3EXP

KRANJI SECONDARY SCHOOL

END-OF-YEAR EXAMINATION 2018

ADDITIONAL MATHEMATICS
PAPER 1

Level : Secondary Three

Stream : Express

Name : ____________________________ (   )

Class : Sec __________

Date : 2 Oct 2018

Duration : 2 hours 30 minutes

Marks : / 100

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the writing papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Set by: **Mr Teh Yiyuan**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1. Express $\dfrac{3(x^2-2x+9)}{(x-3)(x^2+9)}$ in partial fractions. [5]

2. (a) Find the range of values of m for which the straight line $y = mx - 4$ intersects the curve $y = x^2$ at two distinct points. [4]

(b) The quadratic equation $2x^2 + 4x + 7 = 0$ has roots α and β. Find the quadratic equation whose roots are $\alpha + \dfrac{1}{\beta}$ and $\beta + \dfrac{1}{\alpha}$. [4]

3. (a) In the expansion of $(1-8x)^{12}$, the coefficient of x^4 is k times the coefficient of x^2. Evaluate k. [3]

(b) Find the term independent of x in the expansion $\left(x - \dfrac{1}{3x^2}\right)^{18}$. [3]

4. The polynomials $6x - x^3$ and $8 - 3x^2$ leave the same remainder when divided by $(x - m)$. Find the three possible values of m. [6]

5. **Answer the whole of this question on a piece of graph paper.**

The table shows experimental values of two variables x and y.

x	1	2	3	4	5
y	0.48	0.88	1.10	1.36	1.5

It is known that x and y are related by an equation of the form $\dfrac{p}{x} + \dfrac{q}{y} = 1$. By plotting $\dfrac{1}{y}$ against $\dfrac{1}{x}$, draw a straight line for the data given and use it to evaluate p and q. [7]

6. (a) On the same axes, sketch the graph of $y^2 = 2x$ and $y = x^5$. Hence, or otherwise, find the **exact** coordinates of the points of intersection of the two curves. [5]

(b) Calculate the coordinates of the points of intersection of the graph of $y = |3x-2| - 4$ with the coordinate axes. [3]

7. **(a)** A and B are acute angles such that $\sin(A+B)=\dfrac{2}{3}$ and $\cos A \sin B = \dfrac{1}{5}.$ Without using a calculator, find the value of

(i) $\sin A \cos B$, [2]

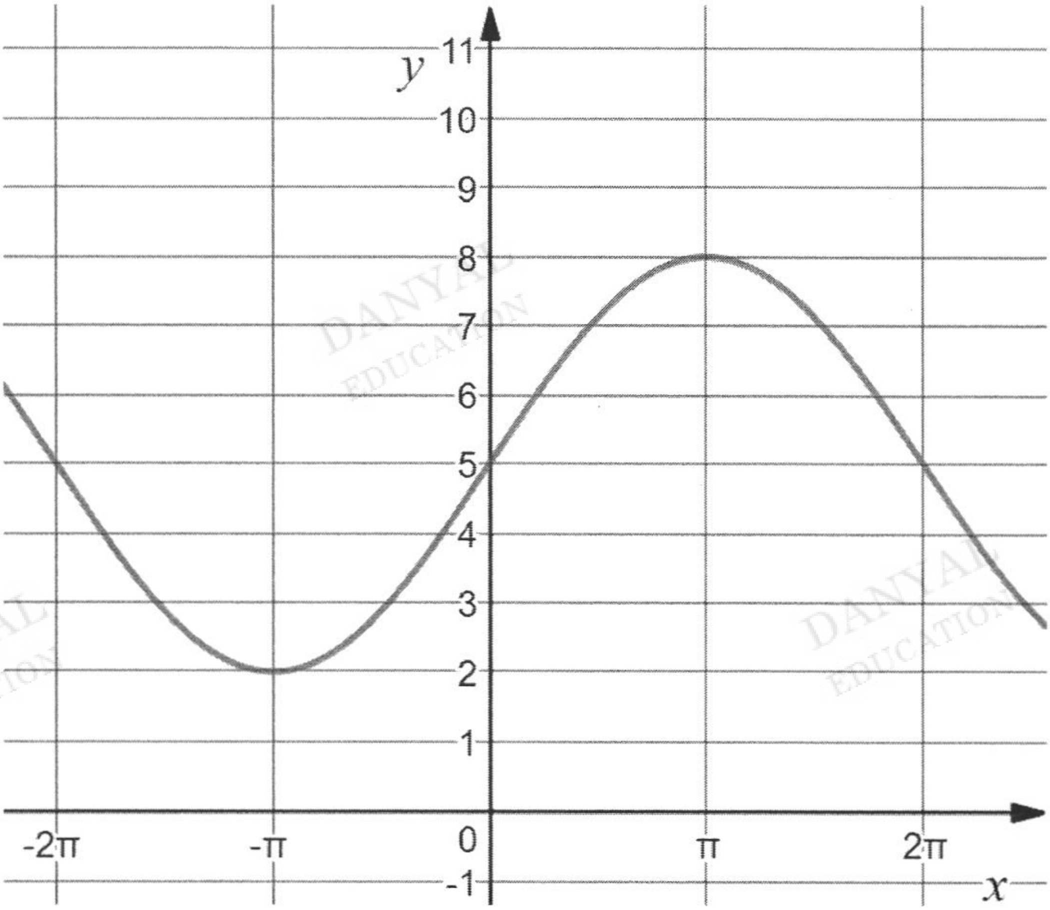
(ii) $\sin(A-B)$, [2]

(iii) $\dfrac{\tan A}{\tan B}$. [2]

(b) Prove that $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \cot x$. [4]

8. **(a)** State the values between which the principal value of $\sin^{-1} x$ must lie. [1]

(b)



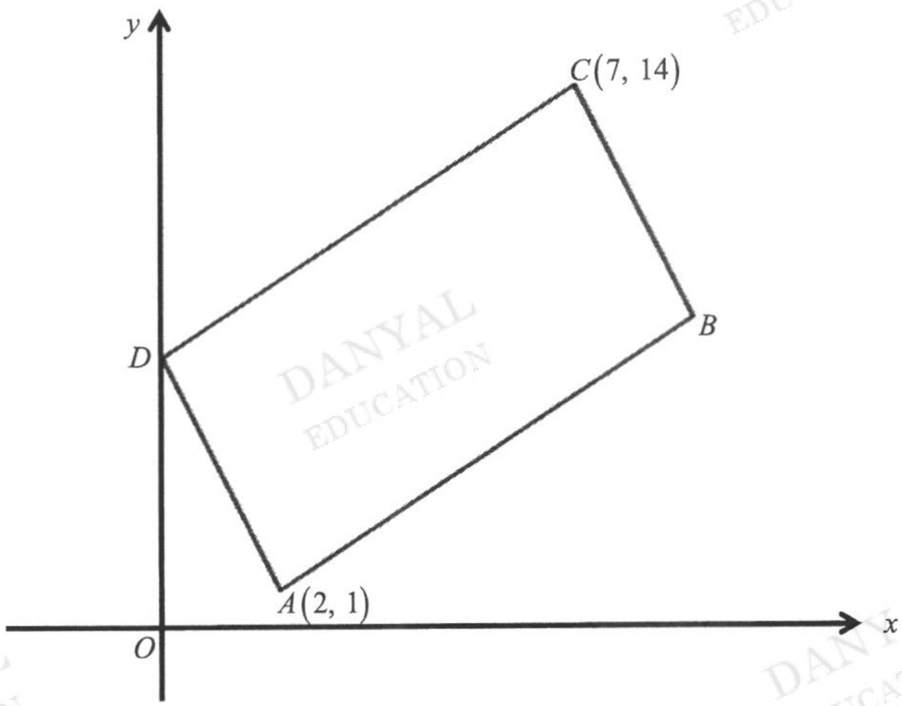
The figure shows part of the graph $y = a\sin\left(\dfrac{x}{b}\right) + c$. Find the value of each of the constants a, b and c. [3]

9. **(i)** Express $2\sin\theta - 3\cos\theta$ in the form of $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ radians. [3]

(ii) The amplitude, h decibels, of a small wave in an experiment is given by $h = 2\sin t - 3\cos t$, where t is the time in seconds after the start of the experiment. After how many seconds does the wave first reach a height of 2 decibels? [3]

10. Solve the equation $2\tan^2 y + 5\sec y - 1 = 0$ for $0 \le y \le 2\pi$. [5]

11.



The diagram shows a parallelogram $ABCD$ in which the coordinates of the points A and C are $(2,\ 1)$ and $(7,\ 14)$ respectively. Given that the point D lies on the y-axis and that the gradient of AD is -3, find

(a) the coordinates of B and of D, [5]

(b) the area of the parallelogram. [2]

12. **(a)** Express $3^{2x+1}-5\left(3^{x}\right)=2$ as a quadratic equation in 3^{x} and hence, find the values of x which satisfies the equation $3^{2x+1}-5\left(3^{x}\right)=2$. [5]

(b) Without using a calculator, find the values of the integers a and b for which the solution of the equation $5x=\sqrt{3}\left(5x-3\right)+1$ is $\dfrac{a+\sqrt{b}}{5}$. [5]

13. **(a)** The moment magnitude scale is a logarithmic function that is used to measure the magnitude of an earthquake. The moment magnitude M_W, is defined by the formula $M_W=\dfrac{2}{3}\log_{10}\left(M_0\right)-10.7$, where M_0 is the seismic moment measured during the earthquake.

(i) Find the value of M_W when $M_0=4.6\times10^{25}$. [1]

On 18th June 2018, an earthquake measuring 5.5 on the moment magnitude scale struck Osaka, Japan.

(ii) Find the seismic moment measured. [3]

(b) Solve the equation $\ln\left(1-2x\right)-1=2\ln x-\ln\left(2-5x\right)$, leaving your answer to 1 decimal place. [5]

14. The positive x- and y-axes are tangents to a circle C.

(i) What can be deduced about the coordinates of the centre of C. [1]

The line T is tangent to C at the point $\left(8,\ 1\right)$ on the circle. Given that the centre of C lies above and to the right of $\left(8,\ 1\right)$, find

(ii) the equation of C, [5]

(iii) the equation of T. [3]

END OF PAPER

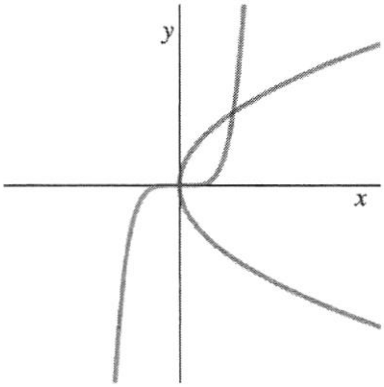
Secondary 3 Express End of Year Examinations
Additional Mathematics Marking Scheme

Question	Solution
1	$\frac{3(x^2-2x+9)}{(x-3)(x^2+9)}=\frac{A}{x-3}+\frac{Bx+c}{(x^2+9)}$ $3(x^2-2x+9)=A(x^2+9)+(Bx+c)(x-3)$ When $x=3$, $A=2$ When $x=0$, $C=-3$ When $x=1$, $B=1$ $\frac{3(x^2-}{(}$

2a	$y = mx - 4$ $y = x^2$ $x^2 = mx - 4$ $x^2 - mx + 4 = 0$ Two distinct roots, $D > 0$, $m^2 - 4(1)(4) > 0$ $(m-4)(m+4) > 0$ $m < -4$ or $m > 4$
2b	$2x^2 + 4x + 7 = 0$ $\alpha + \beta = -2$ $\alpha\beta = \frac{7}{2}$ $\alpha + \frac{1}{\beta} + \beta + \frac{1}{\alpha} = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta}$ $= -2 + \frac{-2}{7/2}$ $= -\frac{18}{7}$ $\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \alpha\beta + 2 + \frac{1}{\alpha\beta}$ $= \frac{7}{2} + 2 + \frac{2}{7}$ $= \frac{81}{14}$ $x^2 + \frac{18}{7}x + \frac{81}{14} = 0$ $14x^2 + 36x + 81 = 0$

3(a)	$(1-8x)^{12} = 1-96x+4224x^2-112640x^3+2027520x^4+\ldots$ $k = \dfrac{2027520}{4224}$ $= 480$
3(b)	$T_{r+1} = \begin{pmatrix} 18 \\ r \end{pmatrix} (x)^{18-r} \left(-\dfrac{1}{3x^2}\right)^r$ $18-r-2r=0$ $r=6$ $T_{6+1} = \begin{pmatrix} 18 \\ 6 \end{pmatrix} (x)^{18-6} \left(-\dfrac{1}{3x^2}\right)^6$ $= \dfrac{6188}{243}$ Independent term is $\dfrac{6188}{243}$

4	$f(x)=6x-x^3$ $g(x)=8-3x^2$ $f(m)=g(m)$ $6m-m^3=8m-3m^2$ $m^3-3m^2-6m+8=0$ $h(m)=m^3-3m^2-6m+8$ $h(1)=1-3-6+8$ $=0$ $\therefore (m-1)$ is a factor of $h(m)$ $h(m)=(m-1)(m^2+km-8)$ by comparing coefficient, $k=-2$ $h(m)=(m-1)(m^2-2m-8)$ $=(m-1)(m+2)(m-4)$ $=0$ $\therefore m=-2,\ 1,\ 4$
5	Refer to graph paper

6a	 $y^2 = 2x$ $y = x^5$ $x^{10} = 2x$ $x(x^9 - 2) = 0$ $x = 0$ or $2^{\frac{1}{9}}$ $y = 0$ or $2^{\frac{5}{9}}$ $(0,\ 0)$ & $\left(2^{\frac{1}{9}},\ 2^{\frac{5}{9}}\right)$
6b	$y = \lvert 3x - 2 \rvert - 4$ When $x = 0$, $y = -2$ When $y = 0$, $4 = \lvert 3x - 2 \rvert$ $3x - 2 = 4$ or $-(3x - 2) = 4$ $x = 2$ or $-\frac{2}{3}$ The intercepts are $(0,\ -2)$, $(2,0)$, $(-\frac{2}{3},0)$

7ai	$\sin(A+B)=\sin A\cos B+\cos A\sin B$ $\frac{2}{3}=\sin A\cos B+\frac{1}{5}$ $\sin A\cos B=\frac{7}{15}$
7aii	$\sin(A-B)=\sin A\cos B-\cos A\sin B$ $=\frac{7}{15}-\frac{1}{5}$ $=\frac{4}{15}$
7aiii	$\frac{\tan A}{\tan B}=\frac{\sin A}{\cos A}\div\frac{\sin B}{\cos B}$ $=\frac{\sin A\cos B}{\cos A\sin B}$ $=\frac{\frac{7}{15}}{\frac{1}{5}}$ $=\frac{7}{3}$
7b	$LHS=(\sec x-\tan x)(\operatorname{cosec} x+1)$ $=\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}+1\right)$ $=\frac{1}{\sin x\cos x}+\frac{1}{\cos x}-\frac{1}{\cos x}-\frac{\sin x}{\cos x}$ $=\frac{1-\sin^2 x}{\sin x\cos x}$ $=\frac{\cos^2 x}{\sin x\cos x}$ $=\frac{\cos x}{\sin x}$ $=\cot x$ $=RHS$ (proven)

8a	$-\dfrac{\pi}{2} \le \sin^{-1} x \le \dfrac{\pi}{2}$
8b	$a = 3$ $b = 2$ $c = 5$
9i	$2\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$ $R = \sqrt{2^2 + 3^2}$ $= \sqrt{13}$ $\alpha = \tan^{-1}\dfrac{3}{2}$ $= 0.98279$ $2\sin\theta - 3\cos\theta = \sqrt{13}\sin(\theta - 0.983)$
9ii	$h = 2\sin t - \cos t$ $= \sqrt{13}\sin(t - 0.98279)$ $= 2$ $\sin(t - 0.98279) = \dfrac{2}{\sqrt{13}}$ $t - 0.98279 = 0.58800$ $t = 1.57$ seconds

10	$2\tan^2 y+5\sec y-1=0$ $2(\sec^2 y-1)+5\sec y-1=0$ $2\sec^2 y+5\sec y-3=0$ $(\sec y+3)(2\sec y-1)=0$ $\sec y=-3$ or $\frac{1}{2}$ $\cos y=-\frac{1}{3}$ or 2 (rejected) Basic angle $=\cos^{-1}\frac{1}{3}$ $=1.23096$ $y=\pi-1.23096, \pi+1.23096$ $y=1.91,\ 4.37$

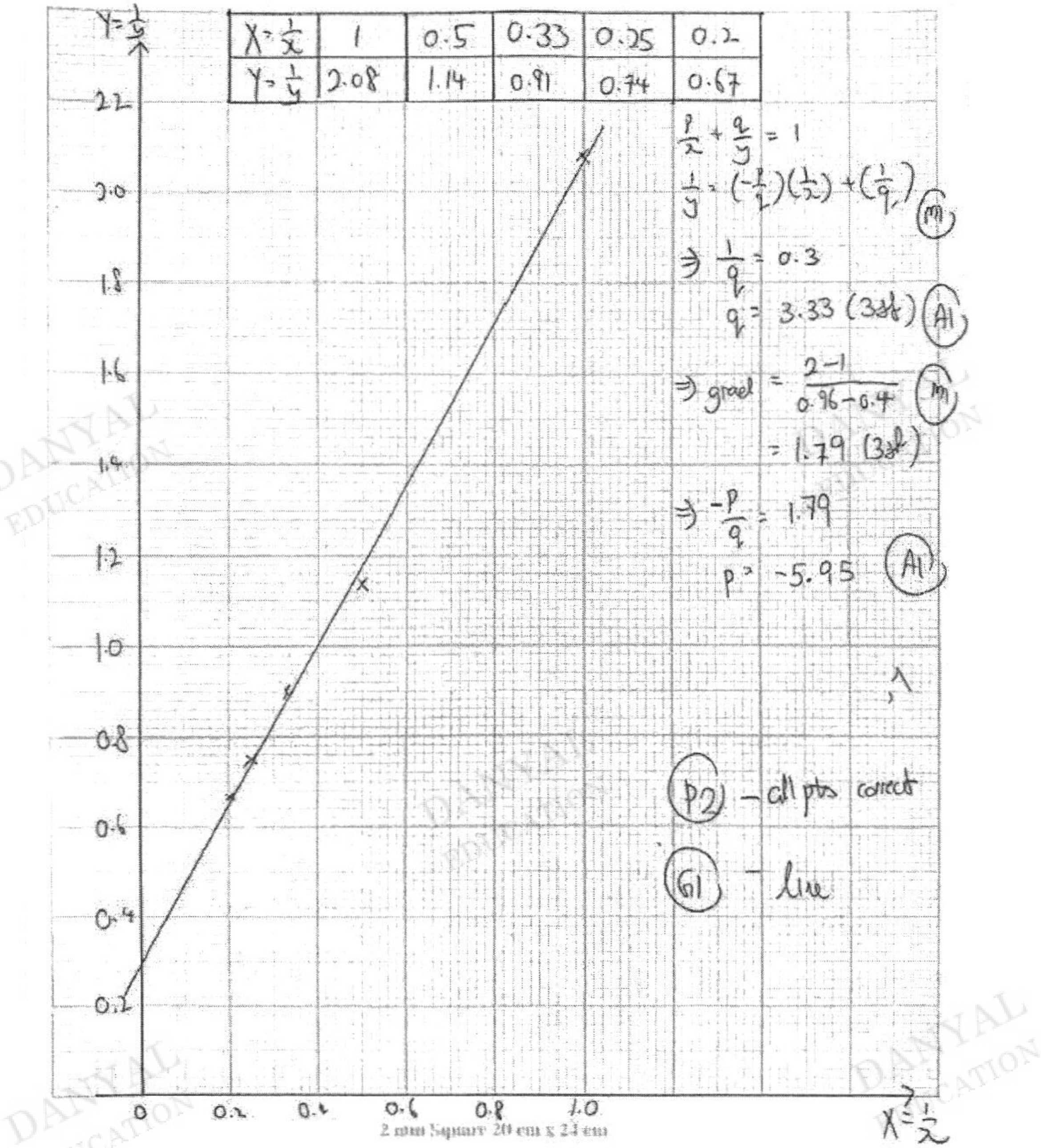
11a	$D(0,d)$ Gradient $AD = \frac{d-1}{0-2} = -3$ $d = 7$ $\therefore D(0,7)$ $B(p,q)$ Midpoint AC = Midpoint BD $\left(\frac{2+7}{2}, \frac{1+14}{2}\right) = \left(\frac{0+p}{2}, \frac{7+q}{2}\right)$ $p = 9$ $q = 8$ $B(9,8)$ Possible to solve by find equation of BC (M1) and equation of CD (M1) and get answer by solving simultaneously. (A1)
11b	$Area = \frac{1}{2}\begin{vmatrix} 2 & 9 & 7 & 0 & 2 \\ 1 & 8 & 14 & 7 & 1 \end{vmatrix}$ $= 56$

12a	$3^{2x+1}-5\left(3^x\right)=2$ $3\left(3^{2x}\right)-5\left(3^x\right)-2=0$ $u=3^x$ $3u^2-5u-2=0$ $(3u+1)(u-2)=0$ $u=-\frac{1}{3}$ or $u=2$ $3^x=-\frac{1}{3}(NA)$ or $3^x=2$ $x\lg 3=\lg 2$ $x=0.631$
12b	$5x=\sqrt{3}\left(5x-3\right)+1$ $(5-5\sqrt{3})x=1-3\sqrt{3}$ $x=\frac{1-3\sqrt{3}}{5-5\sqrt{3}}$ $=\frac{(1-3\sqrt{3})(5+5\sqrt{3})}{(5-5\sqrt{3})(5+5\sqrt{3})}$ $=\frac{-40-10\sqrt{3}}{-50}$ $=\frac{4+\sqrt{3}}{5}$ $a=4$ $b=3$

13i	$M_W=\frac{2}{3}\lg(4.6\times 10^{25})-10.7$ $=6.41$

13ii	$5.5=\frac{2}{3}\lg(M_0)-10.7$ $\lg(M_0)=24.3$ $M_0=10^{24.3}$ $=1.99526\times10^{24}$ $=2.00\times10^{24}$
13b	$\ln(1-2x)-1=2\ln x-\ln(2-5x)$ $\ln(1-2x)-\ln e=\ln x^2-\ln(2-5x)$ $\ln\frac{1-2x}{e}=\ln\frac{x^2}{2-5x}$ $\frac{1-2x}{e}=\frac{x^2}{2-5x}$ $(1-2x)(2-5x)=ex^2$ $(10-e)x^2-9x+2=0$ $x=\frac{-(-9)\pm\sqrt{(-9)^2-4(10-e)(2)}}{2(10-e)}$ $=0.3$ or 0.9 (rejected) $=0.3$

14i	The values of the x and y coordinates are the same.
14ii	Centre is on the line $y = x$, Let centre of C be (a, a), $(x-a)^2 + (y-a)^2 = a^2$ $At\ (8,1)$, $(8-a)^2 + (1-a)^2 = a^2$ $64 - 16a + a^2 + 1 - 2a + a^2 = a^2$ $a^2 - 18a + 65 = 0$ $(a-13)(a-5) = 0$ $a = 13$ or $a = 5$ (NA) Equation of circle, $(x-13)^2 + (y-13)^2 = 13^2$
13iii	Gradient of centre to $(8,1) = \frac{13-1}{13-8}$ $= \frac{12}{5}$ Gradient tangent $= -\frac{5}{12}$ Equation of T, $y - 1 = -\frac{5}{12}(x-8)$ $y = -\frac{5}{12}x + \frac{13}{3}$



3pts correct give P1