FUCHUN SECONDARY SCHOOL Secondary 4 Express / 5 Normal Academic Mid-Year Examination 2021

CANDIDATE NAME

CENTRE NUMBER

| S | 2 | 2 | 4 | 6 |
| :--- | :--- | :--- | :--- | :--- |

INDEX NUMBER


## CLASS

$\square$

6 May 2021
2 hours

## Candidates answer on the Question Paper

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.

The number of marks is given in brackets [ ] at the end of each question or part question. The total number of marks for this paper is 80 .


## Mathematical Formulae

Compound interest
Total amount $=P\left(1+\frac{r}{100}\right)^{n}$
Mensuration $\quad$ Curved Surface area of a cone $=\pi r l$
Surface area of a sphere $=4 \pi r^{2}$
Volume of a cone $=\frac{1}{3} \pi r^{2} h$

Volume of a sphere $=\frac{4}{3} \pi r^{3}$
Area of triangle $A B C=\frac{1}{2} a b \sin C$
Arc length $=r \theta$, where $\theta$ is in radians
Sector area $=\frac{1}{2} r^{2} \theta$, where $\theta$ is in radians

Trigonometry
$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
$a^{2}=b^{2}+c^{2}-2 b c \cos A$

Statistics
Mean $=\frac{\sum f x}{\sum f}$
Standard deviation $=\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}$

1. Write the following numbers in order of size, starting with the smallest.

$$
33 \% \quad \frac{1}{3} \quad\left(\frac{1}{3}\right)^{2} \quad-0.3
$$

Answer $\qquad$ , $\qquad$ , $\qquad$ ,
2. The line $4 y-3 x=21$ crosses the $x$-axis at $A$.

Find
(a) the gradient of the line,

## Answer

(b) the coordinates of point $A$.

Answer $A(\ldots \ldots$
3. A school has 360 students.

The ratio of the number of boys to the number of girls is $3: 5$.
On one day, 15 boys and 15 girls were absent.
Find the ratio of the number of boys to the number of girls on that day.
Give your answer in its simplest form.

Answer : ............................ [2]
4. Sally invested $\$ 20000$ in a savings account paying simple interest at $r \%$ per year.

After 3.5 years, there was $\$ 23150$ in her account.
Calculate the value of $r$.

## Answer $r=$

5. Complete the sentence below.

If $y \mathrm{~cm}^{2}$ is the total surface area of a cube with side of $x \mathrm{~cm}$,
$y$ is
proportional to , where $k=$ $\qquad$
6. A conical paper cup, as shown in the diagram, is filled up to $\frac{1}{2}$ of its maximum height.

Kumar claimed that the paper cup is filled to $50 \%$ of its capacity.
Explain whether his statement is true.

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
7. Sarah knows that one angle of an isosceles triangle is $50^{\circ}$.

She says that one of the other angles must be $\frac{180^{\circ}-50^{\circ}}{2}=65^{\circ}$.
Explain why Sarah is wrong.
$\qquad$
$\qquad$
8. It is given that $x^{2} y=5+4 y$.
(a) Find $y$ when $x=-3$.

Answer $y=$.
(b) Rearrange the formula to make $y$ the subject.

## Answer

9. 100 grams of peanuts contain 25.8 grams of protein.

Calculate the mass of protein in 248.9 kilograms of peanuts, in grams.
Leave your answer in standard form, correct to 3 significant figures.

Answer
grams
10. (a) Expand and simplify $2-3(2-3 x)$.

Answer
(b) Factorise $6 a x+9 b x-4 a-6 b$.

## Answer

11. The total surface area of a cylindrical solid with radius $r$ and height $h$ is twice the total surface area of a sphere with radius $r$.

Find $h$ in terms of $r$.

## Answer

12. A map is drawn to a scale of $1: 50000$.
(a) Two towns $P$ and $Q$ are 5 km apart.

Calculate, in metres, the distance $P Q$ on the map.

Answer
m
(b) A lake has an actual area of $10 \mathrm{~km}^{2}$.

Find the area, in square centimetres, of the lake on the map.
13. Sketch the graph of $y=-(x-8)(x+2)$ on the axes below.

Indicate clearly the coordinates of the maximum point and the values where the graph crosses the axes on the curve.

14. (a) Simplify $\left(\frac{625}{a^{8}}\right)^{-\frac{1}{4}}$, leaving your answer in positive index notation.

## Answer

(b) Given that $9^{n} \times \sqrt{3}=1$, find the value of $n$.
15. (a) Express 720 as the product of its prime factors.

Answer
(b) Given that $720 k=m^{3}$, where $k$ and $m$ are positive integers and $m$ is as small as possible, find the value of $k$ and of $m$.

Answer

(c) The lowest common multiple of two numbers is 720 .

The highest common factor of these two numbers is 24 .
Both numbers are smaller than 150.
Find the two numbers.
16. (a) Use the set notation to describe the shaded region.


## Answer

(b) $\xi=\{$ integers $x$ : $1 \leq x \leq 6\}$
$A=\{$ prime numbers $\}$
$B=\{$ multiples of 3$\}$
(i) List the elements of $A \cap B^{\prime}$.

Answer
(ii) On the Venn diagram below, shade the region which represents $A \cap B^{\prime}$. Answer

(iii) Underline the correct statement from the list below.

$$
5 \subset A \quad\left\} \not \subset B \quad\left(A^{\prime} \cap B^{\prime}\right) \subset \xi\right.
$$

17. The diagram shows the path of a plane from airport $A$ to airport $B$. $C$ is due south of $B, A$ is due west of $C$.


The plane flies at an average speed of $444 \mathrm{~km} / \mathrm{h}$.
It leaves $A$ and flies directly to $B$.
Calculate the time taken for the plane to reach $B$.
Give your answer in hour(s) and minute(s), to the nearest minute.
18. The zoo is setting up a new Panda Park on a plot of land STUV.

There is a waterfall at $W$ and a bamboo forest at $B$, as shown in the diagram below.

(a) Construct the perpendicular bisector of $B W$.

The panda enclosure is to be built in the park such that it is equidistant from the waterfall $(W)$ and the bamboo forest $(B)$.

It must also be nearer to edge $S T$ than to $U T$.
(b) By showing the necessary construction line(s) clearly, label one possible location of the panda enclosure with a $P$.
19. Die $A$ has the numbers $1,2,3$ and 4 engraved on it while die $B$ has the numbers 2,3 and 5 engraved on it. They are rolled one after another.
(a) Complete the possibility diagram.

## Die B

|  | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{5}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ |  |  | $(1,5)$ |
| $\mathbf{2}$ | $(2,2)$ |  |  |
| $\mathbf{3}$ | $(3,2)$ | $(3,3)$ | $(3,5)$ |
| $\mathbf{4}$ |  | $(4,3)$ | $(4,5)$ |

(b) Giving your answer as a fraction in its simplest form, find the probability that
(i) both dice are even numbers,

## Answer

(ii) product of the two numbers is less than 10 .

Answer
(c) Using the experiment above, give one example of an event E , such that $\mathrm{P}(\mathrm{E})$ is $\frac{1}{4}$.
$\qquad$
$\qquad$
20. Alex had 4 tests in a semester and the results were $A, B, C$ and $D$ respectively. He calculated his mean score and standard deviation using a calculator and obtained 67.5 for mean and 9 for standard deviation.
(a) Based on the information, find the value of
(i) $A+B+C+D$,

## Answer

(ii) $A^{2}+B^{2}+C^{2}+D^{2}$.

## Answer

(b) While checking, Alex realised that he had entered one of the scores wrongly.

The score should be 86 instead of 68 .
Calculate the correct mean.
21. The table below shows the number of cups of four brands of ice cream sold at 2 dessert shops on a particular day.

|  | Shop 1 | Shop 2 |
| :---: | :---: | :---: |
| $\operatorname{Brand} A$ | 5 | 3 |
| $\operatorname{Brand} B$ | 11 | 7 |
| Brand $C$ | 8 | 9 |
| Brand $D$ | 5 | 12 |

(a) Write down a $4 \times 2$ matrix $\mathbf{Q}$ that represents the data in the table above.

## Answer $\quad \mathbf{Q}=$

A cup of ice cream costs $\$ 5$ for Brand $A, \$ 10$ for Brand $B, \$ 8$ for Brand $C$ and $\$ x$ for Brand $D$. The information can be represented by the matrix $\mathbf{P}=\left(\begin{array}{llll}5 & 10 & 8 & x\end{array}\right)$.
(b) Find, in terms of $x$, matrix $\mathbf{R}=\mathbf{P Q}$.

$$
\text { Answer } \quad \mathbf{R}=
$$

(c) Explain what each of the elements in $\mathbf{R}$ represent.
$\qquad$
(d) Both shops collected the same amount of money that day.

Calculate the value of $x$.
22. In the diagram, the points $A, B, C, D$ and $E$ lie on a circle, centre $O$. $A D$ is a diameter of the circle, $M$ is the point on $B E$ such that $E M=B M$ and $\angle B C D=121^{\circ}$.

(a) Find
(i) angle $B A D$,

Answer
(ii) angle $A E B$.

Answer
(b) Explain whether line $A E$ is parallel to line $B O$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
23. The diagram shows a regular octagon $A B C D E F G H$.
$A B$ and $D C$ are extended to meet at $P$.

(a) Find angle $C D E$.

## Answer

(b) Show that angle $B P C$ is a right angle.

Answer
(c) Given that $C G=8 \mathrm{~cm}$, find the area of this octagon.
24. The lengths of the sides of an equilateral triangle are $(3 x+y-3) \mathrm{cm},(2 x+3 y) \mathrm{cm}$ and $(x+4 y+5) \mathrm{cm}$.
(a) Write down and simplify two simultaneous equations, in terms of $x$ and $y$, to represent this information.

```
Answer \(1^{\text {st }}\) equation
(b) Solve the simultaneous equations to find the perimeter of the triangle.

Answer Perimeter =
 cm

FUCHUN SECONDARY SCHOOL Secondary 4 Express / 5 Normal Academic Mid-Year Examination 2021

\section*{CANDIDATE NAME}

\section*{CENTRE NUMBER}


CLASS
\begin{tabular}{|l|l|l|l|l|}
\hline S & 2 & 2 & 4 & 6 \\
\hline
\end{tabular}

INDEX NUMBER


MATHEMATICS
Paper 2

4048/02
10 May 2021
2 hours 30 minutes

\section*{Candidates answer on the Question Paper}

\section*{READ THESE INSTRUCTIONS FIRST}

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \(\pi\), use either your calculator value or 3.142 , unless the question requires the answer in terms of \(\pi\).

The number of marks is given in brackets [ ] at the end of each question or part question. The total number of marks for this paper is 100 .


Compound interest

\section*{Mensuration}

Curved Surface area of a cone \(=\pi r l\)
Surface area of a sphere \(=4 \pi r^{2}\)
Volume of a cone \(=\frac{1}{3} \pi r^{2} h\)
Volume of a sphere \(=\frac{4}{3} \pi r^{3}\)
Area of triangle \(A B C=\frac{1}{2} a b \sin C\)
Arc length \(=r \theta\), where \(\theta\) is in radians
Sector area \(=\frac{1}{2} r^{2} \theta\), where \(\theta\) is in radians
\(\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}\)
\(a^{2}=b^{2}+c^{2}-2 b c \cos A\)

Statistics
Mean \(=\frac{\sum f x}{\sum f}\)
Standard deviation \(=\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}\)
1. (a) The first four terms of a sequence are \(2,5,8\) and 11 .
(i) Write down the \(6^{\text {th }}\) term of the sequence.

\section*{Answer}
(ii) Write down an expression for the \(n\)th of this sequence.

Answer
(iii) Explain whether 2021 is a term of this sequence.
\(\qquad\)
\(\qquad\)
(b) The first four terms in a different sequence of numbers are given below.
\[
\begin{aligned}
& T_{1}=2^{2}-1=3 \\
& T_{2}=5^{2}-1=24 \\
& T_{3}=8^{2}-1=63 \\
& T_{4}=11^{2}-1=120
\end{aligned}
\]
(i) Find \(T_{5}\).

\section*{Answer}
(ii) Find and simplify an expression, in terms of \(n\), for \(T_{n}\).

\section*{Answer}
(iii) Hence explain why the terms of the sequence are all multiples of 3 .
\(\qquad\)
\(\qquad\)
2. The diagram shows the speed-time graph for a bus journey between two places.

(a) Find the deceleration after 52 seconds.
\[
\text { Answer } \quad \mathrm{m} / \mathrm{s}^{2}
\]
(b) Describe the motion of the bus between \(t=10\) seconds and \(t=50\) seconds.
(c) Find the distance travelled by the bus in the first 50 seconds.

Answer m

A car started its journey from rest 10 seconds later after the bus.
It took the same route as the bus.
It accelerated uniformly until it met the bus at \(t=50\) seconds.
(d) Calculate the speed of the car at \(t=50\) seconds.

Answer m/s
3. (a) Solve the inequality \(-2<1-2 x \leq 5\).

\section*{Answer}
(b) Alan claimed that \(5^{22}\) is larger than \(3^{33}\).

Explain whether his claim is true without using a calculator.
\(\qquad\)
\(\qquad\)
\(\qquad\)
(c) Simplify \(\frac{3 x^{2}+x-2}{27 x^{2}-12}\).
(d) Express as a single fraction in its simplest form \(\frac{5 x}{(3 x-2)^{2}}+\frac{2}{2-3 x}\).

Answer
(e) (i) Express \(5-9 x+x^{2}\) in the form of \(q+(x-p)^{2}\).

Answer
(ii) Hence find the minimum value of \(5-9 x+x^{2}\).

4 On Saturday, the exchange rate between Euros \((€)\) and Singapore Dollars (S\$) was \(€ 1=\mathrm{S} \$ x\).
(a) Mr Lee changed S\$480 into Euros.

Write down an expression, in terms of \(x\), for the amount of Euros that he received on Saturday.

Answer €

On Sunday, the Euros grew stronger and the exchange rate was \(€ 1=\mathrm{S} \$(x-0.1)\).
(b) Mr Lee changed another \(\mathrm{S} \$ 300\) into Singapore Dollars.

Write down an expression, in terms of \(x\), for the amount of Euros that he received on Sunday.

\section*{Answer €}
(c) Mr Lee realised that he received \(€ 100\) more on Saturday than on Sunday.

Form an equation in \(x\) and show that it reduces to \(50 x^{2}-95 x+24=0\).
Answer
(d) Solve the equation \(50 x^{2}-95 x+24=0\).
(e) Mrs Lim changed \(€ 1000\) into Singapore Dollars on Sunday.

Given that \(x>1\), calculate the amount of Singapore Dollars that she received.

Answer S\$


The diagram shows a sector \(O A B\) of a circle with centre \(O\) and radius \(x \mathrm{~cm}\).
A circle with centre \(C\) and radius 6 cm lies within the sector and touches the sector \(O A B\) at \(P, R\) and \(S\).

It is given that angle \(R O C\) is \(30^{\circ}\).
(a) Show that triangle \(C R O\) is congruent to triangle \(C P O\).
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\square\)
(b) Show that \(x=18\).

Answer
(c) Calculate the perimeter of the unshaded region, \(C P S R C\).

Write your answer in terms of \(a+b \pi\).

Answer
cm
(d) Calculate the total area of the shaded regions.

Answer \(\mathrm{cm}^{2}\)


In the diagram , \(A B C\) is a horizontal triangular plot of land and \(D\) lies on \(B C\).
\(A B=65 \mathrm{~m}, B D=70 \mathrm{~m}, A C=160 \mathrm{~m}\) and angle \(A B D=130^{\circ}\).
(a) Find the distance \(A D\).
(b) Show that angle \(B A C=31.87^{\circ}\).

Answer
(c) Find the bearing of \(A\) from \(C\).

\section*{Answer}
(d) A bird is hovering vertically above \(B\).

The angle of elevation of the bird from \(A\) is \(20^{\circ}\).
Find the angle of elevation of the bird when viewed from \(D\).

Answer

7 (a) 200 students from school \(A\) recorded the time, \(t\) minutes, for their journey from home to school.

The results are shown in the cumulative frequency graph.


Using the graph to find the
(i) number of students who took between 15 and 40 minutes,

\section*{Answer}
(ii) interquartile range.

200 students from school \(B\) also recorded their journey time from home to school and the information could be found in the table.
\begin{tabular}{|c|c|}
\hline Lower Quartile & Upper Quartile \\
\hline 15 minutes & 30 minutes \\
\hline
\end{tabular}
(iii) Susi mentioned that there are more students in school \(A\) than in school \(B\) who took more than 30 minutes to reach school.

Explain whether her statement is correct.
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
(b) A bag contains 5 red balls, 4 blue balls and 3 green balls.

Two balls are taken out at random without replacement.
Find, in its simplest form, the probability that
(i) one red ball and one green ball are selected,

Answer
(ii) both balls are of the same colours.

Answer

A third ball is taken out.
(iii) Find the probability that none of the three balls taken out is red.

Answer

8 The variables \(x\) and \(y\) are connected by the equation \(y=\frac{8}{x^{2}}+2 x-7\).
Some corresponding values of \(x\) and \(y\) are given in the following table.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline\(x\) & 1 & 1.5 & 2 & 3 & 4 & 5 & 6 \\
\hline\(y\) & 3 & -0.44 & -1 & -0.11 & 1.5 & 3.32 & \(p\) \\
\hline
\end{tabular}
(a) Find the value of \(p\).

\section*{Answer}
(b) On the grid opposite, draw the graph of \(y=\frac{8}{x^{2}}+2 x-7\) for \(1 \leq x \leq 6\).
(c) Use the graph to write down the range of values \(x\) where \(\frac{8}{x^{2}}+2 x<8\).

\section*{Answer}
(d) By drawing a tangent, find the gradient of the curve at \(x=1.5\).

\section*{Answer}
(e) (i) On the same axes, draw a line that passes through \((2,0)\) and has gradient of -1 .
(ii) Write down the \(x\)-coordinates of the points where this line intersects the curve.

Answer \(x=\) \(\qquad\) , \(x=\)


9 A beverage factory produces soft drink in a bottle, as shown in figure 1.
The bottle can be modelled as a frustum attached to the top of a cylinder as shown in figure 2 . The measurements are given in the table.

Diameter \(\left(D_{1}\right)\) of the base of the bottle \(=84 \mathrm{~mm}\).

Diameter \(\left(D_{2}\right)\) of bottle opening \(=\) 36 mm

Height \(\left(H_{1}\right)=80 \mathrm{~mm}\)
Height \(\left(H_{2}\right)=120 \mathrm{~mm}\)


Figure 1


Figure 2
(a) Using similarity, show that \(h=60 \mathrm{~mm}\).

\section*{Answer}
(b) Calculate the volume, in cubic centimeters, of the bottle in Figure 2.

Answer
\(\mathrm{cm}^{3}\)
(c) Another bottle is to be made with a volume that is half the volume of the bottle in Figure 2.

Given that the two bottles are geometrically similar, find the height of the smaller bottle.

Answer mm

The table below shows the flag-down rate and charges for SBC taxi.
\begin{tabular}{|l|l|}
\hline \multicolumn{2}{|c|}{ Metered Fare } \\
\hline \begin{tabular}{l} 
Flag down rate (inclusive of \\
first 1 km or less)
\end{tabular} & \(\$ 3.50\) \\
\hline \begin{tabular}{l} 
Distance rate (beyond 1 km) \\
Every 400 m up to 10 km \\
Every 300 m after 10 km
\end{tabular} & \(\$ 0.22\) \\
\hline Traffic waiting time rate & \(\$ 0.25\) \\
\begin{tabular}{l} 
Every 45 seconds of waiting \\
or less
\end{tabular} & \(\$ 0.22\) \\
\hline
\end{tabular}
\begin{tabular}{|l|c|}
\hline \multicolumn{2}{|c|}{ Time-based Additional Surcharges } \\
\hline \begin{tabular}{l} 
Peak period surcharge \\
\((6 \mathrm{am}\) to 9.29 am and 6 pm to \\
\(11.59 \mathrm{pm})\)
\end{tabular} & \(30 \%\) of metered fare \\
\hline \begin{tabular}{l} 
Late night surcharge \\
\((12.00\) am to 5.59 am\()\)
\end{tabular} & \(50 \%\) of metered fare \\
\hline
\end{tabular}
(a) Bryan hailed and got into a taxi during the non-peak period to travel to his workplace. The total distance covered by the taxi is 11 km and the traffic waiting time is about 90 seconds.

Calculate how much Bryan paid for his taxi fare.

Answer \$

Below is some additional information about the payment method.
\begin{tabular}{|l|c|}
\hline Payment by Easy Link Card & \(\$ 0.30\) charge on top of metered fare \\
\hline Payment by Go Card & \(5 \%\) discount of the total fare \\
\hline
\end{tabular}
(b) Lisa hailed a taxi to return to her house.

She boarded the taxi at 7 pm and reached her house at 7.30 pm .
During the journey, the traffic waiting time is about 5 minutes.
She used Go Card and paid \(\$ 27.84\) for the fare.
Suggest what is a likely average speed of the taxi during this journey, in \(\mathrm{km} / \mathrm{h}\).

\section*{Paper 1}
1. \(-0.3,\left(\frac{1}{3}\right)^{2}, 33 \%, \frac{1}{3}\) [B2: all correct, B1 for three numbers in the correct sequence]
2. (a) \(\frac{3}{4}[\mathrm{~B} 1]\)
(b) \(A(-7,0)[\mathrm{B} 1]\)
3. Number of boys \(=135\), number of girls \(=225[\mathrm{~B} 1]\)

New ratio on that day \(=4: 7\) [B1]
4. Amount of interest \(=\$ 3150\)
\(3150=\frac{20000 \times r \times 3.5}{100} \quad\) [M1: correct amount of interest and attempt to use the formula] \(r=4.5\) [A1]

OR interest per year \(=\$ 900\) [M1: correct amount of interest/year]
\(r \%=\frac{900}{20000} \times 100 \%=4.5 \%\).
\(r=4.5\) [A1]
5. \(y\) is directly proportional to \(x^{2}\) with \(k=6\). [B2: all correct, \(\mathrm{B} 1: 2\) correct answers]
6. \(\frac{V_{1}}{V_{2}}=\left(\frac{1}{2}\right)^{3}[\mathrm{~B} 1]\) or attempt to say that the volume of upper half is not equal to volume of lower half. However, correct terminology must be used.
\(\frac{V_{1}}{V_{2}}=0.125\)
His statement is not true as it is only filled to \(\mathbf{1 2 . 5 \%}\) of its capacity. [B1]
7. Sarah is wrong as there is other possible angle.
[B1]
OR \(50^{\circ}\) could be one of the base angles.
The other angle could also be \(180^{\circ}-50^{\circ}-50^{\circ}=80^{\circ}\). [B1: s.o.i.]
8. (a) 1 [B1]
(b) \(x^{2} y-4 y=5[\mathrm{M1}]\)
\(y\left(x^{2}-4\right)=5\)
\(y=\frac{5}{x^{2}-4}[\mathrm{~A} 1]\)
9. 100 grams peanuts rep. 25.8 grams protein \(248.9 \times 1000\) grams peanuts [B1: correct conversion]

248900 grams peanuts rep. \(\frac{248900}{100} \times 25.8\) [M1]
\(=6.42 \times 10^{4}\) grams of protein [A1]
10. (a) \(9 x-4\) [B1]
(b) \(6 a x+9 b x-4 a-6 b\)
\(=3 x(2 a+3 b)-2(2 a+3 b)[\mathrm{M} 1]\)
\(=(3 x-2)(2 a+3 b)[\mathrm{A} 1]\) No mark is awarded if the method is wrong.
For a slip and correct answer, award B1 only
11. Total surface area of cylinder \(=2 \pi r^{2}+2 \pi r h[\mathrm{~B} 1]\)

Surface area of sphere \(=4 \pi r^{2}\)
\[
\begin{align*}
& 2 \pi r^{2}+2 \pi r h=2\left(4 \pi r^{2}\right)[\mathrm{M} 1 \sqrt{ } \text { : total SA of cylinder }=2(\text { surface area of sphere })] \\
& h=3 r \tag{A1}
\end{align*}
\]
12. (a) 0.1 m [B1]
(b) 1 cm rep. 0.5 km
\(1 \mathrm{~cm}^{2}\) rep. \(0.25 \mathrm{~km}^{2}\) [M1: must square and indicate the correct unit] Area on map \(=40 \mathrm{~cm}^{2}\) [A1]
13.


B1: correct turning point - coordinates of the point must be written
B1: correct shape (concave downward, symmetrical, cuts \(x\) - and \(y\)-axis)
14. (a) \(\left(\frac{625}{a^{8}}\right)^{-\frac{1}{4}}\)
\[
\begin{aligned}
& =\left(\frac{a^{8}}{625}\right)^{\frac{1}{4}}[\mathrm{M} 1] \quad \text { OR } \quad\left(\frac{625^{-\frac{1}{4}}}{a^{-2}}\right) \\
& =\frac{a^{2}}{5}[\mathrm{~A} 1]
\end{aligned}
\]
(b) \(9^{n} \times \sqrt{3}=1\)
\[
9^{n}=\frac{1}{\sqrt{3}}
\]
\[
3^{2 n}=\frac{1}{3^{0.5}}[\mathrm{M} 1]
\]
\[
3^{2 n}=3^{-0.5}
\]
\[
n=-0.25[\mathrm{~A} 1]
\]

OR \(\quad 9^{n} \times \sqrt{3}=1\)
\(3^{2 n} \times 3^{0.5}=3^{0}[\mathrm{M} 1\) : at least 2 correct \(]\)
\(2 n+0.5=0\)
\(n=-0.25[\mathrm{~A} 1]\)

For guess and check, only B1 is awarded.
15. (a) \(720=2^{4} \times 3^{2} \times 5[\mathrm{~B} 1]\)
(b) \(k=300\) and \(m=60[\mathrm{~B} 1]\)
(c) \(720=2^{4} \times 3^{2} \times 5\)
\(24=2^{3} \times 3\)
\(1^{\text {st }}\) number \(=2^{3} \times 3 \times 5=120\)
\(2^{\text {nd }}\) number \(=2^{4} \times 3^{2}=144\)
[B1: finding prime factorisation of 24 and one correct number]
[B2: both correct numbers]
16. (a) \(B \cap A^{\prime}[\mathrm{B} 1]\)
(b) (i) 2 and \(5[\mathrm{~B} 1]\)
(ii) Shade the correct region [B1]
(iii) \(\left(A^{\prime} \cap B^{\prime}\right) \subset \xi[\mathrm{B} 1]\)

17. \(\begin{aligned} A B & =600 \mathrm{~km}[\mathrm{~B} 1] \\ \text { time } & =\frac{600}{444} \text { hour }[\mathrm{M} 1: \text { distance } / \text { speed }] \\ \text { time } & =1.351 \text { hour }[\mathrm{A} 1 \sqrt{ }]\end{aligned}\)

1 hour and 21 minutes [A1 \(\sqrt{ }\) if it doesn't change level of difficulty]
18. (a) line is drawn correctly [B1]
(b) Attempt to draw any angle bisector [B1]

Angle bisector of UTS drawn correctly [B1]
Correct point plotted and labelled [B1]
19. (a) \((1,2),(1,3),(2,3),(2,5)\) and \((4,2)[B 1]\)
\(\begin{array}{ll}\text { (b) (i) } \frac{1}{6}[\mathrm{~B} 1] & \text { (ii) } \frac{2}{3}[\mathrm{~B} 1]\end{array}\)
(if both answers correct but not simplified, awards B1)
(c) Any logical answer, where \(\mathrm{n}(\mathrm{E})=3\).

For example, probability that the first die shows 1 [B1]
20. (a) (i) 270 [B1]
(ii) \(9=\sqrt{\frac{\text { sum of } x^{2}}{4}-67.5^{2}}[\mathrm{M} 1]\)

Sum of \(x^{2}=18549\) [A1]
(b) \((72[\mathrm{~B} 1]\)
21. (a) \(\left(\begin{array}{cc}5 & 3 \\ 11 & 7 \\ 8 & 9 \\ 5 & 12\end{array}\right)\) [B1] \(\quad\) (b) \((199+5 x \quad 157+12 x)\) [B1]
(c) Amount of money collected, in dollars, from selling four brands of ice-cream in shop A and B respectively. [B1] or in each shop respectively. Do not accept in both shops respectively.
(d) \(\$ 6[\mathrm{~B} 1]\)
22. (a) (i) \(59^{\circ}[\mathrm{B} 1]\)
(ii) \(\angle B E D=59^{\circ}[\mathrm{B} 1]\) \(\angle A E B=90^{\circ}-59^{\circ}=31^{\circ}[\mathrm{B} 1]\)

\section*{OR}
\[
\angle A O B=180^{\circ}-59^{\circ}-59^{\circ}=62^{\circ}[\mathrm{B} 1]
\]
\[
\angle A O B=\frac{1}{2} \times 62^{\circ}=31^{\circ}[\mathrm{B} 1]
\]

For students who assume \(\angle A E M=\angle A B M, 1\) mark is awarded for correct answer
(iii) \(\angle M B O=180^{\circ}-90^{\circ}-62^{\circ}=28^{\circ}[\mathrm{B} 1]\)

Since \(\angle M B O \neq \angle A E B, A E\) is NOT parallel to line \(B O\) [B1] 0 mark if angle not found.
23. (a) \(\angle C D E=135^{\circ}[\mathrm{B} 1]\)
(b) \(\angle B C P=45^{\circ}\) [ B 1 : must write down what angle is found \(]\)

Therefore, \(\angle B P C=180^{\circ}-45^{\circ}-45^{\circ}=90^{\circ}[\mathrm{B} 1]\)
(c) Area of triangle \(B O C=\frac{1}{2} \times 4 \times 4 \times \sin 45^{\circ}=5.657\) [M1]

Area of octagon \(=45.3 \mathrm{~cm}^{2}[\mathrm{Al}]\)
24. (a) \((3 x+y-3),(2 x+3 y),(x+4 y+5)\)

Any of these two equations or equivalent [B1 each: must simplify]
\(x-2 y=3\)
\(x-y=5\)
\(2 x-3 y=8\)
(b) [M1]: any method to solve the two equations
[A1, A1] \(x=7, y=2\)
\([B 1 \sqrt{ }] 20 \times 3=60 \mathrm{~cm}^{2}\)

\section*{Paper 2}
1. (a) (i) \(17[\mathrm{~B} 1]\)
(ii) \(-1+3 n[\mathrm{~B} 1]\)
(iii) \(-1+3 n=2021[\mathrm{M} 1 \sqrt{ }\) : part (ii) answer \(=2021]\)
\(n=674\)
Since \(n\) is a positive integer, 2021 is a term of the sequence. [A1]
(b) (i) \(T_{5}=195[\mathrm{~B} 1]\)
(ii) \(T_{n}=(3 n-1)^{2}-1[\mathrm{M} 1]\)
\(T_{n}=9 n^{2}-6 n\) [A1]
(iii) \(T_{n}=3\left(3 n^{2}-2 n\right)\). Therefore, it is a multiple of \(3[\mathrm{~B} 1 \sqrt{ } \rightarrow\) must show factor of 3\(]\)
2. (a) \(2.5[\mathrm{~B} 1]\)
(b) The bus moves at a constant speed. [B1]
(c) \(\frac{1}{2} \times 10 \times 15+40 \times 15\) [M1: finding the area under the graph. May award M1 if students accidentally find the distance for all journey instead]
\[
=675 \mathrm{~m}[\mathrm{~A} 1]
\]
(d) \(\frac{1}{2} \times 40 \times v=675[\mathrm{M1} \sqrt{ }]\)
\[
v=33.75[\mathrm{~A} 1]
\]
3. (a) \(-2<1-2 x \leq 5\)
\(-3<-2 x \leq 4\)
\(1.5>x \geq-2\) [ B 1 : for one correct inequality]
[B2: correct inequalities and combined]
(b) \(5^{22}=\left(5^{2}\right)^{11}\)
\(3^{33}=\left(3^{3}\right)^{11} \quad[\) B1: attempt to change to same power 11]
\(27^{11}>25^{11}\). Therefore, Alan's claim is wrong [B1]
(c) \(\frac{(3 x-2)(x+1)}{3(3 x-2)(3 x+2)}\) [M1, M1 each for correct factorization]
\(=\frac{(x+1)}{3(3 x+2)}\) or \(\frac{(x+1)}{9 x+6}[\mathrm{~A} \backslash \sqrt{ }\) : cancelling the common factor from numerator and denominator]
(d) \(\frac{5 x}{(3 x-2)^{2}}-\frac{2}{3 x-2}\) [M1]

OR \(\frac{5 x(2-3 x)+2(3 x-2)^{2}}{(3 x-2)^{2}(2-3 x)}[M 1]\)
\(=\frac{5 x}{(3 x-2)^{2}}-\frac{2(3 x-2)}{(3 x-2)^{2}}\) [M1: equalise denominator]
\(\frac{3 x^{2}-14 x+8}{(3 x-2)^{2}(2-3 x)}[\) M1]
\(=\frac{5 x-6 x+4}{(3 x-2)^{2}}\)
\(\frac{(2-3 x)(-x+4)}{(3 x-2)^{2}(2-3 x)}\)
\(=\frac{-x+4}{(3 x-2)^{2}}[\mathrm{~A} 1]\)
(e) (i) \((x-4.5)^{2}-15.25[\mathrm{~B} 1, \mathrm{~B} 1\) for correct \(p\) and \(q\) each]
(ii) \(-15.25[\mathrm{~B} 1 \sqrt{ }--\) the value of \(q]\)
4. (a) \(\frac{480}{x}[\mathrm{~B} 1]\)
(b) \(\frac{300}{x-0.1}[\mathrm{~B} 1]\)
(c) \(\frac{480}{x}-\frac{300}{x-0.1}=100[\mathrm{M} 1 \sqrt{ }\) : part (a) \(-\operatorname{part}\) (b) \(]\)
\(480(x-0.1)-300 x=100 x(x-0.1)[\mathrm{M} 1 \sqrt{ }\) : get rid of the denominator without changing the level of difficulty]
\(100 x^{2}-190 x+48=0 \rightarrow 50 x^{2}-95 x+24=0\) [A1]
(d) \(\frac{95 \pm \sqrt{(-95)^{2}-4(50)(24)}}{2 \times 50}\left[\right.\) M1 for substitution of \(-b\) and \(2 a\), M1 for \(b^{2}-4 a c\) ]
\[
\begin{aligned}
& x=1.6, x=0.3[\mathrm{~A} 1, \mathrm{~A} 1] \quad \text { (if no working shown, B1 each for correct answer) } \\
& \text { OR }(10 x-3)(5 x-8)=0[\mathrm{M} 2]
\end{aligned}
\]
(e) \(\$ 1500[\mathrm{~B} 1]\)
5. (a) In triangle \(C R O\) and triangle \(C P O\),
\(C O=C O\) (common side)
\(\angle C R O=\angle C P O\) (tangent perpendicular to radius)
\(C R=C P\) (radius)
[B1 for two correct statements and reason, B2 for all correct statements and reason]
\(\triangle C R O\) is congruent to \(\triangle C P O\) (RHS) [ B 1 for RHS -- the congruence test must tally with the three statements used]

Accept SAS, SSS, ASA as well when student used tangents from external point concept. However the 3 statements must tally with the congruence test used.
(b) \(\sin 30^{\circ}=\frac{6}{C O}[\mathrm{M} 1]\)
\(\mathrm{CO}=12\)
Therefore \(x=12+6=18\) [A1]
(c) Arc length \(P S R=\frac{240}{360} \times 2 \pi(6)\) [M1]
\[
=8 \pi[\mathrm{~A} 1]
\]

Perimeter \(=12+8 \pi[\mathrm{~B} 1 \sqrt{ }: 12+\) arc length \(P S R\) found \(]\)
(d) Area of sector \(A O B=\frac{60}{360} \times \pi \times 18^{2}\) [M1]

Area of major sector \(C P R S=\frac{240}{360} \times \pi(6)^{2}[\mathrm{M} 1]\)
Shaded area \(=94.2 \mathrm{~cm}^{2}[\mathrm{~A} 1]\)
6. (a) \(A D^{2}=65^{2}+70^{2}-2(65)(70) \cos 130^{\circ}[\mathrm{M} 1]\)
\(=14974.367\) [M1: taking square root or s.o.i. in taking square root] \(A D=122 \mathrm{~m}[\mathrm{~A} 1]\)
(b) \(\frac{\sin 130^{\circ}}{160}=\frac{\sin \angle A C B}{65}\) [M1: using sine rule to find angle \(A C B\) ]
\(\sin \angle A C B=\frac{65 \sin 130^{\circ}}{160}\)
\(\angle A C B=18.13[\mathrm{M} 1 \sqrt{ }\) : making sin angle \(A C B\) as subject and take sin inverse]
\(\angle B A C=180^{\circ}-18.13^{\circ}-130^{\circ}=31.87^{\circ}[\mathrm{A} 1]\)
(c) Draw north line from C. \(\angle N_{2} C A=31.87^{\circ}\) [M1] or any method reaching to this correct angle.

Bearing of \(A\) from \(C=360-31.87^{\circ}=328.1^{\circ}[\mathrm{A} 1]\)
(d) \(\tan 20^{\circ}=\frac{h}{65}\) [M1] OR using correct sine rule \(h=65 \tan 20^{\circ}\)
\(\tan \theta=\frac{65 \tan 20^{\circ}}{70}[\mathrm{M} 1 \sqrt{ }]\) OR using correct sine rule
\(\theta=18.7^{\circ}\) [A1]
7. (a) (i) \(192-40=152\) [M1, A1] - Award M1 if one of the numbers is correct and attempt to subtract both numbers.
(ii) \(26.5-16.5=10\) [M1, A1] - Award M1 if one of the quartiles is correct and attempt to subtract \(\mathrm{Q}_{1}\) from \(\mathrm{Q}_{3}\).
(iii) School B upper quartile is 30 minutes. Therefore there are \(25 \%\) of students (or 50 students) who took more than 30 minutes. [B1]

Her statement is incorrect as \(\mathrm{Q}_{3}\) in school A is smaller than school B which means that fewer students who took more than 30 minutes. [B1]

OR from the curve, there are 28 students in school A who took more than 30 minutes. Therefore, her statement is incorrect. [B1]
(b) (i) \(\left(\frac{5}{12} \times \frac{3}{11}\right) \times 2=\frac{5}{22}[\mathrm{M} 1, \mathrm{~A} 1]\)

M1 is awarded if \(\left(\frac{5}{12} \times \frac{3}{11}\right)\) is seen.
(ii) \(\left(\frac{5}{12} \times \frac{4}{11}\right)+\left(\frac{4}{12} \times \frac{3}{11}\right)+\left(\frac{3}{12} \times \frac{2}{11}\right)\) [M1: at least two correct] \(=\frac{19}{66}[\mathrm{Al}]\)
(iii) \(\frac{7}{12} \times \frac{6}{11} \times \frac{5}{10}=\frac{7}{44}[\mathrm{~B} 1]\)
8. (a) 5.22 [B1]
(b) P2: all points plotted correctly

P1: 4 to 6 points plotted correctly
C 1 : smooth curve, curve passes through all the points, no 'break'
(c) \(y=1\) is seen or implied. [B1]
\(1.2<x<3.7 \quad[\mathrm{~B} 1 \sqrt{ }\) from graph]
(d) Tangent is drawn correctly at \(x=1.5[\mathrm{~B} 1]\)

Gradient \(=-2.75 \pm 0.5\) [B1]
(e) (i) A line with passes through \((2,0)[B 1]\)

A line has gradient of -1 [B1]
(ii) \(x=1.2\) or \(2.6[\mathrm{~B} 1 \sqrt{ }\) from graph \(]\)
9. (a) \(\frac{h}{h+80}=\frac{36}{84}[\mathrm{M} 1]\)
\(84 h=36 h+2880\)
\(h=60\) [A1]
(b) Volume of cylinder \(\left(\right.\) in \(\left.\mathrm{mm}^{3}\right)=\pi(42)^{2}(120)\) [M1]

Volume of big cone (in \(\mathrm{mm}^{3}\) ) \(=\frac{1}{3} \pi(42)^{2}(140)\) [M1]
Volume of small cone (in \(\mathrm{mm}^{3}\) ) \(=\frac{1}{3} \pi(18)^{2}(60)\) [M1]


Can award at most M1 if all workings are correct but students used diameter and not radius

Volume of frustum \(\left(\right.\) in \(\left.\mathrm{mm}^{3}\right)=\) Volume of big cone - volume of small cone [M1 ل]
\[
=\frac{1}{3} \pi(42)^{2}(140)-\frac{1}{3} \pi(18)^{2}(60)
\]

Total volume \(=\) Volume of cylinder + frustum \(=903.27 \mathrm{~cm}^{3}=903 \mathrm{~cm}^{3}[\mathrm{A1}]\)
Penalise overall 1 mark for issue with wrong conversion for this part.
(c) \(\frac{V_{1}}{V_{2}}=\left(\frac{h_{1}}{h_{2}}\right)^{3}\)
\(\frac{1}{2}=\left(\frac{h_{1}}{200}\right)^{3} \quad\) [M1: concept of ratio of volume and height]
\[
h=159 \mathrm{~mm}[\mathrm{~A} 1]
\]
10. (a) Waiting rate \(=2 \times \$ 0.22=\$ 0.44\) [M1]

Distance rate \(=\$ 3.50+\frac{10000}{400} \times \$ 0.22\) [M1: for flag rate and attempt to use rate: every 400 m is \(\$ 0.22\) ]
\[
\text { Total fare }=\$ 9.44[\mathrm{~A} 1]
\]
(b) Price before discount \(=\$ 27.84 \div 0.95[\mathrm{M} 1]=\$ 29.305\)

Price before surcharge \(=\$ 29.305 \div 1.3=[\mathrm{M} 1 \sqrt{ }]=\$ 22.542\)
Waiting rate \(=7 \times \$ 0.22[\mathrm{M} 1]=\$ 1.54\)
Price excluding waiting rate \(=\$ 22.542-\$ 1.54=\$ 21\)
\[
3.50+\frac{10000}{400} \times \$ 0.22+\frac{x}{300} \times \$ 0.25=\$ 21[\mathrm{M} 1 \sqrt{ }]
\]
\[
\frac{x}{300} \times \$ 0.25=\$ 12[\mathrm{M} 1 \sqrt{ }]
\]
\[
x=14400 \text { Accept }
\]

Total distance \(=1+10+14.4[\mathrm{M} 1 \sqrt{ }: 1+10+\) the remaining km\(]=25.4 \mathrm{~km}\)
Average speed \(=\frac{25.4}{0.5}=50.8 \mathrm{~km} / \mathrm{h}\) [A1] (Accept between 50.2 and \(50.8 \mathrm{~km} / \mathrm{h}\) )```

