Name	Register Number	Class

GREENRIDGE SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2022
Secondary 3 Express

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

28 September 2022 **2 h 15 min**
Wednesday 1145 - 1400

Additional Materials: No Additional Materials are Required

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.
Write your answers and working on the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
90

1 A curve has the equation $y = kx^2 + (2k - 4)x + 3k - 2$, where $k > 0$. Find the set of values of k for which the curve lies completely above the x-axis. [3]

2 A rectangular block has a square base. The length of each side of the base is $\left(\sqrt{3}-\sqrt{2}\right)$ m and the volume of the block is $\left(4\sqrt{2}-3\sqrt{3}\right)$ m^3. Find, without using a calculator, the height of the block in the form $\left(a\sqrt{2}+b\sqrt{3}\right)$ m, where a and b are integers. [4]

3 Solve the simultaneous equations.

$$9^x(27)^y = 1$$
$$8^y \div (\sqrt{2})^x = 16\sqrt{2}$$

[4]

4 **(a)** Find the values of the constant c for which the line $2y = x + c$ is a tangent to the curve $y = 2x + \frac{6}{x}$. [4]

(b) If the quadratic equation $m(x^2 + 9) + 2x(x+1) + (6m-2)x = -16$ has 2 real and distinct roots, given that m is a constant, determine the range of values of m. [4]

5 **(a)** Find the value of each of the integers p and q for which $\left(\frac{25}{16}\right)^{-\frac{3}{2}} = 2^p \times 5^q$. [2]

(b) By using the substitution $u = 3^x$, find the values of x such that $3^{2x+1} - 2 = 8 \times 3^{x-1}$. [5]

6 **(a)** Obtain the first four terms in the expansion of $\left(2-\frac{x}{4}\right)^8$ in ascending powers of x. [2]

(b) Hence, find the coefficient of x^3 in the expansion of $(1+x)^2\left(2-\frac{x}{4}\right)^8$. [3]

7 **(a)** Solve the equation $\lg(x+12)=1+\lg(2-x)$. [3]

(b) Given that $\log_2 p = a$, $\log_8 q = b$ and $\frac{p}{q} = 2^c$, express c in terms of a and b. [4]

8 Desmond buys and sells shares in the stock market. The value of the shares he bought is given by the function $y = 3x^2 - 5x + 7$, where y is the value of the shares in thousands of dollars and x is the time in years after it was first bought.

(a) What is the minimum value of the shares and when does it occur? [4]

(b) Sketch the graph of $y = -x^2 - 2x + 6$, showing clearly the coordinates of the minimum point and the intersections with the axes. [3]

9 The function $f(x)=x^3-6x^2+ax+b$, where a and b are constants, is exactly divisible by $x-3$ and leaves a remainder of -55 when divided by $x+2$.

(a) Find the value of a and of b. [4]

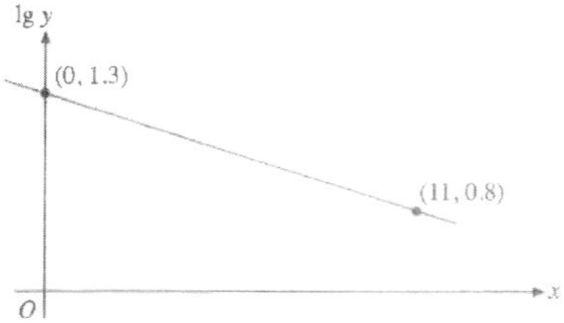
(b) Solve the equation $f(x)=0$. [4]

10 The line $4y = 3x + 1$ intersects the curve $xy = 28x - 27y$ at the points $P(1,1)$ and Q. The perpendicular bisector of PQ intersects the line $y = 4x$ at the point R. Calculate the area of triangle PQR. [9]

11 **(a)** Express $\frac{2x^2+x-3}{(x^2-2)(x+1)}$ in partial fractions. [4]

11 **(b)** Express $\dfrac{x^4+9}{x^3+3x}$ into the form $x+\dfrac{A}{x}+\dfrac{Bx+C}{x^2+3}$, where A, B and C are constants to be determined. [6]

12



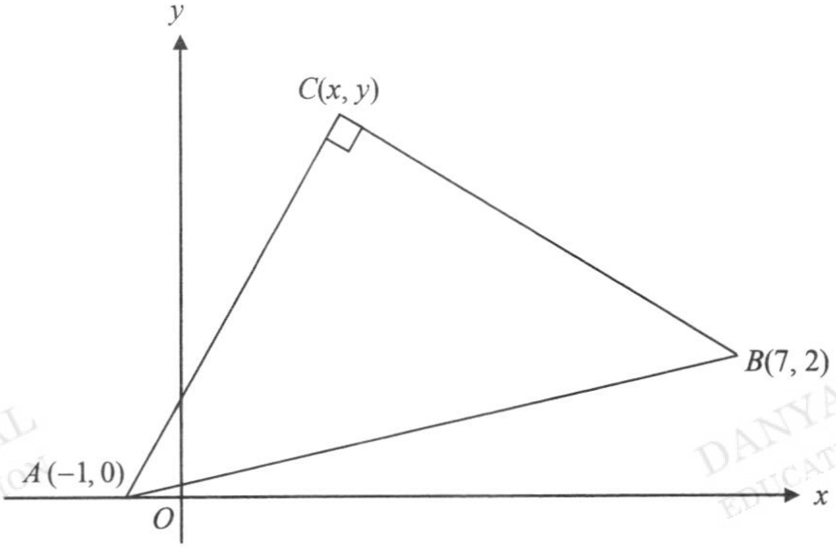
The variables x and y are connected by the equation $y = kb^x$, where k and b are constants. Experimental values of x and y were obtained. The diagram above shows the straight line graph, passing through the points $(0,1.3)$ and $(11,0.8)$, obtained by plotting lg y against x. Estimate

(a) the value of k and of b, corrected to 2 significant figures, [5]

(b) the value of y when $x = 8$. [2]

13 Solutions to this question by accurate drawing will not be accepted.

The diagram shows ΔABC with coordinates $A(-1,0)$, $B(7,2)$ and $C(x,y)$ and $\angle ACB = 90°$. The point $C(x, y)$ lies on the perpendicular bisector of AB.



(a) **(i)** Find the equation of the perpendicular bisector of AB. [3]

(ii) Show that the coordinates of C is $(2,5)$. [4]

13 **(b)** The point *D* is the reflection of point *C* in the line *AB*. Find the coordinates of *D*. [3]

(c) Write down the specific name given to the shape of the quadrilateral *ABCD*. [1]

End of Paper

Name	Register Number	Class

GREENRIDGE SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2022
Secondary 3 Express

ADDITIONAL MATHEMATICS
4049/02

Paper 2

4 October 2022
Wednesday

2 h 15 min
1110 – 1325

Additional Materials: No Additional Materials are Required

READ THESE INSTRUCTIONS FIRST

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For Examiner's Use
90

1 Find the values of k for which the line $x+3y=k$ and the curve $y^2=2x+3$ do not intersect. [4]

2 **(a)** Simplify $3\sqrt{180}+\sqrt{245}-2\sqrt{125}$, leaving your answer in surd form. [4]

(b) Given that $\sqrt{a+b\sqrt{3}}=\frac{13}{4+\sqrt{3}}$, where a and b are integers, find, without using a calculator, the value of a and of b. [4]

3 In the expansion of $\left(3x-\frac{1}{2x}\right)^{10}$, evaluate

(a) the term independent of x, [4]

(b) the term in x^6 , [2]

4 **(a)** On the same graph, sketch the curves $y = e^x$ and $x + y = -2$. [2]

(b) Write down the coordinates of the point where the curve $y = e^x$ cuts the *y*-axis. [1]

(c) Hence, determine the number of solutions of the equation $e^x + x + 2 = 0$. [2]

5 Given that $\log_2 x = p$ and $\log_4 y = q$, express the following in terms of p and/or q.

(a) $\log_2 \sqrt{x}$, [2]

(b) $\log_2 xy^2$, [3]

(c) $\log_4 \dfrac{4x}{y}$. [3]

6 **(a)** Solve the equation $\dfrac{27^{2+x}}{9}=3^{x}\times 81^{2x-1}$. [4]

(b) Given that $25^{x+1}\times 2^{4x-1}=32^{x}\times 5^{3x}$, evaluate 10^{x}. [4]

7 A man buys a new car. After t months, its value $\$C$ is given by $C = 100000e^{-at}$, where a is a constant.

(a) Find the value of the car when the man bought it. [2]

(b) The value of the car after 24 months is expected to be $65000.

(i) Calculate the expected value of the car after 3 years, [3]

(ii) Calculate the age of the car, to the nearest month when its expected value will be $30000, [2]

(iii) After 5 years, a car dealer offers to pay the man $35000 for your car. Based on the equation above, should the man agree to sell it? Explain your answer. [3]

8 Solve.

(a) $\log_7(17y+15) = 2 + \log_7(2y-3)$, [3]

(b) $\log_p 8 \times \log_{16} p$, [3]

(c) $3\log_5 y - \log_y 5 = 2$. [5]

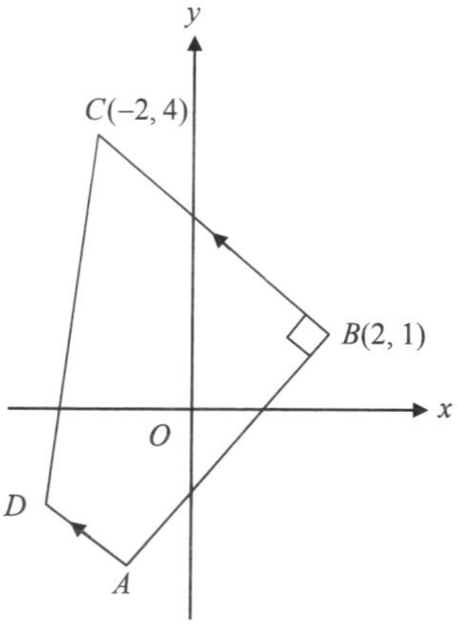
9 Given $f(x) = 2x^3 - 5x^2 - 4x + 12$,

(a) show that $(x-2)$ is a factor of $f(x)$, [1]

(b) factorise $f(x)$ completely, [3]

(c) hence, solve the equation $2(2^{3y}) - 5(2^{2y}) = 4(2^y - 3)$. [4]

10



In the trapezium *ABCD*, *AD* and *BC* are parallel and angle *ABC* is a right angle. The coordinates of the points *B* and *C* are $(2, 1)$ and $(-2, 4)$ respectively.

(a) Find the equation of the line *AB*. [3]

(b) The *y*-intercept of the line *DA* produced is $-\frac{7}{2}$, show that the coordinates of *A* is $(-\frac{22}{25}, -\frac{71}{25})$. [3]

10 **(c)** Given that the midpoint of the line segment BD is $\left(0, -\frac{1}{2}\right)$, find the coordinates of D. [4]

(d) Find the area of the trapezium $ABCD$. [2]

11 The table below shows some experimental values of 2 variables x and y.
It is known that one value of y has been recorded incorrectly.

x	0.5	1.0	1.5	2.0	2.5
y	1.20	1.00	0.92	0.75	0.66

It is known that x and y are related by the equation $y = \frac{a}{x+b}$, where a and b are constants.

(a) Using 2 cm to represent 0.5 units on the horizontal axis and 2 cm to represent 0.2 units on the vertical axis, plot $\frac{1}{y}$ against x for the given data and draw a straight line graph. [4]

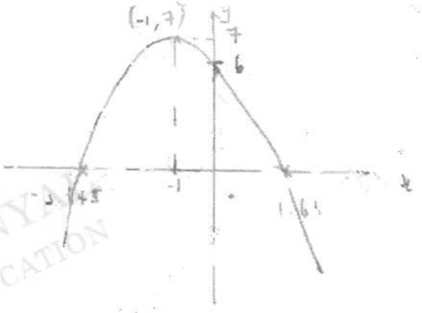
(b) Using your graph,

(i) estimate the value of a and of b, [4]

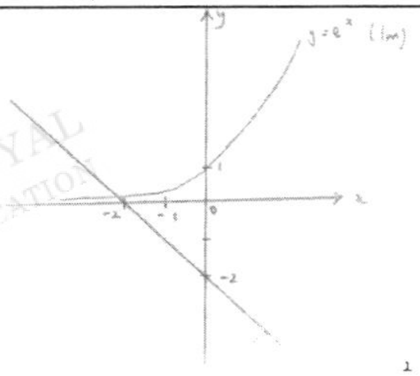
(ii) identify the value of x which has an incorrect value of y and estimate the correct value of y, correct to 2 decimal places. [2]

End of Paper

Greenridge Secondary School
2022 Sec 3Exp EYE P1 A Math

1		$k < -2$ or $k > 1$		9	(a)	$a = 10, b = -3$
2		$h = 2\sqrt{2} + \sqrt{3}$ m			(b)	$x = 3, 2.62, 0.382$
3		$x = -\frac{9}{5}$ and $y = \frac{6}{5}$		10		$Q(9,7)$ Perpendicular bisector $y = -\frac{4}{3}x + \frac{32}{3}$ $R(2,8)$ Area = 25 units^2
4	(a)	$c = \pm 12$		11	(a)	$\frac{1}{x^2-2} + \frac{2}{x+1}$
	(b)	$m < -\frac{16}{17}$			(b)	$x + \frac{3}{x} - \frac{6}{x^2+3}$
5	(a)	$p = 6$ and $q = -3$		12	(a)	b=0.90 k= 20
	(b)	$x = 0.290$			(b)	y=8.59
6	(a)	$256 - 256x + 112x^2 - 28x^3 + ...$		13	(a)(i)	$y = -4x + 13$
	(b)	-60			(a)(ii)	$C(2,5)$
7	(a)	$x = \frac{8}{11}$			(b)	$D(4,-3)$
	(b)	$c = a - 3b$			(e)	Square
8	(a)	Min value=\$4916.67 after $\frac{5}{6}$ years or 10 months				
	(b)					

Greenridge Secondary School
2022 Sec 3Exp EYE P2 A Math

1		Consider $b^2-4ac<0$ $k<-6$		7	(a)	$100000
2	(a)	$15\sqrt{5}$			(b)(i)	$52404.67
	(b)	$a=19, b=-8$			(b)(ii)	67 months
3	(a)	$1913\frac{5}{8}$			(b)(iii)	$34063.04 Yes, because dealer is offering more
	(b)	$73811\frac{1}{4}x^6$		8	(a)	$y=2$
4	(a)				(b)	$\frac{3}{4}$
	(b)	(0,1)			(c)	$y=0.585$ and $y=5$
	(c)	1		9	(a)	Show $f(2)=0$
5	(a)	$\frac{1}{2}p$			(b)	$f(x)=(x-2)^2(2x+3)$
	(b)	$p+4q$			(c)	$2^y=2,-\frac{3}{2}$ $y=1$
	(c)	$1+\frac{p}{2}-q$		10	(a)	$y=\frac{4}{3}x-\frac{5}{3}$
6	(a)	$\frac{4}{3}$			(b)	Equation of DA $y=-\frac{3}{4}x-\frac{7}{2}$
	(b)	$\frac{25}{2}$			(c)	$D(-2,-2)$
					(d)	$15\frac{9}{25}$
				11	(b)(i)	Accept a = 2.5 to 3.0, b=1.5 to 2
					(b)(II)	y=0.85

Name	Register Number	Class

GREENRIDGE SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2022
Secondary 3 Express

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

Oct 2022 **2 h 15 min**
Wednesday

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For Examiner's Use
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*This paper consists of **16** printed pages, including this cover page.*

1 A curve has the equation $y = kx^2 + (2k-4)x + 3k - 2$, where $k > 0$. Find the set of values of k for which the curve lies completely above the x-axis. [3]

$y = kx^2 + (2k-4)x + 3k - 2$

Completely above x axis $\longrightarrow$ $b^2 - 4ac < 0$ (1m)

$(2k-4)^2 - 4k(3k-2) < 0$

$4k^2 - 16k + 16 - 12k^2 + 8k < 0$

$-8k^2 - 8k + 16 < 0$

$k^2 + k - 2 > 0$ (1m)

Roots: $k^2 + k - 2 = 0$

$(k+2)(k-1) = 0$

$k = -2$ or $k = 1$

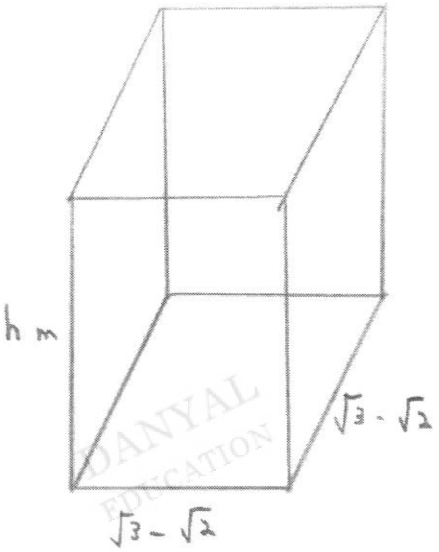
$\therefore k < -2$ or $k > 1$ (1m)

(NA. $\because k > 0$)

(-1m if $k > -2$ is not rejected)

[4]

2 A rectangular block has a square base. The length of each side of the base is $\left(\sqrt{3}-\sqrt{2}\right)$ m and the volume of the block is $\left(4\sqrt{2}-3\sqrt{3}\right)$ m^3. Find, without using a calculator, the height of the block in the form $\left(a\sqrt{2}+b\sqrt{3}\right)$ m, where a and b are integers. [~~5~~]



Vol = $4\sqrt{2}-3\sqrt{3}$ m^3

Let h m be the height of the block

$\left(\sqrt{3}-\sqrt{2}\right)^2 h = 4\sqrt{2}-3\sqrt{3}$ (1m)

$\left(3-2\sqrt{6}+2\right)h = 4\sqrt{2}-3\sqrt{3}$

$h = \dfrac{4\sqrt{2}-3\sqrt{3}}{5-2\sqrt{6}}$ (1m)

$\dfrac{4\sqrt{2}-3\sqrt{3}}{5-2\sqrt{6}} \times \dfrac{5+2\sqrt{6}}{5+2\sqrt{6}}$ (1m)

$\dfrac{20\sqrt{2}+8\sqrt{12}-15\sqrt{3}-6\sqrt{18}}{25-(4\times6)}$

$= \dfrac{20\sqrt{2}+16\sqrt{3}-15\sqrt{3}-18\sqrt{2}}{1}$

$= 2\sqrt{2}+\sqrt{3}$ m (1m)

3 Solve the simultaneous equations.

$$9^x(2\overset{7}{\cancel{9}})^y = 1$$ [4]

$$8^y \div (\sqrt{2})^x = 16\sqrt{2}$$ [5]

$3^{2x}\,(3^3)^y = 3^0$

Consider power of 3

$2x + 3y = 0$ — (1) (1m)

$(2^3)^y \div (2^{\frac{1}{2}})^x = 2^4\, 2^{\frac{1}{2}}$

Consider power of 2

$3y - \frac{1}{2}x = 4\frac{1}{2}$

$\therefore 6y - x = 9$ — (2) 1m

(1) × 2

$4x + 6y = 0$ — (3)

(3) − (2)

$5x = -9$

$x = \frac{-9}{5}$ 1m

Subst $x = \frac{-9}{5}$

$2\left(-\frac{9}{5}\right) + 3$

$3y = \frac{18}{5}$

$y = \frac{6}{5}$ (1m)

4 **(a)** Find the values of the constant c for which the line $2y = x + c$ is a tangent to the curve $y = 2x + \frac{6}{x}$. [4]

$2y = x + c$ —— (1)

$y = 2x + \frac{6}{x}$ —— (2)

Subst $\underline{(2)}$ into (1)

$2\left(2x + \frac{6}{x}\right) = x + c$ (1m)

$4x + \frac{12}{x} - x - c = 0$

$3x^2 - cx + 12 = 0$ (1m)

Tangent $\longrightarrow b^2 - 4ac = 0$ (1m)

$(-c)^2 - 4(3)(12) = 0$

$c^2 = 144$

$c = \pm 12$ (1m)

(b) If the quadratic equation $m(x^2 + 9) + 2x(x+1) + (6m-2)x = -16$ has 2 real and distinct roots, given that m is a constant, determine the range of values of m. [4]

$mx^2 + 9m + 2x^2 + 2x + 6mx - 2x + 16 = 0$

$(m+2)x^2 + 6mx + (9m+16) = 0$ (1m)

2 Real & distinct roots $\longrightarrow b^2 - 4ac > 0$ (1m)

$(6m)^2 - 4(m+2)(9m+16) > 0$

$36m^2 - 36m^2 - 136m - 128 > 0$ (1m)

$-136m - 128 > 0$

$136m + 128 < 0$

$m < -\frac{128}{136}$

$m < -\frac{16}{17}$ (1m)

5 **(a)** Find the value of each of the integers p and q for which $\left(\frac{25}{16}\right)^{-\frac{3}{2}} = 2^p \times 5^q$. [2]

$\left(\frac{16}{25}\right)^{\frac{3}{2}} = 2^p \times 5^q$

$\left(\frac{4}{5}\right)^{3} = 2^p \times 5^q$ (1m)

$2^6 \; 5^{-3} = 2^p \; 5^q$

$\therefore p = 6, \quad q = -3$, 1m.

(b) By using the substitution $u = 3^x$, find the values of x such that $\overset{3^{2x+1}}{\cancel{3^{x+1}}} - 2 = 8 \times 3^{x-1}$. [~~6~~] [5]

$3^{2x} \cdot 3 - 2 = 8 \times \frac{3^x}{3}$ (1m)

Let $u = 3^x$

$3u^2 - 2 = \frac{8}{3}u$ (1m)

$9u^2 - 6 = 8u$

$9u^2 - 8u - 6 = 0$

$u = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(9)(-6)}}{2(9)}$ (1m)

$= \frac{8 \pm \sqrt{64 + 216}}{18}$

$= \frac{8 \pm \sqrt{280}}{18}$

$3^x = 1.37407$ or -0.48518 (1m)

$x \lg 3 = \lg 1.37407$ or $x \lg 3 = \lg -0.48518$

$x = 0.28917$ NA

$= 0.290$ (3sf)

(1m)

(-1m if solution is not rejected)

6 **(a)** Obtain the first four terms in the expansion of $\left(2-\frac{x}{4}\right)^8$ in ascending powers of x. [2]

$\left(2-\frac{x}{4}\right)^8 = 2^8 + {}^8C_1\left(-\frac{x}{4}\right)^1(2)^7 + {}^8C_2\left(-\frac{x}{4}\right)^2(2)^6 + {}^8C_3\left(-\frac{x}{4}\right)^3(2)^5 + \dots$ (1m)

$= 256 + 8\left(-\frac{x}{4}\right)(128) + 28\left(\frac{x^2}{16}\right)(64) + 56\left(-\frac{x^3}{64}\right)(32) + \dots$

$= 256 - 256x + 112x^2 - 28x^3 +$ (1m)

(b) Hence, find the coefficient of x^3 in the expansion of $(1+x)^2\left(2-\frac{x}{4}\right)^8$. [3]

$(1+2x+x^2)(256 - 2$[illegible] [illegible]$)$

Consider x^3

$+1(-28)$ [illegible] 2 [illegible] x^2 [illegible]

$\dots + -$ [illegible] $+$ [illegible] (1m)

[illegible]

[illegible] 0 (1m)

7 **(a)** Solve the equation $\lg(x+12) = 1 + \lg(2-x)$. [3]

$\lg(x+12) = \lg 10 + \lg(2-x)$ (1m)

$\lg(x+12) = \lg 10(2-x)$ (1m)

$\therefore x + 12 = 20 - 10x$

$11x = 8$

$x = \frac{8}{11}$ (1m)

→ **(b)** Given that $\log_2 p = a$, $\log_{\underset{8}{\textcircled{8}}} q = b$ and $\frac{p}{q} = 2^c$, express c in terms of a and b. [4]

$p = 2^a$, $q = 8^b$

(1m) (1m)

$\frac{p}{q} = \frac{2^a}{8^b}$

$= \frac{2^a}{2^{3b}}$ (1m)

$= 2^{a-3b}$

$c = a - 3b$

8 Desmond buys and sells shares in the stock market. The value of the shares he bought is given by the function $y = 3x^2 - 5x + 7$, where y is the value of the shares in thousands of dollars and x is the time in years after it was first bought.

(a) What is the minimum value of the shares and when does it occur? $\cancel{[2]}$ 4

$$y = 3\left(x^2 - \frac{5}{3}x + \frac{7}{3}\right)$$
$$= 3\left[x^2 - \frac{5}{3}x + \left(\frac{-5}{6}\right)^2 - \left(\frac{-5}{6}\right)^2 + \frac{7}{3}\right]$$
$$= 3\left[\left(x - \frac{5}{6}\right)^2 - \frac{25}{36} + \frac{7}{3}\right] \quad (1m)$$
$$= 3\left[\left(x - \frac{5}{6}\right)^2 + \frac{59}{36}\right] \quad 3\left(x - \frac{5}{6}\right)^2 + \frac{59}{12}$$

$\text{min pt} = \left(\frac{5}{6}, \frac{59}{12}\right)$ (1m)

$\therefore$ min value = \$ 491.67 (1m) after $\frac{5}{6}$ yrs or 10 mths (1m)

(b) Sketch the graph of $y = -x^2 - 2x + 6$, showing clearly the coordinates of the minimum point and the intersections with the axes. [3]

$y = -x^2 - 2x + 6$

$a = -2 < 0 \rightarrow$ max pt

x intercept $\rightarrow y = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4[illegible]}}{2(-1)}$$

$= 2$ [illegible]

$= -3.[illegible]4$ or 1.646 (1m)

y intercept $\rightarrow x = 0$

$y = 6$ (1m)

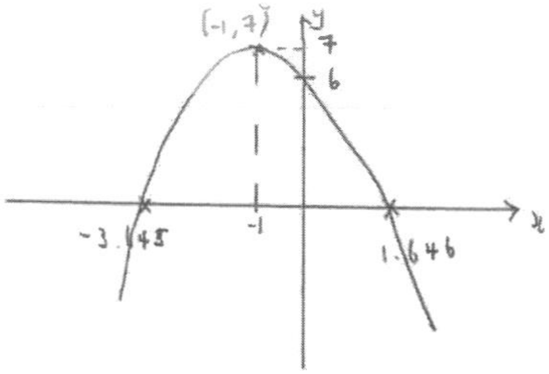
$y = -\left[x^2 + 2x - 6\right]$

$= -\left[x^2 + 2x + (1)^2 - (1)^2 - 6\right]$

$= -\left[(x+1)^2 - 7\right]$

$= -(x+1)^2 + 7$

$\therefore$ Turning pt $= (-1, 7)$ (1m)



9 The function $f(x)=x^3-6x^2+ax+b$, where a and b are constants, is exactly divisible by $x-3$ and leaves a remainder of -55 when divided by $x+2$.

(a) Find the value of *a* and of *b*. [4]

$f(3) = 3^3 - 6(3)^2 + a(3) + b = 0$

$27 - 54 + 3a + b = 0$

$3a + b = 27 \quad — (1) \qquad (1m)$

$f(-2) = (-2)^3 - 6(-2)^2 + a(-2) + b = -55$

$-8 - 24 - 2a + b = -55$

$-2a + b = -23 \quad — (2) \qquad (1m)$

$\underline{(1) - (2)}$

$5a = 50$

$a = 10 \qquad (1m)$

Subst $\underline{a = 10 \text{ into}}$ (1)

$3(10) + \quad = 27$

$b = -3 \qquad (1m)$

37

(b) Solve the equation $f(x)=0$. [4]

$$\begin{array}{r|l} & x^2 - 3x + 1 \\ \hline x-3 & x^3 - 6x^2 + 10x - 3 \\ & \underline{x^3 - 3x^2} \\ & -3x^2 + 10x \\ & \underline{-3x^2 + 9x} \\ & x - 3 \\ & \underline{x - 3} \\ & 0 \end{array} \qquad (1m)$$

$\therefore f(x) = (x-3)(x^2 - 3x + 1) = 0 \qquad (1m)$

Consider $\underline{x^2 - 3x + 1 = 0}$

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$= \dfrac{3 \pm \sqrt{5}}{2} \qquad (1m)$

$\therefore x = 3, \ \dfrac{3+\sqrt{5}}{2}, \ \dfrac{3-\sqrt{5}}{2} \qquad (1m)$

OR $x = 3, \ 2.62, \ 0.382$

10 The line $4y = 3x + 1$ intersects the curve $xy = 28x - 27y$ at the point $P(1,1)$ and at the point Q. The perpendicular bisector of PQ intersects the line $y = 4x$ at the point R. Calculate the area of triangle PQR. 9 [8]

$4y = 3x + 1$

$y = \frac{3}{4}x + \frac{1}{4}$

Subst $\underline{y = \frac{3}{4}x + \frac{1}{4}}$ into $xy = 28x - 27y$

$x\left(\frac{3}{4}x + \frac{1}{4}\right) = 28x - 27\left(\frac{3}{4}x + \frac{1}{4}\right) = 0$ (1m)

$\frac{3}{4}x^2 + \frac{1}{4}x - 28x + \frac{81}{4}x + \frac{27}{4} = 0$

$3x^2 - 30x + 27 = 0$

$x^2 - 10x + 9 = 0$

$(x - 9)(x - 1) = 0$

$x = 9$ or $x = 1$ (pt ?) (1m)

When $x = 9$, $y = \frac{3(9)}{4} + \frac{1}{4}$

$= 7$

$\therefore Q(9, 7)$ (1m)

mid pt of PQ $= \left(\frac{1+9}{2}, \frac{1+7}{2}\right)$

$= (5, 4)$ (1m)

$m_{PQ} = \frac{7-1}{9-1}$

$= \frac{3}{4}$

m of ⊥ bisector of PQ $= -\frac{4}{3}$ (1m)

Consider mid pt $(5, 4)$

$\frac{y-4}{x-5} = -\frac{4}{3}$

$y = -\frac{4}{3}(x-5) + 4$

$y = -\frac{4}{3}x + \frac{32}{3}$ (1m)

Subst $\underline{y = -\frac{4}{3}x + \frac{32}{3}}$ into $y = 4x$

$-\frac{4}{3}x + \frac{32}{3} = 4x$

$x = 2$

$y = 4(2) = 8$

$\therefore R(2, 8)$ (1m)

Area △PQR

$= \frac{1}{2}\begin{vmatrix} 1 & 9 & 2 & 1 \\ 1 & 7 & 8 & 1 \end{vmatrix}$ (1m)

$= \frac{1}{5}(50)$

$= 25$ units^2 (1m)

11 **(a)** Express $\frac{2x^2+x-3}{(x^2-2)(x+1)}$ in partial fractions. [4]

$= \frac{Ax+B}{x^2-2} + \frac{C}{x+1}$ (1m)

$2x^2+x-3 = (Ax+B)(x+1) + C(x^2-2)$

When $x=-1$, $2(-1)^2+(-1)-3 = C(1-2)$

$-C = -2$

$C = 2$

2m (-1m for any 1 error)

When $x=0$, $-3 = B-2C$

$B = -3+4$

$= 1$

Comparing coefficient of x^2

$2 = A+C$

$\therefore A = 0$

$\therefore \frac{2x^2+x-3}{(x^2-2)(x+1)} = \frac{1}{x^2-2} + \frac{2}{x+1}$

(1m)

(b) Express $\frac{x^4+9}{x^3+3x}$ into the form $x+\frac{A}{x}+\frac{Bx+C}{x^2+3}$, where A, B and C are constants to be determined. [6]

$$\begin{array}{r|l} & x \\ \hline x^3+3x & x^4+0x^3+0x^2+9 \\ & x^4+0x^2+3x^2 \\ \hline & -3x^2+9 \end{array}$$

(1m)

$\therefore \frac{x^4+9}{x^3+3x} = x + \frac{-3x^2+9}{x^3+3x}$

Consider $\frac{-3x^2+9}{x^3+3x}$ $\quad \frac{-3x^2+9}{x(x^2+3)}$ (1m)

$= \frac{A}{x} + \frac{Bx+C}{x^2+3}$ (1m)

$\therefore -3x^2+9 = A(x^2+3) + (Bx+C)x$

2m (-1m for any 1 error)

When $x=0$

$9 = 3A$

$A = 3$

When $x=1$

$6 = 3(4) + B + C$

$B+C = -6$ — (1)

Comparing coefficient of x^2 — (2)

$-3 = A+B$

(1) − (2)

$C-3 = -6-(-3)$

$C = 0$

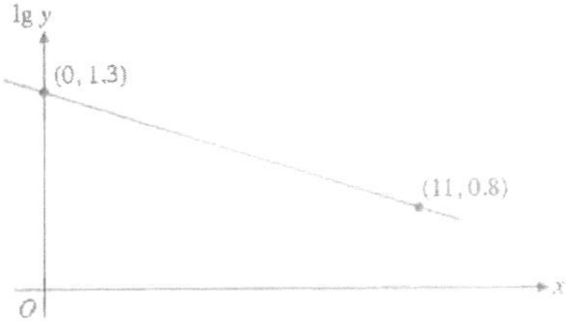
Subst $C=0$ into (1)

$B = -6$

$\therefore \frac{x^4+9}{x^3+3x} = x + \frac{3}{x} - \frac{6x}{x^2+3}$

(1m)

12



The variables x and y are connected by the equation $y = kb^x$, where k and b are constants. Experimental values of x and y were obtained. The diagram above shows the straight line graph, passing through the points $(0, 1.3)$ and $(11, 0.8)$, obtained by plotting lg y against x. Estimate

(a) the value of k and of b, corrected to 2 significant figures, 5 [~~6~~]

$\lg y = \lg kb^x$

$\lg y = x \lg b + \lg k$ (1m)

$m = \lg b = \frac{1.3 - 0.8}{0 - 11}$ (1m)

$= -\frac{1}{22}$

$b = 0.90$ (1m)

Y intercept, $\lg k = 1.3$ (1m)

$k = 19.950$ (2sf) (1m)

(b) the value of y when $x = 8$. [2]

When $x = 8$, $\lg y = 8 \lg 0.90 + \lg 19.952$

$= 0.9339$ (1m)

$y = 8.5887$

$= 8.59$ (3sf) (1m)

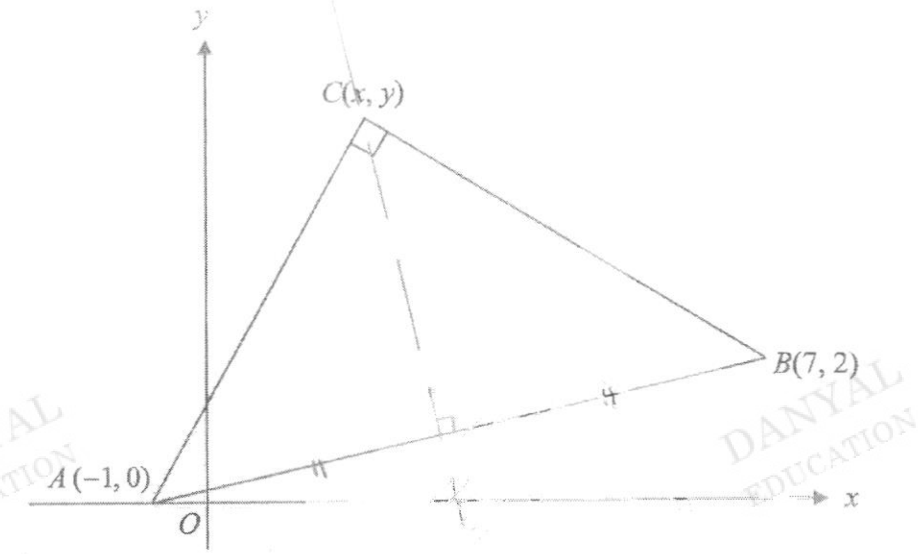
or

$y = 19.952(0.9)^8$

$= 8.58868$

$= 8.59$ (3sf)

13 **Solutions to this question by accurate drawing will not be accepted.**
The diagram shows ΔABC with coordinates $A(-1,0)$, $B(7,2)$ and $C(x,y)$ and $\angle ACB = 90°$. The point $C(x,y)$ lies on the perpendicular bisector of AB.



(a) Find
(i) the equation of the perpendicular bisector of AB, [3]

$m_{AB} = \frac{2-0}{7-(-1)}$

$= \frac{1}{4}$

m of ⊥ bisector of AB $= -4$ (1m)

mid pt$_{AB}$ $= \left(\frac{-1+7}{2}, \frac{0+2}{2}\right)$

$= (3, 1)$

$\therefore 1 = -4(3) + c$ (1m)

$13 = c$

$\therefore$ Equation $y = -4x + 13$ (1m)

(ii) ~~the coordinates of C.~~ Show that co-ordinates of C is (2, 5) [4]

$m_{AC} = \frac{y}{x+1}$

$m_{BC} = \frac{y-2}{x-7}$

} (1m)

$(m_{AC})(m_{BC}) = \left(\frac{y}{x+1}\right)\left(\frac{y-2}{x-7}\right) = -1$ (1m)

$y^2 - 2y = -x^2 + 6x + 7$

$(-4x+13)^2 - 2(-4x+13) = -x^2 + 6x + 7$

$16x^2 - 104x + 169 + 8x - 26 + x^2 - 6x - 7 = 0$

$17x^2 - 102x + 136 = 0$

$x^2 - 6x + 8 = 0$ (1m)

$(x-4)(x-2) = 0$

$x = 4$ or 2

(NA) beyond mid pt (3, 1)

When $x = 2$

$y = -4(2) + 13$

$= 5$

$\therefore C(2, 5)$ (1m)

13 (b) The point D is the reflection of point C in the line AB. Find the coordinates of D. [3]

mid pt $_{CD}$ = mid pt $_{AB}$

Let D be (p, q)

$\left(\frac{2+p}{2}, \frac{5+q}{2}\right) = (3, 1)$ (1m)

$\therefore \frac{2+p}{2} = 3$, $\frac{5+q}{2} = 1$

$p = 3(2) - 2$ $q = 1(2) - 5$

$= 4$ $= -3$ (1m)

$\therefore$ D$(4, -3)$ (1m)

(c) Write down the specific name given to the shape of the quadrilateral $ABCD$. [1]

ABCD is a square - (1m)

Name	Register Number	Class

GREENRIDGE SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2022
Secondary 3 Express

ADDITIONAL MATHEMATICS
Paper 2

4049/02

4 October 2022
Wednesday

2 h 15 min
1110 – 1325

Additional Materials: No Additional Materials are Required

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.
Write your answers and working on the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
90

This paper consists of 17 printed pages, including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\mathrm{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin$$

$$\cos B \mp \sin \quad \sin$$

$$1 - 2\sin^2 A$$

$$) \cos \tfrac{1}{2}(A - B)$$

$$+ B) \sin \tfrac{1}{2}(A - B)$$

$$\tfrac{}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$

$$\cos A \qquad -2 \sin \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta ABC = \frac{1}{2}ab\sin C$$

1 Find the values of k for which the line $x+3y=k$ and the curve $y^2=2x+3$ do not intersect. [4]

$x+3y=k$

$y=-\frac{x}{3}+\frac{k}{3}$ — (1) (1m)

$y^2=2x+3$ — (2)

Subst (1) into (2)

$\left(\frac{-x}{3}+\frac{k}{3}\right)^2=2x+3$

$\frac{x^2}{9}-\frac{2xk}{9}+\frac{k^2}{9}-2x-3=0$

$x^2-2xk+k^2-18x-27=0$

$x^2+(-2k-18)x+(k^2-27)=0$ (1m)

No intersection → $b^2-4ac<0$ (1m)

$(-2k-18)^2-4(1)(k^2-27)<0$

$4k^2+72k+324-4k^2+108$ ..

$72k<-432$

$k<-6$ (1m)

OR

$x+3y=k$

$x=k-3y$ (1m)

subst $x=-3y$ into $y^2=2x+3$

$y^2=2(k-3y)+3$

$=2k-6y+3$

$y^2+6y+(-2k-3)=0$ (1m)

$b^2-4ac<0$ (1m)

$6^2-4(1)(-2k-3)<0$

$36+8k+12<0$

$8k<-48$

$k<-6$ (1m)

2 **(a)** Simplify $3\sqrt{180}+\sqrt{245}-2\sqrt{125}$, leaving your answer in surd form. [4]

2	180
2	90
3	45
3	15
5	5
	1

5	245
7	49
7	7
	1

5	125
5	25
5	5
	1

$$3\sqrt{180}+\sqrt{245}-2\sqrt{125}$$

$$=3\sqrt{2^2\times3^2\times5}+\sqrt{5\times7^2}-2\sqrt{5^2\times5}$$

$$=\underset{(1m)}{3(6)\sqrt{5}}+\underset{(1m)}{7\sqrt{5}}-\underset{1m)}{10\sqrt{5}}$$

$$=15\sqrt{5} \qquad (1m)$$

(b) Given that $\sqrt{a+b\sqrt{3}}=\dfrac{13}{4+\sqrt{3}}$, where a and b are integers, find, without using a calculator, the value of a and of b. [4]

$$\sqrt{a+b\sqrt{3}}=\frac{13}{4+\sqrt{3}}$$

$$a+b\sqrt{3}=\frac{13}{4+\sqrt{3}}\times\frac{13}{4+\sqrt{3}} \quad (1m)$$

$$=\frac{169}{16+8\sqrt{3}+3}$$

$$\frac{169}{19+8\sqrt{3}}\times\frac{19-8\sqrt{3}}{19-8\sqrt{3}} \quad (1m)$$

$$=\frac{169(19-8\sqrt{3})}{361-192}$$

$$=\frac{169(19-8\sqrt{3})}{169}$$

$$a+b\sqrt{3}=19-8\sqrt{3} \quad (1m)$$

$$\therefore a=19,\ b=-8 \quad (1m)$$

OR

$$\sqrt{a+b\sqrt{3}}=\frac{13}{4+\sqrt{3}}\times\frac{4-\sqrt{3}}{4-\sqrt{3}} \quad (1m)$$

$$=\frac{13(4-\sqrt{3})}{16-3}$$

$$=\frac{13(4-\sqrt{3})}{13}$$

$$=4-\sqrt{3} \quad (1m)$$

$$a+b\sqrt{3}=(4-\sqrt{3})^2 \quad (1m)$$

$$=16-8\sqrt{3}+3$$

$$=19-8\sqrt{3} \quad (1m)$$

3 In the expansion of $\left(3x-\frac{1}{2x}\right)^{10}$, evaluate

(a) the term independent of x, [4]

$T_{r+1} = {}^{10}C_r\left(-\frac{1}{2x}\right)^r(3x)^{10-r}$ (1m)

Consider x term

$(x^{-1})^r\,x^{10-r} = x^0$ (1m)

Compare power of x

$-r+10-r=0$

$-2r=-10$

$r=5$ (1m)

$\therefore T_6 = {}^{10}C_5\left(-\frac{1}{2x}\right)^5(3x)^{10-5}$

$= 252\left(-\frac{1}{2}\right)^5 3^5$

$= 252\left(-\frac{1}{32}\right)(243)$

$= 1913\frac{5}{8}$ (1m)

(b) the ~~middle term~~, Term in [illegible] [2]

$(\ \ )^r\ x^{10-r} = x^6$

$-2r = -10$ [illegible]

-2

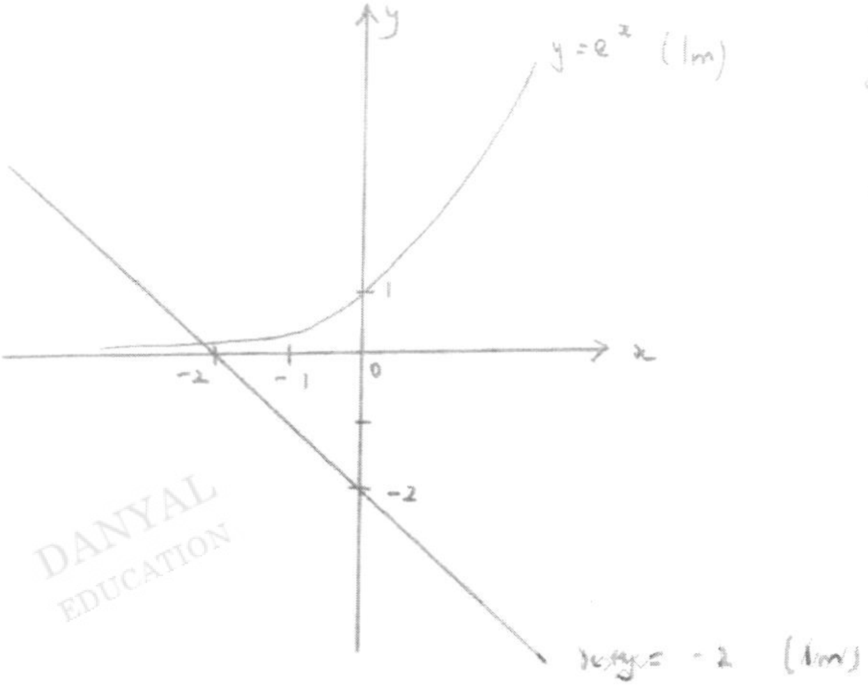
$= 2$ (1m)

$\therefore T_4 = {}^{10}C_2\left(-\frac{1}{2x}\right)^2(3x)^{10-2}$

$= 45\left(\frac{1}{4x^2}\right)(6561x^8)$

$= 73\ 811\frac{1}{4}x^6$ (1m)

4 **(a)** On the same graph, sketch the curves $y = e^x$ and $x + y = -2$. [2]



$x + y = -2$
x intercept $\rightarrow y = 0$
$x = -2$
y intercept $\rightarrow x = 0$
$y = -2$

(b) Write down the coordinates of the point where the curve $y = e^x$ cuts the y-axis. [1]

Pt is $(0, 1)$

(c) Hence, determine the number of solutions of the equation $e^x + x + 2 = 0$. [2]

$x + y = -2$
$y = -x - 2$
$e^x + x + 2 = 0$
$e^x = -x - 2$ (1m)

No. of solutions = no of intersections between
$y = e^x$ + $x + y = -2$

= 1 (1m)

~~6~~ 5 Given that $\log_2 x = p$ and $\log_4 y = q$, express the following in terms of p and/or q.

(a) $\log_2 \sqrt{x}$, [2]

$\log_2 x^{\frac{1}{2}} = \frac{1}{2} \log_2 x$ (1m)

$= \frac{1}{2} p$ (1m)

(b) $\log_2 xy^2$, [3] [~~2~~]

$= \log_2 x + \log_2 y^2$

$= \log_2 x + 2 \log_2 y$

$= p + 2 \left(\frac{\log_4 y}{\log_4 2} \right)$ (1m)

$= p + 2 \left(\frac{q}{\frac{1}{2}} \right) = p + 4q$

(c) $\log_4 \frac{4x}{y}$. [3] [~~2~~]

$= \log_4 4x - \log_4 y$

$= \log_4 4 + \log_4 x - \log_4 y$

$= 1 + \frac{\log_2 x}{\log_2 4} - q$ (1m)

$= 1 + \frac{p}{2 \log_2 2} - q$

$= 1 + \frac{p}{2} - q$ (1m)

6 ~~7~~ **(a)** Solve the equation $\frac{27^{2+x}}{9} = 3^x \times 81^{2x-1}$. [4]

$$\frac{3^{3(2+x)}}{3^2} = 3^x \times 3^{4(2x-1)} \quad (1m)$$

(1m)

$$3^{6+3x-2} = 3^{x+8x-4}$$

Comparing power of 3

$$4 + 3x = 9x - 4 \quad (1m)$$

$$6x = 8$$

$$x = \frac{4}{3} \quad (1m)$$

(b) Given that $25^{x+1} \times 2^{4x-1} = 32^{x} \times 5^{3x}$, evaluate 10^x. [4]

(1m) (1m)

$$5^{2(x+1)} \times 2^{4x-1} = 2^{5x} \times 5^{3x}$$

$$\frac{5^{2x+2}}{5^{3x}} = \frac{2^{5x}}{2^{4x-1}}$$

$$5^{-x+2} = 2^{x+1}$$

$$\frac{5^2}{5^x} = 2^x \times \quad (1m)$$

$$\frac{5^2}{2} = (2^x)(5^x)$$

$$\frac{25}{2} = 10^x \quad (1m)$$