Name:______________________________ ( )	Class:S3 ______

90

SERVE·LEAD·EXCEL

GREENDALE SECONDARY SCHOOL
End-of-Year Examination 2021

Additional Mathematics **4049**

5 October 2021

Secondary 3 Express **2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90.**

Q	1	2	3	4	5	6	7	8	9	10	11	12	13
Strands	A3	A5	A1	A2	A4	A6	A6	G2	A1	A5	G2	A4	G2
Marks													

This document consists of 19 printed pages including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The area of a trapezium is $\left(27+\sqrt{5}\right)$ cm^2. Given that the length of the two parallel sides are $\left(1+\sqrt{5}\right)$ cm and $\left(\sqrt{5}-\ \right)$ cm, express the height of the trapezium in the form $\left(\dfrac{a+b\sqrt{\ }}{c}\right)$ cm, where a, b and c are integers. [4]

2 **(a)** Find the first four terms in the expansion of $(2-3x)^8$ in ascending powers of x. [2]

(b) Hence, estimate the value of 1.7^8 and explain if this is a good estimation? [3]

3 The equation of a curve is $y = x^2 - kx - 5$, where k is a constant, and the equation of a line is $y + 4x = -2$.

(i) In the case where $k = 6$, find the coordinates of the points of intersection of the line with the curve. [3]

(ii) Show that, for all values of k, the line intersects the curve at two distinct points. [2]

4 Solve, for x and y, the simultaneous equations

$$9^x(27)^y = 1,$$

$$(\sqrt{2})^{3y} \div (\sqrt{2})^x = 2\sqrt{2}.$$

[6]

5 **(a)** Explain why $(x + 2)$ is not a factor of $6x^3 - x^2 + 2x + 3$. [1]

(b) Express $\dfrac{6x^3 - x^2 + 2x + 3}{x^2 - 1}$ in the form $Ax + B + \dfrac{C}{x-1} + \dfrac{D}{x+1}$. [5]

6 **(a)** Without using a calculator, show that $\frac{\log_3 5 \times \log_5 9}{\log_{49} 7} = 4$. [3]

(b) Solve the equation $2\log_9 3 + \log_5 (9y-2) = \log_2 8$. [3]

7 **(a)** Sketch the graph of $y = \log_2(x-2)$, for $x > 3$, stating your x-intercept clearly. [2]

(b) A company's share price, $\$P$, has been increasing each year.
The company claims that this increase is exponential and can be modelled by an equation of the form

$$P = 8e^{kt},$$

where k is a constant and t is the time in years since the company was formed.

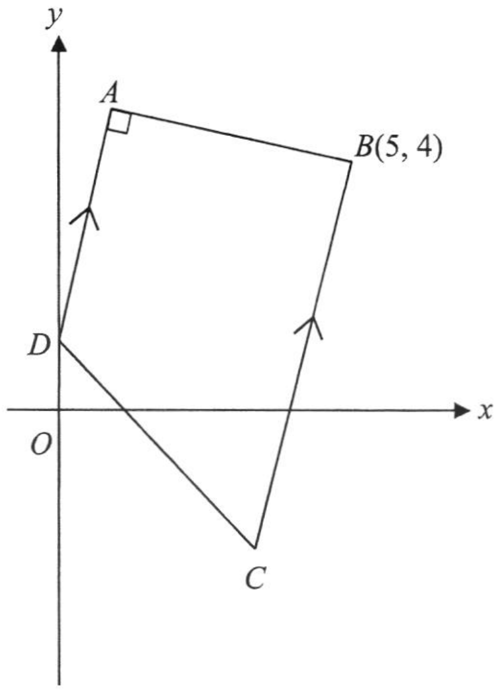
Find

(i) the initial value of the company's share, [1]

(ii) the value of k if, after 5 years, the value of the company's share has doubled, [2]

(iii) Using the value of k in **(bii)**, the value of t when the value of the company's share is 200% more than its original value. [2]

8 The diagram shows a quadrilateral $ABCD$ in which AD is parallel to BC and angle $DAB = 90°$.
The point B is (5, 4) and point D lies on the y-axis .
The equation of AD is $y = 3x + 1$.



(i) Find the coordinates of D. [1]

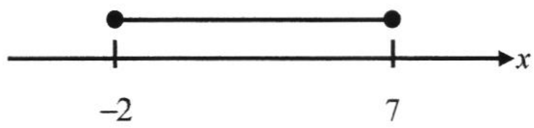
(ii) Show that the coordinates of A are (1.4, 5.2). [4]

Question 8 continued

(iii) It is given that M (4, 1) is the midpoint of BC, find the coordinates of C. [1]

(iv) Find the area of trapezium $ABCD$. [2]

9 **(a)** Find the value of p and of q for which the solution set of $ax^2 + px - q \leq 0$ is represented on the number line below stating any assumption made for a.



[3]

Question 9 continued

(b) Peter throws a stone such that its height, h metres, from the ground at time t seconds, is given by the equation $h = -2t^2 + 3t + 1$.

(i) Express the function in the form $h = a(t-p)^2 + q$. [3]

(ii) Explain why the stone will never reach a height of 2.6 metres. [2]

10 **(a)** Find the value of h for which the coefficient of x^3 in the expansion of $(1+hx)^6+(2-x)^7$ is 720. [4]

Question 10 continued

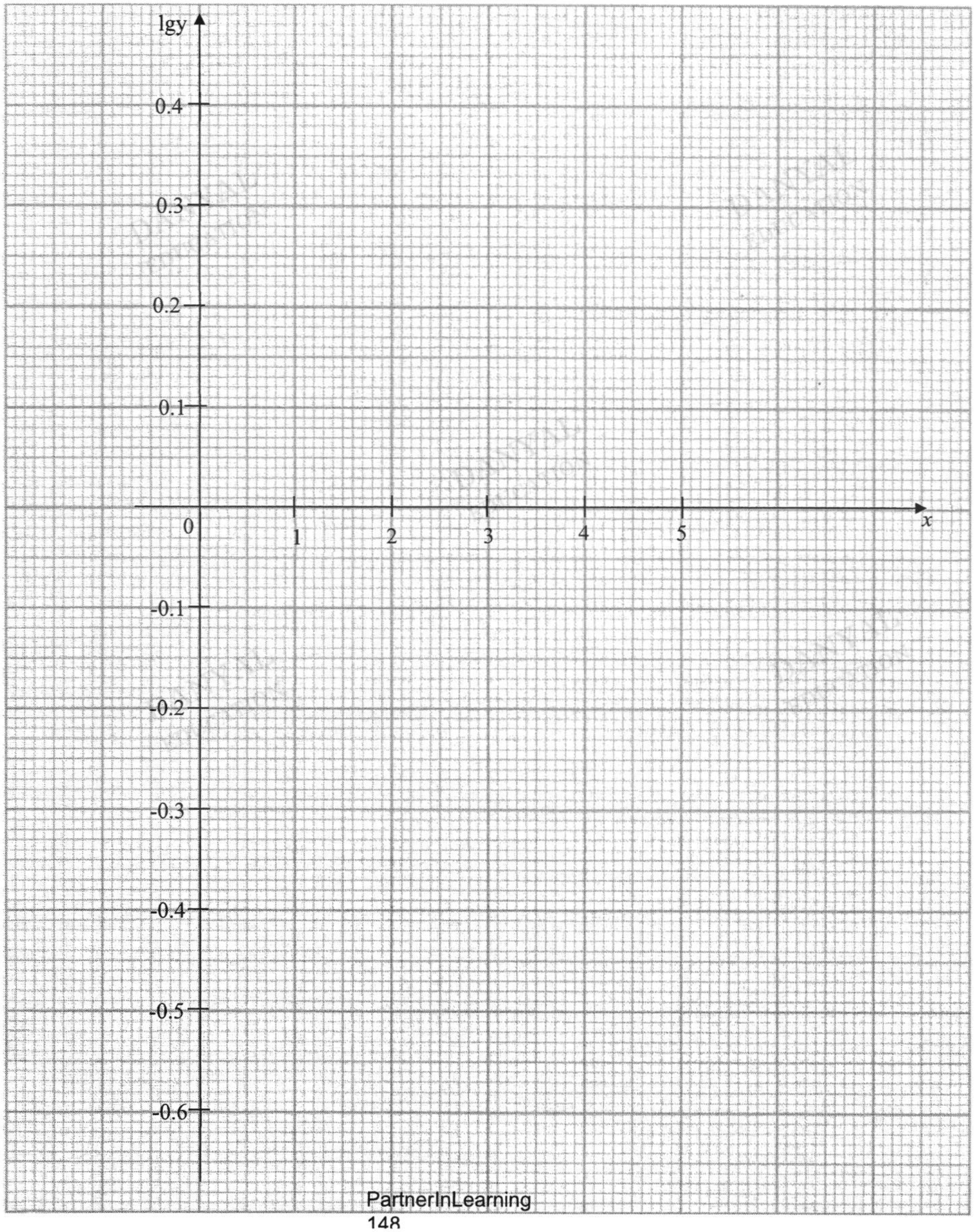
(b) **(i)** Find the general term in the binomial expansion of $\left(x+\frac{4}{x^2}\right)^6$. [1]

(ii) Hence, determine the term independent of x in the expansion of $\left(x+\frac{4}{x^2}\right)^6$. [3]

11 It is known that x and y are related by the equation $my = n(2^{mx})$, where m and n are constants.

x	1	2	3	4	5
y	0.566	0.80	1.13	1.60	2.26

(i) Plot lg y against x and draw a straight line graph. [2]



Question 11 continued

(ii) Use your graph to estimate the value of each of the constants m and of n. [4]

(iii) Use your graph to find the value of y when $x = 2.3$. [3]

12 The function f is defined by $f(x) = x^3 + hx^2 + kx - 3$.

(i) Given that $(x-3)$ is a factor of $f(x)$ and that $f(x)$ gives a remainder of -4 when divided by $(x+1)$, show that the value of $k = -2$ and $h = -2$. [5]

(ii) Hence, factorise $f(x)$ completely and show that $f(x) = 0$ only has one solution. [4]

13 The negative x-axis and the line $y = 10$ are tangents to a circle C_1.

(a) Find the radius of C_1 and the y-coordinate of the centre of C_1. [2]

(b) The line l is a tangent to C_1 at the point P.
The point P has coordinates $(-7, 9)$.
Given that the centre of C_1 lies below and to the right of P, find

(i) the equation of C_1, [4]

(ii) the equation of l. [3]

End of Paper

Marking Scheme

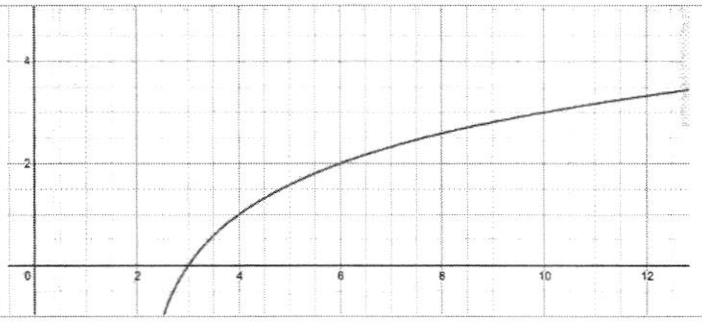
Q	Process	Mark
1	$\left(27+\sqrt{5}\right)=\frac{1}{2}\left(1+\sqrt{5}+3\sqrt{5}-5\right)\times h$	M1
	$\frac{2\left(27+\sqrt{5}\right)}{\left(1+\sqrt{5}+3\sqrt{5}-5\right)}=h$	
	$h=\frac{2\left(27+\sqrt{5}\right)}{\left(-4+4\sqrt{5}\right)}$	
	$=\frac{\left(27+\sqrt{5}\right)\times\left(-2-2\sqrt{5}\right)}{\left(-2+2\sqrt{5}\right)\times\left(-2-2\sqrt{5}\right)}$	M1 (ecf)
	$=\frac{-54-2\sqrt{5}-54\sqrt{5}-10}{4-20}$	
	$=\frac{-64-56\sqrt{5}}{-16}$	M1 (ecf)
	$=\frac{8+7\sqrt{5}}{2}$	A1
		4m
2(a)	$\left(2-3x\right)^8$	
	$=\binom{8}{0}(2)^8(-3x)^0+\binom{8}{1}(2)^7(-3x)^1+\binom{8}{2}(2)^6(-3x)^2+\binom{8}{3}(2)^5(-3x)^3$	M1
	$=256-3072x+16128x^2-48384x^3+\ldots$	A1
2(b)	Sub $x=0.1$	
	1.7^8	
	$=256-3072x+16128x^2-48384x^3+\ldots$	
	$=256-3072(0.1)+16128(0.1)^2-48384(0.1)^3+\ldots$	M1
	$=61.696/\frac{7712}{125}/61\frac{87}{125}$	A1
	This is a bad estimation as the above estimation is not very close to the exact value of $1.7^8=69.7575$. (or AS the powers increases, values gets bigger, therefore the values cannot be ignored….)	B1
		5m

Q	Process	Mark
3(i)	$x^2 - 6x - 5 = -4x - 2$ $x^2 - 2x - 3 = 0$ $x = -1, x = 3$ When $x = -1$, $y = -4(-1) - 2$ $= 2$ When $x = 3$, $y = -4(3) - 2$ $= -14$ $(-1, 2), (3, -14)$	M1 A1, A1
3(ii)	$x^2 - kx - 5 = -4x - 2$ $x^2 - kx + 4x - 3 = 0$ $D = (-k+4)^2 - 4(1)(-3)$ $= (-k+4)^2 + 12$ > 0 Since $(-k+4)^2 + 12 > 0$, the line intersects the curve at two distinct points.	M1 A1
		5m

Q	Process	Mark
4	$9^x(27)^y = 1 ---(1)$	
	$(\sqrt{2})^{3y} \div (\sqrt{2})^x = 2\sqrt{2} ---(2)$	
	From (1):	
	$3^{2x}(3^{3y}) = 3^0$	
	$2x+3y=0 ---(3)$	B1
	From (2):	
	$2^{\frac{3}{2}y} \div 2^{\frac{1}{2}x} = 2^{\frac{3}{2}}$	M1
	$\frac{3}{2}y - \frac{1}{2}x = \frac{3}{2}$	
	$3y - x = 3$	
	$-x+3y=3 ---(4)$	A1
	$(3)-(2)$:	
	$(2x+3y)-(-x+3y)=0-3$	M1 (ecf)
	$3x=-3$	
	$x=-1$	
	When $x=-1$,	
	$-(-1)+3y=3$	
	$3y=2$	
	$y=\frac{2}{3}$	
	$x=-1, y=\frac{2}{3}$	A1, A1
		6m

Q	Process	Mark
5(a)	$f(-2)=6(-2)^3-(-2)^2+2(-2)+3$ $=-53$ Since $f(-2)\neq 0$, so $(x+2)$ is not a factor of $6x^3-x^2+2x+3$.	B1
5(b)	Long division of $6x^3-x^2+2x+3$ by x^2-1: quotient $6x-1$ $-(6x^3+0x^2-6x)$ $-x^2+8x+3$ $-(-x^2+1)$ $8x+2$	M1 (long division)
	$\dfrac{6x^3-x^2+2x+3}{x^2-1}=6x-1+\dfrac{8x+2}{x^2-1}$	A1
	$\dfrac{8x+2}{(x+1)(x-1)}=\dfrac{C}{x-1}+\dfrac{D}{x+1}$	M1
	$8x+2=C(x+1)+D(x-1)$ When $x=1$, $8(1)+2=2C$, $C=5$ When $x=-1$, $8(-1)+2=-2D$, $D=3$	A1
	$\dfrac{6x^3-x^2+2x+3}{x^2-1}=6x-1+\dfrac{5}{x-1}+\dfrac{3}{x+1}$	B1 (ecf)
		6m

Q	Process		Mark
6(a)	$\dfrac{\log_3 5\times\log_5 9}{\log_{49} 7}$	$\dfrac{\log_3 5\times\log_5 9}{\log_{49} 7}$	
	$=\dfrac{\log_3 5\times\dfrac{\log_3 9}{\log_3 5}}{\dfrac{1}{\log_7 49}}$	$=\dfrac{\dfrac{\log_5 5}{\log_5 3}\times\log_5 9}{\dfrac{1}{\log_7 49}}$	M1 (change of base)
	$=\dfrac{\log_3 9}{\dfrac{1}{2}}$	$=\dfrac{\log_5 9}{\dfrac{1}{2}}$	M1 (obtain $\frac{1}{2}$ at denominator)
	$=2\log_3 3^2$	$=2\log_3 3^2$	
	$=4$	$=4$	A1
6(b)	$2\log_9 3+\log_5(9y-2)=\log_2 8$		
	$\log_9 3^2+\log_5(9y-2)=\log_2 2^3$		M1 (power law)
	$\log_9 9+\log_5(9y-2)=3$		
	$1+\log_5(9y-2)=3$		
	$\log_5(9y-2)=2$		
	$9y-2=5^2$		M1
	$9y=27$		
	$y=3$		A1
			6m

Q	Process	Mark
7(ai)		Shape – B1 x-intercept – B1
7(bi)	When $t=0$, $P=8e^{k(0)}$ $P=8$	B1
7(bii)	When $t=5$, $16=8e^{k(5)}$ $2=e^{5k}$ $\ln 2=5k$ $k=\dfrac{\ln 2}{5}$ $k=0.1386...$ $=0.139$	M1 A1
7(biii)	$24=8e^{0.1386t}$ $3=e^{0.1386t}$ $\ln 3=0.1386t$ $t=\dfrac{\ln 3}{0.1386}$ $t=7.9264...$ $=7.93\text{years}$ $24=8e^{\frac{\ln 2}{5}t}$ $3=e^{\frac{\ln 2}{5}t}$ $\ln 3=\dfrac{\ln 2}{5}t$ $t=\dfrac{\ln 3}{\frac{\ln 2}{5}}$ $t=7.92\text{years}$	M1 A1
	Also accept if student round off to 8 years.	
		7m

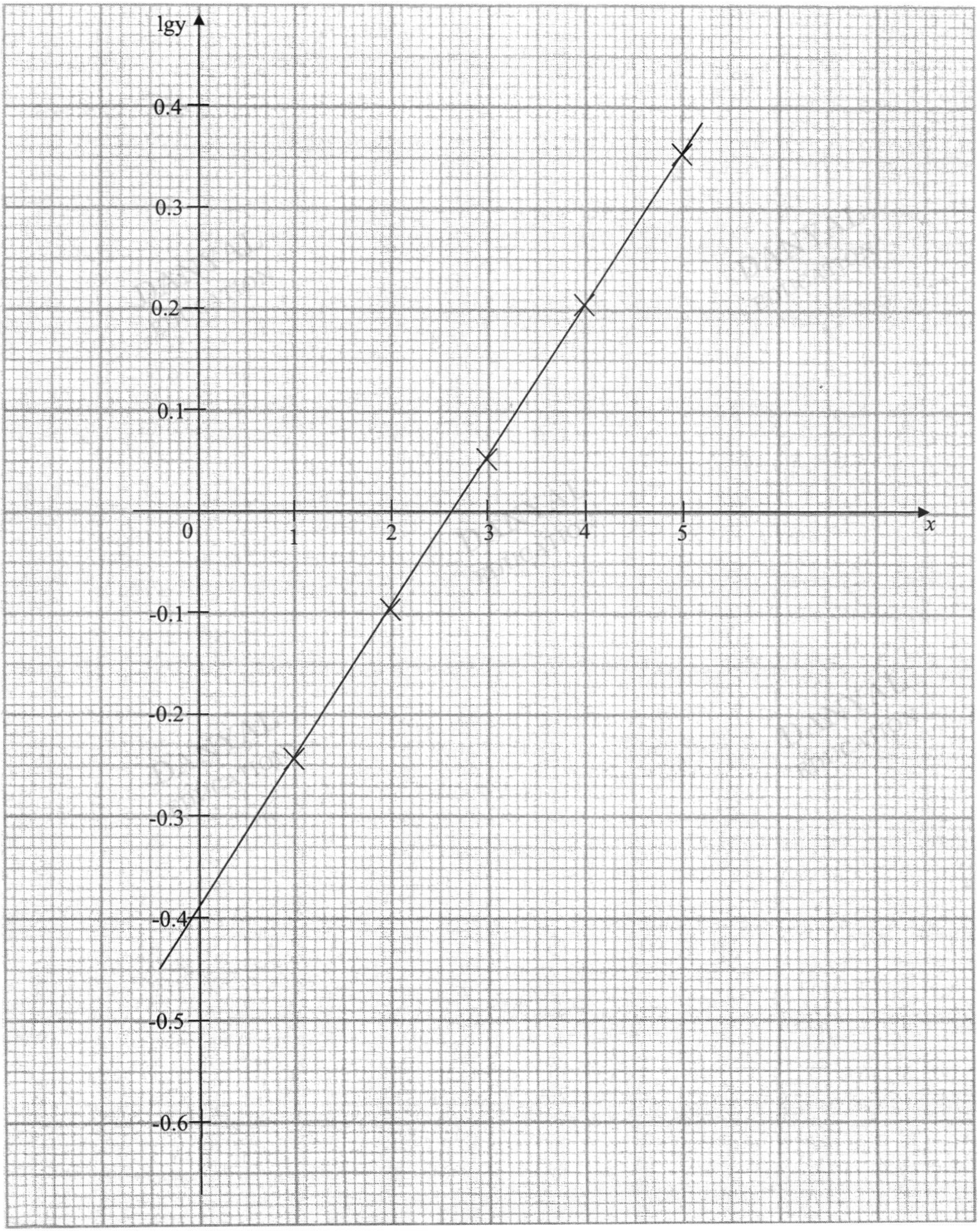
Q	Process	Mark
8(i)	At $D, x=0$ $y=3(0)+1$ $y=1$ $D(0,1)$	B1
8(ii)	$m_{AD}=3$ $m_{AB}=-\frac{1}{3}$ Eqn of AB: $y-4=-\frac{1}{3}(x-5)$ $y-4=-\frac{1}{3}x+\frac{5}{3}$ $y=-\frac{1}{3}x+\frac{17}{3}$ Sub $y=-\frac{1}{3}x+\frac{17}{3}$ into $y=3x+1$ $-\frac{1}{3}x+\frac{17}{3}=3x+1$ $-\frac{10}{3}x=-\frac{14}{3}$ $x=1.4$ Sub $x=1.4$, $y=3(1.4)+1$ $y=5.2$ $A(1.4,5.2)$	M1 (ecf) M1 M1 (either x or y is correct) (ecf) A1
8(iii)	$C(3,-2)$	B1
8(iv)	Area of Trapezium $=\frac{1}{2}\begin{vmatrix} 0 & 1.4 & 5 & 3 & 0 \\ 1 & 5.2 & 4 & -2 & 1 \end{vmatrix}$ $=\frac{1}{2}\left\|(0+5.6-10+3)-(0+12+26+1.4)\right\|$ $=\frac{1}{2}\left\|-\frac{7}{5}-39.4\right\|$ $=20.4\text{units}^2$ Or $\frac{1}{2}\left(\sqrt{19.6}+\sqrt{40}\right)\times\sqrt{14.4}$ $=20.4unit^2$	M1 A1
		8m

Q	Process	Mark
9(a)	For $a > 0$, $(x+2)(x-7) \le 0$ $x^2 - 7x + 2x - 14 \le 0$ $x^2 - 5x - 14 \le 0$ $\therefore a > 0, p = -5, q = 14$	B1, B1, B1 M1 for $(x+2)(x-7)$ seen
9(bi)	$h = -2t^2 + 3t + 1$ $= -2\left(t^2 - \frac{3}{2}t - \frac{1}{2}\right)$ $= -2\left[\left(t - \frac{3}{4}\right)^2 - \frac{1}{2} - \left(-\frac{3}{4}\right)^2\right]$ or $= -2\left[\left(t - \frac{3}{4}\right)^2 - \frac{17}{16}\right]$ $= -2\left(t - \frac{3}{4}\right)^2 + \frac{17}{8}$ $h = -2t^2 + 3t + 1$ $= -2\left(t^2 - \frac{3}{2}t\right) + 1$ $= -2\left[\left(t - \frac{3}{4}\right)^2 - \left(-\frac{3}{4}\right)^2\right] + 1$ $= -2\left[\left(t - \frac{3}{4}\right)^2 - \frac{9}{16}\right] + 1$ $= -2\left(t - \frac{3}{4}\right)^2 + \frac{17}{8}$	M1 M1 A1
9(bii)	Maximum height $= \frac{17}{8}$ $= 2.125\text{m}$ Since the maximum height the stone can reach is 2.125m which is shorter than 2.6m	M1 A1
		8m

Q	Process	Mark
10(a)	$(1+hx)^6+(2-x)^7$	
	$=\ldots\binom{6}{3}(1)^3(hx)^3\ldots+\ldots\binom{7}{3}(2)^4(-x)^3\ldots$	M1, M1
	$=\ldots+20h^3x^3+(35)(16)(-x^3)$	
	$=\ldots20h^3x^3-560x^3\ldots$ $\therefore 20h^3-560=720$	M1(ecf)
	$h^3=64$	
	$h=4$	A1
10(bi)	$T_{r+1}=\binom{6}{r}(x)^{6-r}\left(\frac{4}{x^2}\right)^r$ or $T_{r+1}=\binom{6}{r}(x)^{6-3r}(4)^r$	B1
10(bii)	To find the value r for independent term, $6-r-2r=0$	M1
	$-3r=-6$	
	$r=2$	A1
	Independent term $=\binom{6}{2}(4)^2$	
	$=240$	B1
		8m

Q	Process	Mark
11(i)	x: 1, 2, 3, 4, 5; y: 0.566, 0.80, 1.13, 1.60, 2.26; lgy: -0.2472, -0.0969, 0.0530, 0.204, 0.3541	Points – B1 Line – B1

x	1	2	3	4	5
y	0.566	0.80	1.13	1.60	2.26
lgy	-0.2472	-0.0969	0.0530	0.204	0.3541



Q	Process	Mark
11(ii)	$my = n(2^{mx})$ $y = \frac{n}{m}(2^{mx})$ $\lg y = \lg\left(\frac{n}{m}\right) + (mx)\lg 2$ $\lg y = \lg\left(\frac{n}{m}\right) + m\lg 2(x)$ gradient $= m\lg 2$ $\frac{0.3-(-0.3)}{4.1-0.6} = m\lg 2$ $\frac{0.6}{3.5} \div (\lg 2) = m$ $m = 0.48613...$ $m = 0.486$ y-intercept $= \lg\left(\frac{n}{m}\right)$ $-0.39 = \lg\left(\frac{n}{0.48613}\right)$ $n = 0.19803...$ $n = 0.198$	M1 M1 A1 (accept $m =$ 0.486 to 0.631) A1 (accept $n =$ 0.198 to 0.263)
11(iii)	$x = 2.3$ $\lg y = -0.05$ $y = 10^{-0.05}$ $= 0.891$	B1 (value for lg y) B1 ($10^{\text{their ans}}$) B1 (accept 0.871 to 0.912)
		9m

Q	Process	Mark
12(i)	$f(x) = x^3 + hx^2 + kx - 3$	
	$f(3) = 0$	
	$(3)^3 + h(3)^2 + k(3) - 3 = 0$	M1
	$27 + 9h + 3k - 3 = 0$	
	$9h + 3k = -24$	
	$3h + k = -8$	A1
	$f(-1) = -4$	
	$(-1)^3 + h(-1)^2 + k(-1) - 3 = -4$	M1
	$-1 + h - k - 3 = -4$	
	$h - k = 0$	
	$h = k$	A1
	$\therefore 4h = -8$	
	$h = -2$	
	$k = -2$	A1
12(ii)	$f(x) = x^3 - 2x^2 - 2x - 3$	
	$f(x) = (x-3)(x^2 + bx + 1)$	M1
	By comparing coef of x^2,	
	$-3 + b = -2$	
	$b = 1$	
	$f(x) = (x-3)(x^2 + x + 1)$	M1
	$y = x^2 + x + 1$	
	$D = (1)^2 - 4(1)(1)$	
	$= -3$	
	< 0	M1 (ecf)
	Since $D < 0$, there is only one solution for $f(x)$.	A1
		9m

Q	Process	Mark
13(a)	The value of the radius is 5. The y coordinate of the centre of the circle is 5.	B1 B1
13(bi)	Let the x coordinate of the centre be a. Let the centre be $(a, 5)$ Equation of circle: $(x-a)^2+(y-5)^2=25$ *When* $x=-7,\ y=9$: $(-7-a)^2+4^2=25$ $(-7)^2+14a+a^2=9$ $a^2+14a+40=0$ $a=-4$ *or* $a=-10$ *(rej)* Equation of circle: $(x+4)^2+(y-5)^2=25$ *or* $x^2+8x+y^2-10y+16=0$	M1 M1(ecf) A1 B1 (ecf)
13(bii)	Let the centre of the circle be A. $m_{AP}=\dfrac{9-5}{-7+4}$ $=-\dfrac{4}{3}$ $m_{\tan gent}=\dfrac{3}{4}$ $y=\dfrac{3}{4}x+c$ $9=\dfrac{3}{4}(-7)+c$ $c=\dfrac{57}{4}$ $Equation_{\tan gent}: y=\dfrac{3}{4}x+\dfrac{57}{4}$ *or* $y=\dfrac{3}{4}x+14\dfrac{1}{4}$	 M1 (ecf) M1 (ecf) A1
		9m