| Name: | Index Number: | Class: |
| :--- | :--- | :--- |



## READ THESE INSTRUCTIONS FIRST

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For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.

The number of marks is given in brackets [ ] at the end of each question or part question. The total of the marks for this paper is 80 .


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[Turn Over
Setter: Ms Lee Hui Ling

## Mathematical Formulae

Compound interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

## Mensuration

$$
\begin{gathered}
\text { Curved surface area of a cone }=\pi r l \\
\text { Surface area of a sphere }=4 \pi r^{2} \\
\text { Volume of a cone }=\frac{1}{3} \pi r^{2} h \\
\text { Volume of a sphere }=\frac{4}{3} \pi r^{3} \\
\text { Area of triangle } A B C \frac{1}{2} a b \sin C \\
\text { Arc length }=r \theta, \text { where } \theta \text { is in radians } \\
\text { Sector area }=\frac{1}{2} r^{2} \theta, \text { where } \theta \text { is in radians }
\end{gathered}
$$

Trigonometry

$$
\begin{aligned}
& \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{aligned}
$$

Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

1. Given that $2^{1003}+2^{1000}=2^{m} \times 3^{n}$, find the value of $m$ and of $n$.

$$
\begin{aligned}
& \text { Ans: } m=\square \\
& n= \\
&
\end{aligned}
$$

2. The first four terms of a sequence are $27,24,21$ and 18 .
(a) Write down the $8^{\text {th }}$ term of the sequence.

Ans: $\qquad$ [1]
(b) Find an expression, in terms of $n$, for the $n^{\text {th }}$ term of the sequence.

Ans: $\qquad$
(c) Will -31 be a term in the sequence? Explain.

Answer
$\qquad$
$\qquad$
3. In the diagram, $A B=D C$ and $A C=D B$. The line $B D$ intersect the line $A C$ at $T$.

(a) Prove that $\triangle A B C$ is congruent to $\triangle D C B$.

Answer

Ans:
(b) Explain why $\triangle T B C$ is an isosceles triangle.

Answer
4. Written as a product of its prime factors, $p=3^{3} \times 5 \times 7^{2}$.
(a) Express 450 as a product of its prime factors.

Ans: $\qquad$
(b) Find the highest common factor of $p$ and 450 .

Ans: $\qquad$
(b) Find the smallest integer $m$ such that $450 m$ is a multiple of $p$.

$$
\begin{equation*}
\text { Ans: } m= \tag{1}
\end{equation*}
$$

(c) Find the value of $a$ and $b$ such that $p \times \frac{a}{b}$ is a perfect cube where $a$ and $b$ are prime numbers.

Ans: $a=$ $\qquad$
$b=$
5. The point $\left(\frac{1}{2}, \frac{1}{2}\right)$ is shown on the axes. Sketch the graph of $y=2 x^{3}$.

6. Given that $\left(\frac{1}{7}\right)^{3 y}=7^{\frac{3}{5}}$, find the value of $y$.

$$
\begin{equation*}
\text { Ans: } y= \tag{2}
\end{equation*}
$$

$\qquad$
7. Simplify $\left(\frac{2 p^{3}}{5 q}\right)^{-2} \times \frac{4}{p^{-7} q}$, leaving your answer in positive indices.

Ans:
8. Write an inequality to represent the range of values of $x$ shown on the number line below.


Ans:
9. Solve the inequality $\frac{4 x+3}{-2} \leq 5$.

Ans: $\qquad$
10. Given that $3 v=\sqrt{25-x^{2}}$,
(a) find the value of $v$ when $x=-3$,

Ans:
(b) make $x$ the subject of $3 v=\sqrt{25-x^{2}}$.

Ans:
11. Show that $(3 n+1)^{2}+2$ is always divisible by 3 for all integer values of $n$. Answer
12. The probabilities of Andrew, Ben and Caleb passing the driving test are $\frac{3}{5}, \frac{2}{3}$ and $\frac{3}{4}$ respectively.
(a) Find the probability that only Andrew passes the driving test.

Ans: $\qquad$
(b) Find the probability that at least one of them passes the driving test.

Ans:
13. The diagram shows triangle $A B D$ and $B C D$ is a straight line.

It is also known that $A B=7 \mathrm{~cm}, B C=5 \mathrm{~cm}$ and angle $B A C=$ angle $A D C$.

(a) Prove that triangle $A B C$ is similar to triangle $D B A$.
(b) Find the length of $C D$.

Ans: $\qquad$ cm
14. Two empty paper cups are geometrically similar.

The thickness of the paper used is neligible.
The big cup can be filled by 8 small cups.
The height of the smaller cup is 5 cm and the base area of the big cup is $26 \mathrm{~cm}^{2}$.
(a) Find the height of the big cup.

Ans: $\qquad$ cm
(b) Find the base area of the small cup.

Ans: $\qquad$ $\mathrm{cm}^{2}$
(c) Julie said that the mass of one big cup is 8 times that of the small cups. Do you agree? Explain.

Answer
15. The diagram shows a hexagon. Three interiors angles are as shown. The remaining three interior angles are equal. Find $x$.

$\qquad$
16. In the diagram, $B P$ is parallel to $C Q \cdot A B=A C=C D, \angle A B P=19^{\circ}$ and $\angle Q C D=68^{\circ}$.

Calculate $\angle C D A$. Show workings clearly and give reasons for your workings.

$\qquad$
17. Aladdin invested $\$ 20000$ in a savings account which pays compound interest at the rate of $r \%$ per year. The interest is compounded quarterly. The formula below shows the total amount of the investment at the end of $n$ years.

$$
\text { Total amount }=20000\left(1+\frac{1}{1000}\right)^{20}
$$

Find the value of $r$ and $n$.

Ans: $r=$

$$
n=
$$

18. (a) Sketch the graph of $y=5-(x-2)^{2}$, showing clearly the turning point and the $y$ - intercept.

(b) Using your graph, explain why $5-(x-2)^{2}=7$ will not have a solution.

Answer
$\qquad$
$\qquad$
19. (a) Factorise $27 x^{2}-12$ completely.

Ans:
(b) Hence, simplify $\frac{5}{27 x^{2}-12}+\frac{2}{2-3 x}$.

Ans:
20. A fraction is such that its denominator is 3 more than its numerator. When 1 is added to both the numerator and denominator, the result is $\frac{7}{8}$. Find the original fraction.

Ans: $\qquad$
21. $P Q R$ is a triangle. $\overrightarrow{P Q}=\binom{2}{-3}, \overrightarrow{P R}=\binom{-3}{9}$ and $Q$ is the point $(0,5)$.
(a) Find the coordinates of $P$.

Ans: $P($ $\qquad$ , $\qquad$ [1]
(b) Calculate the length of $Q R$.

Ans: $\qquad$ units [2]
22.

$$
\begin{gathered}
\xi=\{x: x \text { is a positive integer and } x<10\} \\
A=\{2,3,5,7\} \\
B=\{1,4,9\} \\
C=\{2,4,7\}
\end{gathered}
$$

(a) Circle the correct statements from the list below.
$\{7\} \in C$
$B \cup C=\{4\}$
$\{9\} \subset B$
$A \cap B=\{\phi\}$
$5 \notin A^{\prime}$
(b) Find $A^{\prime} \cap C$.

Ans: $\qquad$
(c) Describe in words, the elements in set $B$.

Answer
$\qquad$
$\qquad$
23. (a) There are five numbers. Some information of the numbers are as shown.

$$
\text { Range }=6 \text { mean }=7 \text { median }=8 \quad \text { mode }=9
$$

Find the numbers.

Ans: $\qquad$ , $\qquad$ , $\qquad$ , $\quad$, ,
(b) The average mass of 5 boys is 68 kg and that of 8 girls is 55 kg . To find the average mass of all the children, Caleb takes the average of 68 kg and 55 kg . Explain why Caleb is wrong and find the correct answer.

Answer
$\qquad$
$\qquad$
$\qquad$
$\qquad$
24. In the scale drawing, $P Q R S$ represents a plot of land, which is to be used for a park.

Scale 1 cm to 0.5 km

(a) Find the actual distance between $P$ and $Q$.

Ans: $\qquad$ km
(b) Measure the bearing of $P$ from $S$.

Ans: $\qquad$
(c) Construct the angle bisector of $\angle P Q R$.
(d) Construct the perpendicular bisector of $R S$.
(e) A water fountain is to be built in the park, nearer to $R$ than to $S$ and equidistant to $P Q$ and $Q R$. Mark a possible location for the water fountain to be built and label it as $F$.
25. Emmett is going to Europe for a study exchange.

The exchange rate between the Euro and Singapore dollar changes each day. The graph shows the daily exchange rate in a particular month.

(a) Emmett went to the money changer on the $6^{\text {th }}$ to change $\$ 1200$ to Euros. How much Euros will he get?

Ans: $\qquad$ Euros [1]
(b) The accommodation per night at Europe is 120 Euros. Emmett wanted to convert the accommodation cost to dollars. Use the graph to work out the difference between the greatest and least possible accommodation costs per night in dollars.

Answer

| Name: | Index Number: | Class: |
| :--- | :--- | :--- |

HUA YI SECONDARY SCHOOL

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\end{aligned}
$$

Answer all the questions.

1. John and Peter took part in a marathon race. They each ran 42 km .
(a) John ran at a constant speed of $x$ kilometres per hour.

Write down an expression, in terms of $x$, for the number of hours he took.

Ans: $\qquad$ h [1]
(b) Peter ran at a constant speed which was $\frac{1}{2} \mathrm{~km} / \mathrm{h}$ more than John's speed. Write down an expression, in terms of $x$, for the number of hours he took.

Ans: $\qquad$ h [1]
(c) The difference between their times was 15 minutes.
(i) Write down an equation in $x$ to represent this information, and show that it reduces to $2 x^{2}+x-168=0$.

Answer:
(ii) Solve the equation $2 x^{2}+x-168=0$, giving your answers correct to 3 decimal places.

Ans: $x=$ $\qquad$ or
(iii) Calculate the time that Peter took to complete the race, giving your answer in hours and minutes.

Ans: $\qquad$ h $\qquad$ min
2. (a) In the diagram, $O A B$ is a sector of a circle with centre $O$. Given that $O A=12 \mathrm{~cm}$, $\angle A O B=\theta$ radians, $O C=8 \mathrm{~cm}$ and the length of the arc $A B=15 \mathrm{~cm}$.

(i) Calculate the value of $\theta$.

Ans: $\qquad$
(ii) Find the area of the shaded region.

Ans: $\qquad$ $\mathrm{cm}^{2}$
(iii) Calculate the perimeter of the shaded region.
$\qquad$ cm
(b) In the diagram, $A, B, C$ and $D$ are four points on a circle, centre $O$. $D O$ and $C B$ are parallel.
Find, giving reasons for each answer,

(i) angle $D A B$,

Ans: $\qquad$ -
(ii) angle $D O B$,

Ans: $\qquad$ -
(iii) angle $O D C$.

Ans: $\qquad$。
3.


The diagram shows a solid trapezoidal prism with four rectangular faces. $A B=10 \mathrm{~cm}, B C=12 \mathrm{~cm}, D C=6 \mathrm{~cm}$ and $C T=16 \mathrm{~cm}$.
(a) Show that $A D=12.649 \mathrm{~cm}$, correct to 5 significant figures.

Answer
(b) Calculate the surface area of the prism.

Ans: $\qquad$ $\mathrm{cm}^{2}$
(c) Calculate the volume of the prism.
$\qquad$ $\mathrm{cm}^{3}$
4. The figure shows a triangle $A B C . E$ is the mid-point of $A C . \overrightarrow{A B}=3$ a and $\overrightarrow{A C}=3 \mathbf{c}$. $K$ lies on $E B$ such that $E B=3 E K$.

(a) Express $\overrightarrow{B C}$ in terms of $\mathbf{a}$ and $\mathbf{c}$, as simply as possible.

Ans: $\overrightarrow{B C}=$
(b) Express $\overrightarrow{E B}$ in terms of a and c, as simply as possible.

$$
A n s:=
$$

(c) Given that $\overrightarrow{K P}=\frac{1}{2} \mathbf{a}+\frac{1}{2} \mathbf{c}$, explain why $A, K$ and $P$ lies on a straight line. Answer
(d) Find the value of $\frac{\text { area of } \triangle A E K}{\text { area of } \triangle A K B}$.

Ans: $\qquad$
(e) Find the value of $\frac{\text { area of } \triangle A E K}{\text { area of } \triangle A B C}$.

Ans:
5. The diagram shows the speed-time graph of a motorcyclist.

The shaded area represents the distance travelled.
The distance travelled is 450 m .

(a) Show that $v=12$.

Answer
(b) Find the speed of the motorcyclist after 8 seconds.

Ans: $\qquad$ m/s
(c) Describe the motion of the motorcyclist between 10 and 30 seconds.

Answer
$\qquad$
(d) Find the deceleration of the motorcyclist for the last 25 seconds.

Ans: $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$
(e) Sketch the distance time graph for the journey on the grid.

Answer

6. The variables $x$ and $y$ are connected by the equation $y=\frac{x^{3}}{5}-2 x+1$. Some corresponding values of $x$ and $y$ are given in the table below.

| $\boldsymbol{x}$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | -3.8 | 1.6 | $p$ | 2.8 | 1 | -0.8 | -1.4 | 0.4 | 5.8 |

(a) Find the value of $p$.

Ans: $p=$
(b) On the graph paper on page 11, draw the graph of $y=\frac{x^{3}}{5}-2 x+1$ for $-4 \leq x \leq 4$.
(c) Using your graph, find the value of $m$ such that the graph of $y=m x-2$ intersect the curve at exactly 2 points.

Ans: $m=$ $\qquad$
(d) Using your graph, explain why $\frac{x^{3}}{5}-2 x-3=0$ has only one solution. Answer
$\qquad$
$\qquad$
(e) On the same axes, draw the graph of $2 y+x=4$.
(f) Show that the $x$ coordinates of the points of intersection of the line and the curve give the solutions to the equation $2 x^{3}-15 x-10=0$.

Answer

7. Airport $B$ is 500 km away from airport $A$ on a bearing of $064^{\circ}$. An aircraft leaves airport $A$ at 2245 on Monday to fly to airport $B$. Its speed during the flight is $375 \mathrm{~km} / \mathrm{h}$.

(a) Calculate the time at which the aircraft is expected to arrive at airport $B$.

Ans:

After the aircraft has travelled 200 km at $P$, it is diverted to airport $Q$ because of bad weather at airport $B$. Airport $Q$ is 120 km due to north of airport $B$.
(b) Calculate the distance of $P Q$.

Ans: $\qquad$ km
(c) Calculate the bearing of $Q$ from $P$.
$\qquad$。
8. (a) The waiting time, $t$ in minutes, for 40 customers at two phone service shops, Shop A and Shop B, are as follows.

## Shop A

| $t(\min )$ | $0<t \leq 5$ | $5<t \leq 10$ | $10<t \leq 15$ | $15<t \leq 20$ |
| :--- | :---: | :---: | :---: | :---: |
| Frequency | 5 | 12 | 8 | 15 |

Shop B

| Mean $=13.2 \mathrm{~min}$ |
| :--- |
| Standard Deviation $=2.17 \mathrm{~min}$ |

(i) Calculate the mean waiting time for Shop A.

Ans: $\qquad$ min
(ii) Explain why your answer to (a)(i) is only an estimate.

Answer
$\qquad$
$\qquad$
(iii) Calculate the standard deviation for Shop A.

Ans: $\qquad$ $\min$
(iv) Which shop would you choose to go to? Why?

Answer
$\qquad$
$\qquad$
(v) A customer went to Shop B and his waiting time is 13.2 min . If his waiting time is added to the data, will the mean and standard deviation of the new set of data increase, decrease or remain the same?

Answer
The mean will
The standard deviation will
(b) A teacher recorded the individual time taken by 200 students to complete a crosscountry race. The results are presented in the form of a cumulative frequency curve.


Use the graph to find
(i) the median,

Ans: $\qquad$ $\min$
(ii) the $90^{\text {th }}$ percentile,

Ans: $\qquad$ $\min$ [1]
(iii) the qualifying time to run in the second round if only 30 students qualify for the second round,

Ans: $\qquad$ $\min$ [1]
(iv) the number of students who took more than 50 minutes but no more than 66 minutes.

Ans: $\qquad$
9. A cake shop sells 3 types of cakes.

On Day 1, the shop sold 84 chocolate cakes, 90 vanilla cakes and 56 strawberry cakes.
On Day 2, the shop sold 92 chocolate cakes, 60 vanilla cakes and 61 strawberry cakes.
(a) Represent this information by a $2 \times 3$ matrix, $\mathbf{P}$.

Ans: $\mathbf{P}=$
The table shows the selling price and the profit for each type of cake.

|  | Chocolate | Vanilla | Strawberry |
| :--- | :---: | :---: | :---: |
| Selling Price of each cake | $\$ 7$ | $\$ 8$ | $\$ 9$ |
| Profit made for each cake | $\$ 4$ | $\$ 3$ | $\$ 5$ |

(b) Find the matrix $\mathbf{R}=\mathbf{P}\left(\begin{array}{ll}7 & 4 \\ 8 & 3 \\ 9 & 5\end{array}\right)$.

Ans: $\mathbf{R}=$ $\qquad$
(c) Express the profit for Day 1 as a percentage of the total sale for Day 1 .

Ans: $\qquad$ \% [1]
(d) Find the matrix $\mathbf{M}=\frac{1}{2}\left(\begin{array}{ll}1 & 1\end{array}\right) \mathbf{R}$.

Ans: $\mathbf{M}=$ $\qquad$
(e) Explain what each element in matrix M represents.

Answer
$\qquad$
10. (a) The figure shows $\triangle A B C$.

(i) May said that the line $A C$ intersect the $y$ axis at 3.2. Show how you would check if she is correct.

Answer
(ii) Find the length of $A B$.
$\qquad$ units
(iii) Without calculating the value of any angle, find the value of $\cos A \hat{B} C$.

$$
\begin{equation*}
\text { Ans: } \cos A \hat{B} C= \tag{1}
\end{equation*}
$$

(iv) The point $P$ lies on the line $y=1$. Given that area of $\triangle A B C$ : area of $\triangle A P B=$ 4: 3, find two possible $x$ coordinates of $P$.

Ans: $x=$ $\qquad$ or $\qquad$
(b) Mr Johnson has a plot of field that grows dandelion plants. An average of 70 dandelion plants grow on each square metre. Each plant has an average of 10 flowers. Each flower will produce an average of 150 seeds. It is estimated that these plants will produce 130000000 seeds in total.
(i) Mr Johnson packs the seeds in small packets. A packet consist of about 55 seeds. Find the number of packets of seeds he will get, giving your answer in standard form.

Ans: $\qquad$
(ii) Calculate the area of the plot of field in square metres.

Ans: $\qquad$ $\mathrm{m}^{2}$
11. Mrs Ang runs a small tuition centre for secondary school students. The tables provide information on the cost of running the centre. The centre is closed on Monday and Tuesday.

| Start up cost |  |
| :--- | :--- |
| Renovation | $\$ 40000$ |
| Furniture and IT equipments | $\$ 5100$ |


|  | Cost |
| :--- | :--- |
| Rental <br> (including <br> utilities ) | $\$ 8000$ per <br> month |
| Printing and <br> stationery | $\$ 100$ per <br> week |


|  | Number of <br> staff | Staff cost |
| :---: | :---: | :---: |
| Administrative | 1 | $\$ 1800$ |
| Teaching | 1 per class | $\$ 70 / \mathrm{h}$ ( weekday) |
|  |  | $\$ 90 / \mathrm{h}$ ( weekend) |
| Cleaning | 1 | $\$ 45$ per day $(3 \mathrm{hr})$ |


|  | Total number <br> of classes on <br> Weekdays | Total number <br> of classes on <br> Weekends | Fee per student *per <br> month <br> 1 lesson (2 h)/week |
| :--- | :---: | :---: | :---: |
| Lower <br> Secondary | 6 | 6 | $\$ 200$ |
| Upper <br> Secondary | 6 | 8 | $\$ 300$ |

Number of students per class: 5-8. *1 month has 4 weeks
(a) Mrs Ang took a loan for the renovation cost from a bank that charges $2 \%$ simple interest per annum. She opted to pay in 30 monthly instalments.
Calculate the monthly instalment that Mrs Ang has to pay to the bank.

Ans: \$
(b) Mrs Ang bought the furniture and IT equipment at a $15 \%$ discount. Calculate her saving from the sale.

Ans: \$ $\qquad$
(c) Mrs Ang targets to recover her start up cost and the interest incurred, within 6 months of operating her tuition centre.
She also targets to make a profit of at least $\$ 5000$ per month thereafter.
Will Mrs Ang hit her targets? Justify the decision you made and show your calculation clearly. State any assumptions made clearly.

Answer

| Name: | Index Number: | Class: |
| :--- | :--- | :--- |


| $4 E$ | HUA YI SECONDARY SCHOOL |  |
| :---: | :---: | :---: |
|  | Preliminary Examination 2022 | $4 E$ |
|  | MATHEMATICS | 4048/1 |
|  | Paper 1 | 2022 |
|  | Candidates answer on the Answer Space provided. | $2 \mathrm{~h}$ |

## Mark Scheme

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1.

$$
\begin{aligned}
& 2^{1000}\left(1+2^{3}\right)----M 1 \text { factorise } \\
& =2^{1000} \times 3^{2} \\
& \mathrm{~m}=1000, \mathrm{n}=2---\mathrm{A} 2
\end{aligned}
$$

2. 

(a)

6----B1
(b)
30-3n--------B1
(c)

No as -31 is not a divisible by 3 , or we will get $\mathrm{n}=20.3$, but n must be a whole number. $\qquad$ -B1
3.
(a)

```
                AB=DC(Given )(S)
                AC=DB (Given ) (S)
                BC = CB (S) common side M1
                Hence }\triangleABC\mathrm{ is congruent to }\triangleDCB\mathrm{ .( SSS) A1
```

(b) Since $\triangle A B C$ is congruent to $\triangle D C B, \angle D B C=\angle A C B \quad \angle D B C=\angle A C B$. Hence triangle TBC is an isosceles triangle. ----B1
4.
(a) $450=2 \times 3^{2} \times 5^{2}----\mathrm{B} 1$
(b)

$$
45----\mathrm{B} 1
$$

(c)

$$
147 \text {----B1 }
$$

5. 

B 1 - curve A 1 below the point (If curve is wrong then 0 )

6.

$$
\begin{aligned}
& -3 \mathrm{y}=3 / 5-\mathrm{M} 1\left(\text { convert } 1 / 7 \text { to } 7^{-1}\right) \\
& \mathrm{y}=-1 / 5 \ldots-\mathrm{Al}
\end{aligned}
$$

7. 

$$
\begin{aligned}
& \frac{25 q^{2}}{4 p^{6}} \times \frac{4}{p^{-7} q}-M 1 \\
& =\frac{25 q}{p^{-1}} \\
& =25 p q---A 1
\end{aligned}
$$

8. 

$$
-0.6 \leq x<0.8---B 1
$$

9. 

$$
\begin{aligned}
& 4 x+3 \geq-10---M 1 \\
& x \geq-3.25--A 1
\end{aligned}
$$

10. 

(a)

$$
\mathrm{v}=\frac{4}{3}--\mathrm{B} 1
$$

(b)

$$
\begin{aligned}
& 9 v^{2}=25-x^{2}----M 1 \\
& x^{2}=25-9 v^{2}----M 1 \\
& x= \pm \sqrt{25-9 v^{2}}-----A 1(A 0 \text { if no } \pm)
\end{aligned}
$$

11. 

$$
\begin{aligned}
& 9 n^{2}+6 n+1+2----M 1 \\
& 3\left(3 n^{2}+2 n+1\right)----A 1
\end{aligned}
$$

12. 

(a)

$$
\frac{3}{5} \times \frac{1}{3} \times \frac{1}{4}=\frac{1}{20}---B 1
$$

(b)

$$
\begin{aligned}
& 1-\frac{2}{5} \times \frac{1}{3} \times \frac{1}{4}---M 1 \\
& =\frac{29}{30}---A 1
\end{aligned}
$$

13. 

(a)

Ans:
$\angle B A C=\angle A D B(\mathrm{~A})($ Given $)$
$\angle A B C=\angle A B D(\mathrm{~A})$ (common) -------M 1

Hence
triangle ABC is similar to triangle DBA . ( AA ) ----- A 1
(b)

$$
\begin{aligned}
& \frac{7}{5+C D}=\frac{5}{7}---M 1 \\
& C D=4.8---A 1
\end{aligned}
$$

14. a.
$\frac{V_{\text {small }}}{V_{\text {big }}}=\frac{1}{8}$
$\frac{H_{\text {small }}}{H_{\text {big }}}=\frac{1}{2} \cdots-M 1$
$H_{\text {big }}=10-----A 1$
b.
$\frac{A_{\text {small }}}{A_{\text {big }}}=\frac{1}{4}$
$A_{\text {small }}=\frac{1}{4} \times 26=6.5----A 1$
c. Disagree - because the mass of the cups is equal to mass of the paper. Thus the ratio of their mass should be equal to the ratio of their areas. Mass of big cup should be 4 times that of mass of small cup.
15. Sum of the three equal interior angles $=375----$-M1

One interior angle $=125-----$-M1

$$
x=235----\mathrm{A} 1
$$

16. 

$$
\begin{aligned}
& \angle A B C=68-19(\text { correspondingangles })--M 1 \\
& =49 \\
& \angle A C B=49(\text { isos. } \Delta \text { ) }-----M 1 \\
& \angle C D A=24.5(\text { exteriorangle }, \text { iso } \Delta)--A 1
\end{aligned}
$$

17. 

$$
\mathrm{r}=0.4, \mathrm{n}=5----\mathrm{B} 1 \text { each }
$$

18. (a)

Symmetrical curve -----A1
Coordinates of turning point and y intercept ----A1

(b)

The maximum point is $(2,5)$. The maximum value of $y$ is 5 . Hence there will be no solution if $y$ is more than 5 . ----B1
or
$y=7$ do not the intersect the graph.
19. (a)

$$
\begin{aligned}
& 3\left(9 x^{2}-4\right)---M 1 \\
& 3(3 x-2)(3 x+2)---A 1
\end{aligned}
$$

Ans:
(b)

$$
\begin{aligned}
& \frac{5}{3(3 x-2)(3 x+2)}-\frac{2}{(3 x-2)}----M 1 \\
& =\frac{5}{3(3 x-2)(3 x+2)}-\frac{2(3)(3 x+2)}{3(3 x-2)(3 x+2)}---M 1 \\
& =\frac{-7-18 x}{3(3 x-2)(3 x+2)}---A 1
\end{aligned}
$$

20. Let the fraction be $\frac{x}{x+3}$.
$\frac{x+1}{x+3+1}=\frac{7}{8}---$ Ml
$8 x+8=7 x+28----M 1$
$x=20$
Ans : $\frac{20}{23}----\mathrm{Al}$
21. 

a)

$$
\overrightarrow{Q R}=\overrightarrow{Q P}+\overrightarrow{P R}=\binom{-2}{3}+\binom{-3}{9}=\binom{-5}{12} \ldots-\cdots-\cdots \mathrm{M} 1
$$

Length of $\mathrm{QR}=13$ units
b) $\mathrm{P}(-2,8)$------Bi
22.
(a)

$$
\{7\} \in C \quad B \cup C=\{4\} \quad\{9\} \subset B \quad A \cap B=\{\phi\} \quad 5 \notin A^{\prime}
$$

(b)

$$
\{4\}----\mathrm{B} 1
$$

(c) The elements in set $B$ are perfect squares less than 10. -----B1
23.
(a)

$$
3,6,8,9,9
$$

B1 for getting 3, 6, B1 for getting $8,9,9$
(b)

He is wrong as there are different number of boys and girls. -A1

$$
\frac{5(68)+8(55)}{13}=60 \quad------\mathrm{A} 1
$$

24. 


25.
(a)

$$
1200 \div 1.6=750
$$

(b)

On $23^{\text {rd }}: \$ 1.5775=1$ euro 1200 euro $=\$ 1893$------( Least $)$

On $10^{\text {th }} / 12^{\text {th }}, \$ 1.605=1$ euro 1200 euro $=\$ 1926($ greatest $)$

Difference $=\$ 33$------A1 $($ Difference $)$

| Name: | Index Number: | Class: |
| :--- | :--- | :--- |



## Mark Scheme

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Setter: Ms Lee Hui Ling

## Mathematical Formulae

Compound interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

## Mensuration

$$
\text { Curved surface area of a cone }=\pi r l
$$

$$
\text { Surface area of a sphere }=4 \pi r^{2}
$$

$$
\text { Volume of a cone }=\frac{1}{3} \pi r^{2} h
$$

$$
\text { Volume of a sphere }=\frac{4}{3} \pi r^{3}
$$

$$
\text { Area of triangle } A B C \frac{1}{2} a b \sin C
$$

$$
\text { Arc length }=r \theta, \text { where } \theta \text { is in radians }
$$

$$
\text { Sector area }=\frac{1}{2} r^{2} \theta, \text { where } \theta \text { is in radians }
$$

## Trigonometry

$$
\begin{aligned}
& \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{aligned}
$$

## Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

## Answer all the questions.

1. 

(a)

(b)

$$
\frac{42}{x+0.5}----\mathrm{B} 1
$$

(c)
(i)

$$
\begin{aligned}
& \frac{42}{x}-\frac{42}{x+0.5}=\frac{1}{4}-----M 1 \\
& 84=x^{2}+0.5 x----M 1 \\
& 2 x^{2}+x-168=0-----A 1
\end{aligned}
$$

(ii)

$$
\begin{aligned}
& x=\frac{-1 \pm \sqrt{1^{2}-4(2)(-168)}}{2(2)}---M 1 \\
& x=8.919=8.92(3 s f)-\mathrm{A} 1 \quad \text { or } \quad-9.419=-9.42(3 s f)----A 1
\end{aligned}
$$

(iii) $\frac{42}{8.919+0.5}=4.459 \mathrm{~h}=4.46 \mathrm{~h}(3 \mathrm{sf})-----M 1$

$$
=4 \text { h } 28 \mathrm{~min}-\cdots----\mathrm{Al}
$$

2. (a)
(i)
1.25 $\qquad$
(ii)

Find area of triangle AOC: $0.5(12)(8)(\sin 1.25)=45.55---\mathrm{M} 1$ ecf
Find area of sector AOB: $0.5\left(12^{2}\right)(1.25)=90-----M 1$ ecf
Area of shaded region $=44.45$
(iii)

Find AC using cosine rule $=12.14$
Perimeter $=31.1(3 \mathrm{sf})$---------A1
(b)
(i)
$32+38=70($ isosceles triangle AOD and AOB )
(ii)
$140($ angle at center $=$ twice angle at circumference $)$
(iii)

Find DCB using opposite angle of cyclic quad (M1 ecf)

$$
\begin{aligned}
\text { Angle ODC } & =180-\text { angle } \mathrm{DCB}(\text { interior angles, } / / \text { lines }) \\
& =180-110 \\
& =70-----\mathrm{A} 1
\end{aligned}
$$

3. 

(a)

$$
\begin{aligned}
& \mathrm{AD}^{2}=4^{2}+12^{2}------\mathrm{M} 1 \\
& \mathrm{AD}=12.649(\text { shown })----\mathrm{A} 1
\end{aligned}
$$

(b)

$$
\begin{aligned}
\text { Area of ABCD } & =96------\mathrm{M} 1 \\
\text { Lateral area } & =(10+12+6+12.649) \times 16=650.384-------\mathrm{M} 1 \\
\text { Surface area } & =842.384 \\
& =842(3 \mathrm{sf})----\mathrm{A} 1
\end{aligned}
$$

(c)

$$
\begin{aligned}
\text { Volume of prism } & =96(16)----\mathrm{M} 1 \\
& =1536----\mathrm{A} 1
\end{aligned}
$$

4. 

(a)

$$
3 c-3 a----B 1
$$

(b)

$$
3 \mathbf{a}-1.5 \mathbf{c}----\mathbf{B} 1
$$

(c)

$$
\overrightarrow{A K}=1.5 \mathbf{c}+\frac{1}{3}(3 \mathbf{a}-1.5 \mathbf{c})----M 1
$$

$$
=\mathbf{c}+\mathbf{a}
$$

$$
\overrightarrow{A K}=2 \overrightarrow{K P}-----M 1
$$

This implies that $\mathrm{AK} / / \mathrm{KP}$ and they have a common point K . Hence $\mathrm{A}, \mathrm{K}$ and $P$ lies on a straight line. -----A1
(d)

1/ 2 ------B1
(e)

$$
1 / 6----\mathrm{B} 1
$$

5. 

(a)

Form equation : Area of trapezium $0.5(20+55) v=450-------M 1$

$$
\text { Get } v=12----\mathrm{A} 1
$$

(b)

Acceleration = 1.2 ------M1 or use similar triangle
Speed at $\mathrm{t}=8,1.2(8)=9.6----\mathrm{A} 1$
(c)

The motorcyclist is travelling at a constant speed . ------B1
(d)

$$
0.48 \text {------B1 }
$$

(e)

6.
(a)

$$
p=3.4----\mathrm{B} 1
$$

(b) See graph
(c) Draw any line that cuts $y$ - axis at -2 and intersect the curve at 2 points for $-4 \leq x \leq 4$.

One possible line is the line that passes through $(2,-1.4)$ and $(0,-2)$
$\mathrm{m}=\frac{-1.4-(-2)}{2-0}=0.3-----A 1$
$0.3 \leq \mathrm{m} \leq 1.95$
(d)

Get $\mathrm{y}=4$----M1
The line $\mathrm{y}=4$ intersect the curve at 1 point, hence the equation only has one solution------A1
(e)

B1 for graph
(f)

Form $\frac{x^{3}}{5}-2 x+1=-\frac{x}{2}+2-$-M1
Balance equation -------M1
Get $2 x^{3}-15 x-10=0-----\mathrm{A} 1$
7. (a)

Time taken $=11 / 3 \mathrm{~h}$ or $1 \mathrm{~h} 20 \mathrm{~min}----\mathrm{M} 1$
Time expected to arrive at $\mathrm{B}=0005$ or 12:05am ----A1
(b)

Use cosine rule :
$\mathrm{PQ}^{2}=300^{2}+120^{2}-2(300)(120) \cos 116------\mathrm{M} 1$
$\mathrm{PQ}=368.73$

$$
=369 \text { ( } 3 \mathrm{sf} \text { ) }
$$

(c)

Form sine rule equation or cosine rule to find angle QPB-----M1
find angle QPB------M1 (ecf)
bearing of Q from $\mathrm{P}=064-017.00=047.0---\mathrm{A} 1$
8. (a)
(i)
11.625---B1
(ii)

It was because we do not have the actual timing for each customer. ---B1
(iii)
5.35 -----B1
(iv)

I would go to Shop B although the the mean is slightly higher than shop A.
But the smaller SD suggest that the more consistency in the waiting time. ---
--B1

Or

I would go to Shop A as the mean is smaller, meaning on average I will have a shorter waiting time. ----B1

Any reasoning that is logicial.
(v)

The mean will remain the same and the SD will decrease. B1 each
(b)
(i)

52 ---B1
(ii)

66----B1
(iii)

44 -----B1
(iv)

$$
\begin{aligned}
& 180-75 \text {--------M1 ( Find 75) } \\
& =105----\mathrm{B} 1
\end{aligned}
$$

9. (a)

$$
\left(\begin{array}{lll}
84 & 90 & 56 \\
92 & 60 & 61
\end{array}\right)---\mathrm{B} 1
$$

(b) $\left(\begin{array}{ll}1812 & 886 \\ 1673 & 853\end{array}\right)----$-B1
(c) $48.9 \%----\mathrm{B} 1$
(d) (1742.5 869.5) --------A1
(e) The average sale for the 2 days is $\$ 1742.50$ and the average profit is \$ 869.50. ---B1
10. (a)
(i) Find gradient or y intercept correct ---M1

$$
y=\frac{3}{8} x+3 \frac{1}{4}---A 1
$$

She is not correct as the y intercept should be 3.25 .
(ii) Use Pythagoras' thm or formula ------M1

$$
\mathrm{AB}=5 \text { units }
$$

(iii) -4/5 -----B1
(iv) $x=-1$ or 5
(b) (i)

$$
\begin{aligned}
& \frac{130000000}{55}-\cdots-M 1 \\
& =2.37 \times 10^{6} \ldots
\end{aligned}
$$

(ii)
$\left(1.3 \times 10^{8}\right) \div 70 \div 10 \div 150---$ M1 = 1238 ---- A1
interest $=\frac{2}{100} \times 40000 \times \frac{30}{12}=2000---M 1$
monthly instalment $=42000 \div 30=1400--\mathrm{A} 1$
(b)
$\frac{5100}{85} \times 15---$ M1 $\quad(85 \%$ is $\$ 5100$, find $15 \%)$
= 900------A1
(c) Teaching staff : $12(2)(4)(\$ 70)+14(2)(4)(\$ 90)=(\$ 6720+\$ 10080)=\$ 16800$ M1
Total staff cost : $16800+1800+45(5)(4)=\$ 19500$

Rental and printing cost $=8400$
Total operational cost per month $=\$ 19500+8000+400=27900$ M1
Total fee collected (assuming each class has the minimum number of students) $=12(5)(\$ 200)+14(5)(\$ 300) \quad---\mathrm{M} 1$
$=\$ 33000$
( $\$ 52800$ if they find 8 students per class ) Can find for other number as well but they have to state.

Min Profit per month $=\$ 5100---\mathrm{M} 1$
Her target of a minimum of $\$ 5000$ per month can be reached as her minimum profit per month is $\$ 5100$. -A1 justification
$\$ 5100 \times 6$ months $=\$ 30600<\$ 45100+2000$ interest --- -M 1
She might miss her target of recovering her start up cost within 6 months as the total profit for 6 months assuming she get the minimal number of students per class is less than the start up cost.--_A1

