Name | Reg. No | Class

MAYFLOWER SECONDARY SCHOOL

4E/5N

MATHEMATICS **4052/01**

Paper 1 [90 marks] **PRELIMINARY EXAMINATION**

23 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.
Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **ALL** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is **90**.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three** significant figures. Give answers in degrees to **one** decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

Write the brand and model of your calculator in the space provided below.

Brand/Model of Calculator	For Examiner's Use	
	Total	90

This question paper consists of **18** printed pages.

Mathematical Formulae

Compound interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions

1 Calculate $\sqrt[5]{\frac{13.8^2}{1-0.038}}$. Write your answer correct to the nearest whole number.

Answer ..... [1]

---

2 Simplify.

(a) $3x-5(x-1)$

Answer ..... [1]

(b) $12x^2y \div 3xy^{-5}$

Answer ..... [1]

---

3 A car has an average petrol consumption of 0.0955 kilometres per litre.
Find the petrol consumption in litres per kilometre.

Answer ..... *l* / km [1]

---

4 **(a)** Express 0.00588 in standard form.

Answer ..... [1]

(b) Convert 0.00588 m^3 to cm^3.

Answer .....cm^3 [1]

---

5 Solve $\frac{x}{6}-\frac{2x-1}{4}=1$.

Answer $x =$ ..... [2]

---

6 Ali has 504 one-centimetre cubes. He arranges all the cubes into a cuboid.
If the base area of the cuboid is a square, find the smallest possible height of the cuboid.

Answer .....cm [2]

---

7 The marked price of a computer in a shop is \$ m.
During the National Day Sale, it was sold at a discount of d %.

(a) Express the selling price as a single fraction in terms of m and d.

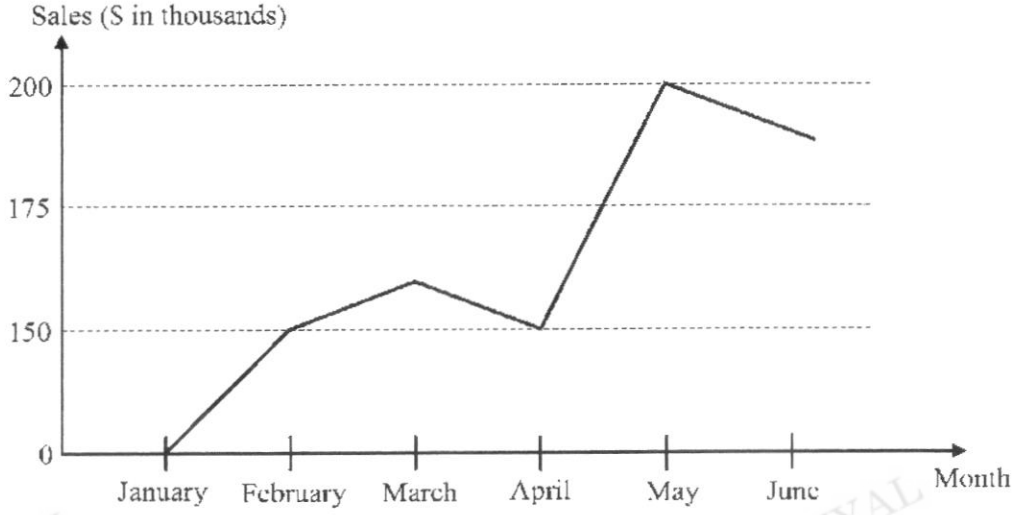
Answer \$ ..... [1]

(b) The shopkeeper made a profit of 20% from the sale of the computer.
Express the cost price as a single fraction in terms of m and d.

Answer \$ ..... [1]

---

8 The graph shows the monthly sales of a newly opened shop from January to June in 2023.



(a) State one misleading feature of the graph.

Answer .....

..... [1]

(b) Explain how this feature affects the reader's interpretation of the graph.

Answer .....

.....

.....

.....

..... [1]

---

9 Written as a product of its prime factors, $20 = 2^2 \times 5$.

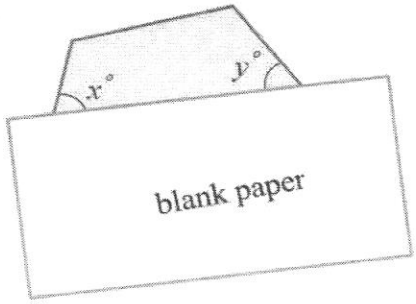
(a) Write 240 as a product of its prime factors.

Answer ..... [1]

(b) The highest common factor (HCF) of two numbers is 20.
The lowest common multiple (LCM) of two numbers is 240.
Both numbers are greater than 50. Find the two numbers.

Answer ..... , ... [2]

10



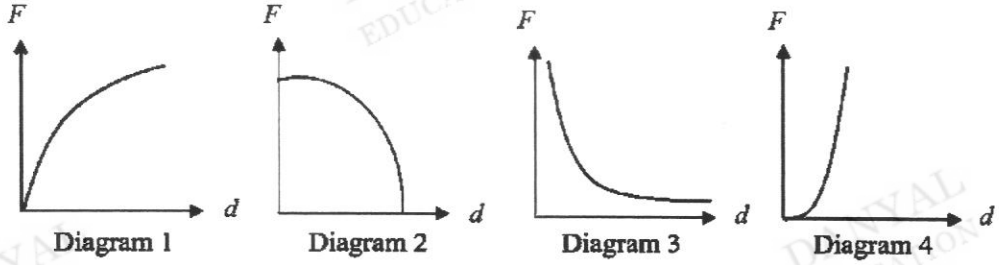
The diagram shows a regular polygon which is partially covered with a sheet of blank paper. The sum of angle $x°$ and $y°$ is 60°. Find the number of sides of the regular polygon.

Answer ..... [3]

---

11 The force, F, between two objects, is inversely proportional to the square of the distance, d, between them.

(a) Which of these diagrams represents the graph of F against d?



Answer Diagram ..... [1]

(b) The distance between two objects is increased by 150%.
Calculate the percentage reduction in the force between the objects.

Answer .....% [3]

---

12 The expression $5-4x-x^2$ can be written as $-(x+2)^2+9$.

(a) Explain why when $x=-2$, the expression $5-4x-x^2$ has its maximum value.

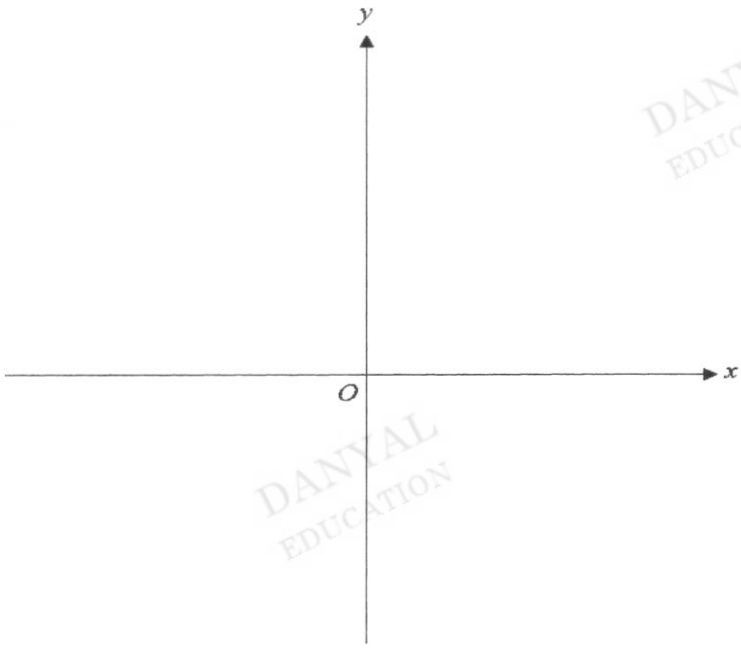
Answer .....

.....

.....

..... [1]

(b) **Sketch** the graph of $y=5-4x-x^2$ on the axis below.
Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point on the curve. [3]



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13 Factorise completely.

(a) $4b+12ab-3a-1$

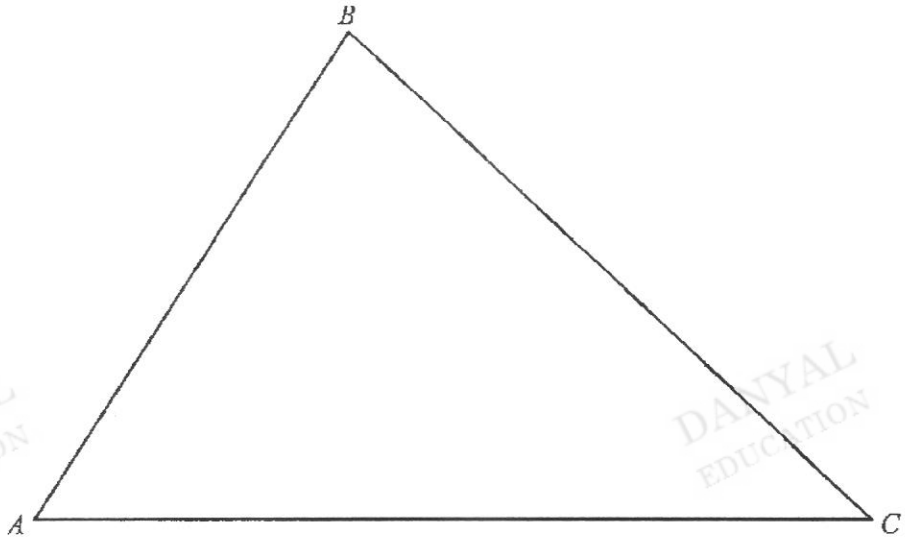
Answer ..... [2]

(b) $a^2+2ax+x^2-4b^2$

Answer ..... [2]

---

14 The diagram shows a triangle ABC.



On the diagram,

(a) Construct the perpendicular bisector of AB. [1]

(b) Construct the bisector of angle BAC. [1]

(c) The two bisectors intersect at the point P.
Complete the statement below.

Answer
The point P is equidistant from the points ..... and .....

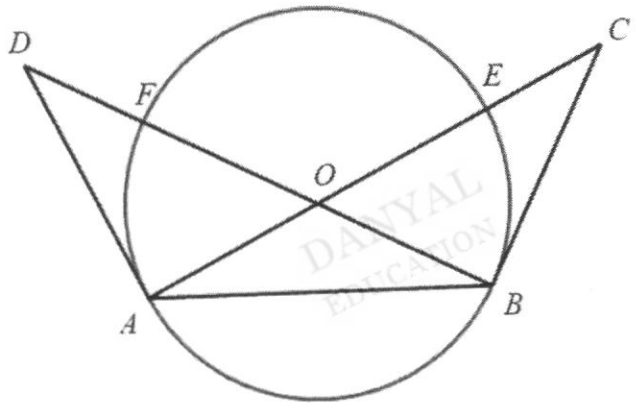
and equidistant from the lines ..... and ..... [1]

15 Given $\left(\frac{81}{9^n}\right)^{-1} = \sqrt{3^m}$.

Find an expression for m in terms of n.

Answer $m =$ ..... [3]

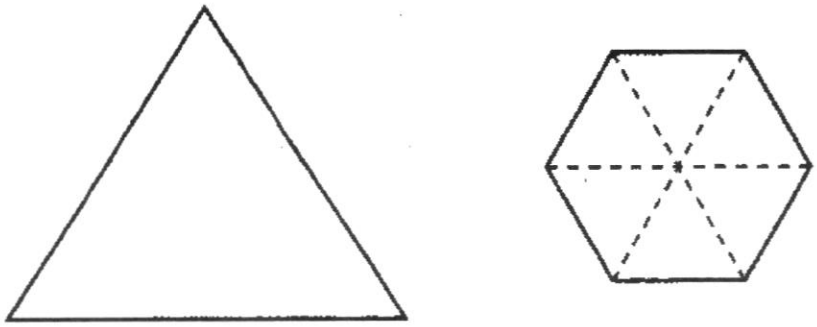
16



In the diagram, AD and BC are tangents to the circle, centre O, at the points A and B respectively. $AOEC$ and $BOFD$ are straight lines.
Show that triangle OAD and triangle OBC are congruent.
Give a reason for each statement you make.

Answer [3]

17



The diagram shows an equilateral triangle and a regular hexagon.
The ratio of the perimeters triangle : hexagon = 3 : 2
Find the ratio of the areas triangle : hexagon .

Answer ..... : ..... [2]

---

18 Cone A has a volume of 400 cm³.

(a) Calculate the volume of cone B with base radius half of cone A and height 5 times of cone A.

Answer .....cm³ [2]

(b) Calculate the volume of cone C that is similar to cone A but has a curved surface area that is $\frac{1}{9}$ of cone A.

Answer .....cm³ [2]

---

19 Ali can paint 7 fence panels in 5 hours.
Cindy can paint 6 fence panels in 4 hours.
Ali and Cindy work together to paint a total of 17 panels.
If they continue to paint at the same rate, how long will it take them to paint the 17 panels ?
Give your answer in hours and minutes, to the nearest minute.

Answer .....hours .....minutes [3]

---

20 Here are the first five terms of a sequence.

$$\frac{1}{2} \quad \frac{4}{4} \quad \frac{7}{6} \quad \frac{10}{8} \quad \frac{13}{10}$$

(a) Find the sixth term of the sequence .

Answer ..... [1]

(b) T_n is the nth term of the sequence.
Find an expression, in terms of n, for T_n.

Answer $T_n =$ ..... [2]

(c) The difference, D, between two consecutive terms of the sequence is $T_{n+1} - T_n$.
Show that $D = \dfrac{1}{n(n+1)}$.

Answer

[3]

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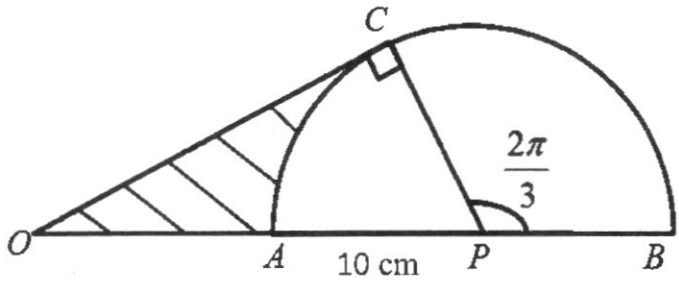
21 The figure shows a semicircle ABC with centre P and radius 10 cm.
OC is a tangent to the circle at C and meets BA produced at O.
Angle $CPB = \frac{2\pi}{3}$ radians.



(a) Find the length OC.

Answer .....cm [2]

(b) Find the area of the shaded region COA.

Answer .....cm² [3]

22 **(a)** $\xi = \{ \text{integers } x : 15 < x < 30 \}$
$A = \{ \text{prime numbers} \}$
$B = \{ \text{multiples of 3} \}$
$C = \{ \text{factors of 30} \}$

List the elements in

(i) A,

Answer ..... [1]

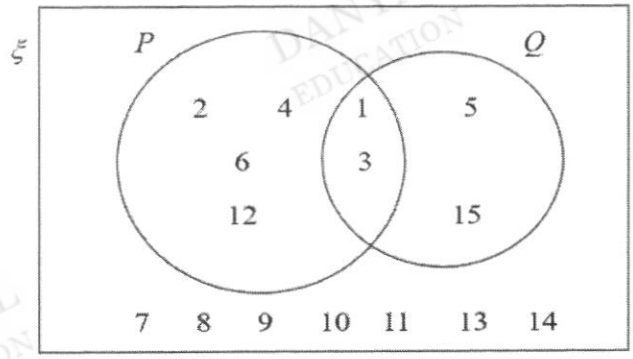
(ii) $(A \cup B)'$.

Answer ..... [1]

(iii) Explain why C is an empty set.

Answer .....
..... [1]

(b) The Venn diagram shows the elements of $\xi = \{ \text{integers } x : 1 \leq x \leq 15 \}$.



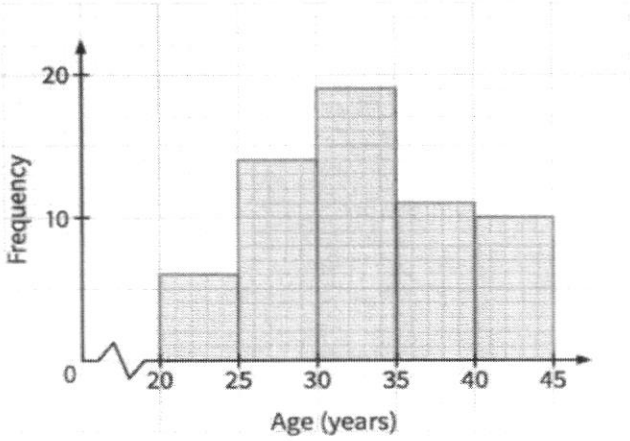
(i) Describe the elements in set P.

Answer .....
..... [1]

(ii) Find the value of $n[(P \cap Q') \cup (P' \cap Q)]$.

Answer ..... [1]

23



The histogram shows the distribution of the ages of 60 members in a club.

(a) Which interval contains the median age?

Answer ..... [1]

(b) Find the estimated mean age of the members.

Answer ..... [1]

(c) Find the estimated standard deviation of the members.

Answer ..... [1]

(d) The members in the club remain unchanged after 5 years.

(i) Write down the new mean age of the members.

Answer ..... [1]

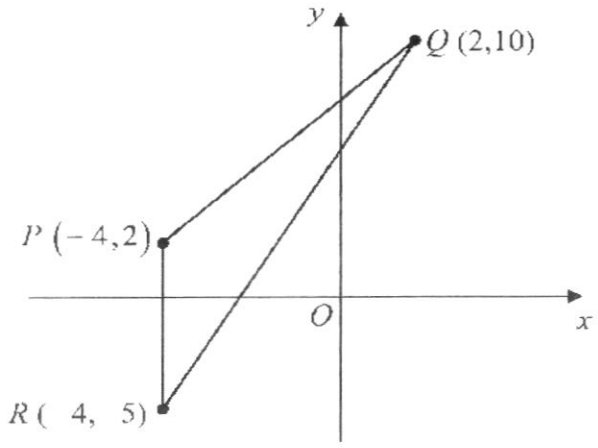
(ii) Without calculating, explain why the standard deviation remains unchanged.

Answer .....

.....

..... [1]

24 The diagram shows three points $P(-4,2)$, $Q(2,10)$ and $R(-4,-5)$.



(a) Find the length of PQ.

Answer .....units [1]

(b) Express as a fraction in its lowest term, find $\cos \angle QPR$.

Answer ..... [1]

(c) Find the area of triangle PQR.

Answer .....units² [1]

(d) The line $mx+2y+3=0$ has the same gradient as line RQ.
Find the value of m.

Answer ..... [3]

25 A zoo is open every day in a week.
The average number of adults, children and seniors visiting the zoo on a weekday is 200, 350 and 150 respectively.
The average number of adults, children and seniors visiting the zoo on a weekend is 500, 750 and 180 respectively.

The information is represented by the matrix $\mathbf{Z} = \begin{pmatrix} 200 & 350 & 150 \\ 500 & 750 & 180 \end{pmatrix}$.

(a) The ticket price for an adult, a child and a senior are \$32, \$21 and \$14 respectively.
Represent the price by a 3×1 matrix **P**.

Answer **P** = ... [1]

(b) Find the matrix $\mathbf{T} = \mathbf{ZP}$.

Answer **T** = ... [2]

(c) State what the elements of **T** represent.

Answer .....
.....
..... [1]

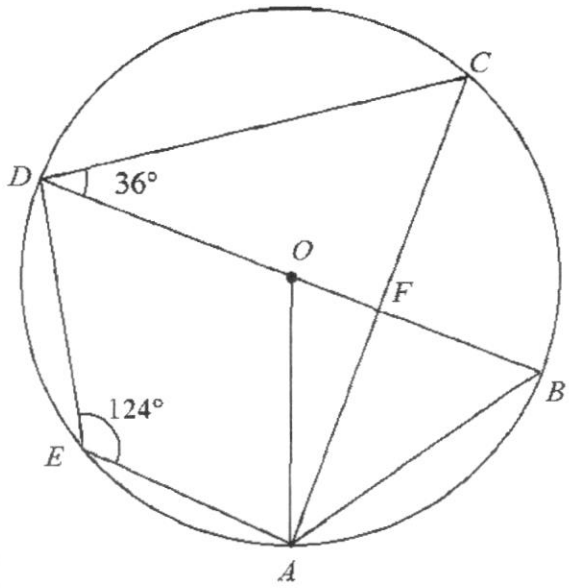
(d) There are 5 weekdays and 2 weekends.
Write down a matrix **D** such that **DT** represent the total revenue of the zoo in a week.

Answer **D** = ... [1]

(e) Find the matrix **DT**.

Answer **DT** = ... [1]

26 **(a)**



A, B, C, D and E are points on a circle, centre O.
BD is the diameter of the circle. BD intersects AC at point F.
Angle $AED = 124°$ and angle $BDC = 36°$.

(i) Complete the statement.

Angle CAB = ... because .....
..... [1]

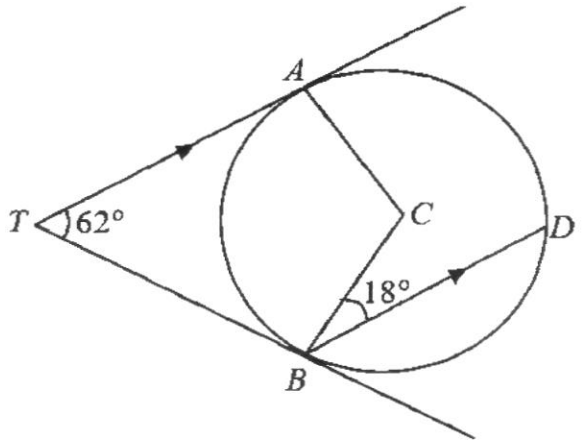
(ii) Complete the statement.

Reflex angle AOD = ...because .....
..... [1]

(iii) Find angle AFB.
Give a reason for each step of your working.

Answer .....° [2]

26 (b)



In the diagram, TA and TB are tangents to the circle at A and B respectively.
D is a point on the circle such that BD is parallel to TA.
C is a point inside the circle such that angle CBD is 18°.
Angle ATB is 62°.

(i) Find angle TAB.
Give a reason of your working.

Answer .....° [1]

(ii) Show that C is not the centre of the circle.

Answer

[2]

Name

Reg. No	Class

MAYFLOWER SECONDARY SCHOOL

4E/5N

MATHEMATICS — 4052/02

Paper 2 [90 marks]

PRELIMINARY EXAMINATION

21 August 2023

2 hours 15 minutes

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Brand/Model of Calculator

For Examiner's Use	
Total	90

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$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions

1 **(a)** $s = \sqrt{\dfrac{p+2r}{p-2q}}$

(i) Find s when $p = 10$, $q = 3$ and $r = 7.5$.

Answer ..... [1]

(ii) Rearrange the formula to make p the subject.

Answer $p =$ ..... [3]

(b) Solve the inequalities $-7 \leq \dfrac{3x-4}{2} < 5-x$.

Answer ..... [3]

1 **(c)** Write as a single fraction in its simplest form $\frac{x+5}{2x^2-5x-3}-\frac{2}{x-3}$.

Answer ..... [3]

2 **(a)** The cash price of a new laptop is \$3369.
Ahmad buys this laptop on hire purchase.
He pays a deposit of one third of the cash price followed by 36 equal monthly instalments.
The total amount Ahmad pays for the laptop is \$3650.20.

(i) Calculate each monthly instalment.

Answer \$ ..... [2]

(ii) Calculate the simple interest rate per annum.

Answer ..... % [2]

(b) Brandon buys an identical laptop.
He borrows \$3369 at a compound interest of 4% per year for 5 years.

Calculate the total amount Brandon pays for the laptop.

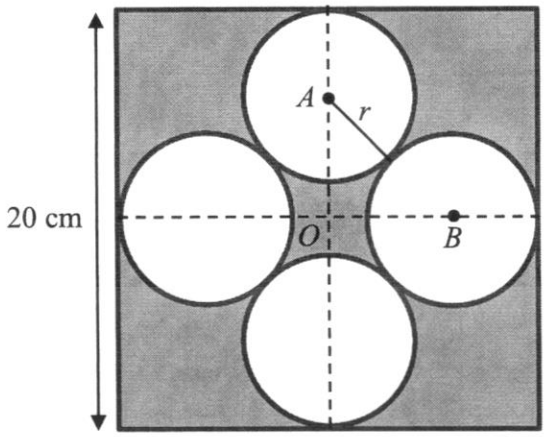
Answer \$ ..... [2]

2 **(c)** Catherine buys the same laptop from an online store selling it for 15,200 Chinese Yuan.
She is charged a 8% tax on the price of the laptop.
The exchange rate between Singapore dollars ($) and Chinese Yuan (¥) is
$1 = ¥5.41.

Calculate the percentage savings by Catherine.

Answer ..... % [3]

3 The diagram shows a square of length 20 cm. O is the centre of the square. Four smaller **identical** circles, with radius r cm are drawn as shown. The circles touch each other and the sides of the square. A and B are centres of two of the smaller circles.



(a) Find and simplify an expression, in terms of r, for

(i) OA,

Answer ..... [1]

(ii) AB.

Answer ..... [1]

(b) Form an equation, in terms of r, and show that it reduces to

$$r^2 + 20r - 100 = 0 .$$

Answer

[3]

3 **(c)** Solve the equation $r^2 + 20r - 100 = 0$.
Give your solutions correct to two decimal places.

Answer $r =$ ..... or ..... [3]

(d) Calculate the area of the shaded region.

Answer ..... cm^2 [2]

4 **(a)** L is the point $(-2, -4)$ and M is the point $(5, 8)$.

(i) Write down the column vector $\overrightarrow{LM}$.

Answer ..... [1]

(ii) Find $|\overrightarrow{LM}|$.

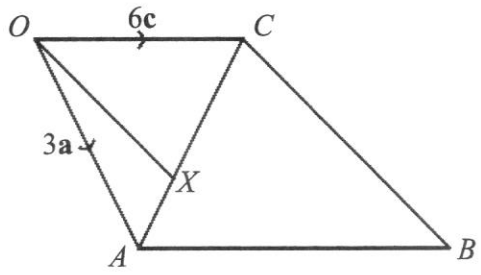
Answer ..... [1]

(iii) Given that $2\overrightarrow{LN} = 3\overrightarrow{LM}$, find the coordinates of N.

Answer (..... , ...) [2]

4 **(b)** $OABC$ is a quadrilateral.

$\overrightarrow{OA} = 3\mathbf{a}$, $\overrightarrow{OC} = 6\mathbf{c}$, $3\overrightarrow{OC} = 2\overrightarrow{AB}$ and $\frac{AX}{XC} = \frac{1}{2}$.



(i) Express $\overrightarrow{AC}$ in terms of **a** and **c**, as simply as possible.

Answer ..... [1]

(ii) Express $\overrightarrow{OX}$ in terms of **a** and **c**, as simply as possible.

Answer ..... [2]

(iii) Express $\overrightarrow{CB}$ in terms of **a** and **c**, as simply as possible.

Answer ..... [2]

(iv) Show that triangle OCX is similar to triangle BAC.
Give a reason for each statement you make.

Answer .....

.....

.....

.....

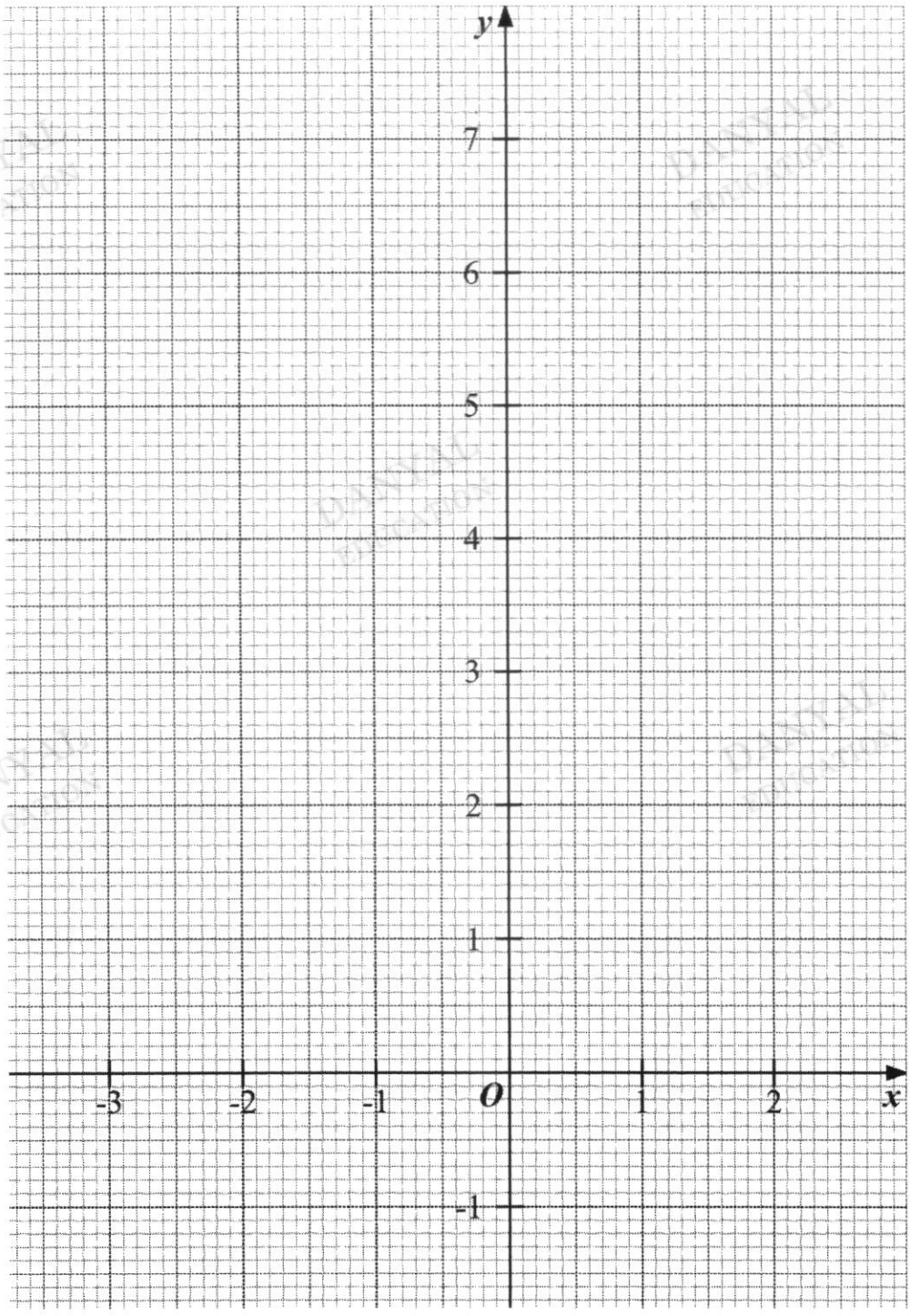
..... [2]

5 **(a)** Complete the table of values for $y = 3x - 1 + \dfrac{10}{x+3}$.

x	–2.25	–2	–1.5	–1	–0.5	0	0.5	1	2
y	5.58	3		1	1.5	2.33	3.36	4.5	7

[1]

(b) On the grid, draw the graph of $y = 3x - 1 + \dfrac{10}{x+3}$ for $-2.25 \leq x \leq 2$. [3]



5 **(c)** Explain why the equation $3x-1+\frac{10}{x+3}=k$ does not have solutions for some values of k.

.....

..... [1]

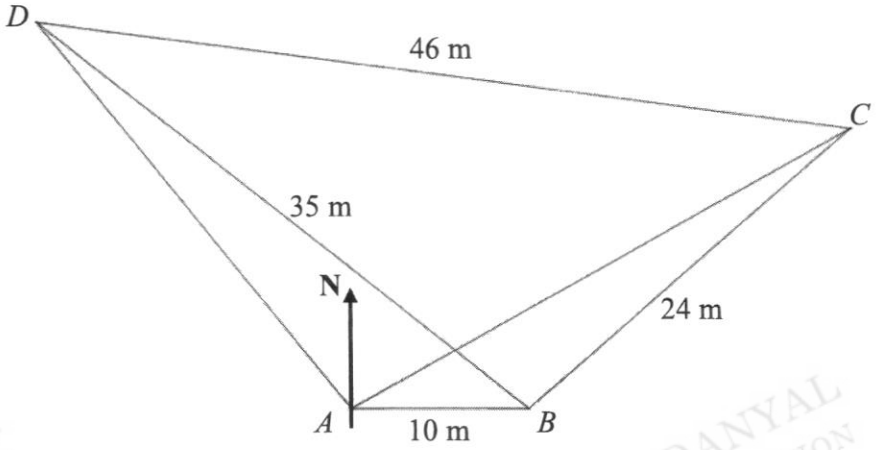
(d) A line $y=\frac{1}{2}x+c$ is a tangent to the curve.
By drawing this tangent, find the value of c.

Answer $c=$ ..... [2]

(e) By drawing a suitable straight line, solve the equation $-\frac{9}{2}x+2=\frac{10}{x+3}$.

Answer $x=$ ..... or ..... [2]

6 *A, B, C* and *D* are four points on level ground.
A is due west of *B* and the bearing of *C* from *A* is 059°.
AB = 10 m, *BC* = 24 m, *BD* = 35 m and *CD* = 46 m.



(a) Calculate
(i) angle *ACB*,

Answer ..... ° [2]

(ii) the bearing of *C* from *B*,

Answer ..... ° [1]

(iii) the area of triangle *BCD*.

Answer ..... m^2 [4]

6 **(b)** A building is located at B such that the angle of depression from the top of the building to D is 49^o.

Calculate

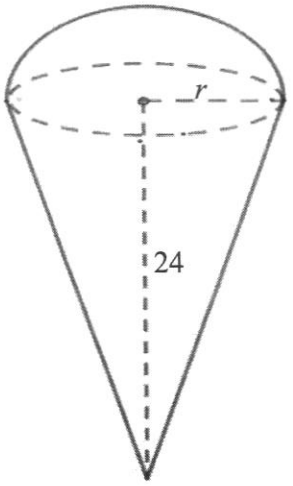
(i) the height of the building,

Answer ..... m [2]

(ii) the greatest angle of elevation of the top of the building when viewed from a point along CD.

Answer ..... ° [3]

7 The diagram shows **Solid A** consisting of a right circular cone attached to a hemisphere with a common circular base of radius r cm. The height of the cone is 24 cm. The volume of the cone is equal to **twice** the volume of the hemisphere.



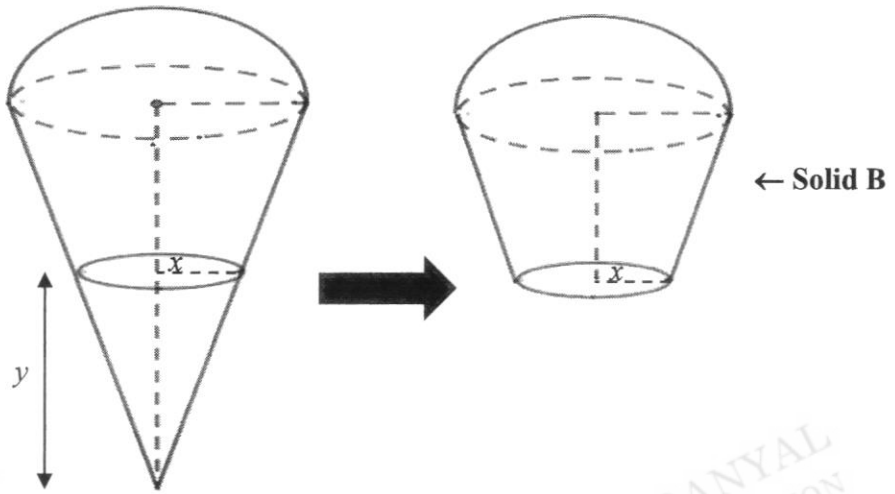
Solid A

(a) Show that $r = 6$.

Answer

[2]

7 **(b)** **Solid B** is constructed by removing a smaller cone of base radius x cm and height y cm.



Solid A

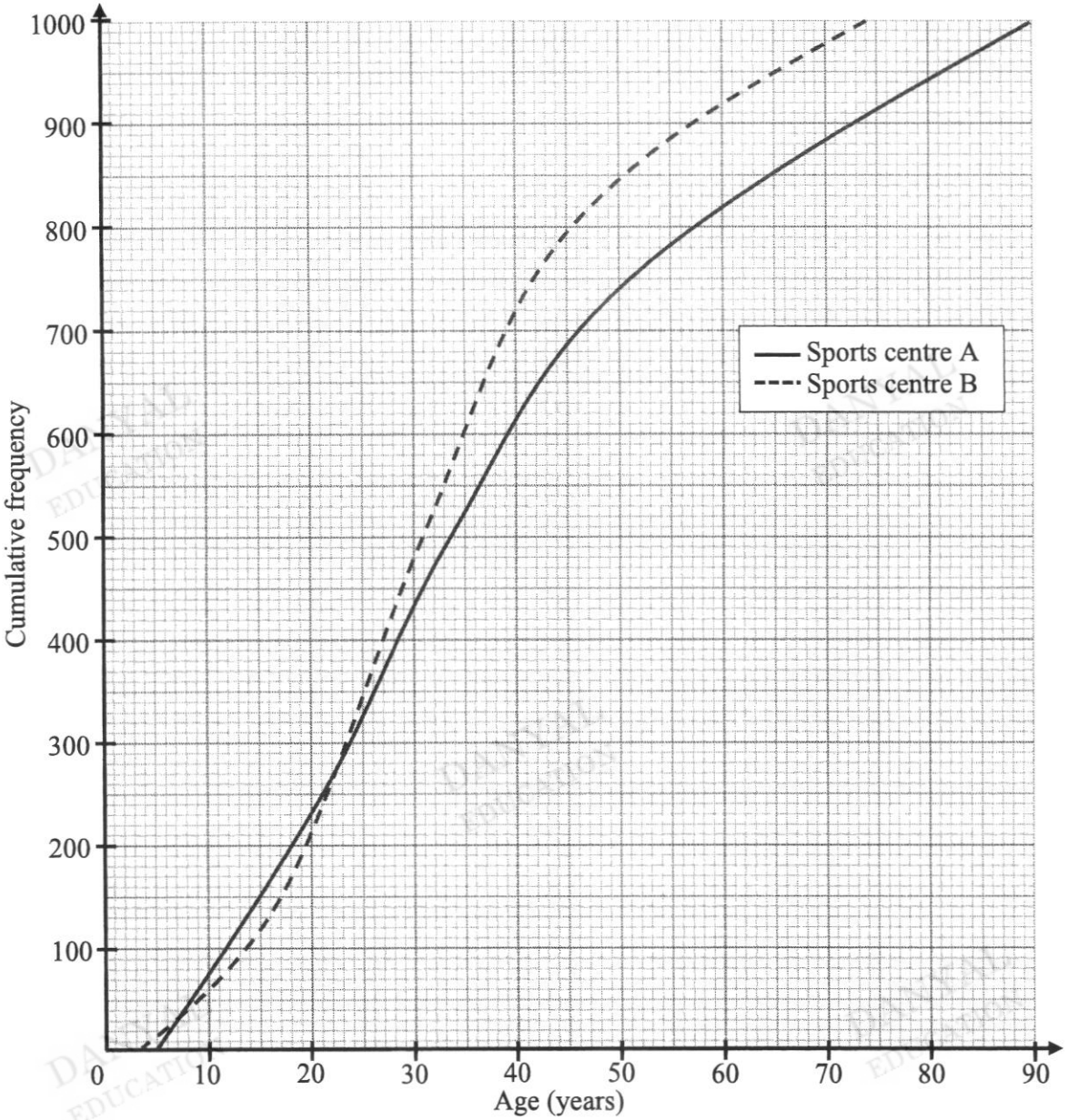
(i) Find the value of $\frac{x}{y}$.

Answer ..... [1]

(ii) Given that the volume of the cone removed is 121.5π cm^3, calculate the total surface area of **Solid B**.

Answer ..... cm^2 [5]

8 **(a)** The ages of 1000 people using sports centre A and sports centre B are summarised in the cumulative frequency diagram below.



(i) Use the curve to estimate

(a) the median age for sports centre A,

Answer ..... [1]

(b) the number of people over the age of 50 for sports centre B,

Answer ..... [1]

(c) the interquartile range for sports centre A.

Answer ..... [2]

8 **(ii)** Make two comparisons between the age distribution in sports centre A and sports centre B.
Use figures to support your answer.

1. .....

.....

2. .....

..... [2]

(b) The table shows information about a group of people using the sports centre on one day.

	Aged under 30	Aged 30 or over
Male	25	15
Female	13	7

(i) One of these people who is a male is selected at random.
Find the probability that this person is aged under 30.

Answer ..... [1]

(ii) Two of the people are selected at random.
Find the probability that
(a) both are female,

Answer ..... [2]

(b) they are both aged 30 or over, but only one is a male.

Answer ..... [2]

9 Steve is planning to start a small business selling bubble tea.
He has read the following **healthy eating guidelines**:

> Our sugar consumption should be no more than 10 percent of our daily caloric* intake (this is equal to 10 teaspoons of sugar based on a 2000-daily caloric intake). This limit includes sugar present in food and drinks.
>
> A further reduction to 5 teaspoons of sugar a day based on a 2000-daily calorie can bring about additional health benefits.

** The average recommended daily caloric intake is 2200 calories for males, and 1800 calories for females (based on an average weight and physical activity of the average male and female Singaporean).*

A cup of bubble tea is made by adding sugar syrup based on sugar level and topping up the remaining amount with flavoured tea.

The tables below give information to calculate the number of calories in a cup of bubble tea.

Sugar Level
Quarter Sugar (25%)
Half Sugar (50%)
Less Sugar (75%)
Full Sugar (100%)

Size	Capacity in ml
Regular	485
Large	705

A regular-sized cup of bubble tea with full sugar (100%) contains 120 ml of sugar syrup.

Flavoured tea	Amount of calories per 500 ml
Black tea	70
Green tea	85
Red tea	100
Milk tea	110

	Amount of calories per 15 ml
Sugar syrup	50

e.g. A regular-sized (485ml) cup of bubble tea with full sugar
= 120 ml of sugar syrup + 365 ml of flavoured tea

(a) Calculate the number of calories in sugar syrup in a regular-sized cup of bubble tea with half sugar.

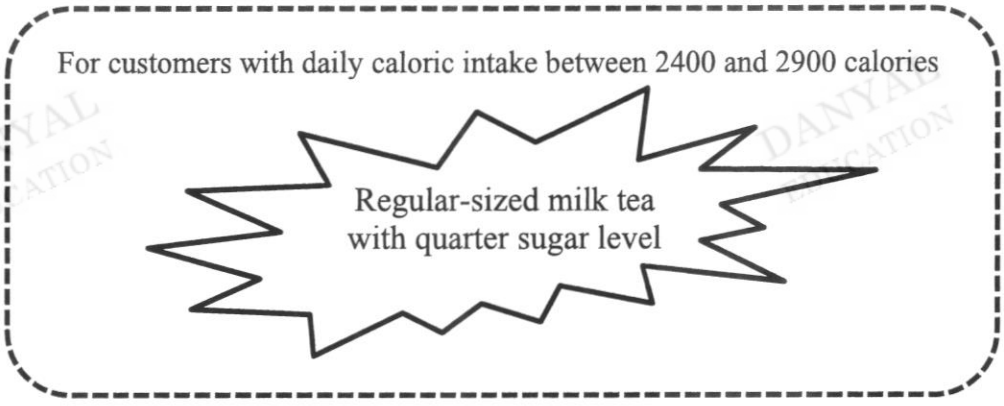
Answer ..... calories [2]

9 Zen's daily caloric intake is about 2800 calories based on his weight and physical activity.

(b) According to the healthy eating guidelines, calculate the maximum number of calories he can consume from sugar.

Answer ..... calories [1]

(c) Steve is offering the following healthier choice option:



Zen decides to select the above healthier choice option.
He thinks that the total calories of this option meets the healthy eating guidelines but exceeds the recommended calories for additional health benefits by 30%.

Is Zen correct?
Justify your decision and show your method clearly.

Answer

.....

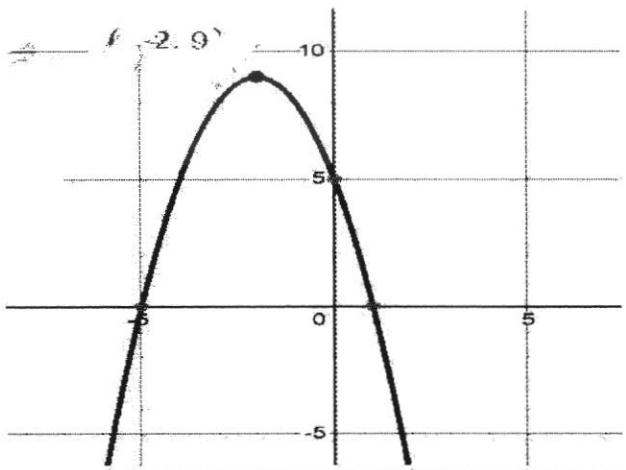
.....

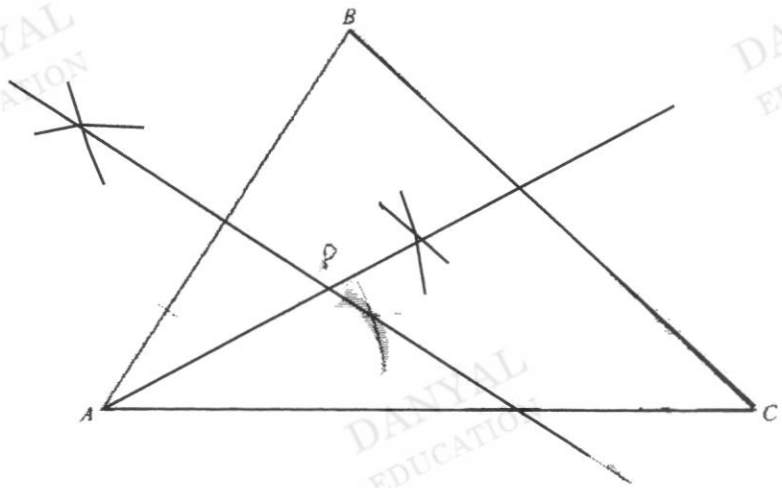
.....

..... [7]

2023 Sec 4E5N Mathematics Preliminary Examination Paper 1 Marking Scheme

1			2.879496932... = 3 (nearest whole number)
2	(a)		$5-2x$
	(b)		$4xy^6$
3			$\frac{1}{0.0955}$ l / km 10.5 l / km (3 s.f)
4	(a)		5.88×10^{-3}
	(b)		$1\ m^3 = 10^6\ cm^3$ $0.00588\ m^3 - 0.00588\times10^6\ cm^3 - 5880\ cm^3$
5			$\frac{x}{6} - \frac{2x-1}{4} = 1$ $2x-6x+3=12$ $x=-2\frac{1}{4}$
6			$504 = 2^3\times3^2\times7$ Greatest square base area $= 2^2\times3^2$ Smallest height $= 2\times7 = 14\ cm$
7	(a)		Selling Price = $\$\frac{m(100-d)}{100}$
	(b)		Cost Price = $\$\frac{m(100-d)}{120}$
8	(a)		The scale / intervals on the vertical axis is not consistent.
	(b)		It may mislead the reader to think that the amount of sales in May ($200000) appears to be three time of that in February / April ($150000). Actually it is only $1\frac{1}{3}$ times. OR It may mislead the reader to think that the increase in sales from April to May ($50000) appears to be twice of that from January to February ($150000). Actually the amount of increase is only one-third.

9	**(a)**		$240 = 2^4 \times 3 \times 5$
	(b)		HCF = 20 = $2^2 \times 5$ LCM = 240 = $(2^2 \times 5) \times 2^2 \times 3$ Two numbers : $(2^2 \times 5) \times 3$ and $(2^2 \times 5) \times 2^2$ = 60 and 80
10			Let the int. $\angle$ of the regular polygon be a° $x + y + 2a = 360$ $60 + 2a = 360$ $a = 150$ ext. $\angle = 180^\circ - 150^\circ = 30^\circ$ OR $(n-2)180^\circ = 150n$ $n = \frac{360}{30} = 12$
11	**(a)**		Diagram 3
	(b)		$d \longrightarrow 2.5d$ $F_1 = \frac{k}{d^2} \longrightarrow F_2 = \frac{k}{(2.5d)^2}$ $F_2 = \frac{1}{2.5^2} F_1$ $F_2 = \frac{4}{25} F_1$ Percentage reduction in the force = $\frac{25-4}{25} \times 100\% = 84\%$
12	**(a)**		As $(x+2)^2 \geq 0$ for all x values $\Rightarrow -(x+2)^2 \leq 0$ for all x values $\Rightarrow 9-(x+2)^2 \leq 9$ for all x values When $x = -2$, $-(x+2)^2$ has its maximum value of 0, hence $9-(x+2)^2$ has its maximum value of 9.
	(b)		

13	**(a)**		$4b+12ab-3a-1$ $=4b(1+3a)-(1+3a)$ $=(1+3a)(4b-1)$ $=(3a+1)(4b-1)$
	(b)		$a^2+2ax+x^2-4b^2$ $=(a+x)^2-(2b)^2$ $=(a+x-2b)(a+x+2b)$
14	**(a)** **(b)**		
	(c)		The point P is equidistant from the points A and B and Equidistant from the lines AB and AC.
15			$\left(\frac{81}{9^n}\right)^{-1}=\sqrt{3^m}$ $\left(\frac{3^4}{3^{2n}}\right)^{-1}=3^{\frac{1}{2}m}$ $\frac{3^{2n}}{3^4}=3^{\frac{1}{2}m}$ $3^{2n-4}=3^{\frac{1}{2}m}$ $2n-4=\frac{1}{2}m$ $m=4n-8$

16

$\underline{\Delta OAD}$ $\underline{\Delta OBC}$

$\angle AOD = \angle BOC$ (vert. opp. $\angle s$)

$OA = OB =$ radius

$\angle OAD = \angle OBC$ (radius $\perp$ tangent)

By *ASA* congruence test, $\Delta OAD \equiv \Delta OBC$

17

Perimeters big triangle : hexagon = 3 : 2
= 9 : 6

Lengths big triangle : small triangle = 3 : 1

Areas big triangle : small triangle = 3^2 : 1^2

Areas big triangle : hexagon = 9 : 6
= 3 : 2

18 **(a)**

Volume of Cone A $= \frac{1}{3}\pi r^2 h = 400 \text{ cm}^3$

Volume of Cone B $= \frac{1}{3}\pi(\frac{1}{2}r)^2(5h)$

$= \frac{1}{4}\times 5 \times \frac{1}{3}\pi r^2 h$

$= \frac{1}{4}\times 5 \times 400 \text{ cm}^3$

$= 500 \text{ cm}^3$

(b)

$\frac{Area_C}{Area_A} = \frac{1}{9}$

$\Rightarrow \frac{l_C}{l_A} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

$\Rightarrow \frac{Volume\ C}{Volume\ A} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$\Rightarrow Volume\ C = \frac{1}{27}\times 400 = 14\frac{22}{27} cm^3 = 14.8 cm^3\ (3s.f)$

19			In one hour, Ali can paint $\frac{7}{5}$ fence panels. Cindy can paint $\frac{6}{4}$ fence panels. Together they can paint $\left(\frac{7}{5}+\frac{6}{4}\right)=\frac{29}{10}$ fence panels per hour. Time taken for them to paint 17 panels $= 17\div\frac{29}{10}h$ =5 h 52 min
20	**(a)**		Sixth term $= \frac{16}{12}$
	(b)		$T_n=\frac{3n-2}{2n}$
	(c)		$T_{n+1}=\frac{3(n+1)-2}{2(n+1)}=\frac{3n+1}{2(n+1)}$ $D=T_{n+1}-T_n$ $=\frac{3n+1}{2(n+1)}-\frac{3n-2}{2n}$ $=\frac{n(3n+1)-(n+1)(3n-2)}{2n(n+1)}$ $=\frac{3n^2+n-(3n^2+n-2)}{2n(n+1)}$ $=\frac{1}{n(n+1)}$
21	**(a)**		$\angle APC=\pi-\frac{2\pi}{3}=\frac{\pi}{3}$ $\tan\frac{\pi}{3}=\frac{OC}{10}$ $\Rightarrow OC=10\tan\frac{\pi}{3}=17.3cm\ (3s.f)$

21	(b)		Area of sector PAC $\quad \frac{1}{2}\times 10^2 \times \frac{\pi}{3} = 52.3598\ cm^2$ Area of $\Delta OPC = \frac{1}{2}\times 10\times 10\tan\frac{\pi}{3} = 86.6025\ cm^2$ Shaded Area $= 86.6025 - 52.3598 = 34.2427 = 34.2\ cm^2\ (3s.f)$
22	(a)	(i)	$A = \{17, 19, 23, 29\}$
		(ii)	$(A\cup B)' = \{16, 20, 22, 25, 26, 28\}$
		(iii)	No elements in ξ are factors of 30, hence $C = \varnothing$
	(b)	(i)	Elements in set P are factors of 12.
		(ii)	$n[(P\cap Q')\cup(P'\cap Q)] = 6$
23	(a)		Interval 30-35
	(b)		Estimated Mean $\frac{\sum fx}{\sum f} = \frac{6\times 22.5+14\times 27.5+19\times 32.5+11\times 37.5+10\times 42.5}{60}$ $= \frac{1975}{60} = 32.9\ (3\ s.f)$
	(c)		$\frac{\sum fx^2}{\sum f} \quad \frac{6\times 22.5^2 + 14\times 27.5^2 + 19\times 32.5^2 + 11\times 37.5^2 + 10\times 42.5^2}{60}$ $= \frac{67225}{60}$ Estimated Standard Deviation $\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \quad \sqrt{\frac{67225}{60} - \left(\frac{1975}{60}\right)^2}$ $= 6.07533 \quad 6.08\ (3s.f)$
	(d)	(i)	New mean age = 32.9 + 5 =37.9
		(ii)	After 5 years, the age of every member is increased by 5 and the new mean age is also increased by 5, hence $\sum f[(x+5)-(\bar{x}+5)]^2 \quad \sum f(x-\bar{x})^2$ remains unchanged. $\therefore SD = \sqrt{\frac{\sum f(x-\bar{x})^2}{60}}$ remains unchanged.

24	**(a)**		$PQ=\sqrt{6^2+8^2}=10\ units$
	(b)		$\cos\angle QPR=-\frac{8}{10}=-\frac{4}{5}$
	(c)		Area of $\Delta PQR=\frac{1}{2}\times 7\times 6=21\ units^2$
	(d)		Gradient of RQ $\frac{15}{6}=\frac{5}{2}$ $mx+2y-3=0 \Rightarrow y=-\frac{m}{2}x-\frac{3}{2}$ $-\frac{m}{2}=-\frac{5}{2} \Rightarrow m=5$
25	**(a)**		$P=\begin{pmatrix}32\\21\\14\end{pmatrix}$
	(b)		$T\ \begin{pmatrix}200 & 350 & 150\\500 & 750 & 180\end{pmatrix}\begin{pmatrix}32\\21\\14\end{pmatrix}\ \begin{pmatrix}15850\\34270\end{pmatrix}$
	(c)		The elements of T represent the revenue of the zoo in a weekday and a weekend respectively.
	(d)		$D=(5\quad 2)$
	(e)		$DT=(5\quad 2)\begin{pmatrix}15850\\34270\end{pmatrix}=(147790)$
26	**(a)**	**(i)**	$\angle CAB=36$ because angles in the same segment are equal.
		(ii)	Reflex $\angle AOD=2\times 124=248$ because angle at centre twice angle at circumference.
		(iii)	$\angle ABF=\angle ABD$ $=180-124=56$ ($\angle s$ in opposite segments) $\angle AFB=180-36-56=88$ ($\angle s$ sum of Δ)
	(b)	**(i)**	$TA=TB$ (equal tangents from external point) $\angle TAB=(180-62)\div 2=59$ (base $\angle s$ of isosceles Δ)
		(ii)	$\angle TBC=180-62-18=100$ (int. $\angle s$, $TA//BD$) By the property 'radius ⊥ tangent', as $\angle TBC\neq 90$, BC is not a radius of the circle. Hence C is not the centre of the circle.

2023 MF Mathematics Preliminary Examination Paper 2 Marking Scheme

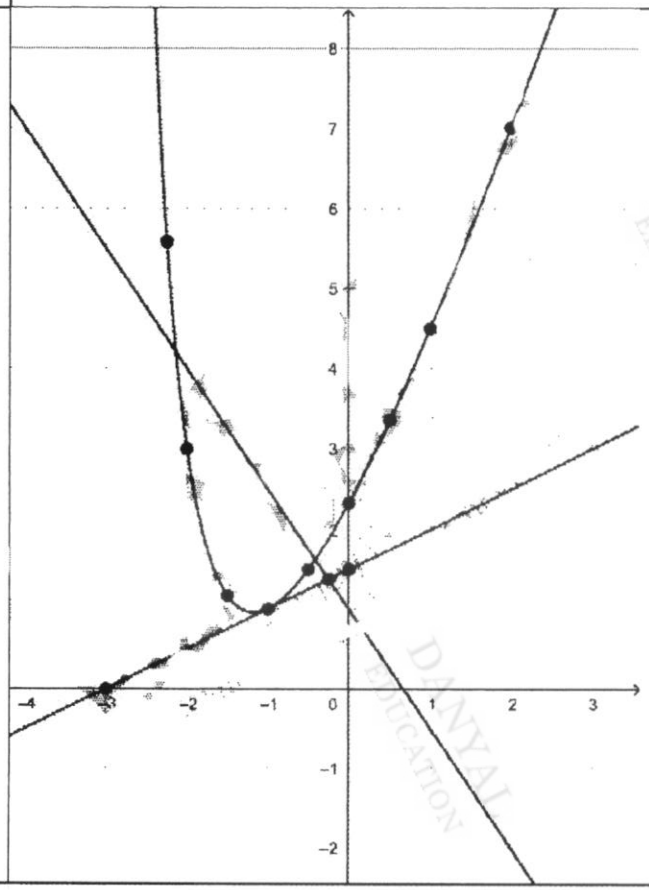
Qn		Solutions	
1	(a)	(i)	$s = 2.5$
1	(a)	(ii)	$s = \sqrt{\dfrac{p+2r}{p-2q}}$
			$s^2 = \dfrac{p+2r}{p-2q}$
			$s^2(p-2q) = p+2r$
			$ps^2 - 2qs^2 = p+2r$
			$ps^2 - p = 2r + 2qs^2$
			$p(s^2-1) = 2r + 2qs^2$
			$p = \dfrac{2r+2qs^2}{s^2-1}$
1	(b)		$-7 \le \dfrac{3x-4}{2} < 5-x$
			$-7 \le \dfrac{3x-4}{2}$ and $\dfrac{3x-4}{2} < 5-x$
			$-14 \le 3x-4$ and $3x-4 < 2(5-x)$
			$-14+4 \le 3x$ and $3x-4 < 10-2x$
			$x \ge -\dfrac{10}{3}$ and $5x < 14$ $x < \dfrac{14}{5}$
			$-\dfrac{10}{3} \le x < \dfrac{14}{5}$

1	**(c)**		$\dfrac{x+5}{2x^2-5x-3}-\dfrac{2}{x-3}$
			$\dfrac{x+5}{(2x+1)(x-3)}-\dfrac{2}{x-3}$
			$\dfrac{x+5}{(2x+1)(x-3)}-\dfrac{2(2x+1)}{(2x+1)(x-3)}$
			$\dfrac{x+5-4x-2}{(2x+1)(x-3)}$
			$\dfrac{-3x+3}{(2x+1)(x-3)}$
2	**(a)**	(i)	Total payable after deposit $=3650.20-\dfrac{3369}{3}$ $=2527.20$
			Monthly instalment $=\dfrac{2527.20}{36}$ $=\$70.20$
2	**(a)**	(ii)	$3650.20-3369=\dfrac{\left(\frac{2}{3}\times 3369\right)R\left(\frac{36}{12}\right)}{100}$
			$R=4.17$ (to 3 s.f.)

2	**(b)**		$A=3369\left(1+\frac{4}{100}\right)^5$
			$A=4098.9036$
2	**(c)**		Price of laptop in Singapore dollars before tax $=\frac{15200}{5.41}$
			$=2809.6118$
			Price after tax $=2809.6118\times1.08$
			$=3034.3807$
			Percentage savings $=\frac{3369-3034.3807}{3369}\times100\%$
			$=9.93\%$ (to 3 s.f.)
3	**(a)**	(i)	$10-r$
3	**(a)**	(ii)	$2r$
3	**(b)**		$(10-r)^2+(10-r)^2=(2r)^2$
			$100-20r+r^2+100-20r+r^2=4r^2$
			$2r^2+40r-200=0$
			$r^2+20r-100=0$
3	**(c)**		$r=\frac{-20\pm\sqrt{20^2-4(1)(-100)}}{2(1)}$
			$r=4.1421$ or $r=-24.1421$
			$r=4.14$ or $r=-24.14$
3	**(d)**		Area of shaded region

		$=20^2-4\pi(4.1421)^2$	
		$=184\ \text{cm}^2$ (to 3 s.f.)	
4	**(a)**	(i)	$\overrightarrow{LM}=\begin{pmatrix}5\\8\end{pmatrix}-\begin{pmatrix}-2\\-4\end{pmatrix}=\begin{pmatrix}7\\12\end{pmatrix}$
4	**(a)**	(ii)	$\left\lvert\overrightarrow{LM}\right\rvert=\sqrt{7^2+12^2}=13.9$
4	**(a)**	(iii)	$2\overrightarrow{LN}=3\overrightarrow{LM}$
			$2\left[\overrightarrow{ON}-\begin{pmatrix}-2\\-4\end{pmatrix}\right]=3\begin{pmatrix}7\\12\end{pmatrix}$
			$2\overrightarrow{ON}=3\begin{pmatrix}7\\12\end{pmatrix}+2\begin{pmatrix}-2\\-4\end{pmatrix}$
			$2\overrightarrow{ON}=\begin{pmatrix}21\\36\end{pmatrix}+\begin{pmatrix}-4\\-8\end{pmatrix}$
			$\overrightarrow{ON}=\begin{pmatrix}\frac{17}{2}\\14\end{pmatrix}$
			$N\left(\frac{17}{2},14\right)$
4	**(b)**	(i)	$\overrightarrow{AC}=6\mathbf{c}-3\mathbf{a}$
4	**(b)**	(ii)	$\overrightarrow{AX}=\frac{1}{3}(6\mathbf{c}-3\mathbf{a})=2\mathbf{c}-\mathbf{a}$
			$\overrightarrow{OX}=3\mathbf{a}+2\mathbf{c}-\mathbf{a}=2\mathbf{a}+2\mathbf{c}$
4	**(b)**	(iii)	$\overrightarrow{AB}=\frac{3}{2}(6\mathbf{c})=9\mathbf{c}$

			$\overrightarrow{CB} = -(6\mathbf{c} - 3\mathbf{a}) + 9\mathbf{c} = 3\mathbf{a} + 3\mathbf{c}$
			$\overrightarrow{CB} = 3\mathbf{a} + 3\mathbf{c}$
4	**(b)**	(iv)	Given $3\overrightarrow{OC} = 2\overrightarrow{AB} \Rightarrow OC$ is parallel to AB and $\frac{OC}{BA} = \frac{2}{3}$
			angle OCX = angle BAC (alt $\angle$s, OC // AB)
			$2\overrightarrow{AX} = \overrightarrow{XC} \Rightarrow \frac{CX}{AC} = \frac{2}{3}$ $\frac{CX}{AC} = \frac{2}{3} = \frac{OC}{BA}$
			By SAS similarity test, triangle OCX is similar to triangle BAC
			OR
			$3\overrightarrow{OC} = 2\overrightarrow{AB} \Rightarrow \frac{OC}{BA} = \frac{2}{3}$
			$2\overrightarrow{AX} = \overrightarrow{XC} \Rightarrow \frac{CX}{AC} = \frac{2}{3}$
			$\overrightarrow{OX} = 2\mathbf{a} + 2\mathbf{c}$ $\overrightarrow{CB} = 3\mathbf{a} + 3\mathbf{c} = \frac{3}{2}(2\mathbf{a} + 2\mathbf{c}) = \frac{3}{2}\overrightarrow{OX} \Rightarrow \frac{OX}{BC} = \frac{2}{3}$
			By SSS similarity test, triangle OCX is similar to triangle BAC
			OR
			Given $3\overrightarrow{OC} = 2\overrightarrow{AB} \Rightarrow OC$ is parallel to AB angle OCX = angle BAC (alt $\angle$s, OC // AB)
			$\overrightarrow{OX} = 2\mathbf{a} + 2\mathbf{c}$

			$\overrightarrow{CB} = 3\mathbf{a} + 3\mathbf{c} = \frac{3}{2}(2\mathbf{a} + 2\mathbf{c}) = \frac{3}{2}\overrightarrow{OX}$ $\Rightarrow$ *CB* is parallel to *OX* angle *OXC* = angle *BCA* (alt $\angle$s, *OX* // *CB*)
			angle *XOC* = angle *CBA* ($\angle$ sum of triangles)
			By AAA similarity test, triangle *OCX* is similar to triangle *BAC*
5	(a)	1.17	
5	(b)		

5	(c)		Since the minimum value of $y=3x-1+\frac{10}{x+3}$ is at $y=0.95$ (accept $y=0.9$, $y=1$), hence <u>when $k<0.95$</u>, the <u>line $y=k$ does not intersect the graph</u> $y=3x-1+\frac{10}{x+3}$ and therefore $3x-1+\frac{10}{x+3}=k$ has no solution.
5	(d)		Draw tangent with gradient $\frac{1}{2}$
			$c=1.5$ (accept 1.4, 1.45, 1.55, 1.6)
5	(e)		$-\frac{9}{2}x+2=\frac{10}{x+3}$ $-\frac{3}{2}x+1=3x-1+\frac{10}{x+3}$
			Plot the line $y=-\frac{3}{2}x+1$
			$x=-2.15$ (accept -2.2, -2.1) $x=-0.4$ (accept -0.45, -0.35)
6	(a)	(i)	$\angle CAB=90°-59°$ $=31°$
			$\frac{\sin\angle ACB}{10}=\frac{\sin 31°}{24}$
			$\angle ACB=\sin^{-1}\left(\frac{10\sin 31°}{24}\right)$
			$\angle ACB=12.392$
			$\angle ACB=12.4°$
6	(a)	(ii)	$\angle ABC=180°-31°-12.392°=136.608°$
			Bearing of C from B $=136.608°-90°$
			$=046.6°$ (to 1 d.p.)

	(a)	(iii)	$24^2 = 35^2 + 46^2 - 2(35)(46)\cos\angle BDC$
			$\angle BDC = \cos^{-1}\left(\dfrac{24^2 - 35^2 - 46^2}{-2(35)(46)}\right)$
			$\angle BDC = 30.829°$
			Area of triangle BCD $= \dfrac{1}{2}(35)(46)\sin 30.829°$
			$= 412.55$
			$= 413$ m^2 (to 3 s.f.)
6	**(b)**	(i)	$\tan 49° = \dfrac{h}{35}$ $h = 35\tan 49° = 40.262$
			$h = 40.3$ m (to 3 s.f.)
6	**(b)**	(ii)	Shortest distance from B to line CD $= \dfrac{412.55}{\frac{1}{2}\times 46}$ $= 17.936$
			Let greatest angle of elevation be θ $\tan\theta = \dfrac{40.262}{17.936}$
			$\theta = 65.987$
			$\theta = 66.0°$ (to 1 d.p.)
7	**(a)**	Volume of cone = 2 × Volume of hemisphere $\dfrac{1}{3}\pi r^2(24) = 2\times\dfrac{2}{3}\pi r^3$	
		$24 = 4r$	
		$r = 6$	

	(b)	(i)	$\frac{x}{y}=\frac{6}{24}=\frac{1}{4}$ or $\left(\frac{x}{6}=\frac{y}{24}\right)$
	(b)	(ii)	$\frac{x}{y}=\frac{1}{4}\Rightarrow y=4x$
			$\frac{1}{3}\pi x^2 y=121.5\pi$
			$\frac{1}{3}\pi x^2(4x)=121.5\pi$
			$x^3=\frac{729}{8}$
			$x=\frac{9}{2}$
			Slant height of big cone $=\sqrt{24^2+6^2}$ $=\sqrt{612}$ Slant height of small cone $=\sqrt{18^2+\left(\frac{9}{2}\right)^2}$ $=\sqrt{344.25}$
			Curved surface area of remaining solid $=\pi(6)\left(\sqrt{612}\right)-\pi\left(\frac{9}{2}\right)\left(\sqrt{344.25}\right)$ $=204.01$
			Curve surface area hemisphere $=2\pi(6)^2$ $=72\pi$
			Total surface area $=72\pi+\pi\left(\frac{9}{2}\right)^2+204.01$
			$=494$ cm^3 (to 3 s.f.)

8	**(a)**	(i)	(a)	33.5 or 34
			(b)	$1000 - 840 = 160$ or 155
			(c)	$51 - 21 = 30$
8	**(a)**	(ii)	The median age for A = 33.5 and the median age for B = 31, thus age of people using sports centre is slightly older for A than B.	
			The IQR for A = 30 and the IQR for B = 42 – 21.5 = 20.5, thus there is wider spread about the median age for A than B.	
8	**(b)**	(i)	$\frac{25}{40}=\frac{5}{8}$	
		(ii)	(a)	$\frac{20}{60}\times\frac{19}{59}$
				$=\frac{19}{177}$
		(ii)	(b)	$\frac{15}{60}\times\frac{7}{59}+\frac{7}{60}\times\frac{15}{59}$
				$=\frac{7}{118}$
9	**(a)**	Volume of half sugar syrup in regular-sized bubble tea $=\frac{1}{2}\times 120$		
		$=60$		
		Amount of calories in regular-sized syrup $=\frac{60}{15}\times 50$		
		$=200$ calories		
9	**(b)**	Max sugar consumption $=\frac{10}{100}\times 2800 = 280$ calories		

9	(c)	Volume of regular-sized = 485 ml
		Volume of quarter sugar syrup $=\frac{1}{4}\times 120$
		$=30$ ml
		Calories in sugar $=\frac{50}{15}\times 30$
		$=100$ calories
		Calories in milk tea $=\frac{(485-30)}{500}\times 110$
		$=100.1$ calories
		Total calories $=100+100.1$
		$=200.1$
		Calories in the bubble tea $=200.1<280$, Yes, he **will** meet the healthy guidelines.
		Percentage exceeding $=\frac{200.1-140}{140}\times 100\%$
		$=42\frac{13}{14}\%$
		$42\frac{13}{14}\%>30\%$, he **will** exceed by 30%.