3E

2022 End of Year Examination

Secondary Three Express

ADDITIONAL MATHEMATICS
4049

10 Oct 2022
2 hour 15 minutes
1000h – 1215h

Name: ______________________________ (   ) **Class:** ________

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR MARKER'S USE		
	Marks Awarded	Max Marks
Total		90

This question paper consists of 17 printed pages including the cover page.

Setter: Mr Ng Chuen Joo

Answer **all** the questions.

1 The function *f* is such that $f(x) = A + 2\cos Bx$, where *A* and *B* are integers.
The minimum value of *f(x)* is -3 and the period of *f* is $180°$.

(i) Find the value of *A* and *B*. [2]

(ii) Hence sketch the graph of $y = f(x)$ for $0° \leq x \leq 180°$. [3]

[Turn Over

2 Express $\dfrac{3x-29}{(2x-3)(x-5)^2}$ as the sum of 3 partial fractions. [5]

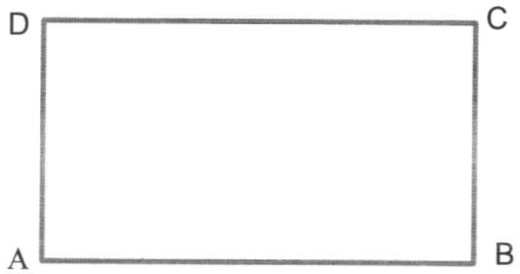
3 **(i)** Express $y=-2x^2+4x+4$ in the form $-2(x-h)^2+k$, where h and k are integers. [2]

(ii) Hence find the coordinates of the turning point and the y-intercept of the graph of $y=-2x^2+4x+4$. [2]

(iii) Find the exact values of x for which $y=0$. [2]

[Turn Over

4



$ABCD$ is a rectangle with length of $3\sqrt{2}-2\sqrt{3}$ and area $3\sqrt{2}+2\sqrt{3}\ cm^2$.

(i) Find the breadth of the rectangle, giving your answer in the form $\left(a+b\sqrt{c}\right)cm$, where a, b and c are integers. [4]

(ii) Find the value of AC^2 in the form $\left(k+h\sqrt{6}\right)cm^2$, where k and h are integers. [3]

5 **(i)** The polynomial $f(x) = x^3 + ax^2 + bx + 9$, where a and b are constants, is exactly divisible by $x + 1$ and leaves a remainder of 24 when divided by $x - 3$. Find the value of a and of b. [4]

(ii) Hence solve the equation $f(x) = 0$. [3]

[Turn Over

6 **(a)** Find the range of values of k for which the line $y = 2x + k$ intersects the curve $xy + 2 = 0$ at two distinct points. [4]

(b) Show that the line $y = 2x + 3$ does not intersect the curve $y = 4x^2 + 7$. [4]

7 **(a)** Solve the equation $2\cos x = \sec x$ for $0° \leq x \leq 360°$. [4]

(b) Prove the identity $\frac{\tan^2 x}{\sec x - 1} = \frac{1 + \cos x}{\cos x}$. [4]

[Turn Over

8 **(a)** Solve the equation $\log_3(x-1)-\log_3(x+1)=1+\log_3\frac{1}{7}$. [4]

(b) Solve the equation $\log_x 100=\ln x$. [4]

9 A circle with centre C passes through the points $A(-1, 7)$ and $B(0, 8)$.

(i) Find the equation of perpendicular bisector of AB. [3]

(ii) Given further that the line $y = 2x - 2$ passes through the centre of the circle, show that the coordinates of C is $(3, 4)$. [3]

(iii) Hence find the equation of the circle. [2]

[Turn Over

10 **(a)** **(i)** Find the first 3 terms in the expansion of $(2x-1)^6$. [2]

(ii) Hence find the coefficient of x^5 in the expansion of $(2x-1)^6(3x+2)$. [3]

(b) Given that the constant term in the binomial expansion of $\left(\frac{x}{2}-\frac{k}{x^2}\right)^9$ is -84,

find the value of the integer k. [4]

[Turn Over

11 Experimental values of two variables x and y are given in the table below.

x	1	2	3	4	5
y	0.5	2.50	3.75	4.75	5.60

It is known that x and y are related by the equation $y - a\sqrt{x} = \frac{b}{\sqrt{x}}$,where a and b are constants.

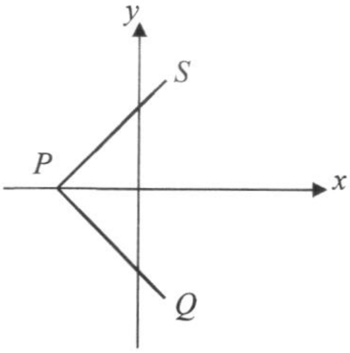
(a) Plot $y\sqrt{x}$ against x for the data and draw a straight line graph on the graph paper. [4]

(b) Use your graph to estimate
(i) the value of a and of b, [3]

(ii) the value of y when $x = 6$. [2]

[Turn Over

12 Solutions to this question by accurate drawing will not be accepted.



PQRS is a square. Three of the vertices *P*, *Q* and *S* are shown in the diagram.
The coordinates of *P*, *Q* and *S* are $(-2, 0)$, $(1, -3)$ and $(1, 3)$ respectively.

(i) Find the equation of *PQ*. [3]

(ii) Find the midpoint of *SQ*, and hence find the coordinates of the vertex *R*. [3]

Another point T has coordinates (7, 3).

(iii) Explain why QT is parallel to PS. [2]

(iv) Find the area of $PQTS$. [2]

- End of paper -

Canberra Secondary School
3E Additional Mathematics End of Year Examination 2022
Marking Scheme

Question	Marking Scheme	Marks
1(a)	A = -1 B = 2	B1 B1
(b)	*[Graph]* B1 (1 complete cycle) B1 (correct minimum and maximum value indicated on graph) B1 (correct axes and shape)	B3
2	Let $\frac{3x-29}{(2x-3)(x-5)^2}=\frac{A}{2x-3}+\frac{B}{x-5}+\frac{C}{(x-5)^2}$	
	$3x-29=A(x-5)^2+B(2x-3)(x-5)+C(2x-3)$	M1
	Let $x=\frac{3}{2}$,	
	$-24\frac{1}{2}=\frac{49}{4}A$	
	$\therefore A=-2$	M1
	Let $x=5$,	
	$-14=7C$	
	$\therefore C=-2$	M1
	Comparing coeff of x^2,	M1
	A + 2B = 0	
	-2 + 2B = 0	
	$\therefore B=1$	
	$\therefore \frac{3x-29}{(2x-3)(x-5)^2}=-\frac{2}{2x-3}+\frac{1}{x-5}-\frac{2}{(x-5)^2}$	A1
3(a)	$y=-2x^2+4x+4$	
	$=-2\left(x^2-2x-2\right)$	
	$=-2\left[x^2-2x+1^2-1^2-2\right]$	M1
	$=-2\left[\left(x-1\right)^2-3\right]$	
	$=-2\left(x-1\right)^2+6$	A1
3(b)	Coord of turning point = (1, 6) When $x=0$, $y=4$	B1 B1

3(c)	When $y = 0$, $-2(x-1)^2 + 6 = 0$ $-2(x-1)^2 = -6$ $(x-1)^2 = 3$ $x - 1 = \pm\sqrt{3}$ $\therefore x = 1+\sqrt{3}$ *or* $x = 1-\sqrt{3}$	M1 A1
4(a)	Breadth $= \dfrac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \dfrac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$ $= \dfrac{9(2)+12\sqrt{6}+4(3)}{(3\sqrt{2})^2-(2\sqrt{3})^2}$ $= \dfrac{30+12\sqrt{6}}{9(2)-4(3)}$ $= 5+2\sqrt{6}$	M1 M1, M1 A1
4(b)	$AC^2 = (3\sqrt{2}-2\sqrt{3})^2 + (5+2\sqrt{6})^2$ $= 9(2) - 12\sqrt{6} + 4(3) + 25 + 20\sqrt{6} + 4(6)$ $= 79 + 8\sqrt{6}$	M1 M1 A1
5(a)	$f(x) = x^3 + ax^2 + bx + 9$ By Factor Theorem, $f(-1) = (-1)^3 + a(-1)^2 + b(-1) + 9 = 0$ $\therefore a - b = -8 \ ------(1)$ By Remainder Theorem, $f(3) = 24$ $3^3 + a(3)^2 + b(3) + 9 = 24$ $9a + 3b = -12$ $3a + b = -4 ------(2)$ $(1)+(2): 4a = -12$ $\therefore a = -3$ $b = 5$	M1 M1 A1 A1

5(b)	Show working of long division (Alternative: compare coefficients) $f(x) = 0$ $(x+1)(x^2-4x+9)=0$ $x=-1 \text{ or } x^2-4x+9=0$ For $x^2-4x+9=0$, $b^2-4ac=(-4)^2-4(1)(9)<0$ (no real roots) $\therefore x=-1$	M1 M1 A1 (Require to state no real roots)
6(a)	$y = 2x + k$ ------(1) $xy + 2 = 0$ ------(2) Subst (1) into (2) $x(2x + k) + 2 = 0$ $2x^2 + kx + 2 = 0$ Roots are real and distinct $b^2 - 4ac > 0$ $k^2 - 4(2)(2) > 0$ $k^2 - 16 > 0$ (Need to show the factorization) $(k-4)(k+4) > 0$ $\therefore k < -4 \text{ or } k > 4$	M1 M1 M1 A1
6(b)	$y = 2x + 3$ ---------(1) $y = 4x^2 + 7$---------(2) Subst (2) into (1) $4x^2 + 7 = 2x + 3$ $4x^2 - 2x + 4 = 0$ $2x^2 - x + 2 = 0$ $b^2 - 4ac = (-1)^2 - 4(2)(2)$ $= -15 < 0$ There are no real roots. $\therefore$ the line $y = 2x + 3$ does not intersect the curve $y = 4x^2 + 7$.	M1 M1 M1(need to show <0) A1

7(a)	$2\cos x = \sec x, \quad 0^\circ \le x \le 360^\circ$	
	$2\cos x = \dfrac{1}{\cos x}$	M1
	$2\cos^2 x = 1$	
	$\cos^2 x = \dfrac{1}{2}$	
	$\cos x = \pm\sqrt{\dfrac{1}{2}}$	M1
	Basic acute angle = 45°	M1
	$x = 45^\circ, \ 135^\circ, \ 225^\circ, \ 315^\circ$	A1
7(b)	$LHS = \dfrac{\tan^2 x}{\sec x - 1}$	
	$= \dfrac{\sec^2 x - 1}{\sec x - 1}$	M1
	$= \dfrac{(\sec x - 1)(\sec x + 1)}{\sec x - 1}$	M1
	$= \sec x + 1$	
	$= \dfrac{1}{\cos x} + 1$	M1
	$= \dfrac{1 + \cos x}{\cos x}$	A1
	$= RHS$	
8(a)	$\log_3(x-1) - \log_3(x+1) = 1 + \log_3 \dfrac{1}{7}$	
	$\log_3 \dfrac{x-1}{x+1} = \log_3 3 + \log_3 \dfrac{1}{7}$	M1
	$\log_3 \dfrac{x-1}{x+1} = \log_3 \dfrac{3}{7}$	M1
	$\dfrac{x-1}{x+1} = \dfrac{3}{7}$	M1
	$7x - 7 = 3x + 3$	
	$x = 2.5$	A1

8(b)	$\log_x 100 = \ln x$ $\frac{\lg 100}{\lg x} = \frac{\lg x}{\lg e}$	M1
	$(\lg x)^2 = (\lg e) \times 2$ $(\lg x)^2 = 0.8685889638$	
	$\lg x = \pm\sqrt{0.8685889638}$ (reject negative value)	M1
	$x = 10^{0.931981203}$	M1
	$x = 8.55$	A1
9(a)	Mid point of $AB = \left(\frac{-1+0}{2}, \frac{7+8}{2}\right) = \left(-\frac{1}{2}, \frac{15}{2}\right)$	M1
	Gradient of $AB = \frac{8-7}{0-(-1)} = 1$ Gradient of perpendicular bisector of AB $= -1$	M1
	When $x = -\frac{1}{2}$ and $y = \frac{15}{2}$, $\frac{15}{2} = -\left(-\frac{1}{2}\right) + c \Rightarrow c = 7$ $\therefore y = -x + 7$	A1
	Alternative: Use $y - \frac{15}{2} = -1\left(x + \frac{1}{2}\right)$ $y = -x + 7$	
9(b)	$y = -x + 7 \quad ---(1)$ $y = 2x - 2 \quad ---(2)$	
	Sub (1) into (2), $-x + 7 = 2x - 2$	M1
	$3x = 9$ $x = 3$	A1
	Sub $x = 3$ into (1), $y = -3 + 7 = 4$ Coord of C = (3, 4) (Shown)	A1
9(c)	Radius $= \sqrt{(0-3)^2 + (8-4)^2} = 5$ units	M1
	Equation of circle:	A1

<table>
<tr><td></td><td>$(x-3)^2+(y-4)^2=5^2$
$(x-3)^2+(y-4)^2=25$ or $x^2-6x+y^2-8y=0$</td><td></td></tr>
<tr><td>10(a)(i)</td><td>$(2x-1)^6=(2x)^6+\begin{pmatrix}6\\1\end{pmatrix}(2x)^5(-1)^1+\begin{pmatrix}6\\2\end{pmatrix}(2x)^4(-1)^2+\dots$
$=64x^6-192x^5+240x^4+\dots$</td><td>M1
A1</td></tr>
<tr><td>10(a)(ii)</td><td>$(2x-1)^5(3x+2)=(64x^6-192x^5+240x^4+\dots)(3x+2)$
For coefficient of x^5,
$-192x^5(2)+240x^4(3x)=336x^5$
Therefore, coefficient is 336.</td><td>M1
M1
A1</td></tr>
<tr><td>10(b)</td><td>$\begin{pmatrix}9\\r\end{pmatrix}\left(\frac{x}{2}\right)^{9-r}\left(-\frac{k}{x^2}\right)^r=-84x^0$
$(x)^{9-r}(x^{-2})^r=x^0$
$(x)^{9-r-2r}=x^0$
$9-3r=0$
$r=3$
$\begin{pmatrix}9\\r\end{pmatrix}\left(\frac{1}{2}\right)^{9-r}(-1)^r(k)^r=-84$
$\begin{pmatrix}9\\3\end{pmatrix}\left(\frac{1}{2}\right)^{9-3}(-1)^3(k)^3=-84$
$\begin{pmatrix}9\\3\end{pmatrix}\left(\frac{1}{2}\right)^{6}(-1)^3(k)^3=-84$
$-84\left(\frac{k^3}{2^6}\right)=-84$
$k^3=64$
$k=4$</td><td>M1
M1
M1
A1</td></tr>
<tr><td>11(a)</td><td><table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>0.5</td><td>2.50</td><td>3.75</td><td>4.75</td><td>5.60</td></tr><tr><td>$y\sqrt{x}$</td><td>0.50</td><td>3.54</td><td>6.50</td><td>9.50</td><td>12.52</td></tr></table>$y=a\sqrt{x}+\frac{b}{\sqrt{x}}$
$y\sqrt{x}=ax+b$</td><td>M1
M1</td></tr>
</table>

	Graph	A2
11(b)(i)	From the graph, $b = -2.2$ $a = \dfrac{8-(-2.2)}{3.5-0}$ $= 2.91$	B1 M1 A1
11(b)(ii)	When $x = 6$, $y\sqrt{6} = 15.4$ $y = \dfrac{15.4}{\sqrt{6}}$ $= 6.29$	M1 A1
12(a)	Gradient of PQ = $\dfrac{0-(-3)}{-2-1} = -1$ Eqn of PQ is $y - 0 = -\left[x - (-2)\right]$ i.e. $y = -x - 2$	M1 M1 A1
12(b)	Midpt of SQ = $\left(\dfrac{1+1}{2}, \dfrac{3+(-3)}{2}\right)$ $= (1, 0)$ Let the coord of R be (x, y). $\left(\dfrac{-2+x}{2}, \dfrac{0+y}{2}\right) = (1,\ 0)$ x = 4, y = 0 Coord of R = (4, 0)	A1 M1 A1
12(c)	Gradient of PS = 1 Gradient of QT = $\dfrac{3-(-3)}{7-1} = 1$ Since gradient of PS = gradient of QT, QT is parallel to PS. (Shown)	M1 A1
12(d)	Area of PQTS = $\dfrac{1}{2}\begin{vmatrix} -2 & 1 & 7 & 1 & -2 \\ 0 & -3 & 3 & 3 & 0 \end{vmatrix} = 27\ units^2$	M1, A1