Class	Index Number

Name : _____

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2021
Secondary 4

Tuesday	**MATHEMATICS**	4048/01
3 August 2021	**Paper 1**	2 hours

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give your answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

80

For Examiner's Use

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{\text{r}}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's Use

Answer **all** the questions

1 Express the ratio $3\frac{2}{3}$ kg : 450 g : 3×10^2 g in its simplest form.

Answer ..... : ..... : ..... [2]

---

2 Given that x is 35% of y, find the value of $\frac{2x}{5y}$, expressing your answer as a fraction in its lowest terms.

Answer ..... [2]

---

3 If $10^m = 2$ and $10^n = 3$, find the value of 10^{3m-n}.

Answer ..... [2]

For Examiner's Use

4 Simplify $5a^2-(3a-2b)^2$.

.

Answer ..... [2]

---

5 Write as a single fraction in its simplest form $\frac{-4}{x^2-4}+\frac{1}{x-2}$.

Answer ..... [2]

---

6 Solve the inequality $x-5<\frac{2x}{5}\leq\frac{x}{2}+\frac{1}{5}$.

Answer ..... [3]

For Examiner's Use

7 Solve the equation $\left(\frac{1}{5}\right)^{-2} \times 125^{x} = \sqrt{25^{x}}$.

Answer $x =$..... [2]

8 **(a)** Madeline deposited \$6 000 into a 3-month fixed deposit which pays a simple interest. If she receives a total amount of \$6 013.50 at the end of the 3 months, what is the rate of interest per annum?

Answer .....% [2]

(b) Another bank offers an interest rate of 0.6% per annum, compounded monthly. What would be the total amount that Madeline receives, correct to the nearest dollar, at the end of 3 months if she were to put the \$6 000 in this bank?

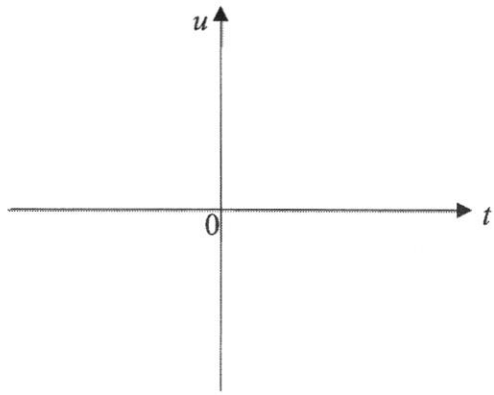
Answer \$..... [2]

For Examiner's Use

9 u is inversely proportional to the square of t.

(a) Sketch the graph of $u = \frac{2}{t^2}$.

Answer: [1]



(b) Find the percentage change in u when t is tripled.

Answer .....% [2]

---

10 These are the first four terms of a sequence.

$$\frac{5}{2} \quad \frac{20}{3} \quad \frac{45}{4} \quad \frac{80}{5}$$

(a) Write down the next two terms.

Answer ..... , ... [1]

(b) Find an expression. in terms of n, for the nth term of this sequence.

Answer ..... [1]

(c) Explain why $71\frac{3}{16}$ is not a term of this sequence.

Answer .....

.....

.....[1]

For Examiner's Use

11 The exchange rate between pounds (£) and Singapore dollars ($) is £1 = $1.86.

The exchange rate between Singapore dollars ($) and euros (€) is $1 = €0.62.

Megan is comparing day tour packages on a website.

Paris 2-Day Tour €280 per person
London 2-Day Tour £280 per person

Megan claims that the tour package in Paris costs less than the tour package in London.

Is Megan's claims correct? Justify your answer with calculations.

Answer .....

.....

.....

.....

..... [2]

---

12 **(a)** **(i)** Express 1008 as a product of prime factors, leaving your answer in index notation.

Answer 1008 = ...[1]

(ii) p and q are numbers such that q is a prime number.

Find the values of p and q such that $\frac{1008p}{q}$ is the smallest possible cube.

Answer p = ..., q =..... [2]

(b) The lowest common multiple of x and 1008 is $2^5 \times 3^2 \times 5 \times 7$. Find the smallest possible value of x.

Answer x =..... [1]

For Examiner's Use

13 **(a)** **(i)** Express $4-2x-x^2$ in the form $a-(x+b)^2$.

Answer ..... [1]

(ii) Explain, without solving, why there is no solution of x when $4-2x-x^2=6$.

Answer

.....

.....

.....

.....

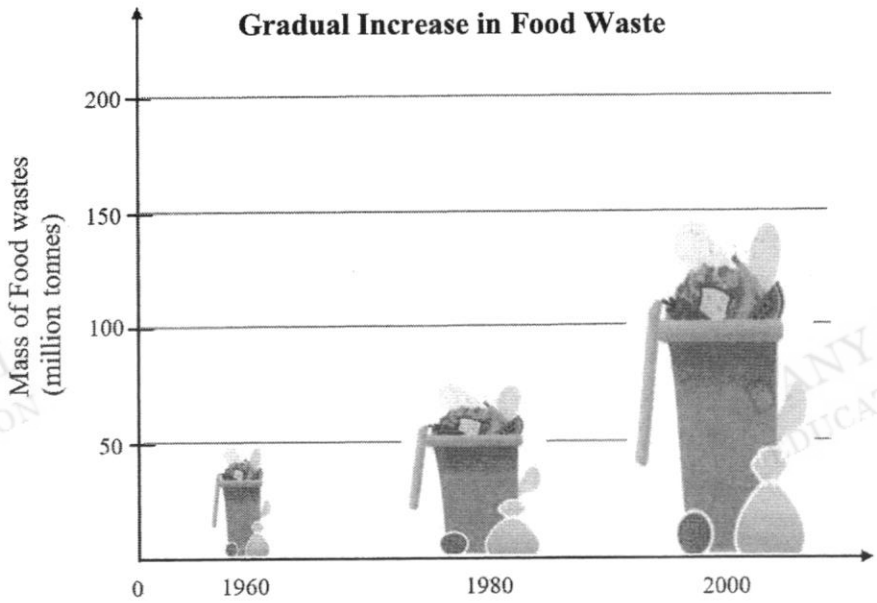
..... [1]

(b) Using part (a)(i), solve the equation $x^2+2x=8$.

Answer $x=$.....or ..... [3]

For Examiner's Use

14 The graph shows the total mass of food waste collected around a particular neighbourhood at the end of each given year.



(a) State one misleading feature of the graph.

Answer .....

.....

.....

.....

..... [1]

(b) Explain how this feature affects the reader's interpretation of the graph.

Answer .....

.....

.....

.....

..... [1]

For Examiner's Use

15 The following diagram represents the stem-and-leaf scores of sixteen students who took a Mathematics test.

2	1
3	
4	
5	4 5 6 6 7 8 9
6	***k*** 9
7	1 3 5 6 9
8	0

Key: 2 | 1 means 21 marks

(a) If the median mark is 62, find the possible value of k.

Answer k =..... [1]

(b) Explain why the mean may not be appropriate average to use to summarise the results of the Mathematics test.

Answer.....

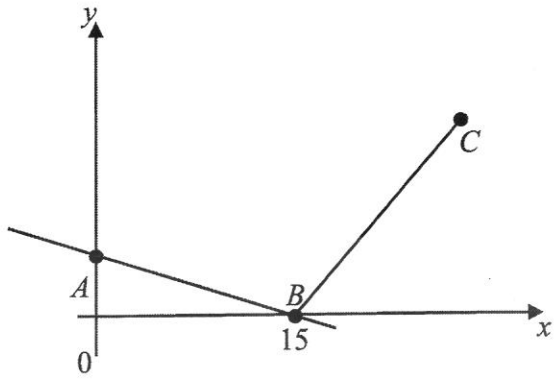
.....

.....

.....[1]

For Examiner's Use

16 In the diagram, C is the point $(30,15)$, A is a point on the y-axis and B is a point on the x-axis.



(a) Given that the equation of the line AB is $3y+x=15$, find the length of AB.

Answer .....units [1]

(b) Find the equation of the line which passes through $C(30,15)$ and is parallel to the line AB.

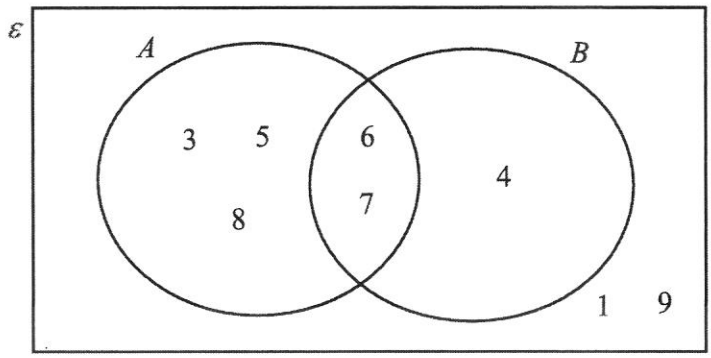
Answer ..... [2]

(c) If $ABCD$ is a parallelogram, find the coordinates of D.

Answer D (..... , ...) [1]

For Examiner's Use

17 **(a)**



Use the Venn diagram to answer the following.

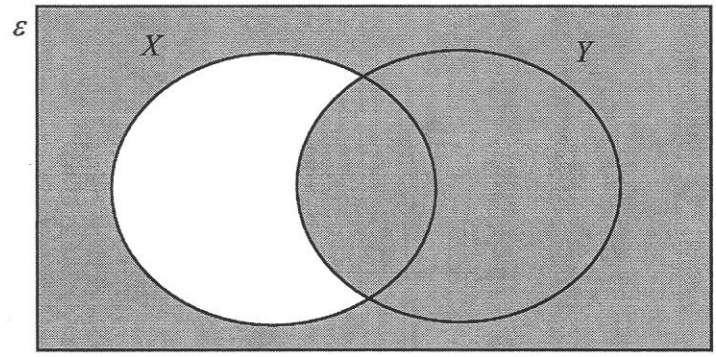
(i) List all the elements in the set $(A \cup B) \cap B'$.

Answer {.....} [1]

(ii) Fill in a set notation symbol to complete the statement below.

Answer {3, 5, 8} ..... A [1]

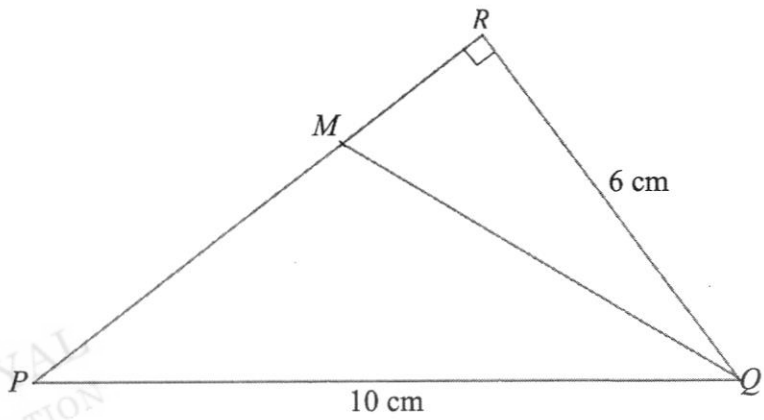
(b) Use set notation to describe the shaded region.



Answer ..... [1]

18 Triangle PQR is a right-angled triangle with $PQ = 10$ cm, $QR = 6$ cm and $\angle PRQ = 90°$.

M is a point on PR such that $11MR = 5PM$.



Find, giving your answer as a fraction in its simplest form, the value of

(a) $\tan \angle MQR$,

Answer ..... [2]

(b) $\cos \angle PMQ$,

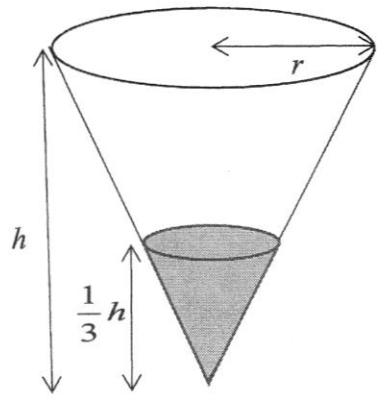
Answer ..... [1]

(c) $\dfrac{\text{area of } \Delta PQM}{\text{area of } \Delta PQR}$.

Answer ..... [1]

For Examiner's Use

19 The diagram shows an inverted hollow paper cone with a base of radius r cm and a height of h cm. The cone is filled with water, represented by the shaded region, to a depth of $\frac{1}{3}h$ cm.



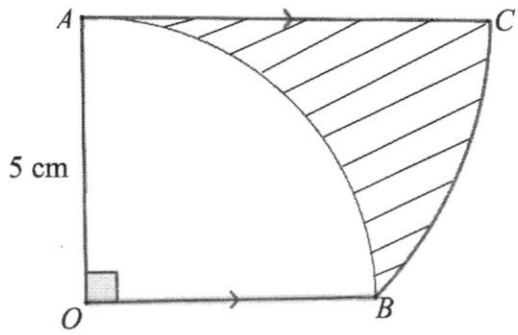
The internal surface area of the cone is 106 cm^2.

Find the surface area of the inside of the cone that is in contact with the water.

Answer .....cm^2 [2]

20 AB is an arc of a circle with the centre O, $\angle AOB = \frac{\pi}{2}$ radians and $OA = 5$ cm.

AC is parallel to OB and BC is an arc of a circle with centre A.



(a) Find the length of arc BC.

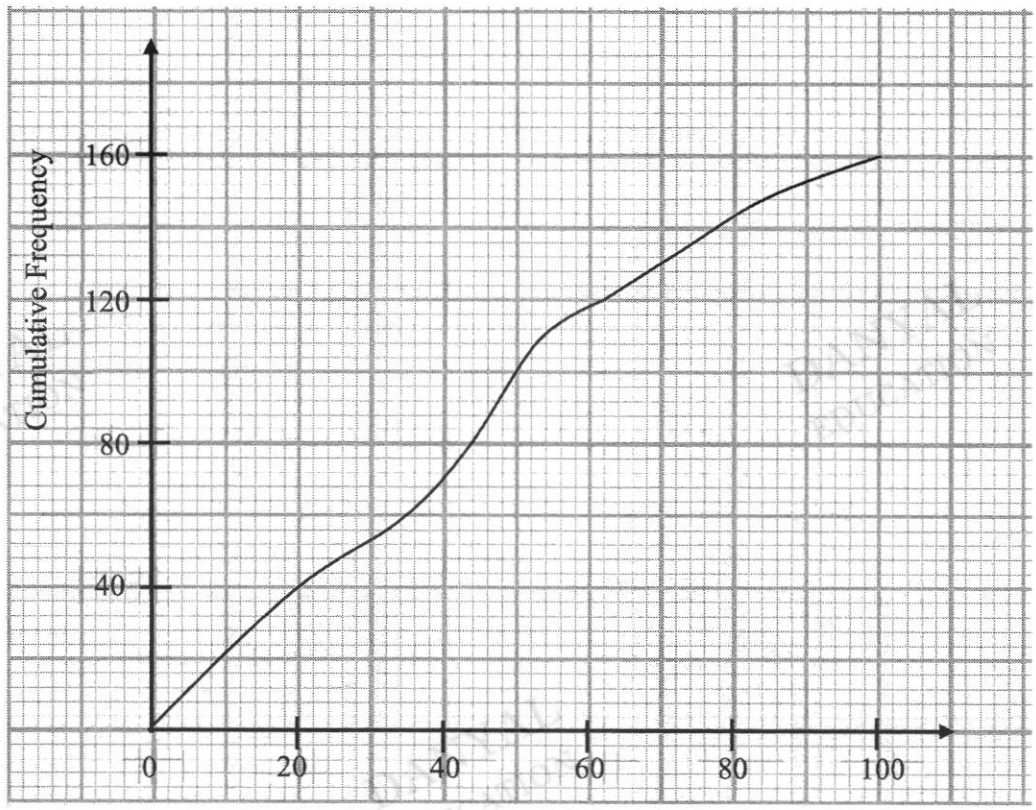
Answer ..... cm [3]

(b) Find the percentage of the figure $OACB$ that is **not** shaded.

Answer .....% [3]

For Examiner's Use

21 The cumulative frequency curve shows the weekly expenditure on home deliveries of 160 households from Block *W*.



(a) Use the cumulative frequency curve to find

(i) the median of the distribution,

Answer $..... [1]

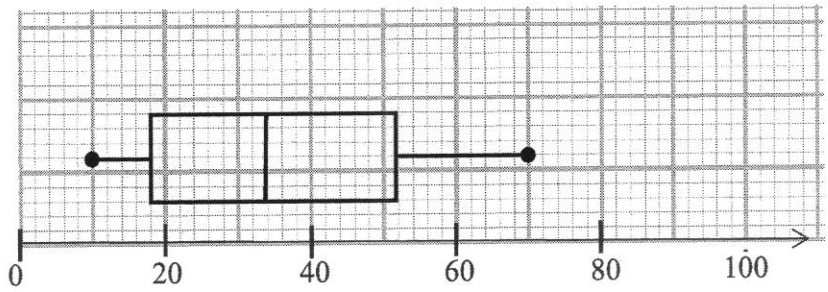
(ii) the interquartile range of the distribution,

Answer $..... [1]

(iii) the value of p, given that p% of the residents spent more than $84 weekly on home deliveries.

Answer p =..... [1]

(b) The weekly expenditure on home deliveries of a group of 160 residents from Block *K* are displayed in the box and whisker plot below.



Weekly expenses on home deliveries ($)

Here are two statements comparing the weekly expenditure on home deliveries of the residents from Block *W* and Block *K*.

For each statement, state whether you agree or disagree.

Give a reason for each answer, stating clearly which statistic you used to make your decision.

(i) On average, the residents in Block *W* has lower weekly expenses on home deliveries than the residents in Block *K*.

Answer I .....because.....

.....

.....

.....

..... [1]

(ii) The weekly expenditure on home deliveries of the residents in Block *K* is closer to the median than the weekly expenditure on home deliveries of the residents from Block *W*.

Answer I .....because.....

.....

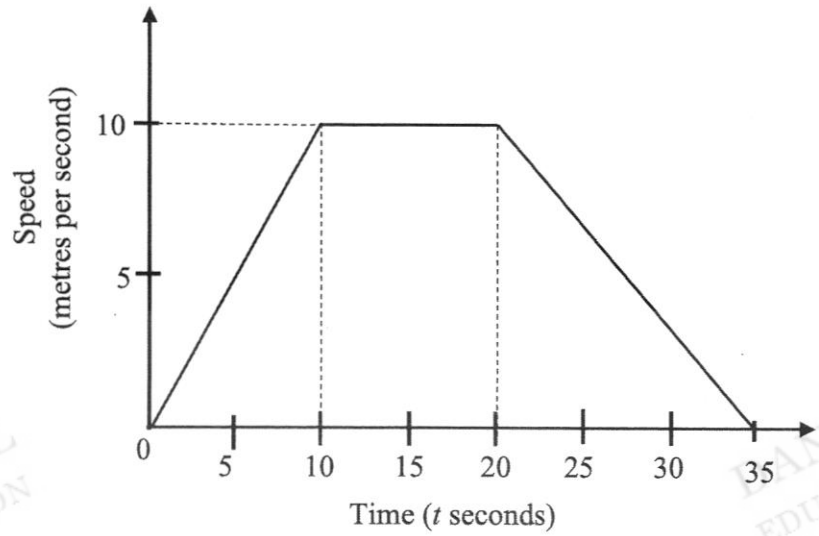
.....

.....

..... [1]

For Examiner's Use

22



The speed-time graph of a particle is shown in the diagram above.

(a) Find

(i) the speed when $t = 25$,

Answer .....**m/s [2]**

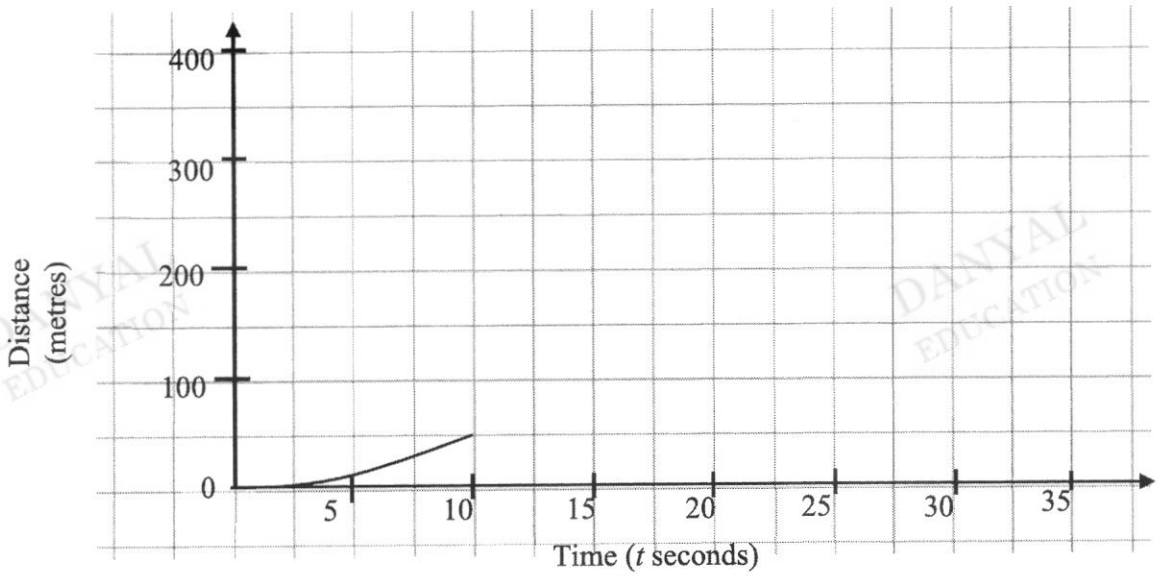
(ii) the distance covered in the first 35 seconds.

Answer .....**m [2]**

For Examiner's Use

(b) Part of the distance-time graph for the same particle is shown in the answer space.

Complete this graph. [1]

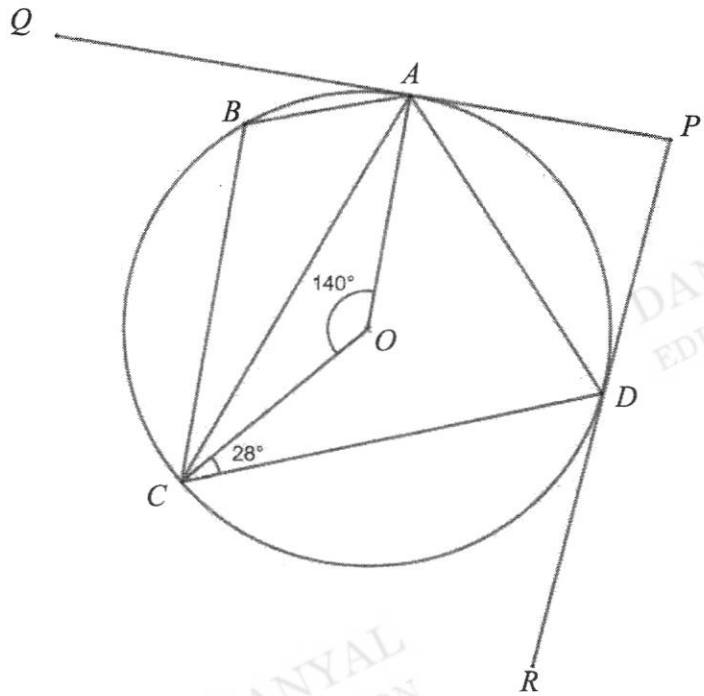


For Examiner's Use

23 In the diagram, A, B C and D lie on a circle, centre O.

PQ and PR are tangents to the circle at A and D respectively.

It is given that $\angle AOC = 140°, \angle DCO = 28°$ and OA is parallel to CB.



(a) Show, stating your reasons clearly, that CA bisects $\angle BCO$.

Answer .....

.....

.....

.....

.....

.....

.....

.....

.....

.....

..... [2]

For Examiner's Use

(b) Find, stating your reasons clearly,

(i) $\angle BAC$,

Answer $\angle BAC$ =……° [2]

(ii) $\angle PDA$,

Answer $\angle PDA$ =……° [2]

(c) Lizzy claims that $QADR\ldots$ forms a part of a regular n-sided polygon.

Justify, with reasons, if you agree or disagree with Lizzy.

Answer ……

……

……

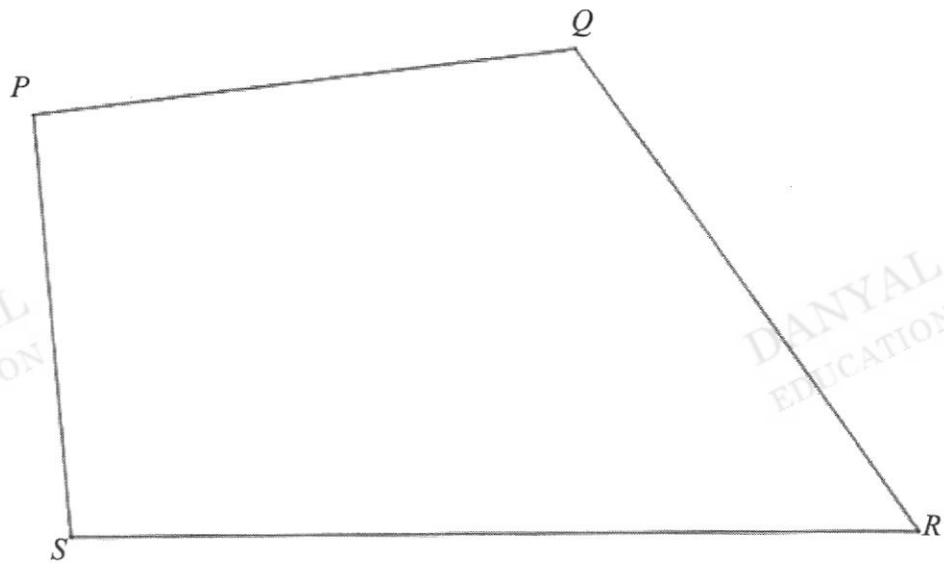
……

…… [1]

For Examiner's Use

24 On the scale drawing, P, Q, R and S are the positions of four garden statues in a park. $PQRS$ is a quadrilateral.

Scale: 1 : 25 000



(a) A water-cooler is to be built at A where it is equidistant from the line segments PS and PQ and the points P and S.

On the scale drawing, mark the point A. [2]

(b) The garden statue at R is due east of S.

The gardener plans to build a pavilion at H.

The bearing of H from R is $315°$ and 750 m from Q.

(i) Calculate the distance HQ on the scale drawing.

Answer HQ =.....cm [1]

(ii) Mark the point H on the diagram above. [1]

~ End of Paper ~

Name : _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2021
Secondary 4

Friday
4 August 2021

MATHEMATICS
Paper 2

4048/02
2 h 30 min

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

/100

This question paper consists of 21 printed pages and 3 blank pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{\text{r}}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Simplify $\dfrac{25m^2-4}{4-4m-15m^2}$.

Answer ..... [3]

(b) Given that $r=3-\sqrt{\dfrac{p^2}{4}-q}$,

(i) evaluate r when $p=-8$ and $q=7$,

Answer ..... [1]

(ii) express p in terms of q and r.

Answer ..... [3]

(c) The value of a 2-digit number can be written as $10x + y$ where x is the tens digit of the number and y is its units digit.

(i) When the digits x and y are reversed, the value is increased by 27.
Show that $y - x = 3$. [1]

(ii) The sum of the original number and the number with reverse digits is 121.
Show that $x + y = 11$. [1]

(iii) Hence, find the original number.

Answer ..... [3]

2 Bag A has 4 green counters, 6 red counters and 2 yellow counters.

(a) Alice takes a counter at random from Bag A, notes its colour and replaces it in the bag. She repeated this process n times. Write down an expression, in terms of n, for the probability that she takes only green counter.

Answer ..... [1]

(b) Bill takes a counter at random from Bag A, notes its colour and replaces it back. He then takes a second counter at random from Bag A.

Calculate the probability that Bill takes only one yellow counter.

Answer ..... [2]

Bag B has 3 green counters and 3 red counters.

(c) Charles takes a counter at random from Bag A and places it into Bag B.
He then takes a counter at random from Bag B and places it into Bag A.
Find the probability that after these two moves, Bag A has 4 green counters, 6 red counters and 2 yellow counters.

Answer ..... [3]

3 **(a)** The table shows the height, h cm, of 60 plants.

Height (h cm)	$10 < h \leq 20$	$20 < h \leq 30$	$30 < h \leq 40$	$40 < h \leq 50$
Frequency	7	14	27	12

(i) Calculate an estimate of the standard deviation of the height of the plants.

Answer ..... cm [1]

(ii) Two plants are chosen at random. Find the probability that both plants have height greater than 30 cm.

Answer ..... [2]

(b) A toothpaste supplier delivers tubes of toothpaste to 3 different shops A, B and C. The number of tubes of toothpaste supplied per delivery to each shop, the sizes and sale prices of the tubes, together with the number of deliveries made to each shop over a 6-month period are shown in the table below.

		Number of tubes of toothpaste per delivery			Number of deliveries over 6 months
Size of tube		small	medium	large	
Name of shop	A	68	85	105	4
	B	75	–	62	7
	C	–	130	98	5
Sale price per tube		\$2.50	\$4.20	\$6.00	

It is given that $\mathbf{T} = \begin{pmatrix} 68 & 85 & 105 \\ 75 & 0 & 62 \\ 0 & 130 & 98 \end{pmatrix}$.

(i) Write down the matrix **P** such that **TP** gives the total number of tubes of toothpaste delivered to each shop per delivery.

Answer **P** = ... [1]

(ii) **(a)** Write down a 1×3 matrix **D** to represent the number of deliveries over 6 months to each shop.

Answer **D** = ... [1]

(ii) **(b)** Evaluate **DT**.

Answer **DT** = ... [1]

(c) State what the elements of **DT** represent.

Answer

.....

.....

..... [1]

(iii) **Q** is a matrix that represents the total amount of money that would be obtained from each shop from the sale of all the tubes of toothpaste per delivery.
By matrix multiplication, find **Q**.

Answer **Q** = ... [2]

4 **(a)** Alan drove a distance of x km and his car used 7.5 litres of fuel.
Write down an expression, in terms of x, for the rate of fuel used by his car in litres per 100 km.

Answer ..... litres per 100 km [1]

(b) Ben drove a distance of $(x + 30)$ km and his car used 8 litres of fuel.
Write down an expression, in terms of x, for the rate of fuel used by his car in litres per 100 km.

Answer ..... litres per 100 km [1]

(c) The rate of fuel used by Alan's car is 1.5 litres per 100 kilometre more than the rate of fuel used by Ben's car.
Form an equation to represent this information and show that it can be simplified to $3x^2 + 190x - 45000 = 0$.

Answer [3]

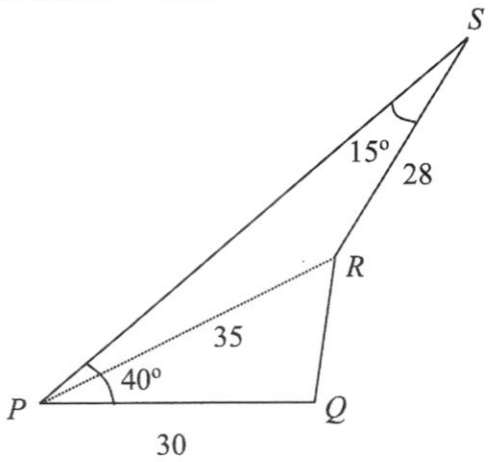
(d) Solve the equation $3x^2 + 190x - 45000 = 0$, giving both answers correct to two decimal places.

Answer $x =$ ..... or ..... [3]

(e) Each litre of fuel costs \$2.42.
Calculate the amount Alan paid for the petrol if he drove 120 km.

Answer \$ ..... [2]

5 The diagram shows four points P, Q, R and S on level ground.
Q is due east of P.
Angle $SPQ = 40°$ and angle $PSR = 15°$.
$PQ = 30$ m, $PR = 35$ m and $RS = 28$ m.



(a) Calculate

(i) the bearing of R from P,

Answer ..... ° [3]

(ii) QR,

Answer ..... m [2]

(iii) the area of triangle PRS.

Answer ..... m^2 [2]

A vertical mast stands at R and the angle of elevation of the top of the mast from P is 18°.

(b) (i) Calculate the height of the mast.

Answer ..... m [2]

(ii) A bird lands on the top of the mast.
Find its largest angle of depression of a point along PQ produced.

Answer ..... $^\circ$ [3]

6 **(a)** It is given that $\overrightarrow{PQ} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$ and the position vector of P is $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$. Find

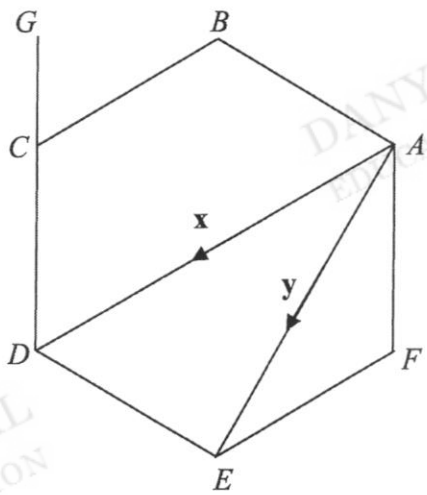
(i) $\left|\overrightarrow{PQ}\right|$,

Answer ..... units [1]

(ii) the coordinates of Q.

Answer (..... , ...) [2]

(b)



$ABCDEF$ is a regular hexagon.
G is a point on DC extended such that $2DG = 3DC$.
$\overrightarrow{AD} = \mathbf{x}$ and $\overrightarrow{AE} = \mathbf{y}$.

(i) Express, as simply as possible, in terms of **x** and/or **y**,

(a) $\overrightarrow{AB}$,

Answer ..... [1]

(b) $\overrightarrow{AF}$,

Answer ..... [1]

(c) $\overrightarrow{BG}$,

Answer ..... [2]

(ii) M is the mid-point of AF.
H is a point on BE where $4HE = 3BE$.

(a) Find $\overrightarrow{HG}$.

Answer ..... [2]

(b) Using vectors, determine whether H lies on MG. [2]

7 Figure **I** shows a piece of wood is in the form of a right circular cone with a base diameter of 18 cm.

(a) The curved surface area of the cone is 135π cm^2.
Find the height of the cone.

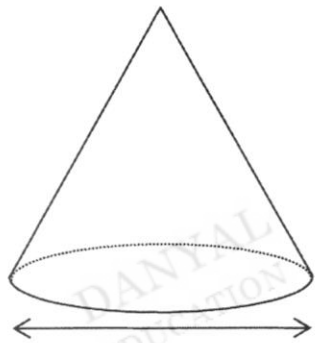


Figure **I**

Answer ..... cm [3]

The cone in Figure **I** is cut into two portions by a plane parallel to its base.
The upper portion is a cone of base diameter 6 cm and the lower portion, as shown in Figure **II**, is a frustum of height x cm.

(b) Find the value of x.

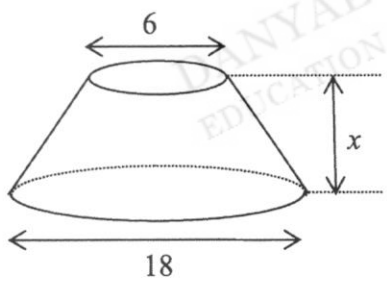


Figure **II**

Answer $x =$ ..... [2]

(c) A right cylindrical hole of diameter 6 cm is drilled through the frustum as shown in Figure **III**.
The wood weighs 810 kg per cubic metres.
Find the mass, in grams, of the solid which remains in the frustum.

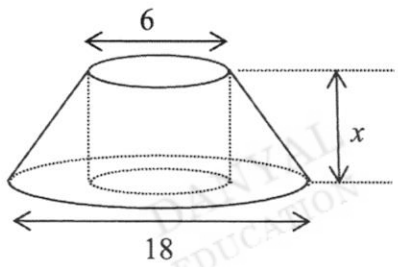


Figure **III**

Answer ..... g [3]

8 The table shows some values for $y = \frac{3}{10}x^3 - x + 1$ for $-3 \le x \le 3$.

x	–3	–2	–1	0	1	2	3
y	p	0.6	1.7	1	0.3	1.4	6.1

(a) Find the value of p.

Answer $p =$ ..... [1]

(b) Using a scale of 2 cm to 1 unit on both axes, draw a horizontal axis for $-3 \le x \le 3$ and a vertical axis for $-4.5 \le y \le 6.5$ on the grid opposite.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve at $x = 2$.

Answer ..... [2]

(d) The equation $\frac{3}{10}x^3 - x = k - 1$, where k is a positive integer, has exactly one solution for $-3 \le x \le 3$. Write down an inequality for k.

Answer ..... [1]

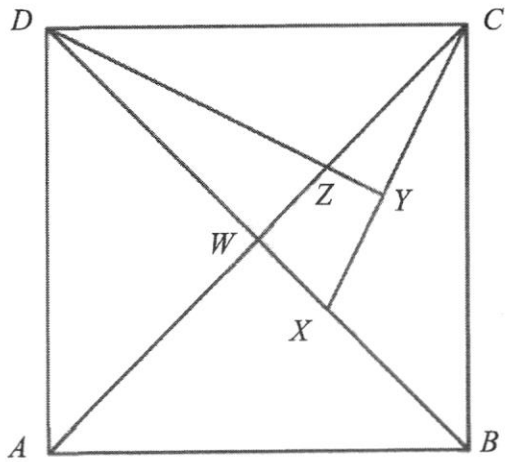
(e) **(i)** The line $y = ax + b$ can be drawn on the same grid to solve the equation $\frac{3}{10}x^3 - 1 = \frac{3}{2}x$. Find the value of a and the value of b.

Answer $a =$ ..... and $b =$ ..... [2]

(ii) Draw the line $y = ax + b$, for $-3 \le x \le 3$, to solve the equation $\frac{3}{10}x^3 - 1 = \frac{3}{2}x$.

Answer $x =$ ....., ... or ..... [3]

9 The diagram shows a square $ABCD$.
The diagonals of the square intersect at W and X is a point on the diagonal BD.
Y lies on CX such that DY is perpendicular to CX and DY meets AC at Z.



(a) Show that triangles DWZ and CYZ are similar.

.....

.....

.....

.....

.....

.....

.....

..... [2]

(b) Write down another pair of triangles that are similar but not congruent.

Answer Δ..... and Δ..... [1]

(c) Prove that triangle BCX and triangle CDZ have the same area.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

..... [4]

(d) It is given that $YZ : WZ = \sqrt{2} : \sqrt{5}$ and $3CZ = 2CW$.

(i) Find $\dfrac{\text{area of } \Delta CYZ}{\text{area of } \Delta DWZ}$.

Answer ..... [1]

(ii) Calculate the area of the square $ABCD$ if the area of triangle CYZ is 3 cm^2.

Answer ..... cm^2 [2]

10 John owns a cleaning company that cleans office blocks.
The number of employees in his company and their salaries in 2020 are as follows.

11 handymen, each receiving a yearly salary of $45 936.
272 cleaners, each receiving $5.20 per hour and working 8 hours per day for a total of 20 days per month.
12 supervisors, men and women in the ratio 1 : 3, each receiving a monthy salary that is 235% more than a cleaner's monthy salary.
11 drivers, each receiving $594.50 per week, for 4 weeks in a month.

In addition, both employer and employee must contribute a portion of the employee's monthly salary to the Central Provident Fund (CPF).
The table shows the CPF contribution and allocation rates for Singapore citizens.

	Contribution rates (for monthly salary ≥ $750)	
Employee's age (years)	By employer (% of salary)	By employee (% of salary)
55 and below	17	20
Above 55 to 60	13	13
Above 60 to 65	9	7.5
Above 65	7.5	5

(a) The yearly salary of each handyman in 2020 is 2.5 % more than in 2017.
Find the yearly salary of each handyman in 2017.

Answer $ ..... [2]

(b) Mdm Lee is a 65 year old Singapore citizen who works as a cleaner in John's cleaning company. Calculate the total amount of contribution to Mdm Lee's CPF account every month in 2020.

Answer $ ..... [1]

(c) Every month, John randomly selects an 'Employee of the Month' from all his employees. Mr Tan comments that the chance of randomly selecting a male supervisor as the employee of the month is most unlikely.
Do you agree with Mr Tan's comment? Justify your answer. [2]

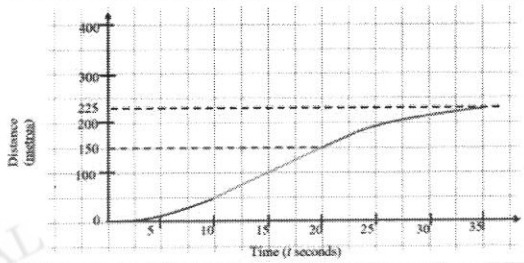
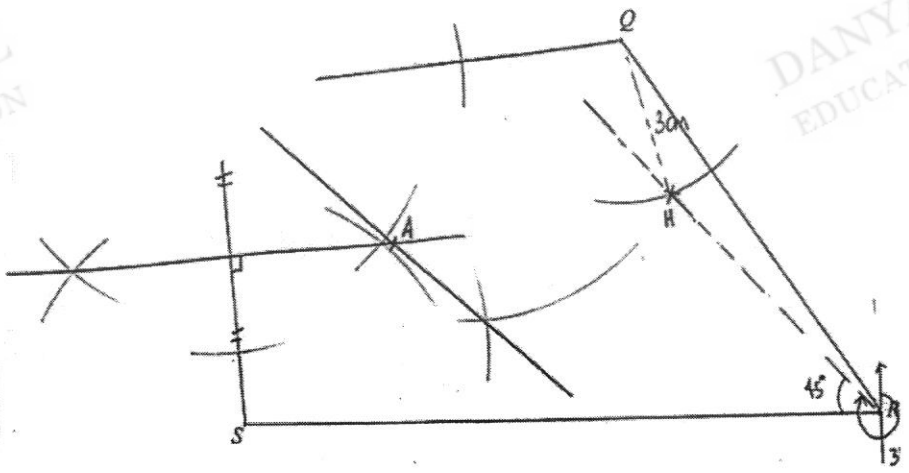
(d) John states that the difference between the salary of the lowest paid employee and the mean salary of all the employees as a percentage of the mean salary of all the employees is less than 25%.
Verify, showing all calculations, whether his statement is valid. [5]

For Examiner's Use

Answer Key

1	220:27:18	13b	2, –4
2	$\frac{7}{50}$	14a	It is not clear if the **height or the area** of the image is used to determine the mass of food waste collected.
3	$\frac{8}{3}$ or $2\frac{2}{3}$	14b	The height and the area of the image is not proportional. The readers may think that there is **more** than 100 million tonnes increase in mass from 1960-2000.
4	$-4a^2+12ab-4b^2$	15a	5
5	$\frac{1}{(x+2)}$	15b	The mean is not appropriate as there is an outlier (21 mark) which will decrease/affect the mean /skew the mean value.
6	$-2 \le x < 8\frac{1}{3}$	16a	15.8
7	–1	16b	$y=-\frac{1}{3}x+25$
8a	0.9	16c	(15, 20)
8b	\$6009	17ai	{3, 5, 8}
9a		17aii	$\subset$
9b	–88.9	17b	$Y \cup X'$
10a	$\frac{125}{6}, \frac{180}{7}$	18a	$\frac{5}{12}$
10b	$\frac{5n^2}{n+1}$	18b	$-\frac{5}{13}$
10c	$71\frac{3}{16}=\frac{1139}{16}$, since 1139 is not a multiple of 5, $71\frac{3}{16}$ cannot be a term	18c	$\frac{11}{16}$
11	£1 = \$1.86 €1$=\frac{1}{0.62}$ = \$1.61 Since it costs less to buy €1 compared to £1, it would be cheaper to go on the tour in Paris.	19	11.8
12ai	$2^4 \times 3^2 \times 7$	20a	5.55
12aii	$p=12, q=7$	20b	61.1%
12b	160	21ai	\$44
13ai	$5-(x+1)^2$	21aii	\$42
13aii	The maximum value of $4-2x-x^2=5$, hence there are no solution of x when $4-2x-x^2=6$.	21aiii	7.5

For Examiner's Use

21bi	I DISAGREE because residents in Block W has a higher median expenditure (\$44) than the residents in Block K (\$34)
21bii	I AGREE because the interquartile range of Block K (\$36) is lower than the interquartile range Block W (\$42) showing that the data is less widely spread than Block W's
22ai	6.67
22aii	225
22b	
23a	$\angle OCA = \angle OAC$ (base $\angle$ of isos.Δ) $= \frac{180° - 140°}{2}$ $= 20°$ $\angle BCA = \angle OAC$ (alt. $\angle$s) or $\angle BCA = 180° - 140° - 20°$ (int. $\angle$) $= 20°$ Since $\angle OCA = \angle BCA = 20°$, CA bisects $\angle BCO$.
23bi	50
23bii	48
23c	ext. $\angle$ of polygon $\angle PAD = 48°$ no. of sides $= \frac{360}{48} = 7.5$ Since the number of sides is not an integer, $QADR$ is not a regular polygon Hence I disagree with Lizzy.
24a,bii	
24bi	3

Name : MARK SCHEME

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2021
Secondary 4

TUESDAY	**MATHEMATICS**	4048/01
3 August 2021	**Paper 1**	2 hours

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give your answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

80

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's Use

Answer **all** the questions

1 Express the ratio $3\frac{2}{3}$ kg : 450 g : 3×10^2 g in its simplest form.

$\frac{11}{3}:0.45:0.3$ M1

$220:27:18$ A1

Answer ...220... : ...27... : ...18.... [2]

---

2 Given that x is 35% of y, find the value of $\frac{2x}{5y}$, expressing your answer as a fraction in its lowest terms.

$x=0.35y$

$\frac{x}{y}=0.35$ M1

$\frac{2x}{5y}=\frac{2}{5}\times0.35$

$=\frac{7}{50}$ A1

Answer ..... $\frac{7}{50}$ ..... [2]

---

3 If $10^m=2$ and $10^n=3$, find the value of 10^{3m-n}.

$10^{3m-n}=\frac{10^{3m}}{10^n}$ M1

$=\frac{2^3}{3}$

$=\frac{8}{3}$ or $2\frac{2}{3}$ A1

Answer ..... $\frac{8}{3}$ or $2\frac{2}{3}$ ..... [2]

For Examiner's Use

4 Simplify $5a^2-(3a-2b)^2$.

$5a^2-(3a-2b)^2$

$=5a^2-(9a^2-12ab+4b^2)$ M1

$=5a^2-9a^2+12ab-4b^2$

$=-4a^2+12ab-4b^2$ A1

Answer $\ldots\ldots -4a^2+12ab-4b^2 \ldots\ldots$ [2]

---

5 Write as a single fraction in its simplest form $\dfrac{-4}{x^2-4}+\dfrac{1}{x-2}$.

$\dfrac{-4}{x^2-4}+\dfrac{1}{x-2}$

$=\dfrac{-4}{(x-2)(x+2)}+\dfrac{1}{(x-2)}$ M1

$=\dfrac{-4+x+2}{(x-2)(x+2)}$

$=\dfrac{-2+x}{(x-2)(x+2)}$

$=\dfrac{1}{(x+2)}$ A1

Answer $\ldots\ldots \dfrac{1}{(x+2)} \ldots$ [2]

---

6 Solve the inequality $x-5<\dfrac{2x}{5}\le\dfrac{x}{2}+\dfrac{1}{5}$.

$x-5<\dfrac{2x}{5}$

$5x-25<2x$

$3x<25$

$x<\dfrac{25}{3}$ or $8\dfrac{1}{3}$ M1

$\dfrac{2x}{5}\le\dfrac{x}{2}+\dfrac{1}{5}$

$4x\le 5x+2$

$x\ge -2$ M1

$-2\le x<8\dfrac{1}{3}$ A1

Answer $\ldots -2\le x<8\dfrac{1}{3} \ldots\ldots$ [3]

For Examiner's Use

7 Solve the equation $\left(\frac{1}{5}\right)^{-2} \times 125^{x} = \sqrt{25^{x}}$.

$$\left(\frac{1}{5}\right)^{-2} \times 125^{x} = \sqrt{25^{x}}$$

$$5^{2} \times 5^{3x} = \left(5^{2}\right)^{\frac{x}{2}} \quad \text{M1}$$

$$2 + 3x = x$$

$$2x = -2$$

$$x = -1 \quad \text{A1}$$

Answer $x =$ ..... -1 ..... [2]

---

8 **(a)** Madeline deposited \$6 000 into a 3-month fixed deposit which pays a simple interest. If she receives a total amount of \$6 013.50 at the end of the 3 months, what is the rate of interest per annum?

$$13.50 = \frac{6000(R)\left(\frac{3}{12}\right)}{100} \quad \text{M1}$$

$$R = \frac{13.50 \times 100}{6000 \times \left(\frac{3}{12}\right)}$$

$$= 0.9\% \quad \text{A1}$$

Answer .....0.9.....% [2]

(b) Another bank offers an interest rate of 0.6% per annum, compounded monthly. What would be the total amount that Madeline receives, correct to the nearest dollar, at the end of 3 months if she were to put the \$6 000 in this bank?

$$T = 6000\left(1 + \frac{\frac{0.6}{12}}{100}\right)^{12\left(\frac{3}{12}\right)} \quad \text{M1}$$

$$= 6009.00$$

$$= \$6009 \text{ (nearest \$)} \quad \text{A1}$$

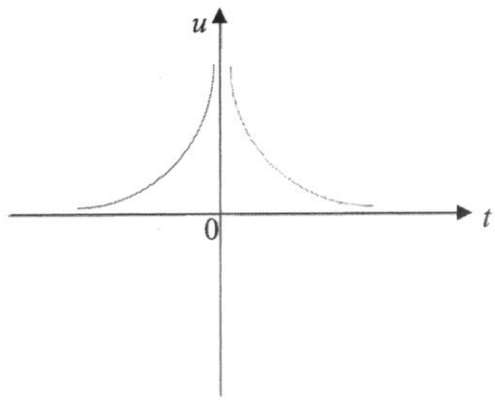
Answer \$...6009..... [2]

For Examiner's Use

9 u is inversely proportional to the square of t.

(a) Sketch the graph of $u = \frac{2}{t^2}$.

Answer: [1]



(a) Find the percentage change in u when t is tripled.

$u = \frac{k}{t^2}$

$u_1 = \frac{k}{9t^2}$ M1 A1

Answer .....–88.9.....% [2]

---

10 These are the first four terms of a sequence.

$$\frac{5}{2} \quad \frac{20}{3} \quad \frac{45}{4} \quad \frac{80}{5}$$

(a) Write down the next two terms. B2

Answer ..... $\frac{125}{6}$ ..... , ... $\frac{180}{7}$ .... [1]

(b) Find an expression. in terms of n, for the nth term of this sequence.

Answer ..... $\frac{5n^2}{n+1}$ ..... B1 [1]

(c) Explain why $71\frac{3}{16}$ is not a term of this sequence.

Answer $71\frac{3}{16} = \frac{1139}{16}$, since **1139 is not a multiple of 5,** $71\frac{3}{16}$ cannot be a term.

.....[1]

For Examiner's Use

11 The exchange rate between pounds (£) and Singapore dollars ($) is £1 = $1.86.

The exchange rate between Singapore dollars ($) and euros (€) is $1 = €0.62.

Megan is comparing day tour packages on a website.

Paris 2-Day Tour €280 per person
London 2-Day Tour £280 per person

Megan claims that the tour package in Paris costs less than the tour package in London.

Is Megan's claims correct? Justify your answer with calculations.

Answer [2]

£1 = $1.86

$€1 = \frac{1}{0.62} = \$1.61$ B1

Since it costs less to buy €1 compared to £1, it would be cheaper to go on the tour in Paris. B1

Cost of tour in Paris

$= S\$\frac{280}{0.62}$

$= 451.61$ B1

Cost of tour in London

$= S\$1.86 \times 280$

$= S\$520.80$ B1

It costs S$69.19 less in Paris than in London.

For Examiner's Use

12 **(a)** **(i)** Express 1008 as a product of prime factors, leaving your answer in index notation.

3	1008
3	336
2	112
2	56
7	28
2	4
2	2
	1

Answer $1008 = \ldots 2^4 \times 3^2 \times 7$ ..... [1]

(ii) p and q are numbers such that q is a prime number.

Find the values of p and q such that $\frac{1008p}{q}$ is the smallest possible cube.

$$\frac{1008p}{q} = \frac{2^4 \times 3^2 \times 7p}{q}$$
$$= 2^6 \times 3^3$$
$$p = 2^2 \times 3 = 12$$
$$q = 7$$

B2

Answer $p =$ .....12...., $q =$.....7..... [2]

(b) The lowest common multiple of x and 1008 is $2^5 \times 3^2 \times 5 \times 7$. Find the smallest possible value of x.

$$x = 2^5 \times 5 = 160$$

Answer $x =$.....160..... [1]

For Examiner's Use

13 **(a)** **(i)** Express $4-2x-x^2$ in the form $a-(x+b)^2$.

$4-2x-x^2$
$=-(x^2+2x+1^2-4-1^2)$
$=-(x+1)^2+5$
$=5-(x+1)^2$

Answer $\ldots\ldots 5-(x+1)^2 \ldots\ldots$ [1]

(ii) Explain, without solving, why there is no solution of x when $4-2x-x^2=6$.

Answer [1]

The maximum value of $4-2x-x^2=5$, **hence there are no solution of x when $4-2x-x^2$=6.**

(b) Using part (a)(i), solve the equation $x^2+2x=8$.

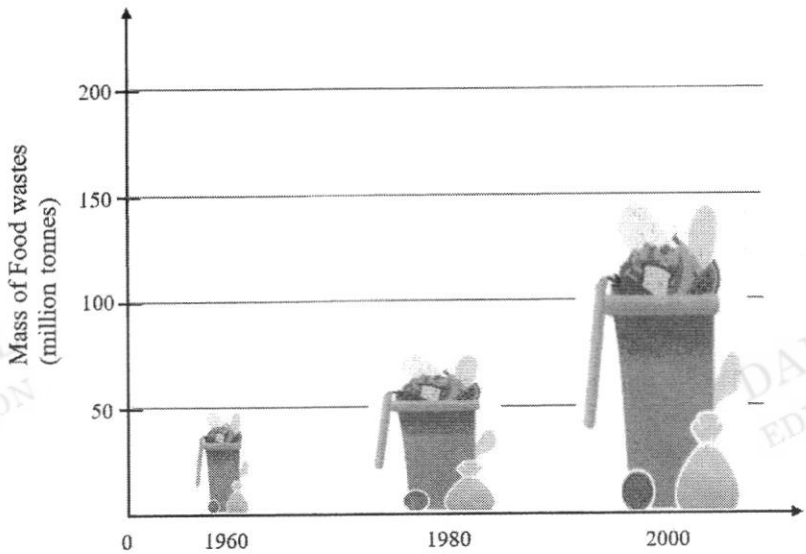
$x^2+2x=8$
$-x^2-2x=-8$
$-x^2-2x+4=-4$
$5-(x+1)^2=-4$ M1
$(x+1)^2=9$
$x+1=\pm 3$
$x=2$ or $x=-4$

A2

Answer $x=$……2…or ……–4…… … [3]

For Examiner's Use

14 The graph shows the total mass of food waste collected around a particular neighbourhood at the end of each given year.



(a) State one misleading feature of the graph. *Answer* [1]	**(b)** Explain how this feature affects the reader's interpretation of the graph. *Answer* [1]
It is not clear if the **height or the area** of the image is used to determine the mass of food waste collected. The widths of the icons are not the same.	The height and the area of the image is not proportional. The readers may think that there is more than 200% increase in mass from 1960-2000.

Not Accepted

1) The title is biased.
2) The images are of different sizes/inconsistent.→ You will need to elaborate further.
3) The horizontal axis does not show all the years.

For Examiner's Use

15 The following diagram represents the stem-and-leaf scores of sixteen students who took a Mathematics test.

2	1
3	
4	
5	4 5 6 6 7 8 9
6	***k*** 9
7	1 3 5 6 9
8	0

Key: 2 | 1 means 21 marks

(a) If the median mark is 62, find the possible value of k.

$$\frac{59+m}{2} = 62$$

$$m = 2(62) - 59$$

$$= 65$$

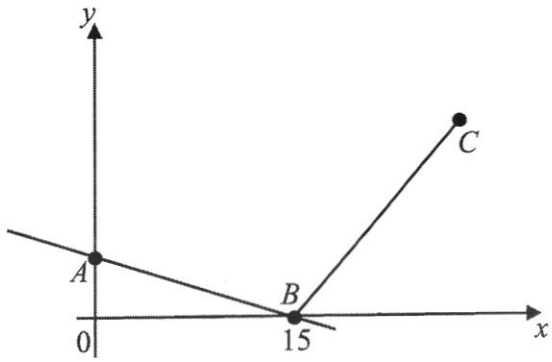
Answer k =.....5..... [1]

(b) Explain why the mean may not be appropriate average to use to summarise the results of the Mathematics test.

Answer [1]

The mean is not appropriate as there is an **outlier** (21 mark) which will **decrease the mean /skew the mean value.**

16 In the diagram, C is the point $(30,15)$, A is a point on the y-axis and B is a point on the x-axis.



(a) Given that the equation of the line AB is $3y+x=15$, find the length of AB.

$A(0,5), B(15,0)$

$AB=\sqrt{(15-0)^2+(0-5)^2}$

$=15.811...$

$=15.8$ units

Answer .....15.8.....units [1]

(b) Find the equation of the line which passes through $C(30,15)$ and is parallel to the line AB.

$m=-\frac{1}{3}$

$y-15=-\frac{1}{3}(x-30)$

$y=-\frac{1}{3}x+10+15$

$y=-\frac{1}{3}x+25$

Answer ..... $y=-\frac{1}{3}x+25$ ..... [1]

(c) If $ABCD$ is a parallelogram, find the coordinates of D.

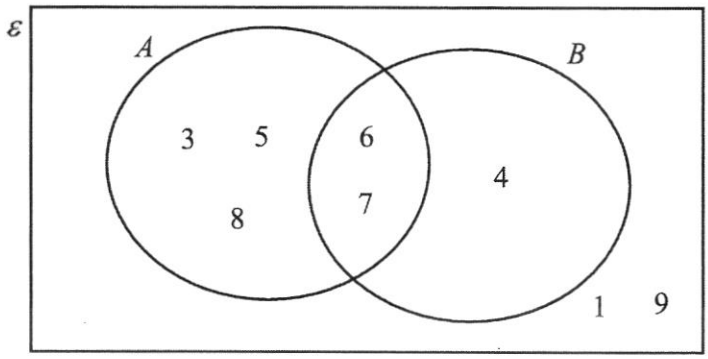
$D(30-15,15+5)$

$=D(15,20)$

Answer D (.....15... , ...20....) [1]

For Examiner's Use

17 **(a)**



Use the Venn diagram to answer the following.

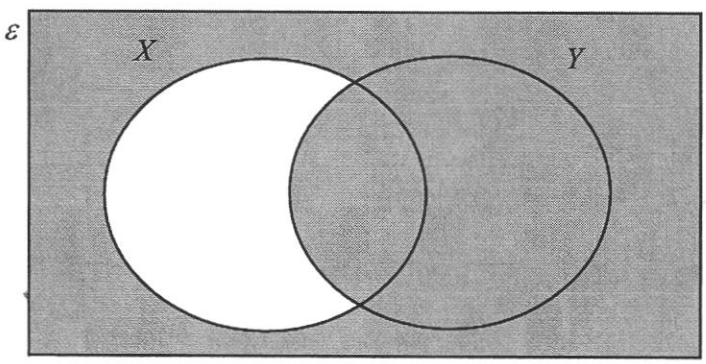
(i) List all the elements in the set $(A \cup B) \cap B'$.

Answer {.....3,5, 8.....} [1]

(ii) Fill in a set notation symbol to complete the statement below.

Answer {3, 5, 8} ..... ⊂ ..... A [1]

(b) Use set notation to describe the shaded region.

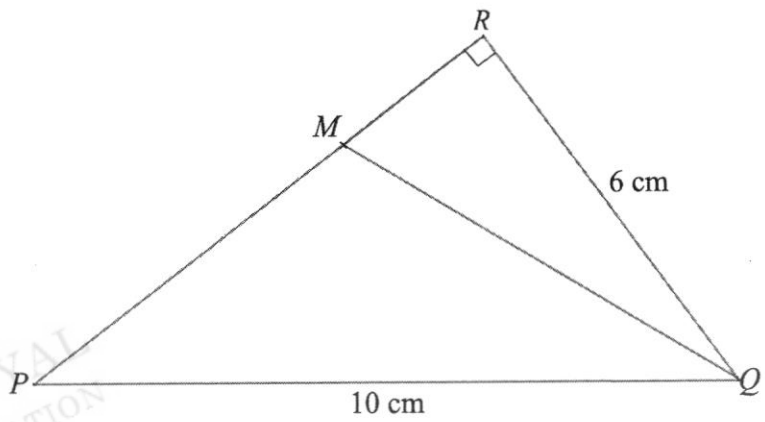


Answer ..... $Y \cup X'$ ..... [1]

For Examiner's Use

18 Triangle PQR is a right-angled triangle with $PQ = 10$ cm, $QR = 6$ cm and $\angle PRQ = 90°$.

M is a point on PR such that $11MR = 5PM$.



Find, giving your answer as a fraction in its simplest form, the value of

(a) $\tan \angle MQR$,

$$MR = \frac{5}{16}PR$$
$$= \frac{5}{16}\sqrt{10^2 - 6^2}$$
$$= 2.5 \text{ cm} \quad \text{M1}$$
$$\tan \angle MQR = \frac{2.5}{6}$$
$$= \frac{5}{12} \quad \text{A1}$$

Answer ..... $\frac{5}{12}$ ..... [2]

(b) $\cos \angle PMQ$,

$$\cos \angle PMQ = -\cos \angle RMQ$$
$$= -\frac{2.5}{\sqrt{2.5^2 + 6^2}}$$
$$= -\frac{5}{13}$$

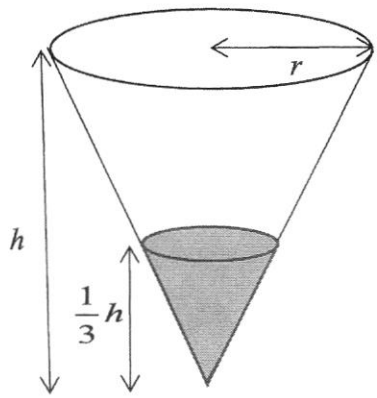
Answer ..... $-\frac{5}{13}$ ..... [1]

(c) $\dfrac{\text{area of } \Delta PQM}{\text{area of } \Delta PQR}$.

Answer ..... $\frac{11}{16}$ ..... [1]

For Examiner's Use

19 The diagram shows an inverted hollow paper cone with a base of radius r cm and a height of h cm. The cone is filled with water, represented by the shaded region, to a depth of $\frac{1}{3}h$ cm.



The internal surface area of the cone is 106 cm^2.

Find the surface area of the inside of the cone that is in contact with the water.

$$\frac{106}{A_2} = \left(\frac{3}{1}\right)^2 \quad \text{M1}$$

$$A_2 = \frac{106}{9}$$

$$= 11.7777...$$

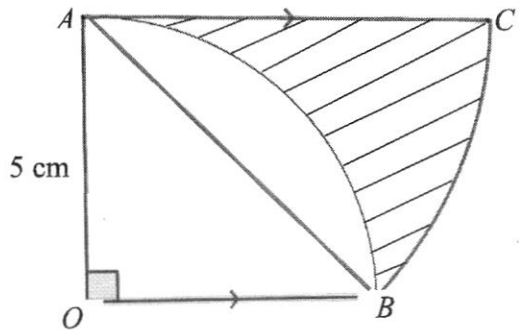
$$= 11.8 \text{ cm}^2 \text{ (3sf)} \quad \text{A1}$$

Answer .....11.8.....cm^2 [2]

For Examiner's Use

20 AB is an arc of a circle with the centre O, $\angle AOB = \frac{\pi}{2}$ radians and $OA = 5$ cm.

AC is parallel to OB and BC is an arc of a circle with centre A.



(a) Find the length of arc BC.

$AB = \sqrt{5^2 + 5^2} = 50$ cm

M1 for either AB or $\angle CAB$

$\angle CAB = \frac{\pi}{4}$ radians

$BC = \sqrt{50}\left(\frac{\pi}{4}\right)$ M1

$= 5.5536...$

$= 5.55$ cm (3sf) A1

Answer .....5.55..... cm [3]

(b) Find the percentage of the figure $OACB$ that is **not** shaded.

Area of $OACB = \frac{1}{2}(5)(5) + \frac{1}{2}\left(\sqrt{50}\right)^2\left(\frac{\pi}{4}\right) = 32.1349...$ cm^2 M1

Percentage of figure not shaded $= \frac{\frac{1}{4}\pi(5)^2}{32.1349...} \times 100\%$ M1

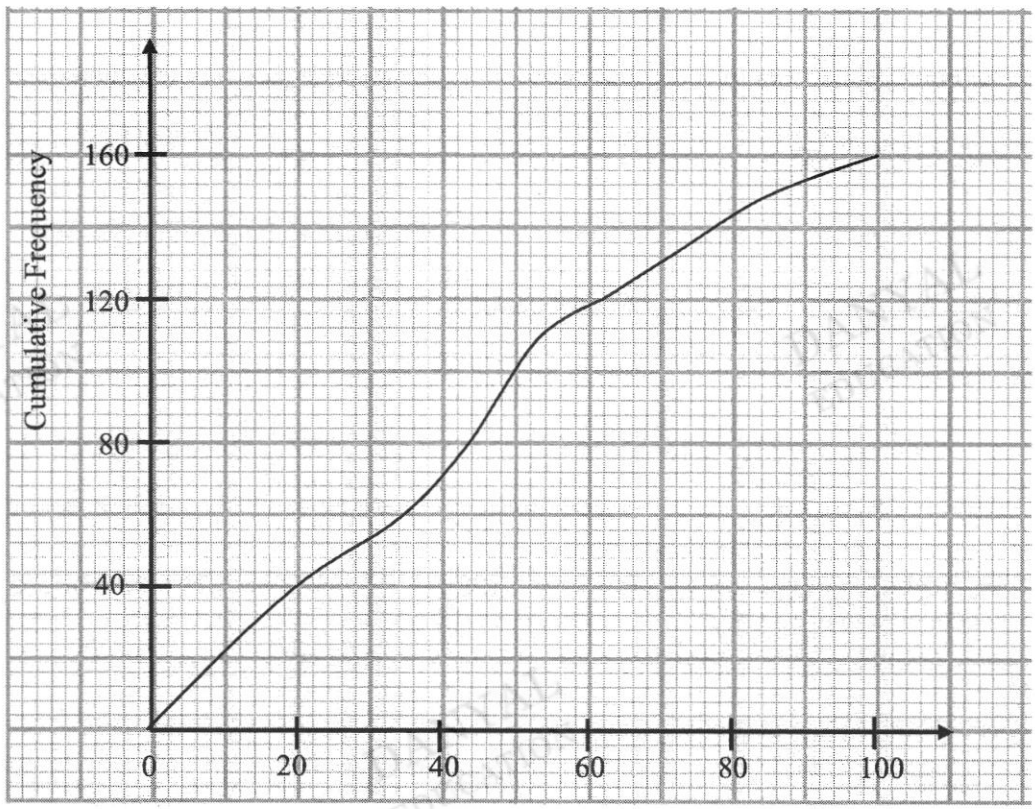
$= 61.1016...\%$

$= 61.1\%$ (3sf) A1

Answer .....61.1.....% [3]

For Examiner's Use

21 The cumulative frequency curve shows the weekly expenditure on home deliveries of 160 households from Block *W*.



(a) Use the cumulative frequency curve to find

(i) the median of the distribution,

Answer $.....44..... [1]

(ii) the interquartile range of the distribution,

Interquartile range = $ (62 – 20) = $42

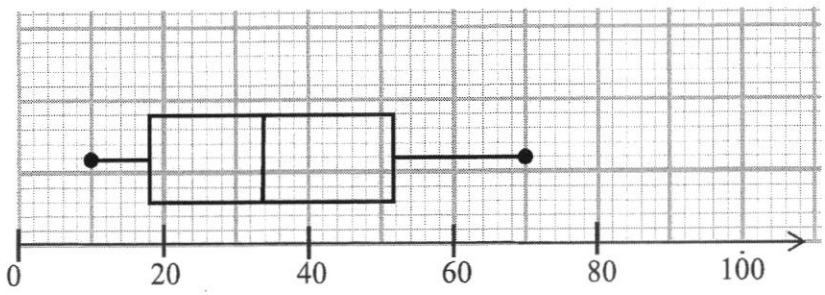
Answer $.....42..... [1]

(iii) the value of p, given that p% of the residents spent more than $84 weekly on home deliveries.

$$p = \frac{160-148}{160} \times 100\%$$
$$= 7.5\%$$

Answer p =.....7.5..... [1]

(b) The weekly expenditure on home deliveries of a group of 160 residents from Block K are displayed in the box and whisker plot below.



Here are two statements comparing the weekly expenditure on home deliveries of the residents from Block W and Block K.

For each statement, state whether you agree or disagree.

Give a reason for each answer, stating clearly which statistic you used to make your decision.

(i) On average, the residents in Block W has lower weekly expenses on home deliveries than the residents in Block K.

Answer

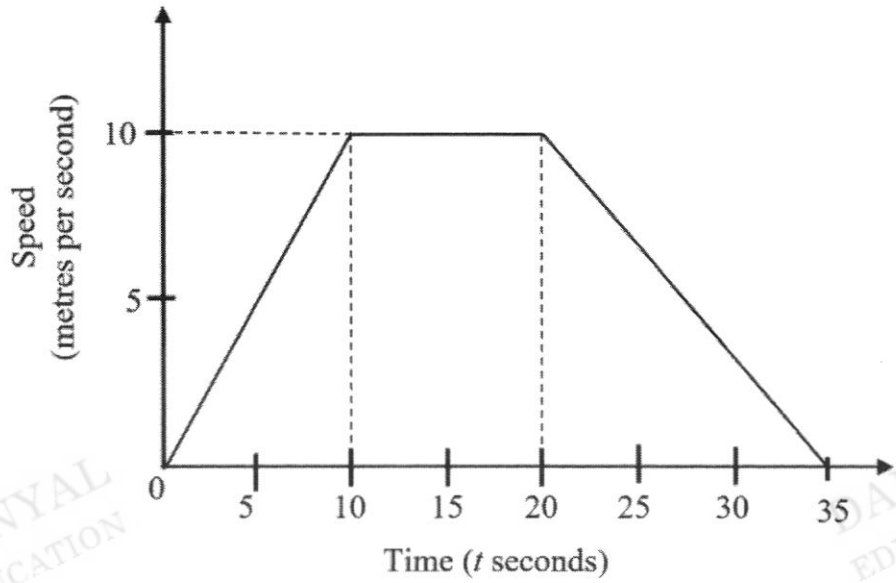
I **DISAGREE** because residents in **Block W has a higher median expenditure** of \$44 than the residents in Block K (\$34) [1]

(ii) The weekly expenditure on home deliveries of the residents in Block K is closer to the median than the weekly expenditure on home deliveries of the residents from Block W.

Answer I **AGREE** because **the interquartile range of Block K (\$36) is lower than the interquartile range Block W (\$42)** showing that the data is less widely spread than Block W's [1]

For Examiner's Use

22



The speed-time graph of a particle is shown in the diagram above.

(a) Find

(i) the speed when $t = 25$,

$(35,0),(20,10),(25,v)$

$\frac{v-0}{25-35}=\frac{10-0}{20-35}$ M1

$v=\frac{-10(10)}{-15}$

$=6.67$ (3sf) A1

Answer ..... 6.67 .....m/s [2]

(ii) the distance covered in the first 35 seconds.

$\text{Dist}=\frac{1}{2}(10)(10)+10(10)+\frac{1}{2}(10)(15)$ M1

$=50+100+75$

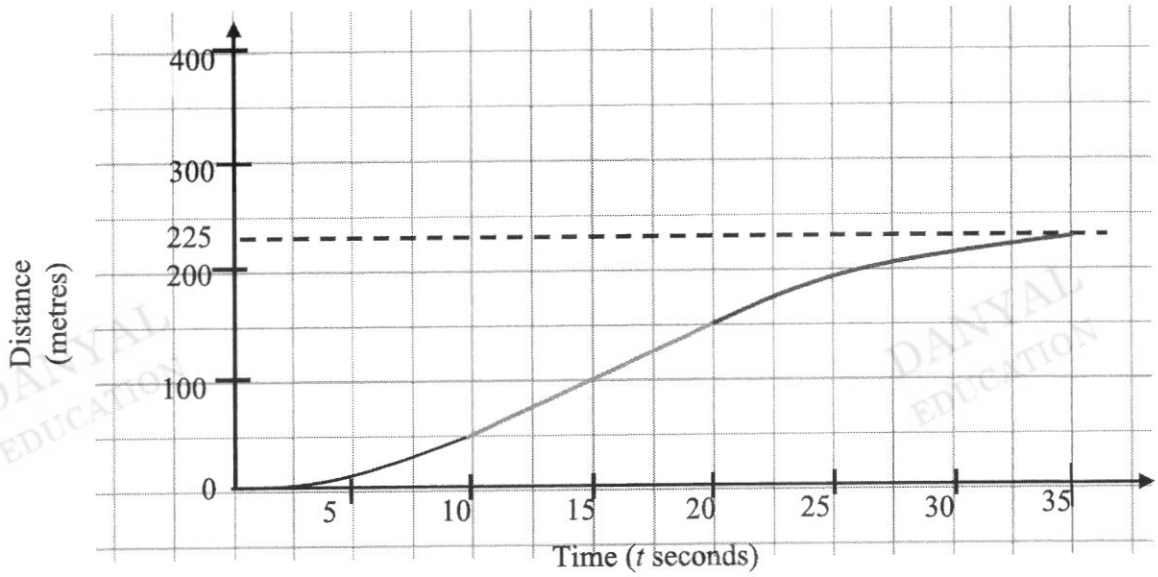
$=225$ m A1

Answer .....225.....m [2]

For Examiner's Use

(b) Part of the distance-time graph for the same particle is shown in the answer space.

Complete this graph. [1]

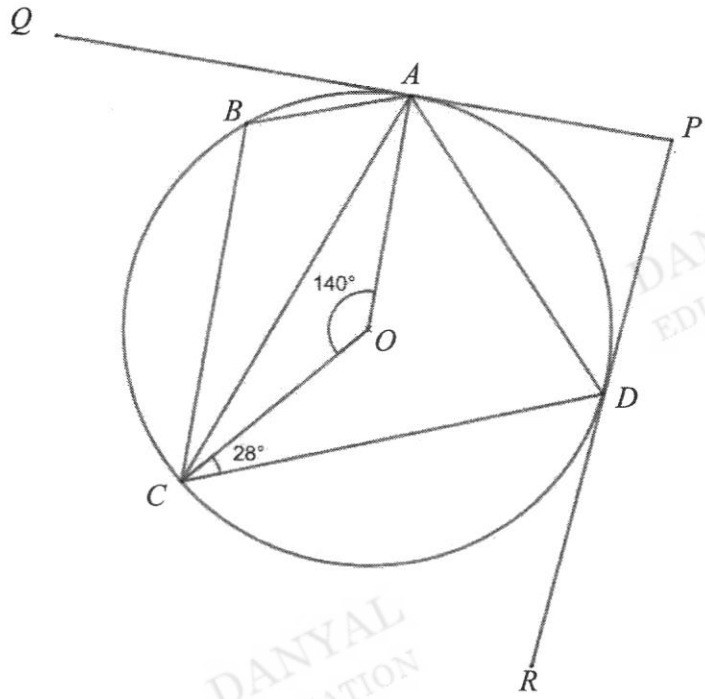


For Examiner's Use

23 In the diagram, A, B C and D lie on a circle, centre O.

PQ and PR are tangents to the circle at A and D respectively.

It is given that $\angle AOC = 140°, \angle DCO = 28°$ and OA is parallel to CB.



(a) Show, stating your reasons clearly, that CA bisects $\angle BCO$.

Answer [2]

$\angle OCA = \angle OAC$ (base $\angle$ of isos.Δ)

$$= \frac{180° - 140°}{2}$$

$= 20°$ M1

$\angle BCA = \angle OAC$ (alt. $\angle$s) or $\angle BCA = 180° - 140° - 20°$ (int. $\angle$)

$= 20°$ M1

Since $\angle OCA = \angle BCA = 20°$,

CA bisects $\angle BCO$.

(b) Find, stating your reasons clearly,

(i) $\angle BAC$,

Reflex $\angle AOC = 360° - 140°$ ($\angle$s at a point)

$= 220°$

$\angle ABC = \frac{220°}{2}$ ($\angle$ at centre $= 2\angle$ at circumference)

$= 110°$ M1

$\angle BAC = 180° - 20° - 110°$ ($\angle$s sum of Δ)

$= 50°$ A1

Answer $\angle BAC$ =.....50.....° [2]

(ii) $\angle PDA$,

$\angle PDA = 20° + 28°$ ($\angle$ in alt segment)

$= 48°$

OR

$\angle OAP = 90°$ (radius $\perp$ tangent)

$\angle OAD = 180° - 20° - 20° - 28° - 50° - 20°$($\angle$ in opp segment) M1

$= 42°$

$\angle PAD = 90° - 42°$

$= 48°$

$= \angle PDA$ (base $\angle$ of isos. Δ) A1

Answer $\angle PDA$ =.....48.....° [2]

(c) Lizzy claims that $QADR\ldots$ forms a part of a regular n-sided polygon.

Justify, with reasons, if you agree or disagree with Lizzy.

Answer [1]

ext. $\angle$ of polygon $\angle PAD = 48°$

no. of sides $= \frac{360}{48} = 7.5$

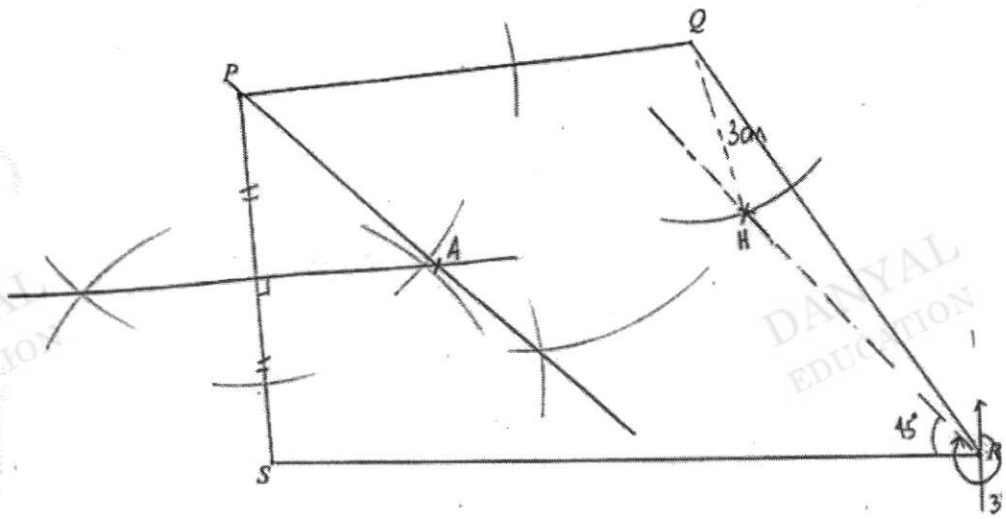
Since the **number of sides is not an integer**, $QADR$ is **not** a regular polygon

Hence I **disagree** with Lizzy.

For Examiner's Use

24 On the scale drawing, P, Q, R and S are the positions of four garden statues in a park. $PQRS$ is a quadrilateral.

Scale: 1 : 25 000



(a) A water-cooler is to be built at A where it is equidistant from the line segments PS and PQ and the points P and S.

On the scale drawing, mark the point A. [2]

(b) The garden statue at R is due east of S.

The gardener plans to build a pavilion at H.

The bearing of H from R is 315° and 750 m from Q.

(i) Calculate the distance HQ on the scale drawing.

Scale:
1 cm : 25 000 cm
1 cm : 250 m
3 cm : 750 m

Answer HQ =.....3.....cm [1]

(ii) Mark the point H on the diagram above. [1]

~ End of Paper ~

1	(a)	$-\dfrac{5m+2}{2+3m}$		
	(b)	(i)	0	
		(ii)	$p=\pm 2\sqrt{(3-r)^2+q}$	
	(c)	(iii)	47	
2	(a)	$\left(\dfrac{1}{3}\right)^n$		
	(b)	$\dfrac{5}{18}$		
	(c)	$\dfrac{1}{2}$		
3	(a)	(i)	9.10 (3sf)	
		(ii)	$\dfrac{247}{590}$	
	(b)	(i)	$\begin{pmatrix}1\\1\\1\end{pmatrix}$	
		(ii)	(a)	$\begin{pmatrix}4 & 7 & 5\end{pmatrix}$
			(b)	$\begin{pmatrix}797 & 990 & 1344\end{pmatrix}$
			(c)	It represents the total number of tubes of toothpaste of each size delivered to all the shops over 6 months.
		(iii)	$\begin{pmatrix}1157\\559.5\\1134\end{pmatrix}$	
4	(a)	$\dfrac{750}{x}$		
	(b)	$\dfrac{800}{x+30}$		
	(d)	94.84, −158.17		
	(e)	\$22.97		
5	(a)	(i)	061.9°	
		(ii)	16.5 m	
		(iii)	222 m^2	
	(b)	(i)	11.4 m	
		(ii)	shortest distance $= 35\sin 28.0502^\circ$ $= 16.4585$ m angle of depression $= \tan^{-1}\dfrac{11.3721}{16.4585}$ $= 34.6^\circ$ (1dp)	

6	(a)	(i)	7.81	
		(ii)	$(-3, 2)$	
	(b)	(i)	(a)	$-\mathbf{y}+\mathbf{x}$
			(b)	$\mathbf{y}-\frac{1}{2}\mathbf{x}$
			(c)	$\frac{3}{4}\mathbf{x}-\frac{1}{2}\mathbf{y}$
		(ii)	(a)	$\mathbf{x}-\mathbf{y}$
			(b)	Since $\overrightarrow{MG}=2\overrightarrow{HG}$ and G is a common point, H lies on BG.
7	(a)	12 cm		
	(b)	8		
	(c)	611 g		
8	(a)	-4.1		
	(c)	2.6		
	(d)	$2 \le k \le 6$ or $1 < k < 7$		
	(e)	(i)	$a=\frac{1}{2}$ and $b=2$	
		(ii)	$-1.75, \ -0.75, \ 2.5$	
9	(b)	ΔCXW and ΔDXY		
	(d)	(i)	$\frac{2}{5}$	
		(ii)	90 cm^2	
10	(a)	\$44815.61		
	(b)	\$137.28		
	(c)	I agree with Mr Tan's claim.		
	(d)	22.4% < 25%, John's statement is valid.		

Class	Index Number

Name : ___*Solutions*_____

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2021
Secondary 4

Wednesdaday 4 August 2021	**MATHEMATICS** **Paper 2**	**4048/02** 2 h 30 min

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For , use either your calculator value or 3.142, unless the question requires the answer in terms of

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

100

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Simplify $\dfrac{25m^2-4}{4-4m-15m^2}$.

$$\frac{25m^2-4}{4-4m-15m^2}$$
$$=\frac{(5m-2)(5m+2)}{(2-5m)(2+3m)}$$
$$=\frac{(5m-2)(5m+2)}{-(5m-2)(2+3m)}$$
$$=-\frac{5m+2}{2+3m}$$

Answer ..... [3]

(b) Given that $r=3-\sqrt{\dfrac{p^2}{4}-q}$,

(i) evaluate r when $p=-8$ and $q=7$,

$$r=3-\sqrt{\frac{(-8)^2}{4}-7}$$
$$=0$$

Answer ..... [1]

(ii) express p in terms of q and r.

$$r=3-\sqrt{\frac{p^2}{4}-q}$$
$$\frac{p^2}{4}-q=(3-r)^2$$
$$p=\pm 2\sqrt{(3-r)^2+q}$$

Answer ..... [3]

(c) The value of a 2-digit number can be written as $10x + y$ where x is the tens digit of the number and y is its units digit.

(i) When the digits x and y are reversed, the value is increased by 27.
Show that $y - x = 3$. [1]

$10y + x - (10x + y) = 27$
$9y - 9x = 27$
$y - x = 3$

(ii) The sum of the original number and the number with reverse digits is 121.
Show that $x + y = 11$. [1]

$10y + x + 10x + y = 121$
$11y + 11x = 121$
$y + x = 11$

(iii) Hence, find the original number.

$y - x + y + x = 14$
$y = 7$
$x = 4$
The number is 47.

Answer ..... [3]

2 Bag A has 4 green counters, 6 red counters and 2 yellow counters.

(a) Alice takes a counter at random from Bag A, notes its colour and replaces it in the bag. She repeated this process n times. Write down an expression, in terms of n, for the probability that she takes only green counter.

$$\left(\frac{1}{3}\right)^n$$

Answer ..... [1]

(b) Bill takes a counter at random from Bag A, notes its colour and replaces it back. He then takes a second counter at random from Bag A.

Calculate the probability that Bill takes only one yellow counter.

$$\frac{1}{6}\times\frac{5}{6}\times 2$$
$$=\frac{5}{18}$$

Answer ..... [2]

Bag B has 3 green counters and 3 red counters.

(c) Charles takes a counter at random from Bag A and places it into Bag B.
He then takes a counter at random from Bag B and places it into Bag A.
Find the probability that after these two moves, Bag A has 4 green counters, 6 red counters and 2 yellow counters.

$$\frac{1}{3}\times\frac{4}{7}+\frac{1}{2}\times\frac{4}{7}+\frac{1}{6}\times\frac{1}{7}$$
$$=\frac{1}{2}$$

Answer ..... [3]

3 **(a)** The table shows the height, h cm, of 60 plants.

Height (h cm)	$10 < h \leq 20$	$20 < h \leq 30$	$30 < h \leq 40$	$40 < h \leq 50$
Frequency	7	14	27	12

(i) Calculate an estimate of the standard deviation of the height of the plants.

9.10 (3sf)

Answer ..... cm [1]

(ii) Two plants are chosen at random. Find the probability that both plants have height greater than 30 cm.

$$\frac{39}{60} \times \frac{38}{59}$$
$$= \frac{247}{590}$$

Answer ..... [2]

(b) A toothpaste supplier delivers tubes of toothpaste to 3 different shops A, B and C. The number of tubes of toothpaste supplied per delivery to each shop, the sizes and sale prices of the tubes, together with the number of deliveries made to each shop over a 6-month period are shown in the table below.

		Number of tubes of toothpaste per delivery			Number of deliveries over 6 months
Size of tube		small	medium	large	
Name of shop	A	68	85	105	4
	B	75	–	62	7
	C	–	130	98	5
Sale price per tube		\$2.50	\$4.20	\$6.00	

It is given that $\mathbf{T} = \begin{pmatrix} 68 & 85 & 105 \\ 75 & 0 & 62 \\ 0 & 130 & 98 \end{pmatrix}$.

(i) Write down the matrix **P** such that **TP** gives the total number of tubes of toothpaste delivered to each shop per delivery.

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Answer **P** = ... [1]

(ii) **(a)** Write down a 1×3 matrix **D** to represent the number of deliveries over 6 months to each shop.

$\begin{pmatrix} 4 & 7 & 5 \end{pmatrix}$

Answer **D** = ... [1]

(ii) **(b)** Evaluate **DT**.

$\begin{pmatrix} 797 & 990 & 1344 \end{pmatrix}$

Answer **DT** = ... [1]

(c) State what the elements of **DT** represent.

Answer

It represents the total number of tubes of toothpaste of each size delivered to all the shops over 6 months.

[1]

(iii) **Q** is a matrix that represents the total amount of money that would be obtained from each shop from the sale of all the tubes of toothpaste per delivery.
By matrix multiplication, find **Q**.

$$\begin{pmatrix} 68 & 85 & 105 \\ 75 & 0 & 62 \\ 0 & 130 & 98 \end{pmatrix}\begin{pmatrix} 2.5 \\ 4.2 \\ 6 \end{pmatrix}$$
$$= \begin{pmatrix} 1157 \\ 559.5 \\ 1134 \end{pmatrix}$$

Answer **Q** = ... [2]

4 **(a)** Alan drove a distance of x km and his car used 7.5 litres of fuel.
Write down an expression, in terms of x, for the rate of fuel used by his car in litres per 100 km.

$$\frac{750}{x}$$

Answer ..... litres per 100 km [1]

(b) Ben drove a distance of $(x + 30)$ km and his car used 8 litres of fuel.
Write down an expression, in terms of x, for the rate of fuel used by his car in litres per 100 km.

$$\frac{800}{x+30}$$

Answer ..... litres per 100 km [1]

(c) The rate of fuel used by Alan's car is 1.5 litres per 100 kilometre more than the rate of fuel used by Ben's car.
Form an equation to represent this information and show that it can be simplified to $3x^2 + 190x - 45000 = 0$.

Answer [3]

$$\frac{750}{x} - \frac{800}{x+30} = \frac{3}{2}$$
$$2[750(x+30) - 800x] - = 3x(x+30)$$
$$45000 - 100x = 3x^2 + 90x$$
$$3x^2 + 190x - 45000 = 0$$

(d) Solve the equation $3x^2 + 190x - 45000 = 0$, giving both answers correct to two decimal places.

$$\frac{-190 \pm \sqrt{190^2 - 4(3)(-45000)}}{2(3)}$$
$$= 94.8354 \quad \text{or} \quad -158.1687$$
$$= 94.84 \text{ (2dp)} \quad = -158.17 \text{ (2dp)}$$

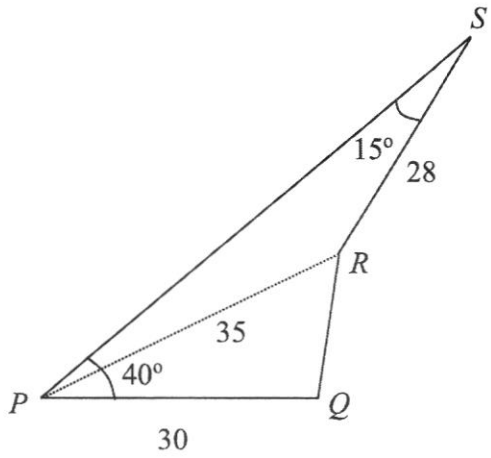
Answer $x =$ ..... or ..... [3]

(e) Each litre of fuel costs \$2.42.
Calculate the amount Alan paid for the petrol if he drove 120 km.

$$\frac{7.5}{94.8354} \times 120 \times 2.42$$
$$= \$22.97$$

Answer \$ ..... [2]

5 The diagram shows four points P, Q, R and S on level ground.
Q is due east of P.
Angle $SPQ = 40°$ and angle $PSR = 15°$.
$PQ = 30$ m, $PR = 35$ m and $RS = 28$ m.



(a) Calculate

(i) the bearing of R from P,

$$\frac{\sin \angle SPR}{28} = \frac{\sin 15^\circ}{35}$$
$$\angle SPR = 11.9498^\circ$$
$$\angle RPQ = 40^\circ - 11.9498^\circ = 28.0502^\circ$$
$$\text{Bearing} = 90^\circ - 28.0502^\circ = 061.9^\circ \text{ (1dp)}$$

Answer ..... ° [3]

(ii) QR,

$$QR^2 = 30^2 + 35^2 - 2(30)(35)\cos 28.0502^\circ$$
$$QR = \sqrt{30^2 + 35^2 - 2(30)(35)\cos 28.0502^\circ}$$
$$QR = 16.5 \text{ m (3sf)}$$

Answer ..... m [2]

(iii) the area of triangle PRS.

$\frac{1}{2} \times 35 \times 28 \times \sin 153.0502^\circ$

$= 222 \text{ m}^2$ (3sf)

Answer ..... m^2 [2]

A vertical mast stands at R and the angle of elevation of the top of the mast from P is 18°.

(b) (i) Calculate the height of the mast.

$35 \tan 18^\circ$

$= 11.4$ m (3sf)

Answer ..... m [2]

(ii) A bird lands on the top of the mast.
Find its largest angle of depression of a point along PQ produced.

shortest distance $= 35 \sin 28.0502^\circ$

$= 16.4585$ m

angle of depression $= \tan^{-1} \frac{11.3721}{16.4585}$

$= 34.6^\circ$ (1dp)

Answer ..... $^\circ$ [3]

6 **(a)** It is given that $\overrightarrow{PQ} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$ and the position vector of P is $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$. Find

(i) $\left|\overrightarrow{PQ}\right|$,

7.81

Answer ..... units [1]

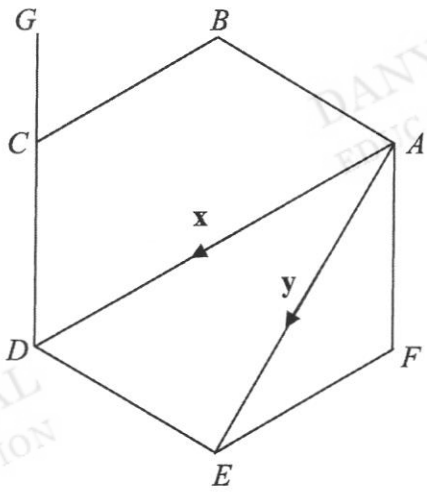
(ii) the coordinates of Q.

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$$
$$= \begin{pmatrix} 2 \\ -4 \end{pmatrix} + \begin{pmatrix} -5 \\ 6 \end{pmatrix}$$
$$= \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$Q(-3, 2)$

Answer (..... , ...) [2]

(b)



$ABCDEF$ is a regular hexagon.
G is a point on DC extended such that $2DG = 3DC$.
$\overrightarrow{AD} = \mathbf{x}$ and $\overrightarrow{AE} = \mathbf{y}$.

(i) Express, as simply as possible, in terms of **x** and/or **y**,

(a) $\overrightarrow{AB}$,

$-\mathbf{y} + \mathbf{x}$

Answer ..... [1]

(b) $\overrightarrow{AF}$,

$$\mathbf{y} - \frac{1}{2}\mathbf{x}$$

Answer ..... [1]

(c) $\overrightarrow{BG}$,

$$\frac{1}{2}\mathbf{x} + \frac{1}{2}\left(\frac{1}{2}\mathbf{x} - \mathbf{y}\right)$$
$$= \frac{3}{4}\mathbf{x} - \frac{1}{2}\mathbf{y}$$

Answer ..... [2]

(ii) M is the mid-point of AF.
H is a point on BE where $4HE = 3BE$.

(a) Find $\overrightarrow{HG}$.

$$\overrightarrow{HG}$$
$$= \overrightarrow{HB} + \overrightarrow{BG}$$
$$= \frac{1}{2}\left(-\mathbf{y} + \frac{1}{2}\mathbf{x}\right) + \frac{3}{4}\mathbf{x} - \frac{1}{2}\mathbf{y}$$
$$= \mathbf{x} - \mathbf{y}$$

Answer ..... [2]

(b) Using vectors, determine whether H lies on MG. [2]

$$\overrightarrow{MG}$$
$$= \overrightarrow{MA} + \overrightarrow{AB} + \overrightarrow{BG}$$
$$= \frac{1}{2}\left(-\mathbf{y} + \frac{1}{2}\mathbf{x}\right) + \mathbf{x} - \mathbf{y} + \frac{3}{4}\mathbf{x} - \frac{1}{2}\mathbf{y}$$
$$= 2\mathbf{x} - 2\mathbf{y}$$
$$= 2(\mathbf{x} - \mathbf{y})$$

Since $\overrightarrow{MG} = 2\overrightarrow{HG}$ and G is a common point, H lies on BG.

7 Figure **I** shows a piece of wood is in the form of a right circular cone with a base diameter of 18 cm.

(a) The curved surface area of the cone is 135π cm^2.
Find the height of the cone.

$\pi(9)l = 135\pi$

$l = 15$

height $= \sqrt{15^2 - 9^2}$

$= 12$ cm

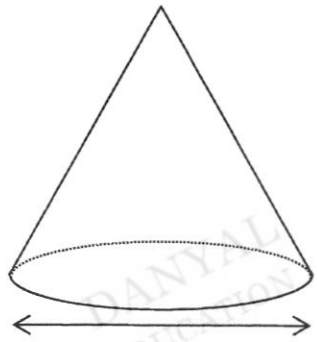


Figure **I**

Answer ..... cm [3]

The cone in Figure **I** is cut into two portions by a plane parallel to its base.
The upper portion is a cone of base diameter 6 cm and the lower portion, as shown in Figure **II**, is a frustum of height x cm.

(b) Find the value of x.

$\frac{12-x}{12} = \frac{6}{18}$

$12 - x = 4$

$x = 8$

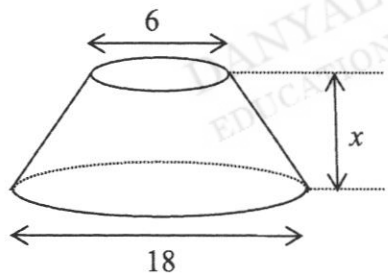


Figure **II**

Answer $x =$ ..... [2]

(c) A right cylindrical hole of diameter 6 cm is drilled through the frustum as shown in Figure **III**.
The wood weighs 810 kg per cubic metres.
Find the mass, in grams, of the solid which remains in the frustum.

volume remaining

$$= \frac{1}{3}\pi(9)^2 12 - \frac{1}{3}\pi(3)^2 4 - \pi(3)^2 8$$

$$= 240\pi$$

$$\text{mass} = \frac{810000}{10^6} \times 240\pi$$

$$= 611 \text{ g (3sf)}$$

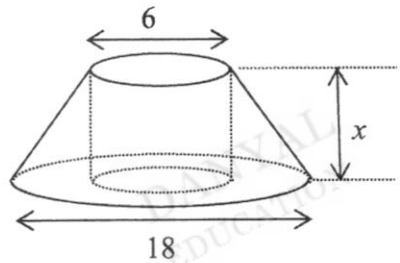


Figure **III**

Answer ..... g [3]

8 The table shows some values for $y = \frac{3}{10}x^3 - x + 1$ for $-3 \le x \le 3$.

x	–3	–2	–1	0	1	2	3
y	p	0.6	1.7	1	0.3	1.4	6.1

(a) Find the value of p.

–4.1

Answer $p =$ ..... [1]

(b) Using a scale of 2 cm to 1 unit on both axes, draw a horizontal axis for $-3 \le x \le 3$ and a vertical axis for $-4.5 \le y \le 6.5$ on the grid opposite.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve at $x = 2$.

Draw tangent and calculate gradient

2.6 ± 0.2

Answer ..... [2]

(d) The equation $\frac{3}{10}x^3 - x = k - 1$, where k is a positive integer, has exactly one solution for $-3 \le x \le 3$. Write down an inequality for k.

$2 \le k \le 6$ or $1 < k < 7$

Answer ..... [1]

(e) **(i)** The line $y = ax + b$ can be drawn on the same grid to solve the equation $\frac{3}{10}x^3 - 1 = \frac{3}{2}x$. Find the value of a and the value of b.

$$\frac{3}{10}x^3 = \frac{3}{2}x + 1$$

$$\frac{3}{10}x^3 - x + 1 = \frac{3}{2}x + 1 - x + 1$$

$$y = \frac{1}{2}x + 2$$

$$a = \frac{1}{2} \quad \text{and} \quad b = 2$$

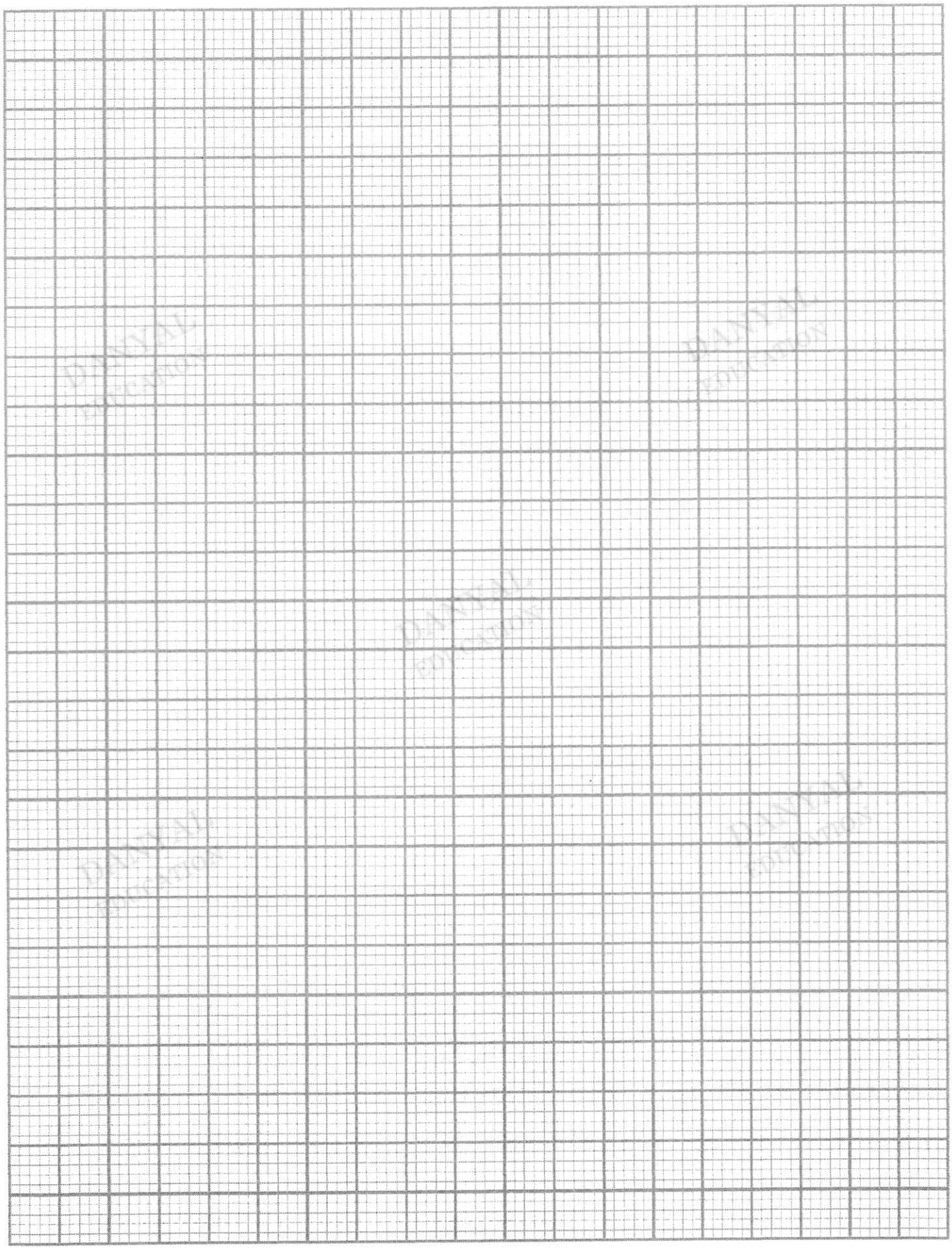
Answer $a =$ ..... and $b =$ ..... [2]

(ii) Draw the line $y = ax + b$, for $-3 \le x \le 3$, to solve the equation $\frac{3}{10}x^3 - 1 = \frac{3}{2}x$.

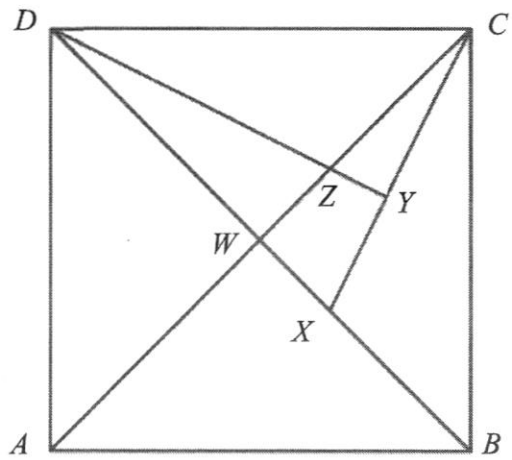
Draw line for $-3 \le x \le 3$

-1.75, -0.75, 2.5 (± 0.05)

Answer $x =$ ....., ... or ..... [3]



9 The diagram shows a square $ABCD$.
The diagonals of the square intersect at W and X is a point on the diagonal BD.
Y lies on CX such that DY is perpendicular to CX and DY meets AC at Z.



(a) Show that triangles DWZ and CYZ are similar.

$\angle DWZ = 90°$ (diagonals of square are perpendicular)
$\angle CYZ = 90°$ (given) $= \angle DWZ$
$\angle DZW = \angle CZY$ (vertically opposite angles)
$\therefore$ Triangles DWZ and CYZ are similar.

[2]

(b) Write down another pair of triangles that are similar but not congruent.

ΔCXW and ΔDXY
OR
ΔDAB and ΔDWA

Answer Δ..... and Δ..... [1]

(c) Prove that triangle BCX and triangle CDZ have the same area.

$BC = CD$ (sides of square)
$\angle CBX = \angle DCZ$ (diagonal of square bisects interior angle)
$\angle YCZ = \angle WDZ$ (ΔDWZ and ΔCYZ are similar)
$\angle BCW = \angle CDW$ (diagonal of square bisects interior angle)
$\angle BCX = \angle BCW - \angle YCZ$
$= \angle CDW - \angle WDZ$
$= \angle CDZ$
$\therefore \Delta BCX$ is congruent to ΔCDZ (AAS) and they have the same area.

[4]

(d) It is given that $YZ : WZ = \sqrt{2} : \sqrt{5}$ and $3CZ = 2CW$.

(i) Find $\dfrac{\text{area of } \Delta CYZ}{\text{area of } \Delta DWZ}$.

$\dfrac{2}{5}$

Answer ..... [1]

(ii) Calculate the area of the square $ABCD$ if the area of triangle CYZ is 3 cm^2.

area of $\Delta DWZ = \dfrac{3}{2} \times 5 = 7.5$
area of square $ABCD = 7.5 \times 12$
$= 90$ cm^2

Answer ..... cm^2 [2]

10 John owns a cleaning company that cleans office blocks.
The number of employees in his company and their salaries in 2020 are as follows.

11 handymen, each receiving a yearly salary of $45 936.
272 cleaners, each receiving $5.20 per hour and working 8 hours per day for a total of 20 days per month.
12 supervisors, men and women in the ratio 1 : 3, each receiving a monthly salary that is 235% more than a cleaner's monthly salary.
11 drivers, each receiving $594.50 per week, for 4 weeks in a month.

In addition, both employer and employee must contribute a portion of the employee's monthly salary to the Central Provident Fund (CPF).
The table shows the CPF contribution and allocation rates for Singapore citizens.

	Contribution rates (for monthly salary ≥ $750)	
Employee's age (years)	By employer (% of salary)	By employee (% of salary)
55 and below	17	20
Above 55 to 60	13	13
Above 60 to 65	9	7.5
Above 65	7.5	5

(a) The yearly salary of each handyman in 2020 is 2.5 % more than in 2017.
Find the yearly salary of each handyman in 2017.

$$\frac{45936}{102.5}\times 100$$
$$= \$44815.61$$

Answer $ ..... [2]

(b) Mdm Lee is a 65 year old Singapore citizen who works as a cleaner in John's cleaning company. Calculate the total amount of contribution to Mdm Lee's CPF account every month in 2020.

$$\frac{16.5}{100}\times 5.2\times 8\times 20$$
$$= \$137.28$$

Answer $ ..... [1]

(c) Every month, John randomly selects an 'Employee of the Month' from all his employees. Mr Tan comments that the chance of randomly selecting a male supervisor as the employee of the month is most unlikely.
Do you agree with Mr Tan's comment? Justify your answer. [2]

$$\text{P(male supervisor)}=\frac{3}{11+272+12+11}=\frac{1}{102}$$

$\frac{1}{102}=0.009803$ (3sf) is very close to 0.
Thus, I agree with Mr Tan's claim.

(d) John states that the difference between the salary of the lowest paid employee and the mean salary of all the employees as a percentage of the mean salary of all the employees is less than 25%.
Verify, showing all calculations, whether his statement is valid. [5]

monthly salary of each handyman $=\frac{45936}{12}=\$3828$

monthly salary of each cleaner $=5.2\times8\times20=\$832$

monthly salary of each supervisor $=\frac{335}{100}\times832=\$2787.20$

monthly salary of each driver $=594.5\times4=\$2378$

mean monthly salary of all employees

$$=\frac{11(3828)+272(832)+12(2787.2)+11(2378)}{11+272+12+11}$$

$=1071.94902$

Percentage difference $=\frac{1071.94902-832}{1071.94902}\times100=22.4\%$ (3sf)

$22.4\% < 25\%$, John's statement is valid.