Candidate Name	Class	Register Number

Through School To Nation

CHANGKAT CHANGI SECONDARY SCHOOL

End of Year Examination 2018

Subject	:	**Additional Mathematics**
Paper	:	**4047/01**
Level	:	**Secondary 3 Express**
Date	:	**9 Oct 2018**
Duration	:	**2 hours**
Setters	:	**Ms Huang Yaling**

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiners' Use	Marks
Additional Mathematics	/ 80
Personal Target	Actual Grade
Parent's / Guardian's signature	

This document consists of **6** printed pages including the cover page

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}bc \sin A$$

1 The curve $y=\dfrac{2x+1}{x-1}$ intersects the line $y+x=7$ at the points P and Q.
Find the coordinates of P and of Q. [4]

2 The quadratic equation $-x^2-4x+5=0$ has roots $\dfrac{1}{\alpha}$ and β, where $\alpha>0$.

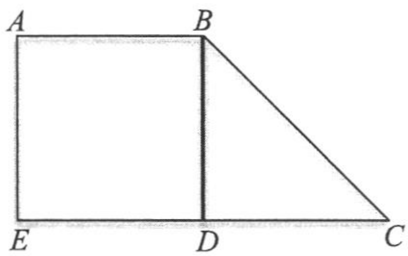
(i) Find the value of α and of β. [3]

(ii) Hence, or otherwise, form a quadratic equation whose roots are $\dfrac{\beta}{\alpha^2}$ and $\dfrac{\beta^2}{\alpha}$. [3]

3 (a) Find the values of p for which the line $y=2x-3$ is a tangent to the curve
$y=px^2+6x+p-6$. [3]

(b) A prism with volume $3(x^2-5)$ cm^3 has a base area of $(x-1)$ cm^2. Calculate the range of values of x for which the height of the prism is greater than 10 mm. [4]

4 The polygon consists of a square $ABDE$ of side length $(4-\sqrt{2})$ m and a triangle BCD with $CD=(\sqrt{2}+4)$ m.



Find

(i) the perimeter of the polygon, expressing your answer in the form of $a+b\sqrt{2}$, where a and b are integers, [2]

(ii) the area of the polygon. [2]

[Turn Over

5 **(a)** Show that the largest prime factor of $125(5^n)-5^n-100(5^{n-2})$ is 5 for all positive integer values of n. [2]

(b) Given that $\frac{8^x}{5^x}=\frac{5^{3-x}}{27^x}$, find the value of 6^x. [3]

6 The population of a new town is given by $P=250342e^{0.012t}$, where $t=0$ represents the population in the year 2000.

(i) Find the population of the new town in the year 2010. Round off the answer to the nearest whole number. [1]

(ii) Find the year in which the population will be 320,000. [2]

(iii) Find the minimum number of years required for the population of the new town to be at least doubled from the year 2000. [3]

7 Solve the following equations:

(a) $\log_4 64=2\log_3(2x)-\log_3(x-1)$, [4]

(b) $\frac{1}{\log_x e}-2=\ln(x-e)$. [3]

8 Variables x and y are connected by the equation $y=a^{x+b}$, where a and b are constants.

When a graph of $\lg y$ is plotted against x, a straight line passing through the points (3, 1) and (6, 4) is obtained.

Find

(i) the value of a and of b, [4]

(ii) the coordinates of the point on the line at which $\lg y=2x-4$. [3]

9 Express $\dfrac{x^3 - 5x^2 - 7x - 6}{x^2(x-3)}$ in partial fractions. [5]

10 A function f defined by $f(x) = 2x^3 + px^2 + qx + 15$, where p and q are constants, has a factor of $x - 5$ and leaves a remainder of 12 when divided by $x + 1$.

(i) Find the value of p and of q. [3]

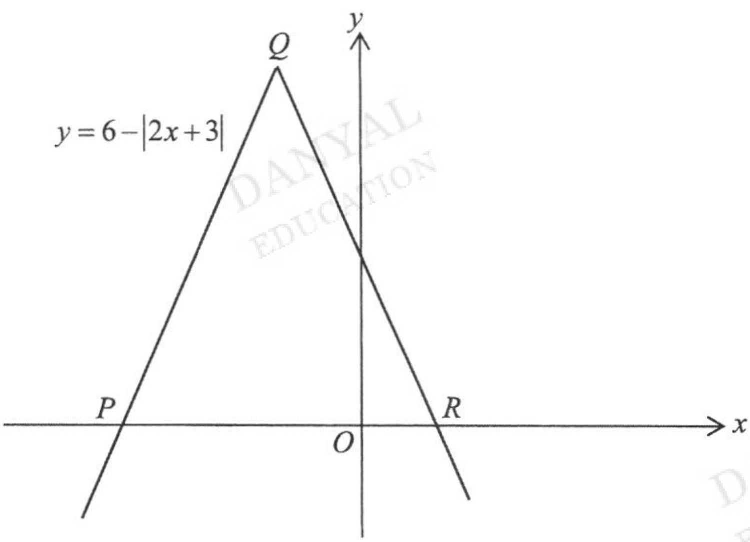
(ii) Find the remainder when $f(x)$ is divided by $2x - 3$. [1]

11 Given that $g(x) = 3x^3 - 4x^2 - 18x + 9$,

(i) show that $(x - 3)$ is a factor of $g(x)$, [1]

(ii) hence solve the equation $g(x) = 0$. [3]

12



The diagram shows part of the graph $y = 6 - |2x + 3|$.

(a) Find the coordinates of P, Q and R. [4]

(b) On separate diagrams, copy and use your graph to determine the number of solutions of the equation $6 - |2x + 3| = mx - 1$ when

(i) $m = 2$, [2]

(ii) $m = -\dfrac{1}{2}$. [2]

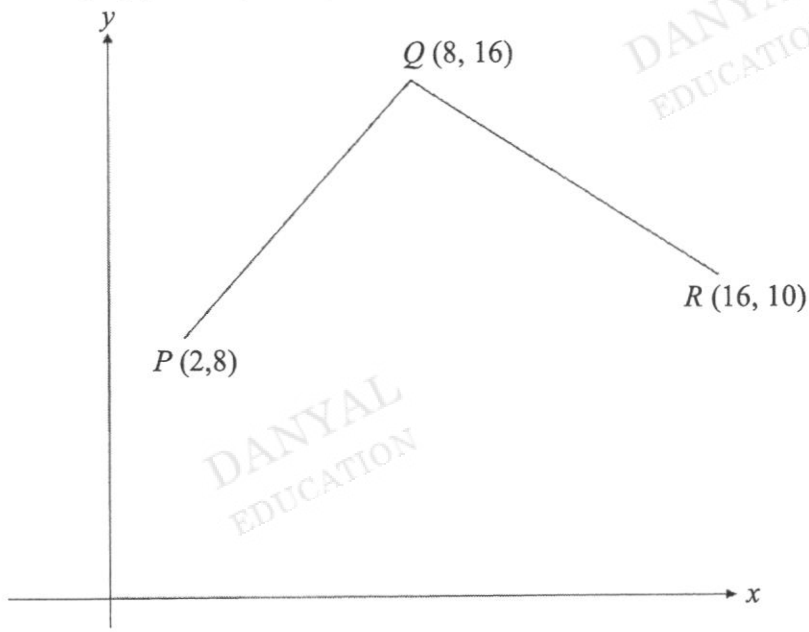
[Turn Over

13 **(i)** Find in descending powers of x, up to and including the x^3 term, the terms in the expansion of $\left(x-\dfrac{3}{x}\right)^7$. [1]

(ii) Find the term independent of x^4 in the expansion of $\left(4x^3-\dfrac{3}{x^2}\right)^5$. [3]

14 The diagram below shows part of a polygon.

The three vertices of the polygon are given by $P\,(2, 8)$, $Q\,(8, 16)$ and $R\,(16, 10)$.



(i) Show that $\angle PQR = 90°$. [2]

(ii) Find the equation of the perpendicular bisector of PQ. [2]

The perpendicular bisector of PQ intersects the line $3y = 4x - 9$ at point S.

(iii) Show that the coordinate of S *is* $(9,9)$. [2]

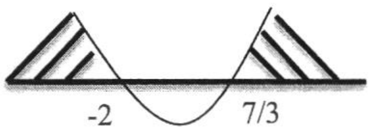
(iv) Determine if points P, R and S are collinear. [2]

(v) Find the area of PQS. [1]

End of paper

3E A Maths 2018 EOY Marking Guide

1	$\frac{2x+1}{x-1}+x=7$ $2x+1+x(x-1)=7(x-1)$ $2x+1+x^2-x=7x-7$ $x^2-6x+8=0$ $(x-4)(x-2)=0$ $x=4 \qquad x=2$ $y=3$ or $y=5$ $(4,3) \qquad (2,5)$	**M1** **M1** **A1, A1**	**[4]**
2i	$\frac{1}{\alpha}+\beta=-4$ $\frac{\beta}{\alpha}=-5$ $\beta=-5\alpha$ $\frac{1}{\alpha}-5\alpha=-4$ Sub $b=-5a$, $1-5\alpha^2=-4\alpha$ $5\alpha^2-4\alpha-1=0$ $(5\alpha+1)(\alpha-1)=0$ $\alpha=1$ or $\alpha=-\frac{1}{5}$ (rej, $\alpha>0$) $\beta=-5$	**M1** **B1 (must reject)** **B1**	**[3]**
2ii	New sum: $\frac{\beta}{\alpha^2}+\frac{\beta^2}{\alpha}=\frac{-5}{1^2}+\frac{(-5)^2}{1}=20$ New product: $\frac{\beta}{\alpha^2}\times\frac{\beta^2}{\alpha}=\frac{-5}{1^2}\times\frac{(-5)^2}{1}=-125$ New Equation: $x^2-20x-125=0$	**M1** **M1** **B1**	**[3]**
3a	$2x-3=px^2+6x+p-6$ $px^2+4x+p-3=0$ $4^2-4(p)(p-3)=0$ $4p^2-12p-16=0$ $p^2-3p-4=0$ $(p+1)(p-4)=0$ $p=-1$ or $p=4$	 **M1** **M1** **A1 (both ans)**	

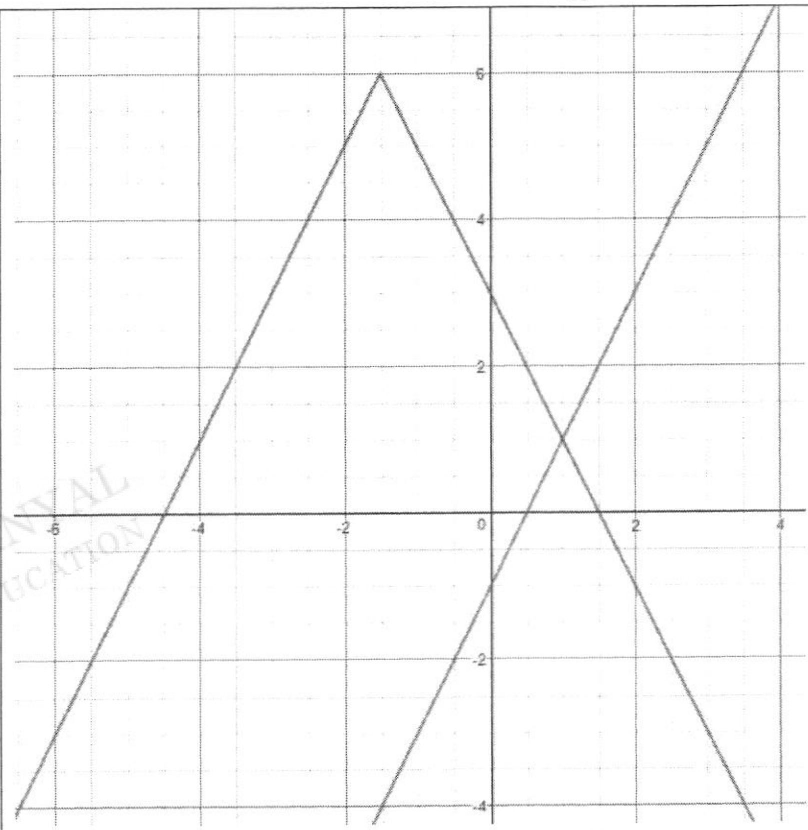
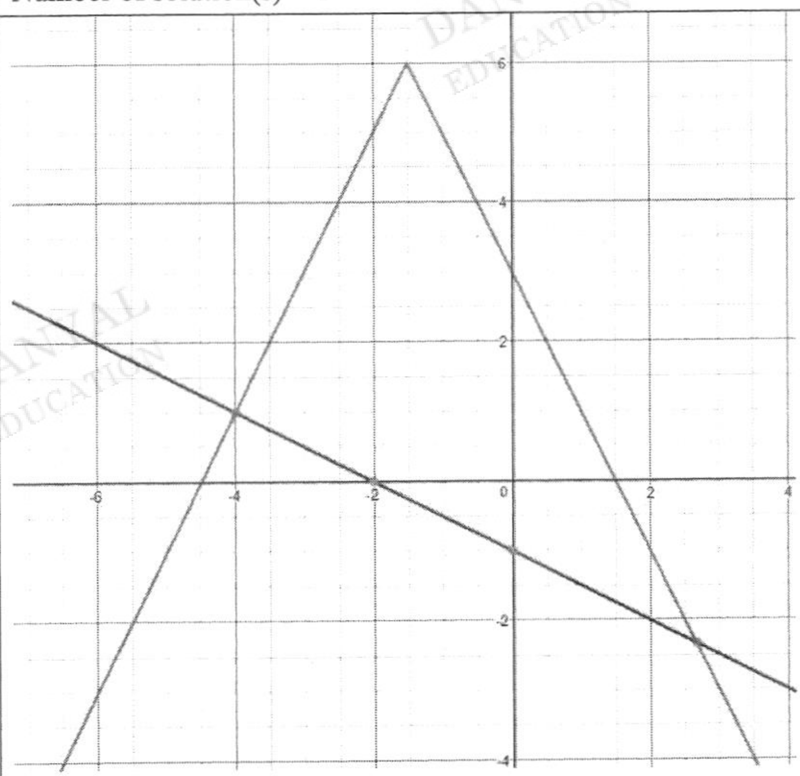
3b	$\frac{3(x^2-5)}{x-1}>1$ $3x^2-15>x-1$ $3x^2-x-14>0$ $(3x-7)(x+2)>0$  $x<-2$ (rej) or $x>\frac{7}{3}$	**M1** **M1** **B1, B1**	**[7]**
4a	BC= $\sqrt{(4-\sqrt{2})^2+(\sqrt{2}+4)^2}$ m $=\sqrt{16-8\sqrt{2}+2+2+8\sqrt{2}+16}$ $=\sqrt{36}$ $=6$ Perimeter = $3(4-\sqrt{2})+6+(\sqrt{2}+4)$ $=22-2\sqrt{2}$	**M1** **A1**	
b	Area = $(4-\sqrt{2})^2+\frac{1}{2}(\sqrt{2}+4)(4-\sqrt{2})$ $=25-8\sqrt{2}$	**M1** **A1**	**[4]**
5a	$=5^n(125-1-4)$ $=5^n(120)$ $=5^n(2^3\times3\times5)$ Largest Prime Factor: 5	 **B1** **A1**	
b	$\frac{8^x}{5^x}=\frac{5^{3-x}}{27^x}$ $8^x\times27^x=5^{3-x}\times5^x$ $216^x=5^3$ $6^{3x}=5^3$ $6^x=5$	 **M1** **B1** **A1**	**[5]**
6i	$P=250342e^{0.012t}$ $=282259.82$ $=282260$	 **B1**	

6ii	$320000 = 250342e^{0.012t}$		
	$In\left(\frac{320000}{250342}\right) = 0.012t$	**M1**	
	$t = \frac{In\left(\frac{320000}{250342}\right)}{0.012}$		
	$= 20.46$		
	Year 2020	**B1**	
6iii	282259.82×2		
	$= 564519.64$	**M1**	
	$564519.64 = 250342e^{0.012t}$		
	$In\left(\frac{564519.64}{250342}\right) = 0.012t$		
	$t = \frac{In\left(\frac{564519.64}{250342}\right)}{0.012}$	**M1**	
	$= 67.76$	**A1**	
	$= 68$		**[6]**
7a	$\log_4 64 = 2\log_3(2x) - \log_3(x-1)$		
	$\log_4 4^3 = \log_3(2x)^2 - \log_3(x-1)$	**M1**	
	$3 = \log_3\left(\frac{4x^2}{x-1}\right)$	**M1**	
	$27 = \frac{4x^2}{x-1}$		
	$27x - 27 = 4x^2$		
	$4x^2 - 27x + 27 = 0$		
	$x = \frac{-(-27) \pm \sqrt{(-27)^2 - 4(4)(27)}}{2(4)}$	**M1**	
	$x = 1.22$ or $x = 5.53$	**A1 (both ans)**	
b	$\frac{1}{\log_x e} - 2 = \ln(x-e)$		
	$\log_e x = \ln(x-e) + 2$	**M1**	
	$\ln x = \ln(x-e) + \ln e^2$		
	$x = e^2(x-e)$		
	$x = e^2 x - e^3$		
	$x(e^2 - 1) = e^3$	**M1**	**[7]**

	$x=\frac{e^3}{e^2-1}$ $x=3.14$	A1	
8i	$\lg y=(x+b)\lg a$ $\lg y=x\lg a+b\lg a$ $\lg y=(x+b)\lg a$ $\lg a=\frac{4-1}{6-3}$ $\lg a=1$ $a=10$ Sub a = 0 and (3, 1) $1=3+b$ $b=-2$	M1 A1 M1 A1	
ii	$2x-4=x-2$ $x=2$ Sub x = 2, $\lg y=2(2)-4$ $y=10^0$ $=1$ $(2,0)$	M1 M1 A1	[7]
9	$\frac{x^3-5x^2-7x-6}{x^2(x-3)}=1-\frac{2x^2+7x+6}{x^2(x-3)}$ $\frac{2x^2+7x+6}{x^2(x-3)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-3}$ $2x^2+7x+6=Ax(x-3)+B(x-3)+Cx^2$ Sub x = 0 $6=-3B(x-3)+Cx^2$ $B=-2$ Sub x = 3 $2(3)^2+7(3)+6=9C$ $45=9C$ $C=5$ Sub x = 1, B = -2, C = 5	B1 B1 B1	[5]

	$2+7+6=A(-2)+-2(-2)+5$ $15=-2A+9$ $6=-2A$ $A=-3$ $\frac{x^3-5x^2-7x-6}{x^2(x-3)}=1-\left(\frac{-3}{x}+\frac{-2}{x^2}+\frac{5}{x-3}\right)$ $=1+\frac{3}{x}+\frac{2}{x^2}-\frac{5}{x-3}$	B1 B1	
10i	By factor theorem, $f(5)=0$ $2(5)^3+p(5)^2+q(5)+15=0$ $25p+5q=-265$ $5p+q=-53$ $q=-53-5p$ ———— (1) By remainder theorem, $f(-1)=12$ $2(-1)^3+p(-1)^2+q(-1)+15=12$ $p=-1+q$ ———— (2) Sub (1) into (2): $q=-53-5(-1+q)$ $q=-53+5-5q$ $6q=-48$ $q=-8$ $\therefore p=-9$	M1 M1 A1 (both ans)	
ii	By remainder theorem, $f(\frac{3}{2})=2(\frac{3}{2})^3-9(\frac{3}{2})^2-8(\frac{3}{2})+15$ $=-10.5$	A1	4
11i	$g(3)=3(3)^3-4(3)^2-18(3)+9$ $=81-36-48+9$ $=0$ since $g(3)=0$, $x-3$ is a factor by factor theorem.	B1	

11ii	$3x^2+5x-3$ $x-3\overline{)3x^3-4x^2-18x+9}$ $-\quad 3x^3-9x^2$ $5x^2-18x$ $-\quad 5x^2-15x$ $-3x+9$ $-\quad -3x+9$ $\therefore g(x)=(x-3)(3x^2+5x-3)$ Given $g(x)=0$ $(x-3)(3x^2+5x-3)=0$ $x=3$ or $x=\dfrac{-5\pm\sqrt{5^2-4(3)(-3)}}{2(3)}$ $x=0.468$ or $x=-2.14$	**M1** **M1** **A1 (all 3 ans)**	**[4]**
12a	At P and R, y = 0 $0=6-\|2x+3\|$ $\|2x+3\|=6$ $2x+3=6$ or $2x+3=-6$ $x=1.5$ or $x=-4.5$ $P(-4.5,0)$, $R(1.5,0)$ At Q, $2x+3=0$ $x=-1.5$ $y=6$ $Q(-1.5,6)$	**M1** **A1, A1** **A1**	

bi	 Since there is only 1 point of intersection, Number of solution(s) = 1	M1 A1	
bii	 Since there are 2 points of intersection, Number of solution(s) = 2	M1 A1	[8]

13i	$\left(x-\frac{3}{x}\right)^7 = x^7 + \binom{7}{1}x^6\left(-\frac{3}{x}\right)^1 + \binom{7}{2}x^5\left(-\frac{3}{x}\right)^2 + \ldots$ $= x^7 - 21x^5 + 189x^3 + \ldots$	B1	
13ii	$T_{r+1} = \binom{5}{r}\left(4x^3\right)^{5-r}\left(-\frac{3}{x^2}\right)^r$ $= \binom{5}{r}(4)^{5-r}x^{15-3r}(-3)^r\left(\frac{1}{x^{2r}}\right)$ $= \binom{5}{r}(4)^{5-r}(-3)^r x^{15-5r}$ Comparing powers: $15-5r=0 \rightarrow r=3$ $T_4 = \binom{5}{3}\left(4x^3\right)^2\left(-\frac{3}{x^2}\right)^3$ $=-4320$	M1 M1 A1	
14i	$m_{PQ} \times m_{QR} = \frac{16-8}{8-2} \times \frac{16-10}{8-16}$ $= \frac{8}{6} \times \frac{6}{-8}$ $=-1$	M1 A1	
14ii	$M_{PQ} = (5,12)$ $12 = -\frac{6}{8}(5) + c$ $c = 15\frac{3}{4}$ $y = -\frac{3}{4}x + 15\frac{3}{4}$	M1 A1	
14iii	$-\frac{3}{4}x + 15\frac{3}{4} = \frac{4}{3}x - 3$ $18\frac{3}{4}x = 2\frac{1}{12}$ $x = 9$ $y = 9 \rightarrow S(9,9)$	M1 B1	
14iv	Since they have the same gradient , $m_{PR} = \frac{1}{7}$ $m_{RS} = \frac{1}{7} = m_{PR}$ And they share a common point R, they must be collinear.	M1 B1	

14(v)	Area of triangle $PQR = \frac{1}{2} \times \begin{vmatrix} 2 & 9 & 8 & 2 \\ 8 & 9 & 16 & 8 \end{vmatrix}$ $= 25 units^2$	**B1**	