
14a	$(2x + 3)(x - 5)$
14b	$2(y - 2)(6y + 1)$
15a	125
15b	55
16a	129
16b	53.2
16c	No, the calculation is based on an estimation of even distribution in the grouped data

17a	215
17b	$287 - 9n$
17c	-1
18	8000
19	3420
20a	990
20b	55.44
21a	$\frac{36}{25}$
21b	$\pm \sqrt{\frac{ac+44}{a-1}}$
22a	210
22b	$\angle AOB = 180^\circ - 75^\circ \times 2 = 30 = \angle OBC$ By converse of alternate angles, AO is parallel to BC .
23a	$2^2 \times 3^2 \times 5^2 \times 7$
23b	$p = 98 \quad q = 210$
23c	126 and 300 Accept 150 and 252
24a	8
24b	27 : 26
24c	210π
25a(i)	$\frac{3}{5} \sqrt[3]{q}$
25a(ii)	$\frac{1}{27}$
25b(i)	2a
25b(ii)	
26a	$\begin{pmatrix} 68 & 15 & 70 \\ x & 0 & 88 \end{pmatrix}$
26b	$\begin{pmatrix} 1150 \\ 10x + 440 \end{pmatrix}$
26c	The amount of money collected from sales of tickets through ticket counter and online respectively.
26d	72
26e	$\begin{pmatrix} 0.85 & 0 & 0 \\ 0 & 0.75 & 0 \\ 0 & 0 & 0.8 \end{pmatrix}$

2023 4E5N Prelim Paper 2

Answer Key

1a	$\frac{5}{3x^2y}$
b	$\frac{p^7}{4r^5q^3}$
c	$x \leq 11$
d	$\frac{9x^2 - 43x}{2(x-5)^2}$
e	$x = 1$ or $4\frac{1}{3}$
2ai	10.2
aii	$y = 5x - 22$
aiii	$y = 3$
b	Agree with Max's claim
3a	029.9
b	81.9
c	34.1
d	312
e	24.0
f	11.1
4a	After 1 year, value of car = $\$525 \times 0.8$ After 2 years, value = $\$525 \times 0.8 \times 0.8$ After t years, value, $v = 525(0.8)^t$
b	$a = 215$
d	After 2.3 years
ei	-60 (accept -54 to -66)
eii	The rate at which the value of the car drops in the third year is \$58 300 per year.
5ai	$\overline{OB} = a + \frac{1}{2}d$
aii	$\overline{CF} = a - d$
aiii	$\overline{OE} = \frac{4}{3}a + \frac{2}{3}d$
b	$\overline{OE} = \frac{4}{3}\overline{OB}$ O is a common point
5c	AA similarity test
d	1:4
6ai	11.8
aii	65.8
aiii	89.8
b	189
7ai	$\frac{1}{2x}$

aii	$\frac{1}{3x-4}$
c	$x = 4.806$ or 0.694
d	5 h 26 mins
e	No external environmental sources of water is involved, e.g. raining
8aia	21
aib	10
aii	8.33
aiii	The graph is steeper/ shift to the left
bia	$\frac{1}{12}$
bib	$\frac{7}{48}$
bii	$\frac{395}{1287}$
9a	\$24.30
b	13.70
c	Mrs Lee should leave at 8 pm as the cab fare with T cab would be the cheapest, \$17.38.

Name:	Index No.:	Class:
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PRESBYTERIAN HIGH SCHOOL



MATHEMATICS PAPER 1

4052/01

14 August 2023

Monday

2 hours 15 minutes

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2023 SECONDARY FOUR EXPRESS PRELIMINARY EXAMINATION

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES:

Write your name, index number and class on the spaces provided above.

Write in dark blue or black pen. You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

Note that all the diagrams in this paper are not drawn to scale.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90.

<i>For Examiner's Use</i>				
Category	Accuracy	Notations	Others	Marks Deducted
Question No.				

Total Marks
90

This question paper consists of **23** printed pages (including this cover page) and **1** blank page.

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

3

Answer **all** the questions.

1 Solve $7x = 18 + 3x$.

Answer $x = \dots\dots\dots$ [1]

2 (a) Calculate $\frac{26.18^3}{\sqrt{4.52 - 0.4^2}}$.

Write your answer correct to 5 significant figures.

Answer $\dots\dots\dots$ [1]

(b) Write your answer to **part (a)** in standard form.

Answer $\dots\dots\dots$ [1]

4

3 (a) Express 784 as the product of prime factors.

Answer [1]

(b) It is given that a and b are prime numbers.

Find the smallest values of a and b such that $784 \times \frac{a}{b}$ is a perfect cube.

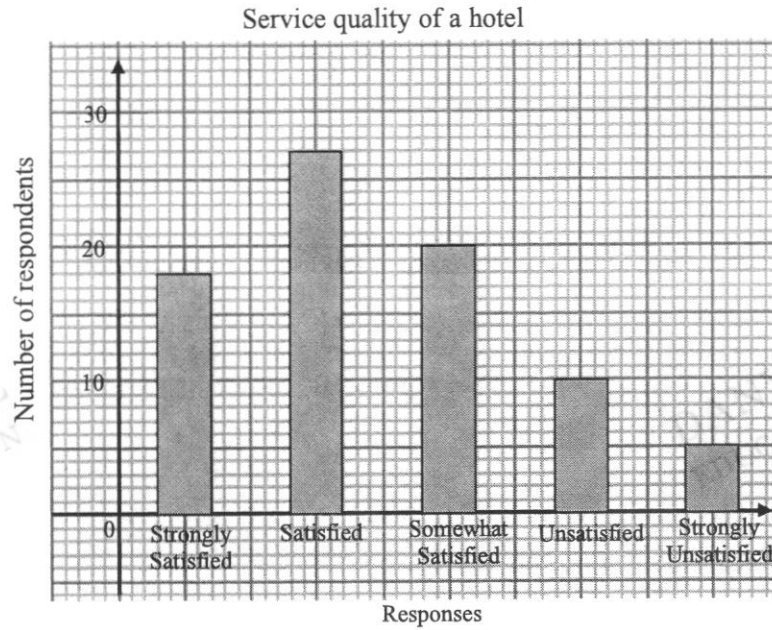
Answer $a =$ [1]

$b =$ [1]

4 Expand and simplify $(w + 5)(1 - w)$.

Answer [2]

5 The bar graph below shows the results of a survey conducted on the service quality of a hotel.



(a) Find the percentage of respondents who answered ‘Strongly Satisfied’ and ‘Satisfied’.

Answer% [1]

(b) Suggest the use of another statistical diagram to represent the results of the survey conducted, that can show the relative size of a part in relation to the whole.

Answer [1]

6

- 6 Find the largest integer that satisfies $2y - 3 < 4$.

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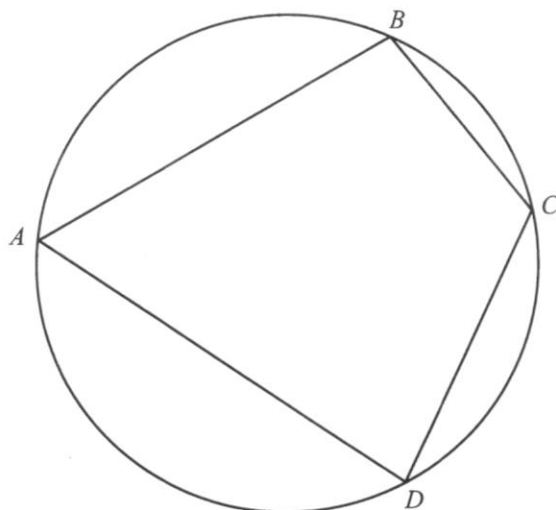
Answer [2]

- 7 P is directly proportional to Q^3 .
When $Q = 2$, $P = 64$.
When the value of Q is halved, the value of P changes by a factor of m .
Find the value of m .

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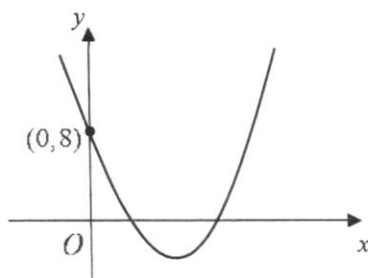
Answer $m =$ [2]

- 8 The diagram shows a quadrilateral playground $ABCD$.
A circular fence is constructed around the playground such that the vertices, A , B , C and D of the playground touch the circumference of the fence.



- (a) Construct the perpendicular bisector of AB . [1]
- (b) Construct the bisector of angle ADC . [1]
- (c) A sand pit is to be constructed inside the circular fence but outside the quadrilateral playground. The sand pit is nearer to AD than CD and nearer to B than A .
Shade the region for the sand pit to be constructed. [1]

9 The diagram below shows the graph of $y = 3(x-h)^2 - 4$.



(a) Find the value of h .

Answer $h = \dots\dots\dots$ [2]

(b) Explain why the graph of $y = 3(x-h)^2 + 1$ does not cut the x -axis.

Answer

.....

[1]

10 A group of six students took a Mathematics quiz and the marks were recorded below.

8 10 9 13 10 9

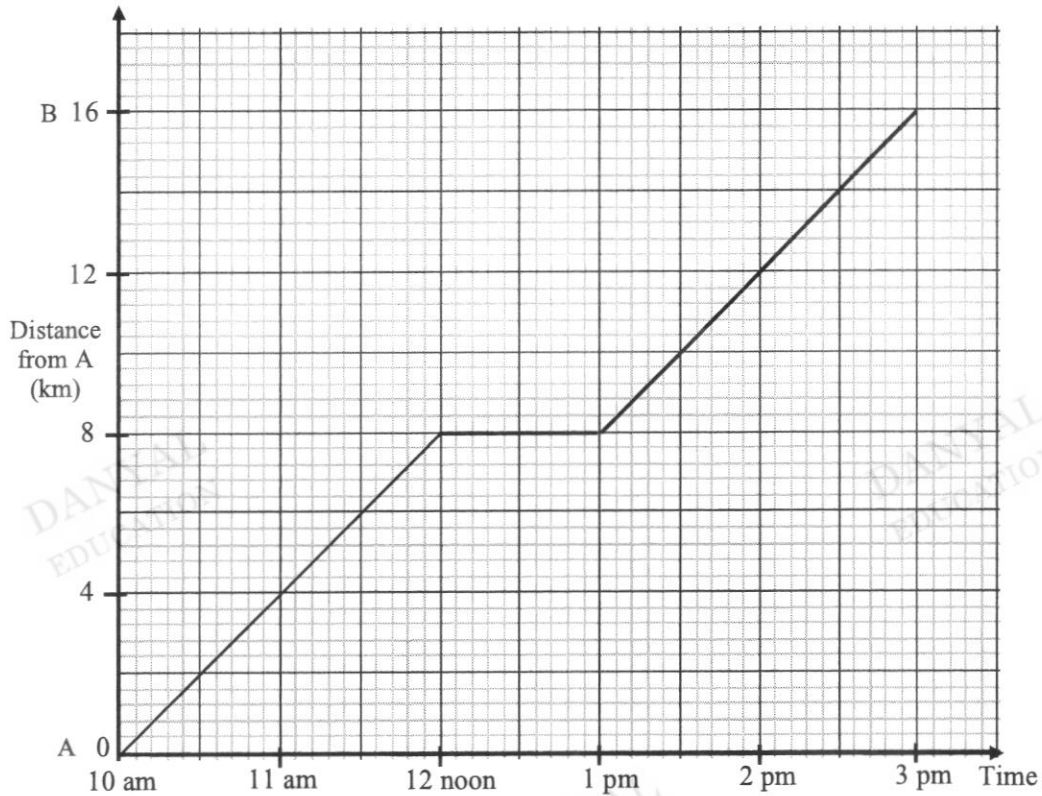
(a) Calculate the standard deviation.

Answer [1]

(b) Two other students also took the quiz, and their marks were recorded.
 Given that the mean mark obtained by the eight students was 10 and the mode was also 10,
 find the marks of these two students.

Answer and [2]

11



The distance-time graph shows the journey Tan took to run from town A to B.

- (a) Find the distance Tan ran in the first two hours.

Answerkm [1]

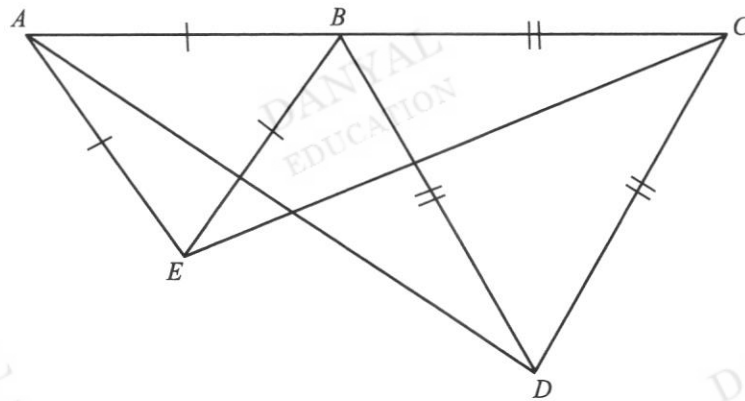
- (b) Calculate the average speed, in m/s, for the whole journey Tan ran.

Answerm/s [2]

12 Simplify $\frac{2y^2 + y - 3}{4y^2 - 9}$.

Answer [3]

13



In the diagram, ABC is a straight line and triangles ABE and BCD are equilateral triangles.

Show that triangle ABD and triangle EBC are congruent.

Give a reason for each statement you make.

Answer

.....

.....

.....

.....

[3]

14 The first three terms in a sequence of numbers, T_1, T_2, T_3, \dots are given below.

$$T_1 = 1 - \frac{1}{2}$$

$$T_2 = \frac{1}{2} - \frac{1}{3}$$

$$T_3 = \frac{1}{3} - \frac{1}{4}$$

⋮
⋮
⋮

(a) Write down T_4 .

Answer [1]

(b) Show that the total sum of $T_1, T_2, T_3, \dots, T_n$ in the above sequence is $1 - \frac{1}{n+1}$.

Answer

[2]

15 A, B and C are points $(-1, 0)$, $(3, 8)$ and $(2, 1)$ respectively.

(a) Find the length of AB .

Answer $AB = \dots\dots\dots$ units [2]

(b) Find the equation of the line that passes through B and has the same gradient as AC .

Answer $\dots\dots\dots$ [2]

16 (a) Find the interior angle of a regular 18-sided polygon.

Answer $\dots\dots\dots^\circ$ [2]

(b) An n -sided polygon has two of its exterior angles at 45° and 75° .

If the remaining exterior angles are each 20° , calculate the value of n .

Answer $n = \dots\dots\dots$ [2]

- 17 (a) Simplify $\left(\frac{a^{-6}}{b^9}\right)^{\frac{1}{3}}$ and leave your answer in positive index notation.

Answer [2]

- (b) Given that $2^{4x} \div 2^x = \sqrt[3]{2}$, find x .

Answer $x =$ [2]

- 18 (a) Given that $m^2 - 8mn + 16n^2 = 0$, find the value of $\frac{m}{n}$.

Answer [2]

- (b) Factorise completely $3ac - 7c + 18ab - 42b$.

Answer [2]

- 19 A florist sells three types of bouquets, Bliss, Love and Commitment. The number of stalks for each type of flower in each type of bouquet is shown in the table.

		Type of Flower			
		Rose	Lily	Gerbera	Sunflower
Type of Bouquet	Bliss	2	0	7	3
	Love	3	1	5	1
	Commitment	8	2	4	0

- (a) Represent the above information in a 3×4 matrix, **F**.

$$\text{Answer } \mathbf{F} = \begin{pmatrix} 2 & 0 & 7 & 3 \\ 3 & 1 & 5 & 1 \\ 8 & 2 & 4 & 0 \end{pmatrix} \quad [1]$$

- (b) The cost of each stalk of Rose, Lily, Gerbera and Sunflower is \$6, \$7.80, \$2.50 and \$3 respectively.

- (i) Represent this information in a 4×1 column matrix, **H**.

$$\text{Answer } \mathbf{H} = \begin{pmatrix} 6 \\ 7.80 \\ 2.50 \\ 3 \end{pmatrix} \quad [1]$$

- (ii) Evaluate the matrix $\mathbf{J} = \mathbf{FH}$.

$$\text{Answer } \mathbf{J} = \quad [1]$$

- (iii) State what the elements of **J** represent.

Answer

.....

[1]

15

- 20** Box X contains 5 balls numbered 2, 3, 4, 7 and 9.
 Box Y contains another 5 balls numbered 1, 5, 6, 8, and 10.
 In a game, Ming drew a ball at random from each box, and the sum of both numbers is obtained.

(a) Complete the possibility diagram below.

		Box Y				
		1	5	6	8	10
Box X	2	3			10	12
	3		8			
	4			10		
	7				15	
	9	10	14			19

[2]

(b) Find the probability that

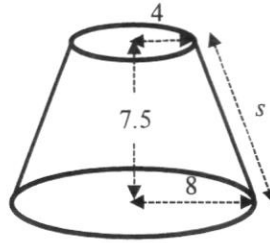
- (i) the sum of both numbers is an odd number,

Answer [1]

- (ii) the sum is a multiple of **one** of the two numbers drawn.

Answer [1]

- 21 The upper part of a solid wooden right circular cone was cut off leaving the frustum as shown in the diagram. The frustum has top radius 4 cm, base radius 8 cm and height 7.5 cm.



- (a) Show that the slant height, s , is 8.5 cm.

Answer

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[1]

- (b) Find the curved surface area of the frustum.

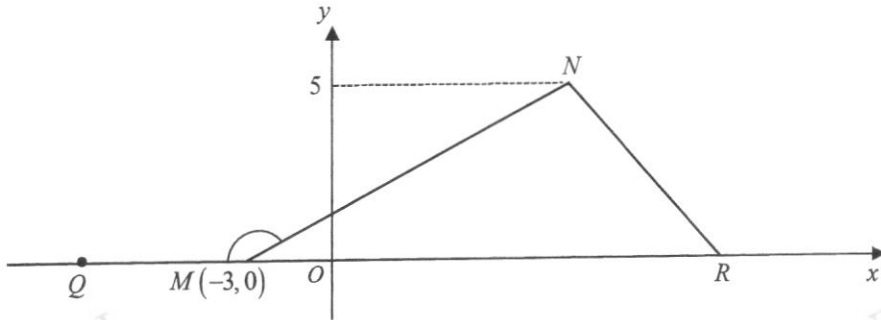
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Answer cm² [3]

- 22 In triangle MNR , point M is $(-3, 0)$ and $\sin \angle NMR = \frac{5}{13}$.
 Q is a point on the negative x -axis.



- (a) Express the following as a fraction
 (i) $\cos \angle NMQ$,

Answer [2]

- (ii) $\tan \angle NMR$.

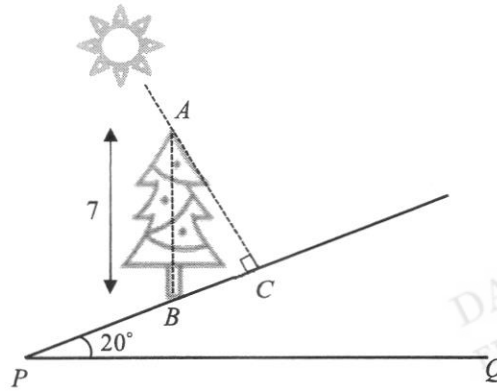
Answer [1]

- (b) The area of triangle MNR is 50 square units.
 Find the coordinates of R .

Answer (.....,.....) [2]

23 The diagram below shows a tree AB of height 7 m that stands vertically on a slope inclined at 20° with the horizontal PQ .

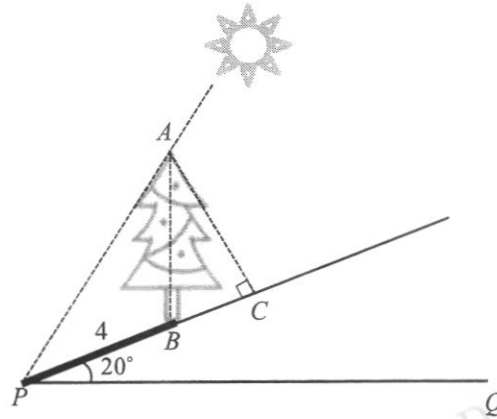
At a particular time in the morning, the tree casts a shadow, BC , on the slope. AC is perpendicular to the slope.



(a) Calculate the length of the shadow, BC .

Answer m [2]

After some time, the sun goes into a position as shown below.



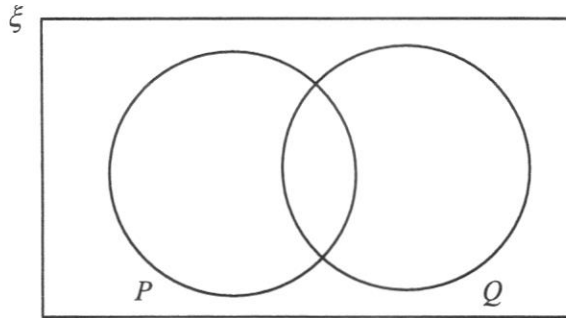
- (b) If the shadow, BP , of the tree on the slope is 4 m, find the angle that the sunray makes with the horizontal PQ .

Answer ° [3]

- 24 (a) $\xi = \{\text{integers } x : 1 \leq x \leq 12\}$
 $P = \{\text{prime numbers}\}$
 $Q = \{\text{multiples of } 3\}$

(i) Represent the above information on the Venn diagram shown in the answer space below.

Answer



[2]

(ii) List the elements in $(P' \cap Q') \cup (P \cap Q)$.

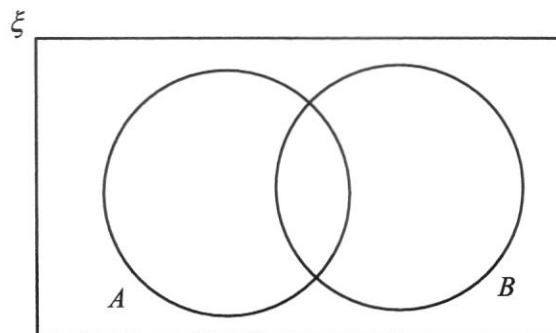
Answer [1]

(iii) $R = \{x : x \text{ is a multiple of } 6\}$

Use set notation to describe the relationship between Q and R .

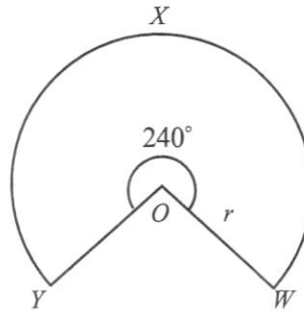
Answer [1]

(b) On the Venn diagram, shade the region which represents the set $A \cap B'$.



[1]

- 25 $OWXY$ is a sector of a circle, centre O , of radius r cm and reflex angle 240° .



The sector $OWXY$ has an area of 150π cm².

- (a) Express 240° in terms of π radians.

Answerrad [2]

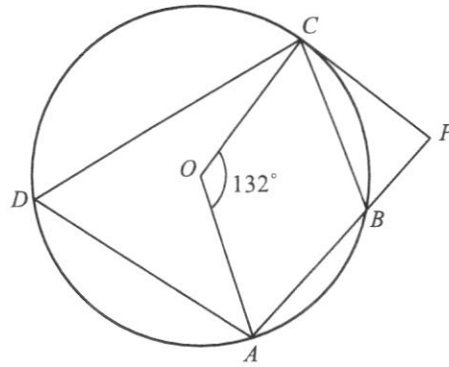
- (b) Show that $r = 15$.

Answer

- (c) The radii, OW and OY , are joined together to form a cone.
Find the base radius of the cone.

Answer cm [2]

26



In the diagram above, O is the centre of the circle, such that angle $COA = 132^\circ$.
 PC is a tangent to the circle at C and PBA is a straight line.

By giving a reason for each step of your working, find

(a) $\angle CDA$,

Answer^o [2]

(b) $\angle CBP$.

Answer^o [2]

- (c) If the radius of the circle is 3.55 cm, calculate the area of triangle AOC .

Answer cm^2 [2]

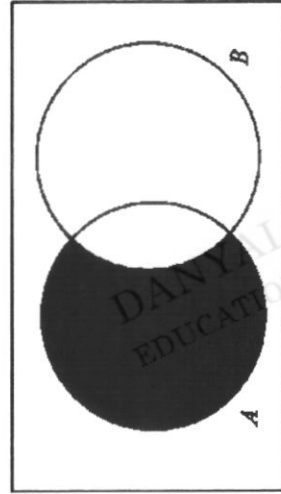
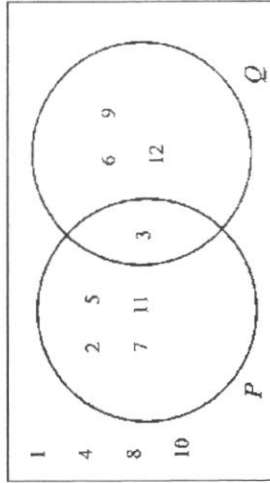
END OF PAPER

		2. The equation $(x-h)^2 = -\frac{1}{3}$ has no solution for x
10a	1.57	
10b	10 and 11	
11a	8 km	
11b	$\frac{8}{9}$ m/sec or 0.889 m/sec	
12	$\frac{y-1}{2y-3}$	
13	1) $AB = EB$ (sides of an equilateral triangle / given) 2) $BD = BC$ (sides of an equilateral triangle / given) 3) $\angle ABD = 180^\circ - 60^\circ$ (adj. \angle on a st. line) $= \angle EBC$ $= 120^\circ$	
14a	\therefore triangle ABD is congruent to triangle EBC (SAS) $T_4 = \frac{1}{4} - \frac{1}{5}$	
14b	$\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1}\right) = 1 - \frac{1}{n+1}$	
15a	8.94 units	
15b	$y = \frac{1}{3}x + 7$	
16a	160°	
16b	$n = 14$	
17a	$\frac{1}{a^2 b^3}$	
17b	$x = \frac{1}{9}$	
18a	$\frac{m}{n} = 4$	

1	$x = \frac{9}{2}$
2a	8593.4
2b	8.5934×10^3
3a	$784 = 2^4 \times 7^2$
3b	$a = 7$ and $b = 2$
4	$-w^2 - 4w + 5$
5a	56.25%
5b	Pie chart
6	3
7	$m = \frac{1}{8}$
8	
9a	$h = 2$
9b	<p>Either one</p> <p>1. The minimum point of the graph $y = 3(x-h)^2 + 1$ is $(h, 1)$ or $(2, 1)$.</p>

18b	$(c + 6b)(3a - 7)$																																				
19a	$F = \begin{pmatrix} 2 & 0 & 7 & 3 \\ 3 & 1 & 5 & 1 \\ 8 & 2 & 4 & 0 \end{pmatrix}$																																				
19bi	$H = \begin{pmatrix} 6 \\ 7.80 \\ 2.50 \\ 3 \end{pmatrix}$																																				
19bii	$\begin{pmatrix} 38.50 \\ 41.30 \\ 73.60 \end{pmatrix}$																																				
19biii	The elements of J represent the total cost of the four types of flowers - Rose, Lily, Gerbera and Sunflower in bouquet Bliss, Love and Commitment respectively.																																				
20a	<table border="1"> <tr> <td>+</td> <td>1</td> <td>5</td> <td>6</td> <td>8</td> <td>10</td> </tr> <tr> <td>2</td> <td>3</td> <td>7</td> <td>8</td> <td>10</td> <td>12</td> </tr> <tr> <td>3</td> <td>4</td> <td>8</td> <td>9</td> <td>11</td> <td>13</td> </tr> <tr> <td>4</td> <td>5</td> <td>9</td> <td>10</td> <td>12</td> <td>14</td> </tr> <tr> <td>7</td> <td>8</td> <td>12</td> <td>13</td> <td>15</td> <td>17</td> </tr> <tr> <td>9</td> <td>10</td> <td>14</td> <td>15</td> <td>17</td> <td>19</td> </tr> </table>	+	1	5	6	8	10	2	3	7	8	10	12	3	4	8	9	11	13	4	5	9	10	12	14	7	8	12	13	15	17	9	10	14	15	17	19
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2	3	7	8	10	12																																
3	4	8	9	11	13																																
4	5	9	10	12	14																																
7	8	12	13	15	17																																
9	10	14	15	17	19																																
20bi	$\frac{13}{25}$																																				
20bii	$\frac{2}{5}$																																				
21a	Show																																				

21b	320 cm ²
22ai	$-\frac{12}{13}$
22aii	$\frac{5}{12}$
22b	Coordinates of R = (17, 0)
23a	2.39 m
23b	65.8°
24ai	ξ
24aai	$(P' \cap Q') \cup (P \cap Q) = \{1, 3, 4, 8, 10\}$
24aiii	$R \subset Q$
24b	ξ
25a	$\frac{4}{3}\pi$ rad



26

25b	Show
25c	10
26a	$\angle CDA = 132^\circ \div 2$ $= 66^\circ$ (\angle at the centre = twice \angle at circumference)
26b	$\angle CBA = 180^\circ - 66^\circ$ $= 114^\circ$ (\angle s in opp. segment) $\angle CBP = 180^\circ - 114^\circ$ (adj. \angle s on a st. line) $= 66^\circ$
26c	4.68 cm ²