2022 End of Year Examination

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Secondary Three Express

MATHEMATICS

4052/01

Name: ____

06 Oct 2022 1 hour 45 minutes 1130h - 1315h

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in. Write in dark blue or black pen. You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question. The total number of marks for this paper is 70. EDUCATIO

1.011		00-
	Marks	Max
	Awarded	Marks
Total		70

FOR MARKER'S USE

This question paper consists of 14 printed pages including the cover page.

Setter: Mr Tan Qi Guang

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5	\mathbb{D}		J
0	21	L	٦

Class:)

1 Solve the following simultaneous equations.

$$2x - y = 2$$
$$x + 3y = 15$$



(b) $6pq \times 4p^3q$



Answer[1]

DANYAL

[Turn Over



(a) $y = x^3$,

Answer Figure [1]



(c) $y = \frac{1}{x}$?

Answer Figure [1]

4 (a) Solve the inequality $5-3x \le 1-2x < 3x+7$.

(b) Hence, write down the smallest prime number that satisfies $5-3x \le 1-2x < 3x+7$. $5-3x \le 1-2x < 3x+7$.

5 In 2020, there were 5.626 million people living in Country A.
9% of the population was elderly.
The number of youths was twice the number of elderly.
By expressing your answers in standard form, correct to 2 significant figures, calculate

(a) the number of elderly,

DANYAL

(b) the number of youths.

PartnerInLearning 112 [Turn Over



(a)
$$3^4 \times 3^q = \frac{1}{27}$$

(b) $8^r \pm 4 = 1$ (b) $8^r \pm 4 = 1$ Answer $r = \dots [2]$ 7 Given that $-5 \le x \le 4$ and $-3 \le y \le 2$, find (a) the smallest possible value of xy, Answer $\dots [1]$ (b) the greatest possible value of $x^2 - y^2$.

8 The figure below shows a quadratic curve.



Answer (.....) [1]

[Turn Over

9 Two bottles are geometrically similar.

The ratio of the surface areas of the bottles is 9:16.

(a) Write down the ratio of the length of the smaller bottle to the length of the larger bottle.

(b) The volume of the larger bottle is 1440 cm³. Calculate the volume of the smaller bottle.

Answer cm³ [2]

10 (a) Express $x^2 + 8x - 20$ in the form $(x+h)^2 - k$.

(b) Hence, solve the equation $x^2 + 8x - 20 = 0$.

11 In the diagram, DF is parallel to BC and AC is parallel to DE. Given that DB = 5 cm, AD = 10 cm, EC = 12 cm and AF = 18 cm.

A 18 10 D 5 E В 12 C (a) Name a triangle that is similar to triangle ADF. Prove that the two triangles are similar Answer INVATO [3] Given that triangle CEF and triangle DFE are congruent, (b) DANYAL DANYAL EDUCATION calculate the length of BE. (c) Find the Answer cm [3] Find the ratio of _____area of triangle ADF area of parallelogram *DBCF*, express your answer as a fraction in its simplest form.

12 The diagram below shows a figure, where PQ = 12 cm, QR = 9 cm, PR = 15 cm and RS = 7 cm.



(a) Explain why angle $PQR = 90^{\circ}$.

Answer

[2]

(b) Expressing your answer as a fraction in the simplest form, find (i) $\tan \angle QPS$,

(ii) $\cos \angle PRS$,

Answer[1]

(iii) $\sin \angle PSQ$.

10 workers take 90 days to complete building a bus stop.
 The manager claims that by hiring 5 more workers, the completion date can be brought forward by 30 days.
 Explain with calculations whether the manager is correct in his claim.
 State an assumption made.



14 The diagram shows a triangle with AB = 4 cm, BC = 6 cm, $\angle BAC = 79^{\circ}$ and $\angle ACB = 41^{\circ}$.



(a) Calculate the area of triangle ABC.

(b) Hence, or otherwise, find the shortest distance from C to AB.

Answer cm [2]

[Turn Over

15 The diagram below shows a square *ABCD* and a regular hexagon *CEFGHD*.



Calculate angle BEC.



Answer[3]

Zen invests \$3000 in GRC finance company that pays 2.8% compound interest per annum compounded half-yearly.
 Find the total amount at the end of 6 years, giving your answer correct to the nearest dollar.

17 In the diagram, *ABCDEF* is a triangular prism, and *M* is the midpoint of *BC*. It is also given that AB = 10 cm, EF = 30 cm, and $\angle ABF = 33^{\circ}$.



Answer $\angle FMA = \dots [2]$

[Turn Over



18 The diagrams below show the distance-time and speed-time graphs of a van travelling

End of Paper



2022 End of Year Examination

Secondary Three Express

_____(

MATHEMATICS

4052/02

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) Class:

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1 Here are the first four terms of a sequence.

$$T_{1} = 1^{2} + 1 = 2$$

$$T_{2} = 3^{2} + 2 = 11$$

$$T_{3} = 5^{2} + 3 = 28$$

$$T_{4} = 7^{2} + 4 = 53$$

(a) Write down the fifth term of the sequence.

Answer $T_5 = \dots$ [1] DANYALEDUCATION

(b) T_n is the *n*th term of the sequence.

Show that $T_n = 4n^2 - 3n + 1$.

Answer

DANYAL WO CONCLINION

(c) The difference, D, between two consecutive terms of the sequence is $T_{n+1} - T_n$. Find an expression, in terms of n, for D.

Answer $D = \dots$ [3]

(d) Explain why the difference between two consecutive terms of the sequence is always odd.

[3]



The diagram shows a triangle ABC with vertices A(-2, 4) and C(10, -4). AB is parallel to the y-axis. (a) Calculate the length of the line segment AC

(a) Calculate the length of the line segment AC.

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Answer units [2]

Given that the gradient of the line BC = -1. **(b)**

(i) the coordinates of B,

Answer B (.....) [2]

(ii) the equation of the line BC.

(d) Find the coordinates of point D, given that ABCD is a parallelogram.

 $D_{EDUCATION}^{Answer D(\dots)} [1]$



3 *ABCD* represents a plot of land.



A is 24 m due north of B. AC = 21 m, CD = 5 m and angle $ADC = 70^{\circ}$. The bearing of C from A is 140°.

(a) Calculate

(i) *BC*,



Answer m [2]

(ii) the bearing of D from A.

- E is a point at the top of a tree at point C. **(b)** The angle of elevation of E from D is 78°. Calculate
 - the height of the tree, (i)

[2]

(ii) the greatest possible angle of elevation of *E* from a point on *AB*.

Answer [2] DANYAL

4 Aida and Bernard took part in a marathon race.

They each ran 70 km.

Aida ran at an average speed of x km/h. (a)

Write down an expression, in terms of x, for the number of hours she took.

(b) Bernard ran at an average speed which was 0.5 km/h more than Aida's speed. DANYAL EDUCATION Find an expression, in terms of x, for the number of hours he took. DANYAL

The difference between their timings was 50 minutes. (c)

> Write down an equation in x to represent this information, and show that it DANYATH reduces to $2x^2 + x - 84 = 0$.

Answer

(d) Solve the equation $2x^2 + x - 84 = 0$, giving your solutions correct to 1 decimal place.

Answer $x = \dots$ or [4]

(e) Hence calculate the time that Aida took to complete the race.Give your answer in hours and minutes, correct to the nearest minute.

Answer h min [2]

5



The diagram shows a circle *ABCDE*, with centre *O*. *AE* is parallel to *CD*. Angle $COD = 46^{\circ}$ and angle $CEB = 14^{\circ}$. *EF* and *DF* are tangents to the circle at *E* and *D* respectively.

(a) Find angle DBE.

Give a reason for each step of your working.

Answer Angle DBE = [2]

(b) Find angle AEB.

Give a reason for each step of your working.

Answer Angle $AEB = \dots$ [2]

(c) Find angle *BAE*.

Give a reason for each step of your working.

(d) Find angle *EFO*.Give a reason for each step of your working.

DANYAL

Answer Angle $EFO = \dots$ [2]

(e) A point G lies on the same side of DE as B such that angle $DGE = 90^{\circ}$.

State whether point G lies inside, outside or on the circle. Explain your answer.

EDUC: [1]

(f) The radius of the circle is 5 cm.Calculate the length of the major arc *EABCD*.

6 A rectangular sheet of paper has length x cm and an area of 36 cm².

Four identical squares of side 1 cm are cut from the four corners of the paper and the paper is folded to make an open box.



(a) Show that the volume of the open box, $y \text{ cm}^3$, is given by $y = 2(x-2)\left(\frac{18}{x}-1\right)$. Answer



[2]

(b) Complete the table of values for $y = 2(x-2)\left(\frac{18}{x}-1\right)$.

Values are given to 1 decimal place where appropriate.

x	2	3	4	5	7	9	12	16
y	0	10	14	15.6	15.7	14	10	





[2]

(d) Use your graph to estimate the range of values of x for which the volume of the box is more than 7 cm^3 .

- (e) Explain how the graph shows that the volume cannot be equal to 18 cm³.

 [1]

 [1]
- (f) By drawing a tangent, find the gradient of the curve at the point x = 5.

(g) By drawing a suitable straight line on your graph, find the solutions of the equation $2x^2 + 5x = 4(x-2)(18-x)$.



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The diagrams show a kids' pool in a refurbished hotel.

The cross-section of the pool is a major segment of a circle, centre O and radius 4 m. The depth of the pool is 0.5 m.

- (a) The area of the minor sector OPQ is 11.2 m².
 - (i) Find angle *POQ* in radians.

DANYAL

Answer Angle POQ = rad [1]

(ii) Show that the capacity of the pool is 23.47 m³, correct to 2 decimal places. Answer



(b) For safety reasons, a flight of stairs made from 1.05 m³ cement was placed along PQ inside the pool.

Mr Tan, the manager, needs to fill the pool with water, up to 95% capacity.

Company GRC provides water-filling service for swimming pools.

Water pump	8 gallons per minute
Cost of water pump	\$17 per 100 gallons of water
Labour cost	\$30 per hour

1 gallon = 3.785 litres $1 \ litre = 0.001 \ m^3$

Mr Tan thinks that the pool can be filled in 10 hours, within a budget of \$1200. DANIAN

Is Mr Tan correct? Justify your answer.

Answer

End of Paper

PartnerInLearning 137

1.		2x - y = 2 (1)	
		x + 3y = 15(2)	
	(2):	$\times 2 2x + 6y = 30 (3)$	
	(1	(-3) - 7y = -28	[M1] Reduce
		y = 4	to linear equation with
	Sub $y = 4$ into equation (2)		one variable. Substitution
		x = 15 - 12	Method is
		x = 3	also
			accepted.
		r - 3 v - 4	NAM
	T.	x = 3, y = 4	[A2]
7	DT3	CATIL	
2.	(a)	$(3a^3)^2 = 9a^6$	[B1]
	(b)	$6 pa \times 4 p^3 a - 24 p^4 a^2$	[B1]
		$opq \wedge \tau p q - 2\tau p q$	[~~]
	(c)		
		$\sqrt[3]{a^4 \div a^3 \times a^0}$	
		$\frac{4}{3}$, $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$	[M1]
		$=a^{3} \div a^{3} \times a^{3}$	FA 13
			[AI]
3	(2)	Figure 5	[B1]
5.	(a) (b)	Figure 6	[B1]
	(c)	Figure 1	[B1]
			NAL
4.	(a)	$5 - 3x \le 1 - 2x < 3x + 7$	MOD AGO
	N	$5-3x \le 1-2x$ $1-2x \le 3x+7$	DUCAL
	Db	-2x - 3x < 7 - 1	EL
	ED	and $-5x < 6$	
		$-x \ge -4$ 6	[M1]
		$x \ge 4$ $x > -\frac{1}{5}$	[M1]
			For solving
			inequalities.
		Solution: $x \ge 4$	[A 1]
	(h)	5	[AI] [B1]
		-	

2022 SA2 3E Mathematics Paper 1 Marking Scheme

5.	(a)	number of elderly $=\frac{9}{100} \times 5.626 \times 10^6$	
		$\approx 5.063 \times 10^{5}$	
		$\approx 5.1 \times 10^5$	1011
	(b)	number of youths $=2 \times 5.063 \times 10^5$	
		$\approx 1.0 \times 10^6$	[B1]
6.	(a)	$3^4 \times 3^q = \frac{1}{27}$	
		$3^{4+q} = \frac{1}{3^3}$	J.
		$3^{4+q} = 3^{-3}$	[M1] Same
	A	4 + q = -3	base
7	DUG	q = -7	EV [A1]
	5		
	(b)	$8^r \div 4 = 1$	
		$2^{3r} \div 2^2 = 2^0$	[M1] Same
		$2^{3r-2} = 2^0$	base
		3r - 2 = 0	
		$r = \frac{2}{3}$ DANYAU	[A1]
		EDDE	
7.	(a)	xy = 4(-3)	[121]
	(h)	=-12	[D1]
	(0)	x - y = (-5) - 0	[B1]
		= 25	NY NY
8.	(a)	y = -(x+1)(x-3)	[M1]
	DP	$=-(x^2-3x+x-3)$	EDU
	ED	$=-(x^2-2x-3)$	
		$= -x^2 + 2x + 3$	[A1]
	(b)	<i>x</i> = 1	[B1]
	(c)	(1,4)	[B1]
9.	(a)	3:4	[B1]

	(b)	$\frac{V_1}{V_2} = \left(\frac{3}{4}\right)^3$	
		$=\frac{27}{64}$	[M1]
		Volume of smaller bottle $=\frac{27}{64} \times 1440$	
		$= 607.5 \text{ cm}^3$	[A1]
10.	(a)	$x^2 + 8x - 20$	
		$=(x+4)^2-16-20$	(B1)
		$=(x+4)^2-36$	[B1]
	(b)	$x^2 + 8x - 20 = 0$	ATION
1	122	$(x+4)^2 - 36 = 0$	BC
	EDU	$(x+4)^2 = 36$	[M1]
		$x + 4 = \pm 6$	[A1]
		x = 2 or -10	
11.	(a)	$\triangle ABC$ is similar to $\triangle ADF$.	
		$\angle BAC = \angle DAF$ (common $\angle s$) $\angle ABC = \angle ADF$ (corresponding $\angle s$) By AA similarity test, $\triangle ABC$ is similar to $\triangle ADF$.	[B1] [B1] with angle properties stated
		or ΔDBE is similar to ΔADF .	[A1] conclude with AA similarity test
	DA	$\angle DBE = \angle ADF \text{ (corresponding } \angle s)$ $\angle DEB = \angle AFD \text{ (corresponding } \angle s)$	or AA test.
		By AA similarity test, ΔDBE is similar to ΔADF .	
	(b)	DF = EC = 12 cm $\frac{BC}{12} = \frac{15}{10}$	[M1] Ratio
		$BC = \frac{15}{10} \times 12$ $= 18 \text{ cm}$	[M1] Finding

	1		150
		BE = 18 - 12	BC
		= 6 cm	[A1]
		or	
			[M1]
		$\frac{BE}{12} = \frac{5}{10}$	[MI]
		ng 5 10	D/11
		$BE = \frac{10}{10} \times 12$	
		= 6 cm	[A1]
			NAL
		LAY DAS	MON
D	(c)	area of triangle ADF = $\left(\frac{10}{15}\right)^2$	[M1]
1	DU	area of triangle ABC (15)	Mark is not
		$=\frac{4}{9}$	awarded if students
		area of triangle $ADF = 4$	attempt to
		area of $DBCF$ 9–4	find the areas
		$=\frac{4}{5}$	vertical
		ANTION	height given. Hence the
		DUCALL	areas
			calculated will not be
			correct.
			[41]
			L'II A A
	~	ST Press) prestric.
12.	(a)	$PR^2 = 15^2$	£N.
	ED	= 225	
		$DO^2 + OD^2 = 10^2 + 0^2$	
		$\frac{1}{2} \frac{1}{2} + \frac{1}{2} \frac{1}{2} + \frac{1}{2} \frac{1}{2} + \frac{1}{2} \frac{1}{2$	[M1] Mark is not
		- 225	awarded if
		Since $PR^2 = PQ^2 + QR^2$, by converse of Pythagoras' Theorem,	students write
		triangle PQR is a right-angled triangle. Thus $\angle PQR = 90^{\circ}$.	$15^2 = 12^2 + 9^2$
			[A1] Students

			must state the theorem used.
	(b)	(i) $\tan \angle QPS = \frac{16}{12}$	
		$=\frac{4}{3}$	[B1]
		(ii) $\cos \angle PRS = -\cos \angle PRQ$	
		$=-\frac{9}{15}$	
		$=-\frac{3}{5}$	[B1]
		(iii) $PS = \sqrt{12^2 + 16^2}$	MIL
	N	= 20 cm	CATION
2	DD U	$\sin \angle PSQ = \frac{12}{20}$	00.
	Fr	$=\frac{3}{5}$	[A1]
13	10 v	vorkers $\rightarrow 90 \text{days}$	
101	1	worker \rightarrow 900 days	0.00
	15	workers $\rightarrow 60$ days	
	Yes, requ	the manager is correct in his claim as 15 workers will only ire 60 days to complete building the bus stop.	[A1]
	The	assumption made is that all workers work at the same rate.	[B1]
14.	(a)	$\angle ABC = 180^\circ - 79^\circ - 41^\circ$. N.
		$=60^{\circ}$	
	DA	area of $\triangle ABC = \frac{1}{2} \times 4 \times 6 \times \sin 60^{\circ}$	[M1]
	EL	≈ 10.392	
		$=10.4 \text{ cm}^2$	[A1]
	(b)	Let the shortest distance from C to AB be d .	
		$\frac{1}{2} \times 4 \times d = 10.392$	[M1]
		$d \approx 5.20$	[A1]

15		360°		
	1 exteri	for $\angle = \frac{288}{6}$		
		= 60°		[M1]
	Z	$BCE = 180^{\circ} - (90^{\circ} - 60^{\circ})$		D.(1)
		=150°		[M1]
		1909 1509		
	Zi	$BEC = \frac{180^{\circ} - 150^{\circ}}{2}$		
		=15°		[A1]
				NAL
	A TA		- 5	40. A
16.	Total a	mount = $3000 \left(1 + \frac{2.8 \div 2}{100} \right)^{6 \times 2}$	EDI	[M1]
	E.F.	≈ \$3545		[A1]
17.	(a) co	$2533^{\circ} = \frac{10}{10}$		[M1]
		BF		awarded if
		$BF = \frac{10}{\cos^2 3^\circ}$		sine rule too
		≈11.924		
		$\approx 11.9 \text{ cm}$ DB CATIO		[A1]
	(b) <i>B</i> .	$M = 30 \div 2$		
		=15 cm		[M1]
	F	$M = \sqrt{11.924^2 + 15^2}$		WAL
	. 5	≈19.162	5	DELCATION
	DA	≈ 19.2 cm		[AI]

Mathematics Paper 1 Secondary 3 Express BP~147

	(c)	$\tan 33^\circ = \frac{FA}{10}$	
		$FA \approx 6.4941$	[M1]
		$\sin \angle FMA = \frac{6.4941}{19.162}$	
		$\angle FMA = \sin^{-1}\left(\frac{6.4941}{19.162}\right)$ $\approx 19.8^{\circ}$	[A1]
		or	
	The second secon	$AM = \sqrt{10^2 + 15^2}$ ≈ 18.028	[M1]
3	EDU	$\cos \angle FMA = \frac{18.028}{19.162}$	
		$\angle FMA = \cos^{-1}\left(\frac{18.028}{19.162}\right)$	[A1]
		≈ 19.8°	1
		A P	
		AN TON	
18.	(a)	$a = \frac{25 - 0}{20 - 0}$	
		$=1.25 \text{ m/s}^2$	[В1]
	(b)	Distance for first p seconds	
		$\frac{1}{2} \times 20 \times 25 + 25(p - 20) = 625$	[M1]
		250 + 25p - 500 = 625	DESCATIO
	DA	p=35	[A1]
	ED	or	
		25(p-20) = 375	
		p=35	

(c)	Total distance = $\frac{1}{2} \times [52 + (35 - 20)] \times 25$	[M1]
	2 = 837.5 m	
	<i>q</i> = 837.5	[A1]

Mathematics Paper 1 Secondary 3 Express

PartnerInLearning 147



CANBERRA SECONDARY SCHOOL

2022 End of Year Examination

Secondary Three Express

MATHEMATICS 4052/02

Name:

11 October 2022 1 hour 45 minutes 1130 – 1315

marking scheme

Class:

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This question paper consists of 16 printed pages including the cover page.

Setter: Ms Sim Xiu Jun

Here are the first four terms of a sequence. 1

- $T_1 = 1^2 + 1 = 2$ $T_2 = 3^2 + 2 = 11$ $T_3 = 5^2 + 3 = 28$ $T_{\rm c} = 7^2 + 4 = 53$
- Write down the fifth term of the sequence. (a)
- $9^2 + 5 = 86$ [B1] (b) T_n is the *n*th term of the sequence. Show that $T_n = 4n^2 - 3n + 1$. Answer $T_n = \left(2n - 1\right)^2 + n$ [M1] form the formula $=4n^{2}-4n+1+n$ [M1] expand $=4n^2-3n+1 \quad \text{(Shown)}$

The difference, D, between two consecutive terms of the sequence is $T_{n+1} - T_n$. (c) Find an expression, in terms of n, for D.

$$D = T_{n+1} - T_n$$

$$= \left[4(n+1)^2 - 3(n+1) + 1 \right] - (4n^2 - 3n + 1)$$
 [M1] substitute T_n and T_{n+1}

$$= 4(n^2 + 2n + 1) - 3n - 3 + 1 - 4n^2 + 3n - 1$$
 [M1] expand
$$= 4n^2 + 8n + 4 - 3 - 4n^2$$

$$= 8n + 1$$
 [A1]

8n + 1Answer D =

- Explain why the difference between two consecutive terms of the sequence is
- always odd. $D = 8n + 1 = 2(4n) + 1 \implies 8n$ is always a multiple of $2 \implies D$ always odd [1]

(d)

[3]

[3]

4

2



The diagram shows a triangle ABC with vertices A(-2, 4) and C(10, -4). AB is parallel to the y-axis. (a) Calculate the length of the line segment AC

(a) Calculate the length of the line segment AC.

$$AC = \sqrt{(-2-10)^{2} + [4-(-4)]^{2}}$$
[M1]
= $\sqrt{208}$
= 14.4 units (3 s. f.) [A1]
DAMATION 14.4
Environment 14.4
[2]

Given that the gradient of the line BC = -1. (b)

Find

(i) the coordinates of B,

EDUCAT Let the coordinates of B be (-2, b).

$$\frac{b - (-4)}{-2 - 10} = -1$$
 [M1]

$$b + 4 = 12$$

$$b = 8$$

$$B = (-2, 8)$$
 [A1]

-2 8 Answer B (.....) [2]

14.4

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(ii) the equation of the line BC.

$$y = mx + c$$

-4 = (-1)(10) + c [M1]
$$c = 6$$

Equation of BC is y = -x + 6 [A1] reject y = -1x + 6



- (d) Find the coordinates of point D, given that ABCD is a parallelogram.

	10	8	
Answer D		,)	[1]

3 *ABCD* represents a plot of land.



A is 24 m due north of B. AC = 21 m, CD = 5 m and angle $ADC = 70^{\circ}$. The bearing of C from A is 140°.

(a) Calculate

(i) *BC*,

Using cosine rule, for BC > 0.

$$BC = \sqrt{24^2 + 21^2 - 2(24)(21)\cos(180^\circ - 140^\circ)}$$
[M1]
= 15.647 (5 s. f.)
= 15.6 m (3 s. f.) [A1]

(ii) the bearing of D from A. Using sine rule, $\frac{\sin \angle CAD}{5} = \frac{\sin 70^{\circ}}{21}$ [M1] $\angle CAD = \sin^{-1} \left(\frac{5 \sin 70^{\circ}}{21}\right)$ $= 12.929^{\circ}$ (3 d. p.) [M1] Bearing of D from $A = 140^{\circ} - 12.929^{\circ}$ $= 127.1^{\circ}$ (1 d. p.) [A1]

.1° (1 d. p.) [A1] 127.1° Answer

[3]

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- E is a point at the top of a tree at point C. **(b)** The angle of elevation of E from D is 78° . Calculate
 - (i) the height of the tree,

DANYAL the greatest possible angle of elevation of E from a point on AB.

Shortest distance from C to $AB = 21\sin(180^\circ - 140^\circ)$

$$= 13.499 \quad (5 \text{ s. f.}) \qquad [M1]$$

Greatest possible angle of elevation
$$= \tan^{-1} \left(\frac{23.523}{13.499} \right)$$
$$= 60.2^{\circ} \quad (1 \text{ d. p.}) \qquad [A1]$$

60.2° DANYAL

4 Aida and Bernard took part in a marathon race.

They each ran 70 km.

(a) Aida ran at an average speed of x km/h.

Write down an expression, in terms of x, for the number of hours she took.

$$\frac{70}{x}$$
 [B1]
Answer h [1]

(b) Bernard ran at an average speed which was 0.5 km/h more than Aida's speed. Find an expression, in terms of *x*, for the number of hours he took.

The difference between their timings was 50 minutes. (c)

Write down an equation in x to represent this information, and show that it DANYAL reduces to $2x^2 + x - 84 = 0$.

Answer

x

$$\frac{70}{x} - \frac{140}{2x+1} = \frac{5}{6}$$
[70(6)(2x+1)-140(6)(x) = 5(x)(2x+1)
[70(6)(2x+1)-140(6)(x) = 5(x)(2x+1)]
[70(6)(2x+1)-140(x) = 10x^2 + 5x]
[70(6)(2x+1)-140(x) = 10x^2 + 5x - 420]
[70(2x^2 + x - 84 = 0] (Shown)]
[70(6)(2x+1)-140(x) = 10x^2 + 5x]
[70(6)(2x+1)-140(x) = 10x^2 +

M1] forming equation

[A1]

M1] removing denominator DANVAL

[3]

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- (d) Solve the equation $2x^2 + x 84 = 0$, giving your solutions correct to 1 decimal place.

$$2x^{2} + x - 84 = 0$$

$$x = \frac{-(1) \pm \sqrt{(1)^{2} - 4(2)(-84)}}{2(2)}$$
[M1] applying
quadratic formula
$$= \frac{-1 \pm \sqrt{673}}{4}$$
[M1]
$$= -6.736 (3 \text{ d. p.}) \text{ or } 6.236 (3 \text{ d. p.})$$

$$= -6.7 (1 \text{ d. p.}) \text{ or } 6.2 (1 \text{ d. p.})$$
[A2] correct to 1 d. p.

Multiplying
$$\frac{67}{62} = \frac{62}{4}$$
Hence calculate the time that Aida took to complete the race.
[4]

(e) Hence calculate the time that Aida took to complete the race. Give your answer in hours and minutes, correct to the nearest minute.

Aida's time taken =
$$\frac{70}{6.236}$$
 [M1]
⁴ = 11.225 h
= 11 h 14 min (to the nearest minute) [A1]



(c) Find angle BAE.

Give a reason for each step of your working.

$\angle CBE = 90^{\circ}$	(angle in semicircle)
$\angle BCE = 180^{\circ} - 90^{\circ} - 14^{\circ}$	(angle sum of triangle) [M1]
= 76°	
$\angle BAE = 180^\circ - 76^\circ$	(opposite angles in cyclic quadrilateral)
=104°	[A1] with correct reason

* one reason one mark

(d) Find angle EFO.

 104°

Give a reason for each step of your working.

$\angle FOE = \frac{134^\circ}{2}$	(tangent from external point)		
= 67°			
$\angle OEF = 90^{\circ}$	(angle between tangent and radius)	[M1]	
$\angle EFO = 180^\circ - 90^\circ - 67^\circ$	(angle sum of triangle)		
= 23°	[A1] with correct reason		
* one reason one mark		23°	
	Answer Angle $EFO = \dots$		[2]

(e) A point G lies on the same side of DE as B such that angle $DGE = 90^\circ$.

State whether point G lies inside, outside or on the circle. Explain your answer. G lies inside the circle. Angle $DGE = 90^\circ > 67^\circ =$ angle DBE.

EDU-

(f) The radius of the circle is 5 cm.

Calculate the length of the major arc EABCD.

Arc length =
$$\frac{180^\circ + 46^\circ}{360^\circ} \times 2\pi (5)$$
 [M1]

$$=19.7 \text{ cm} (3 \text{ s. f.})$$
 [A1]

Canberra Secondary School 2022 End of Year Examination 6 A rectangular sheet of paper has length x cm and an area of 36 cm².

Four identical squares of side 1 cm are cut from the four corners of the paper and the paper is folded to make an open box.



Answer

$$y = (x - 1 - 1) \left(\frac{36}{x} - 1 - 1 \right) (1) \quad [M1]$$

$$= (x - 2) \left(\frac{36}{x} - 2 \right)$$

$$= 2(x - 2) \left(\frac{18}{x} - 1 \right) \quad (Shown) \quad [A1]$$

[2]

(b) Complete the table of values for $y = 2(x-2)\left(\frac{18}{x}-1\right)$.

ANYAL

Values are given to 1 decimal place where appropriate.

x	2	3	4	5	7	9	12	16
y	0	10	14	15.6	15.7	14	10	3.5

(a)

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Correct plotting of all 8 points [G1] Smooth curve passing through all 8 points [G1]

[Turn Over

[2]

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(d) Use your graph to estimate the range of values of x for which the volume of the box is more than 7 cm^3 .

- (e) Explain how the graph shows that the volume cannot be equal to 18 cm³. The maximum volume is 16 cm³, which is less than 18 cm³.
 [1]
- (f) By drawing a tangent, find the gradient of the curve at the point x = 5.



(g) By drawing a suitable straight line on your graph, find the solutions of the equation $2x^2 + 5x = 4(x-2)(18-x)$.

$$2x^{2} + 5x = 4(x-2)(18-x)$$

$$\frac{2x^{2} + 5x}{2x} = \frac{4(x-2)(18-x)}{2x}$$

$$x + 2\frac{1}{2} = 2(x-2)\left(\frac{18}{x} - 1\right)$$

$$x + 2\frac{1}{2} = y$$

[M1] correct equation of straight line

Plot the line $y = x + 2\frac{1}{2}$.

x	0	6	12
у	21/2	81/2	141/2

[M1] correct (labelled) straight line drawn

From the graph,

 $x = 2.4 (\pm 0.1)$ or $x = 10.1 (\pm 0.1)$

[A1]

2.4 10.1 Answer $x = \dots$ or [3]

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The diagrams show a kids' pool in a refurbished hotel.

The cross-section of the pool is a major segment of a circle, centre O and radius 4 m. The depth of the pool is 0.5 m.

- (a) The area of the minor sector OPQ is 11.2 m².
 - (i) Find angle POQ in radians.

$$\frac{1}{2}(4)^2 \theta = 11.2$$
$$\theta = 1.4$$

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DAMYALEDUCATION 1.4 [B1]Answer Angle POQ =rad [1]

(ii) Show that the capacity of the pool is 23.47 m³, correct to 2 decimal places.
 Answer

Area of triangle
$$POQ = \frac{1}{2}(4)^2 \sin 1.4$$
 [M1]
= 7.8836 m² (4 d. p.)
Area of major sector $POQ = \frac{1}{2}(4)^2 (2\pi - 1.4)$ [M1]
= 39.0655 m² (4 d. p.)

$$Volume = (7.8836 + 39.0655) \times 0.5$$

$$= 23.47 \text{ m}^3$$
 (2 d. p.) (Shown) [A1]

[3]

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[Turn Over

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- (b) For safety reasons, a flight of stairs made from 1.05 m³ cement was placed along PQ inside the pool.

Mr Tan, the manager, needs to fill the pool with water, up to 95% capacity.

Company GRC provides water-filling service for swimming pools.

Water pump	8 gallons per minute
Cost of water pump	\$17 per 100 gallons of water
Labour cost	\$30 per hour

1 gallon = 3.785 litres $1 \ litre = 0.001 \ m^3$

Mr Tan thinks that the pool can be filled in 10 hours, within a budget of \$1200. DANILON

Is Mr Tan correct? Justify your answer.

Answer

Capacity to be filled =
$$\frac{95}{100}(23.47 - 1.05)$$

= 21.299 m³ [M1] vol in m³
= 21299 litres
= 5627.213 gallons (3 d. p.) [M1] vol in gallons
Cost of water pump = \$17 × 56
= \$952 (2 d. p.) [M1] cost of pump

= \$952 (2 d. p.)

 $=\frac{5627.213}{8}$ $= 703.402 \min (3 d. p.)$ =11.72 h (2 d. p.) =12 h (nearest hour)

Total cost = $$952 + 30×12

= \$1312

..... Mr Tan is incorrect as it will take about 12 h at a cost of \$1312. [A1]

[M1] time in min

[M1] labour cost

[6]

End of Paper

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