

NAME: _____ ()

MARKS: _____ /90

CLASS: _____

**ADDITIONAL MATHEMATICS****MONDAY****3 OCTOBER 2022****2 HOUR 15 MINUTES**

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JUYING SECONDARY SCHOOL**END-OF-YEAR EXAMINATION****SECONDARY THREE EXPRESS****Instructions to students:**

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Read these notes carefully.

1. Answer **all** questions.
2. Write in dark blue or black pen in the spaces provided.
3. You may use an HB pencil for any diagrams or graphs.
4. Omission of essential working will result in loss of marks.
5. The use of an approved calculator is expected, where appropriate.
6. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
7. The number of marks is given in brackets [] at the end of each question or part question.
8. The total number of marks for this paper is 90.

This question paper consists of 19 printed pages, including this page.

Setter: Ms Yeo Yee Teng
 Vetter: Mdm Norhafiani A Majid

1. Solve $\sqrt{7-6x} + 2 = x$.

[4]

2. The curved surface area of a cylinder is $(6\sqrt{3} + 16)\pi \text{ cm}^2$. The radius of the base of the cylinder is $(1 + \sqrt{3}) \text{ cm}$. Find, without using a calculator, the height of the cylinder in the form $a + b\sqrt{3}$, where a and b are rational numbers. Show your steps clearly. [4]

3. Solve the simultaneous equations

[5]

$$3^y = \frac{1}{3}(3^{x-7}) ,$$

$$\log_9(14 - 2y) - \log_9 x = 0.5 .$$

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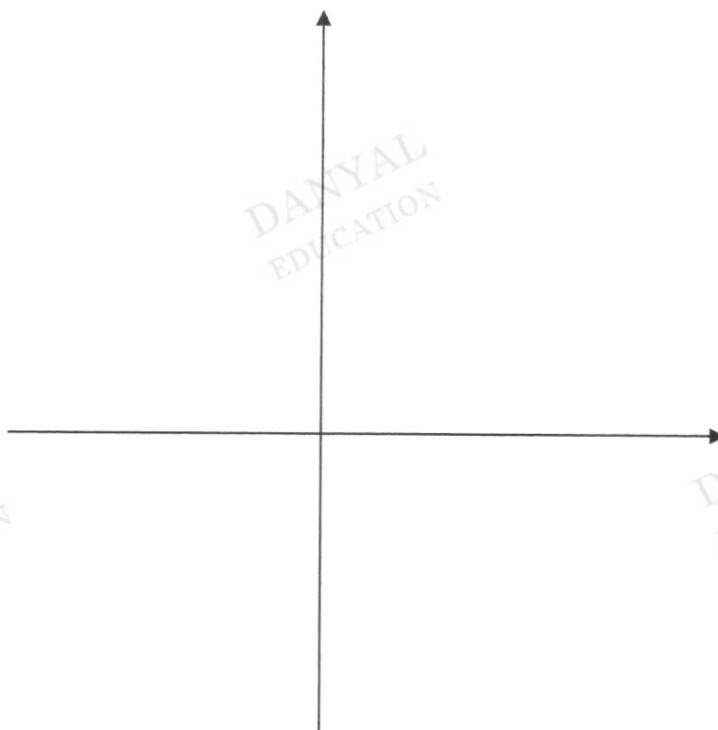
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4. (a) Express $y = -3x^2 + 6x + 24$ in the form of $y = a(x + b)^2 + c$, where a , b and c are constants. [2]

- (b) Sketch the graph of $y = -3x^2 + 6x + 24$. Label the turning point and y -intercept. [2]



- (c) Hence, explain why y cannot exceed 27. [1]

5. The cubic polynomial $f(x) = 3x^3 + hx^2 - 14x + k$ leaves a remainder of 40 when divided by $x - 3$ and it is exactly divisible by $x + 2$.

(a) Show that $h = 1$ and $k = -8$.

[4]

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(b) Hence, solve the cubic equation $f(x) = 0$.

[4]

6. Find the term independent of x in the expansion of $\left(x^3 - \frac{3}{x}\right)^8$.

[3]

7. (a) Write down and simplify the first three terms in the expansion of $(2 - 3x)^6$. [2]

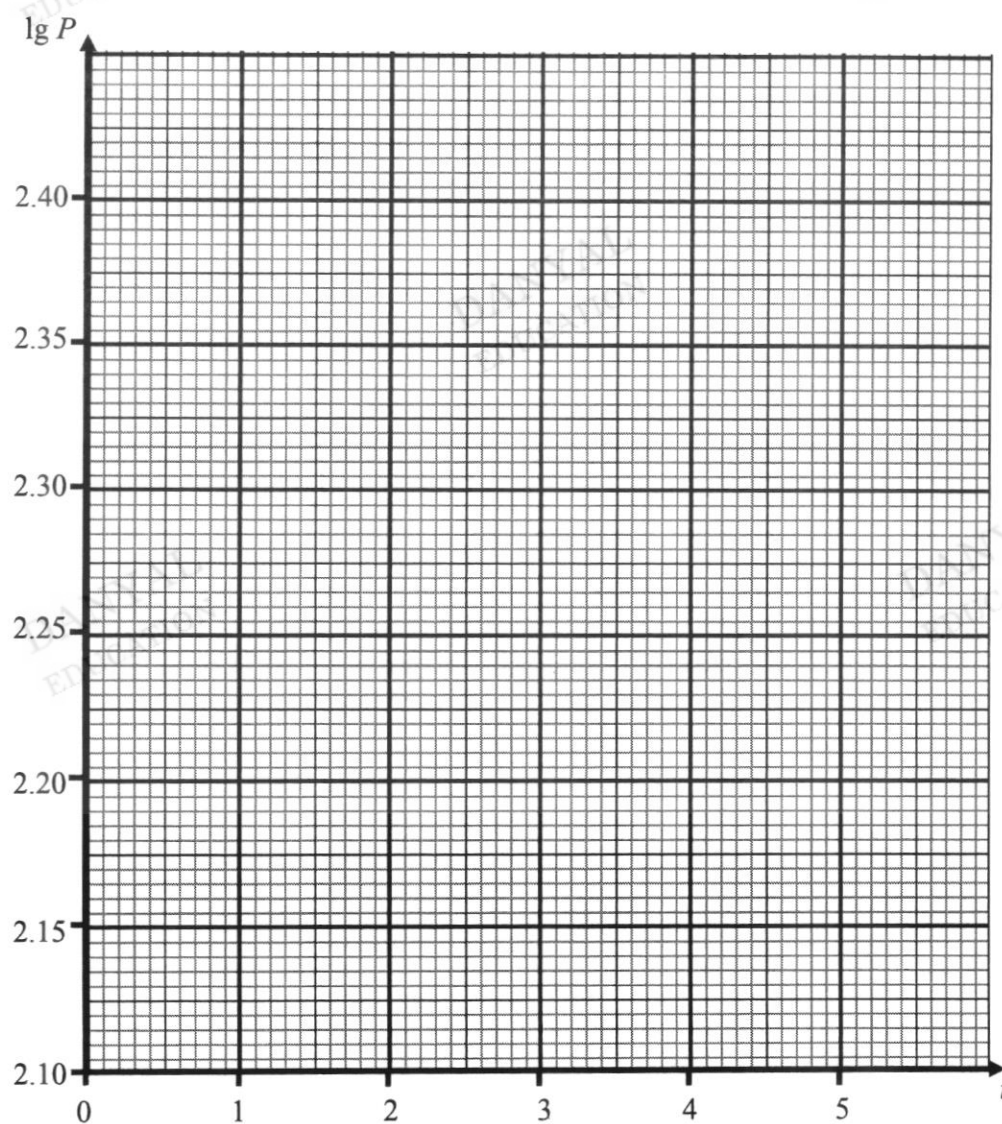
- (b) Hence, find the value of p if the coefficient of the $\frac{1}{x}$ term in the expansion $\left(\frac{1}{x} - \frac{p}{x^2}\right)(2 - 3x)^6$ is 1216. [3]

8. The population of a species is measured daily. The results are tabulated in the table below.

Number of days, t	1	2	3	4	5
Population, P	174	192	211	232	255
$\lg P$	2.24	2.28	2.32	2.37	2.41

- (a) It is known that t and P are related by the equation $P = ab^t$, where a and b are constants. Express this equation in a form suitable for plotting a straight line graph of $\lg P$ against t . [2]

- (b) On the grid below plot $\lg P$ against t and draw a straight line graph. [2]



(c) Explain what the value of a represents. [1]

(d) (i) Use your graph to estimate the value of a . [2]

(ii) Use your graph to estimate the value of b . [2]

9. (a) State the values between which the principal values of $\tan^{-1}x$ must lie. [1]

(b) Express the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ in radians as a multiple of π . [1]

10. It is given that $y_1 = 3 \cos bx + c$ and $y_2 = -1 + 2 \sin 3x$.

(a) Find the value of c , given that the maximum value of y_1 is 4. [1]

(b) Find the value of b , given that the period of y_1 is 180° . [1]

(c) Sketch the graph of $y_2 = -1 + 2 \sin 3x$ for $0 \leq x \leq 180^\circ$. [3]



(d) Explain how the graphical method can be used to find the solutions of $2 \sin 3x - 3 \cos bx = c + 1$ for $0 \leq x \leq 360^\circ$. [1]

11. Without using a calculator, find the exact value of $\frac{\sin 45^\circ}{\cos 60^\circ}$.
Show all workings clearly. [1]

12. (a) Prove that $\frac{1+\cos 2x}{\sin 2x} = \cot x$. [2]

- (b) Hence, solve $\frac{1+\cos 4x}{\sin 4x} = -\frac{1}{3}$ for $0 \leq x \leq 2\pi$. [3]

13. Express $\frac{x^2 + 4x - 2}{x^2 - x}$ in partial fractions.

[5]

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14. A circle has the equation $x^2 + y^2 - 2x - 6y = 22$ with centre C .

- (a) Find the coordinates of the centre and radius of the circle. [2]

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- (b) The equation of the circle represents the area of Wi-Fi signal coverage where the centre of the circle is the location of the Wi-Fi modem.

Explain if an individual standing at a point $X(4,2)$ is able to receive Wi-Fi signal on his electronic device. Show your working. [2]

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- (c) A point $P(5, q)$ lies on the circle where $q < 0$. A line AB passing through point P is perpendicular to the line CP . Find the equation of AB . [3]

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15. Find the range of k where the line $y = x + k$ and the curve $x^2 + y^2 = 2$ do no intersect. [4]

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16. Given that $\sin A = -\frac{3}{5}$ and $\cos B = -\frac{24}{25}$ where A and B are in the same quadrant.

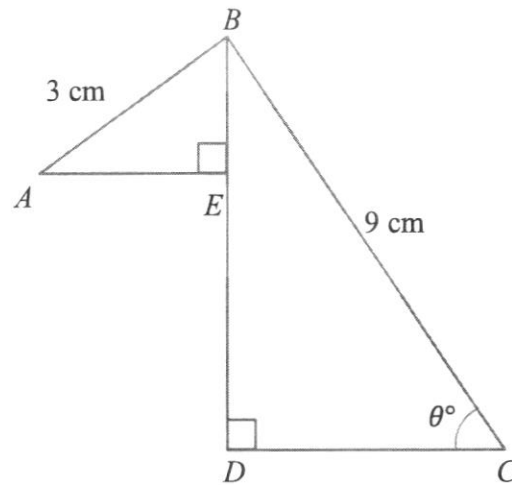
Calculate the value of

(a) $\sec A$, [2]

(b) $\sin (A + B)$, [2]

(c) $\tan 2A$. [2]

17. The figure shows two right angled triangles ABE and BCD . Line AB is perpendicular to the line BC . It is given that $\angle BCD = \theta^\circ$, $AB = 3$ cm and $BC = 9$ cm.



- (a) Show that the horizontal distance, L , between point A and C , in terms of θ is given by

$$L = 3\sin\theta + 9\cos\theta. \quad [2]$$

- (b) Express $3\sin\theta + 9\cos\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and α is acute. [3]

- (c) State the maximum value of L and the corresponding value of θ . [3]

- (d) Find the value(s) of θ when $L = \sqrt{55}$ cm. [3]

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1. Solve $\sqrt{7-6x} + 2 = x$. [4]

$$\sqrt{7-6x} + 2 = x$$

$$\sqrt{7-6x} = x - 2$$

$$7 - 6x = (x - 2)^2 \quad [M1]$$

$$7 - 6x = x^2 - 4x + 4$$

$$x^2 + 6x - 4x + 4 - 7 = 0$$

$$x^2 + 2x - 3 = 0 \quad [M1]$$

$$(x - 1)(x + 3) = 0 \quad [M1]$$

$$x - 1 = 0 \text{ or } x + 3 = 0$$

$$x = 1 \quad x = -3 \quad [A1 \text{ deduct if not even 1 ans is rejected}]$$

Both x values do not satisfy the equations, there are no solution.

2. The curved surface area of a cylinder is $(6\sqrt{3} + 16)\pi \text{ cm}^2$. The radius of the base of the cylinder is $(1 + \sqrt{3}) \text{ cm}$. Find, without using a calculator, the height of the cylinder in the form $a + b\sqrt{3}$, where a and b are rational numbers. Show your steps clearly. [4]

$$2\pi rh = (6\sqrt{3} + 16)\pi$$

$$2\pi(1 + \sqrt{3})h = (6\sqrt{3} + 16)\pi \quad [M1]$$

$$h = \frac{3\sqrt{3} + 8}{1 + \sqrt{3}}$$

$$h = \frac{3\sqrt{3} + 8}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}} \quad [M1]$$

$$h = \frac{(3\sqrt{3} + 8)(1 - \sqrt{3})}{1 - 3}$$

$$h = \frac{3\sqrt{3} - 9 + 8 - 8\sqrt{3}}{1 - 3} \quad [M1]$$

$$h = \frac{-1 - 5\sqrt{3}}{-2}$$

$$h = \frac{1}{2} + \frac{5}{2}\sqrt{3} \quad [A1]$$

3. Solve the simultaneous equations

[5]

$$3^y = \frac{1}{3}(3^{x-7}),$$

$$\log_3(14 - 2y) - \log_3 x = 0.5.$$

$$3^y = \frac{1}{3}(3^{x-7})$$

$$3^y = 3^{x-8}$$

$$y = x - 8 \quad \text{--- eqn 1} \quad \text{[M1]}$$

$$\log_3(14 - 2y) - \log_3 x = 0.5$$

$$\log_3 \frac{(14-2y)}{x} = 0.5 \log_3 9 \quad \text{[M1]}$$

$$\log_3 \frac{(14-2y)}{x} = \log_3 9^{0.5}$$

$$\frac{(14 - 2y)}{x} = 3$$

$$14 - 2y = 3x \quad \text{--- eqn 2} \quad \text{[M1]}$$

Sub (1) into (2),

$$14 - 2(x - 8) = 3x$$

$$14 - 2x + 16 = 3x$$

$$30 = 5x$$

$$x = 6 \quad \text{[M1]}$$

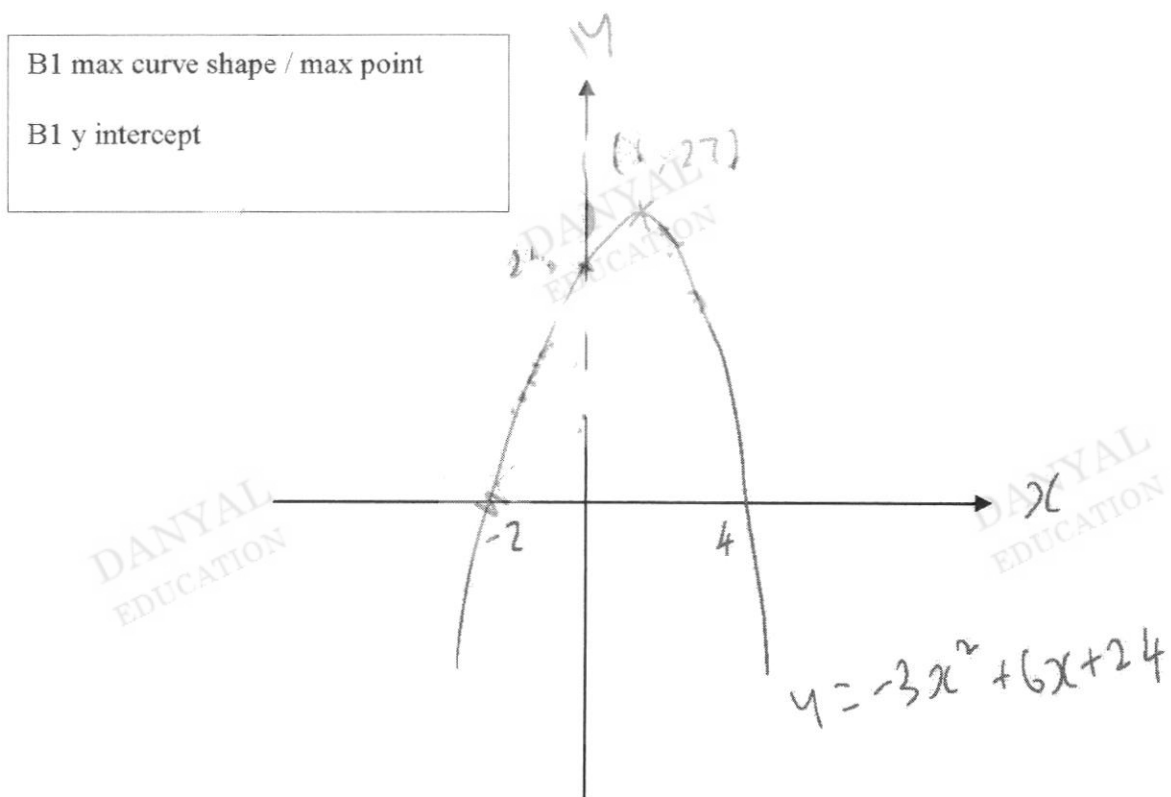
Sub $x = 6$ into (1),

$$y = -2 \quad \text{[A1]}$$

4. (a) Express $y = -3x^2 + 6x + 24$ in the form of $y = a(x + b)^2 + c$, where a , b and c are constants. [2]

$$\begin{aligned}
 y &= -3x^2 + 6x + 24 \\
 &= -3(x^2 - 2x) + 24 \\
 &= -3(x^2 - 2x + 1 - 1) + 24 \\
 &= -3[(x - 1)^2 - 1] + 24 \\
 &= -3(x - 1)^2 + 27 \quad \text{[B1/B1]}
 \end{aligned}$$

- (b) Sketch the graph of $y = -3x^2 + 6x + 24$. Label the turning point and y-intercept. [2]



- (c) Hence, explain why y cannot exceed 27. [1]

Since $(x - 2)^2 \geq 0$ for all real values of x , OR the maximum point is $(1, 27)$, the maximum value of $y = 27$ and thus y cannot exceed 27 [B1]

5. The cubic polynomial $f(x) = 3x^3 + hx^2 - 14x + k$ leaves a remainder of 40 when divided by $x - 3$ and it is exactly divisible by $x + 2$.

- (a) Show that $h = 1$ and $k = -8$ [4]

$$f(3) = 40$$

$$3(3)^3 + h(3^2) - 14(3) + k = 40$$

$$81 + 9h - 42 + k = 40$$

$$9h + k = 1 \quad (\text{eqn 1}) \quad [\text{M1}]$$

$$f(-2) = 0$$

$$3(-2)^3 + h(-2)^2 - 14(-2) + k = 0$$

$$-24 + 4h + 28 + k = 0$$

$$k = -4 - 4h \quad (\text{eqn 2}) \quad [\text{M1}]$$

$$\text{Sub (2) into (1), } 9h - 4 - 4h = 1$$

$$5h = 5$$

$$h = 1 \quad [\text{A1}]$$

$$\text{sub } h = 1 \text{ into (2), } k = -8 \quad [\text{A1}]$$

- (b) Hence, solve the cubic equation $f(x) = 0$. [4]

$$3x^3 + x^2 - 14x - 8 = 0$$

$$f(-2) = 0$$

$(x + 2)$ is a factor

$$(x + 2)(3x^2 - 5x - 4) = 0 \quad [\text{M1 for quadratic factor}]$$

$$x + 2 = 0 \text{ or } 3x^2 - 5x - 4 = 0$$

$$x = -2 \quad x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-4)}}{2(3)} \quad [\text{M1}]$$

$$x = -2, \quad x = 2.26, \quad x = -0.591 \text{ (3sf)} \quad [\text{A1/A1}]$$

6. Find the term independent of x in the expansion of $\left(x^3 - \frac{3}{x}\right)^8$. [3]

$$T_{r+1} = \binom{8}{r} (x^3)^{8-r} \left(-\frac{3}{x}\right)^r \quad [\text{M1}]$$

$$T_{r+1} = \binom{8}{r} x^{24-3r} (-3)^r \left(\frac{1}{x}\right)^r$$

$$T_{r+1} = \binom{8}{r} (-3)^r x^{24-4r}$$

$$24 - 4r = 0$$

$$r = 6 \quad [\text{M1}]$$

$$\text{Term independent of } x = 20412 \quad [\text{A1}]$$

7. (a) Write down and simplify the first three terms in the expansion of $(2 - 3x)^6$. [2]

$$= 2^6 + \binom{6}{1} 2^5(-3x) + \binom{6}{2} 2^4(-3x)^2 + \dots \quad [\text{M1}]$$

$$= 64 - 576x + 2160x^2 + \dots \quad [\text{M1}]$$

- (b) Hence, find the value of p if the coefficient of the $\frac{1}{x}$ term in the expansion

$$\left(\frac{1}{x} - \frac{p}{x^2}\right)(2 - 3x)^6 \text{ is } 1216. \quad [3]$$

$$\left(\frac{1}{x} - \frac{p}{x^2}\right)(64 - 576x + 2160x^2 + \dots)$$

$$= \frac{64}{x} - \frac{p}{x^2}(-576x) + \dots$$

$$= \frac{64}{x} + \frac{576p}{x} + \dots$$

$$\text{Coefficient of } \frac{1}{x} = 1216$$

$$64 + 576p = 1216$$

$$[\text{M1, M1} - \text{for each coefficient of } \frac{1}{x}]$$

$$576p = 1152$$

$$p = 2$$

$$[\text{A1}]$$

8. The population of a species is measured daily. The results are tabulated in the table below.

Number of days, t	1	2	3	4	5
Population, P	174	192	211	232	255
$\lg P$	2.24	2.28	2.32	2.37	2.41

- (a) It is known that t and P are related by the equation $P = ab^t$, where a and b are constants.

Express this equation in a form suitable for plotting a straight line graph of $\lg P$ against t .

[2]

$$P = ab^t$$

$$\lg P = \lg ab^t$$

[M1]

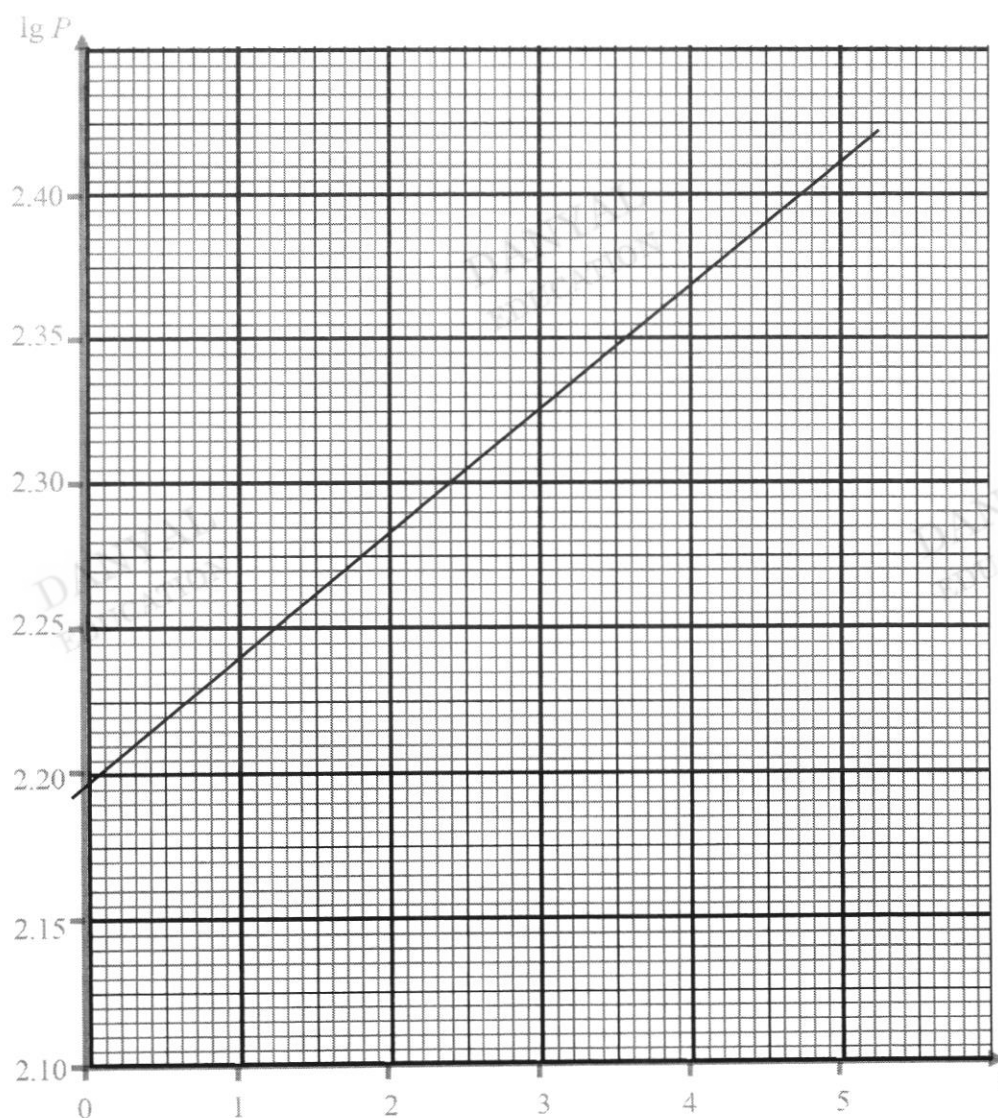
$$\lg P = \lg a + \lg b^t$$

$$\lg P = t \lg b + \lg a$$

[A1]

- (b) On the grid below plot $\lg P$ against t and draw a straight line graph

[2]



B1 cut y axis

B1 join 5 points with straight line

- (c) Explain what the value of a represents. [1]

It is the initial population of the species.

It is the population when $t = 0$.

[B1]

It is the population at the beginning.

- (d) (i) Use your graph to estimate the value of a . [2]

$$y\text{-intercept} = 2.195$$

[M1]

$$\lg a = 2.195$$

$$\lg a = 2.195 \lg 10$$

$$a = 10^{2.195}$$

$$a = 156.67$$

$$a = 157 \text{ (3sf)}$$

[A1]

- (ii) Use your graph to estimate the value of b . [2]

$$\text{gradient} = \frac{2.41 - 2.28}{5 - 2}$$

[M1]

$$= \frac{13}{300}$$

$$\lg b = \frac{13}{300} \lg 10$$

$$b = 10^{\frac{13}{300}}$$

[A1]

$$b = 1.10 \text{ (3sf)}$$

9. (a) State the values between which the principal values of $\tan^{-1}x$ must lie. [1]

$$-90^\circ < \tan^{-1}x < 90^\circ$$

[B1] accept $\frac{\pi}{2}$

- (b) Express the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ in radians as a multiple of π . [1]

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

[B1]

10. It is given that $y_1 = 3 \cos bx + c$ and $y_2 = -1 + 2 \sin 3x$.

- (a) Find the value of c , given that the maximum value of y_1 is 4. [1]

$$a + c = 4$$

$$c = 1$$

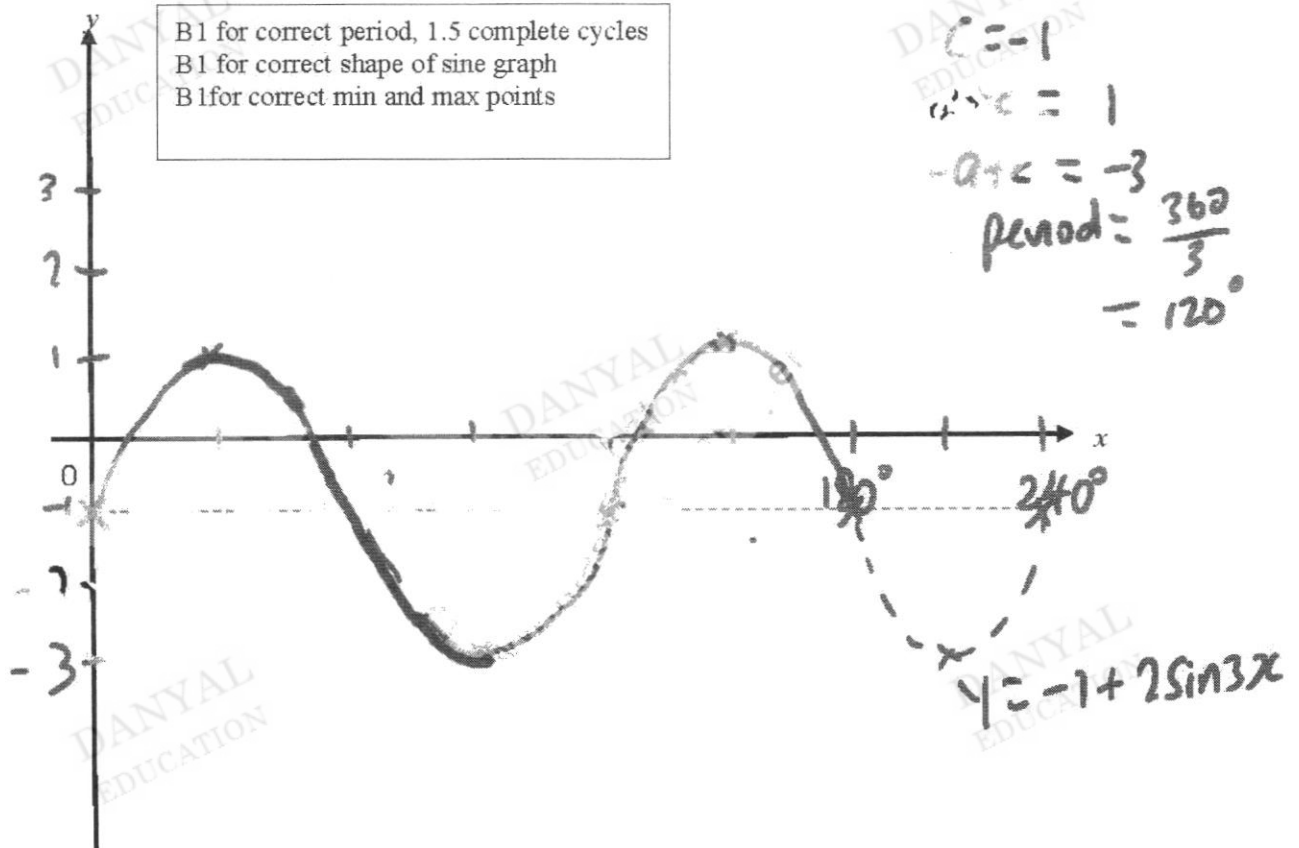
[B1]

- (b) Find the value of b , given that the period of y is 180° . [1]

$$b = 2$$

[B1]

- (c) Sketch the graph of $y_2 = -1 + 2 \sin 3x$ for $0 \leq x \leq 180^\circ$. [3]



- (d) Explain how the graphical method can be used to find the solutions of $2 \sin 3x - 3 \cos bx = c + 1$ for $0 \leq x \leq 360^\circ$. [1]

Sketch the graph of $y = 3 \cos bx + c$ on the graph in (b) and find the points of intersections of the 2 curves. [B1]

11. Without using a calculator, find the exact value of $\frac{\sin 45^\circ}{\cos 60^\circ}$.
Show all workings clearly. [1]

$$\begin{aligned}\frac{\sin 45^\circ}{\cos 60^\circ} &= \frac{\sqrt{2}}{2} \div \frac{1}{2} \\ &= \sqrt{2}\end{aligned}\quad [\text{B1}]$$

12. (a) Prove that $\frac{1+\cos 2x}{\sin 2x} = \cot x$. [2]

$$\begin{aligned}\frac{1+\cos 2x}{\sin 2x} &= \frac{1+2\cos^2 x-1}{2\sin x\cos x} & [\text{M1 for either}] \\ &= \frac{2\cos^2 x}{2\sin x\cos x} \\ &= \cot x & [\text{A1}]\end{aligned}$$

- (b) Hence, solve $\frac{1+\cos 4x}{\sin 4x} = -\frac{1}{3}$ for $0 \leq x \leq 2\pi$. [3]

$$\begin{aligned}\cot 2x &= -\frac{1}{3} \\ \tan 2x &= -3 & [\text{M1}]\end{aligned}$$

$$\begin{aligned}\alpha &= \tan^{-1}(3) \\ \alpha &= 1.1071487\end{aligned}$$

$$2\theta = 1.8925, 5.0341, 8.1756, 11.317 \quad [\text{A1, A1}]$$

$$\theta = 0.946, 2.52, 4.09, 5.66$$

13. Express $\frac{x^2 + 4x - 2}{x^2 - x}$ in partial fractions.

[5]

$$\begin{aligned}\frac{x^2 + 4x - 2}{x^2 - x} &= 1 + \frac{5x - 2}{x^2 - x} \\ &= 1 + \frac{5x - 2}{x(x - 1)}\end{aligned}$$

M1 by long division

$$\frac{5x - 2}{x(x - 1)} = \frac{A}{x} + \frac{B}{x - 1}$$

[M1, M1]

$$5x - 2 = A(x - 1) + Bx$$

$$\text{Sub } x = 1$$

$$B = 3$$

[M1]

$$\text{Sub } x = 0$$

$$-2 = -A$$

$$A = 2$$

[M1]

$$\frac{x^2 + 4x - 2}{x^2 - x} = 1 + \frac{2}{x} + \frac{3}{x - 1}$$

14. A circle has the equation $x^2 + y^2 - 2x - 6y = 22$ with centre C .

(a) Find the coordinates of the centre and radius of the circle.

[2]

$$\begin{aligned}\text{Centre} &= \left(\frac{-2}{-2}, \frac{-6}{-2}\right) & [\text{B1}] \\ &= (1, 3)\end{aligned}$$

$$\begin{aligned}\text{Radius} &= \sqrt{1 + 9 - (-22)} \\ &= \sqrt{32} \\ &= 5.66 \text{ units} & [\text{B1}]\end{aligned}$$

- (b) The equation of the circle represents the area of Wi-Fi signal coverage where the centre of the circle is the location of the Wi-Fi modem.

Explain if an individual standing at a point $X(4,2)$ is able to receive Wi-Fi signal on his electronic device. Show your working.

[2]

$$\begin{aligned}\text{Length of } CX &= \sqrt{(4-1)^2 + (2-3)^2} \\ &= \sqrt{10} \\ &= 3.16 & \text{M1}\end{aligned}$$

Since the length of CX is less than the radius of the circle, the person is standing inside the circle and is able to receive Wi-Fi signal. A1

- (c) A point $P(5, q)$ lies on the circle where $q < 0$. A line AB passing through point P is perpendicular to the line CP . Find the equation of AB . [3]

$$5^2 + y^2 - 2(5) - 6y = 22$$

$$y^2 - 6y - 7 = 0$$

$$(y - 7)(y + 1) = 0$$

$$y = 7 \text{ (rej) or } y = -1 \quad [\text{M1}]$$

$$P(5, -1)$$

$$\begin{aligned} \text{Gradient of } CP &= \frac{3 - (-1)}{1 - 5} \\ &= -1 \end{aligned}$$

[M1]

$$\text{Gradient of } AB = 1$$

Equation of AB :

$$y - (-1) = 1(x - 5)$$

$$y + 1 = x - 5$$

$$y = x - 6 \quad [\text{A1}]$$

15. Find the range of k where the line $y = x + k$ and the curve $x^2 + y^2 = 2$ do no intersect. [4]

$$x^2 + y^2 = 2 \text{ - eqn (1)}$$

$$y = x + k \text{ - eqn (2)}$$

Sub (2) into (1),

$$x^2 + (x + k)^2 = 2$$

$$x^2 + x^2 + 2kx + k^2 = 2$$

$$2x^2 + 2kx + k^2 - 2 = 0 \quad [\text{M1}]$$

$$b^2 - 4ac < 0$$

$$(2k)^2 - 4(2)(k^2 - 2) < 0 \quad [\text{M1}]$$

$$4k^2 - 8k^2 + 16 < 0$$

$$-4k^2 + 16 < 0$$

$$-k^2 + 4 < 0$$

$$k^2 - 4 > 0 \quad [\text{M1}]$$

$$(k + 2)(k - 2) > 0$$



$$k < -2 \text{ or } k > 2$$

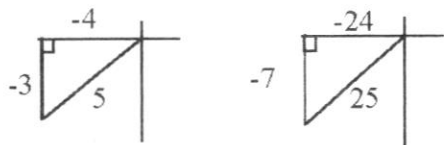
[A1]

16. Given that $\sin A = -\frac{3}{5}$ and $\cos B = -\frac{24}{25}$ where A and B are in the same quadrant.

Calculate the value of

(a) $\sec A$,

[2]



$$\begin{aligned}\cos A &= -\frac{4}{5} \\ \sec A &= \frac{1}{\cos A} \\ &= -\frac{5}{4}\end{aligned}$$

M1

A1

(b) $\sin(A + B)$,

[2]

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B \\ &= \left(-\frac{3}{5}\right)\left(-\frac{24}{25}\right) + \left(-\frac{4}{5}\right)\left(-\frac{7}{25}\right) \\ &= \frac{4}{5}\end{aligned}$$

M1

A1

(c) $\tan 2A$.

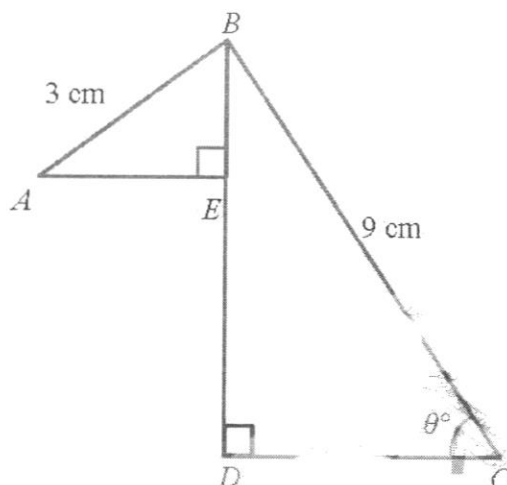
[2]

$$\begin{aligned}\tan 2A &= \frac{2 \tan A}{1 - \tan^2 A} \\ &= \frac{2\left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2} \\ &= \frac{24}{7}\end{aligned}$$

M1

A1

17. The figure shows two right angled triangles ABE and BCD . Line AB is perpendicular to the line BC . It is given that $\angle BCD = \theta^\circ$, $AB = 3$ cm and $BC = 9$ cm.



- (a) Show that the horizontal distance, L , between point A and C , in terms of θ is given by

$$L = 3\sin\theta + 9\cos\theta. \quad [2]$$

$$\begin{aligned} \sin\theta &= \frac{AE}{3} \\ AE &= 3\sin\theta \end{aligned} \quad [B1]$$

$$\begin{aligned} \cos\theta &= \frac{CD}{9} \\ CD &= 9\cos\theta \end{aligned} \quad [B1]$$

$$L = 3\sin\theta + 9\cos\theta \text{ (shown)}$$

- (b) Express $3\sin\theta + 9\cos\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and α is acute. [3]

$$\begin{aligned} 3\sin\theta + 9\cos\theta &= R\cos(x - \alpha) \\ 9\cos\theta + 3\sin\theta &= R\cos(x - \alpha) \end{aligned}$$

$$\begin{aligned} R &= \sqrt{9^2 + 3^2} \\ R &= \sqrt{90} \end{aligned} \quad [B1]$$

$$\begin{aligned} \alpha &= \tan^{-1}\left(\frac{3}{9}\right) \\ \alpha &= 18.435^\circ \end{aligned} \quad [B1]$$

$$9\cos\theta + 3\sin\theta = \sqrt{90}\cos(x - 18.4^\circ) \quad [A1]$$

- (c) State the maximum value of L and the corresponding value of θ . [3]

$$\text{Max value} = \sqrt{90} \quad [\text{B1}]$$

$$\begin{aligned}\sqrt{90}\cos(x - 18.4^\circ) &= \sqrt{90} \\ \cos(x - 18.4^\circ) &= 1\end{aligned}$$

$$\begin{aligned}\alpha &= \cos^{-1}(1) \\ \alpha &= 0^\circ\end{aligned} \quad [\text{M1}]$$

$$\begin{aligned}x - 18.4^\circ &= 0^\circ \\ x &= 18.4^\circ\end{aligned} \quad [\text{A1}]$$

- (d) Find the value(s) of θ when $L = \sqrt{55}$ cm. [3]

$$\begin{aligned}\sqrt{90}\cos(x - 18.4^\circ) &= \sqrt{55} \\ \cos(x - 18.4^\circ) &= \frac{\sqrt{55}}{\sqrt{90}}\end{aligned} \quad [\text{M1}]$$

$$\begin{aligned}\alpha &= \cos^{-1}\left(\frac{\sqrt{55}}{\sqrt{90}}\right) \\ \alpha &= 38.580^\circ\end{aligned} \quad [\text{M1}]$$

$$\begin{aligned}x - 18.4^\circ &= 38.580^\circ, \quad 360^\circ - 38.580^\circ \\ x &= 56.98^\circ, \quad 339.855^\circ \\ x &= 57.0^\circ, \quad 339.9^\circ(\text{ref})\end{aligned} \quad [\text{A1}]$$