3Exp
Session 2

KRANJI SECONDARY SCHOOL

END-OF-YEAR EXAMINATION 2021
ADDITIONAL MATHEMATICS 4049

Level : Secondary Three
Stream : Express
Name : ______________________________ (   )
Class : Secondary ___________

Date : 29 Sep 2021
Duration : 2 hr 15 min
Marks : / 90

READ THESE INSTRUCTIONS FIRST:

Do not open this question paper until you are told to do so.

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

Give non-exact numerical answers correct to three significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

Question	Marks
1	
2	
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11	
12	
13	
Total	

Set by: Ms Cynthia Wong

This Question Paper consists of 20 printed pages, including the cover page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 State the principal value of

(i) $\cos^{-1}\left(\frac{1}{2}\right)$ in degrees, [1]

(ii) $\tan^{-1}\left(-\sqrt{3}\right)$ in radians as a multiple of π. [1]

2 Solve, for x and y, the simultaneous equations

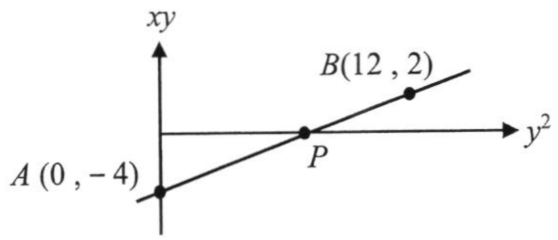
$$3^x\left(9^y\right)=1,$$

$$\left(\sqrt{2}\right)^x \div 16^y = 32.$$ [4]

3 **(a)** Clearly labelling each graph, sketch, on the same axes, the graphs of $y^2 = 8x$ and $y = 2^x$ for $x > 0$. [2]

(b) Express $\dfrac{8}{x\left(x^2+4\right)}$ in partial fraction. [4]

4. The graph shows part of a straight line graph drawn to represent the equation $ay = x + \frac{b}{y}$.



(i) Calculate the value of a and of b. [4]

(ii) Find the coordinates of point P, given that point P lies on the horizontal axis. [2]

5. The heights, in metres, of tides are recorded at a particular beach over time t hours.

It is found that the height, in metres, is given by $H(t) = 1.2\sin 2t + 1.4$.

(i) State the highest and lowest value of $H(t)$. [2]

(ii) State the period of $H(t)$. [1]

(iii) Find the height of the tide when $t = 5$ hours. [1]

(iv) Sketch the graph of $y = H(t)$ for $0 \le t \le 2\pi$. [2]

6. **(a)** Show that $\cos A\left(\dfrac{1}{1+\sin A}-\dfrac{1}{1-\sin A}\right)$ can be written in the form of $k\tan A$.

Find the value of k. [3]

(b) Given that θ is acute and $\sin\theta = c$, express in terms of c,

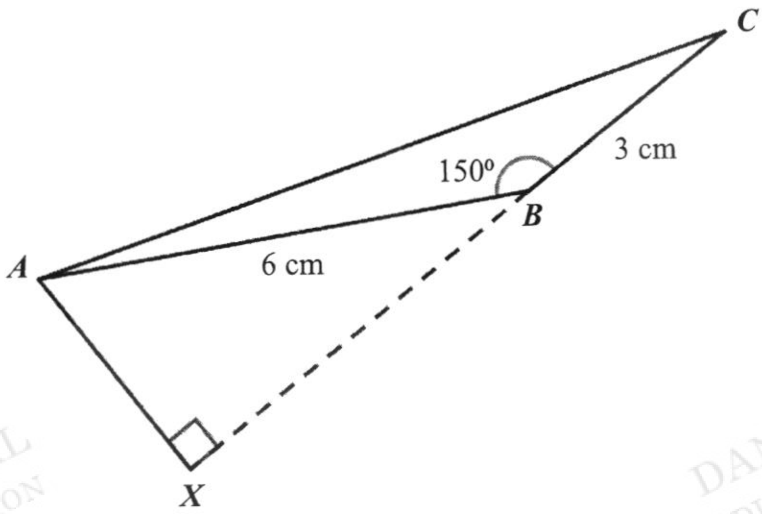
(i) $\sin(90°-\theta)$, [1]

(ii) $\operatorname{cosec}\theta$, [1]

(iii) $\tan\theta$. [2]

7. **(a)** Solve the equation $\sin 2x = \cos 2x$ for $0° \le x \le 360°$. [3]

(b) The diagram shows a triangle ABC in which $AB = 6$ cm, $BC = 3$ cm and the angle $ABC = 150°$.
The line CB is extended to the point X where angle $AXB = 90°$.



(i) Find the exact length of AX. [2]

(ii) Show that angle $ACB = \tan^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$. [3]

8. **(a)** Find the term in x^7 in the expansion of $\left(x^2+\frac{1}{x}\right)^8$. [3]

(b) **(i)** Write down and simplify the first three terms in the expansion, in ascending powers of x, of $\left(1-\frac{x}{2}\right)^7$. [2]

(ii) In the expansion of $\left(4+kx+2x^2\right)\left(1-\frac{x}{2}\right)^7$, there is no term in x.

Find the value of the constant k. [2]

9. **(a)** It is given $2x^3 - 10x^2 + 12x - 27 = Ax(x-1)(x-4) + B(x-4) + C$ for all values of x,

(i) Find the values of A, of B and of C. [3]

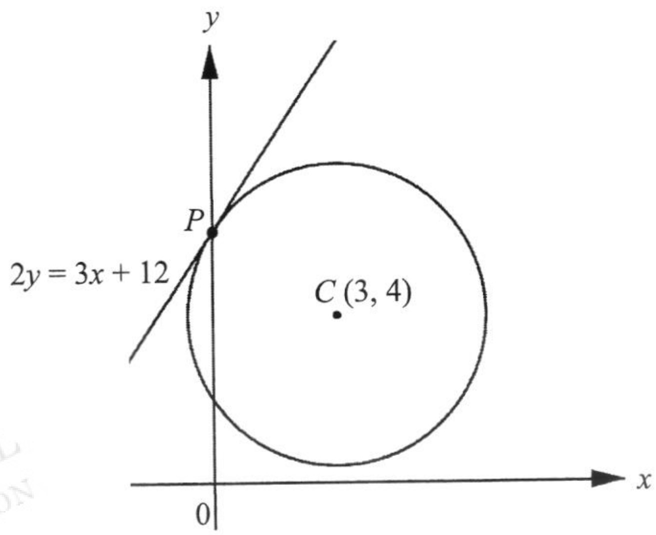
(ii) **Hence**, state the remainder when $2x^3 - 10x^2 + 12x - 27$ is divided by $x^2 - x$.

[1]

(b) When the expression x^3+3x^2-2x+c is divided by $x-2$, the remainder is R. When the expression is divided by $x+2$, the remainder is $2R$.

Evaluate c. [4]

10. The diagram below shows a circle with centre (3, 4) and the equation of tangent at point P of the circle is $2y = 3x + 12$.



(a) Find the coordinates of point P. [1]

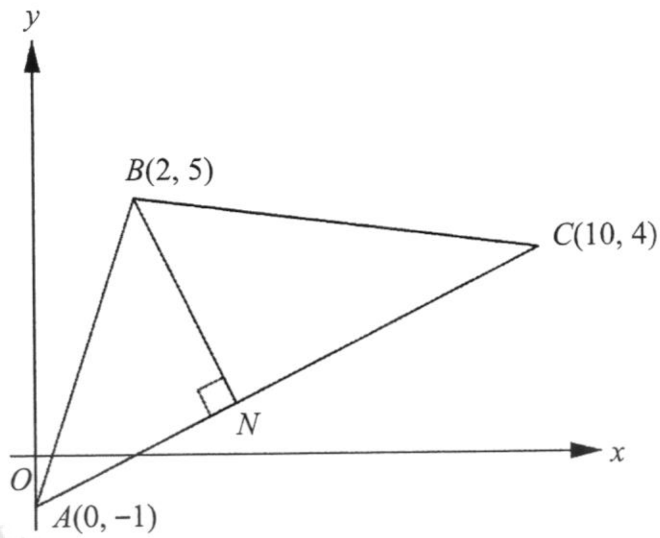
(b) Find the equation of the circle. [2]

(c) Find the coordinates of the point Q, given that PQ is the diameter of the circle. [1]

(d) Determine whether the tangent to the circle at point Q is parallel to the tangent to the circle at point P.

[2]

11.



The diagram shows a triangle ABC with the vertices $A(0, -1)$, $B(2, 5)$ and $C(10, 4)$. The point N lies on the line AC such that BN is perpendicular to AC.

(i) Find the gradient of line AC.

Hence, or otherwise, find the angle that line AC makes with the positive x-axis. [3]

(ii) Find the coordinates of N. [5]

(iii) Find, in the form $n : 1$, the ratio of the area of triangle ABN to the area of triangle CBN. [2]

12. **(a)** Without using a calculator, show that $(\log_2 3)(\log_9 8) = \frac{3}{2}$. [2]

(b) Solve the following:

(i) $\log_m(12+m) = 2$, [2]

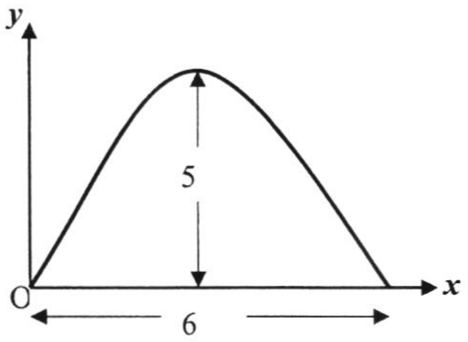
(ii) $\lg(x+8)-\lg 2=1+\lg\left(\dfrac{x}{4}\right).$ [3]

(c) The number of bacteria in a culture doubled every hour.
It is given that N_0 is the number of bacteria present at a particular time and that N is the number of bacteria present t hours later.
Calculate the value of the constant k in the relationship $N=N_0e^{kt}$, giving your answer correct to 2 decimal places. [3]

13. **(a)** Solve the inequality $x^2 - x - 12 < 0$ and represent the solution set on the number line. [3]

(b) The equation of a curve is $y = 2x^2 + cx + 1$, where c is a constant, and the equation of a line is $y + x = 5$.
Show that, for all values of c, the line intersects the curve at two distinct points. [3]

(c) An arched tunnel has the shape of a parabola as shown.



In the diagram, x m is the horizontal distance from one end of the arch and y m is the height of the arch.

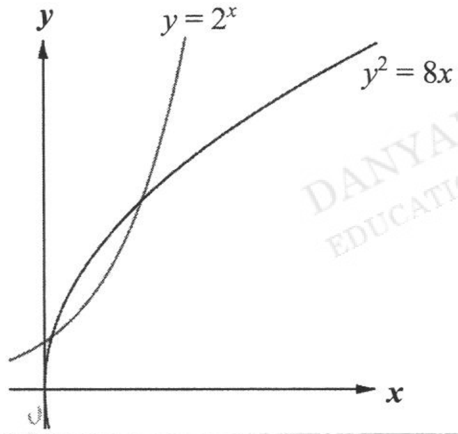
The one-way tunnel passing under the arch is 6 m wide and the maximum height of the arch is 5 m.

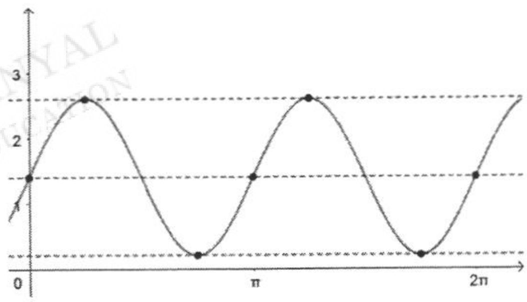
(i) Express the function in the form $y = a(x-h)^2 + k$. [2]

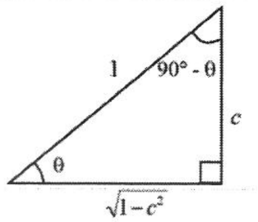
(ii) A tour bus is approximately 12 m long, 4 m tall and 3 m wide. Explain whether the tour bus can pass through the tunnel [2]

- End of Paper -

3E AM SA 2 2021 (Solution for students)

1	**(i)**	60^o
	(ii)	$-\frac{\pi}{3}$
2		$3^x(9)^y=1$ $3^x \times 3^{2y}=3^0$ $x+2y=0 \quad --(1)$ **or** $\left(\sqrt{2}\right)^x \div 16^y=32$ $2^{\frac{1}{2}x} \div 2^{4y}=2^5$ $\frac{1}{2}x-4y=5 \quad --(2)$ Sub. $x=-2y$ into eq (2): $\frac{1}{2}(-2y)-4y=5$ $x=2 \quad y=-1$
3	**(a)**	
	(b)	$\frac{8}{x\left(x^2+4\right)}=\frac{A}{x}+\frac{Bx+C}{x^2+4}$ $8=A\left(x^2+4\right)+\left(Bx+C\right)\left(x\right)$ Comparing coefficient of x : $C=0$ Comparing constant : $4A=8 \quad A=2$ Comparing coefficient of x^2: $0=A+B$ $B=-A=-2$ $\frac{8}{x\left(x^2+4\right)}=\frac{2}{x}-\frac{2x}{x^2+4}$

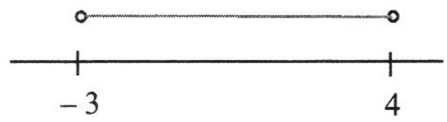
4	**(i)**	Gradient of line = $\frac{2-(-2)}{12}=\frac{1}{2}$ Y- intercept = –4 Equation of line: $Y = mX + c$ $xy=\frac{1}{2}y^2-4$ $\frac{1}{2}y^2=xy+4$ $\frac{1}{2}y=x+\frac{4}{y}$ $a=\frac{1}{2}$ and $b=4$
	(ii)	Gradient = $\frac{2-(-4)}{12}=\frac{1}{2}$ Since points A, P and B are collinear and at x-axis, $y = 0$, let P be $(x_P, 0)$: $\frac{0-(-4)}{x_p-0}=\frac{1}{2}$ $x_P = 8$; hence P is (8, 0)
5	**(i)**	Highest value = $1.2(1)+1.4=2.6$ m Lowest = $1.2(-1)+1.4=0.2$ m
	(ii)	π
	(iii)	$H(t)=1.2\sin 2(5)+1.4=0.747175$ = 0.747 m (3 s.f)
	(iv)	

6	(a)		$\cos A\left(\dfrac{1}{1+\sin A}-\dfrac{1}{1-\sin A}\right)$ $=\cos A\left[\dfrac{1-\sin A-(1+\sin A)}{1-\sin^2 A}\right]=\dfrac{\cos A}{\cos^2 A}(-2\sin A)=-2\tan A$ Hence, $k=-2$
	(b)		
	(i)	(i)	$\sin(90°-\theta)=\cos\theta=\sqrt{1-}$
		(ii)	$\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}=\dfrac{1}{c}$
		(iii)	$\tan\theta=\dfrac{c}{\sqrt{1-c^2}}$
7	(a)		$\sin 2x=\cos 2x$ $\tan 2x=1$ Basic angle $=45^{o}$ $2x=45°,\ 225°, 405°, 585°$ $x=22.5°,\ 112.5°, 202.5°, 292.5°$
	(b)	(i)	$\sin 30°=\dfrac{AX}{6}$ $AX=3$ cm
		(ii)	For triangle ABX: $XB=\sqrt{6^2-3^2}=\sqrt{27}=\sqrt{}$ $\tan\angle ACB=\dfrac{3}{3\sqrt{3}+3}$ $\tan\angle ACB=\dfrac{}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-}{\sqrt{3}-1}$ $\angle ACB=\tan^{-1}\left(\dfrac{\sqrt{3}-1}{2}\right)$

8	**(a)**		To find term in x^7: $$\binom{8}{r}\left(x^2\right)^{8-r}\left(\frac{1}{x}\right)^r = x^7$$ $$16-2r-r=7$$ $$r=3$$ Term in $x^7 = \binom{8}{3}\left(x^2\right)^{8-3}\left(\frac{1}{x}\right)^3 = 56x^7$
	(b)	**(i)**	$\left(1-\frac{x}{2}\right)^7 = 1-\frac{7}{2}x+\frac{21}{4}x^2+\ldots$
		(ii)	$\left(4+kx+2x^2\right)\left(1-\frac{x}{2}\right)^7$ $$=\left(4+kx+2x^2\right)\left(1-\frac{7}{2}x+\frac{21}{4}x^2+\ldots\right)$$ Coefficient of x term: $4\left(-\frac{7}{2}\right)+k=0$ $$k=14$$
9	**(a)**	**(i)**	Sub. $x=4$: $C=-11$ Sub. $x=0$: $-27=-4B-11$ $B=4$ Comparing coefficient of x^3: $A=2$
		(ii)	Remainder = $4(x-4)-11=4x-27$
	(b)		Let $f(x)=x^3+3x^2-2x+c$ When divided by $x-2$, $f(2)=(2)^3+3(2)^2-2(2)+c=R$ $$c=R-16 \text{ -- (1)}$$ When divided by $x+2$, $$f(-2)=(-2)^3+3(-2)^2-2(-2)+c=2R$$ $$c=2R-8 \text{ -- (2)}$$ Solving eq (1) & eq (2): $R=-8$ $$c=R-16=-8-16=-24$$

10	**(a)**	At y–axis, sub. $x = 0$: $y = 6$ P is (0, 6)
	(b)	Radius = $CP = \sqrt{(\qquad -4)^2} = \sqrt{13}$ Hence equation of circle with centre (3, 4): $(x-3)^2 + (y-4)^2 = \left(\sqrt{13}\right)^2$ $(x-3)^2 + (y-4)^2 = 13$
	(c)	Midpoint of PQ = centre of circle = (3, 4) $\left(\frac{0+x_Q}{2}, \frac{6+y_Q}{2}\right) = (3, 4)$ Q is (6, 2)
	(d)	Since <u>$PC = CQ$ = radius of circle</u>, and both the tangents to the circle at point P and Q are perpendicular to the radius (tangent $\perp$ radius), hence the tangents have the <u>same gradient</u>.
11	**(i)**	$m_{AC} = \frac{4-(-1)}{10-0} = \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26.5651°$ AC makes an angle of 26.6° (1 dp) with the positive x-axis.
	(ii)	Equation of AC: $y = \frac{1}{2}x - 1$ -- (1) $m_{BN} \times m_{AC} = -1$ $m_{BN} = -2$ Equation of BN: $y - 5 = -2(x-2)$ $y = -2x + 9$ -- (2) Solving (1) & (2): $x = 4$; $y = 1$ Hence, N is (4, 1)
	(iii)	Area of $\Delta ABN = \begin{vmatrix} 0 & 4 & 2 & 0 \\ -1 & 1 & 5 & -1 \end{vmatrix} =$ 10 units² Area of $\Delta CBN = \begin{vmatrix} 4 & 10 & 2 & 4 \\ 1 & 4 & 5 & 1 \end{vmatrix} =$ 15 units² Hence ratio of area = 10 : 15 = $\frac{2}{3}$: 1

12	(a)		$(\log_2 3)(\log_9 8) = \log_2 3 \times \frac{3\log_2 2}{2\log_2 3} = \frac{3}{2}$ (shown)
	(b)	(i)	$\log_m(12+m) = 2$ $12+m = m^2$ $m^2 - m - 12 = 0$ $(m+3)(m-4) = 0$ Since $m > 0$, $m = 4$
		(ii)	$\lg(x+8) - \lg 2 = 1 + \lg\left(\frac{x}{4}\right)$ $\lg(x+8) - \lg 2 = 1g10 + \lg\left(\frac{x}{4}\right)$ $\lg\frac{x+8}{2} = \lg\left(\frac{10x}{4}\right)$ $4x + 32 = 20x$ $x = 2$
	(c)		When $t=0$: $N = N_0$ When $t=1$: $N = 2N_0$ $N_0 e^{k(1)} = 2N_0$ $e^{k(1)} = 2$ $k \ln e = \ln 2$ $k = \ln 2 = 0.69\ (2\text{ dp})$

13	**(a)**		$x^2-x-12<0$ $(x-4)(x+3)<0$ $-3<x<4$ 
	(b)		$y=2x^2+cx+1$ -- (1) & $y=5-x$ -- (2) Sub (1) into (2): $2x^2+cx+1=5-x$ $2x^2+(c+1)x-4=0$ $b^2-4ac=(c+1)^2-4(2)(-4)=(c+1)^2+32$ Since $(c+1)^2\geq 0$, $b^2-4ac=(c+1)^2+32>0$, thus line intersects curve at two distinct points.
	(c)	**(i)**	Max point is (3, 5), $y=a(x-3)^2+5$ Since at $x=0$ m, $y=0$ m: $y=a(x-3)^2+5$ $\quad a=-\frac{5}{9}$ Hence, the function is $y=-\frac{5}{9}(x-3)^2+5$
		(ii)	When the width of the bus is 3 m, $x=3-\frac{3}{2}=1.5$ m $y=3.75$ m < 4 m Since the height of the tour bus, 4 m > permissible height of 3.75 m, it cannot pass through the tunnel.