

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1. Factorise fully $2x^2 + y^2 - 2x - xy^2$.

Answer : [2]

2. It takes p workers q days to build r houses.

If the number of days is halved and $5r$ houses are to be built, how many workers must be hired for the job?

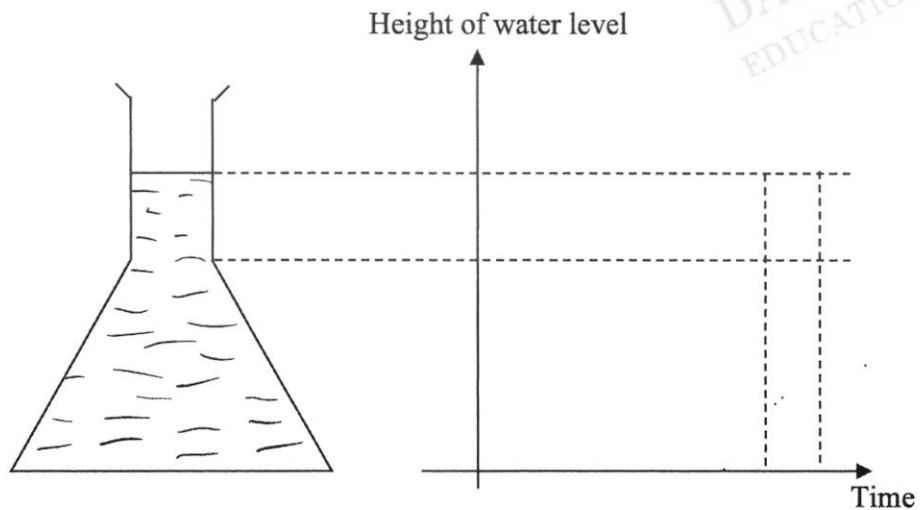
Express your answer in terms of p .

Answer : [2]

3. Water is poured at a constant rate into the conical flask as shown below.

Draw on the axes provided, the change in the water level of the conical flask over time.

Answer



[2]

4. Solve the equation $(1-x)^2 = \frac{9}{4}$.

Answer : $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

5. A group of 15 students recorded their timings (to the nearest minute), for a 5-km run. The results are represented in the stem and leaf diagram below.

Stem	Leaf
2	6 8 8 9
3	0 0 1 3 4 4 4 5 7 9
4	
5	
6	1

Key: 2|6 means 26 minutes

- (a) Find the percentage of students who took at least 35 minutes to complete the run.

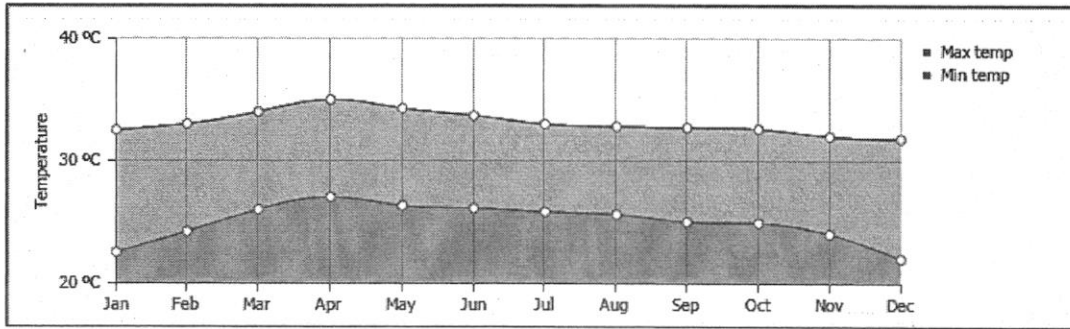
Answer : $\dots\dots\dots$ % [1]

- (b) Explain why the mean may not be an appropriate measure of average time taken by these students to complete the 5-km run.

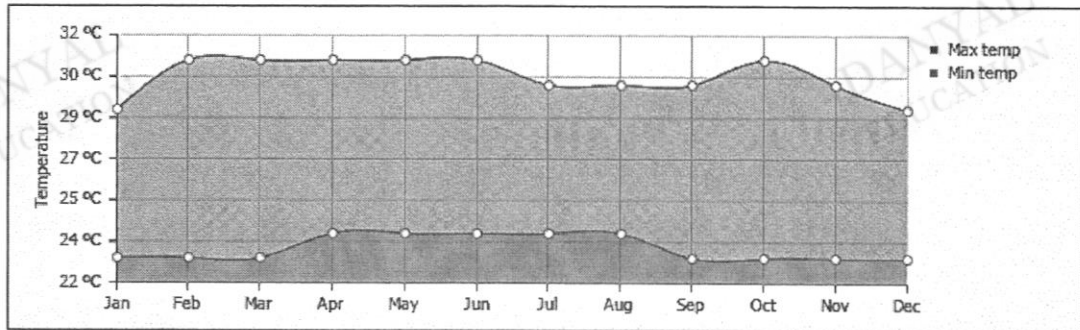
Answer : $\dots\dots\dots$

$\dots\dots\dots$ [1]

6. The line graphs below show the monthly average minimum and maximum temperatures of Bangkok (Thailand) and Singapore, for the year 2020.



Average min and max temperature, Bangkok Thailand Copyright 2020 www.weather-and-climate.com



Average min and max temperature, Singapore Copyright 2020 www.weather-and-climate.com

- (a) Huda claims that Singapore experienced wider differences between the maximum and minimum temperatures.

Explain why the data presented above may have been misleading for Huda.

Answer :

.....

..... [1]

- (b) Give a suggestion on how the data above can be presented in a clearer way.

Answer :

..... [1]

7. A polygon with n sides has two exterior angles 100° and 50° .
The remaining $(n - 2)$ exterior angles are 14° each.

Find n .

Answer : $n = \dots\dots\dots$ [2]

8. Written as the product of its prime factors, $504 = 2^x \times 3^y \times 7$.

(a) Find the values of x and y .

Answer : $x = \dots\dots\dots$, $y = \dots\dots\dots$ [1]

(b) The highest common factor and the lowest common multiple of 18 and z are 6 and 504 respectively.

Find the smallest possible value of z .

Answer : smallest $z = \dots\dots\dots$ [2]

9. Simplify $\left(\frac{x^6}{64}\right)^{-\frac{2}{3}} \div \frac{y^3}{x^6}$.

Leave your answer in positive index.

Answer : [3]

10. Mr Png bought a massage chair with a down payment of 15% of the cash price and a monthly instalment of \$195.60 for 18 months.

If he paid a total of \$3970.80, find the cash price of the massage chair.

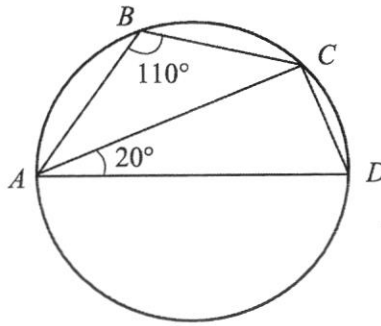
Answer : \$ [3]

11. Given that $m = \sqrt{\frac{2m+1+k}{4k}}$, express k in terms m , in its simplest form.

Answer : $k =$ [3]

12. The diagram shows a circle with points A , B , C and D on its circumference. Angle $ABC = 110^\circ$ and angle $CAD = 20^\circ$.

Explain why AD is a diameter of the circle.



Answer :

[3]

13. n is a positive integer.

(a) Show that $(3n-2)^2 - n^2$ is a multiple of 4.

[2]

(b) Hence or otherwise, factorise $(3n-2)^2 - n^2$ fully.

Answer : [1]

14. A bean bag is filled with small polystyrene balls of radius 3 millimetres.

- (a) Write the radius of the ball in metres, leaving your answer in standard form.

Answer : m [1]

- (b) A bean bag of volume 2.88 m^3 is to be 80% filled with the spherical polystyrene balls.

Find the number of polystyrene balls required, giving your answer to the nearest million.

Answer million [3]

15. The scores for a Mathematics Test sat by six students are as follows:

$$a, b, 59, c, 65, 90$$

The range of the scores is 48.

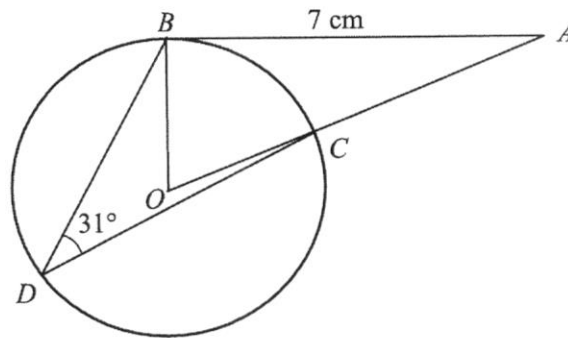
The median score is 62.

The mean score is 61.

Find the values of a , b and of c .

Answer : $a = \dots\dots\dots$, $b = \dots\dots\dots$, $c = \dots\dots\dots$ [4]

16.



In the diagram, B , C and D are points on a circle.

O is the centre, OCA is a straight line and BA is tangent to the circle at B .
Angle $BDC = 31^\circ$ and $AB = 7$ cm.

(a) State the reason why angle $OBA = 90^\circ$.

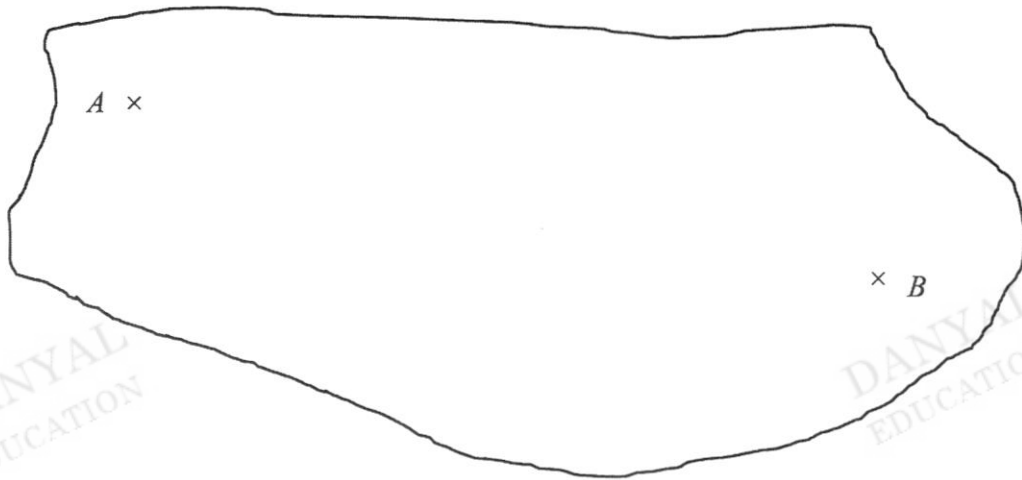
Reason : [1]

(b) Find the length of the radius of the circle.

Answer : cm [3]

17. (Diagram is drawn to scale)

The scale of the map below is 1 : 50 000.



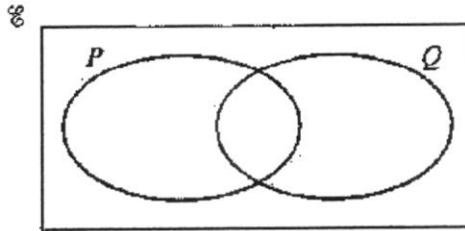
Asher cycles from Point *A* to Point *B* at an average speed of 13 km/h.

If Asher has to reach point *B* by 4.30 pm, suggest the latest time he should set off from point *A*.

Show your working clearly.

Answer : He should leave point *A* latest by pm [4]

18. (a) On the Venn diagram, shade the region which represents $P \cup Q'$.



[1]

(b) $\varepsilon = \{x : x \text{ is an integer such that } 5 \leq x \leq 9\}$

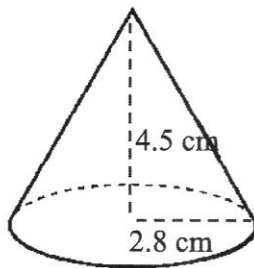
$$A = \left\{x : 5 - \frac{x}{2} \geq 1\right\}$$

$$B = \{x : 4x - 1 > 19\}$$

List the element(s) contained in the set $(A \cap B)'$.

Answer : $(A \cap B)' = \{ \hspace{10em} \}$ [3]

19. The diagram shows a paper cup in a shape of a cone with radius 2.8 cm and vertical height 4.5 cm.



(a) Show that the curved surface area is $14.84 \pi \text{ cm}^2$.

Answer :

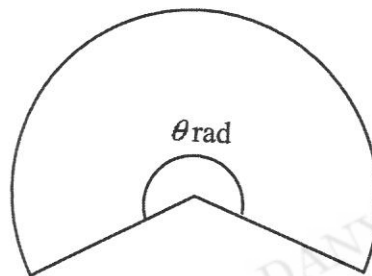
DANYAL
EDUCATION

DANYAL
EDUCATION

[2]

(b) The paper cup is cut open to form a sector of a circle with angle θ radians.

Find angle θ .

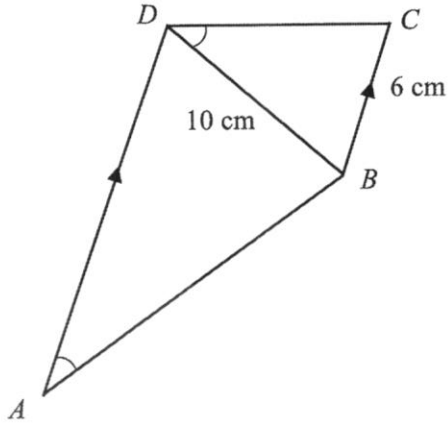


DANYAL
EDUCATION

DANYAL
EDUCATION

Answer : $\theta = \dots\dots\dots \text{ rad}$ [2]

20. In the diagram, the line AD is parallel to BC and $\angle BAD$ is equal to $\angle CDB$.
 $BC = 6$ cm and $BD = 10$ cm.



- (a) Explain why $\triangle ABD$ is similar to $\triangle DCB$.

Answer

.....

.....

..... [2]

- (b) Find the length of AD .

Answer : cm [2]

21. The equation of a curve is given by $y = x(10 - x)$.

(a) Explain why the maximum value of y is 25.

Answer

.....

.....

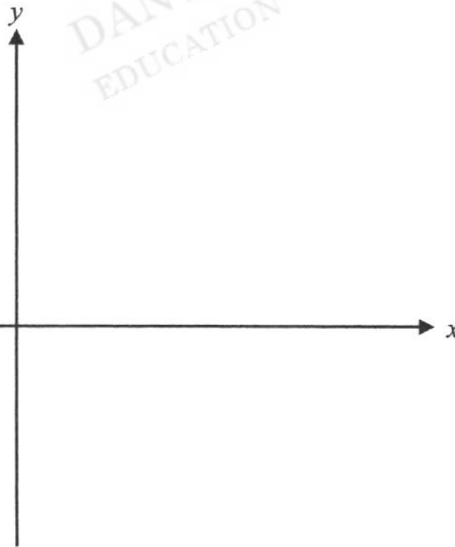
.....

.....

.....

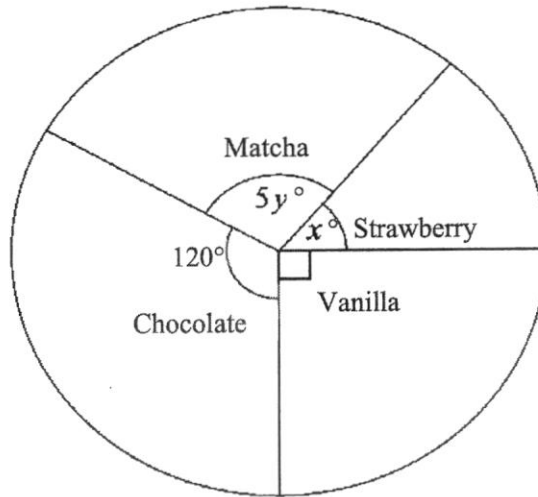
..... [3]

(b) Sketch the curve $y = x(10 - x)$, showing clearly the intercepts and the maximum point.



[2]

22. The pie chart represented the number of people who chose their favourite milkshake flavour.



- (a) Form an equation in terms of x and y .

Answer [1]

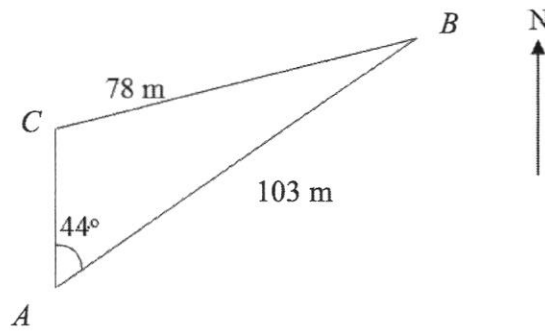
- (b) The ratio of people who chose Strawberry to Matcha flavour was 4 : 11.
Show that $11x - 20y = 0$.

[1]

- (c) Using your equations from (a) and (b), solve them simultaneously to find the values of x and of y .

Answer $x = \dots\dots\dots$, $y = \dots\dots\dots$ [3]

23.



A , B and C are 3 points on level ground.

$BC = 78$ m, $AB = 103$ m and angle $BAC = 44^\circ$.

C is due north of A .

(a) Calculate the bearing of A from B .

Answer : $^\circ$ [1]

(b) Find the area of triangle ABC .

Answer : m^2 [4]

24. A line passes through the points A and B whose coordinates are $(5, -13)$ and $(-2, 8)$ respectively.

(a) Find the equation of the line AB .

Answer : [2]

(b) A point P lies on the y -axis, such that it is the same distance from A as it is from B .

Find the coordinates of point P .

Answer : $P = (\dots\dots\dots, \dots\dots\dots)$ [3]

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

DANYAL
EDUCATION

DANYAL
EDUCATION

[Turn over for Question 1]

DANYAL
EDUCATION

DANYAL
EDUCATION

1 (a) Write as a single fraction in its simplest form

(i) $\frac{9a^3}{b} \div \frac{81a}{b^7}$,

Answer [1]

(ii) $\frac{5}{(y-3)^2} - \frac{7}{3-y}$.

Answer [2]

(b) Simplify $\frac{8x^2-18}{2x^2-x-6}$.

Answer [3]

(c) Solve the equation $\frac{32}{x-5} = 3x-5$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [3]

- 2 (a) From 2019 to 2020 the total number of international visitors in Singapore decreased by 85.9%.
In 2020, the total number was 2.7×10^6 .
Calculate the number of international visitors in 2019, giving your answer in standard form.

Answer $\dots\dots\dots$ [2]

- (b) Siti invested some money in a saving accounts for 4 years.
The rate of interest was fixed at 1.08% per annum compounded annually.
At the end of 4 years, there was \$8351.24 in her account.

How much did Siti invest in the account?
Give your answer correct to the nearest cent.

DANYAL
EDUCATION

DANYAL
EDUCATION

DANYAL
EDUCATION

DANYAL
EDUCATION

DANYAL
EDUCATION

Answer \$..... [2]

- (c) The exchange rate between Singapore dollars (\$) and Euros (€) is $\$1 = \text{€}0.63$.

Andy is shopping online for a pair of boots.

He finds these prices online for the same model of boots.

Website	Price
Zalora	\$195 (before 7% GST)
Footshopping	€130 Nett

- (i) Both websites offer free shipping to Singapore.
 GST needs to be paid if he buys from Zalora.
 It is also known that Andy's credit card charges a foreign currency transaction fee of 3.25%.

Which website offers a better deal?
 Show your working clearly.

Answer [3]

- (ii) A certain debit card offers $x\%$ foreign currency transaction fee.
 x must be less than a certain value so that buying from Footshopping will be a better deal.

Find that value.

Answer [2]

- 3 Gardens by the Bay Flower Dome operates for 340 days a year. The matrix, \mathbf{M} , shows the number of different types of tickets (**in thousands**) sold per day in 2019.

$$\mathbf{M} = \begin{matrix} & \begin{matrix} \text{Child} & \text{Adult} & \text{Senior} \end{matrix} \\ \begin{pmatrix} 1.2 & 2 & 5 \\ 2.1 & 4.5 & 9 \end{pmatrix} & \begin{matrix} \text{Residents} \\ \text{Non-residents} \end{matrix} \end{matrix}$$

- (a) Evaluate the matrix $\mathbf{P} = 340\mathbf{M}$.

Answer [1]

- (b) The tickets cost \$10, \$16 and \$14 for child, adult and senior respectively.

Represent these amounts in a 3×1 matrix \mathbf{N} .

Answer $\mathbf{N} =$ [1]

- (c) Evaluate the matrix $\mathbf{T} = \mathbf{PN}$.

Answer $\mathbf{T} =$ [2]

(d) State what each of the elements of **T** represents.

Answer

.....

..... [1]

(e) Calculate the total amount of ticket sales in 2019.
Give your answer correct to the nearest million.

Answer \$..... million [1]

(f) In 2020, the number of tickets sold for residents increased by 60% across the different types of tickets.
In the same year, the number of tickets sold for non-residents dropped to 20% across the different types of tickets.

Calculate the percentage change in the amount of ticket sales from 2019 to 2020.

State whether this change is an increase or a decrease.

Answer%

increase / decrease
(circle the appropriate answer) [3]

DANYAL
EDUCATION

DANYAL
EDUCATION

DANYAL
EDUCATION

DANYAL
EDUCATION

- 4 (a) The first three terms in a sequence of numbers, T_1 , T_2 , T_3 , ... are given below.

$$T_1 = 1^2 + 1 = 2$$

$$T_2 = 2^2 + 3 = 7$$

$$T_3 = 3^2 + 5 = 14$$

- (i) Find T_4 .

Answer [1]

- (ii) Find an expression, in terms of n , for T_n .

Answer [2]

- (b) The first four terms in a different sequence are -55 , -51 , -47 , -43 .

- (i) Find an expression, in terms of n , for the n th term, P_n , of this sequence.

Answer [2]

- (ii) Explain why 222 is not a term of this sequence.

Answer

.....

.....

.....

..... [1]

- (iii) Find the least value of n for which $P_n > 1$.

Answer [2]

- 5 The variables x and y are connected by the equation $y = \frac{x^3}{5} - \frac{3x}{2} + 1$.
Some corresponding values of x and y are given in the table below.

x	-4	-3	-2	-1	0	1	2	3	4
y	p	0.1	2.4	2.3	1	-0.3	-0.4	1.9	7.8

- (a) Find the value of p .

Answer $p = \dots\dots\dots$ [1]

- (b) Draw the graph of $y = \frac{x^3}{5} - \frac{3x}{2} + 1$ for $-4 \leq x \leq 4$ in the given grid. [3]

- (c) (i) On the same grid, draw the graph of $2y - 3x = 2$ for $-4 \leq x \leq 4$. [2]

- (ii) Show that the points of intersection of the line and the curve give the solutions of the equation $x^3 - 15x = 0$.

Answer

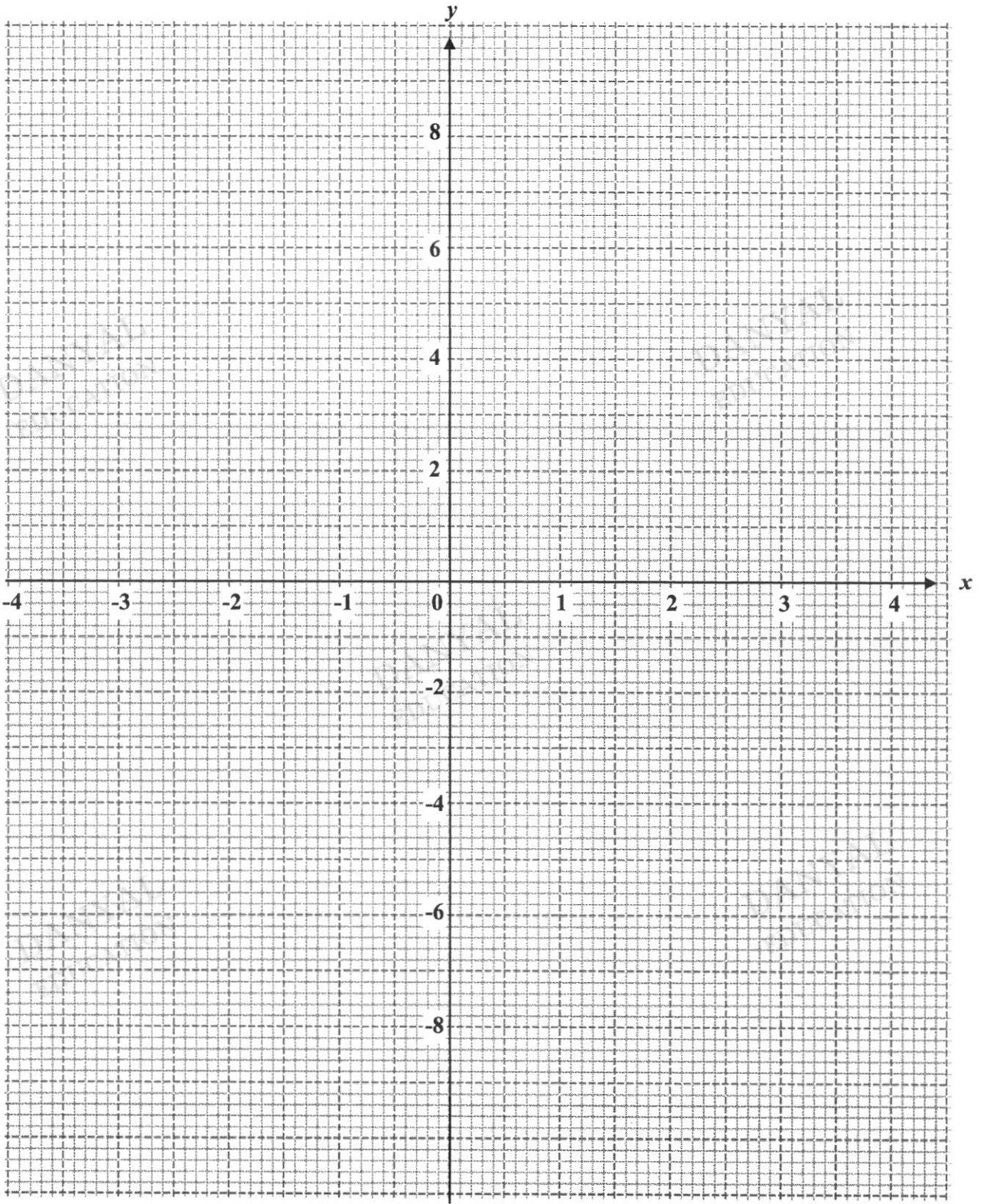
[2]

- (iii) Hence, solve the equation $x^3 - 15x = 0$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ or $\dots\dots\dots$ [2]

- (d) By drawing a tangent, find the gradient of the curve at (3, 1.9).

Answer $\dots\dots\dots$ [2]



6 Jacelyn and Patrine went for a 21 km hike.

- (a) Jacelyn walked at a constant speed of x kilometres per hour.
Write down an expression, in terms of x , for the number of hours she took.

Answerh [1]

- (b) Patrine walked at a constant speed which was $\frac{1}{3}$ km/h more than Jacelyn's speed.

Write down an expression, in terms of x , for the number of hours she took.

Answerh [1]

- (c) The difference between their times was 15 **minutes**.

Write down an equation in x to represent this information, and show that it reduces to $3x^2 + x - 84 = 0$.

- (d) Solve the equation $3x^2 + x - 84 = 0$, giving the answers correct to **three** decimal places.

DANYAL
EDUCATION

DANYAL
EDUCATION

DANYAL
EDUCATION

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [4]

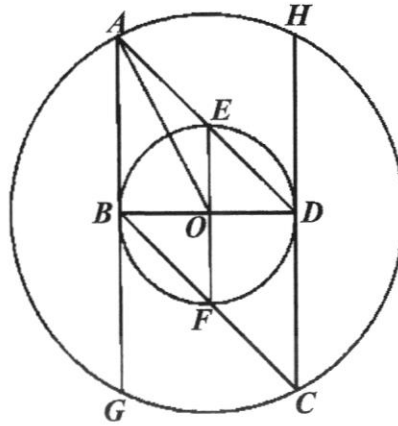
- (e) Calculate the time that Jacelyn took to complete the hike, giving your answer in hours, minutes and seconds.

DANYAL
EDUCATION

DANYAL
EDUCATION

Answer $\dots\dots\dots$ h $\dots\dots\dots$ min $\dots\dots\dots$ sec [2]

7



The centers of the two circles are at O .
 BD is a diameter of the smaller circle.
 AB and CD are tangents to the smaller circle.

- (a) Show that triangle ABD is congruent to triangle CDB .
 Give a reason for each statement you make.

Answer

[3]

- (b) Suppose the diameter of the smaller circle is 8 cm, angle $BAD = 45^\circ$ and BD is perpendicular to EF .
 (i) Calculate the length of OA .

Answercm [2]

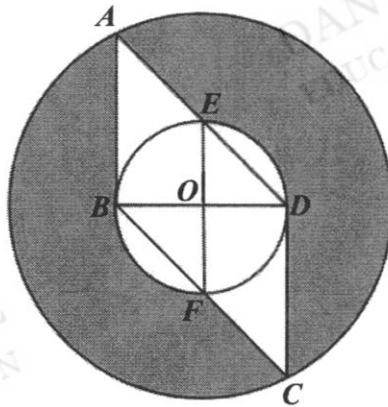
(ii) Calculate the area of triangle ABD .

Answercm² [1]

(iii) Calculate the area of sector OBE , giving your answer in terms of π .

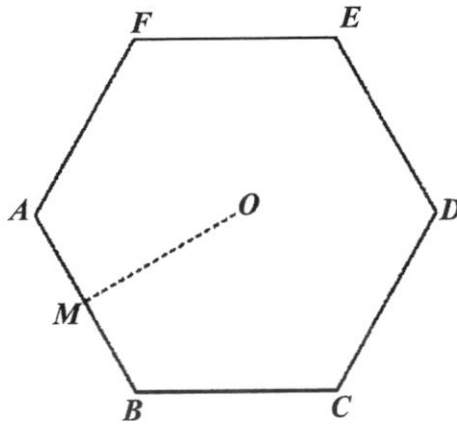
Answercm² [1]

(iv) Calculate the shaded area.



Answercm² [3]

8



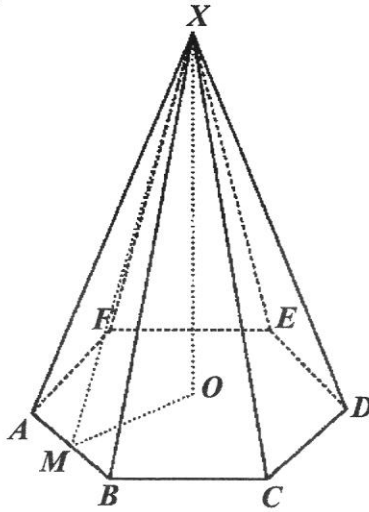
A regular hexagon, $ABCDEF$, has sides of 5 cm.

M is the midpoint of AB and O is the centre of the hexagon.

- (a) Show that the area of the hexagon $ABCDEF$ is 64.95 cm^2 , correct to 4 significant figures.

Answer

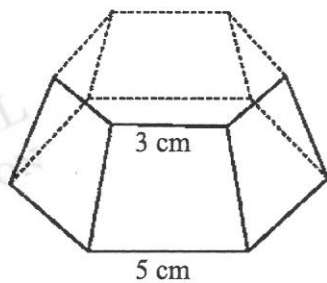
Hexagon $ABCDEF$ forms the base of a pyramid.
 The vertex, X , is directly above O .
 The height, OX , of the pyramid is 12 cm.



(b) Calculate the volume of the pyramid.

Answercm³ [2]

(c) The top part of the pyramid is cut off leaving the bottom portion as shown.
 The smaller hexagon has sides of 3 cm.
 Find the volume of the remaining bottom portion.



Answercm³ [2]

(d) Calculate the slant height, MX , of the pyramid.

DANYAL
EDUCATION

DANYAL
EDUCATION

Answercm [2]

(e) Calculate the **total** surface area of the pyramid.

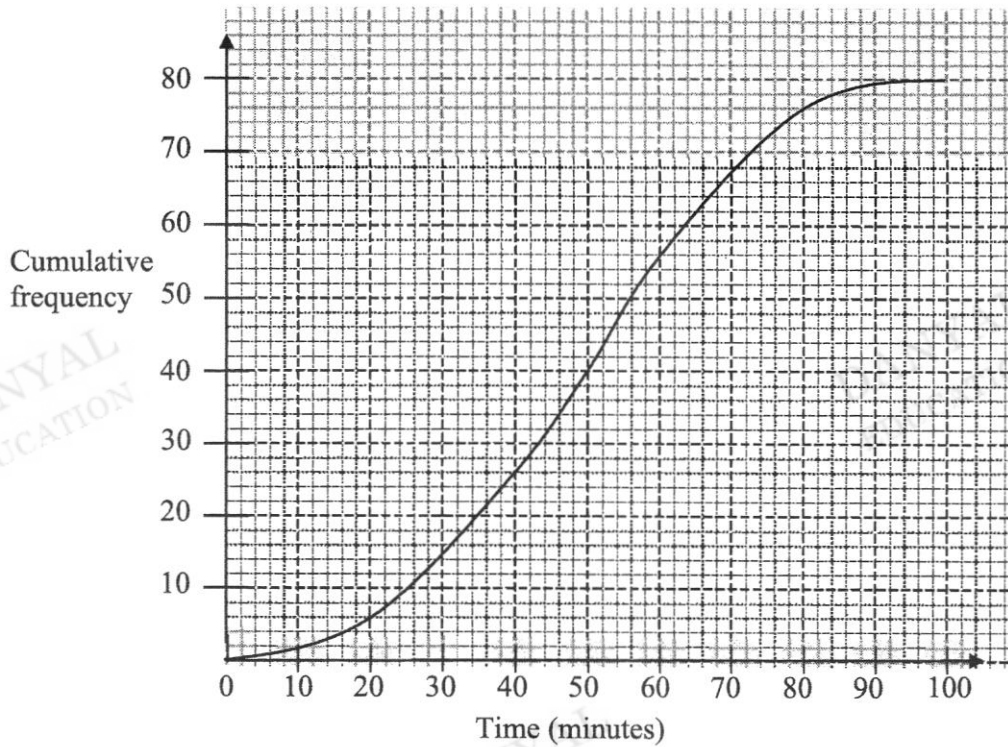
DANYAL
EDUCATION

DANYAL
EDUCATION

DANYAL
EDUCATION

Answercm² [2]

- 9 The amount of time 80 secondary school students spent on social media in a day are recorded.
The cumulative frequency curve below shows the distribution of their times.



- (a) Use the curve to estimate
(i) the median,

Answer min [1]

- (ii) the interquartile range of the times.

Answer min [2]

- (b) Estimate the percentage of secondary students who spent more than 70 min on social media per day.

Answer% [2]

- (c) Complete the grouped frequency table for the time spent on social media.

Time (min)	$0 \leq x < 20$	$20 \leq x < 40$	$40 \leq x < 60$	$60 \leq x < 80$	$80 \leq x < 100$
Frequency	6	20			

[1]

- (d) Calculate an estimate of the mean time spent on social media.

Answer min [1]

- (e) Calculate an estimate of the standard deviation.

Answer min [1]

- (f) Explain why the mean and standard deviation are estimates.

Answer

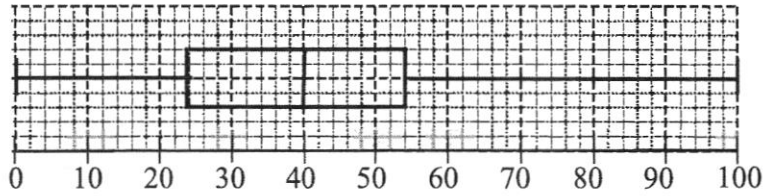
.....

.....

[1]

- (g) The amount of time 80 **primary** school students spent on social media in a day are also recorded.

The box-and-whisker plot shows the distribution of the times (in minutes).



Make two comments comparing the amount of time primary school students and secondary school students spent on social media.

Answer

.....

.....

.....

.....

.....

.....

.....

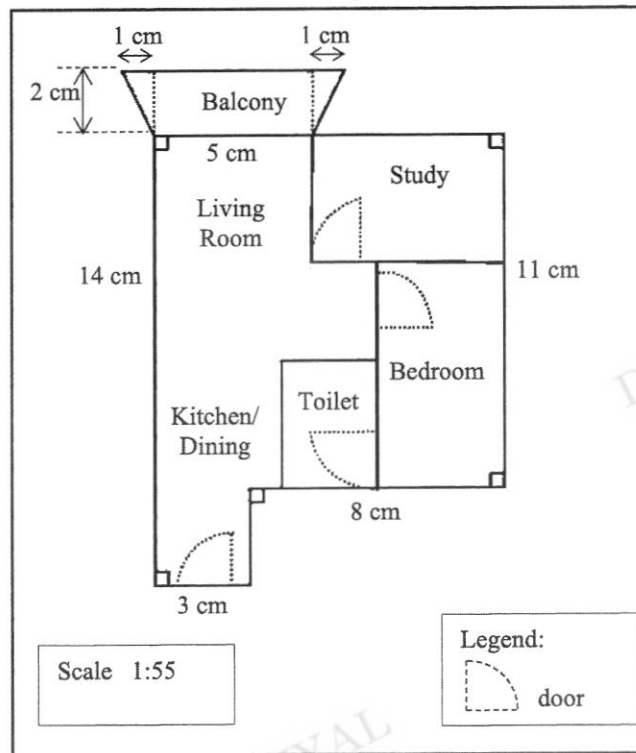
.....

.....

.....

[2]

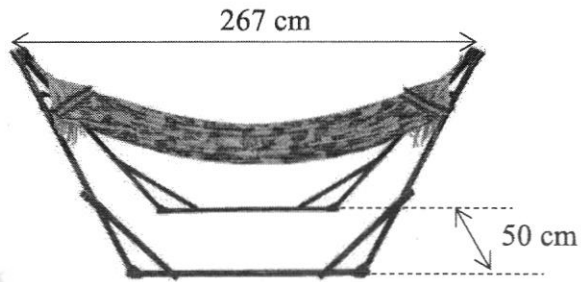
- 10 Xavier and his wife are planning to buy an apartment.
The brochure below shows the layout of the apartment they are interested in.



- (a) Find the **ratio of area** of the floor plan to the actual area of the apartment in $1 : n$.

Answer 1 : [1]

- (b) Xavier wanted to buy the following hammock to be placed at the trapezium-shaped balcony. He wanted a walking space of at least 30 cm to be all around when the hammock is placed at the balcony.



Show, with appropriate working, if he should buy this hammock.

Answer

He *should* / *should not* buy the hammock.
(circle the appropriate answer)

[3]

- (c) Xavier and his wife wanted to change the flooring for the whole apartment. They wanted the whole apartment to have the same flooring. The cost of different types of flooring materials and the cost of installation are found in the tables below.

Cost of different types of flooring materials

Type of flooring	Cost per square foot
Vinyl flooring	\$4 - \$7
Porcelain tiles	\$3 - \$5

<https://www.homerenoguru.sg/articles/renovation-essentials/flooring-singapore/>

Cost of flooring installation by material

Type of flooring	Cost per square foot
Vinyl flooring	\$4 - \$8
Porcelain tiles	\$7 - \$11

<https://www.homeadvisor.com/cost/flooring/install-flooring/>

Which type of flooring should they go for if they have limited budget? Suggest a suitable budget Xavier and his wife should set aside for changing the flooring.

Show your working clearly stating your assumption(s).

$$1 \text{ m}^2 = 10.7639 \text{ ft}^2$$

DANYAL
EDUCATION

DANYAL
EDUCATION

DANYAL
EDUCATION

DANYAL
EDUCATION

DANYAL
EDUCATION

Type of Flooring

[6]

Budget \$.....

Assumption(s)

.....

-----**End of Paper**-----

1. Factorise fully $2x^2 + y^2 - 2x - xy^2$.

$2x^2 + y^2 - 2x - xy^2$ $= 2x(x-1) + y^2(1-x)$ $= 2x(x-1) - y^2(x-1)$ $= (2x - y^2)(x-1)$	M1 (first level factorization) A1
--	--

Answer : $(2x - y^2)(x-1)$ [2]

2. It takes p workers q days to build r houses.
If the number of days is halved and $5r$ houses are to be built, how many workers must be hired for the job?

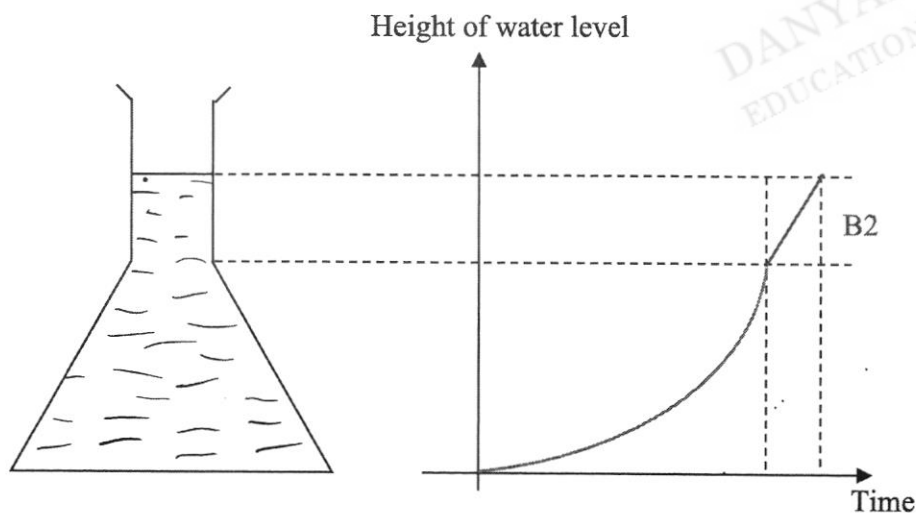
Express your answer in terms of p .

Workers : Days : Houses p : q : r <div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">$5p$</div> : q : <div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">$5r$</div> <div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">$10p$</div> : $\frac{q}{2}$: $5r$	OR Workers : Days : Houses p : q : r <div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">$2p$</div> : <div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">$\frac{q}{2}$</div> : r <div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">$10p$</div> : $\frac{q}{2}$: <div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">$5r$</div>	M1 (direct proportion W and H or Inverse proportion W and D) A1 ($10p$)
---	--	---

Answer : **10p** [2]

3. Water is poured at a constant rate into the conical flask as shown below.
Draw on the axes provided, the change in the water level of the conical flask over time.

Answer



[2]

4. Solve the equation $(1-x)^2 = \frac{9}{4}$.

$(1-x)^2 = \frac{9}{4}$ $1-x = \pm\sqrt{\frac{9}{4}}$ $1-x = \frac{3}{2} \quad \text{or} \quad 1-x = -\frac{3}{2}$ $x = 1 - \frac{3}{2} \qquad x = 1 + \frac{3}{2}$ $x = -\frac{1}{2} \qquad x = 2\frac{1}{2}$	<p>M1 (or if alternative M1 Quadratic Formula Step)</p> <p>A1</p>
--	---

Answer : $x = -\frac{1}{2}$, $x = 2\frac{1}{2}$ [2]

5. A group of 15 students recorded their timings (to the nearest minute), for a 5-km run. The results are represented in the stem and leaf diagram below.

Stem	Leaf
2	6 8 8 9
3	0 0 1 3 4 4 4 5 7 9
4	
5	
6	1

Key: 2|6 means 26 minutes

- (a) Find the percentage of students who took at least 35 minutes to complete the run.

$\frac{4}{15} \times 100 = 26\frac{2}{3}\%$	B1 (accept 26.7%)
---	----------------------

Answer : $26\frac{2}{3}\%$ [1]

- (b) Explain why the mean may not be an appropriate measure of average time taken by these students to complete the 5 km run.

There is an outlier / extreme value of 61. B1

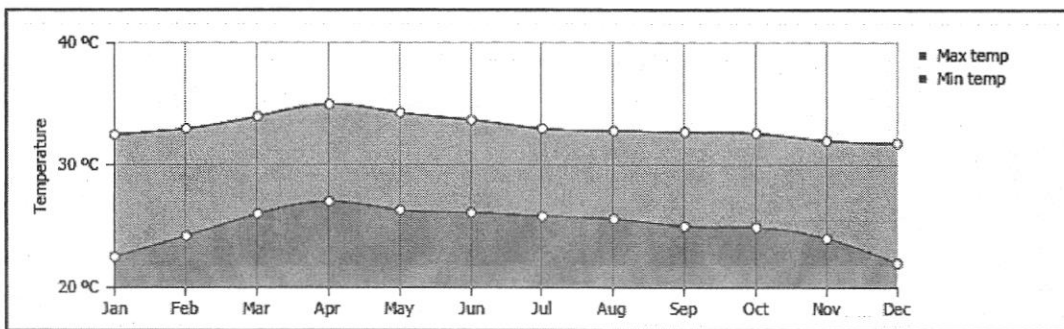
Answer :

Accept : Range is too wide because of 61/ anomaly
Any equivalent description of outlier

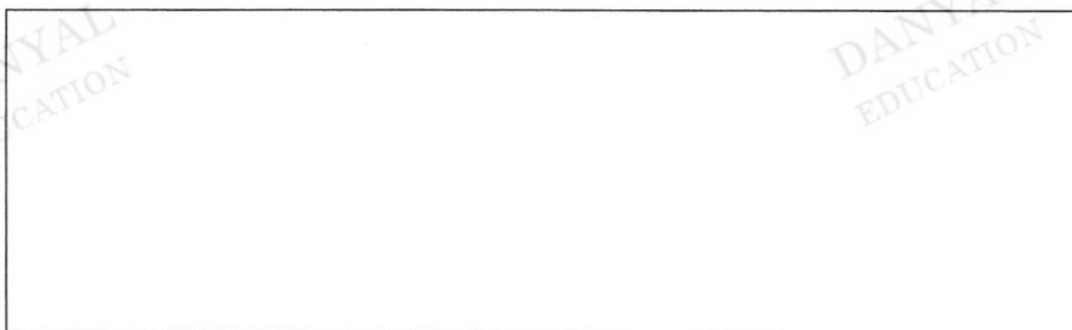
..... [1]

MARKING SCHEME

6. The line graphs below show the monthly average minimum and maximum temperatures of Bangkok (Thailand) and Singapore, for the year 2020.



Average min and max temperature, Bangkok Thailand Copyright 2020 www.weather-and-climate.com



Average min and max temperature, Singapore Copyright 2020 www.weather-and-climate.com

- (a) Huda claims that Singapore experienced wider differences between the maximum and minimum temperatures.

Explain why the data presented above may have been misleading for Huda.

Answer : The gap between lowest(min) and highest (max) temperatures appear greater for Singapore. This is due to the **different scale** used for the vertical axis. B1

The range on y-axis for Bangkok is 20-40 whereas for spore is 22-32 hence difficult to compare. [1]

- (b) Give a suggestion on how the data above can be presented in a clearer way.

Answer : 1. Use the same scale for vertical axis or
 2. Draw both on the same grid/same graph/same axes B1 [1]
 3. use Comparative Bar Graph/ Comparative line graph

Accept any equivalent to SCALE (unit per degree etc)

Do not accept if 'did not start from zero' without mentioning scale

MARKING SCHEME

7. A polygon with n sides has two exterior angles 100° and 50° .
The remaining $(n - 2)$ exterior angles are 14° each.

Find n .

$100 + 50 + 14(n - 2) = 360$ $14(n - 2) = 210$ $n - 2 = 15$ $n = 17$	<p>M1 (application sum of ext angles)</p> <p>A1</p>
--	--

Answer : $n = \dots 17 \dots$ [2]

8. Written as the product of its prime factors, $504 = 2^x \times 3^y \times 7$.

(a) Find the values of x and y .

$\begin{array}{r l} 2 & 504 \\ \hline 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$ $504 = 2^3 \times 3^2 \times 7$ $x = 3, y = 2$	<p>B1</p>
--	-----------

Answer : $x = \dots 3 \dots, y = \dots 2 \dots$ [1]

(b) The highest common factor and the lowest common multiple of 18 and z are 6 and 504 respectively. Find the smallest possible value of z .

$\begin{aligned} \text{HCF} &= 6 = 2 \times 3 \\ \text{LCM} &= 504 = 2^3 \times 3^2 \times 7 \end{aligned}$ $\begin{aligned} 18 &= 2 \times 3^2 \\ z &= 2^3 \times 3 \times 7 = 168 \end{aligned}$	<p>M1 (prime factorization of 18)</p> <p>A1</p>
---	---

Answer : smallest $z = \dots 168 \dots$ [2]

9. Simplify $\left(\frac{x^6}{64}\right)^{\frac{2}{3}} \div \frac{y^3}{x^6}$. Leave your answer in positive index.

$\begin{aligned} \left(\frac{x^6}{64}\right)^{\frac{2}{3}} \div \frac{y^3}{x^6} &= \left(\frac{64}{x^6}\right)^{\frac{2}{3}} \div \frac{y^3}{x^6} \\ &= \frac{16}{x^4} \times \frac{x^6}{y^3} \\ &= \frac{16x^6}{x^4y^3} \\ &= \frac{16x^2}{y^3} \end{aligned}$	<p>M1 (negative index to positive with reciprocal O.E)</p> <p>A1 ($16/y^3$) A1 (x^2) [allow A1 if only '16' is not seen]</p>
---	---

Answer : $\frac{16x^2}{y^3}$ [3]

10. Mr Png bought a massage chair with a down payment of 15% of the cash price and a monthly instalment of \$195.60 for 18 months.
If he paid a total of \$3970.80, find the cash price of the massage chair.

<p>Total monthly installment = \$195.60 × 18 = \$3520.80</p> <p>Downpayment = \$3970.8 – 3520.80 = \$450</p> <p>∴ Cash price = $\frac{450}{15} \times 100$ = \$3000</p>	<p>M1 (mthly x 18)</p> <p>M1 ($\sqrt{\text{'their' downpayment / 15} \times 100}$) A1</p>
--	---

Answer : \$... 3000 ... [3]

11. Given that $m = \sqrt{\frac{2m+1+k}{4k}}$, express k in terms m , in its simplest form.

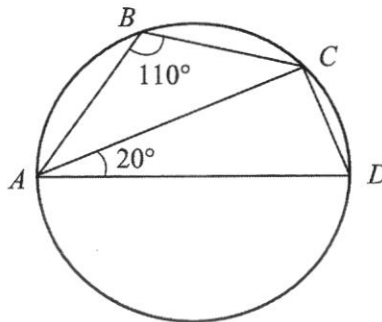
$\begin{aligned} m &= \sqrt{\frac{2m+1+k}{4k}} \\ m^2 &= \frac{2m+1+k}{4k} \\ 4km^2 &= 2m+1+k \\ 4km^2 - k &= 2m+1 \\ k(4m^2 - 1) &= 2m+1 \\ k &= \frac{2m+1}{4m^2 - 1} \\ k &= \frac{2m+1}{(2m+1)(2m-1)} \\ k &= \frac{1}{2m-1} \end{aligned}$	<p>M1 (square both sides)</p> <p>M1 (isolating k)</p> <p>A1</p>
---	--

Answer : $k = \frac{1}{2m-1}$ [3]

MARKING SCHEME

12. The diagram shows a circle with points A, B, C and D on its circumference. $AB = BC$, angle $ABC = 110^\circ$, and angle $CAD = 20^\circ$.

Explain why AD is a diameter of the circle.



Answer :

$\angle ADC = 180^\circ - 110^\circ \text{ (angles in opp. segment)}$ $= 70^\circ$	M1 (with reason)
$\angle ACD = 180^\circ - 70^\circ - 20^\circ \text{ (angle sum of } \Delta)$ $= 90^\circ$	M1 (accept if no reason provided above have)
<p>Since $\angle ACD = 90^\circ$, using right-angle in a semicircle property, AD is a diameter of the circle.</p>	A1

[3]

13. n is a positive integer.

(a) Show that $(3n - 2)^2 - n^2$ is a multiple of 4.

$(3n - 2)^2 - n^2 = 9n^2 - 12n + 4 - n^2$ $= 8n^2 - 12n + 4$ $= 4(2n^2 - 3n + 1)$	<p>M1 (expansion)</p> <p>A1 (accept if student able to explain that 4 is a factor of each term)</p> <p>Give B1 if $8n^2 - 12n + 4$ is evident in (b) but not in (a)</p>
---	--

[2]

(b) Hence or otherwise, factorise $(3n - 2)^2 - n^2$ fully.

$(3n - 2)^2 - n^2 = 4(2n^2 - 3n + 1)$ $= 4(2n - 1)(n - 1)$	B1
--	----

Answer : $4(2n - 1)(n - 1)$ [1]

MARKING SCHEME

14. A bean bag is filled with small polystyrene balls of radius 3 millimetres.

(a) Write the radius of the ball in metres, leaving your answer in standard form.

$3 \text{ mm} = 3 \div 1000 \text{ m}$ $= 3 \times 10^{-3} \text{ m}$	B1 (do not accept 3.0)
---	------------------------

Answer : 3×10^{-3} m [1]

(b) A bean bag of volume 2.88 m^3 is to be 80% filled with the spherical polystyrene balls.

Find the number of polystyrene balls required, giving your answer to the nearest million.

$\frac{80}{100} \times 2.88 = 2.304 \text{ m}^3$	M1
No. of polystyrene balls (vol. sphere) $= \frac{2.304}{\frac{4}{3} \pi (3 \times 10^{-3})^3} \text{ m}$	M1 (√ 'their' volume divide by vol sphere)
$= 20371832.72$	A1
$= \mathbf{20 \text{ million}}$	

Answer ... 20... million [3]

15. The scores for a Mathematics Test sat by six students are as follows:

$a, b, 59, c, 65, 90$

The range of the scores is 48.

The median score is 62.

The mean score is 61.

Find the values of a, b and of c .

a	b	59	c	65	90	
		↙				Median : $\frac{59+c}{2} = 62$
						$59+c = 124$
						$c = \mathbf{65}$ ----- B1

Using range :
 $a = 90 - 48 = \mathbf{42}$ -----B1

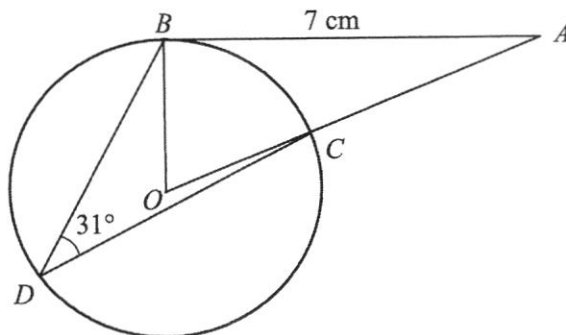
Mean : $\frac{42 + b + 59 + 65 + 65 + 90}{6} = 61$ ----- M1 (using 'their' 6 values)

$321 + b = 366$

$b = \mathbf{45}$ ----- A1

Answer : $a = \dots 42 \dots, b = \dots 45 \dots, c = \dots 65 \dots$ [4]

16.



In the diagram, B , C and D are points on a circle.

O is the centre, OCA is a straight line and BA is tangent to the circle at B .

Angle $BDC = 31^\circ$ and $AB = 7$ cm.

*Accept any full explanation describing
Tangent and radius, hence 90°*

(a) State the reason why angle $OBA = 90^\circ$.

Reason : Tangent is perpendicular to radius. B1 (write in full) [1]

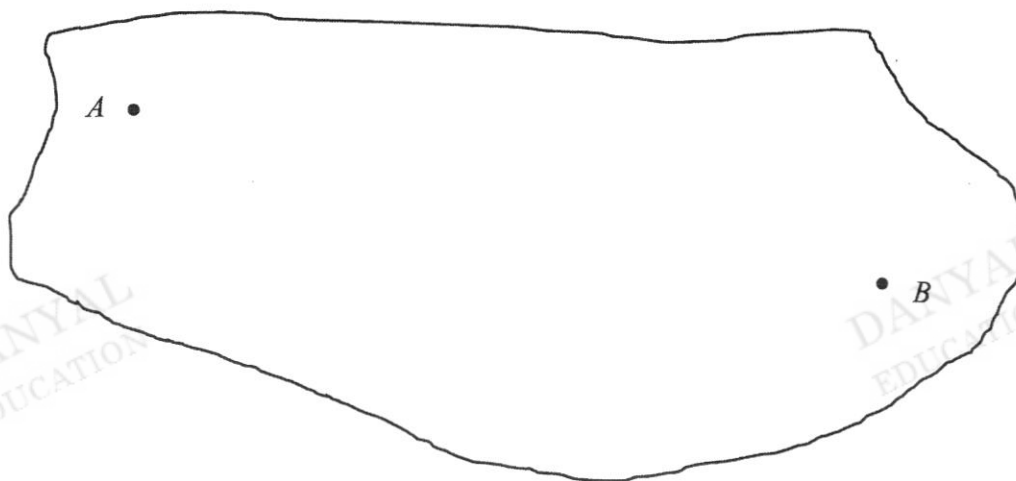
(b) Find the length of the radius of the circle.

$\angle BOC = 31^\circ \times 2$ (angle at centre = 2 angle circumference) $= 62^\circ$	M1
$\tan 62^\circ = \frac{7}{OB}$	M1 (correct trig ratio)
$OB = \frac{7}{\tan 62^\circ}$	
$OB = 3.72$ cm	A1

Answer : 3.72 cm [3]

17. (Diagram is drawn to scale)

The scale of the map below is 1 : 50 000.



Asher cycles from Point *A* to Point *B* at an average speed of 13 km/h.

If Asher has to reach point *B* by 4.30 pm, suggest the latest time he should set off from point *A*.

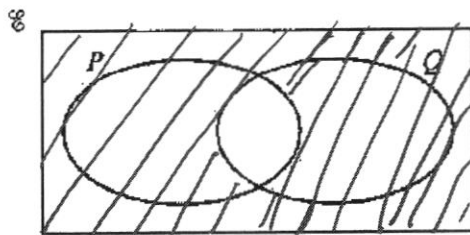
Show your working clearly.

<p>1 cm : 50 000 cm 1 cm : 0.5 km</p> <p>10.6 cm (measured) $\rightarrow 0.5 \times 10.6$ = 5.3 km</p> <p>Time = $\frac{5.3}{13}$ h = 0.407 h = 24 min 27 sec</p> <p>He should leave house at by 4.05pm latest.</p>	<p>M1 (using scale with or w/o conversion) (0.5 x 10.6 or 10.6 x 50000)</p> <p>M1 (dist in km over speed) M1 [Correct conversion to min] A1</p>
--	---

Answer : He should leave point *A* latest by ... **4.05** ... pm [4]

MARKING SCHEME

18. (a) On the Venn diagram, shade the region which represents $P' \cup Q'$.



B1 [1]

(b) $\varepsilon = \{x : x \text{ is an integer such that } 5 \leq x \leq 9\}$

$$A = \left\{x : 5 - \frac{x}{2} \geq 1\right\}$$

$$B = \{x : 4x - 1 > 19\}$$

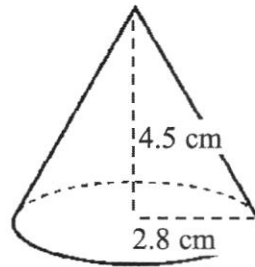
List the element(s) contained in the set $(A \cap B)'$.

$5 - \frac{x}{2} \geq 1$	$4x - 1 > 19$	
$5 - 1 \geq \frac{x}{2}$	$4x > 20$	
$4 \geq \frac{x}{2}$	$x > 5$ ---- M1 (or $B = \{6, 7, 8, 9\}$ B1)	
$8 \geq x$		
$x \leq 8$ ----- M1 (or $A = \{5, 6, 7, 8\}$ --- B1)		
$(A \cap B) = \{6, 7, 8\}$		
$(A \cap B)' = \{5, 9\}$		A1 (ecf that is appropriate)

Answer : $(A \cap B)' = \{ 5 , 9 \}$ [3]

MARKING SCHEME

19. The diagram shows a paper cup in a shape of a cone with radius 2.8 cm and vertical height 4.5 cm.



- (a) Show that the curved surface area is $14.84 \pi \text{ cm}^2$.

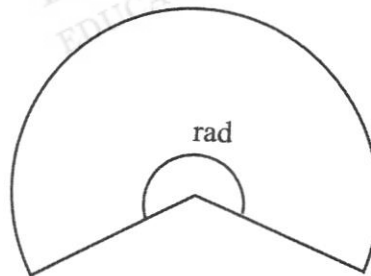
Answer :

<p>To find slant height (using Pythagoras' Theorem)</p> $l^2 = 2.8^2 + 4.5^2$ $l = 5.3$ <p>Curved Surface Area = $\pi r l$</p> $= \pi \times 2.8 \times 5.3$ $= 14.84 \pi \text{ cm}^2. \text{ (shown)}$	<p>M1/ B1</p> <p>B1</p>
---	-----------------------------

[2]

- (b) The paper cup is cut open to form a sector of a circle with angle θ radians.

Find angle θ .

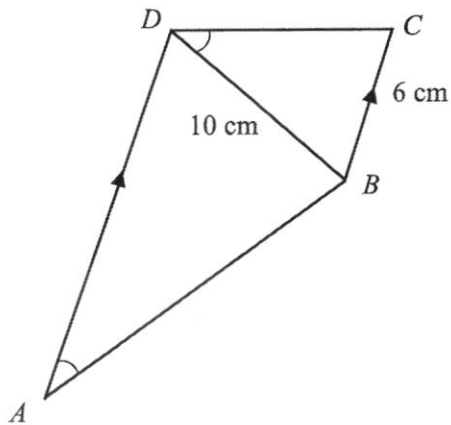


<p>Area Sector = Area Curved Surface Cone</p> $\frac{1}{2} r^2 \theta = 14.84 \pi$ $\frac{1}{2} (5.3)^2 \theta = 14.84 \pi$ $\theta = \frac{14.84 \pi}{\frac{1}{2} (5.3)^2}$ $\theta = 3.32 \text{ rad}$	<p>M1 (form equation connecting area sector and cone S.A)</p> <p>Accept equation connecting arc length + $2r$ = conebase circumference.</p> <p>A1</p>
--	--

Answer : $\theta = \dots\dots\dots 3.32 \dots\dots \text{ rad}$ [2]

MARKING SCHEME

20. In the diagram, the line AD is parallel to BC and $\angle BAD$ is equal to $\angle CDB$. $\angle ABD = 81^\circ$, $BC = 6$ cm and $BD = 10$ cm.



- (a) Explain why $\triangle ABD$ is similar to $\triangle DCB$.

Answer $\angle ADB = \angle CBD$ (alt angles) B1(both correct)
 $\angle BAD = \angle CDB$ (given)
 $\therefore \triangle ABD$ is similar to $\triangle DCB$ (AA Similarity Test) B1
 [2]

- (b) Find the length of AD .

$\frac{AD}{10} = \frac{10}{6}$ $6 AD = 100$ $AD = 16\frac{2}{3} \text{ cm}$	<p>M1 (correct ratio)</p> <p>A1 (accept 16.7)</p>
---	---

Answer : $16\frac{2}{3}$ cm [2]

21. The equation of a curve is given by $y = x(10 - x)$.

(a) Explain why the maximum value of y is 25.

The x -intercepts are 0 and 10. [B1]
Answer
 The line of symmetry is $x = \frac{0+10}{2} = 5$ [B1]

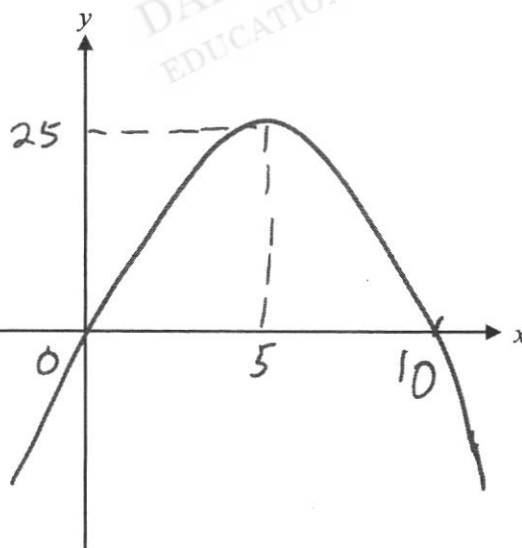
 Maximum occur at line of symmetry.

 When $x = 5, y = 5(10 - 5) = 25$ [B1 (sub $x=5$)]

 [3]

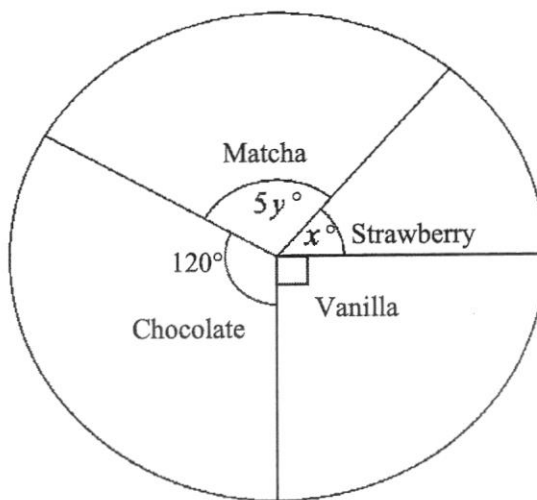
(b) Sketch the curve $y = x(10 - x)$, showing clearly the intercepts and the maximum point.

B1 open downwards, cutting at 0 and 10
 B1 max point (5,25)



[2]

22. The pie chart represented the number of people who chose their favourite milkshake flavour.



- (a) Form an equation in terms of x and y .

$5y + x + 120 + 90 = 360$ $5y + x = 150$	B1 (accept non-simplified) Answer $5y + x = 150$ [1]
--	---

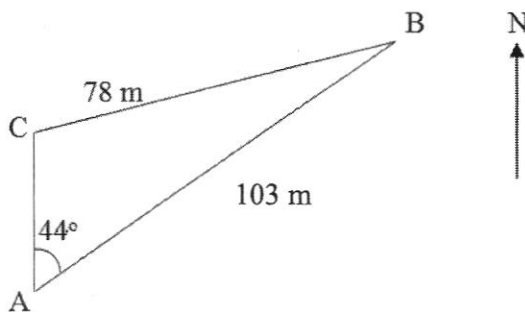
- (b) The ratio of people who chose Strawberry to Matcha flavour was 4 : 11.
Show that $11x - 20y = 0$.

$\frac{x}{5y} = \frac{4}{11}$ $11x = 20y$ $11x - 20y = 0$	B1 (starting with ratio form) [1]
---	--------------------------------------

- (c) Using your equations from (a) and (b), solve them simultaneously to find the values of x and of y .

$5y + x = 150$ $x = 150 - 5y \text{ ----- [1]}$ $11x - 20y = 0 \text{ ----- [2]}$ Sub [1] into [2] $11(150 - 5y) - 20y = 0$ $1650 - 55y - 20y = 0$ $1650 - 75y = 0$ $75y = 1650$ $y = 22$ And $x = 150 - 5(22) = 40$	M1 [elimination or substitution step] A1 A1
--	---

23.



A, B and C are 3 points on level ground.
 $BC = 78$ m, $AB = 103$ m and angle $BAC = 44^\circ$. C is due north of A .

(a) Calculate the bearing of A from B .

$180^\circ + 44^\circ = 224^\circ$	B1
------------------------------------	----

Answer : $\dots\dots\dots 224\dots\dots\dots^\circ$ [1]

(b) Find the area of triangle ABC .

<p>To find $\angle ACB$, using sine rule” $\frac{\sin \hat{ACB}}{103} = \frac{\sin 44^\circ}{78}$ $\hat{ACB} = 66.5352^\circ$ Taking Obtuse angle $= 180^\circ - 66.5352^\circ$ $= 113.5^\circ$ $\angle CBA = 180^\circ - 113.5^\circ - 44^\circ$ $= 22.535^\circ$ Area of $\Delta = \frac{1}{2} \times 78 \times 103 \times \sin 22.535^\circ$ $= 1539.5228$ $= 1540 \text{ m}^2$</p>	<p>M1 (correct application of sine rule) A1 M1 (using ‘their’ angle CBA) A1</p>
---	--

Answer : $\dots\dots\dots 1540 \dots\dots \text{m}^2$ [4]

BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME

1	(a)	Write as a single fraction in its simplest form	
	(i)	$\frac{9a^3}{b} \div \frac{81a}{b^7}$,	
		$\frac{a^2b^6}{9}$ [B1]	
	(ii)	$\frac{5}{(y-3)^2} - \frac{7}{3-y}$.	
		$\frac{5}{(y-3)^2} + \frac{7}{y-3}$ [M1 – changing denominator to $y-3$] $= \frac{5+7(y-3)}{(y-3)^2}$ $= \frac{7y-16}{(y-3)^2}$ [A1]	
	(b)	Simplify $\frac{8x^2-18}{2x^2-x-6}$.	
		$\frac{2(4x^2-9)}{(2x+3)(x-2)}$ [M1 for factorising denominator] $= \frac{2(2x+3)(2x-3)}{(2x+3)(x-2)}$ [M1 for factorising numerator - at least twice] $= \frac{2(2x-3)}{(x-2)}$ or $= \frac{4x-6}{(x-2)}$ [A1]	
	(c)	Solve the equation $\frac{32}{x-5} = 3x-5$.	
		$32 = 3x^2 - 20x + 25$ [M1 for multiplying $x-5$ on both sides and expanding RHS] $(3x+1)(x-7) = 0$ [M1 for all terms on one side and factorising/quadratic formula] $x = -\frac{1}{3}$ or $x = 7$ [A1 for both answers]	

BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME

2	<p>(a) From 2019 to 2020 the total number of international visitors in Singapore decreased by 85.9%.</p> <p>In 2020, the total number was 2.7×10^6.</p> <p>Calculate the number of international visitors in 2019, giving your answer in standard form.</p>							
	$\frac{2.7 \times 10^6}{14.1} \times 100 \quad [\text{M1 for finding } 1\%]$ $= 1.91 \times 10^7 \quad [\text{A1}]$							
	<p>(b) Siti invested some money in a saving accounts for 4 years.</p> <p>The rate of interest was fixed at 1.08% per annum compounded annually.</p> <p>At the end of 4 years, there was \$8351.24 in her account.</p> <p>How much did Siti invest in the account?</p> <p>Give your answer correct to the nearest cent.</p>							
	$8351.24 = P\left(1 + \frac{1.08}{100}\right)^4 \quad [\text{M1}]$ $P = \$8000.00 \quad [\text{A1 - to nearest cent}]$							
	<p>(c) The exchange rate between Singapore dollars (\$) and euros is $\\$1 = \text{€}0.63$.</p> <p>Andy is shopping online for a pair of boots.</p> <p>He finds these prices online for the same model of boots.</p> <table border="1" style="width: 100%; margin: 10px 0;"> <thead> <tr> <th style="text-align: left;">Website</th> <th style="text-align: left;">Price</th> </tr> </thead> <tbody> <tr> <td>Zalora</td> <td>\$195 (before 7% GST)</td> </tr> <tr> <td>Footshopping</td> <td>€130 Nett</td> </tr> </tbody> </table>	Website	Price	Zalora	\$195 (before 7% GST)	Footshopping	€130 Nett	
Website	Price							
Zalora	\$195 (before 7% GST)							
Footshopping	€130 Nett							
	<p>(i) Both websites offer free shipping to Singapore.</p> <p>GST needs to be paid if he buys from Zalora.</p> <p>It is also known that Andy's credit card charges a foreign currency transaction fee of 3.25%.</p> <p>Which website offers a better deal?</p> <p>Show your working clearly.</p>							
	<p><u>Zalora</u></p> $1.07 \times 195 = \$208.65 \quad [\text{M1 for calculating cost including GST}]$ <p><u>Footshopping</u></p> $\text{€}130 \times 1.0325 = \text{€}134.225 = \213.06 <p>[M1 for calculating cost inclusive of transaction fee and converting to \$]</p> <p>[accept if students convert Zalora to euros instead]</p> <p><u>Zalora</u> offers a better deal. [A1]</p>							

BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME

	(ii)	A certain debit card offers $x\%$ foreign currency transaction fee. x must be less than a certain value so that buying from Footshopping will be a better deal. Find that value.																
		$\text{€}130 = \$206.35$ [M1 for converting €130 to \$] $\frac{\$208.65 - \$206.35}{206.35} \times 100\% = 1.11\%$ [A1 for 1.11 without % accept 1.12]																
3		Gardens by the Bay Flower Dome operates for 340 days a year. The matrix, \mathbf{M} , shows the number of different types of tickets (in thousands) sold per day in 2019. <div style="text-align: center;"> <table style="margin: auto;"> <tr> <td></td> <td>Child</td> <td>Adult</td> <td>Senior</td> <td></td> </tr> <tr> <td>$\mathbf{M} =$</td> <td>$\begin{pmatrix} 1.2 & 2 & 5 \\ 2.1 & 4.5 & 9 \end{pmatrix}$</td> <td>Residents</td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td>Non-residents</td> <td></td> <td></td> </tr> </table> </div>		Child	Adult	Senior		$\mathbf{M} =$	$\begin{pmatrix} 1.2 & 2 & 5 \\ 2.1 & 4.5 & 9 \end{pmatrix}$	Residents					Non-residents			
	Child	Adult	Senior															
$\mathbf{M} =$	$\begin{pmatrix} 1.2 & 2 & 5 \\ 2.1 & 4.5 & 9 \end{pmatrix}$	Residents																
		Non-residents																
	(a)	Evaluate the matrix $\mathbf{P} = 340\mathbf{M}$.																
		$\mathbf{P} = \begin{pmatrix} 408 & 680 & 1700 \\ 714 & 1530 & 3060 \end{pmatrix}$ [B1]																
	(b)	The tickets cost \$10, \$16 and \$14 for child, adult and senior respectively. Represent these amounts in a 3×1 matrix \mathbf{N} .																
		$\mathbf{N} = \begin{pmatrix} 10 \\ 16 \\ 14 \end{pmatrix}$ [B1]																
	(c)	Evaluate the matrix $\mathbf{T} = \mathbf{PN}$.																
		$\mathbf{T} = \begin{pmatrix} 38760 \\ 74460 \end{pmatrix}$ [B2] or [M1 for either 38760 or 74460. Allow ecf]																
	(d)	State what each of the elements of \mathbf{T} represents.																
		Total amount of money in thousands collected from ticket sales from residents and non-residents respectively. [B1]																

**BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME**

	(e)	Calculate the total amount of ticket sales in 2019. Give your answer correct to the nearest million.	
		113220×10^3 $= 113.22 \times 10^6$ 113 [B1]	
	(f)	In 2020, the number of tickets sold for residents increased by 60% across the different types of tickets. In the same year, the number of tickets sold for non-residents dropped to 20% across the different types of tickets. Calculate the percentage change in the amount of ticket sales from 2019 to 2020. State whether this change is an increase or a decrease.	
		<u>Amount taken in tickets sales for 2020</u> $38760 \times 1.6 + 74460 \times 0.2$ $= 76908$ [M1] accept other methods e.g. $\begin{pmatrix} 652.8 & 1088 & 2720 \\ 142.8 & 306 & 612 \end{pmatrix} \begin{pmatrix} 10 \\ 16 \\ 14 \end{pmatrix}$ <u>Percentage change</u> $\frac{113220 - 76908}{113220} \times 100\%$ $= 32.1\%$ [A1] decrease [A1]	
4	(a)	The first three terms in a sequence of numbers, T_1, T_2, T_3, \dots are given below. $T_1 = 1^2 + 1 = 2$ $T_2 = 2^2 + 3 = 7$ $T_3 = 3^2 + 5 = 14$	
	(i)	Find T_4 .	
		$T_4 = 23$ [B1]	
	(ii)	Find an expression, in terms of n , for T_n .	
		$T_n = n^2 + 2n - 1$ [B1 for n^2 ; B1 for $2n - 1$] [M1 for correct expression without simplification]	

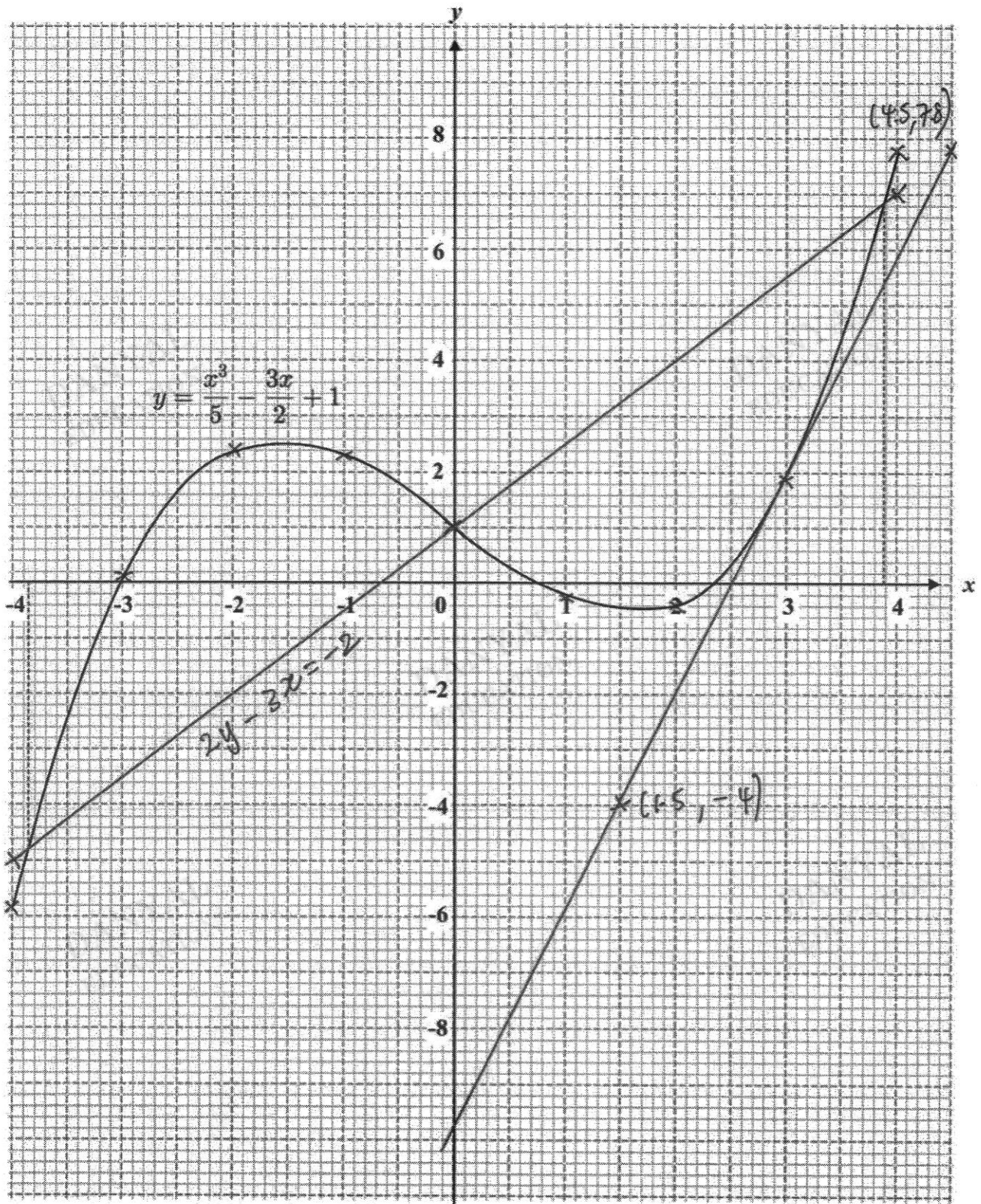
BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME

	(b)	The first four terms in a different sequence are $-55, -51, -47, -43$.	
	(i)	Find an expression, in terms of n , for the n th term, P_n , of this sequence.	
		$P_n = 4n - 59$ [B1 for $4n$; B1 for -59]	
	(ii)	Explain why 222 is not a term of this sequence.	
		$222 = 4n - 59$ $n = 70.25$ Since n is not an integer, 222 is not a term of this sequence. [B1]	
	(iii)	Find the least value of n for which $P_n > 1$.	
		$4n - 59 > 1$ $n > 15$ [M1] Least value of n is <u>16</u> . [A1] or [B2]	

BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME

5	<p>The variables x and y are connected by the equation $y = \frac{x^3}{5} - \frac{3x}{2} + 1$.</p> <p>Some corresponding values of x and y are given in the table below.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">-4</td> <td style="padding: 5px;">-3</td> <td style="padding: 5px;">-2</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">4</td> </tr> <tr> <td style="padding: 5px;">y</td> <td style="padding: 5px;">p</td> <td style="padding: 5px;">0.1</td> <td style="padding: 5px;">2.4</td> <td style="padding: 5px;">2.3</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">-0.3</td> <td style="padding: 5px;">-0.4</td> <td style="padding: 5px;">1.9</td> <td style="padding: 5px;">7.8</td> </tr> </table>		x	-4	-3	-2	-1	0	1	2	3	4	y	p	0.1	2.4	2.3	1	-0.3	-0.4	1.9	7.8	
x	-4	-3	-2	-1	0	1	2	3	4														
y	p	0.1	2.4	2.3	1	-0.3	-0.4	1.9	7.8														
	(a)	Find the value of p .																					
		$p = -5.8$ [B1]																					
	(b)	Draw the graph of $y = \frac{x^3}{5} - \frac{3x}{2} + 1$ for $-4 \leq x \leq 4$ in the given grid.	[3]																				
		Points – [B2 for all points correctly plotted; B1 for 1 mistake] Curve – [B1 for smooth curve]																					
	(c)	(i) On the same grid, draw the graph of $2y - 3x = 2$ for $-4 \leq x \leq 4$.	[2]																				
		Points – [B1 for at least 2 points] Curve – [B1 for line]																					
		(ii) Show that the points of intersection of the line and the curve give the solutions of the equation $x^3 - 15x = 0$.																					
		$\frac{x^3}{5} - \frac{3x}{2} + 1 = \frac{3}{2}x + 1$ [M1 or equivalent e.g. substitution] $\frac{x^3}{5} - 3x = 0$ $x^3 - 15x = 0$ [A1]																					
			[2]																				
	(iii)	Hence, solve the equation $x^3 - 15x = 0$.																					
		$x = -3.9 \pm 0.1$ or $x = 0$ or $x = 3.9 \pm 0.1$ [B2] [B1 for any one correct answer] Do not accept $x = -3.87$ or $x = 3.87$																					
	(d)	By drawing a tangent, find the gradient of the curve at $(3, 1.9)$.																					
		[M1 for tangent line drawn] $m = 3.93 \pm 0.5$ [A1]																					

BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
 MARKING SCHEME



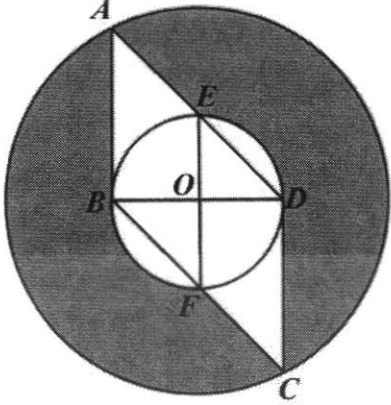
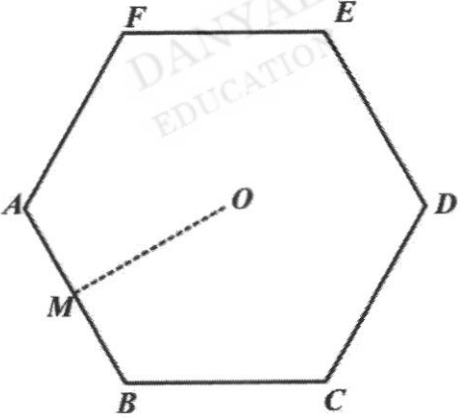
BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME

6	Jacelyn and Patrine went for a 21 km hike.	
(a)	Jacelyn walked at a constant speed of x kilometres per hour. Write down an expression, in terms of x , for the number of hours she took.	
	$\frac{21}{x}$ [B1]	
(b)	Patrine walked at a constant speed which was $\frac{1}{3}$ km/h more than Jacelyn's speed. Write down an expression, in terms of x , for the number of hours she took.	
	$\frac{21}{x + \frac{1}{3}}$ [B1] accept $\frac{63}{3x+1}$	
(c)	The difference between their times was 15 minutes. Write down an equation in x to represent this information, and show that it reduces to $3x^2 + x - 84 = 0$.	
	$\frac{21}{x} - \frac{21}{x + \frac{1}{3}} = \frac{15}{60}$ [M1] $\frac{21(x + \frac{1}{3}) - 21x}{x(x + \frac{1}{3})} = \frac{1}{4}$ [M1 for putting as common denominator] $3x^2 + x - 84 = 0$ [A1]	
(d)	Solve the equation $3x^2 + x - 84 = 0$, giving the answers correct to three decimal places.	
	$x = \frac{-1 \pm \sqrt{1^2 - 4(3)(-84)}}{2(3)}$ [M1] $x = \frac{-1 \pm \sqrt{1009}}{6}$ $x = 5.127$ [A1] or $x = -5.461$ [A1] [M1 for 3 decimal places]	

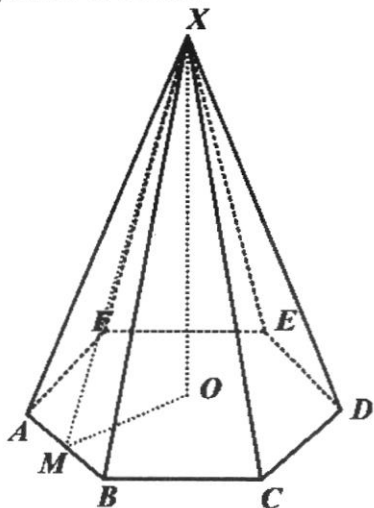
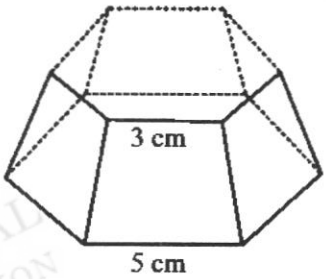
**BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME**

	(e)	Calculate the time that Jacelyn took to complete the hike, giving your answer in hours, minutes and seconds.	
		$Time = \frac{21}{5.127}$ [M1]	
		= 4h 5min 44sec/ 45 sec [A1]	
7		<p>The centers of the two circles are at O. BD is a diameter of the smaller circle. AB and CD are tangents to the smaller circle.</p>	
	(a)	Show that triangle ABD is congruent to triangle CDB . Give a reason for each statement you make.	
		$BD = DB$ (common side) $\angle ABD = \angle CDB$ (tangent \perp radius)	
		Since $BO = OD$ and $\angle ABO = \angle CDO = 90^\circ$ Therefore $AG = CH$ (Equal chords) $AB = BG$ and $CD = DH$ (\perp bisectors of chord) Therefore $AB = CD$	B2/ B1/ 0
		Triangle ABD is congruent to triangle CDB (SAS property)	[B1]
	(b)	Suppose the diameter of the smaller circle is 8 cm, angle $BAD = 45^\circ$ and BD is perpendicular to EF .	
	(i)	Calculate the length of OA .	
		$OA = \sqrt{8^2 + 4^2}$ [M1 – accepts other correct method] $OA = 8.94$ [A1]	
	(ii)	Calculate the area of triangle ABD .	
		32/ 32.0 [B1]	
	(iii)	Calculate the area of sector OBE , giving your answer in terms of π .	
		4π [B1]	

BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME

	(iv) Calculate the shaded area.	
		
	<p>Area of $ABE = 32 - 4\pi - \frac{1}{2} \times 4 \times 4$ $= 24 - 4\pi$ [M1]</p> <p>Area of bigger circle $= \pi\sqrt{80}^2$ [M1 for finding area of bigger circle]</p> <p>Shaded area $= \pi\sqrt{80}^2 - \pi \times 4^2 - 2(24 - 4\pi)$ $= 178 \text{ cm}^2$ [A1]</p>	
8		
	<p>A regular hexagon, $ABCDEF$, has sides of 5 cm. M is the midpoint of AB and O is the centre of the hexagon.</p>	
	(a) Show that the area of the hexagon $ABCDEF$ is 64.95 cm^2 , correct to 4 significant figures.	
	<p>Area of triangle $OAB = \frac{1}{2} \times 5 \times 5 \times \sin 60^\circ$ [M1 for finding area of 1 Δ]</p> <p>Area of hexagon $= 6 \times \frac{1}{2} \times 5 \times 5 \times \sin 60^\circ$ [M1 for multiplying by 6]</p> <p>Area of hexagon $= 64.95 \text{ cm}^2$ [A1]</p>	

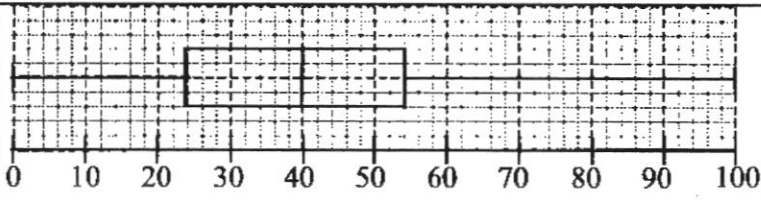
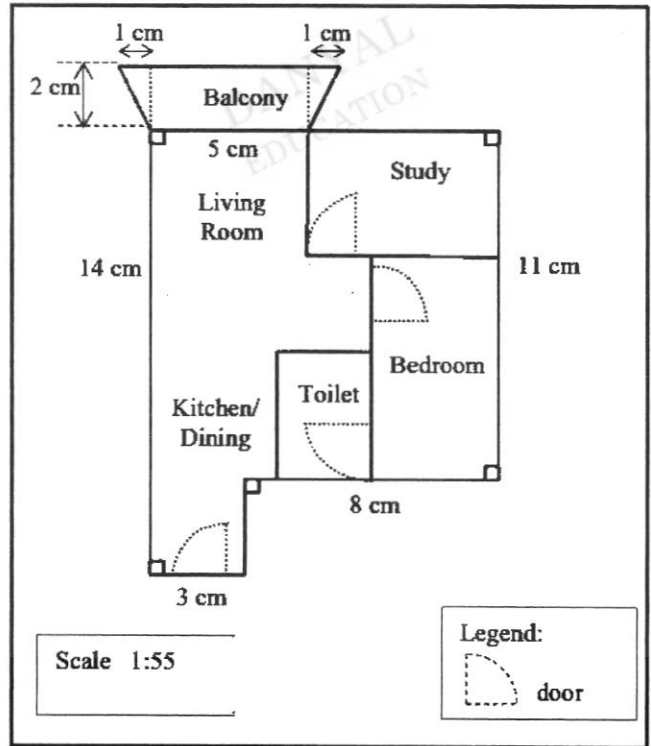
BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME

	<p>Hexagon $ABCDEF$ forms the base of a pyramid. The vertex, X, is directly above O. The height, OX, of the pyramid is 12 cm.</p> 	
(b)	<p>Calculate the volume of the pyramid.</p>	
	<p>$Vol = \frac{1}{3} \times 64.95 \times 12$ [M1] $Vol = 260 \text{ cm}^3$ [A1]</p>	
(c)	<p>The top part of the pyramid is cut off leaving the bottom portion as shown. The smaller hexagon has sides of 3 cm. Find the volume of the remaining bottom portion.</p> 	
	<p>Ratio of vol = 27: 125 [M1 for finding ratio of volume. Accept other method] Remaining units = 125 - 27 = 98 125 units - 259.8 cm^3 98 units - 204 cm^3 [A1]</p>	
(d)	<p>Calculate the slant height, MX, of the pyramid</p>	
	<p>$\tan 30^\circ = \frac{2.5}{OM}$ $OM = \frac{2.5}{\tan 30^\circ}$ [M1 for finding length of OM] $MX = 12.8 \text{ cm}$ [A1]</p>	

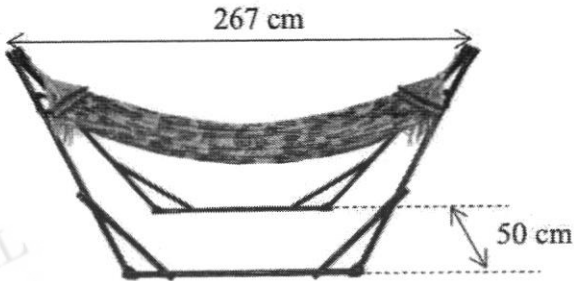
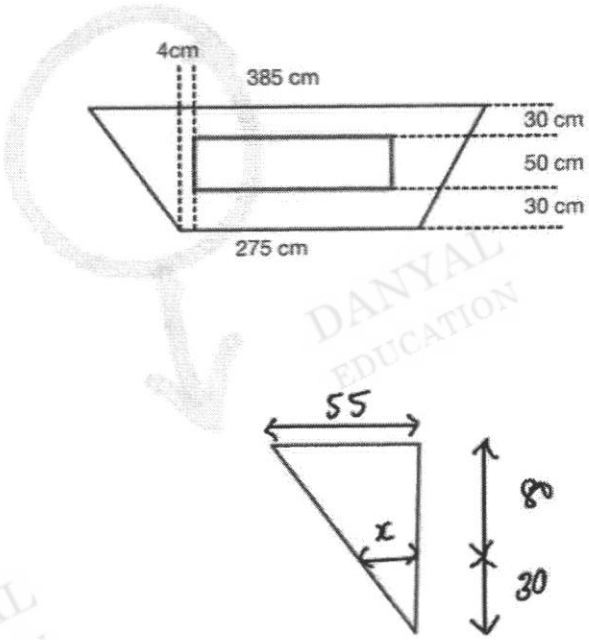
**BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME**

	(e)	Calculate the total surface area of the pyramid.					
		Area of 6 lateral triangles = $6 \times \frac{1}{2} \times 5 \times 12.7573\dots$ [M1] TSA = 256 cm ² [A1]					
9	The amount of time 80 secondary school students spent on social media in a day are recorded. The cumulative frequency curve below shows the distribution of their times.						
	(a)	Use the curve to estimate					
	(i)	the median,					
		50 min [B1]					
	(ii)	the interquartile range of the times.					
		IQR = 63 – 35 [M1 accept 64 – 35] IQR = 28 [A1 accept 29]					
	(b)	Estimate the percentage of secondary students who spent more than 70 min on social media per day.					
		Number of students who spent more than 70min = 80 – 68 [M1 accept 80 – 67] Percentage = 15% [A1 accept 16.25%]					
	(c)	Complete the grouped frequency table for the time spent on social media.					
		Time (min)	$0 \leq x < 20$	$20 \leq x < 40$	$40 \leq x < 60$	$60 \leq x < 80$	$80 \leq x < 100$
		Frequency	6	20	30	20	4
		[B1 for all 3 correct values]					
	(d)	Calculate an estimate of the mean time spent on social media.					
		Mean = $\frac{10 \times 6 + 30 \times 20 + 50 \times 30 + 70 \times 20 + 90 \times 4}{80}$ Mean = 49 [B1/ ecf]					
	(e)	Calculate an estimate of the standard deviation.					
		SD = 20.0 [B1/ ecf]					
	(f)	Explain why the mean and standard deviation are estimates.					
		We do not know the exact time each student spent on social media. [B1] or Mid-values are used in the calculation. [B1]					

**BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME**

	<p>(g) The amount of time 80 primary school students spent on social media in a day are also recorded. The box-and-whisker plot shows the distribution of the times (in minutes).</p>	
		
	<p>Make two comments comparing the amount of time primary school students and secondary school students spent on social media.</p>	
	<p>The median time for primary school students is 10 min lower than secondary school students => On average, secondary school students spend more time on social media.</p>	[B1]
	<p>The interquartile range of the primary school students is higher than secondary school students => higher spread among the primary school students.</p>	[B1]
<p>10</p>	<p>Xavier and his wife are planning to buy an apartment. The brochure below shows the layout of the apartment they are interested in.</p> 	
	<p>(a) Find the ratio of area of the floor plan to the actual area of the apartment in $1 : n$.</p>	
	<p>1: 3025 [B1]</p>	

BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME

<p>(b)</p>	<p>Javier wanted to buy the following hammock to be placed at the trapezium-shaped balcony. He wanted a walking space of at least 30 cm to be all around when the hammock is placed at the balcony.</p>
	
	<p>Show, with appropriate working, if he should buy this hammock.</p>
	 <p>[M1 for evidence of 30 cm, 50 cm, 30 cm or 110 cm]</p> $\frac{x}{55} = \frac{30}{110}$ <p>$x = 15$ cm [M1 for applying similar triangles]</p> <p>narrowest distance = $15 + 4 = 19$ cm</p> <p>He <u>should not</u> buy the hammock. [A1 - ecf]</p>

BVSS MYE 2021 4E5N E. MATHEMATICS PAPER 2 (4048/02)
MARKING SCHEME

(c)	<p>Xavier and his wife wanted to change the flooring for the whole apartment. They wanted the whole apartment to have the same flooring. The cost of different types of flooring materials and the cost of installation are found in the tables below.</p>												
	<p>Cost of different types of flooring materials</p> <table border="1" data-bbox="351 357 890 467"> <thead> <tr> <th>Type of flooring</th> <th>Cost per square foot</th> </tr> </thead> <tbody> <tr> <td>Vinyl flooring</td> <td>\$4 - \$7</td> </tr> <tr> <td>Porcelain tiles</td> <td>\$3 - \$5</td> </tr> </tbody> </table> <p>https://www.homerenoguru.sg/articles/renovation-essentials/flooring-singapore/</p> <p>Cost of flooring installation by material</p> <table border="1" data-bbox="351 566 1273 675"> <thead> <tr> <th>Type of flooring</th> <th>Cost per square foot</th> </tr> </thead> <tbody> <tr> <td>Vinyl flooring</td> <td>\$4 - \$8</td> </tr> <tr> <td>Porcelain tiles</td> <td>\$7 - \$11</td> </tr> </tbody> </table> <p>https://www.homeadvisor.com/cost/flooring/install-flooring/</p>	Type of flooring	Cost per square foot	Vinyl flooring	\$4 - \$7	Porcelain tiles	\$3 - \$5	Type of flooring	Cost per square foot	Vinyl flooring	\$4 - \$8	Porcelain tiles	\$7 - \$11
Type of flooring	Cost per square foot												
Vinyl flooring	\$4 - \$7												
Porcelain tiles	\$3 - \$5												
Type of flooring	Cost per square foot												
Vinyl flooring	\$4 - \$8												
Porcelain tiles	\$7 - \$11												
	<p>Which type of flooring should they go for if they have limited budget? Suggest a suitable budget for Xavier and his wife. Show your working clearly stating your assumption(s). $1 \text{ m}^2 = 10.7639 \text{ ft}^2$</p>												
	<p>Area of floor plan = $11 \times 11 + \frac{1}{2}(5 + 7) \times 2 + 3 \times 3$ Area of floor plan = 142 cm^2 [M1]</p> <p>$142 \text{ cm}^2 : 42.955 \text{ m}^2$ [M1 for finding actual area in either m^2 or cm^2] $42.955 \text{ m}^2 = 463 \text{ ft}^2$ (rounded up) [M1 for converting to square feet and rounding up]</p> <p><u>Vinyl</u> Cost = $\\$7 \times 463 + \\$8 \times 463 = \\$6945$ [assume highest in range] Cost = $\\$5.5 \times 463 + \\$6 \times 463 = \\$5324.50$ [assume mid-point] Cost = $\\$4 \times 463 + \\$4 \times 463 = \\$3704$ [assume lowest in range]</p> <p><u>Procelain</u> Cost = $\\$5 \times 463 + \\$11 \times 463 = \\$7408$ [assume highest in range] Cost = $\\$4 \times 463 + \\$9 \times 463 = \\$6019$ [assume highest in range] Cost = $\\$3 \times 463 + \\$7 \times 463 = \\$4630$ [assume lowest in range] [M1 for calculating total cost for Vinyl and Procelain]</p> <p>Type of flooring: <u>Vinyl</u></p> <p>Budget: <u>\$6945</u> [Highest value in range.] Or <u>\$5324.50</u> [Mid-point] or <u>\$3704</u> [lowest range] Accept other reasonable budget provided working is clear [A1 – for correct type of flooring and corresponding budget] Assumption: Assume the highest value in the range for materials and installation. [B1 - ok to assume mid-point or lowest]</p>												