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FAJAR SECONDARY SCHOOL
2021 END-OF-YEAR EXAMINATIONS
SECONDARY 3 EXPRESS

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4049

30 September 2021

Setter: Miss Lily

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Total	**90**

Do not open this document till permission is given.

This document consists of **18** printed pages and **0** blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 **(a)** Express each of the following expressions $2x^2 - 4x + 7$ and $-x^2 - 4x - 1$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [4]

(b) Use your answer from part **(a)** to explain why the curves with equations $y = 2x^2 - 4x + 7$ and $y = -x^2 - 4x - 1$ will not intersect. [3]

[Turn over

2 **Do not use a calculator in this question.**

(a) It is given that $x - \sqrt{\ } = x\sqrt{3} +$. Find x in the form $a + b\sqrt{\ }$ where a and b are constants. [3]

(b) Solve the equation $\sqrt{x-3} + x =$. [3]

3 **(a)** Find the set of values of b for which the curve $y = 2x^2 + bx + 17$ intersects the line $y - 7x - 9 = 0$ at two distinct points. [4]

(b) Find the range of values of k for which $x^2 + k(2x+1) + 12$ is always positive. [3]

[Turn over

4 **(a)** The expression x^2+7x+3 has the same remainder when divided by $(x-p)$ and $(x+q)$, where p and q are constants and $p \neq q$. Find the value of $p-q$. [4]

(b) Given that $f(x) = 2x^3 + x^2 - 10x - 8$, show that $x + 2$ is a factor and hence solve the equation $f(x) = 0$. [5]

[Turn over

5 Express $\dfrac{x^3+2x^2-6}{x^2-9}$ in partial fractions. [5]

6 The coefficient of x^2 in the expansion of $(a+x)^4 + \left(2 - \frac{x}{4}\right)^5$ is 29.

(a) Find the value of the positive constant a. [4]

(b) Hence calculate the coefficient of x in the expansion of $(a+x)^4 + \left(2 - \frac{x}{4}\right)^5$. [2]

[Turn over

7 **(a)** Given that $\log_2 x = p$ and $\log_2 y = q$, express $\log_x 4y$ in terms of p and q. [3]

(b) Express $2\log_2 x - \log_2 (x-5) = 2$ as a quadratic equation in x and explain why there are no real solutions. [5]

8 A returning resident is suspected to have contracted influenza from his recent travel. Suppose that all the residents who come into contact with him are susceptible to influenza, an epidemic is expected to break out in Hugetown following the expression,

$$N = \frac{15000}{1 + 9999e^{-0.2t}},$$

where N is the number of residents who contracted influenza, t days after first contact.

(a) Find the number of residents who are likely to have contracted influenza after 7 days. [1]

(b) Explain why the number of infected residents can never exceed 15 000. [1]

The town would be labelled as an influenza red-zone if at least 3000 residents were infected.

(c) Suppose the spread of influenza continues following the given expression, after how many days would the town be labelled as a red zone? [3]

[Turn over

9 Solve the simultaneous equations

$$8^y = \frac{1}{4^x}$$

$$^{y} \div \left(\sqrt{3}\right)^x = \sqrt{\ }$$

[5]

10 A circle, C, has equation $x^2 + y^2 - 27x = -41$.

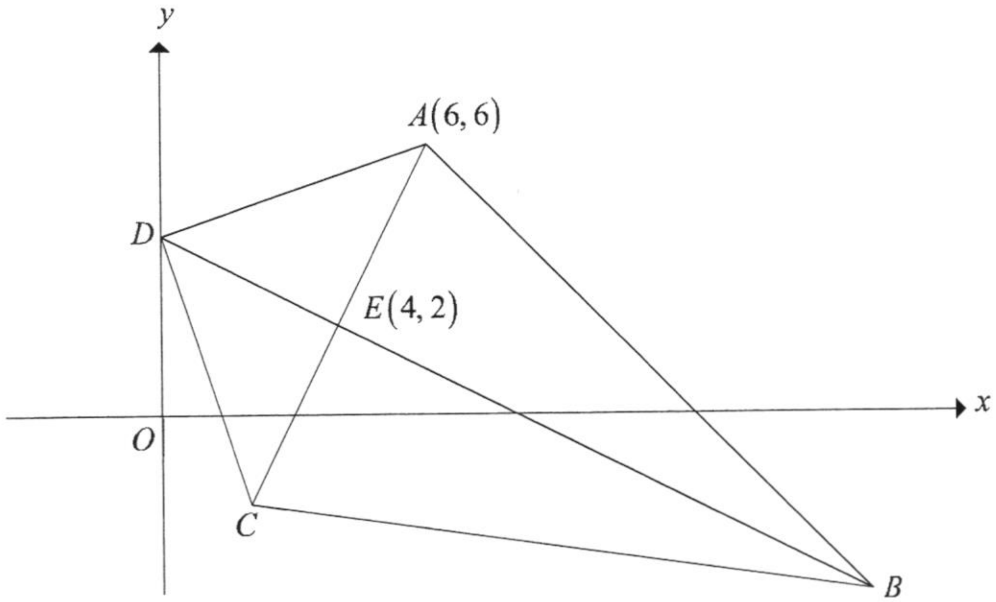
(a) Find the radius and the coordinates of the centre of C. [4]

A straight line that cuts the y-axis at -3 intersects the circle $x^2 + y^2 - 27x = -41$ at $P(2, 3)$.

(b) Find the coordinates of the point Q at which the line meets the circle again. [4]

[Turn over

11



The diagram shows a quadrilateral $ABCD$ in which BD is the perpendicular bisector of AC and cuts AC at the point E. The coordinates of A and E are (6, 6) and (4, 2) respectively.

(a) Find the coordinates of C. [2]

AB is parallel to the line passing through the point C and the origin. The point D lies on the y-axis.

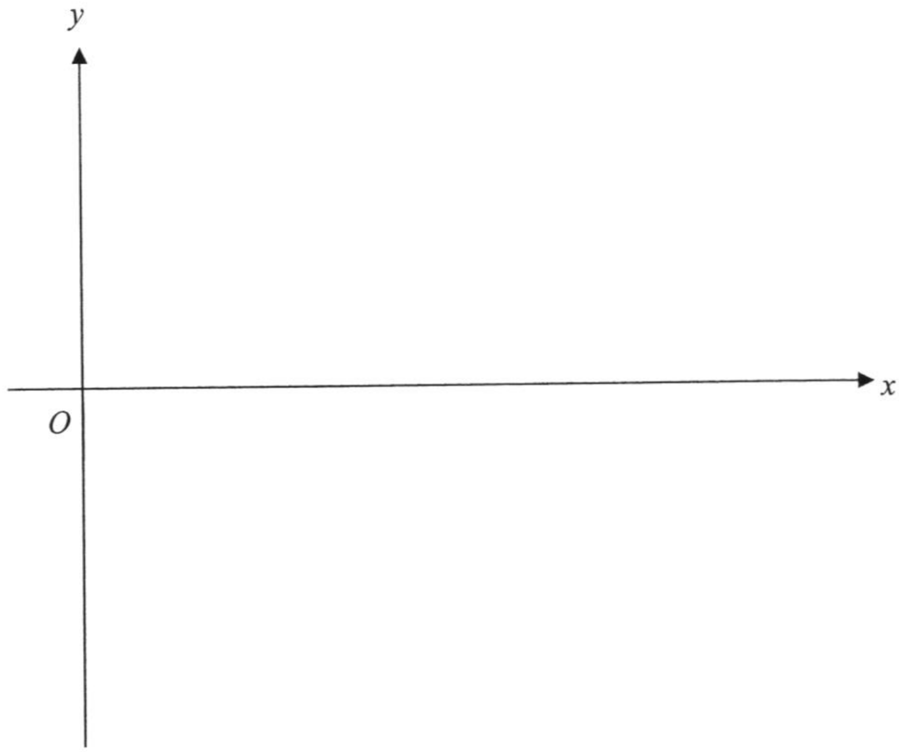
Find

(b) the coordinates of B and of D, [7]

(c) the area of $ABCD$. [2]

12 **(a)** State the period and amplitude of $y = 2 + 3\cos 2x$. [2]

(b) Sketch the graph of $y = 2 + 3\cos 2x$ for $0 \le x \le 2\pi$. [2]



13 The acute angles A and B are such that $\sin A = \frac{12}{13}$ and $\tan B = \frac{3}{4}$.

Without using a calculator, find the exact value of

(a) $\cos A$, [1]

(b) $\sin(A+B)$. [2]

[Turn over

14 **(a)** Show that $\frac{\sec x - \operatorname{cosec} x}{\sec x + \operatorname{cosec} x} = \frac{\tan x - 1}{\tan x + 1}$. [3]

(b) Hence find, for $0 \le x \le 2\pi$, the values of x in radians for which

$$\frac{\sec x - \operatorname{cosec} x}{\sec x + \operatorname{cosec} x} = \frac{4}{9}.$$ [3]

END OF PAPER

FSS 2021 3Express EOY

Additional Mathematics (90 marks)

Qn. #	Solution	Mark Allocation
1a	$2x^2-4x+7$ $=2\left(x^2-2x+\frac{7}{2}\right)$ $=2\left[x^2-2x+\left(-\frac{2}{2}\right)^2-\left(-\frac{2}{2}\right)^2+\frac{7}{2}\right]$ $=2\left[(x-1)^2+\frac{5}{2}\right]$ $=2(x-1)^2+5$ $-x^2-4x-1$ $=-(x^2+4x+1)$ $=-\left[x^2+4x+\left(\frac{4}{2}\right)^2-\left(\frac{4}{2}\right)^2+1\right]$ $=-\left[(x+2)^2-3\right]$ $=-(x-1)^2+3$	M1 (take out factor 2) A1 ($a(x+b)^2+c$ form) M1 (take out factor -1) A1 ($a(x+b)^2+c$ form)
1b	Minimum value of $2x^2-4x+7=5$ Maximum value of $-x^2-4x-1=3$ Since the minimum value of $y=2x^2-4x+7$ is higher than the maximum value of $y=-x^2-4x-1$, they will not intersect.	B1 (min value) B1 (max value) B1 (conclusion)

Qn. #	Solution	Mark Allocation
2a	$x - \sqrt{\ } = x\sqrt{3} +$ $x - x\sqrt{3} = \ + \sqrt{\ }$ $x\left(\ -\sqrt{3}\right) = \ + \sqrt{\ }$ $x = \frac{5+7\sqrt{3}}{4-\sqrt{3}}\left(\times\frac{4+\sqrt{3}}{4+\sqrt{3}}\right)$ $= \frac{20+5\sqrt{3}+28\sqrt{3}+21}{16-3}$ $= \frac{41+33\sqrt{3}}{13}$ $= \frac{41}{13}+\frac{33}{13}\sqrt{3}$	M1 (factorise; $\left(4-\sqrt{3}\right)$ seen) M1 (rationalise surds) A1 (in the form $a+b\sqrt{3}$)
2b	$2\sqrt{x-3}+x=11$ $\sqrt{x-3} = \quad -x$ $4(x-3)=(11-x)^2$ $4x-12=121-22x+x^2$ $x^2-26x+133=0$ $(x-7)(x-19)=0$ $x=7$ or $x=19$ (rejected)	M1 (square both sides) M1 (factorise) A1 (reject $x = 19$)
3a	$2x^2+bx+17-7x-9=0$ $2x^2+(b-7)x+8=0$ Since curve and line intersect at two distinct points, $b^2-4ac>0$ $(b-7)^2-4(2)(8)>0$ $b^2-14b+49-64>0$ $b^2-14b-15>0$ $(b-15)(b+1)>0$ $\therefore b<-1$ or $b>15$	M1 (substitution method) M1 (correct discriminant) M1 (factorise) A1

Qn. #	Solution	Mark Allocation
3b	$x^2+k(2x+1)+12=x^2+2kx+k+12$ Since curve is always positive, $b^2-4ac<0$ $(2k)^2-4(1)(k+12)<0$ $4k^2-4k-48<0$ $k^2-k-12<0$ $(k-4)(k+3)<0$ $\therefore -3<k<4$	M1 (correct discriminant) M1 (factorise) A1
4a	Let $f(x)=x^2+7x+3$ $f(p)=(p)^2+7(p)+3$ $f(p)=p^2+7p+3$ - (1) $f(-q)=(-q)^2+7(-q)+3$ $f(-q)=q^2-7q+3$ - (2) (1) = (2) $p^2+7p+3=q^2-7q+3$ $p^2-q^2+7p+7q=0$ $(p+q)(p-q)+7(p+q)=0$ $(p+q)(p-q+7)=0$ $p+q=0$ or $p-q+7=0$ (reject) $p-q=-7$	M1 (either equation seen) M1 (substitution method) M1 (factorise) A1 (no mark award if student did not reject $p+q=0$)

Qn. #	Solution	Mark Allocation
4b	$f(-2)=2(-2)^3+(-2)^2-10(-2)-8=0$ Therefore, $(x+2)$ is a factor of $f(x)=2x^3+x^2-10x-8$. (shown) $2x^3+x^2-10x-8=(x+2)(ax^2+bx+c)$ Comparing coefficient of x^3, $a=2$ Comparing constant, $2c=-8$ $c=-4$ Comparing coefficient of x, $c+2b=-10$ $-4+2b=-10$ $b=-3$ $2x^3+x^2-10x-8=(x+2)(2x^2-3x-4)$ $2x^3+x^2-10x-8=0$ $(x+2)(2x^2-3x-4)=0$ $x=-2$ or $x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-4)}}{2(2)}$ Therefore $x=-2$, $x=2.35$, or $x=-0.851$.	B1 M1 (compare coefficients / long division) A1 ($2x^2-3x-4$ seen) M1 (solve quadratic equation) A1

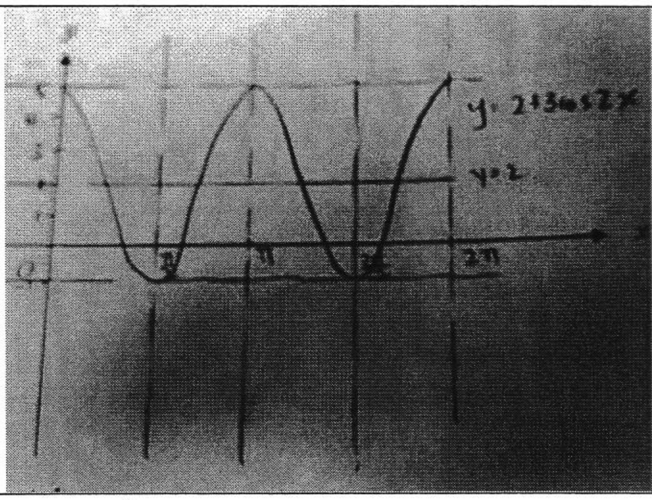
Qn. #	Solution	Mark Allocation
5	By long division, $\dfrac{x^3+2x^2-6}{x^2-9}=x+2+\dfrac{9x+12}{x^2-9}$	M1 (long division)
	$\dfrac{9x+12}{x^2-9}=\dfrac{9x+12}{(x-3)(x+3)}=\dfrac{A}{x-3}+\dfrac{B}{x+3}$	
	$9x+12=A(x+3)+B(x-3)$	M1 (factorise x^2-9 $=(x-3)(x+3)$)
	When $x=3$, $9(3)+12=A(3+3)$	
	$A=\dfrac{13}{2}$	M1 (A found, ecf)
	When $x=-3$, $9(-3)+12=B(-3-3)$	
	$B=\dfrac{5}{2}$	M1 (B found, ecf)
	$\dfrac{9x+24}{x^2-9}=\dfrac{13}{2(x-3)}+\dfrac{5}{2(x+3)}$	
	$\dfrac{x^3+2x^2-6}{x^2-9}=x+2+\dfrac{13}{2(x-3)}+\dfrac{5}{2(x+3)}$	A1
6a	$(a+x)^4+\left(2-\dfrac{x}{4}\right)^5$	M1 $a^4+\binom{4}{1}a^3x+\binom{4}{2}a^2x^2+...$ seen
	$=a^4+\binom{4}{1}a^3x+\binom{4}{2}a^2x^2+...+2^5+\binom{5}{1}2^4\left(-\dfrac{x}{4}\right)+\binom{5}{2}2^3\left(-\dfrac{x}{4}\right)^2+...$	M1 $2^5+\binom{5}{1}2^4\left(-\frac{x}{4}\right)+\binom{5}{2}2^3\left(-\frac{x}{4}\right)^2+...$ seen
	$=a^4+4a^3x+6a^2x^2+80-20x+5x^2+...$	
	Comparing coefficient of x^2,	
	$6a^2+5=29$	M1
	$6a^2=24$	
	$a^2=4$	
	$a=2$ (reject $a=-2$)	A1 (rejected $a=-2$)
6b	Coefficient of x $=4a^3-20$	M1
	$=4(2)^3-20$	
	$=12$	A1

Qn. #	Solution	Mark Allocation
7a	$\log_x 4y = \dfrac{\log_2 4y}{\log_2 x}$ $= \dfrac{\log_2 4 + \log_2 y}{\log_2 x}$ $= \dfrac{2+q}{p}$	M1 (change base to 2) M1 (product law) A1
7b	$2\log_2 x - \log_2(x-5) = 2$ $\log_2 \dfrac{x^2}{(x-5)} = 2$ $\dfrac{x^2}{(x-5)} = 2^2$ $x^2 = 4(x-5)$ $x^2 - 4x + 20 = 0$ Discriminant $= b^2 - 4ac$ $= (-4)^2 - 4(1)(20)$ $= -64 < 0$ Since discriminant < 0, the equation has no real solutions.	M1 (quotient law) M1 (change log to exponential) M1 (form quadratic equation) M1 (find value of discriminant / used quadratic formula) A1 (show that when discriminant < 0, there are no real solutions / concluded that no answer from quadratic formula)
8a	$N = \dfrac{15000}{1+9999e^{-0.2(7)}} = 6.080942101 = 6$ 6 residents are likely to have contracted influenza by Day 7.	B1
8b	As $t \to \infty$, $9999e^{-0.2t} \to 0$ $\dfrac{15000}{1+9999e^{-0.2t}} \to 15000$ The population of residents in Hugetown is only 15 000.	B1

Qn. #	Solution	Mark Allocation
8c	$\frac{15000}{1+9999e^{-0.2t}}=3000$ $1+9999e^{-0.2t}=5$ $e^{-0.2t}=\frac{4}{9999}$ $-0.2t=\ln\frac{4}{9999}$ $t=39.1197300293$ $\therefore$Day 40	M1 (3000 seen) M1 (take ln) A1
9	$2^{3y}=2^{-2x}$ $3y=-2x$ $y=-\frac{2}{3}x$ - (1) $3^{3y}\div 3^{\frac{1}{2}x}=3^4\left(3^{\frac{1}{2}}\right)$ $3^{3y-\frac{1}{2}x}=3^{4+\frac{1}{2}}$ $3y-\frac{1}{2}x=\frac{9}{2}$ - (2) Sub (1) into (2), $3\left(-\frac{2}{3}x\right)-\frac{1}{2}x=\frac{9}{2}$ $-\frac{5}{2}x=\frac{9}{2}$ $x=-\frac{9}{5}$ Sub $x=-\frac{9}{5}$ into (1), $y=-\frac{2}{3}\left(-\frac{9}{5}\right)$ $y=\frac{6}{5}$ Therefore, $x=-\frac{9}{5}$ and $y=\frac{6}{5}$.	M1 (law of indices) M1 (both equations) M1 (substitution method) A1 A1

Qn. #	Solution	Mark Allocation
10a	$x^2+y^2+2gx+2fy+c=0$, $r=\sqrt{g^2+f^2-c}$ Given $x^2+y^2-27x+41=0$, Comparing coefficient of x, $2g=-27$ $g=-\frac{27}{2}$ Comparing coefficient of y, $-2f=0$ $f=0$ Comparing constant, $c=41$ Centre $\left(\frac{27}{2},0\right)$ and $r=\sqrt{\left(-\frac{27}{2}\right)^2-41}$ $=11.9$ units^2 (3 s.f.)	M1 (either g, f or c found) A1 (centre) M1 (radius) A1
10b	Let the equation of the straight line be $y=mx+c$. Line passes through $(0,-3)$ and $P(2,3)$. $m=\frac{3+3}{2-0}$ $=3$ $y=3x-3$ $x^2+y^2-27x+41=0$ - (1) $y=3x-3$ - (2) Sub (2) into (1). $x^2+(3x-3)^2-27x+41=0$ $x^2+9x^2-18x+9-27x+41=0$ $10x^2-45x+50=0$ $2x^2-9x+10=0$ $(2x-5)(x-2)=0$ $x=\frac{5}{2}$ or $x=2$ (reject; x-coordinate of P) $y=\frac{9}{2}$ Therefore, $Q\left(\frac{5}{2},\frac{9}{2}\right)$	M1 (equation of line) M1 (substitution method) M1 (quadratic equation form) A1 (coordinates of Q found)

Qn. #	Solution	Mark Allocation
11a	Let the coordinates of $C(x,y)$. E is the midpoint of AC. $\left(\frac{x+6}{2},\frac{y+6}{2}\right) \quad =(4,2)$ $\frac{x+6}{2}=4$ or $\frac{y+6}{2}=2$ $x=2$ $\quad y=-2$ Therefore, $C(2,-2)$.	M1 (Midpoint) A1
11b	Let the equation of AB be $y=m_{AB}+c$. $m_{OC} =\frac{-2-0}{2-0}$ $=-1$ $m_{AB}=m_{OC}=-1$ Sub $A(6,6)$ and $m_{AB}=-1$, $y-6=-(x-6)$ $y=-x+12.$ Let the equation of BD be $y=m_{BD}+c$. $m_{AC} =\frac{6+2}{6-2}$ $=2$ $m_{BD}=-\frac{1}{m_{AC}}=-\frac{1}{2}$ E lies on line BD $y-2=-\frac{1}{2}(x-4)$ $y=-\frac{1}{2}x+4.$ B lies on AB and BD, $y=-x+12.$ - (1) $y=-\frac{1}{2}x+4.$ - (2) $(1)=(2)$,	M1 (parallel gradient) M1 (equation of AB) M1 (perpendicular gradient) M1 (equation of BD)

Qn. #	Solution	Mark Allocation
	$-x+12=-\frac{1}{2}x+4$ $-\frac{1}{2}x=-8$ $x=16$ $y=-16+12=-4$ $B(16,-4)$. D lies on y axis and BD, When $x=0$, $y=-\frac{1}{2}(0)+4=4$ $D(0,4)$.	M1 (substitution) A1 (B) A1 (D)
11c	Area of $ABCD$ $=\frac{1}{2}\begin{vmatrix} 0 & 2 & 16 & 6 & 0 \\ 4 & -2 & -4 & 6 & 4 \end{vmatrix}$ $=\frac{1}{2}\lvert 112-(-48)\rvert$ $=80$ units^2.	M1 (Simplification) A1
12a	Period $=\frac{2\pi}{2}$ $=\pi$ amplitude $=3$	B1 B1
12b		B1 correct shape and correct max/min value B1 correct number of cycles
13a	$\cos A=\frac{5}{13}$	B1
13b	$\sin(A+B)=\sin A\cos B+\cos A\sin B$ $=\frac{12}{13}\left(\frac{4}{5}\right)+\frac{5}{13}\left(\frac{3}{5}\right)$ $=\frac{63}{65}$	M1 A1

Qn. #	Solution	Mark Allocation
14a	$\text{LHS} = \dfrac{\sec x - \operatorname{cosec} x}{\sec x + \operatorname{cosec} x}$	
	$= \dfrac{\dfrac{1}{\cos x} - \dfrac{1}{\sin x}}{\dfrac{1}{\cos x} + \dfrac{1}{\sin x}}$	M1 (reciprocal for either sec x or cosec x)
	$= \dfrac{\dfrac{\sin x - \cos x}{\sin x \cos x}}{\dfrac{\sin x + \cos x}{\sin x \cos x}}$	M1 (common denominator)
	$= \dfrac{\dfrac{\sin x - \cos x}{\cos x}}{\dfrac{\sin x + \cos x}{\cos x}}$	
	$= \dfrac{\tan x - 1}{\tan x + 1}$	A1 (shown)
14b	$\dfrac{\tan x - 1}{\tan x + 1} = \dfrac{4}{9}$	
	$9\tan x - 9 = 4\tan x + 4$	
	$5\tan x = 13$	
	$\tan x = \dfrac{13}{5}$	M1
	$\alpha = 1.2036...$	M1 (basic angle)
	$x = 1.2036...\ ,\ \pi + 1.2036...$	A1 (both values)
	$x \approx 1.20\ ,\ 4.35$	