BP~525

O

JURONG SECONDARY SCHOOL
2023 GRADUATION EXAMINATION
SECONDARY 4 EXPRESS/
SECONDARY 5 NORMAL (ACADEMIC)

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/01

PAPER 1
Candidates answer on the Question Paper.
Additional Materials: Writing Paper (1 sheet)

22 August 2023
2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **19** printed pages including this page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 A curve has an equation $y = 2x^2 - x + 5$.

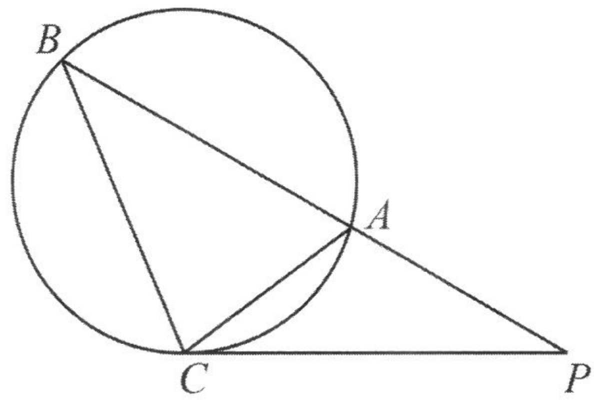
(a) Express $y = 2x^2 - x + 5$ in the form of $a(x-b)^2 + c$. Hence state the coordinates of the turning point. [3]

(b) The line $y = 2x + 7$ intersects the curve at points A and B. Find the distance AB. [3]

2 Express $\dfrac{3x^3+10x^2+x+1}{x^3+3x^2}$ in partial fractions. [6]

3 A curve has equation $y = \frac{1}{3}x^3 + x^2 + kx$, where k is a constant and $k > 1$. Explain why the curve does not have a stationary point. [4]

4 The diagram shows a circle. The line PC is the tangent to the circle at P. A and B are points on the circle such that PAB is a straight line.



Prove that

(a) triangle BPC is similar to triangle CPA, [3]

(b) $PA \times PB = PC^2$. [2]

5 The equation of a curve is $y = -\frac{4}{3}x^3 - (k+1)x^2 - k^2x$, where k is a constant.

(a) Find the range of values of k for which y is always decreasing. [4]

(b) Given that y has three distinct roots, find the range of values of k. [2]

6 A curve is such that $\frac{d^2y}{dx^2} = 3\sin x - 4\cos 2x$. The curve passes through $A(0,1)$ and $B(\pi,3)$. Find the equation of the curve. [7]

7 For the curve $y = 2x^2$, the tangent at point P where $x = a$, intersect the y-axis at A. The normal to the curve at point P intersects the y-axis at B.

Given that $a > 0$, show that the area of triangle ABP is $\dfrac{a\left(16a^2+1\right)}{8}$. [7]

8 **(a)** The equation of a curve is $y = a\sin bx + c$, $a > 0$. The curve attains maximum and minimum values of 4 and 2 respectively, and the period is π radians. Show that $a = 1$, $b = 2$ and show that $c = 3$. [3]

(b) **(i)** Sketch, on the same diagram, the curves $y = \sin 2x + 3$ and $y = 3\cos x$ for $0 \leq x \leq 2\pi$ radians. [4]

(ii) Find the number of solutions to the equation $\sin 2x + 3 - 3\cos x = 0$ for $0 \leq x \leq 2\pi$ radians. [1]

9 A container of liquid was heated to a temperature of $90°C$. It was then left to cool in a chiller such that its temperature, $T°C$, t minutes after the heat was removed, is given by $T = Ae^{-qt}$, where A and q are constants.
Measured values of t and T are given in the following table.

t (minutes)	2	4	6	8
T°C	66.674	49.393	36.591	27.107

(a) Explain why $A = 90$. [1]

(b) Plot $\ln T$ against t and draw a straight line to illustrate the information. [3]

(c) Use the graph to estimate the value of q. [3]

(d) Use your graph to estimate the temperature of the liquid 5 minutes after it was left to cool. [2]

10 The length, breadth and height of a cuboid is $3p$ cm , p cm and $(1-p)$ cm respectively. The volume of the cuboid is $\frac{4}{9}$ cm^3.

(a) Show that $27p^3-27p^2+4=0$. [2]

(b) Show that $3p-2$ is a factor to $27p^3-27p^2+4$. [2]

(c) Hence, find p and compute the surface area of the cuboid. [5]

11 **(a)** **(i)** Find the first 4 terms, in ascending powers of x, of the expansion of $(2-kx)^6$ where k is a non-zero constant. [2]

(ii) Given that the coefficient of x^3 is 30 times the coefficient of x, find the possible value(s) of k. [2]

(iii) Hence, show that there is no term in x^2 in the expansion of $(1-135x^2)(2-kx)^6$. [2]

(b) Explain why there is no odd powers of x in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ for $n \in \mathbb{N}$.

[4]

12 A particle travels in a straight line so that, at time t seconds after leaving a fixed point O, its displacement from O is s metres and its velocity is v ms^{-1} , where $v = 3e^{t} - 60e^{-3t}$.

Find

(a) the initial velocity of the particle, [1]

(b) the value of t when the particle is instantaneously at rest, [3]

(c) the acceleration of the particle when $t = \ln 8$, [2]

(d) an expression for s in terms of t, [4]

(e) the total distance travelled in the first 5 seconds. [3]

BP~544

O

JURONG SECONDARY SCHOOL
2023 GRADUATION EXAMINATION
SECONDARY 4 EXPRESS/
SECONDARY 5 NORMAL ACADEMIC

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS — 4049/02

PAPER 2 — **24 August 2023**
Candidates answer on the Question Paper. — **2 hours 15 minutes**
Additional Materials : Writing Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 At the beginning of a virus outbreak, the number of cases of infected people increased with time. After t days, the number of recorded cases was N. It was observed that N can be modelled by the equation $N = 1200e^{kt}$.

(a) Write down the initial number of cases recorded. [1]

The number of cases recorded after 6 days rose to 4800.

(b) Estimate the number of cases recorded after 10 days. [4]

A pandemic is declared if the number reaches 20 000 cases.

(c) Assuming the trend continues, estimate after how many days will it take for a pandemic to be declared. [2]

2 The expression $x^3 + px^2 + qx + r$ is divisible by both x and $x - 2$ and it leaves a remainder of 8 when divided by $x + 2$.

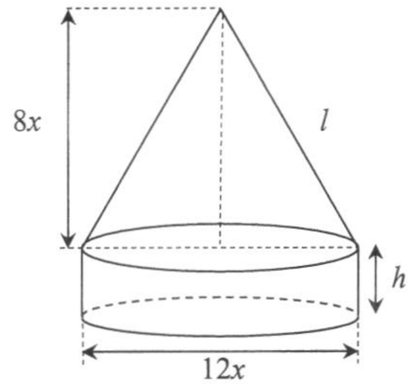
(a) Find the values of p, q and r. [4]

(b) Hence, find the remainder when it is divided by $x^2 + 2x - 3$. [2]

3 (a) Show that $2\cos\theta + \cot\theta - 1 = 2\cos\theta\cot\theta$ can be written as $(2\cos\theta - 1)(\sin\theta - \cos\theta) = 0$. [3]

(b) Hence, solve the equation $2\cos 2x + \cot 2x - 1 = 2\cos 2x\cot 2x$ for $0° < x < 180°$. [4]

4 The diagram below shows a mould made of a cylinder and a right circular cone. The diameter of the cylinder is $12x$ cm and its height is h cm. The vertical height of the cone is $8x$ cm.



(a) Find an expression, in terms of x, for the slant height l of the cone. [1]

(b) Given that the entire mould is covered with a plastic sheet whose area is 240π cm^2, express h in terms of x. [2]

(c) Show that the volume, V cm^3, of the mould is given by $V = 720\pi x - 192\pi x^3$. [3]

(d) Hence, find the value of x for which the volume has a stationary value and determine whether this value for the volume is a maximum or minimum. [4]

5 **(a)** Without using a calculator, show that $\cos 15° = \dfrac{\sqrt{2}+\sqrt{6}}{4}$. [3]

(b) Hence, state the value of $\cos(-15°)$. [1]

(c) Using your answer from **part (a)**, find the exact value of $\sec(15°)$ [3]

6 **(a)** Solve $6^x = \frac{10}{3} - 6^{-x}$. [4]

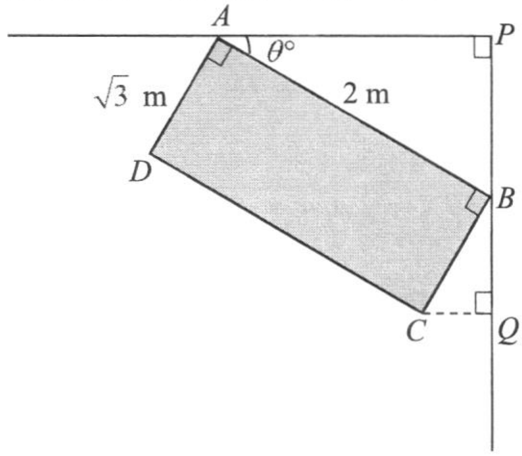
(b) Sketch the graph of $y = \ln x$, showing any points of intersection with the axes. [2]

(c) To solve $e^{1-2x} = x^3$, a straight line can be drawn on the same axes as the graph in **part (b)**.

(i) Determine the equation of the straight line to be drawn. [2]

(ii) Hence, state the number of solutions for $e^{1-2x} = x^3$. [1]

7 The diagram below shows a rectangular table, $ABCD$ placed at the corner of a classroom. It is given that the table has length $AB = 2$ m and width $AD = \sqrt{3}$ m. It is also given that $\angle APB = 90°$ and $\angle PAB = \theta°$.



(a) Show that the length of PQ, L, can be expressed as $L = 2\sin\theta + \sqrt{3}\cos\theta$. [2]

(b) Express L in the form $R\sin(\theta + \alpha)$ where $0° < \alpha < 90°$ and $R > 0$. [3]

(c) Find the value of θ for $L = 2.3$ m. [2]

(d) Find the maximum value of L and the corresponding value of θ. [2]

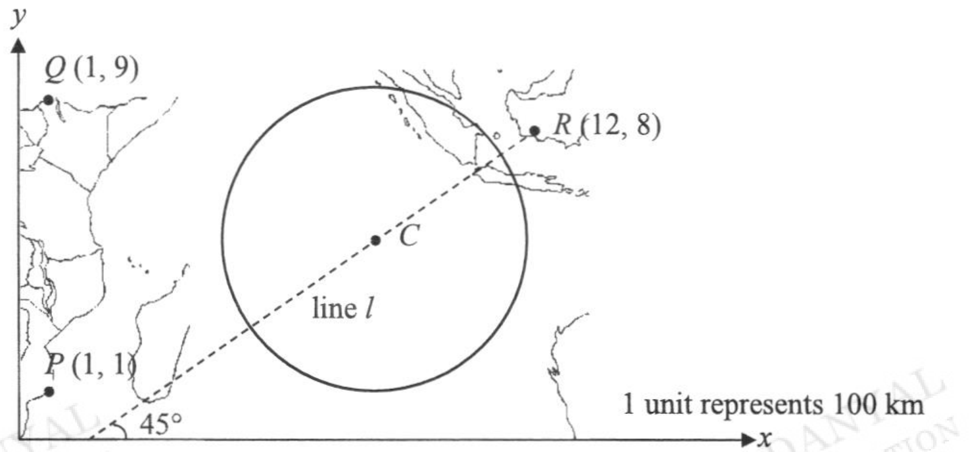
8 A curve has the equation $y = (3-x)\sqrt{2x+5}$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{ax+b}{\sqrt{2x+5}}$, where a and b are constants to be determined. [3]

(b) A point (x, y) moves along the curve. When the y-coordinate is increasing at the same rate as the x-coordinate, find the x-coordinate.
Explain why you need to reject the positive value. [4]

(c) Using your answer in **(a)**, evaluate $\int_{-2}^{2} \frac{-3x}{\sqrt{2x+5}}\,\mathrm{d}x$. [4]

9 The map below shows part of the Indian Ocean.
Geological stations $P(1, 1)$, $Q(1, 9)$ and $R(12, 8)$ detected an earthquake and a geologist is attempting to locate the epicentre, C of the earthquake.



Instruments at P and Q detected the earthquake at exactly the same time, indicating that the epicentre, C is equidistant from P and Q. Instrument at R detected it in the direction indicated by the line l, which makes an angle of $45°$ with the positive x-axis.

(a) Show that the line l can be represented by the equation $y = x - 4$. [2]

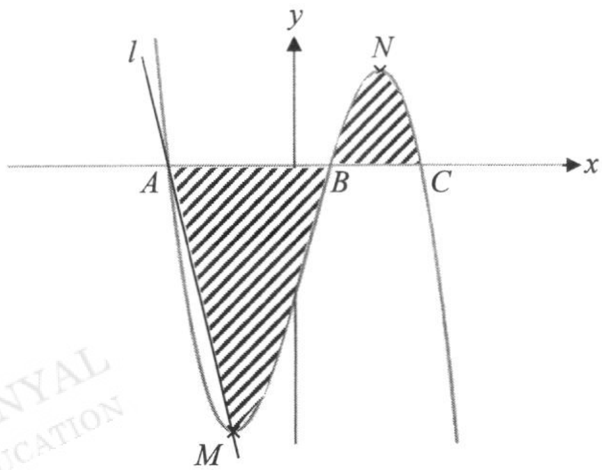
(b) Find the coordinates of C. [3]

(c) It is given that the earthquake detected can be felt at places as far as 450 km from the epicentre.
Find the equation of the circle that represents the places affected. [2]

(d) Hence, or otherwise, determine if geological station R is inside the circle. [2]

(e) Explain why it is not possible to draw such a circle that passes through all three geological stations P, Q and R, where PR is the diameter.
Support your answer with mathematical calculations. [3]

10 The diagram below shows part of the curve $y = (2x - 1)(3 - x^2)$. The curve has a minimum point at M and a maximum point at N. The curve intersects the x-axis at A, B and C respectively. The line l pass through A and M.



(a) Find the coordinates of A, B and C. [3]

(b) Find the coordinates of M. (You are not required to prove that it is the minimum point.) [3]

(c) Hence, find the area of the shaded region. [6]

---END OF PAPER---

BP~562

O

JURONG SECONDARY SCHOOL
2023 GRADUATION EXAMINATION
SECONDARY 4 EXPRESS/
SECONDARY 5 NORMAL (ACADEMIC)

CANDIDATE NAME	

CLASS			INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/01**

PAPER 1 **22 August 2023**

Candidates answer on the Question Paper. **2 hours 15 minutes**

Additional Materials: Writing Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **19** printed pages including this page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 A curve has an equation $y = 2x^2 - x + 5$.

(a) Express $y = 2x^2 - x + 5$ in the form of $a(x-b)^2 + c$. Hence state the coordinates of the turning point. [3]

$y = 2x^2 - x + 5$

$= 2\left(x^2 - \frac{x}{2}\right) + 5$ **M1**: Factor out the 2

$= 2\left[x^2 - \frac{x}{2} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right] + 5$

$= 2\left[x^2 - \frac{x}{2} + \left(\frac{1}{4}\right)^2\right] - 2\left(\frac{1}{4}\right)^2 + 5$

$= 2\left(x - \frac{1}{4}\right)^2 + \frac{39}{8}$ **A1**

Turning point: $\left(\frac{1}{4}, \frac{39}{8}\right)$ **B1**: FT from wrong completed square form

(b) The line $y = 2x + 7$ intersects the curve at points A and B. Find the distance AB. [3]

$y = 2x^2 - x + 5 --- (1)$

$y = 2x + 7 --- (2)$

$\therefore 2x^2 - x + 5 = 2x + 7$

$2x^2 - 3x - 2 = 0$

$(2x+1)(x-2) = 0$ **M1**: Solve quadratic equation (FT)

$\therefore x = -\frac{1}{2}$ OR $x = 2$

When $x = -\frac{1}{2}$, $y = 2\left(-\frac{1}{2}\right) + 7 = 6$

When $x = 2$, $y = 2(2) + 7 = 11$

$\therefore$ Coordinates of intersection are $\left(-\frac{1}{2}, 6\right)$ or $(2, 11)$ **A1**

Required distance $= \sqrt{\left(2 + \frac{1}{2}\right)^2 + (11-6)^2} = \frac{5\sqrt{5}}{2}$ units **A1**: Accept 5.59

2 Express $\dfrac{3x^3+10x^2+x+1}{x^3+3x^2}$ in partial fractions. [6]

By long division, $\dfrac{3x^3+10x^2+x+1}{x^3+3x^2}=3+\dfrac{x^2+x+1}{x^3+3x^2}$ **B1**: With long division working

$\dfrac{x^2+x+1}{x^3+3x^2}=\dfrac{x^2+x+1}{x^2(x+3)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+3}$ **M1**: Correct form (FT)

$\therefore x^2+x+1=Ax(x+3)+B(x+3)+Cx^2$ **M3**: Substitution or comparing coefficients correctly per unknown

When $x=0$:

$1=B(3)$

$\therefore B=\dfrac{1}{3}$

$x^2+x+1=Ax(x+3)+\dfrac{1}{3}(x+3)+Cx^2$

When $x=-3$:

$9-3+1=C(-3)^2$

$7=9C$

$\therefore C=\dfrac{7}{9}$

$x^2+x+1=Ax(x+3)+\dfrac{1}{3}(x+3)+\dfrac{7}{9}x^2$

When $x=1$:

$1+1+1=A(1+3)+\dfrac{1}{3}(1+3)+\dfrac{7}{9}(1)^2$

$3=4A+\dfrac{4}{3}+\dfrac{7}{9}$

$\therefore A=\dfrac{2}{9}$

$\therefore \dfrac{x^2+x+1}{x^2(x+3)}=\dfrac{2}{9x}+\dfrac{1}{3x^2}+\dfrac{7}{9(x+3)}$

$\therefore \dfrac{3x^3+10x^2+x+1}{x^3+3x^2}=3+\dfrac{2}{9x}+\dfrac{1}{3x^2}+\dfrac{7}{9(x+3)}$ **A1**

3 A curve has equation $y=\frac{1}{3}x^3+x^2+kx$, where k is a constant and $k>1$. Explain why the curve does not have a stationary point. [4]

$y=\frac{1}{3}x^3+x^2+kx$

$\frac{dy}{dx}=x^2+2x+k$ — **M1**: Find $\frac{dy}{dx}$

Method 1:

To find stationary point, $\frac{dy}{dx}=0$:

$x^2+2x+k=0$ — **M1**: Find discriminant of $\frac{dy}{dx}=0$

Assume on the contrary that curve has at least one stationary point.

$\therefore$ There are real roots to $x^2+2x+k=0$.

$\therefore (2)^2-4(1)(k)\geq 0$

$4-4k\geq 0$

$\therefore k\leq 1$ — **A1**: $k\leq 1$

However, $k>1$. — **A1**: Conclusion

$\therefore$ Curve does not have a stationary point

Method 2:

$x^2+2x+k=(x+1)^2+(k-1)$ — **M1**: Completing the square

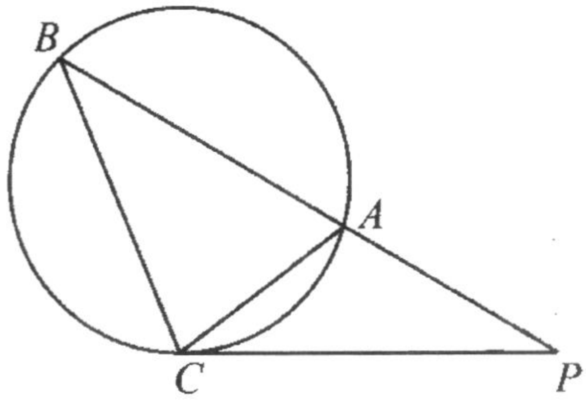
$(x+1)^2+(k-1)\geq k-1>0$ $(\because k>1)$ — **M1**: Establishing the inequality

$\therefore \frac{dy}{dx}>0$ for all values of x

$\therefore$ Graph is strictly increasing for all values of x

$\therefore$ Curve does not have a stationary point — **A1**: Conclusion

4 The diagram shows a circle. The line PC is the tangent to the circle at P. A and B are points on the circle such that PAB is a straight line.



Prove that

(a) triangle BPC is similar to triangle CPA, [3]

$\angle BPC = \angle CPA$ (Common angle) **B1**

$\angle CBP = \angle ACP$ (Angles in alternate segment) **B1**

$\therefore$ By AA similarity test, ΔBPC is similar to ΔCPA. **B1**

(b) $PA \times PB = PC^2$. [2]

$\frac{BPC}{CPA}: \frac{BP}{CP} = \frac{PC}{PA} = \frac{BC}{CA}$ **M1**: Ratio of corresponding sides of similar triangles

$\therefore \frac{BP}{CP} = \frac{PC}{PA}$

$\therefore BP \times PA = CP \times PC$

$\therefore PA \times PB = PC^2$ **A1**

5 The equation of a curve is $y = -\frac{4}{3}x^3 - (k+1)x^2 - k^2x$, where k is a constant.

(a) Find the range of values of k for which y is always decreasing. [4]

$y = -\frac{4}{3}x^3 - (k+1)x^2 - k^2x$

$\frac{dy}{dx} = -4x^2 - 2(k+1)x - k^2$ **M1**: Differentiation and set $\frac{dy}{dx} < 0$

For y to be strictly decreasing,

$\frac{dy}{dx} < 0$

$\therefore -4x^2 - 2(k+1)x - k^2 < 0$

$4x^2 + 2(k+1)x + k^2 > 0$

For $4x^2 + 2(k+1)x + k^2$ to be always positive,

$b^2 - 4ac < 0$ **B1**: $b^2 - 4ac < 0$

$[2(k+1)]^2 - 4(4)(k^2) < 0$

$(k+1)^2 - 4k^2 < 0$

$(k+1)^2 - (2k)^2 < 0$

$(k+1+2k)(k+1-2k) < 0$

$(3k+1)(-k+1) < 0$

$(3k+1)(k-1) > 0$ **M1**: Solve quadratic inequality

$\therefore k < -\frac{1}{3}$ OR $k > 1$ **A1**

(b) Given that y has three distinct roots, find the range of values of k. [2]

$\because$ y to have three distinct roots,

y has two turning points.

$\frac{dy}{dx} = 0$ has two real roots.

$-4x^2 - 2(k+1)x - k^2 = 0$ has two real roots.

$\therefore 4x^2 + 2(k+1)x + k^2 = 0$ has two real roots.

$\therefore b^2 - 4ac > 0$ **M1**: $b^2 - 4ac > 0$

$[2(k+1)]^2 - 4(4)(k^2) > 0$

$(k+1)^2 - 4k^2 > 0$

$(3k+1)(k-1) < 0$

$\therefore -\frac{1}{3} < k < 1$ **A1**

6 A curve is such that $\frac{d^2y}{dx^2} = 3\sin x - 4\cos 2x$. The curve passes through $A(0,1)$ and $B(\pi,3)$. Find the equation of the curve. [7]

$$\frac{d^2y}{dx^2} = 3\sin x - 4\cos 2x$$

$$\frac{dy}{dx} = \int(3\sin x - 4\cos 2x)dx$$

M1: Integration

$$= -3\cos x - \frac{4\sin 2x}{2} + c$$

$$= -3\cos x - 2\sin 2x + c$$

M1: Integration

$$y = \int(-3\cos x - 2\sin 2x + c)dx$$

$$= -3\sin x - \frac{(-2\cos 2x)}{2} + cx + d$$

$$= -3\sin x + \cos 2x + cx + d$$

A2: Minus one mark per mistake

M2: Substitute the two conditions

When $x = 0,\ y = 1$:

$$1 = -3\sin 0 + \cos 0 + c(0) + d$$

$$1 = 1 + d$$

$$\therefore d = 0$$

$$y = -3\sin x + \cos 2x + cx$$

When $x = \pi,\ y = 3$:

$$3 = -3\sin \pi + \cos 2\pi + c\pi$$

$$3 = 1 + c\pi$$

$$c = \frac{2}{\pi}$$

$$\therefore y = -3\sin x + \cos 2x + \frac{2}{\pi}x$$

A1

7 For the curve $y=2x^2$, the tangent at point P where $x=a$, intersect the y-axis at A. The normal to the curve at point P intersects the y-axis at B.

Given that $a>0$, show that the area of triangle ABP is $\dfrac{a\left(16a^2+1\right)}{8}$. [7]

$P=\left(a,2a^2\right)$ — **B1**: Coordinate of P

$y=2x^2$

$\dfrac{dy}{dx}=4x$

When $x=a$, $\dfrac{dy}{dx}=4a$ — **M1**: Differentiate to find gradient of tangent at $x=a$

$\therefore$ Gradient of tangent at $P=4a$

$\therefore$ Gradient of normal at $P=-\dfrac{1}{4a}$ — **B1**: Don't give if one of the gradients is wrong

Finding equation of tangent at P: $y=4ax+c$ — **M1**: Finding equation of tangent

When $x=a$, $y=2a^2$:

$2a^2=4a(a)+c$

$\therefore c=-2a^2$

$\therefore$ Equation of tangent at P: $y=4ax-2a^2$

$\therefore A=\left(0,-2a^2\right)$ — **A1**: SOI

Finding equation of tangent at P: $y=-\dfrac{1}{4a}x+c$

When $x=a$, $y=2a^2$:

$2a^2=-\dfrac{1}{4a}(a)+c$

$\therefore c=2a^2+\dfrac{1}{4}$

$\therefore$ Equation of normal at P: $y=-\dfrac{1}{4a}x+2a^2+\dfrac{1}{4}$

$\therefore B=\left(0,2a^2+\dfrac{1}{4}\right)$ — **A1**: SOI

$\therefore$ Area of $\Delta ABP=\dfrac{1}{2}\left[2a^2+\dfrac{1}{4}-\left(-2a^2\right)\right](a)=\dfrac{a\left(16a^2+1\right)}{8}$ units^2 — **A1**

8 **(a)** The equation of a curve is $y = a\sin bx + c$, $a > 0$. The curve attains maximum and minimum values of 4 and 2 respectively, and the period is π radians. Show that $a = 1$, $b = 2$ and show that $c = 3$. [3]

$-1 \le \sin bx \le 1$

$-a \le a\sin bx \le a$

$a + c < a\sin bx + c < a + c$

$\therefore a + c = 4,\ -a + c = 2$

$\therefore a = 1$ AND $c = 3$

$\frac{2\pi}{b} = \pi$

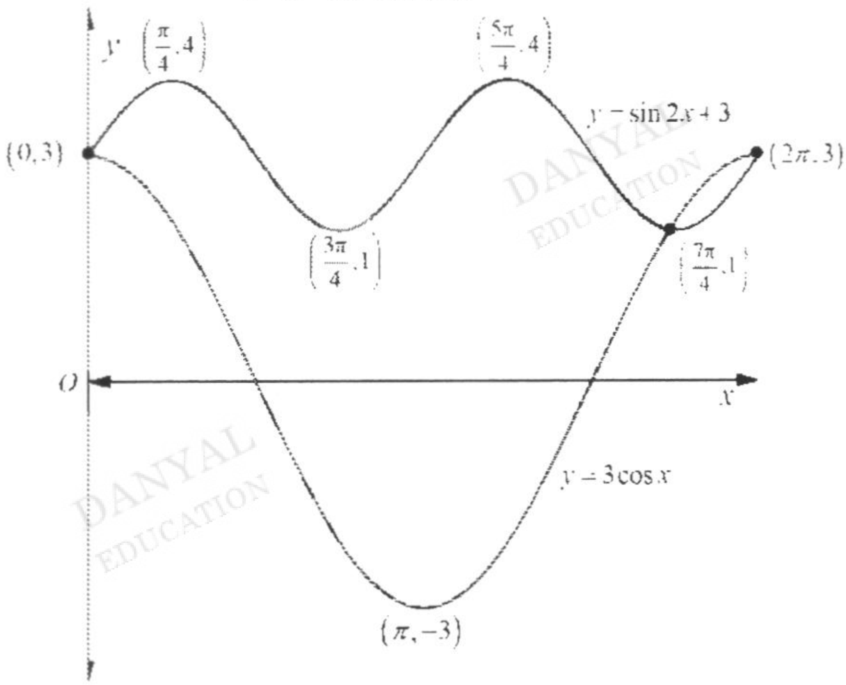
$\therefore b = 1$

M1: Constructing simultaneous equations with the max and min value.

A1

B1: Use period formula

(b) **(i)** Sketch, on the same diagram, the curves $y = \sin 2x + 3$ and $y = 3\cos x$ for $0 < x < 2\pi$ radians. [4]



B1: Correct shape for $y = \sin 2x + 3$

B1: Correct shape for $y = 3\cos x$

B2: Correct turning points and end points labelled, minus one for any missing two points.

(ii) Find the number of solutions to the equation $\sin 2x+3-3\cos x=0$ for $0 \le x \le 2\pi$ radians. [1]

$\sin 2x+3-3\cos x=0$

$\sin 2x+3=3\cos x$

Number of solutions corresponds to the number of intersections between the curves $y=\sin 2x+3$ and $y=3\cos x$.

From the graphs in part **(i)**, there are three intersections.
Hence, there will be three solutions to the given equation.

B1: Three solutions

9 A container of liquid was heated to a temperature of $90°C$. It was then left to cool in a chiller such that its temperature, $T°C$, t minutes after the heat was removed, is given by $T = Ae^{-qt}$, where A and q are constants.
Measured values of t and T are given in the following table.

t (minutes)	2	4	6	8
$T°C$	66.674	49.393	36.591	27.107

(a) Explain why $A = 90$. [1]

When $t = 0$, $T = 90$:

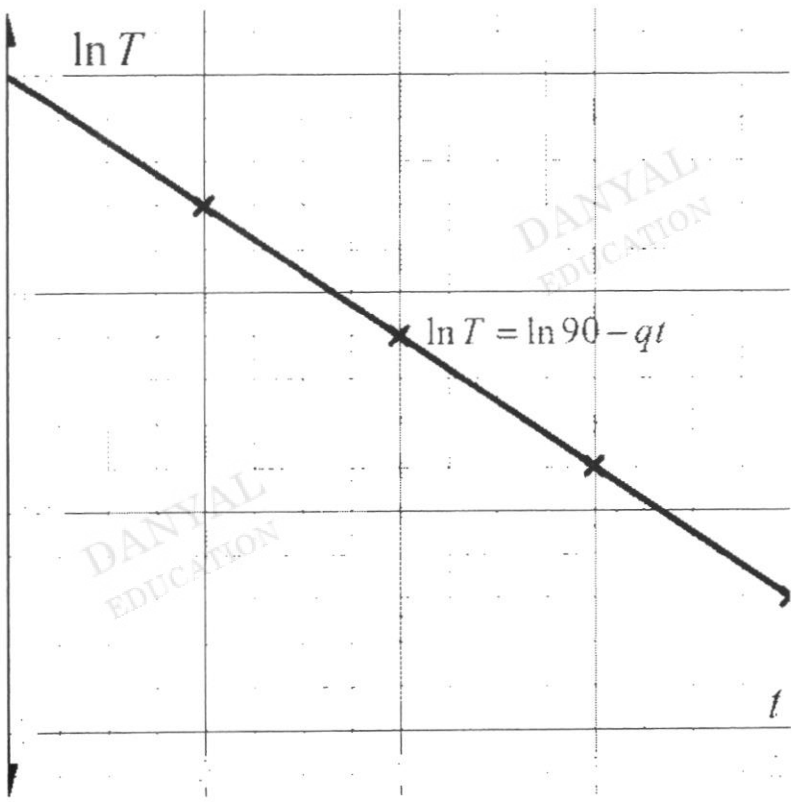
$90 = Ae^{0}$

$\therefore A = 90$ **B1**

(b) Plot $\ln T$ against t and draw a straight line to illustrate the information. [3]

t	2	4	6	8
$\ln T$	4.20	3.90	3.60	3.30

B1



B1: Straight line drawn passing through the $\ln T$ axis (not exceeding)

B1: Correct axes labelled and scale

(c) Use the graph to estimate the value of q. [3]

$T = 90e^{-qt}$

$\ln T = \ln 90e^{-qt}$

$\ln T = \ln 90 + \ln e^{-qt}$

$\ln T = \ln 90 - qt$ — **M1**: Take ln both sides

$-q$ is the gradient of the straight line — **M1**: Find gradient of straight line

$\therefore -q = -0.150$

$q = 0.150$ — **A1**

(d) Use your graph to estimate the temperature of the liquid 5 minutes after it was left to cool. [2]

From the graph,

when $t = 5$, $\ln T = 3.75$ — **M1**: Locate $t = 5$ to find the $\ln T$ coordinate

$\therefore T = e^{3.75} = 42.5$

$\therefore$ Required temperature $= 42.5°C$ — **A1**

10 The length, breadth and height of a cuboid is $3p$ cm, p cm and $(1-p)$ cm respectively. The volume of the cuboid is $\frac{4}{9}$ cm^3.

(a) Show that $27p^3-27p^2+4=0$. [2]

$(3p)(p)(1-p)=\frac{4}{9}$

$27p^2(1-p)-4$ **M1**: $(p)(p)(1-p)$

$27p^3-27p^2-4=0$

$\therefore 27p^3-27p^2+4-0$ (Shown)

A1

(b) Show that $3p-2$ is a factor to $27p^3-27p^2+4$. [2]

Method 1 **M1**: Long division

$$\begin{array}{r} 9p^2-3p-2 \\ 3p-2\overline{)\,27p^3-27p^2+4} \\ -(27p^3-18p^2) \\ \hline -9p^2+4 \\ -(-9p^2+6p) \\ \hline -6p+4 \\ -(-6p+4) \\ \hline 0 \end{array}$$

$\therefore 3p-2$ is a factor to $27p^3-27p^2+4$ **A1**

Method 2

Note that $27\left(\frac{2}{3}\right)^3-27\left(\frac{2}{3}\right)^2+4=0$. **M1**: Apply factor theorem

Hence, by Factor Theorem, $3p-2$ is a factor of $27p^3-27p^2+4$. **A1**: Must see "Factor Theorem"

(c) Hence, find p and compute the surface area of the cuboid. [5]

$27p^3-27p^2+4=0$

$(3p-2)(9p^2-3p-2)=0$

M1: $(3p-2)(9p^2-3p-2)$ (Can be shown without working from long division working in the previous part)

$\therefore p=\frac{2}{3}$ OR $9p^2-3p-2=0$

$\therefore p=\frac{2}{3}$ OR $p=\frac{-(-3)+\sqrt{(-3)^2-4(9)(-2)}}{2(9)}$

M1: Factorisation or quadratic formula

$\therefore p=\frac{2}{3}$ OR $p=\frac{2}{3}$ OR $p=-\frac{1}{3}$

$\because p>0$

$\therefore p=\frac{2}{3}$

A1: With reject (No reason required)

Required surface area

$=2\left[(2)\left(\frac{2}{3}\right)+\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)+(2)\left(\frac{1}{3}\right)\right]=\frac{40}{9}\text{cm}^2$

B1

11 **(a)** **(i)** Find the first 4 terms, in ascending powers of x, of the expansion of $(2-kx)^6$ where k is a non-zero constant. [2]

$$(2-kx)^6 = 2^6+\binom{6}{1}(2)^5(-kx)^1+\binom{6}{2}(2)^4(-kx)^2+\binom{6}{3}(2)^3(-kx)^3+\ldots$$

M1: Correct expansion

$$= 64-192kx+240k^2x^2-160k^3x^3+\ldots$$

A1

(ii) Given that the coefficient of x^3 is 30 times the coefficient of x, find the possible value(s) of k. [2]

$$\frac{-160k^3}{-192k}=30$$

M1: Relevant ratios (FT)

$$\frac{5k^2}{6}=30$$

$$k^2=36$$

$$k=\pm 6$$

A1

(iii) Hence, show that there is no term in x^2 in the expansion of $(1-135x^2)(2-kx)^6$. [2]

$$(1-135x^2)(2-kx)^6 = (1-135x^2)(64-192kx+240k^2x^2+\ldots)$$

$$=\ldots+240k^2x^2-8640x^2+\ldots$$

$$=\ldots+(240k^2-8640)x^2+\ldots$$

M1: Selective expansion to get coefficient of x^2 (FT)

When $k=\pm 6$, coefficient of $x^2 = 240(36)-8640 = 0$

$\therefore$ Required coefficient of $x^2 = 0$

$\therefore$ Expansion has no term in x^2.

A1: (With conclusion)

(b) Explain why there is no odd powers of x in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ for $n \in \mathbb{N}$.

[4]

General term $= \binom{2n}{r}(x)^{2n-r}\left(\frac{1}{x}\right)^{r}$

M1: Consider general term

$= \binom{2n}{r}(x)^{2n-r}\left(x^{-1}\right)^{r}$

$= \binom{2n}{r}(x)^{2n-2r}$

A1: Powers combined (must see simplification)

Note that $2n-2r = 2(n-r)$ is always even since n and r are whole numbers.
Hence the general terms in the given expansion always have even powers.
Therefore, there is no odd powers of x.

M1: Argue that powers are even **since n and r are whole numbers** (Must see)

A1: With conclusion

12 A particle travels in a straight line so that, at time t seconds after leaving a fixed point O, its displacement from O is s metres and its velocity is v ms^{-1}, where $v = 3e^{t} - 60e^{-3t}$.

Find

(a) the initial velocity of the particle, [1]

For $t = 0$, **B1**

$v = 3e^{0} - 60e^{0} = -57$ ms^{-1}

$\therefore$ Initial velocity $= -57$ ms^{-1}

(b) the value of t when the particle is instantaneously at rest, [3]

For $v = 0$, **M1**: $v = 0$

$0 = 3e^{t} - 60e^{-3t}$

$3e^{t} = 60e^{-3t}$

$e^{4t} = 20$

$\ln e^{4t} = \ln 20$ **M1**: Solve exponential equations

$4t = \ln 20$

$t = \frac{1}{4}\ln 20 = 0.749$ s **A1**: Accept $\frac{1}{4}\ln 20$

(c) the acceleration of the particle when $t = \ln 8$, [2]

$v = 3e^{t} - 60e^{-3t}$ **M1**: $a = \frac{dv}{dt}$

$a = \frac{dv}{dt} = 3e^{t} - 60(-3)e^{-3t}$

$a = 3e^{t} + 180e^{-3t}$

When $t = \ln 8$,

$a = 3e^{\ln 8} + 180e^{-3\ln 8} = 24.4$ ms^{-2} **A1**

(d) an expression for s in terms of t, [4]

$v = 3e^t - 60e^{-3t}$

$s = \int \left(3e^t - 60e^{-3t}\right) dt$

$= 3e^t - \frac{60}{(-3)}e^{-3t} + c$

$= 3e^t + 20e^{-3t} + c$

When $t = 0,\ s = 0$:

$0 = 3e^0 + 20e^0 + c$

$c = -23$

$\therefore s = 3e^t + 20e^{-3t} - 23$

M1: Integrate v to get s

A1

M1: Substitute initial conditions

A1

(e) the total distance travelled in the **first** 5 seconds. [3]

Total distance travelled in the first 5 seconds

$= 2(-1)\left(3e^{\frac{1}{4}\ln 20} + 20e^{-\frac{3}{4}\ln 20} - 23\right) + 3e^5 + 20e^{-15} - 23$

$= 451$ m

M1: Substitute $t = \frac{1}{4}\ln 20$ (FT from (b)) or $t = 5$ into part (d) (FT from (d))

M1: $\times 2$

A1

THE END

BP~582

O

JURONG SECONDARY SCHOOL
2023 GRADUATION EXAMINATION
SECONDARY 4 EXPRESS/
SECONDARY 5 NORMAL ACADEMIC

CANDIDATE NAME	

CLASS			INDEX NUMBER	

ADDITIONAL MATHEMATICS — 4049/02

PAPER 2 — 2023

Candidates answer on the Question Paper. — 2 hours 15 minutes

Additional Materials : Writing Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **17** printed pages including this page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 At the beginning of a virus outbreak, the number of cases of infected people increased with time. After t days, the number of recorded cases was N. It was observed that N can be modelled by the equation $N = 1200e^{kt}$.

(a) Write down the initial number of cases recorded. [1]

Solution	Marks
1200	B1

The number of cases recorded after 6 days rose to 4800.

(b) Estimate the number of cases recorded after 10 days. [4]

Solution	Marks
$4800 = 1200e^{6k}$	M1 – substitute values correctly
$e^{6k} = \frac{4800}{1200}$	
$6k = \ln\frac{4800}{1200}$	
$k = 0.231049$	M1 – value of k – accept rounded off values.
$N = 1200e^{10(0.231049)}$	M1
$N = 12095.2$	
≈ 12100	A1

A pandemic is declared if the number reaches 20 000 cases.

(c) Assuming the trend continues, estimate after how many days will it take for a pandemic to be declared. [2]

Solution	Marks
$20000 = 1200e^{0.231049t}$	M1 – substitute values correctly
$t = 12.1766$	
≈ 13 days	A1 – Do not accept 12 days

2 The expression $x^3 + px^2 + qx + r$ is divisible by both x and $x-2$ and it leaves a remainder of 8 when divided by $x+2$.

(a) Find the values of p, q and r. [4]

Solution	Marks
$f(0)=0$ $r=0$	[B1]
$f(2)=0$ $8+4p+2q=0$......(1)	[M1] – Forming correct equations (either one)
$f(-2)=8$ $-8+4p-2q=8$ $-16+4p-2q=0$......(2)	
$(1)+(2)$ $8p=8$ $p=1$ $q=-6$	[A1] [A1]

(b) Hence, find the remainder when it is divided by x^2+2x-3. [2]

Solution	Marks
Long division $x^2+2x-3 \;\overline{)\; x^3+x^2-6x}$ quotient $x-1$ $-(x^3+2x^2-3x)$ $-x^2-3x$ $-(-x^2-2x+3)$ $-x-3$	[M1 – Long division]
Remainder $= -x-3$	A1 – Writing out remainder

Alternative Method $x^2+2x-3=(x+3)(x-1)$ $x^3+x^2-6x=Q(x)(x^2+2x-3)+R$ $x^3+x^2-6x=Q(x)(x^2+2x-3)+(ax+b)$ When $x=1$, $1+1-6=a+b$ $a=-4-b......(1)$ When $x=-3$, $-27+9-6(-3)=-3a+b$ $0=-3a+b......(2)$ Sub (1) into (2), $0=-3(-4-b)+b$ $0=12+3b+b$ $b=-3$ $a=-1$ Remainder = $-x-3$	[M1] – Forming of equations [A1]

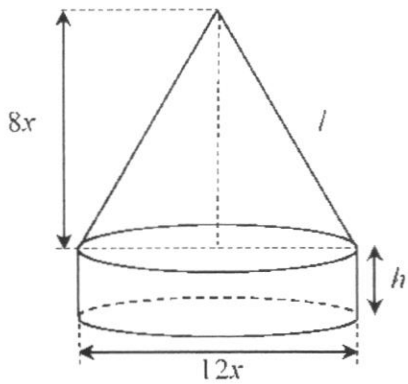
3 **(a)** Show that $2\cos\theta+\cot\theta-1=2\cos\theta\cot\theta$ can be written as $(2\cos\theta-1)(\sin\theta-\cos\theta)=0$. [3]

Solution	Marks
$2\cos\theta+\cot\theta-1=2\cos\theta\cot\theta$	
$2\cos\theta+\cot\theta-1-2\cos\theta\cot\theta=0$	
$2\cos\theta-1+\cot\theta(1-2\cos\theta)=0$	M1 – factorisation
$(2\cos\theta-1)-\cot\theta(2\cos\theta-1)=0$	
$(2\cos\theta-1)(1-\cot\theta)=0$	M1 – conversion of $\cot\theta$
$(2\cos\theta-1)\left(1-\dfrac{\cos\theta}{\sin\theta}\right)=0$	A1
$(2\cos\theta-1)(\sin\theta-\cos\theta)=0$	
Alternatively	
$2\cos\theta+\dfrac{\cos\theta}{\sin\theta}-2\cos\theta\times\dfrac{\cos\theta}{\sin\theta}-1=0$	M1 – conversion of cot
$2\sin\theta\cos\theta+\cos\theta-2\cos^2\theta-\sin\theta=0$	M1 – simplification
$2\cos\theta(\sin\theta-\cos\theta)-(\sin\theta-\cos\theta)=0$	A1 – factorise by grouping
$(\sin\theta-\cos\theta)(2\cos\theta-1)=0$	

(b) Hence, solve the equation $2\cos 2x+\cot 2x-1=2\cos 2x\cot 2x$ for $0°<x<180°$. [4]

Solution	Marks
$\cot 2x+2\cos 2x-1=2\cos 2x\cot 2x$	
$(2\cos 2x-1)(\sin 2x-\cos 2x)=0$	M1 – With $2x$
$\cos 2x=\dfrac{1}{2}$ *or* $\sin 2x=\cos 2x$	M1, M1 – Correct Equations
$\tan 2x=1$	
Basic$\angle=60°$ Basic$\angle=45°$	
$2x=60°,300°$ $2x=45°,225°$	
$x=30°,150°$ $x=22.5°,112.5°$	
$x=30°,22.5°,112.5°,150°$	A1

4 The diagram below shows a mould made of a cylinder and a right circular cone. The diameter of the cylinder is $12x$ cm and its height is h cm. The vertical height of the cone is $8x$ cm.



(a) Find an expression, in terms of x, for the slant height l of the cone. [1]

Solution	Marks
$l = \sqrt{(8x)^2 + (6x)^2}$ $= 10x$	B1

(b) Given that the entire mould is covered with a plastic sheet whose area is 240π cm^2, express h in terms of x. [2]

Solution	Marks
$\pi rl + 2\pi rh + \pi r^2 = 240\pi$	
$\pi(6x)(10x) + 2\pi(6x)h + \pi(6x)^2 = 240\pi$	M1
$60x^2 + 12xh + 36x^2 = 240$	
$12xh = 240 - 96x^2$	
$h = \dfrac{240 - 96x^2}{12x}$	
$h = \dfrac{20 - 8x^2}{x}$	A1

(c) Show that the volume, V cm^3, of the mould is given by $V = 720\pi x - 192\pi x^3$. [3]

Solution	Marks
Volume $= \frac{1}{3}\pi r^2 h_{cone} + \pi r^2 h$ $= \frac{1}{3}\pi(6x)^2(8x) + \pi(6x)^2(\frac{20-8x^2}{x})$ $= 96\pi x^2 + 720\pi x - 288\pi x^3$ $= 720\pi x - 192\pi x^3$ (shown)	M2 – Cone, cylinder A1

(d) Hence find the value of x for which the volume has a stationary value and determine whether this value for the volume is a maximum or minimum. [4]

Solution	Marks
Volume $V = 720\pi x - 192\pi x^3$ $\frac{dV}{dh}$ $720\pi - 576\pi x^2$ $\frac{dV}{dh}$ 0 $720\pi - 576\pi x^2 = 0$ $x^2 = \frac{720\pi}{576\pi}$ $x = 1.11803$, -1.11803(rej) $x = 1.12$ $\frac{d^2V}{dh^2} = -1152\pi x$ $\frac{d^2V}{dh^2} < 0$ since x is positive. Hence, maximum volume.	M1 A1 M1 A1

5 **(a)** Without using a calculator, show that $\cos 15° = \dfrac{\sqrt{2}+\sqrt{6}}{4}$. [3]

Solution	Marks
$\cos 15° = \cos(60° - 45°)$ $= \cos 60 \cos 45 + \sin 60 \sin 45$ $= \frac{1}{2}(\frac{\sqrt{2}}{2}) + \frac{\sqrt{3}}{2}(\frac{\sqrt{2}}{2})$ $= \frac{\sqrt{2}+\sqrt{6}}{4}$ (shown)	M1 – use of correct formulae with appropriate values M1 – either term A1

(b) Hence, state the value of $\cos(-15°)$. [1]

Solution	Marks
$\cos(-15°) = \frac{\sqrt{2}+\sqrt{6}}{4}$	[B1]

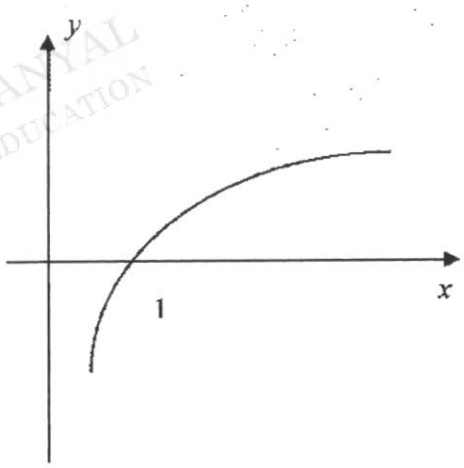
(c) Using your answer from **part (i)**, find the exact value of $\sec(15°)$ [3]

Solution	Marks
$\sec 15° = \frac{1}{\frac{1}{4}(\sqrt{2}+\sqrt{6})}$ $= \frac{4}{(\sqrt{2}+\sqrt{6})} \times \frac{(\sqrt{2}-\sqrt{6})}{(\sqrt{2}-\sqrt{6})}$ $= \frac{4\sqrt{2}-4\sqrt{6}}{2-6}$ $= \frac{4\sqrt{2}-4\sqrt{6}}{-4}$ $= \sqrt{6}-\sqrt{2}$	[M1] Rationalising [M1] Either denominator or numerator. [A1]

6 **(a)** Solve $6^x = \frac{10}{3} - 6^{-x}$. [4]

Solution		Marks
$6^x = \frac{10}{3} - 6^{-x}$		
$6^x = \frac{10}{3} - \frac{1}{6^x}$		[M1]
Let $u = 6^x$		
$u = \frac{10}{3} - \frac{1}{u}$		
$3^2 u - 10u + 3 = 0$		[M1]
$(3u-1)(u-3) = 0$		
$u = \frac{1}{3}$	*or* $u = 3$	[M1]
$6^x = \frac{1}{3}$	$6^x = 3$	
$x = \frac{\lg\frac{1}{3}}{\lg 6} = -0.613$	$x = \frac{\lg 3}{\lg 6} = 0.613$	[A1]

(b) Sketch the graph of $\ln x$, showing any points of intersection with the axes. [2]

Solution	Marks
 	[G1 – Shape] [G1 – Intersection]

(c) To solve $e^{1-2x} = x^3$, a straight line can be drawn on the same axes as the graph in **part (b)**.

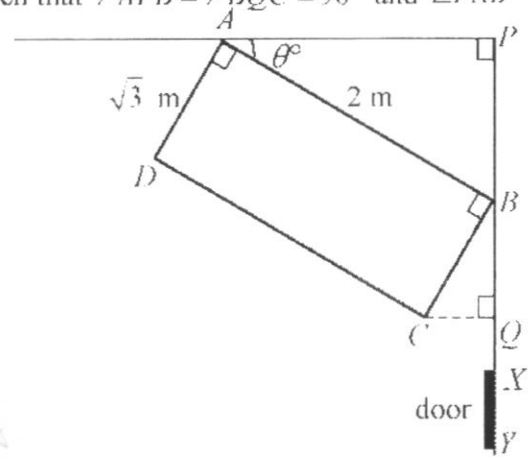
(i) Determine the equation of the straight line to be drawn. [2]

Solution	Marks
$e^{1-2x} = x^3$	
$\ln e^{1-2x} = \ln x^3$	[M1]
$1-2x = 3\ln x$	
$\frac{1-2x}{3} = \ln x$	
$y = \frac{1-2x}{3}$	[A1]

(ii) Hence, state the number of solutions for $e^{1-2x} = x^3$. [1]

Solution	Marks
Number of solutions = 1	[B1]

7 The diagram below shows a rectangular table, $ABCD$ placed at the corner of a classroom. It is given that the table has length $AB = 2$ m and width $AD = \sqrt{3}$ m. It is also given that $\angle APB = \angle BQC = 90°$ and $\angle PAB = \theta°$.



(a) Show that the length of PQ, L can be expressed as $L = 2\sin\theta + \sqrt{3}\cos\theta$. [2]

Solution	Marks
$\frac{PB}{2} = \sin\theta, \frac{BQ}{\sqrt{3}} = \cos\theta$	M1
$L = PB + BQ = 2\sin\theta + \sqrt{3}\cos\theta$	A1

(b) Express L in the form $R\sin(\theta + \alpha)$ where $0° < \alpha < 90°$ and $R > 0$. [3]

Solution	Marks
$R = \sqrt{2^2 + 3}$	M1
$\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$	M1
$L = \sqrt{7}\sin(\theta + 40.9°)$	A1

(c) Find the value of θ for $L = 2.3$ m. [2]

Solution	Marks
$\sin(\theta + 40.894°) = 0.869318$	
$b.a. = 60.3795°$	M1
$\theta = 19.5°$ (1 d.p.)	A1

(d) Find the maximum value of L and the corresponding value of θ. [2]

Solution	Marks
max $L = \sqrt{7}$ m	B1
corr value of $\theta = 49.1°$	B1

8 A curve has the equation $y=(3-x)\sqrt{2x+5}$.

(a) Show that $\frac{dy}{dx}=\frac{ax+b}{\sqrt{2x+5}}$, where a and b are constants to be determined. [3]

Solution	Marks
$\frac{dy}{dx}=(3-x)\times\frac{1}{2}(2x+5)^{-\frac{1}{2}}\times 2+(2x+5)^{\frac{1}{2}}\times(-1)$	M1 product rule
$=\frac{3-x-(2x+5)}{\sqrt{2x+5}}$	M1 combine into a single fraction
$=\frac{-3x-2}{\sqrt{2x+5}}$	A1

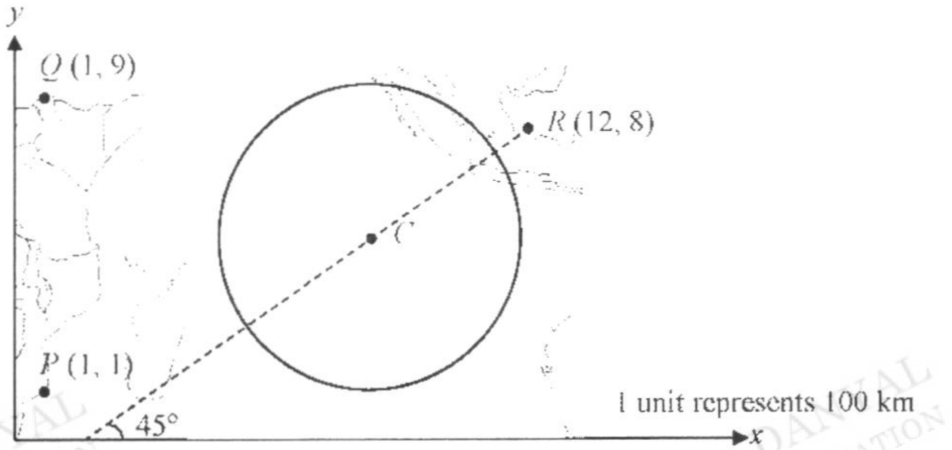
(b) A point (x, y) moves along the curve. Find the value of x when the y-coordinate is increasing at the same rate as the x-coordinate. [3]

Solution	Marks
$\frac{dy}{dx}=1$	M1
$-3x-2=\sqrt{2x+5}$	
$9x^2+12x+4=2x+5$	M1 square both sides
$9x^2+10x-1=0$	
$x=\frac{-10\pm\sqrt{10^2-4(9)(-1)}}{2(9)}$	
$=0.0923$ (rej) or -1.20 (3 s.f.)	A1
x cannot be 0.0923 as $-3x-2\geq 0$	B1

(c) Using your answer in **(a)**, evaluate $\int_{-2}^{2}\frac{-3x}{\sqrt{2x+5}}\,dx$. [4]

Solution	Marks
$\int_{-2}^{2}\frac{-3x}{\sqrt{2x+5}}\,dx=\left[(3-x)\sqrt{2x+5}\right]_{-2}^{2}+\int_{-2}^{2}\frac{2}{\sqrt{2x+5}}\,dx$	M1
$=(3-5)+\left[\frac{2\sqrt{2x+5}}{2\times\frac{1}{2}}\right]_{-2}^{2}$	M1 correct integral M1 substitution
$=2$	A1

9 The map below shows part of the Indian Ocean.
Geological stations $P(1, 1)$, $Q(1, 9)$ and $R(12, 8)$ detected an earthquake and a geologist is attempting to locate the epicentre, C of the earthquake.



Instruments at P and Q detected the earthquake at exactly the same time, indicating that the epicentre, C is equidistant from P and Q. Instrument at R detected it in the direction indicated by the line l, which makes an angle of 45° with the positive x-axis.

(a) Show that the line l can be represented by the equation $y = x - 4$. [2]

Solution	Marks
Gradient of $l = 1$ $y - 8 = x - 12$ $y = x - 4$ (shown)	M1 correct gradient A1 with substitution

(b) Find the coordinates of C. [2]

Solution	Marks
y coordinate $= \frac{1+9}{2} = 5$ $5 = x - 4$ $x = 9, C = (9,5)$	M1 M1 A1

(c) It is given that the earthquake detected can be felt at places as far as 450 km from the epicentre.
Find the equation of the circle that represents the places affected. [2]

Solution	Marks
radius $= \frac{450}{100} = 4.5$ units $(x-9)^2 + (y-5)^2 = \frac{81}{4}$	M1 A1

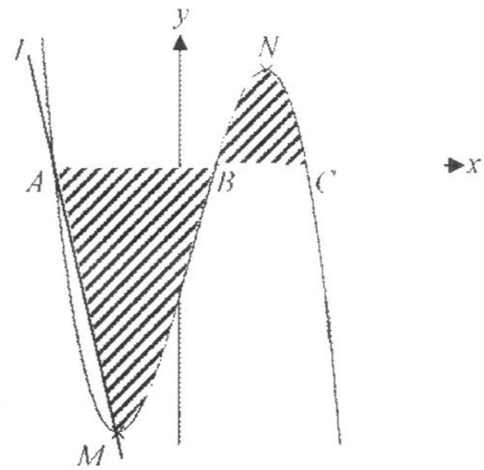
(d) Hence, or otherwise, determine if geological station R is inside the circle. [2]

Solution	Marks
$CR=\sqrt{(12-9)^2+(8-5)^2}$	M1
$=4.24<4.5$	
R lies inside the circle	A1

(e) Explain why it is not possible to draw such a circle that passes through all three geological stations P, Q and R.
Support your answer with mathematical calculations. [3]

Solution	Marks
Gradient of $PR=\dfrac{8-1}{12-1}=\dfrac{7}{11}$	M1 find gradients (or using Pythagoras' Theorem)
Gradient of $RQ=\dfrac{8-9}{12-1}=-\dfrac{1}{11}$	
Since gradient of $PR\times$ gradient of $RQ\neq-1$, $\angle QRP\neq 90°$	M1 conclusion
PQ is a vertical line and neither PR nor QR are horizontal.	
PQR is not a right-angled triangle, by right angle in a semicircle, P, Q and R do not lie on the same circle.	A1 with circle property stated

10 The diagram below shows part of the curve $y = (2x - 1)(3 - x^2)$. The curve has a minimum point at M and a maximum point at N. The curve intersects the x-axis at A, B and C. The line l pass through A and M.



(a) Find the coordinates of A, B and C. [3]

Solution	Marks
$(2x-1)(3-x^2) = 0$ $A = \left(-\sqrt{3}, 0\right)$ $B = (\frac{1}{2}, 0)$ $C = (\sqrt{3}, 0)$	M1 A1 three correct coordinates (accept 3 sf) A1 correct reference to the diagram -1 if not in coordinate form

(b) Find the coordinates of M. (You are not required to prove that it is the minimum point.) [3]

Solution	Marks
$y = 2x^3 + x^2 + 6x - 3$ $\frac{dy}{dx} = -6x^2 + 2x + 6$ $6x^2 + 2x + 6 = 0$ $x = \frac{2 + \sqrt{4 \quad 4(\quad 6)(6)}}{12}$ $= 0.847$ or 1.18 coordinates of M $(-0.847, 6.15)$	M1 correct derivative M1 '= 0' A1

(c) Hence, find the area of the shaded region. [6]

Solution	Marks
Area of the shaded region $=\frac{1}{2}\times 6.1493\times 0.88505$ $-\int_{0.847}^{0.5}\left(-2x^3+x^2+6x-3\right)\,dx$ $+\int_{0.5}^{\sqrt{3}}\left(-2x^3+x^2+6x-3\right)\,dx$	M1,M1,M1 each part
$=2.72122-\left[-\frac{1}{2}x^4+\frac{1}{3}x^3+3x^2-3x\right]_{0.847}^{0.5}+\left[-\frac{1}{2}x^4+\frac{1}{3}x^3+3x^2-3x\right]_{0.5}^{\sqrt{3}}$	M1 correct integral
$=2.72122-\left(-\frac{71}{96}-4.23334\right)+\left(1.03589+\frac{71}{96}\right)$	M1 correct substitution
$=9.47$ units^2 (3s.f.)	A1