Name:____________________________________ Class:_____________

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2022

MATHEMATICS
Higher 2

9758/01

16 September 2022

Paper 1

3 hours

Candidates answer on the Question Paper.

Additional materials: List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given by [ ] at the end of each question or part question.

For Candidate's Use	For Examiner's Use
Question Number	**Marks Obtained**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total Marks	/ 100

This document consists of **5** printed pages.

[Turn over

1 A function f is defined by $\mathrm{f}(x)=ax^3+bx^2+cx+d$. The graph of $y=\mathrm{f}(x)$ passes through $(1,-19)$ and has a maximum point $(-1,\ 13)$. Given that $\int_{-1}^{0}\mathrm{f}(x)\,\mathrm{d}x=9.5$, find the values of a, b, c and d. [5]

2 Given that $\mathrm{f}(x)=\dfrac{ax^2+3ax+10}{x+2}, x\in\mathbb{R}, x\neq -2$, where a is a constant.

(i) Given that $a=3$, solve $\mathrm{f}(x)\geq 2x+6$. [3]

(ii) Find the set of values of a such that $\mathrm{f}'(x)>0$ for all real values of x, $x\neq -2$. [4]

3 A curve C has equation $y=\dfrac{1}{x^2-6ax}$, where $a>0$.

(i) Sketch the curve C and give the equations of any asymptotes and the coordinates of any turning points in terms of a where appropriate. [4]

(ii) Describe the transformation that maps the graph of C onto the graph of $y=\dfrac{1}{x^2-9a^2}$. [2]

4 **(i)** Express $\dfrac{1}{(2r+1)(2r+3)}$ in the form $\dfrac{A}{2r+1}+\dfrac{B}{2r+3}$ where A and B are constants to be determined. [2]

(ii) Hence find the sum of the series

$$\frac{1}{(3)(5)}+\frac{1}{(5)(7)}+\frac{1}{(7)(9)}+\frac{1}{(9)(11)}+\frac{1}{(11)(13)}+\ldots\frac{1}{(2n+1)(2n+3)},$$

giving your answer in the form $k-\mathrm{f}(n)$, where k is a constant and $\mathrm{f}(n)$ is a function in n to be determined. [3]

(iii) Give a reason why the series

$$\frac{1}{(3)(5)}+\frac{1}{(5)(7)}+\frac{1}{(7)(9)}+\frac{1}{(9)(11)}+\frac{1}{(11)(13)}+\ldots$$

converges and write down the value of the sum to infinity. [2]

(iv) Hence find

$$\frac{1}{(10)(14)}+\frac{1}{(14)(18)}+\frac{1}{(18)(22)}+\frac{1}{(22)(26)}+\ldots.$$ [3]

5 **(a)** Find $\int 3\sin x\cos 3x \, dx$. [2]

(b) Use the substitution $\theta = \sqrt{x}$ to find the exact value of $\int_{\sqrt{\frac{\pi}{4}}}^{\sqrt{\pi}} \theta^3 \sin\left(\theta^2\right) d\theta$. [5]

6 **(i)** By means of the substitution $u = xy$, express the differential equation

$$x\frac{dy}{dx} + y - 2(xy)^2 = 0$$

into the form $\frac{du}{dx} = f(u)$, where f(u) is a function in u to be found. [2]

(ii) Hence find the general solution of y in terms of x. [3]

(iii) Find the equation of the solution curve that passes through $\left(1, \frac{1}{2}\right)$. [1]

(iv) State a particular solution for which the solution curve has no stationary point. [1]

7 The equation of a curve C is $2x^3 + y^3 - 3xy = k$, where k is a constant.

(i) Find $\frac{dy}{dx}$ in terms of x and y. [2]

It is given that C has a tangent which is parallel to the y-axis.

(ii) Show that the y-coordinate of the point of contact of the tangent with C must satisfy

$$ay^6 + by^3 - k = 0,$$

where the constants a and b are to be determined. [3]

(i) Hence, find the values of k when the line $x = 1$ is a tangent to the curve C. [3]

8 Given that $y = \frac{\ln\sqrt{1-x}}{2+x}$, where $-1 \le x < 1$, show that

$$2y + 2(2+x)\frac{dy}{dx} + \frac{1}{1-x} = 0.$$ [2]

(i) By further differentiation, find the Maclaurin's series for y up to and including the term in x^3. [5]

(ii) Verify that the same result can be obtained if the standard series expansions are used. [3]

(iii) By substituting $x = -1$ to your result, find an approximate value for ln 2, giving your answer to 4 decimal places. [2]

9 **(a)** The functions f and g are defined by

$$\mathrm{f} : x \to \frac{x-2}{x+2}\ ,\ x \in \mathbb{R},\ x \neq -2\ ,$$
$$\mathrm{g} : x \to\ -x^2\ ,\ x \in \mathbb{R},\ x < -\sqrt{2}.$$

(i) Find $\mathrm{f}^{-1}(x)$ and state its domain. [3]

(ii) Find an expression for fg(x) and hence, or otherwise, find $(\mathrm{fg})^{-1}(3)$. [4]

(b) It is given that

$$\mathrm{h}(x) = \begin{cases} -4x-12 & \text{for } -4 \leq x \leq -2, \\ x|x| & \text{for } -2 < x \leq 2 \end{cases}$$

and that $\mathrm{h}(x) = \mathrm{h}(x+6)$ for all real values of x.

(i) Evaluate $\mathrm{h}(-4)$ and $\mathrm{h}(12)$. [2]

(ii) Sketch the graph of $y = \mathrm{h}(x)$ for $-4 \leq x \leq 6$ and explain why h has no inverse. [4]

10 A customer owes a bank \$15 000. In the middle of every month, the customer pays \$$x$ to the bank where $x \leq 1000$. At the end of every month, the bank adds interest at a rate of 4% of the remaining amount still owed. This process continues every month until the money owed is repaid in full.

(i) Find the value of x, for which the customer still owes \$15 000 at the end of the first month. [3]

(ii) Show that the amount owed at the end of the nth month is

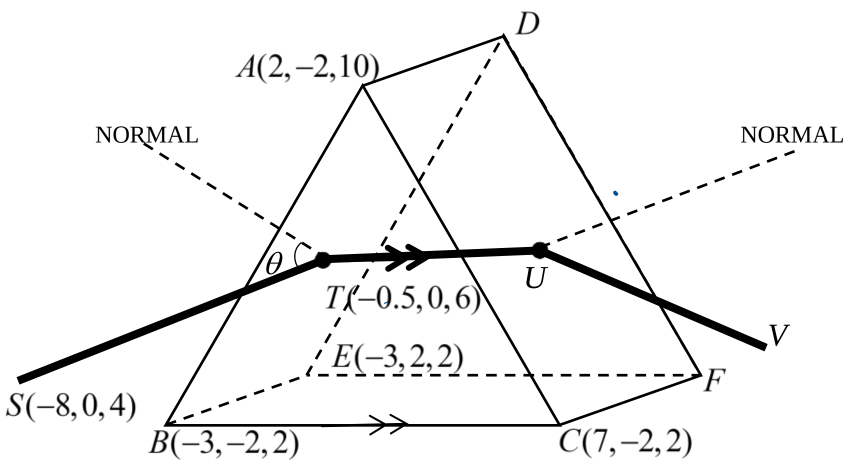
$$(1.04)^n(15\ 000) - kx(1.04^n - 1),$$

where k is a constant to be determined. [4]

(iii) Find the amount that the customer still owes the bank at the beginning of the 13th month if x = 1000. [2]

(iv) Find the least number of months required to repay the loan, given that x = 800. [3]

11



A light ray passes from air into a material made into a triangular prism *ABCFDE* with triangular sides *ABC* and *DEF* and rectangular sides *ABED*, *ACFD* and *BCFE*. The coordinates of the vertices *A*, *B*, *C* and *E* are shown in the diagram. A ray of light is sent from a monochromatic light source at point $S(-8, 0, 4)$ to enter the prism at point $T(-0.5, 0, 6)$. It then emerges at point *U* and is picked up by a sensor at point *V*. The refracted ray *TU* is parallel to the side *BC* and the equation of the plane *ADFC* is given by $8x + 5z = 66$.

(i) Find a vector equation of the plane *ABED* in scalar product form. [3]

(ii) Find the angle of incidence θ, the acute angle *ST* makes with the normal of the plane *ABED*. [3]

(iii) Find the coordinates of *U*. [4]

(iv) Find the shortest distance from *T* to the plane *ADFC*. [3]

Name:____________________________________ **Class:**_____________

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2022

MATHEMATICS
Higher 2

Paper 2

9758/02
20 September 2022
3 hours

Candidates answer on the Question Paper.

Additional materials: List of Formulae (MF 26)

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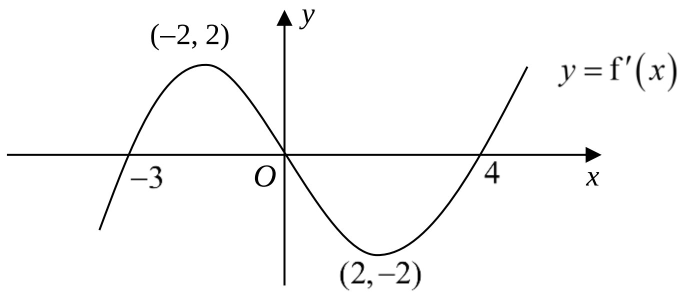
Section A: Pure Mathematics [40 marks]

1 **(a)** The diagram shows the **derivative** graph of $y = \mathrm{f}(x)$.



Justifying your answers, find the range of values of x for which the graph $y = \mathrm{f}(x)$ is

(i) decreasing, [2]

(ii) increasing and concave downwards. [2]

(b) [It is given that the volume and surface area of a sphere of radius r is $\frac{4}{3}\pi r^3$ and $4\pi r^2$ respectively.]

Air is pumped into a spherical balloon at a constant rate of $12\pi\,\text{cm}^3$ per second.

(i) Find the rate of increase of the balloon's surface area when the volume is $\frac{256}{3}\pi\ \text{cm}^3$. [4]

(ii) Show that the rate of increase of the balloon's radius is inversely proportional to the surface area of the balloon. [1]

2 **(a)** With reference to the origin O, the points A, B and X are $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$ and $\overrightarrow{OX}=\frac{1}{8}\mathbf{a}+\frac{3}{8}\mathbf{b}$. The point Y lies on AB such that O, X and Y are collinear. Express $\overrightarrow{OY}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ and find the ratio of $AY:YB$. [5]

(b) The points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively. P and Q are fixed and R varies. Describe geometrically the set of possible positions of the point R such that

(i) $(\mathbf{r}-\mathbf{p})\times\mathbf{q}=\mathbf{0}$, [2]

(ii) $(\mathbf{r}-\mathbf{p})\bullet\mathbf{q}=0$. [2]

3 **Do not use a calculator in answering this question.**

(a) If $z = 1 + \mathrm{i}$ is a root of the equation $z^4+4z^2-8z+12=0$, find the other roots. [4]

(b) By expressing in the exponential form or otherwise, show that

$$\frac{1+\sin\frac{3\pi}{8}+\mathrm{i}\cos\frac{3\pi}{8}}{1+\sin\frac{3\pi}{8}-\mathrm{i}\cos\frac{3\pi}{8}}=\cos\frac{\pi}{8}+\mathrm{i}\sin\frac{\pi}{8}.$$ [3]

Hence find the two smallest positive integer values of n for which

$$\left(\frac{1+\sin\frac{3\pi}{8}+\mathrm{i}\cos\frac{3\pi}{8}}{1+\sin\frac{3\pi}{8}-\mathrm{i}\cos\frac{3\pi}{8}}\right)^n-\mathrm{i}=0.$$ [3]

4 A curve C is given by the parametric equations

$$x = 2 + 2\sin\theta, \quad y = 2\cos\theta + \sin 2\theta, \quad \text{for } -\pi < \theta \le \pi .$$

(i) Sketch the curve, indicating clearly the coordinates of the axial intercepts. [2]

(ii) Find the exact area bounded by the curve. [5]

(iii) Verify that $y = x\cos\theta$.

Deduce that the Cartesian equation of the curve C is

$$4y^2 = 4x^3 - x^4 .$$ [3]

(iv) Find the volume of the solid of revolution formed when the curve C is rotated π radians about the x-axis. [2]

Section B: Probability and Statistics [60 marks]

5 A bag contains 2 fair tetrahedral dice. The first die has faces labelled 1, 1, 2, and 3 and the second die has faces labelled 1, 2, 3 and 3. A die is taken at random from the bag and thrown. The score, W is defined as follows:

If the first die is picked and thrown, the score is defined as twice the number which appears on its base.

If the second die is picked and thrown, the score is the number which appears on its base.

(i) Show that $P(W = 2) = \frac{3}{8}$ and find the probability distribution of W. [3]

(ii) Find $E(W)$ and $Var(W)$. [4]

6 A computer is used to generate codes consisting of four letters followed by two digits. Each of the four letters generated is equally likely to be any of the twenty-six letters of the alphabet "*A* – *Z* ". Each of the two digits generated is equally likely to be any of the ten digits "0 – 9".

Find the probability that a randomly chosen code has

(i) four different letters and two different digits, [2]

(ii) two different consonants and two different vowels, where the consonants and vowels alternate, [3]

(iii) two letters the same, two letters different and two digits the same. [3]

7 **(a)** Draw separate scatter diagrams, each with 8 data points, all in the first quadrant which represent the situation where the product moment correlation coefficient between variables x and y is

(i) between -0.8 and -0.5,

(ii) 0. [2]

(b) In a chemical reaction, the concentration, y grams/ litre of a particular reactant at time x minutes is given in this table. The product moment correlation coefficient for this data is –0.9811.

x (min)	5	10	15	20	25	30	35	40
y (g/ litre)	6	5.75	5.66	5.51	5.39	5.31	5.26	5.15

(i) Sketch a scatter diagram of y against x for the data given in the table. [1]

(ii) A student attempts to model the relationship between y and x with a straight line, explain whether this is likely to provide a good model. [1]

(iii) By using the model $\frac{1}{y} = ax + b$, where a and b are constants to be found, write down the equation for the relationship between x and y. State the product moment correlation coefficient for this model. [3]

(iv) Using the equation found in **(iii)**, estimate the time taken for the concentration to reach 5.4 g / litre. Comment on whether we should use an equation of the form $x = \frac{c}{y} + d$ to find the estimate instead. [2]

8 On average, 72 out of 100 students in a school complete online Mathematics homework by the deadline. The number of students in a class of n students who complete online Mathematics homework by the deadline is denoted by M. State, in context, two assumptions needed for M to be well-modelled by a binomial distribution. [2]

Assume now that M has a binomial distribution.

(a) By taking $n = 25$,

(i) find the probability that fewer than 15 students in a class complete the online Mathematics homework by the deadline. [2]

(ii) 10 classes with n students each are chosen at random from the school. Find the probability that exactly 3 of these classes have fewer than 15 students who complete the online Mathematics homework by the deadline. [2]

(iii) 30 classes with n students each are chosen at random from the school. Find the probability that the mean number of students who complete the online Mathematics homework by the deadline per class is more than 19. [3]

(b) Find the least value of n such that there is greater than 70% chance that at least 20 students in a class complete the online Mathematics homework by the deadline.

[3]

9 A company administers two aptitude tests to the job applicants. One test measures verbal ability while the other measures written ability. Based on past experience, the verbal ability score V and the written ability score W are independent and normally distributed with means and standard deviations shown in the following table:

Test score	Mean	Standard deviation
V	56	8
W	60	12

(i) A female applicant and a male applicant are randomly chosen. Find the probability that the female's verbal score is less than 55 and the male's written score is more than 55. [2]

(ii) Three females and one male are randomly chosen. Find the probability that the females' total verbal score is within 15 marks of thrice the male's written score. [4]

(iii) Five job applicants' verbal scores and six job applicants' written scores are observed. Given that M is the average score of these applicants, find $\mathrm{E}(M)$ and $\mathrm{Var}(M)$. Hence find the probability that M is less than 60. [3]

(iv) The company's manager set a criterion for an interview based on a combined score T where $T = 2V + W$. Find the value of t if 35% of the job applicants have a combined score exceeding t. [3]

10 The Electronic Road Pricing (ERP) system is the primary method of regulating traffic in Singapore. ERP rates are determined based on traffic conditions. If the average traffic speed rises above 45 km/h on expressways, ERP charges at that gantry will be reduced. Conversely, ERP rates will be increased if traffic moves slower than the average speed of 45 km/h on expressways.

(a) The authority reviewed the traffic conditions on a particular expressway by measuring the speeds of 150 randomly selected cars as they pass a speed camera. The speed, x km/h was recorded. The results are summarised by

$$\sum(x-40) = 585, \qquad \sum(x-40)^2 = 10998.$$

(i) Find the unbiased estimates of the population mean and variance of the speed of a car on the expressway. [3]

(ii) Test, at the 5% level of significance, whether the ERP rate needs to be increased for this expressway. [4]

(iii) Explain the meaning of 'p-value' in the context of the question in **(ii)**. [1]

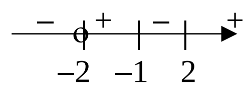
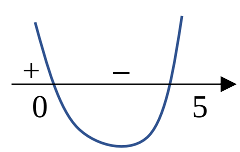
(b) A new speed camera is installed along the expressway and it is now known that the population standard deviation of the speed of a car along this expressway is 7.79 km/h. A large random sample of n cars is observed and the average speed of 45.9 km/h is recorded. A new test is carried out at 5% level of significance. Find the least value of n that results in the reduction of the ERP rate along this expressway. [4]

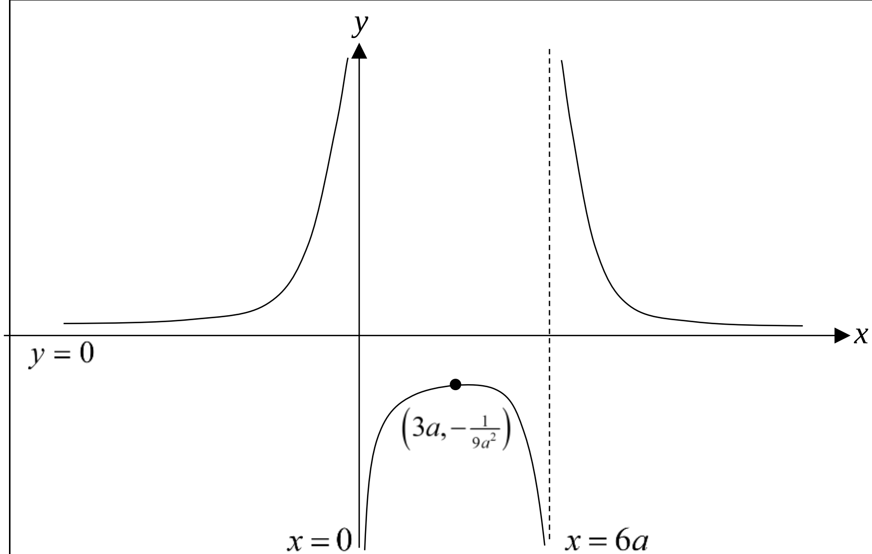
2022 J2 H2 Mathematics Preliminary Examination P1 (Worked Solutions)

<table>
<tr><th>Qn</th><th>Solution</th><th>Notes</th></tr>
<tr><td rowspan="3">1</td><td>$a+b+c+d=-19----(1)$
$a(-1)^3+b(-1)^2+c(-1)+d=13$
$-a+b-c+d=13----(2)$

$\mathrm{f}'(x)=3ax^2+2bx+c$
$0=3a(1)+2b(-1)+c$
$3a-2b+c=0----(3)$</td><td></td></tr>
<tr><td>$\int_{-1}^{0} ax^3+bx^2+cx+d \ \mathrm{d}x=9.5$
$\left[\frac{ax^4}{4}+\frac{bx^3}{3}+\frac{cx^2}{2}+dx\right]_{-1}^{0}=9.5$
$0-\left(\frac{1}{4}a-\frac{1}{3}b+\frac{1}{2}c-d\right)=9.5$
$-\frac{1}{4}a+\frac{1}{3}b-\frac{1}{2}c+d=9.5----(4)$</td><td></td></tr>
<tr><td>Using GC,
$a=2, \quad b=-6 \quad c=-18, \quad d=3$</td><td></td></tr>
</table>

2022 J2 H2 Mathematics Preliminary Examination P1 (Worked Solutions)

Qn	Solution	Notes
2(i)	$\dfrac{3x^2+9x+10}{x+2} \geq 2x+6$ $\dfrac{3x^2+9x+10-(2x+6)(x+2)}{x+2} \geq 0$ $\dfrac{3x^2+9x+10-(2x^2+10x+12)}{x+2} \geq 0$ $\dfrac{x^2-x-2}{x+2} \geq 0$ $\dfrac{(x-2)(x+1)}{x+2} \geq 0$  $-2 < x \leq -1$ or $x \geq 2$	
(ii)	$\mathrm{f}(x) = \dfrac{ax^2+3ax+10}{x+2}$ $\mathrm{f}'(x) = \dfrac{(x+2)(2ax+3a)-(ax^2+3ax+10)}{(x+2)^2}$ $= \dfrac{ax^2+4ax+6a-10}{(x+2)^2}$	
	$\dfrac{ax^2+4ax+6a-10}{(x+2)^2} > 0$ Since $(x+2)^2 > 0$ for all real values of x, $x \neq -2$ We need $ax^2+4ax+6a-10 > 0$ for all real values of x, $x \neq -2$ $a > 0$ & Discriminant < 0 $(4a)^2 - 4a(6a-10) < 0$ $16a^2 - 24a^2 + 40a < 0$ $8a^2 - 40a > 0$ $(a)(a-5) > 0$  $a < 0$ or $a > 5$ (N.A. since $a > 0$)	

Qn	Solution	Notes
3(i)	Asymptotes: $y=0$, $x=0$, $x=6a$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\left(x^2-6ax\right)^{-2}\left(2x-6a\right)=-\frac{2x-6a}{\left(x^2-6ax\right)^2}=0$ $2x-6a=0$ $x=3a$ $y=-\frac{1}{9a^2}$	
		
(ii)	$y=\frac{1}{x^2-6ax}=\frac{1}{(x-3a)^2-9a^2}$	
	$y=\frac{1}{(x-3a)^2-9a^2}\rightarrow y=\frac{1}{x^2-9a^2}$ Replace x by $x+3a$ <u>Translation</u> of $y=\frac{1}{x^2-6ax}$ by <u>$3a$ units</u> in the <u>negative x-direction</u> to get $y=\frac{1}{x^2-9a^2}$.	

	Alternatively, $y=\dfrac{1}{x^2-6ax}=\dfrac{1}{x(x-6a)}$ $y=\dfrac{1}{x^2-9a^2}=\dfrac{1}{(x+3a)(x-3a)}$	
	$y=\dfrac{1}{x(x-6a)} \rightarrow y=\dfrac{1}{(x+3a)(x-3a)}$ Replace x by $x+3a$ Translation of $y=\dfrac{1}{x^2-6ax}$ by $3a$ units in the negative x-direction to get $y=\dfrac{1}{x^2-9a^2}$.	

2022 J2 H2 Mathematics Preliminary Examination P1 (Worked Solutions)

Qn	Solutions	Notes
4(i)	$\dfrac{1}{(2r+1)(2r+3)}=\dfrac{A}{2r+1}+\dfrac{B}{2r+3}=\dfrac{A(2r+3)+B(2r+1)}{(2r+1)(2r+3)}$ Subs $r=-\dfrac{3}{2}$ $\quad B=-\dfrac{1}{2}$ $r=-\dfrac{1}{2}$ $\quad A=\dfrac{1}{2}$ $\dfrac{1}{(2r+1)(2r+3)}=\dfrac{\frac{1}{2}}{(2r+1)}-\dfrac{\frac{1}{2}}{(2r+3)}$ $=\dfrac{1}{2(2r+1)}-\dfrac{1}{2(2r+3)}$ $=\dfrac{1}{2}\left[\dfrac{1}{(2r+1)}-\dfrac{1}{(2r+3)}\right]$	
(ii)	$\displaystyle\sum_{r=1}^{n}\frac{1}{(2r+1)(2r+3)}=\frac{1}{2}\sum_{r=1}^{n}\left[\frac{1}{2r+1}-\frac{1}{2r+3}\right]$ $=\dfrac{1}{2}\left[\begin{array}{lcl} & \frac{1}{3} & - & \cancel{\frac{1}{5}} \\ + & \cancel{\frac{1}{5}} & - & \cancel{\frac{1}{7}} \\ + & \cancel{\frac{1}{7}} & - & \cancel{\frac{1}{9}} \\ & & \cdot & \\ & & \cdot & \\ & & \cdot & \\ + & \cancel{\frac{1}{2n-1}} & - & \cancel{\frac{1}{2n+1}} \\ + & \cancel{\frac{1}{2n+1}} & - & \frac{1}{2n+3} \end{array}\right]$ $=\dfrac{1}{2}\left[\dfrac{1}{3}-\dfrac{1}{2n+3}\right]$ $=\dfrac{1}{6}-\dfrac{1}{2}\left(\dfrac{1}{2n+3}\right)$	

(iii)	$\frac{1}{(3)(5)}+\frac{1}{(5)(7)}+\frac{1}{(7)(9)}+\frac{1}{(9)(11)}+\frac{1}{(11)(13)}+\ldots$ $=\sum_{r=1}^{\infty}\frac{1}{(2r+1)(2r+3)}$ As $n\to\infty$, $\frac{1}{2n+3}\to 0$ $\sum_{r=1}^{n}\frac{1}{(2r+1)(2r+3)}\to\frac{1}{6}$ (finite number) Hence the series converges. $\therefore S_\infty=\frac{1}{6}$	
(iv)	$\frac{1}{(10)(14)}+\frac{1}{(14)(18)}+\frac{1}{(18)(22)}+\frac{1}{(22)(26)}+\ldots$ $=\frac{1}{4}\left[\frac{1}{(5)(7)}+\frac{1}{(7)(9)}+\frac{1}{(9)(11)}+\frac{1}{(11)(13)}\ldots\right]$ $=\frac{1}{4}\sum_{r=2}^{\infty}\frac{1}{(2r+1)(2r+3)}$ $=\frac{1}{4}\left[S_\infty-S_1\right]$ $=\frac{1}{4}\left[\frac{1}{6}-\frac{1}{(3)(5)}\right]$ $=\frac{1}{40}$	

Qn	Solution	Notes
5(a)	$\int 3\sin x\cos 3x\ \mathrm{d}x = \int 3\cos 3x\sin x\ \mathrm{d}x$ $= \frac{3}{2}\int(\sin 4x - \sin 2x)\ \mathrm{d}x$ $= -\frac{3}{8}\cos 4x + \frac{3}{4}\cos 2x + c$	
(b)	$\theta = \sqrt{x} \Rightarrow \theta^2 = x$ $2\theta\frac{\mathrm{d}\theta}{\mathrm{d}x} = 1$ when $\theta = \sqrt{\frac{\pi}{4}}$, $x = \frac{\pi}{4}$; when $\theta = \sqrt{\pi}$, $x = \pi$	
	$\int_{\sqrt{\frac{\pi}{4}}}^{\sqrt{\pi}} \theta^3\sin(\theta^2)\ \mathrm{d}\theta$ $= \frac{1}{2}\int_{\sqrt{\frac{\pi}{4}}}^{\sqrt{\pi}} 2\theta(\theta^2)\sin(\theta^2)\ \mathrm{d}\theta$ $= \frac{1}{2}\int_{\frac{\pi}{4}}^{\pi} x\sin x\ \mathrm{d}x$ $u = x \quad \frac{\mathrm{d}v}{\mathrm{d}x} = \sin x$ $\Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1 \quad \Rightarrow v = -\cos x$ $= \frac{1}{2}\left[\left[-x\cos x\right]_{\frac{\pi}{4}}^{\pi} - \int_{\frac{\pi}{4}}^{\pi}(-\cos x)\ \mathrm{d}x\right]$ $= \frac{1}{2}\left(\pi + \frac{\pi}{4\sqrt{2}} + \left[\sin x\right]_{\frac{\pi}{4}}^{\pi}\right)$ $= \frac{1}{2}\left(\pi + \frac{\pi}{4\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$	

Qn	Solution	Notes
6(i)	$u = xy$ $\dfrac{\mathrm{d}u}{\mathrm{d}x} = x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y$ Substituting into $x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y - 2(xy)^2 = 0$ $\dfrac{\mathrm{d}u}{\mathrm{d}x} - 2u^2 = 0 \quad \Rightarrow \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = 2u^2$	
(ii)	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = 2u^2$ $\displaystyle\int \frac{1}{u^2}\ \mathrm{d}u = \int 2\ \mathrm{d}x$ $\Rightarrow -\dfrac{1}{u} = 2x + c$, where c is an arbitrary constant $y = -\dfrac{1}{x(2x+c)}$	
(iii)	When $x = 1$, $y = \dfrac{1}{2}$, $c = -4$ $y = -\dfrac{1}{x(2x-4)} = -\dfrac{1}{2x(x-2)}$	
(iv)	Curve has no stationary point when $c = 0$ i.e. $y = -\dfrac{1}{2x^2}$	

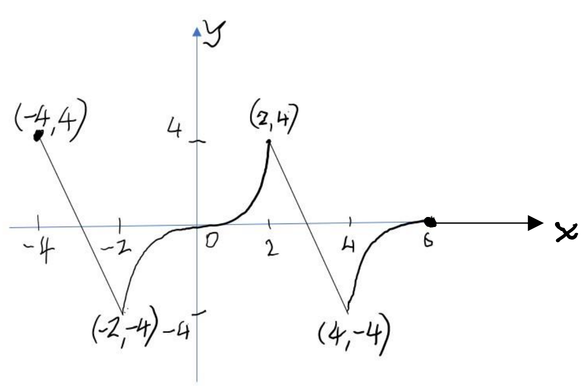
2022 J2 H2 Mathematics Preliminary Examination P1 (Worked Solutions)

Qn	Solution	Notes
7(i)	$2x^3+y^3-3xy=k$ Differentiate w.r.t. x, $6x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}-3x\frac{\mathrm{d}y}{\mathrm{d}x}-3y=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3y-6x^2}{3y^2-3x}=\frac{y-2x^2}{y^2-x}$	
(ii)	When tangent is parallel to the y-axis, $y^2-x=0$	
	Sub $x=y^2$ into $2x^3+y^3-3xy=k$ $2(y^2)^3+y^3-3(y^2)y=k$ $2y^6-2y^3-k=0$ i.e. a = 2, b = −2	
(iii)	line x = 1 is a tangent to the curve C $y^2=1$ $y=\pm1$	
	When $y=1$, $2-2-k=0$ $k=0$	
	When $y=-1$, $2+2-k=0$ $k=4$	

Qn	Solution	Notes
8	$y=\frac{\ln\sqrt{1-x}}{2+x} \quad \Rightarrow \quad y(2+x)=\frac{1}{2}\ln(1-x)$ Differentiate w.r.t. x $y+(2+x)\frac{dy}{dx}=-\frac{1}{2(1-x)}$ $2y+2(2+x)\frac{dy}{dx}+\frac{1}{1-x}=0$	
(i)	Differentiate w.r.t. x $2\frac{dy}{dx}+2\frac{dy}{dx}+2(2+x)\frac{d^2y}{dx^2}+\frac{1}{(1-x)^2}=0$ $4\frac{dy}{dx}+2(2+x)\frac{d^2y}{dx^2}+\frac{1}{(1-x)^2}=0$	
	Differentiate w.r.t. x $4\frac{d^2y}{dx^2}+2\frac{d^2y}{dx^2}+2(2+x)\frac{d^3y}{dx^3}+\frac{2}{(1-x)^3}=0$	
	when x = 0, f(0) = 0, $f'(0)=-\frac{1}{4}$, $f''(0)=0$, $f'''(0)=-\frac{1}{2}$	
	By Maclaurin's series, $y=0+\left(-\frac{1}{4}\right)x+(0)\frac{x^2}{2!}+\left(-\frac{1}{2}\right)\frac{x^3}{3!}+\ldots$ $\approx -\frac{1}{4}x-\frac{1}{12}x^3$	
(ii)	$y=\frac{\ln\sqrt{1-x}}{2+x}=\frac{1}{2}\ln(1-x)\cdot\frac{1}{2}\left(1+\frac{x}{2}\right)^{-1}$ $=\frac{1}{2}\ln(1-x)\cdot\frac{1}{2}\left(1+\frac{x}{2}\right)^{-1}$ $=\frac{1}{4}\left(-x-\frac{x^2}{2}-\frac{x^3}{3}+\ldots.\right)\left(1-\frac{x}{2}+\frac{x^2}{4}-\frac{x^3}{8}+\ldots\right)$ $=\frac{1}{4}\left(-x+\frac{x^2}{2}-\frac{x^3}{4}-\frac{x^2}{2}+\frac{x^3}{4}-\frac{x^3}{3}+\ldots.\right)$ $\approx -\frac{1}{4}x-\frac{1}{12}x^3$	

Qn	Solution	
(iii)	When $x = -1$, LHS $= \ln\sqrt{2} = \frac{1}{2}\ln 2$ RHS $= -\frac{1}{4}(-1) - \frac{1}{12}(-1)^3 = \frac{1}{3}$ $= 0.333333$ $\therefore \ln 2 = 2(0.333333) \approx 0.6667$ (4 d.p.)	

Qn	Solutions	Notes
9(a)(i)	$y = \frac{x-2}{x+2}$ Making x the subject, $x = \frac{2+2y}{1-y}$ $\mathrm{f}^{-1}(x) = \frac{2+2x}{1-x}, \quad x \neq 1$	
(ii)	$(\mathrm{fg})(x) = \mathrm{f}\left(\mathrm{g}(x)\right)$ $= \mathrm{f}\,(-x^2)$ $= \frac{-x^2-2}{-x^2+2}$	
	Method 1: Let $(\mathrm{fg})^{-1}(3) = p$ $\therefore (\mathrm{fg})(p) = 3$ $\frac{-p^2-2}{-p^2+2} = 3$ $-p^2 - 2 = -3p^2 + 6$	
	$p^2 = 4$ $p = \pm 2$ Reject $p = 2$ Hence $(\mathrm{fg})^{-1}(3) = -2$	

	Method 2: $\text{Let } y=\dfrac{-x^2-2}{-x^2+2}$ $-yx^2+2y=-x^2-2$ $x^2(1-y)=-2(y+1)$ $x^2=\dfrac{2(y+1)}{y-1}$ $x=\pm\sqrt{\dfrac{2(y+1)}{y-1}},\quad \text{reject } \sqrt{\dfrac{2(y+1)}{y-1}} \text{ since } x<-\sqrt{2}$ $(\mathrm{fg})^{-1}(x)=-\sqrt{\dfrac{2(x+1)}{x-1}}$ $(\mathrm{fg})^{-1}(3)=-2$	
	$\text{Let } y=\dfrac{-x^2-2}{-x^2+2}$ $-yx^2+2y=-x^2-2$ $x^2(1-y)=-2(y+1)$ $x^2=\dfrac{2(y+1)}{y-1}$ $x=\pm\sqrt{\dfrac{2(y+1)}{y-1}},\quad \text{reject } \sqrt{\dfrac{2(y+1)}{y-1}} \text{ since } x<-\sqrt{2}$ $(\mathrm{fg})^{-1}(x)=-\sqrt{\dfrac{2(y+1)}{y-1}}$ $(\mathrm{fg})^{-1}(3)=-2$	
(b)(i)	$\mathrm{h}(-4) = -4(-4)-12=4$ h(12) = h(6) = h(0) = 0	
(ii)	 The line $y = 0$ cuts $y=\mathrm{h}(x)$ at two or more points. Hence h does not have an inverse.	

2022 J2 H2 Mathematics Preliminary Examination P1 (Worked Solutions)

<table>
<tr><th>Qn</th><th>Solutions</th><th>Notes</th></tr>
<tr><td>10(i)</td><td>At the start of month 1, customer owes $15 000
At mid month 1, customer owes $15 000 – x
At the end of month 1, he owes 1.04 (15000 – x)

$15000 = 1.04(15000 - x)$

$x = 576.923 \approx \$576.92$</td><td></td></tr>
<tr><td>(ii)</td><td>
<table>
<tr><th>Month</th><th>Amount owed in the middle of the month</th><th>Amount owed on the last day of the month</th></tr>
<tr><td>1</td><td>$15000 - x$</td><td>$1.04(15000 - x)$</td></tr>
<tr><td>2</td><td>$1.04(15000 - x) - x$</td><td>$1.04[1.04(15000 - x) - x]$
$= (1.04)^2(15000) - (1.04)^2x - 1.04x$</td></tr>
<tr><td>3</td><td>$(1.04)^2(15000) - (1.04)^2x - 1.04x - x$</td><td>$(1.04)^3(15000) - (1.04)^3x - (1.04)^2x - 1.04x$</td></tr>
</table>

At the nth month, the customer owes

$(1.04)^n(15000) - (1.04)^n x - (1.04)^{n-1}x - \ldots(1.04)x$
$= (1.04)^n(15000) - x[1.04 + 1.04^2 + \ldots + 1.04^n]$

$= (1.04)^n(15000) - \dfrac{x(1.04)[(1.04)^n - 1]}{0.04}$
$= (1.04)^n(15000) - 26x(1.04^n - 1)$

$k = 26$</td><td></td></tr>
<tr><td>(iii)</td><td>From
$(1.04)^n(15000) - 26x(1.04^n - 1)$

Substituting $n = 12$
$(1.04)^{12}(15000) - 26(1000)(1.04^{12} - 1)$
$= \$8388.65$</td><td></td></tr>
</table>

(iv)	Using the amount owed at the end of the nth month, $(1.04)^n(15000)-26x\left[(1.04)^n-1\right]\le 0$ $(1.04)^n(15000)-26(800)\left[(1.04)^n-1\right]\le 0$ $n\ge 32.56$ Least number of months = 33	

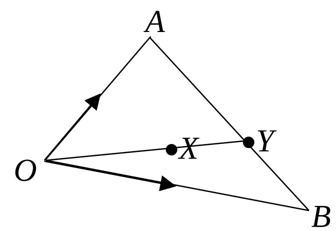
Qn	Solution	Notes
11(i)	$\overrightarrow{AB}=\begin{pmatrix}-3\\-2\\2\end{pmatrix}-\begin{pmatrix}2\\-2\\10\end{pmatrix}=\begin{pmatrix}-5\\0\\-8\end{pmatrix}=-\begin{pmatrix}5\\0\\8\end{pmatrix}$ $\overrightarrow{BE}=\begin{pmatrix}-3\\2\\2\end{pmatrix}-\begin{pmatrix}-3\\-2\\2\end{pmatrix}=\begin{pmatrix}0\\4\\0\end{pmatrix}=4\begin{pmatrix}0\\1\\0\end{pmatrix}$ A normal to the plane $\begin{pmatrix}5\\0\\8\end{pmatrix}\times\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}-8\\0\\5\end{pmatrix}$	
	Plane $ABED$ $\mathbf{r}\bullet\begin{pmatrix}-8\\0\\5\end{pmatrix}=\begin{pmatrix}2\\-2\\10\end{pmatrix}\bullet\begin{pmatrix}-8\\0\\5\end{pmatrix}=34$ $\mathbf{r}\bullet\begin{pmatrix}-8\\0\\5\end{pmatrix}=34$	

(ii)	$\overrightarrow{ST}=\begin{pmatrix}-0.5\\0\\6\end{pmatrix}-\begin{pmatrix}-8\\0\\4\end{pmatrix}=\begin{pmatrix}7.5\\0\\2\end{pmatrix}$ $\cos\theta=\dfrac{\left\|\begin{pmatrix}7.5\\0\\2\end{pmatrix}\bullet\begin{pmatrix}-8\\0\\5\end{pmatrix}\right\|}{\sqrt{7.5^2+2^2}\sqrt{8^2+5^2}}$ $=\dfrac{50}{\sqrt{\frac{241}{4}}\sqrt{89}}=0.682880$ $\theta=46.9^{\circ}$	
(iii)	$\overrightarrow{BC}=\begin{pmatrix}7\\-2\\2\end{pmatrix}-\begin{pmatrix}-3\\-2\\2\end{pmatrix}=\begin{pmatrix}10\\0\\0\end{pmatrix}=10\begin{pmatrix}1\\0\\0\end{pmatrix}$ Line *TU* $\mathbf{r}=\begin{pmatrix}-0.5\\0\\6\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\0\end{pmatrix},\lambda\in\mathbb{R}$	
	Plane *ADFC* is given by $\mathbf{r}\bullet\begin{pmatrix}8\\0\\5\end{pmatrix}=66$. Coordinates of *U*: $\left[\begin{pmatrix}-0.5\\0\\6\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\bullet\begin{pmatrix}8\\0\\5\end{pmatrix}=66$ $-4+8\lambda+30=66$ $\lambda=5$ $\overrightarrow{OU}=\begin{pmatrix}-0.5\\0\\6\end{pmatrix}+5\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}4.5\\0\\6\end{pmatrix}$ $U\left(4.5,0,6\right)$	

(iv)	Plane *ADFC* is given by $\mathbf{r}\bullet\begin{pmatrix}8\\0\\5\end{pmatrix}=66$. $\overrightarrow{TC}=\overrightarrow{OC}-\overrightarrow{OT}=\begin{pmatrix}7\\-2\\2\end{pmatrix}-\begin{pmatrix}-0.5\\0\\6\end{pmatrix}=\begin{pmatrix}15/2\\-2\\-4\end{pmatrix}$ Shortest distance $=\left\|\overrightarrow{TC}\bullet\mathbf{n}\right\|=\dfrac{\left\|\begin{pmatrix}15/2\\-2\\-4\end{pmatrix}\bullet\begin{pmatrix}8\\0\\5\end{pmatrix}\right\|}{\sqrt{8^2+5^2}}$ $=\dfrac{40}{\sqrt{89}}$	

<u>**2022 J2 H2 Mathematics Preliminary Examination P2 (Worked Solutions)**</u>

Qn	**Solution**	**Notes**
1(a) **(i)**	The graph of $y = \mathrm{f}(x)$ is decreasing when $\mathrm{f}'(x) < 0$. $x < -3$ or $0 < x < 4$	
(ii)	The graph of $y = \mathrm{f}(x)$ is increasing and concave downwards when $\mathrm{f}'(x) > 0$ and $\mathrm{f}'(x)$ is decreasing $-2 < x < 0$	
(b) **(i)**	$\frac{256}{3}\pi = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad r = 4$ $\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t}$ When $r = 4$, $12\pi = 4\pi(4)^2 \frac{\mathrm{d}r}{\mathrm{d}t}$ $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{3}{16}$ $A = 4\pi r^2 \quad \Rightarrow \quad \frac{\mathrm{d}A}{\mathrm{d}r} = 8\pi r$ $\frac{\mathrm{d}A}{\mathrm{d}t} = 8\pi r \frac{\mathrm{d}r}{\mathrm{d}t}$ $= 8\pi(4)\frac{3}{16}$ $= 6\pi$ cm^2/s	
(ii)	Since $\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t}$ $12\pi = A\frac{\mathrm{d}r}{\mathrm{d}t}$ $\frac{\mathrm{d}r}{\mathrm{d}t} = 12\pi\left(\frac{1}{A}\right)$ The rate of increase of balloon's radius is inversely proportional to the area of the balloon.	

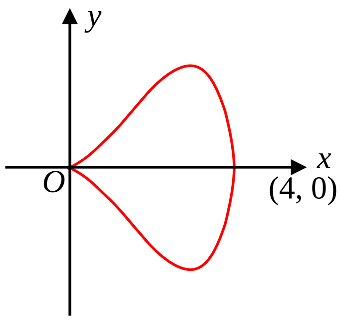
2(a)	O, X and Y are collinear, $\overrightarrow{OY} = \alpha\overrightarrow{OX} = \alpha\left(\frac{1}{8}\mathbf{a}+\frac{3}{8}\mathbf{b}\right)$ --(1)  $\overrightarrow{AY} = \beta\overrightarrow{AB}$ $\overrightarrow{OY} - \mathbf{a} = \beta(\mathbf{b}-\mathbf{a})$ $\overrightarrow{OY} = \beta\mathbf{b}+(1-\beta)\mathbf{a}$ ---(2) $\frac{\alpha}{8}\mathbf{a}+\frac{3\alpha}{8}\mathbf{b} = \beta\mathbf{b}+(1-\beta)\mathbf{a}$ Comparing, $\frac{\alpha}{8} = 1-\beta$ ---(3) $\frac{3\alpha}{8} = \beta$ ---(4) Solving (3) and (4) $\alpha = 2, \quad \beta = \frac{3}{4}$ $\overrightarrow{OY} = 2\left(\frac{1}{8}\mathbf{a}+\frac{3}{8}\mathbf{b}\right) = \frac{1}{4}\mathbf{a}+\frac{3}{4}\mathbf{b}$ $\overrightarrow{AY} = \frac{3}{4}\overrightarrow{AB}$ $AY:YB = 3:1$	
(b) (i)	$(\mathbf{r}-\mathbf{p})\times\mathbf{q} = \mathbf{0}$ $(\mathbf{r}-\mathbf{p}) // \mathbf{q}$ $(\mathbf{r}-\mathbf{p}) = k\mathbf{q}, k\in\mathbb{R}$ $\mathbf{r} = \mathbf{p}+k\mathbf{q}, k\in\mathbb{R}$ R represents the points on the line containing the point P and parallel to $\mathbf{q}$.	
(ii)	$(\mathbf{r}-\mathbf{p})\bullet\mathbf{q} = 0$ $\mathbf{r}\bullet\mathbf{q}-\mathbf{p}\bullet\mathbf{q} = 0$ $\mathbf{r}\bullet\mathbf{q} = \mathbf{p}\bullet\mathbf{q}$ R represents the points on the plane that is perpendicular to $\mathbf{q}$ and containing the point P.	

2022 J2 H2 Mathematics Preliminary Examination P2 (Worked Solutions)

Qn	Solutions	Notes
3(a)	Since all the coefficients of the polynomial are real, another root is $1-\mathrm{i}$ The quadratic factor is $\left[z-(1+\mathrm{i})\right]\left[z-(1-\mathrm{i})\right]$ $=\left[(z-1)-\mathrm{i}\right]\left[(z-1)+\mathrm{i}\right]$ $=z^2-2z+2$	
	$z^4+4z^2-8z+12=(z^2-2z+2)(z^2+2z+6)$ The other quadratic factor is z^2+2z+6 Let $z^2+2z+6=0$ $z=\dfrac{-2\pm\sqrt{4-24}}{2}$ $=-1\pm\sqrt{5}\mathrm{i}$	
	Hence the other roots are $-1+\sqrt{5}\mathrm{i},\ -1-\sqrt{5}\mathrm{i}$ and $1-\mathrm{i}$	
3 (b)	**Method 1:** $\dfrac{1+\mathrm{i}\cos\frac{3\pi}{8}+\sin\frac{3\pi}{8}}{1-\mathrm{i}\cos\frac{3\pi}{8}+\sin\frac{3\pi}{8}}=\dfrac{1+\mathrm{i}\left(\cos\frac{3\pi}{8}-\mathrm{i}\sin\frac{3\pi}{8}\right)}{1-\mathrm{i}\left(\cos\frac{3\pi}{8}+\mathrm{i}\sin\frac{3\pi}{8}\right)}$ $=\dfrac{1+\mathrm{i}e^{-\frac{3\pi}{8}\mathrm{i}}}{1-\mathrm{i}e^{\frac{3\pi}{8}\mathrm{i}}}$ $=\dfrac{1+e^{\mathrm{i}\frac{\pi}{2}}e^{-\frac{3\pi}{8}\mathrm{i}}}{1+e^{-\frac{\pi}{2}\mathrm{i}}e^{\frac{3\pi}{8}\mathrm{i}}}$ $=\dfrac{1+e^{\frac{\pi}{8}\mathrm{i}}}{1+e^{-\frac{\pi}{8}\mathrm{i}}}$ $=\dfrac{\left(1+e^{\frac{\pi}{8}\mathrm{i}}\right)}{e^{-\frac{\pi}{8}\mathrm{i}}\left(1+e^{\frac{\pi}{8}\mathrm{i}}\right)}$ $=e^{\frac{\pi}{8}\mathrm{i}}$ $=\cos\frac{\pi}{8}+\mathrm{i}\sin\frac{\pi}{8}$	

Qn	Solution	Notes
3 (b)	**Method 2:** $\dfrac{1+\sin\frac{3\pi}{8}+\mathrm{i}\cos\frac{3\pi}{8}}{1+\sin\frac{3\pi}{8}-\mathrm{i}\cos\frac{3\pi}{8}}=\dfrac{1+\cos\frac{\pi}{8}+\mathrm{i}\sin\frac{\pi}{8}}{1+\cos\frac{\pi}{8}-\mathrm{i}\sin\frac{\pi}{8}}$ $=\dfrac{1+\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}}{1+\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{8}\right)}}\bullet\dfrac{\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}}{\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}}$	
	$=\dfrac{\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}+\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}}{\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}+1}$ $=\dfrac{\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}\left(1+\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}\right)}{\left(1+\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}\right)}$ $=\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}$ $=\cos\frac{\pi}{8}+\mathrm{i}\sin\frac{\pi}{8}$	
	$\left(\dfrac{1+\sin\frac{3\pi}{8}+\mathrm{i}\cos\frac{3\pi}{8}}{1+\sin\frac{3\pi}{8}-\mathrm{i}\cos\frac{3\pi}{8}}\right)^n=\cos\frac{n\pi}{8}+\mathrm{i}\sin\frac{n\pi}{8}=\mathrm{i}$ Comparing real and imaginary parts, $\cos\frac{n\pi}{8}=0$ and $\sin\frac{n\pi}{8}=1$	
	For $\cos\frac{n\pi}{8}=0$ $\frac{n\pi}{8}=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2},\ldots$ n = 4, 12, 20, 28…	
	For $\sin\frac{n\pi}{8}=1$, When n = 4, $\sin\frac{\pi}{2}=1$ n = 12, $\sin\frac{3\pi}{2}=-1$ n = 20, $\sin\frac{5\pi}{2}=1$ The two smallest positive integer values of n are 4 and 20.	

2022 J2 H2 Mathematics Preliminary Examination P2 (Worked Solutions)

<table>
<tr><th>Qn</th><th>Solution</th><th>Notes</th></tr>
<tr><td>4(i)</td><td></td><td></td></tr>
<tr><td>(ii)</td><td>$x = 2 + 2\sin\theta, \qquad y = 2\cos\theta + \sin 2\theta$
$\frac{dx}{d\theta} = 2\cos\theta$
when $x = 0$, $\sin\theta = -1 \quad \Rightarrow \quad \theta = -\frac{\pi}{2}$
when $x = 4$, $\sin\theta = 1 \quad \Rightarrow \quad \theta = \frac{\pi}{2}$</td><td></td></tr>
<tr><td></td><td>Area bounded by the curve
$= 2\int_0^4 y \, dx$
$= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2\cos\theta + \sin 2\theta)(2\cos\theta) \, d\theta$</td><td></td></tr>
<tr><td></td><td><u>Method 1</u>
$= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2\cos 2\theta + 2 + 4\sin\theta\cos^2\theta) \, d\theta$
$= 2\left[\sin 2\theta + 2\theta - \frac{4\cos^3\theta}{3}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$
$= 4\pi$</td><td></td></tr>
<tr><td></td><td><u>Method 2</u>
$= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2\cos 2\theta + 2 + \sin 3\theta + \sin\theta) \, d\theta$
$= 2\left[\sin 2\theta + 2\theta - \frac{1}{3}\cos 3\theta - \cos\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$
$= 4\pi$</td><td></td></tr>
</table>

Qn	Solution	Notes
4(iii)	$\text{RHS} = (2+2\sin\theta)\cos\theta$ $= 2\cos\theta + 2\sin\theta\cos\theta$ $= 2\cos\theta + \sin 2\theta = y = \text{LHS}$	
	From $x = 2+2\sin\theta$ $\Rightarrow \sin\theta = \dfrac{x-2}{2}$ $y = x\cos\theta \quad \Rightarrow \cos\theta = \dfrac{y}{x}$ Since $\sin^2\theta + \cos^2\theta = 1$ $\left(\dfrac{x-2}{2}\right)^2 + \left(\dfrac{y}{x}\right)^2 = 1$ $y^2 = x^2\left[1-\left(\dfrac{x-2}{2}\right)^2\right]$ $y^2 = x^2\left[\dfrac{4-(x-2)^2}{4}\right]$ $4y^2 = x^2\left[4-(x^2-4x+4)\right]$ $4y^2 = 4x^3 - x^4$	
(iv)	Volume of solid generated $= \pi\int_0^4 \left(x^3 - \dfrac{1}{4}x^4\right) \mathrm{d}x$ $= \dfrac{64\pi}{5}$ or 40.2	

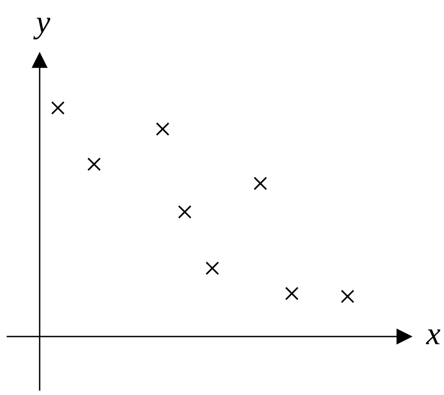
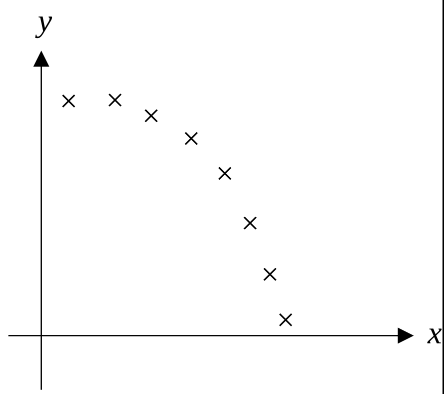
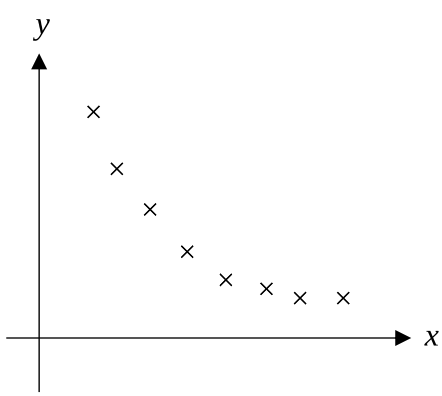
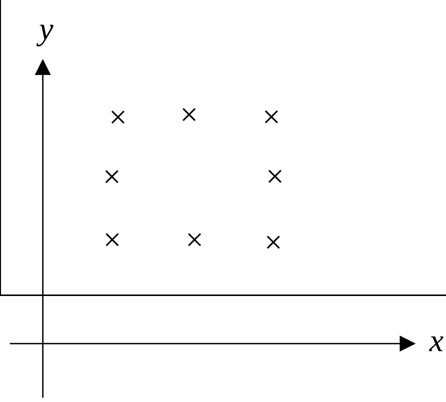
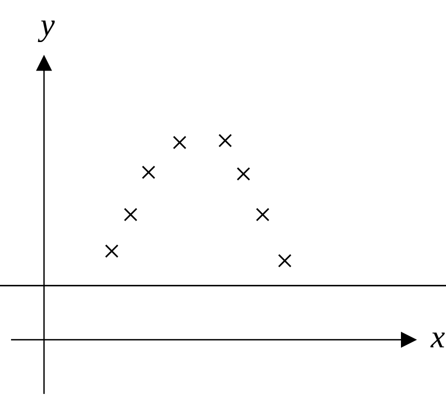
Qn	Solution	Notes
5(i)	P(W = 2) = P(1st die picked & '1' obtained) + P(2nd die picked & '2' obtained) $= \frac{1}{2} \times \frac{2}{4} + \frac{1}{2} \times \frac{1}{4}$ $= \frac{3}{8}$ (shown)	
	P(W = 1) = P(2nd die picked & '1' obtained) $= \frac{1}{2} \times \frac{1}{4}$ $= \frac{1}{8}$ P(W = 3) = P (2nd die picked & '3' obtained) $= \frac{1}{2} \times \frac{2}{4}$ $= \frac{1}{4}$ P(W = 4) = P(1st die picked & '2' obtained) $= \frac{1}{2} \times \frac{1}{4}$ $= \frac{1}{8}$ P(W = 6) = P(1st die picked & '3' obtained) $= = \frac{1}{2} \times \frac{1}{4}$ $= \frac{1}{8}$ The probability distribution of W is:	

w	1	2	3	4	6
P(W = w)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

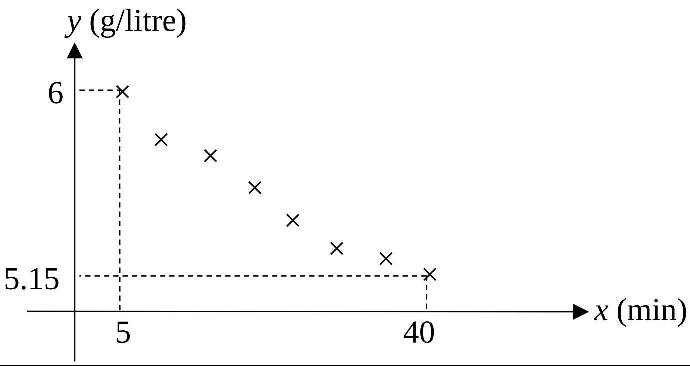
(ii)	E(W) $= 1\left(\frac{1}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{8}\right) + 6\left(\frac{1}{8}\right)$ $= \frac{23}{8} = 2\frac{7}{8}$	
	E(W^2) $= 1^2 \times \frac{1}{8} + 2^2 \times \frac{3}{8} + 3^2 \times \frac{1}{4} + 4^2 \times \frac{1}{8} + 6^2 \times \frac{1}{8}$ $= \frac{83}{8} = 10\frac{3}{8}$ Var(W) = E(W^2) – [E(W)]^2 $= \frac{83}{8} - \left(\frac{23}{8}\right)^2$ $= \frac{135}{64} = 2\frac{7}{64}$	

Qn	Solution	Notes
6 (i)	Without any restriction, no. of codes generated $= 26\times26\times26\times26\times10\times10 = 26^4\times10^2 = 45697600$ No. of codes with four different letters and two different digits $= \left({}^{26}C_4 \times 4!\right)\left({}^{10}C_2 \times 2!\right) = 32292000$ Required probability $= \frac{32292000}{45697600} = \frac{3105}{4394}$	
(ii)	No. of codes with two different consonants, two different vowels where the consonants and vowels alternate $= \left({}^{21}C_2 \times 2! \times {}^{5}C_2 \times 2!\right) \times 2 \times 10^2 = 1680000$ Required probability $= \frac{1680000}{26^4 \times 10^2} = 0.0368$ (3 sf)	

(iii)	No of codes with 2 letters the same , 2 letters different and 2 digits the same $=\left({}^{26}C_3\times 3\times\dfrac{4!}{2!}\right)\times(10\times 1)=936000$ P(2 letters the same , 2 letters different and 2 digits the same) $=\dfrac{936000}{(26)^4(10)^2}$ $=\dfrac{45}{2197}$	

Qn	Solution	Notes
7(a) (i)	Some possible scatter plots  r between -0.8 and -0.5	
(ii)	Some possible scatter plots 	

	$r = 0$	

(b) (i)		
(ii)	No, a linear model is not suitable. From the scatter diagram, as time increases, the concentration decreases by decreasing amounts.	
(iii)	From the GC $\frac{1}{y} = 0.16537 + 0.00074414x$ $\frac{1}{y} = 0.165 + 0.000744x$ $a = 0.165, \quad b = 0.000744$ $r = 0.988$	
(iv)	$\frac{1}{5.4} = 0.16537 + 0.00074414x$ $x = 26.628 \approx 26.6$ Since x is the independent (controlled) variable, we should use the equation $\frac{1}{y} = ax + b$ and not $x = \frac{c}{y} + d$ to find the estimate.	

2022 J2 H2 Mathematics Preliminary Examination P2 (Worked Solutions)

Qn	Solution	Notes
8	The event that a student completes the online Mathematics homework by the deadline is independent of any other student. The probability of each student completing the online Mathematics homework by the deadline is constant.	
(i)	$M \sim \mathrm{B}(25, 0.72)$ $\mathrm{P}(M < 15) = \mathrm{P}(M \le 14)$ $= 0.063632$ ≈ 0.0636	
(ii)	Required probability = $[\mathrm{P}(M<15)]^3[\mathrm{P}(M \ge 15)]^7\left(\dfrac{10!}{3!7!}\right)$ ≈ 0.0195	
	Alternative method: X= Number of classes with fewer than 15 students who complete the online Mathematics homework by the deadline, out of 10 $X \sim \mathrm{B}(10, 0.063632)$ $\mathrm{P}(X=3) \approx 0.0195$	
(iii)	$\mathrm{E}(M) = np = 18$, $\mathrm{Var}(M) = np(1-p) = 5.04$ Since number of classes (Sample size) = 30 is large, by Central Limit Theorem, $\overline{M} \sim N\left(18, \dfrac{5.04}{30}\right)$ approximately $\mathrm{P}(\overline{M} > 19) \approx 0.00735$	
(b)	Let $M \sim \mathrm{B}(n, 0.72)$ $\mathrm{P}(M \ge 20) > 0.7$ $1 - \mathrm{P}(M \le 19) > 0.7$ $\mathrm{P}(M \le 19) < 0.3$	
	From GC, when $n = 28$, $\mathrm{P}(M \le 19) = 0.37986 > 0.3$ when $n = 29$, $\mathrm{P}(M \le 19) = 0.27745 < 0.3$ when $n = 30$, $\mathrm{P}(M \le 19) = 0.19429 < 0.3$ $n \ge 29$ There must be at least 29 students in the class.	

Qn	Solution	Notes
9(i)	$V \sim \mathrm{N}\left(56, 8^2\right)$ and $W \sim \mathrm{N}\left(60, 12^2\right)$ Required probability = P(V <55) . P(W >55) = 0.29787 $\approx$ 0.298	
(ii)	$V_1 + V_2 + V_3 - 3W \sim \mathrm{N}(-12, 1488)$ $\mathrm{P}(\lvert V_1 + V_2 + V_3 - 3W \rvert < 15)$ = 0.28901 $\approx$ 0.289	
(iii)	Let $M = \dfrac{V_1 + ...V_5 + W_1 + ...W_6}{11}$ E(M) = $\dfrac{640}{11}$ and Var(M) = $\dfrac{1184}{121}$ $M \sim \mathrm{N}\left(\dfrac{640}{11}, \dfrac{1184}{121}\right)$ P(M < 60) = 0.71946 $\approx$ 0.719	
(iv)	$T = 2V + W$ $T \sim \mathrm{N}(172, 400)$ P($T > t$) = 0.35 t = 179.71 $\approx$ 180	

2022 J2 H2 Mathematics Preliminary Examination P2 (Worked Solutions)

Qn	Solution	Notes
10 (a) (i)	$\bar{x} = 40 + \frac{1}{150}\left[\sum(x-40)\right] = 43.9$	
	$s^2 = \frac{1}{150-1}\left\{\sum(x-40)^2 - \frac{\left[\sum(x-40)\right]^2}{150}\right\}$ $= 58.5$	
(ii)	X = speed of a car on the expressway μ = average speed of a car on the expressway $H_0: \mu = 45$ $H_1: \mu < 45$ Under H_0, since n = 150 is large, by Central Limit Theorem, $\bar{X} \sim N\left(45, \frac{58.5}{150}\right)$ approximately. Test statistic, $z = \frac{\bar{x}-\mu_0}{\sqrt{\frac{s^2}{n}}} = \frac{43.9-45}{\sqrt{\frac{58.5}{150}}} = -1.76141$ From GC, p-value = 0.039084 $\approx$ 0.0391 Level of significance, α = 0.05 Since p-value < α, we reject H_0. There is sufficient evidence, at the 5% level, to indicate that the average speed of a car is less than 45km/h. Hence, the ERP rate needs to be increased for this particular expressway.	
(iii)	The p-value is 0.0391 and it means that there is a probability of 0.0391 of observing a test statistic, $z < -1.76$, given that the population average speed of a car is 45 km/h.	

(b)	$H_0: \mu = 45$ $H_1: \mu > 45$ Under H_0, since n is large, by Central Limit Theorem, $\overline{X} \sim N\left(45, \frac{7.79^2}{n}\right)$ approximately. Test statistic, $z = \frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{45.9-45}{\frac{7.79}{\sqrt{n}}} = \frac{0.9\sqrt{n}}{7.79}$ Level of significance, $\alpha = 0.05$ Critical region is $z > 1.64485$ Reject H_0 if $z > 1.64485$	
	$\frac{0.9\sqrt{n}}{7.79} > 1.64485$ $n > 202.6$ least $n = 203$	