	90

SERVE·LEAD·EXCEL

GREENDALE SECONDARY SCHOOL
Preliminary Examination 2023

Additional Mathematics **4049/01**

Paper 1 24 August 2023

Secondary 4 EXPRESS/5 NORMAL **2 hour 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The total number of marks for this paper is **90.**
The number of marks is given in brackets [ ] at the end of each question or part question.

Q	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14
Str														
M														

No of additional booklets/ writing paper used		**No of additional graph paper used**	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cos}ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$Area\ of\ \Delta = \frac{1}{2} bc \sin A$$

Answer all questions.

1 A steel cylinder has a radius $\left(2\sqrt{3}-3\sqrt{2}\right)$ cm.

Find the top cross-sectional area of the cylinder in the form $\left(a+b\sqrt{6}\right)\pi$ cm^2, where a and b are integers. [3]

2 Solve the simultaneous equations.

$$2x+y=4$$
$$4^{y^2}=256^x$$

[4]

3 **(i)** Express $y = -2x^2 + 8x - 13$ in the form of $a(x-h)^2 + k$. [2]

(ii) State the coordinates of the turning point. [1]

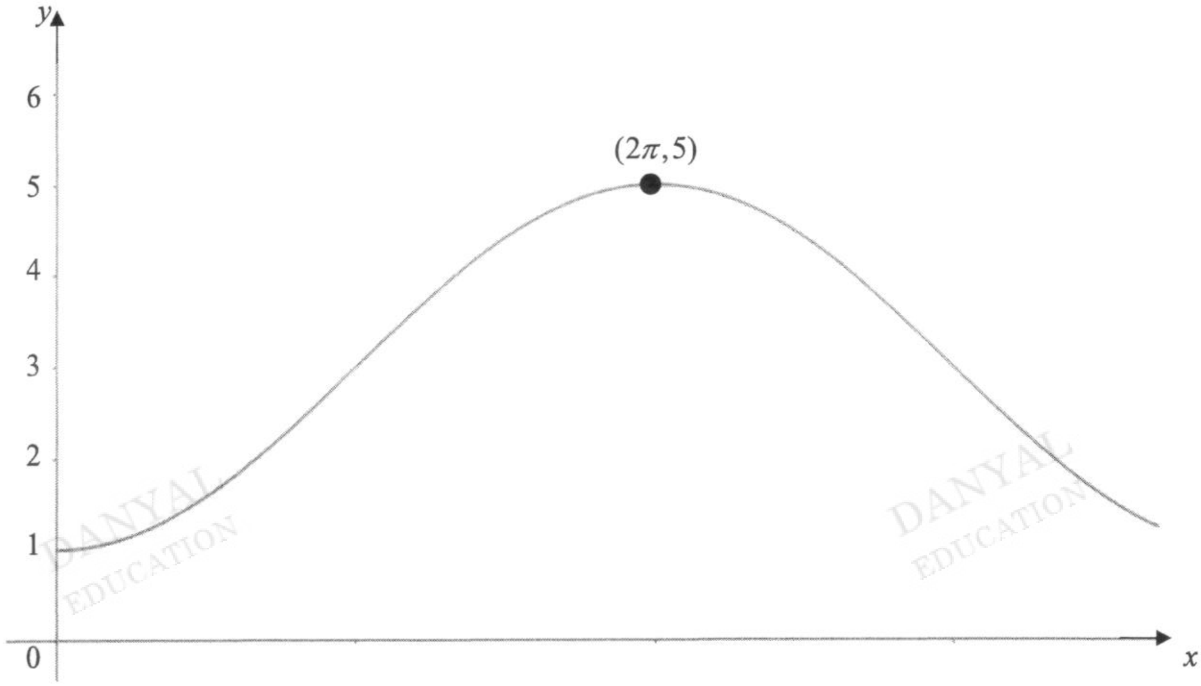
(iii) Hence, find the minimum value of $\dfrac{4}{-2x^2 + 8x - 13}$. [2]

4 Given that $\int_{n}^{8} \frac{x-5}{x^2-2x-15}\,dx = \ln\frac{33}{13}$, find the value of n. [4]

5 Express $\dfrac{x^4+2x+1}{x^3+2x}$ in partial fractions. [6]

6 When the expression $3x^3 + px^2 + qx + 8$ is divided by $x^2 - 3x + 2$, the remainder is $2x + 3$. Find the value of p and of q. [5]

7



The diagram shows part of the curve $y = a\cos bx + c$. The y-intercept of the curve is 1 and the curve has a maximum point at $(2\pi, 5)$.

(i) Find the equation of the curve. [4]

(ii) It is given that $x = \dfrac{4\pi}{3}$ is a solution of the equation $a\cos bx + c = 4$.
Write down the next solution of $a\cos bx + c = 4$. [2]

8 It is given that $f(x) = x\sin x$.

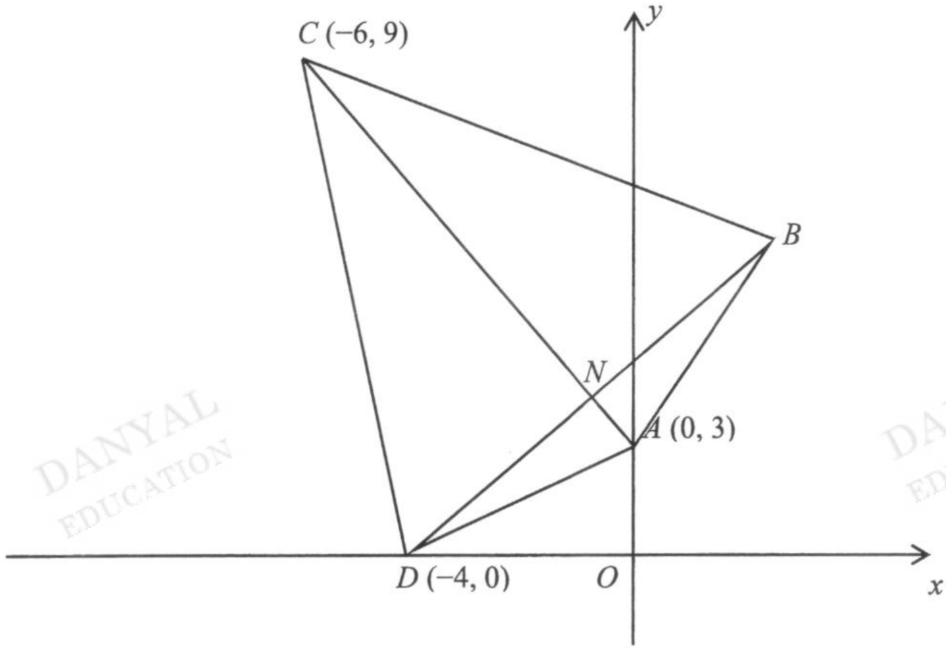
(a) Find $f'(x)$. [2]

(b) Hence,

(i) show that $f''(x) + f(x) = 2\cos x$. [2]

(ii) find the value of $\int_0^3 x\cos x \, dx$. [3]

9 The diagram shows a kite $ABCD$ in which the coordinates of A, C and D are (0, 3), (−6, 9) and (−4, 0) respectively. AC and BD intersect at point N.



(i) Find the equation of BD. [3]

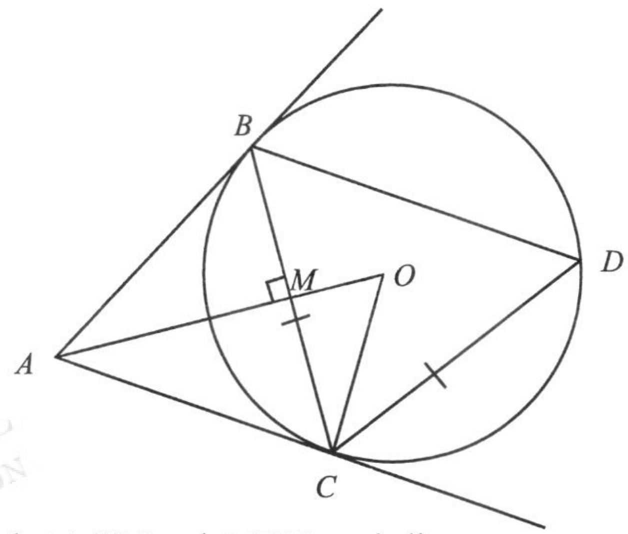
(ii) Find the coordinates of B. [4]

(iii) Find the area of kite $ABCD$. [2]

10 **(i)** Prove the identity $7\cos^4 x - 7\sin^4 x = 7\cos 2x$. [3]

(ii) Hence, solve the equation $7\cos^4 x - 7\sin^4 x = \dfrac{5}{\operatorname{cosec} 2x}$, for $0 \le x \le 2\pi$. [5]

11 In the diagram, AB and AC are tangents to the circle with centre O. M is the midpoint of BC and $BC = DC$.



(i) Prove that ΔABM and ΔCOM are similar. [3]

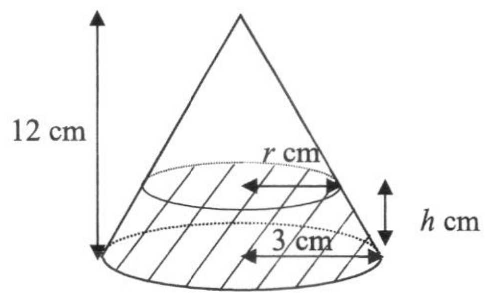
(ii) Tim says that a circle can be drawn through points B, A, C and O. Is he correct? Explain with clear workings. [2]

(iii) Prove that BD is parallel to AC. [2]

12 **(i)** Given that $a = 3^x$ and $b = 3^y$, express $\log_3 \frac{a^4 b}{81}$ in terms of x and y. [3]

(ii) Solve the equation $\log_2 x^2 - 3\log_x 4 = \log_4 2 \times \log_4 16$. [4]

13 A container in the shape of a right circular cone has a height of 12 cm and radius 3 cm. The container is initially filled to the brim with water at first. Due to a leak, the water level is dropping at a constant rate of 6 $cm^3\ s^{-1}$. The depth of water after t seconds is h cm.

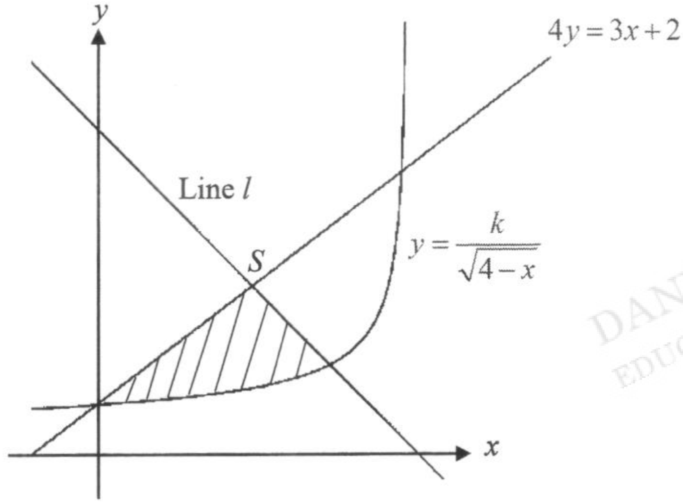


(i) Show that the volume of water in the container, V cm^3, at time t seconds is given by $V = 36\pi - \frac{1}{48}\pi(12-h)^3$. [4]

(ii) Find the rate of change of the depth when the depth of water in the container is 7 cm. [3]

(iii) Showing your workings clearly, state what happens to the rate of change of the depth as t increases. [1]

14 The diagram shows part of the curve $y = \frac{k}{\sqrt{4-x}}$, where k is a constant. The line $4y = 3x + 2$ intersects the curve at $\left(0, \frac{1}{2}\right)$. Line l is the equation of the normal to the curve at $x = 3$.



(i) Find the equation of line l. [4]

(ii) Find the coordinates of the intersection point S. [2]

(iii) Find the area of the shaded region. [5]

END OF PAPER

90

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GREENDALE SECONDARY SCHOOL
Preliminary Examination 2023

Additional Mathematics **4049/02**

Paper 2 29 August 2023

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Strand										
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$$\text{Area of } \Delta = \frac{1}{2} bc \sin A$$

Answer all questions.

1 Solve the equation $3-2e^{x}=e^{-x}$. [3]

2 The term containing the highest power of x in the polynomial $\mathrm{f}(x)$ is $3x^3$. It is given that x^2-2x+3 is a quadratic factor of $\mathrm{f}(x)$ and one of the roots of $\mathrm{f}(x)$ is 1.

(i) Find an expression for $\mathrm{f}(x)$ in descending powers of x. [2]

(ii) Find the number of real roots of the equation $\mathrm{f}(x)=0$, justifying your answer. [2]

(iii) Find the remainder when $\mathrm{f}(x)$ is divided by $3x-2$. [2]

3 A particle P is moving on the x-axis and its displacement from point O, s m, after t seconds, is given by $s = t^2(2t-3) + At + B$, where A and B are constants.

(i) Find, in terms of t, the velocity of P, v m/s, after t seconds. [1]

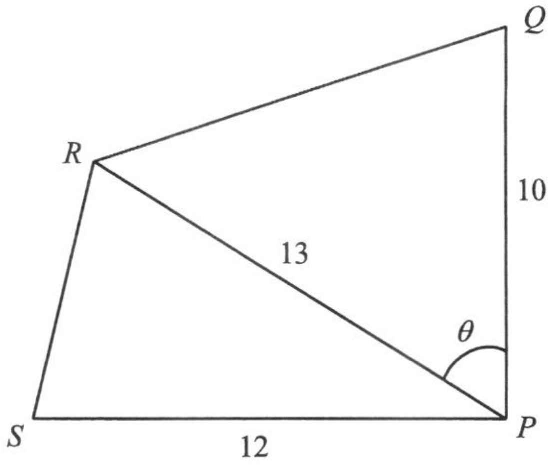
When $t = 1.5$, P is passing through O, and is moving in the negative x-direction with speed 7.5 m/s.

(ii) Find the values of A and B. [3]

(iii) Explain what the value of B represents. [1]

(iv) When P is instantaneously at rest, find the acceleration of P. [4]

4 The diagram shows a plot of land *PQRS* formed by two triangles, triangle *PQR* and triangle *PRS*. The lengths of *PQ*, *PR* and *PS* are 10 m, 13 m and 12 m respectively. Angle $RPQ = \theta$ rad and *QP* is perpendicular to *SP*.



(i) Show that the area of *PQRS*, A m^2, is $A = 78\cos\theta + 65\sin\theta$. [3]

(ii) Express A in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

(iii) John claims the area of *PQRS* can be 120 m^2.
Do you agree? Explain your answer. [1]

(iv) Find the value of θ when the area of the field is 90 m^2. [2]

5 (a) (i) Determine the set of values of p for which the equation

$x^2 + 2x + p = 3px - 1$ has no real roots. [4]

(ii) Hence, state what can be deduced about the curve $y = (x+1)^2$ and the line $y = 3x - 1$. Justify your answer. [2]

(b) Show that for the curve $y = hx^2 + kx + 3k$ to meet the line $y = -h(1+3x)$ at two distinct points, the condition that must apply to the constants h and k is $k < h$ or $k > 5h$, given that $h > 0$. [4]

6 **(a)** Find the term independent of x in the expansion of $\left(x^3 - \frac{1}{3x^2}\right)^{10}$. [3]

(b) In the expansion of $\left(2 + \frac{x}{4}\right)^n$, where n is a positive integer, the ratio of the coefficient of x to the coefficient of x^2 is 16 : 5.

(i) Show that the second term in the expansion is $2^{n-3}nx$. [2]

(ii) Find the value of n. [4]

7 **(a)** A cuboid of volume V cm^3 has a square base of sides x cm and a height of $(px + q)$ cm. Values of V for different values of x have been collected. Explain how a straight line graph can be drawn to represent the formula, and state how the values of p and q can be obtained from the line. [4]

(b) A liquid in a container was left to cool. The difference between its temperature and the room temperature at time t minutes was T°C. The table shows some recorded values of t and T.

t min	5	10	15	20	25
T°C	16.1	8.6	4.6	2.5	1.3

These figures can be modelled by the formula $T = ae^{bt}$, where a and b are constants.

(i) Draw a straight line graph to show that the model is reasonable. [4]

(ii) Given that the room temperature is $28°C$, what is the initial temperature of the liquid? [2]

(iii) Explain why the value of T cannot be zero. [1]

8 A curve has the equation $y = xe^{2x}$.

(a) Find the coordinates of the stationary point. [4]

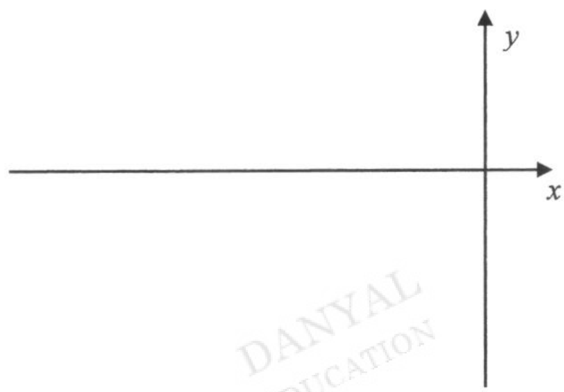
(b) By finding $\frac{d^2y}{dx^2}$, determine the nature of the stationary point. [3]

(c) Complete the table, leaving your answer in standard form.

x	0	-1	-5	-10
y				

[1]

(d) Using your answers in **(a)-(c)**, sketch the graph $y = xe^{2x}$, $x \leq 0$. [1]



9 The circle, C_1, intersects the y-axis at 3 and $y = 1$ is a tangent to the circle at $(4, 1)$.

(i) Show that the coordinates of the centre of C_1 is $(4, 6)$. [3]

(ii) Find the equation of circle, C_1. [2]

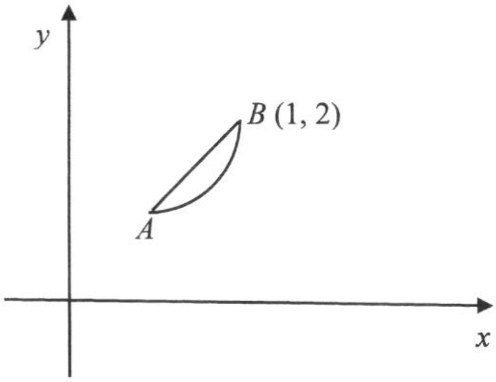
Another circle, C_2, has equation $x^2 + y^2 + 8x - 12y + 27 = 0$.

(iii) Find the radius and the coordinates of the centre of C_2. [3]

(iv) Explain the relationship between C_1 and C_2. [1]

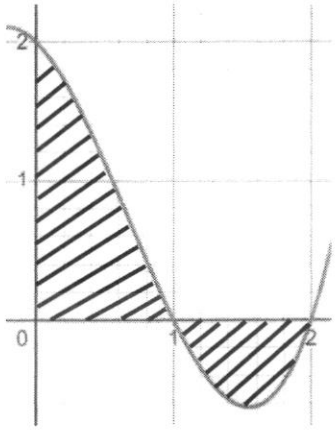
(v) The point $(p, 6)$ lies within only one of the circles.
Find the range of values of p. [3]

10 **(a)** The graph shows part of the graph $y = x^3 + \frac{1}{x}$, where A is the minimum point. The line cuts the curve at points A and $B(1, 2)$.



Find the area bounded by the line AB and the curve. [10]

(b) Kandy is asked to find the shaded area as shown in the diagram.
The curve $y = x^3 - 2x^2 - x + 2$ cuts the x-axis at 1 and 2.



Kandy incorrectly claims that
Area of region $= \int_0^2 x^3 - 2x^2 - x + 2 \ \mathrm{d}x \approx 2.67$units .

(i) Explain what 2.67 units represents in this context. [1]

(ii) Without finding the exact value, describe how Kandy should find the correct area of the region. [1]

- End of Paper -

2023 4Exp AMath Paper 1 Marking Scheme

Q	Process
1 **AO1**	*Area* $=\pi\left(2\sqrt{3}-3\sqrt{2}\right)^2$ $=\pi\left(12-12\sqrt{6}+18\right)$ $=\pi\left(30-12\sqrt{6}\right)$
2 **AO1**	$y=4-2x$............(1) $4^{y^2}=256^x$ $4^{y^2}=4^{4x}$ $y^2=4x$.........(2) *Sub.* (1) int *o* (2) $\left(4-2x\right)^2=4x$ $16-16x+4x^2-4x=0$ $4x^2-20x+16=0$ $x^2-5x+4=0$ $\left(x-1\right)\left(x-4\right)=0$ $x=1$ *or* $x=4$ $y=2$ *or* $y=-4$ $x=1, y=2$ $x=4, y=-4$
3(i) **AO1**	$y=-2x^2+8x-13$ $=-2\left(x^2-4x\right)-13$ $=-2\left[x^2-4x+\left(-2\right)^2-\left(2\right)^2\right]-13$ $=-2\left[\left(x-2\right)^2-4\right]-13$ $=-2\left(x-2\right)^2-5$
3(ii)	$\left(2,-5\right)$
3(iii) **AO2**	Minimum value $=\dfrac{4}{-5}=-\dfrac{4}{5}$

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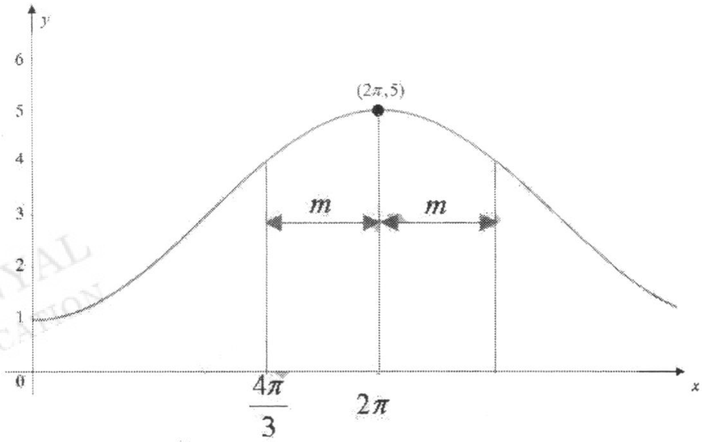
4 **AO1**	$\int_n^8 \frac{x-5}{x^2-2x-15}\,dx = \ln\frac{33}{13}$ $\int_n^8 \frac{x-5}{(x-5)(x+3)}\,dx = \ln\frac{33}{13}$ $\int_n^8 \frac{1}{x+3}\,dx = \ln\frac{33}{13}$ $\left[\ln(x+3)\right]_n^8 = \ln\frac{33}{13}$ $\ln(8+3)-\ln(n+3) = \ln\frac{33}{13}$ $\ln\frac{11}{n+3} = \ln\frac{33}{13}$ $\frac{11}{n+3} = \frac{33}{13}$ $143 = 33n+99$ $33n = 44$ $n = \frac{4}{3}$
5 **AO2**	$\frac{x^4+2x+1}{x^3+2x} = x + \frac{1+2x-2x^2}{x(x^2+2)}$ $Let\ \frac{1+2x-2x^2}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{(x^2+2)}$ *When* $x=0$, $1+2(0)-2(0)^2 = A(0+2)+0(Bx+C)$ $1 = 2A$ $A = \frac{1}{2}$ *When* $x=1$, $1+2(1)-2(1)^2 = A(3)+1(B+C)$ $1 = 3\left(\frac{1}{2}\right)+B+C$ $B+C = -\frac{1}{2}$ ....(1)

2023 4Exp AMath Paper 1 Marking Scheme

	When $x=2$, $1+4-2(2)^2=\frac{1}{2}(6)+2(2B+C)$ $5-8=3+4B+2C$ $4B+2C=-6$ $2B+C=-3$...............(2) (2) − (1): $B=-\frac{5}{2}$ $C=2$ $\frac{x^4+2x+1}{x^3+2x}=x+\frac{1}{2x}+\frac{4-5x}{2(x^2+2)}$
6 **AO2**	$3x^3+px^2+qx+8=(x^2-3x+2)Q(x)+(3x+2)$ $3x^3+px^2+qx+8=(x-2)(x-1)Q(x)+(3x+2)$ *When* $x=1$, $3(1)^3+p(1)^2+q(1)+8=2(1)+3$ $3+p+q+8=5$ $p+q=-6$..............(1) *When* $x=2$, $3(2)^3+p(2)^2+q(2)+8=2(2)+3$ $24+4p+2q+8=7$ $4p+2q=-25$................(2) (2) − (1): $2p=-13$ $p=-\frac{13}{2}$ $q=-6+\frac{13}{2}=\frac{1}{2}$
7i **AO1**	Period = 4π $\frac{2\pi}{b}=4\pi$ $b=\frac{1}{2}$

2023 4Exp AMath Paper 1 Marking Scheme

	$c=\frac{1+5}{2}=3$ $a=-2$ Equation: $y=-2\cos\frac{1}{2}x+3$
7ii **AO2**	 $m=2\pi-\frac{4\pi}{3}=\frac{2\pi}{3}$ By symmetry of graph, next solution of graph $=2\pi+\frac{2\pi}{3}$ $=\frac{8\pi}{3}$ Alternative solution: $-2\cos\frac{1}{2}x+3=4$ $\cos\frac{1}{2}x=-\frac{1}{2}$ $\alpha=\frac{\pi}{3}$ $\frac{1}{2}x=\pi-\frac{\pi}{3},\pi+\frac{\pi}{3}$ $x=\frac{4\pi}{3},\frac{8\pi}{3}$ Ans: $\frac{8\pi}{3}$
8a **AO1**	

2023 4Exp AMath Paper 1 Marking Scheme

	$f'(x) = x\cos x + \sin x$
8bi **AO1**	Show that $f''(x) + f(x) = 2\cos x$. $f''(x) = -x\sin x + \cos x + \cos x$ $f''(x) + f(x) = 2\cos x$
8bii **AO2**	$\int_0^3 x\cos x + \sin x \, dx = [x\sin x]_0^3$ $\int_0^3 x\cos x \, dx$ $= 3\sin 3 - \int_0^3 \sin x \, dx$ $= 3\sin 3 + [\cos x]_0^3$ $= 3\sin 3 + \cos 3 - 1$ $= -1.57$
9i **AO2**	$m_{AC} = \dfrac{9-3}{-6-0} = -1$ $m_{AC} \times m_{BD} = -1$ $m_{BD} = 1$ $y = x + c$, sub $(-4,0)$ $c = 4$ Equation of BD: $y = x + 4$
9ii **AO2**	Equation of AC: $y = -x + 3$ $x + 4 = -x + 3$ $2x = -1$ $x = -\dfrac{1}{2}, y = \dfrac{7}{2}$ $N\left(-\dfrac{1}{2}, \dfrac{7}{2}\right)$ N is the midpoint of BD. Let coordinates of B be (x, y)

2023 4Exp AMath Paper 1 Marking Scheme

	$\left(\frac{-4+x}{2},\frac{0+y}{2}\right)=\left(-\frac{1}{2},\frac{7}{2}\right)$ $\frac{-4+x}{2}=-\frac{1}{2}$ $x=3$ $\frac{0+y}{2}=\frac{7}{2}$ $y=7$ $\therefore$ Coordinates of B : (3, 7)
9iii **AO2**	Area of kite $ABCD = 2\times$ Area of ΔACD $=2\times\frac{1}{2}\begin{vmatrix}0 & -6 & -4 & 0\\ 3 & 9 & 0 & 3\end{vmatrix}$ $=[(-12)-(-36-18)]$ $=42$ units^2 Or Area of kite $ABCD=\frac{1}{2}\begin{vmatrix}0 & 3 & -6 & -4 & 0\\ 3 & 7 & 9 & 0 & 3\end{vmatrix}$ $=(27-12)-(-36-42+9)=$ $=42$ units^2
10(i) **AO3**	$7\cos^4 x-7\sin^4 x$ $=7(\cos^4 x-\sin^4 x)$ $=7(\cos^2 x+\sin^2 x)(\cos^2 x-\sin^2 x)$ $=7(\cos^2 x-\sin^2 x)$ $=7\cos 2x$

2023 4Exp AMath Paper 1 Marking Scheme

(ii) **AO2**	$7\cos^4 x - 7\sin^4 x = \frac{5}{\operatorname{cosec} 2x}$ $7\cos 2x = \frac{5}{\operatorname{cosec} 2x}$ $7\cos 2x \operatorname{cosec} 2x = 5$ $7\frac{\cos 2x}{\sin 2x} = 5$ $\tan 2x = \frac{7}{5}$ basic angle $= 0.9505$ $2x = 0.9505, \pi + 0.9505, 2\pi + 0.9505, 3\pi + 0.9505$ $x = 0.475, 4.09, 7.23, 10.4$
11i **AO2**	$\angle AMB = \angle CMO = 90°$ (vertically opposite angles) Let $\angle ABM = \angle ACM = x°$ (tan from ext pt) $\angle OCA = 90°$ (tan $\perp$ rad) $\angle OCM = (90 - x)°$ $\angle BAM = (180 - 90 - x)°$ ($\angle$ sum of Δ) $= (90 - x)°$ $= \angle OCM$ $\therefore \Delta ABM$ and ΔCOM are similar. (AA)
11ii **AO3**	Since ΔABM and ΔCOM are similar, $\angle BAM = \angle OCM$ $\angle ABM = \angle COM$ $\therefore$ Yes he is correct because by the property of angles in the same segment, a circle can be drawn through points B, A, C and O.
11iii **AO3**	$\angle BCA = \angle BDC$ (alt seg thm) $\angle CBD = \angle BDC$ (base $\angle$ of isos Δ) $= \angle BCA$ By alternate angles, BD is parallel to AC.

2023 4Exp AMath Paper 1 Marking Scheme

12(i)	$a = 3^x$ $x = \log_3 a$ $b = 3^y$ $y = \log_3 b$ $\log_3 \frac{a^4 b}{81}$ $= \log_3 a^4 + \log_3 b - \log_3 81$ $= 4\log_3 a + \log_3 b - \log_3 3^4$ $= 4x + y - 4$
12(ii)	$\log_2 x^2 - 3\log_x 4 = \log_4 2 \times \log_4 16$ $\log_2 x^2 - 3\left(\frac{\log_2 4}{\log_2 x}\right) = \frac{\log_2 2}{\log_2 4} \times \frac{\log_2 16}{\log_2 4}$ $Sub\ \log_2 x = y$ $2y - \frac{6}{y} = 1$ $2y^2 - y - 6 = 0$ $y = 2\ or\ y = -\frac{3}{2}$ $\log_2 x = 2$ $x = 2^2 = 4$ $\log_2 x = -\frac{3}{2}$ $x = 2^{-\frac{3}{2}}\ or \sqrt{\frac{1}{8}}$

2023 4Exp AMath Paper 1 Marking Scheme

13(i) **AO2**	$\frac{r}{3}=\frac{12-h}{12}$ $r=\frac{12-h}{4}$ $V=\frac{1}{3}\pi(3)^2(12)-\frac{1}{3}\pi\left(\frac{12-h}{4}\right)^2(12-h)$ $=36\pi-\frac{1}{3}\pi\frac{(12-h)^2}{16}(12-h)$ $=36\pi-\frac{1}{48}\pi(12-h)^3$
13(ii) **AO1**	$V=36\pi-\frac{1}{48}\pi(12-h)^3$ $\frac{dV}{dh}=-\frac{1}{48}\pi(3)(12-h)^2(-1)$ $=\frac{\pi(12-h)^2}{16}$ $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$ $=\frac{16}{\pi(12-h)^2}\times 6$ $=\frac{96}{\pi(12-h)^2}$ $\left.\frac{dh}{dt}\right\|_{h=7}=\frac{96}{\pi(12-7)^2}$ $=1.22\text{ cm s}^{-1}$
13(iii) **AO3**	Since $\frac{dh}{dt}=\frac{96}{\pi(12-h)^2}$, as ***t* increases, *h* decreases**, hence $(12-h)^2$ **increases**, so $\frac{dh}{dt}$ decreases. $\therefore$ the **rate of change of the depth will decrease.**

2023 4Exp AMath Paper 1 Marking Scheme

14(i) **AO1**	$\frac{1}{2}=\frac{k}{\sqrt{4-0}}$ $k=1$ Eqn of curve: $y=\frac{1}{\sqrt{4-x}}$ $\frac{dy}{dx}=-\frac{1}{2}(4-x)^{-\frac{3}{2}}(-1)$ $=\frac{1}{2(4-x)^{\frac{3}{2}}}$ $\left.\frac{dy}{dx}\right\|_{x=3}=\frac{1}{2(4-3)^{\frac{3}{2}}}$ $=\frac{1}{2}$ gradient of $l=-1\div\frac{1}{2}$ $=-2$ at $x=3$, $y=1$ Equation of line l: $\frac{y-1}{x-3}=-2$ $y-1=-2(x-3)$ $=-2x+6$ $y=-2x+7$
14(ii) **AO2**	$4(-2x+7)=3x+2$ $11x=26$ $x=2\frac{4}{11}$ $y=2\frac{3}{11}$ Point of intersection of 2 lines $=\left(2\frac{4}{11},2\frac{3}{11}\right)$
14(iii) **AO2**	

2023 4Exp AMath Paper 1 Marking Scheme

	Area under $4y = 3x + 2$ from $x = 0$ to $x = 2\frac{4}{11}$ $= \frac{1}{2}\left(\frac{1}{2} + 2\frac{3}{11}\right)\left(2\frac{4}{11}\right)$ $= 3\frac{67}{242}$ or 3.27685 Area under line l from $x = 2\frac{4}{11}$ to $x = 3$ $= \frac{1}{2}\left(1 + 2\frac{3}{11}\right)\left(3 - 2\frac{4}{11}\right)$ $= 1\frac{5}{121}$ or 1.041322 Area under curve $= \int_0^3 \frac{1}{\sqrt{4-x}}\,dx$ $= \int_0^3 (4-x)^{-\frac{1}{2}}\,dx$ $= \left[\frac{(4-x)^{\frac{1}{2}}}{\frac{1}{2}(-1)}\right]_0^3$ $= \left[-2(4-x)^{\frac{1}{2}}\right]_0^3$ $= 2$ Area of shaded region $= 3\frac{67}{242} + 1\frac{5}{121} - 2$ $= 2\frac{7}{22}$ or 2.32

2023 4Exp AMath Paper 2 Marking Scheme

Q	Process
1 **AO1**	$3-2e^{x}=e^{-x}$ $3-2e^{x}=\frac{1}{e^{x}}$ $3e^{x}-2e^{2x}=1$ $2e^{2x}-3e^{x}+1=0$ Let $y=e^{x}$ $2y^{2}-3y+1=0$ $(2y-1)(y-1)=0$ $y=\frac{1}{2}$ or $y=1$ $e^{x}=\frac{1}{2}$ or $e^{x}=1$ $x=\ln\frac{1}{2}$ or $x=\ln 1$ $x=-0.693$ or $x=0$
2(i) **AO2**	$f(x)=3(x-1)(x^{2}-2x+3)$ $=3x^{3}-6x^{2}+9x-3x^{2}+6x-9$ $=3x^{3}-9x^{2}+15x-9$
2(ii) **AO2**	$f(x)=0$ $x^{2}-2x+3=0$ $b^{2}-4ac=(-2)^{2}-4(1)(3)=-8<0$ Since $x^{2}-2x+3$ *has no real roots*, $f(x)$ *has only* 1 *real root*, $x=1$.
2(iii) **AO1**	$f(\frac{2}{3})=3\left(\frac{2}{3}\right)^{3}-9\left(\frac{2}{3}\right)^{2}+15\left(\frac{2}{3}\right)-9$ $=\frac{8}{9}-4+1$ $=-\frac{19}{9}$
3(i) **AO1**	$s=t^{2}(2t-3)+At+B$ F $s=2t^{3}-3t^{2}+At+B$ $v=6t^{2}-6t+A$

2023 4Exp AMath Paper 2 Marking Scheme

3(ii) AO2	$t=1.5$ $-7.5=6(1.5)^2-6(1.5)+A$ $A=-12$ $0=1.5^2(2\times1.5-3)-12(1.5)+B$ $B=18$
3(iii) AO3	P started moving **18m** to the **right** of O.
3(iv) AO2	$v=6t^2-6t-12$ $v=0,$ $(t-2)(t+1)=0$ $t=2$ $a=12t-6$ $a=18\text{m/s}^2$
4i AO2	Area of $\Delta PRQ=\frac{1}{2}(10)(13)\sin\theta$ $=65\sin\theta$ $\measuredangle RPS=\frac{\pi}{2}-\theta$ Area of $\Delta RPS=\frac{1}{2}(13)(12)\sin\left(\frac{\pi}{2}-\theta\right)$ $=78\sin\left(\frac{\pi}{2}-\theta\right)$ $=78\cos\theta$ Area of $PQRS, A=78\cos\theta+65\sin\theta$ (shown)
4ii AO1	$A=78\cos\theta+65\sin\theta$ $R=\sqrt{78^2+65^2}$ $=\sqrt{10309}$ $=101.533$ $\alpha=\tan^{-1}\frac{65}{78}$ $=0.69473$ $A=102\cos(\theta-0.695)$ or $A=\sqrt{10309}\cos(\theta-0.695)$ or $A=13\sqrt{61}\cos(\theta-0.695)$
4iii AO3	Maximum $A=13\sqrt{61}$ or $101.533<120$, hence I disagree with John.

2023 4Exp AMath Paper 2 Marking Scheme

4iv **AO1**	$13\sqrt{61}\cos(\theta-0.69473)=90$ $\cos(\theta-0.69473)=0.8864$ $\theta=0.48126+0.69473$ $\theta=1.176$ $=1.18$
5(ai) **AO1**	$x^2+2x+p=3px-1$ $x^2+(2-3p)x+(p+1)=0$ *For no real roots,* $b^2-4ac<0$ $(2-3p)^2-4(1)(p+1)<0$ $4-12p+9p^2-4(p+1)<0$ $9p^2-16p<0$ $p(9p-16)<0$ $0<p<\frac{16}{9}.$
5(aii) **AO3**	$x^2+2x+1=3x-1$ By comparing with (i), p=1 When p=1, it is within the set of values for which the curve has no real roots. Hence, the curve not meet/cut the line.
5(b) **AO2**	$hx^2+kx+3k=-h(1+3x)$ $hx^2+kx+3k=-h-3hx$ $hx^2+kx+3hx+3k+h=0$ $hx^2+(k+3h)x+(3k+h)=0$ $b^2-4ac>0$ $(k+3h)^2-4(h)(3k+h)>0$ $k^2+6hk+9h^2-12hk-4h^2>0$ $k^2-6hk+5h^2>0$ $(5h-k)(h-k)>0$ $5h<k \quad or \quad h>k$
6(a) **AO2**	T_{r+1} term $=\binom{10}{r}(x^3)^{10-r}\left(-\frac{1}{3x^2}\right)^r$

2023 4Exp AMath Paper 2 Marking Scheme

	term in $x = \left(x^3\right)^{10-r}\left(\frac{1}{x^{2r}}\right)$ $= x^{30-3r-2r}$ $= x^{30-5r}$ $x^{30-5r} = x^0$ $r = 6$ term independent of $x = \binom{10}{6}\left(-\frac{1}{3}\right)^6$ $= \frac{70}{243}$
6(bi) **AO2**	$\left(2+\frac{x}{4}\right)^n = 2^n + \binom{n}{1}2^{n-1}\left(\frac{x}{4}\right) + \binom{n}{2}2^{n-2}\left(\frac{x}{4}\right)^2 + ...$ 2nd term $= \binom{n}{1}2^{n-1}\left(\frac{x}{4}\right)$ $= n2^{n-1}\left(\frac{x}{2^2}\right)$ $= 2^{n-1-2}nx$ $= 2^{n-3}nx$
6(bii) **AO1**	$\left(2+\frac{x}{4}\right)^n = 2^n + \binom{n}{1}2^{n-1}\left(\frac{x}{4}\right) + \binom{n}{2}2^{n-2}\left(\frac{x}{4}\right)^2 + ...$ 2nd term $= \binom{n}{1}2^{n-1}\left(\frac{x}{4}\right)$ $= n2^{n-1}\left(\frac{x}{2^2}\right)$ $= 2^{n-1-2}nx$ $= 2^{n-3}nx$

2023 4Exp AMath Paper 2 Marking Scheme

	3rd term $=\binom{n}{2}2^{n-2}\left(\frac{x}{4}\right)^2$ $=\frac{n(n-1)}{2}2^{n-2}\left(\frac{x^2}{2^4}\right)$ $=n(n-1)2^{n-2-5}x^2$ $=n(n-1)2^{n-7}x^2$ $\frac{n2^{n-3}}{n(n-1)2^{n-7}}=\frac{16}{5}$ $\frac{1}{n-1}2^4=\frac{16}{5}$ $\frac{1}{n-1}=\frac{1}{5}$ $n-1=5$ $n=6$
7a **AO3**	$V=(x)(x)(px+q)$ $V=px^3+qx^2$ Divide by x^2 throughout, $\frac{V}{x^2}=px+q$ A straight line can be obtained by drawing $\frac{V}{x^2}$ against x. The value of p can be obtained from the gradient of the graph. The value of q can be obtained from the vertical intercept of the graph.
7bi **AO2**	$T=ae^{bt}$ $\ln T=\ln a+bt$ Plot $\ln T$ against t.

t min	5	10	15	20	25
T°C	16.1	8.6	4.6	2.5	1.3
$\ln T$	2.78	2.15	1.53	0.92	0.26

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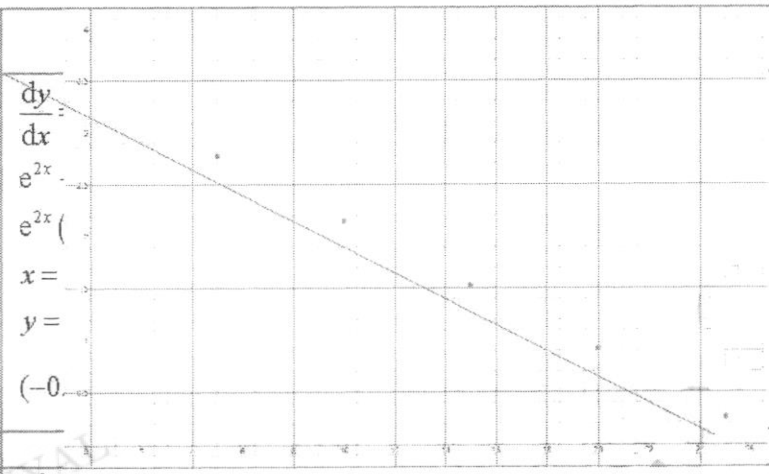
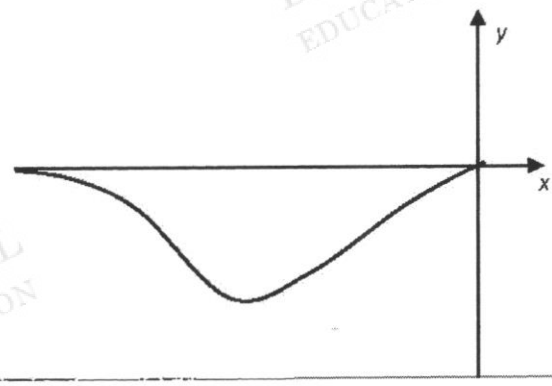
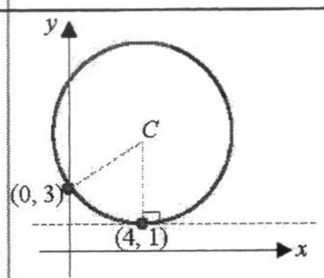
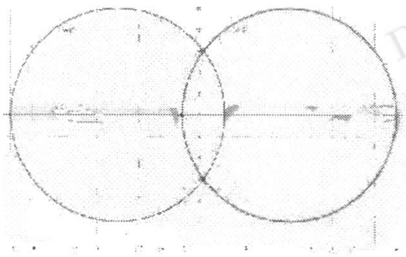
		
8(a) **AO2**	$\frac{dy}{dx}$ = … e^{2x} … $e^{2x}($ … $x =$ … $y =$ … $(-0$ …	M1- correct differentiation M1- =0 seen A1- x-value A1- coordinates
8(b) / **7b(i)** **AO2**	When $t = 0$, $\ln T = 3.4$ $T = e^{3.4} - 0.5$ $T = 29.96$ (accept 27.1 to 33.1) Initial temperature $= 28 + 29.96$ $= 58.0\ °C$ (accept 55.1 to 61.1) $\frac{dy}{dx} = e^{2x}(2) + (1+2x)e^{2x}.2$ $x = -0.5$ $\frac{d^2y}{dx^2} = 2e^{-1}$ $= 0.7357$ Minimum point	M1 A1 A1
8(c) / **7b(iii)** **AO3**	$\ln T$ is undefined when $T = 0$. Hence T cannot be zero. As $x \to -\infty$, y **approaches/ tends to** 0.	B1
8(d) **AO3**	~~Using your answers in (a)-(c), sketch the graph $y = xe^{-2x}$, $x \le 0$.~~ 	B1
		Total: 9 marks
9i **AO2**	 Coordinates of C: (4, y)	M1

	$\sqrt{(0-4)^2+(3-y)^2}=\sqrt{(y-1)^2}$ $16+9-6y+y^2=y^2-2y+1$ $-4y=-24$ $y=6$ Coordinates of C: (4, 6) [shown]	M1 A1
9ii **AO1**	Radius = 5 units Equation of circle: $(x-4)^2+(y-6)^2=25$	M1 A1
9iii	$x^2+y^2+8x-12y+27=0$ $(x+4)^2-16+(y-6)^2-36+27=0$ $(x+4)^2+(y-6)^2=25$ Centre: $(-4,6)$ Radius: 5	 M1 A1 A1
9iv **AO3**	C_2 is a reflection of C_1 on the y-axis (or $x=0$).	B1
9v **AO2**	Intersection of C_1 and $y=6$: (4–5 , 6) and (4+5, 6) (-1, 6) and (9, 6) Intersection of C_2 and $y=6$: (1, 6) and (-9, 6)  Range of p: $-9<p<-1$ or $1<p<9$	 M1 M1 A1
		Total: 12 marks

2023 4Exp AMath Paper 2 Marking Scheme

10(a) AO2	$y = x^3 + \frac{1}{x}$ $\frac{dy}{dx} = 3x^2 - \frac{1}{x^2} = 0$ $3x^4 - 1 = 0$ $x = 0.75983$ $y = 1.7548$ $A(0.760,\ 1.75)$ Area of trapezium $= \frac{1}{2}(1-0.75983)(2+1.7548)$ $= 0.45090$ Area under curve $= \int_{0.75983}^{1} x^3 + \frac{1}{x}\ dx$ $= \left[\frac{x^4}{4} + \ln x\right]_{0.75983}^{1}$ $= 0.4413297$ Area = 0.00957units	M1, M1 A1 A1 M1-follow thru their x, y-values A1 M1- integration A1 M1- area of trap – area of curve A1
10b(i) AO3	Explain what 2.67 units represents in this context. The **difference** in the areas between the region above and the region below x-axis.	B1
10b(ii) AO3	$\int_1^2 x^3 - 2x^2 - x + 2\ dx - \int_0^1 x^3 - 2x^2 - x + 2\ dx$	Any explanations, including in words- the need to split the 2 regions and to find the positive value of the area of both regions.
		Total: 12 marks