Class	Register Number	Name

Bukit Batok Secondary School
Mid-Year Examination 2021
Secondary 4 Express/ 5 Normal Academic

MATHEMATICS **4048/01**

Paper 1

6 May 2021 Thursday
0800 - 1000
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen in the spaces provided on the Question Paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

Habits of Mind: Striving for Accuracy and Precision

For Examiner's Use

This document consists of **16** printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1 + \frac{r}{100})^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 Calculate $40.78^2 + \sqrt{\frac{4(48500)-25}{40.78}}$.

Answer ..... [1]

---

2 It is given that y is directly proportional to the cube of x.

Which of these diagrams represents the graph of y against x?

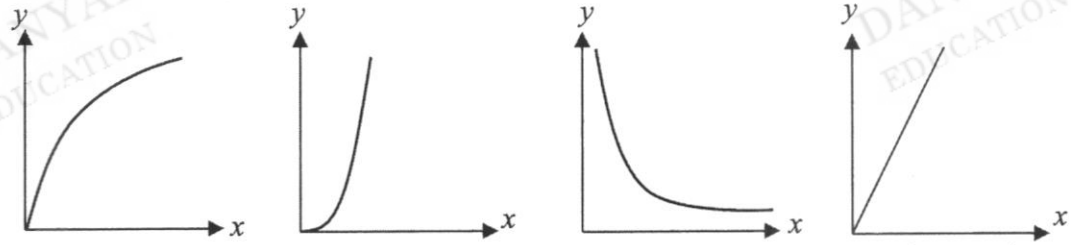
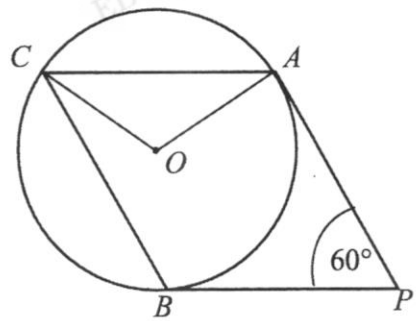


Diagram 1 Diagram 2 Diagram 3 Diagram 4

Answer Diagram..... [1]

3



PA and *PB* are tangents to a circle, centre *O*.
CAPB is a parallelogram.
Angle *APB* = 60°.

Find angle *OAC*.

Answer ..... [2]

---

4 **(a)** Write $\sqrt{32}$ as a power of 2.

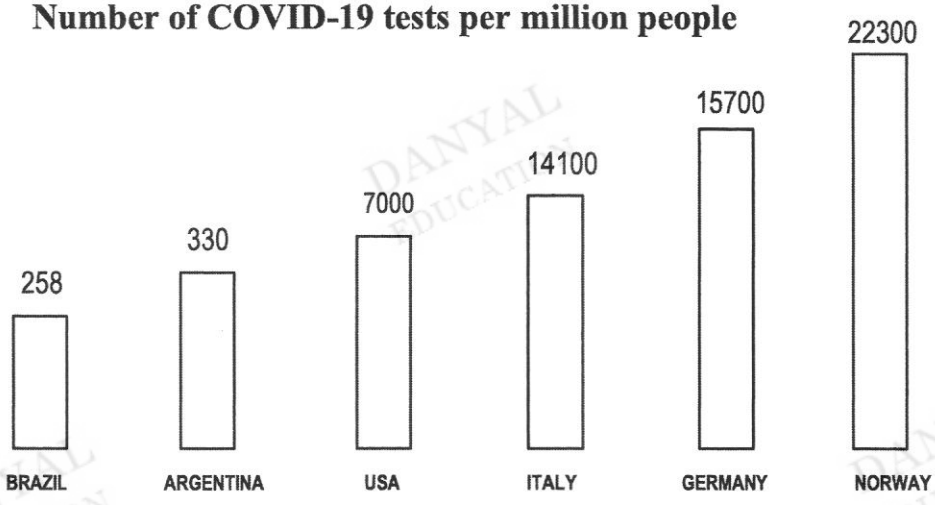
Answer ..... [1]

(b) Simplify $\frac{3x}{4y^2} \div \frac{15x^2y}{16}$.

Answer ..... [2]

---

5 The graph shows the number of COVID-19 tests per million people by a TV channel in Argentina.



(a) State one aspect of the graph that may be misleading.

.....

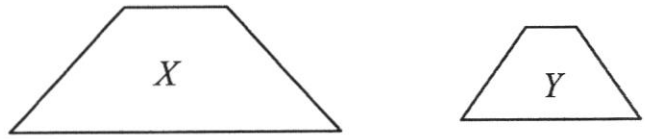
..... [1]

(b) Explain how this may lead to the viewer's misinterpretation of the graph.

.....

..... [1]

6



The diagram shows trapezium X whose parallel sides are in the ratio 1 : 3.
The lengths of trapezium Y is 45% of the corresponding lengths of trapezium X.

(a) Find the ratio of their respective areas X : Y.

Answer ..... : ..... [2]

(b) If the length of the shorter of the parallel sides of trapezium X is 6 cm, find the length of the longer parallel side of trapezium Y.

Answer .....cm [1]

---

7 The times, to the nearest minute, taken for 19 bus drivers to complete a specific journey, are represented in the stem-and-leaf diagram below.

8	2	3	8				
9	1	3	4	4	5		
10	3	3	6	6	6	8	8
11	0	1	7	9			

Key 9 | 3 means 93 minutes

(a) Write down the modal time taken.

Answer .....minutes [1]

(b) When the time for a 20th driver is added to the diagram, the median time and the range remains unchanged.

Write down a possible time taken for this driver.

Answer .....minutes [1]

---

8 Solve the equation $\frac{2x+1}{7} - \frac{x-1}{3} = -1$.

Answer $x =$..... [3]

---

9 **(a)** Express $x^2 + 5x - 5$ in the form $(x + h)^2 + k$ where h and k are constants.

Answer ..... [2]

(b) Write down the equation of the line of symmetry of the graph $y = x^2 + 5x - 5$.

Answer ..... [1]

---

10 There are some boys and girls in a group.
The probability of choosing a boy randomly from the group is 0.42.

(a) Find the probability of choosing a girl randomly from the group.

Answer ..... [1]

(b) If there are 8 more girls than boys, find the total number of children in the group.

Answer ..... [2]

---

11 **(a)** These are the first four terms of a sequence.

1 6 11 16

Find an expression, in terms of n, for the nth term of the sequence.

Answer ..... [1]

(b) Which term in the sequence has a value of 616?

Answer ..... term [2]

---

12 Mr Tan, a restaurant owner, pays his 3 employees, a chef and two kitchen helpers, a total of $4880 as salary.
The ratio of the amount his chef receives to the total amount his two kitchen helpers receive is 5:3.
Between the two kitchen helpers, Kim, the more experienced one receives $90 more than the new kitchen helper, Yong.

Calculate how much salary each employee receives.

Answer Chef $.....

Kim $.....

Yong $..... [3]

---

13 It is given that $y = \frac{5x + 3}{6 - 7x}$.

(a) Find y when $x = -2$.

Answer ..... [1]

(b) Rearrange the formula to make x the subject.

Answer ..... [2]

---

14 Adam manages a performing troupe.
58 of the members are dancers and 37 are singers.
His aim is that at least 70% of the members should be dancers.

Work out the smallest number of dancers that would need to join the troupe in order to achieve his aim.

Answer ..... dancers [3]

---

15 Mr Lim received an interest of \$6159 from an investment that offers compound interest of 3.5% per year after 10 years.

What is the initial investment amount?
(Correct your answer to the nearest dollar.)

DANYAL EDUCATION

Answer \$..... [3]

---

16 **(a)** $\xi = \{$ integers $x : 1 \leq x \leq 10\}$.
$A = \{$ prime numbers$\}$.
$B = \{$integers divisible by 3$\}$.

Find

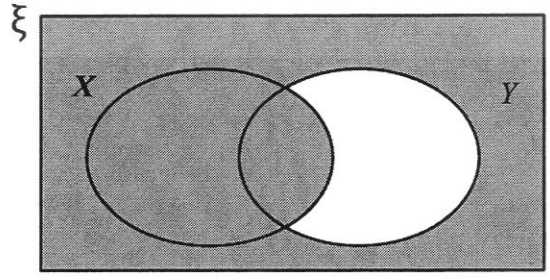
(i) $A \cap B$,

Answer ..... [1]

(ii) $(A \cup B)'$.

Answer ..... [1]

(b) Use set notation to describe the shaded region.



Answer ..... [1]

---

17 **(a)** Simplify $3(5x + 4y) - 2(x - 3y)$.

Answer ..... [1]

(b) Factorise completely $3pq - 6pr + 2r - q$.

DANYAL EDUCATION DANYAL EDUCATION

Answer ..... [2]

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18 Erik bought a watch for \$280 in Singapore.

(a) When he sold it, he made a profit of 135% of his cost.

Find the selling price.

DANYAL EDUCATION

Answer \$..... [2]

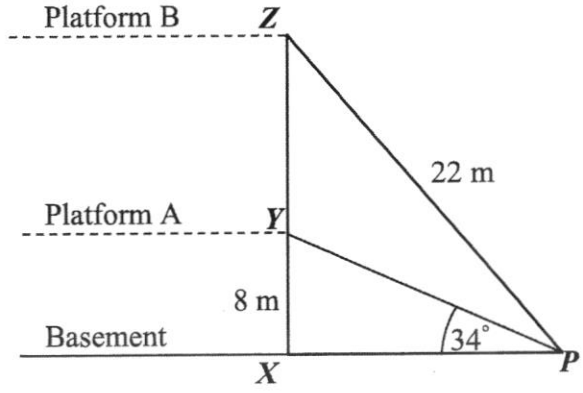
(b) The same watch costs 22 900 yen in Japan.

Given that the exchange rate is 100 yen = \$1.21, does the watch cost more or less in Japan? Justify your answer.

Answer

[2]

19



The diagram shows two escalators PY and PZ that lead up from the basement to the Platforms A and B in a new MRT station respectively.
Platform A is 8 m above the basement.
Escalator PZ has a length of 22 m.
The angle between the basement and the escalator $PY = 34°$.

Calculate the angle between the escalator PZ and the basement.

Answer .....° [4]

20 **(a)** Express 504 as a product of its prime factors.

Answer ..... [1]

(b) Find the lowest common multiple (LCM) of 504 and 392.

Answer ..... [2]

(c) Find the highest common factor (HCF) of 504 and 392.

Answer ..... [1]

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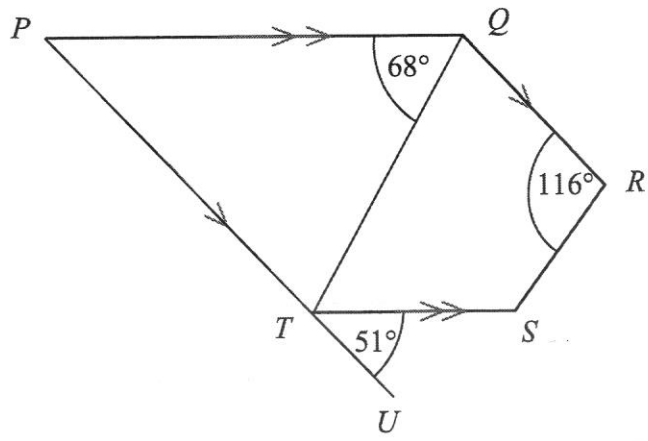
21



In the diagram, PQ is parallel to TS and PTU is parallel to QR.
Angle $PQT = 68°$, angle $QRS = 116°$ and angle $STU = 51°$.

(a) Find, giving reasons for each answer,

(i) angle QPT,

Answer ..... ° [1]

(ii) angle TQR,

Answer ..... ° [1]

(iii) angle RST.

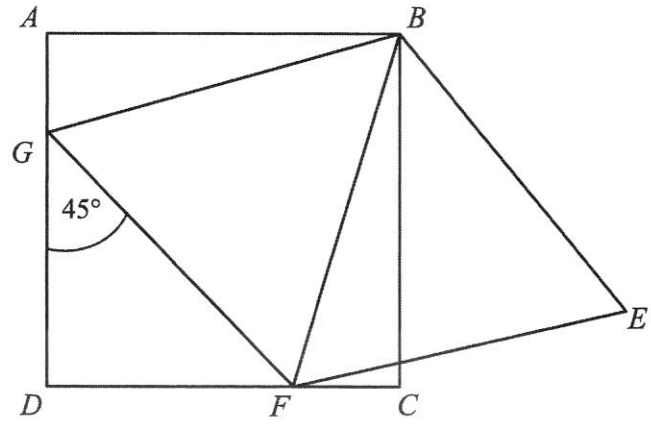
Answer ..... ° [2]

(b) State, with reasons, whether QT is or is not parallel to RS.

Answer .....

..... [1]

22



In the diagram, $ABCD$ is a square and angle $DGF = 45°$.

(a) Show that

(i) $DG = DF$,

Answer

[1]

(ii) triangle BGA is congruent to triangle BFC.

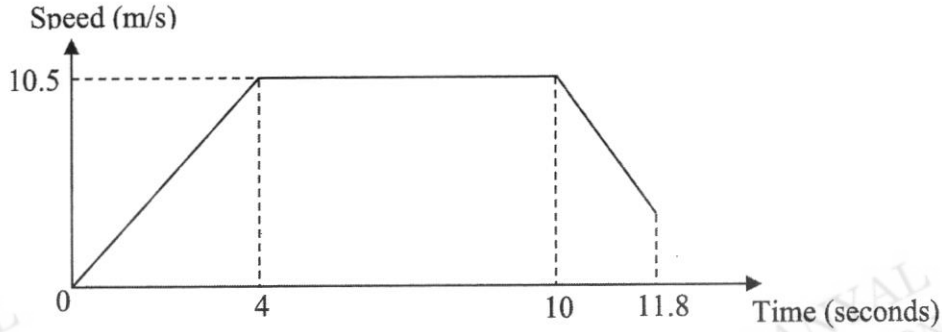
Answer

[3]

(b) Given that $EFGB$ is a rhombus, find angle AGB, giving reasons for your calculations.

Answer .....° [2]

23 During a sports meet, an athlete accelerated to a speed of 10.5 m/s in 4 seconds. She ran at the same speed for the next 6 seconds and slowed down over the last 1.8 seconds as shown in the speed-time graph.
She crossed the finishing line at 11.8 seconds.



(a) Find the acceleration of the athlete in the first 3 seconds.

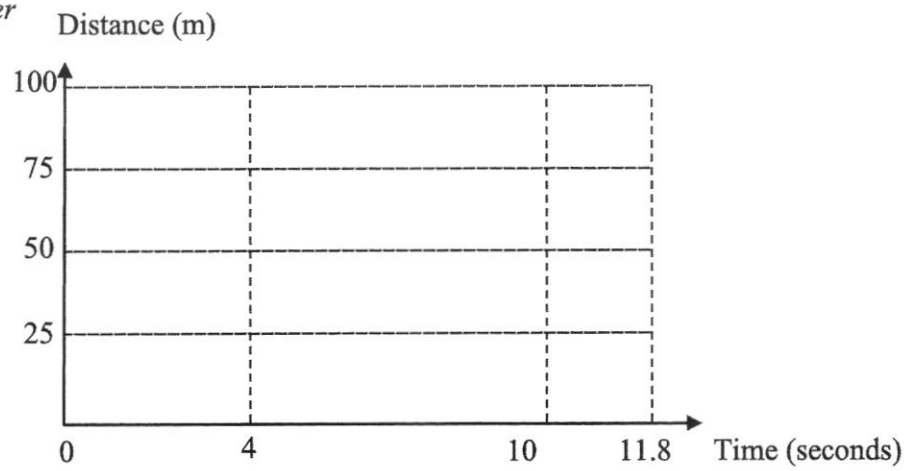
Answer ..... m/s^2 [1]

(b) It is given that distance travelled = area under the speed-time graph.
If the total distance she ran was 100 m, calculate her speed when she crossed the finishing line.

Answer ..... m/s [3]

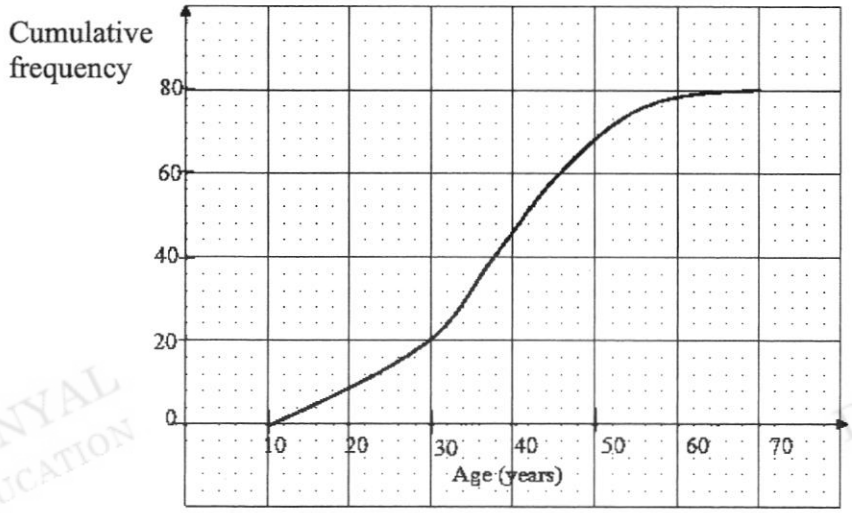
(c) Use the grid below to sketch the distance-time graph for the first 11.8 seconds.

Answer



[2]

24 The cumulative frequency curve shows the age distribution of 80 members in a community enrichment club.



(a) Use the graph to find the

(i) median age,

Answer ..... [1]

(ii) interquartile range.

Answer ..... [2]

(b) 5% of the club members are entitled for a special senior citizen discount.

Use the graph to find the minimum age to qualify for the discount.

Answer ..... [2]

(c) One of the members is chosen at random.

Find the probability that the member is between 20 and 30 years old.

Answer ..... [2]

End of Paper

HOM: Persist for "Character consists of what you do on the third and fourth tries." ***James A. Michener***

Class	Register Number	Name

Bukit Batok Secondary School
Mid-Year Examination 2021
Secondary 4 Express/ 5 Normal Academic

MATHEMATICS **4048/02**

Paper 2

10 May 2021 Monday
08 00 – 10 30
2 hours 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen in the spaces provided on the Question Paper.
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For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

If you need additional writing space, you may request for writing/graph papers from the invigilators. At the end of the examination, candidates will insert the additional writing/graph papers into their Question Paper.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

For Examiner's Use

Have you communicated with Clarity & Precision?

This document consists of **24** printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

DANYAL EDUCATION

1 Henry is a yoga instructor.
He offers sessions for Basic and Intermediate students on weekdays and at weekends.
Each student has a 10-week block of sessions with one session per week.
The matrix **S** shows the number of students he coaches each week in one 10-week block.

$$\mathbf{S} = \begin{matrix} \text{Basic} & \text{Intermediate} \\ \begin{pmatrix} 6 & 9 \\ 3 & 5 \end{pmatrix} & \begin{matrix} \text{Weekday} \\ \text{Weekend} \end{matrix} \end{matrix}$$

(a) Evaluate the matrix $\mathbf{T} = 10\mathbf{S}$.

Answer $\mathbf{T} =$ ..... [1]

(b) Henry charges \$50 for each basic session and \$80 for each Intermediate session.

Represent the session charges in a 2×1 column matrix **F**.

Answer $\mathbf{F} =$ ..... [1]

(c) Evaluate the matrix $\mathbf{Q} = \mathbf{TF}$.

Answer $\mathbf{Q} =$ ..... [2]

(d) State what the elements of **Q** represent.

.....

.....

.....[1]

(e) Henry wants to attract more students, so in the next 10-week block he reduces his prices by 10%.
For this block of sessions, on weekdays he has 10 Basic students and 5 intermediate students.
On weekends he has 6 basic students and 4 Intermediate students.

Calculate the total amount of money he earns for this 10-week block of sessions.

DANYAL EDUCATION

Answer $..... [3]

2 **(a)** Solve the inequality $3 - 5x > 8$.

DANYAL EDUCATION

Answer ..... [2]

(b) Solve these simultaneous equations.

$$x - 2y = 7$$
$$3x = 11 + 4y$$

DANYAL EDUCATION

Answer x = ...

y = ... [3]

(c) Express as a single fraction in its simplest form $\frac{6}{(3x-1)^2} - \frac{1}{(1-3x)}$.

Answer ..... [2]

(d) Simplify $\left(\frac{4x^{-1}}{xy^2}\right)^{\frac{1}{2}}$.

Answer ..... [2]

(e) Simplify $\frac{49-p^2}{21-3p}$.

Answer ..... [3]

3 **(a)** Complete the table of values for $y = \frac{x^2}{3} + \frac{3}{x}$, where $x \neq 0$.

x	0.5	1	1.5	2	3	3.5	4	4.5
y	6.08	3.33	2.75	2.83		4.94	6.08	7.42

[1]

(b) Answer the whole part of **part (b)** on the grid provided on **page 9**.
Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $0 \leq x \leq 5$.
Using a scale of 2 cm to 1 unit, draw a vertical y-axis for $0 \leq y \leq 8$.
On your axes, plot the points given in the table and join them in a smooth curve. [3]

(c) Use the graph to find the solutions of the equation $\frac{x^2}{3} + \frac{3}{x} = 5$ in the range $0 \leq x \leq 5$.

Answer $x =$ ..... or ..... [2]

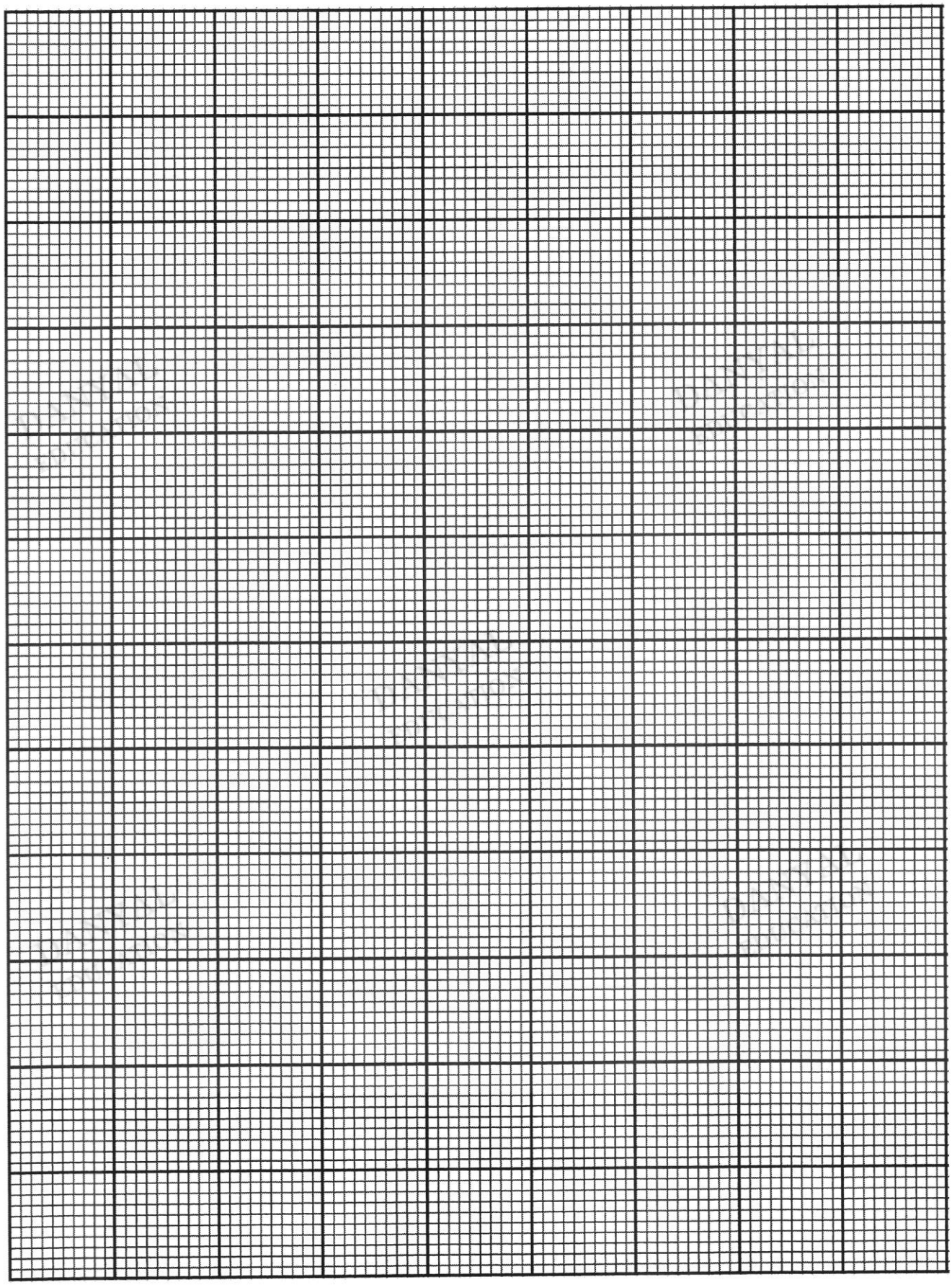
(d) By drawing a tangent, find the gradient of the curve at the point (2, 2.83).

Answer ..... [2]

(e) By drawing a suitable straight line, solve the equation $x^3 + 9 = 6x^2 + 3x$.

DANYAL EDUCATION

Answer $x =$..... [3]



4 Billy was out on a night cycling trip from Jurong Point to Changi Airport.
He cycled a total distance of 36 km.
For the first $\frac{2}{3}$ of the journey, his average speed was x km/h.

(a) Write down an expression, in terms of x, for the time taken in hours, for the first $\frac{2}{3}$ of his journey.

Answer .....hours [1]

For the remaining journey, he decided to travel slower by reducing his average speed by 4 km/h.

(b) Write down an expression, in terms of x, for the time taken in hours, for the remaining part of his journey.

Answer .....hours [1]

(c) It is given that the total time taken for the whole journey was 3 hours.
Write down an equation in x to represent this information, and show that it reduces to

$$x^2 - 16x + 32 = 0.$$ [3]

DANYAL EDUCATION

(d) **(i)** Solve the equation $x^2 - 16x + 32 = 0$, giving your solutions correct to 2 decimal places.

Answer $x =$ ..... or ..... [3]

(ii) Explain why one of the value of x must be rejected.

.....

.....

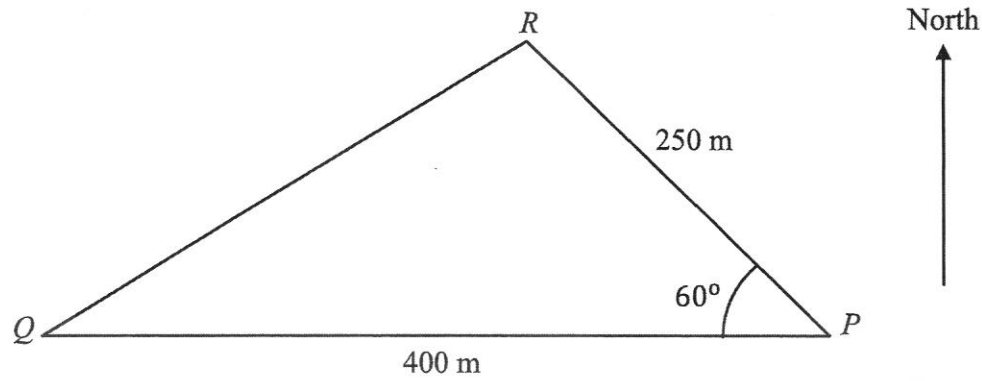
.....

..... [1]

(e) Calculate the time taken by Billy for the remaining part of his journey, correct to the nearest minute.

Answer .....minutes [2]

5



The base, R. of a lighthouse is at sea level.
Yacht P is 250 m from R.
Yacht Q is 400 m due west of yacht P.
Angle $RPQ = 60^{\circ}$.

(a) Calculate

(i) RQ,

Answer .....m [3]

(ii) the area of triangle RPQ,

Answer ..... m^2 [2]

(iii) angle RQP,

Answer .....° [2]

(iv) the bearing of Q from R.

Answer .....° [1]

(b) The angle of elevation of the top of the lighthouse seen from P is 9^o.

Calculate the angle of elevation of the top of the lighthouse from Q.

Answer .....° [3]

6 The speeds of 400 cars making an entry to the Pan Island Expressway (PIE) from Lornie Road were recorded on a weekday morning.
The table below shows the distribution of the speeds of the cars.

Speed (s km/h)	$20 < s \leq 40$	$40 < s \leq 50$	$50 < s \leq 60$	$60 < s \leq 70$	$70 < s \leq 90$
Frequency	65	80	115	130	10

(a) Which class interval is the upper quartile of the distribution in?

Answer ..... [1]

(b) Estimate the mean and standard deviation of the distribution.

Answer mean = ...km/h [2]

Standard deviation = ...km/h [2]

The speeds of 400 cars making an entry to the Pan Island Expressway (PIE) from Lornie Road were also recorded on a Saturday morning.

The results are summarised in the table.

Mean	57.6 km/h
Standard deviation	7.5

(c) Make two comparisons between the speeds of cars making entry to the PIE from Lornie Road on both mornings.

1. .....

.....

.....

.....

.....

2. .....

.....

.....

.....

..... [2]

DANYAL EDUCATION

7 A bag contains six counters, numbered 2, 3, 5, 7, 11 and 13.
A counter is drawn and replaced.
A second counter is then drawn.

(a) Draw a possibility diagram to represent the outcomes. [2]

DANYAL EDUCATION

Find, in its simplest form, the probability that

(b) **(i)** both counters are even numbers,

DANYAL EDUCATION

Answer ..... [1]

(ii) the sum of the two numbers is more than 16,

DANYAL EDUCATION

Answer ..... [1]

(iii) at least one of the numbers is a multiple of 5.

Answer ..... [1]

(c) At the end of a company's training workshop, participants have to pass a test to gain employment.

The probability of passing the test at the first attempt is $\frac{5}{7}$.

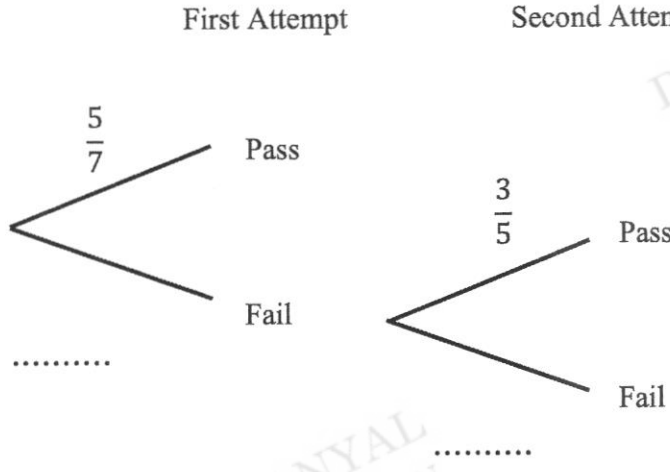
Those who fail are allowed to take a re-test.

The probability of passing the re-test is $\frac{3}{5}$.

No further attempts are allowed.

The tree diagram below shows all the possible outcomes.

(i) Complete the tree diagram. [1]



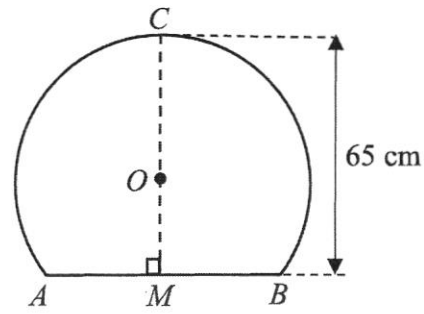
(ii) Find the probability that a participant gains employment.

Answer ..... [2]

(iii) If 70 participants sign up for this company's training workshop, what is the expected number of participants that will gain employment?

Answer ..... [1]

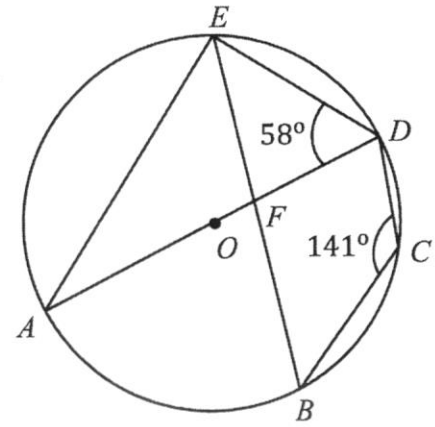
8 **(a)** $AMBC$ is a major segment of a circle, centre O and diameter 90 cm.
$CM = 65$ cm.
Angle $AMC = 90°$.
Find the area of the segment.



DANYAL EDUCATION

Answer .....cm^2 [5]

(b) The diagram shows a circle $ABCDE$, with centre O.
The straight line DA passes through O.
Points A, B, C, D and E lie on the circumference.
Angle $ADE = 58^\circ$ and angle $BCD = 141^\circ$.



Find, giving reason(s) for each answer,

(i) angle AED,

Answer .....° [1]

(ii) angle EAD,

Answer .....° [1]

(iii) angle EBD,

Answer .....° [1]

(iv) angle DEB,

Answer .....° [1]

(v) angle EFA.

Answer .....° [1]

9

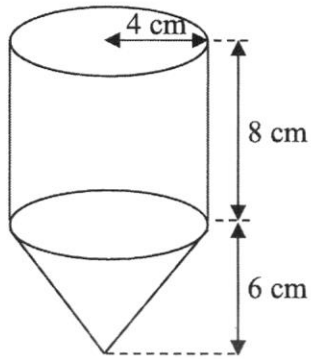


Diagram I

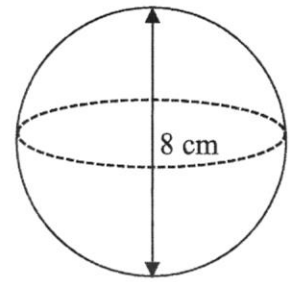


Diagram II

Diagram I shows an open container made by joining a cylinder of radius 4 cm and height 8 cm and a cone of height 6 cm.

(a) Calculate the total external surface area of the container.

Answer ..... cm^2 [3]

Diagram II shows a spherical ball of diameter 8 cm.
The ball is completely filled with water through a small opening on its top.

(b) Calculate the volume of water in the spherical ball.

Answer ..... cm^3 [2]

All the water from the ball is poured into the container in Diagram I.
The water level in the container from the bottom tip of the cone is measured to be h cm.

(c) Find the value of h.

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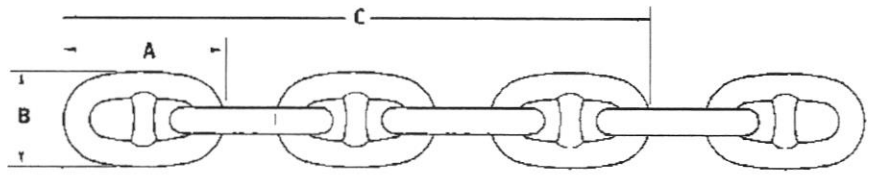
Answer .....cm [4]

Suppose the water starts to leak through a small hole at the tip of the cone and it leaks at a constant rate of 2 cm^3 per second.

(d) Find the time taken to drain the water completely from the container, leaving your answer in minutes and seconds.

Answer ..... [2]

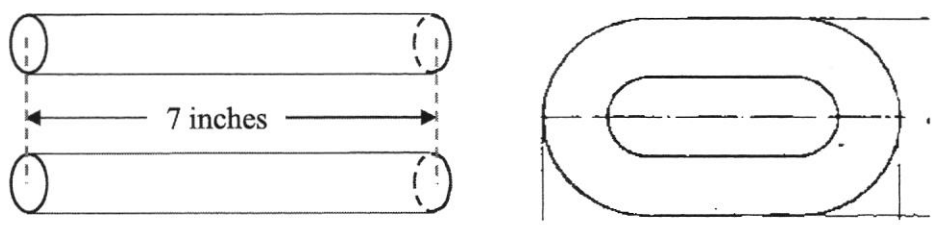
10 A chain, like the one shown below, is commonly used to drop anchors into the sea.



STUD LINK ANCHOR CHAIN DIMENSIONS

Diameter CHAIN SIZE		DIMENSIONS (INCHES)			WEIGHT PER 15 FATHOM SHOT	GRADE 2		GRADE 3		OIL RIG QUALITY		LINKS PER SHOT
						PROOF TEST	BREAK TEST	PROOF TEST	BREAK TEST	PROOF TEST	BREAK TEST	
INCHES	MM	A	B	C	#	#	#	#	#	#	#	(15 FATHOMS)
5/8	16	3 3/4	2 1/4	13 3/4	365	23,745	33,220	33,220	47,465			432
3/4	19	4 1/2	2 5/8	16 1/2	480	34,000	47,600	47,600	68,000			357
13/16	21	4 7/8	2 7/8	17 7/8	570	39,800	55,700	55,700	79,500			329
7/8	22	5 1/4	3 1/8	19 1/4	660	46,000	64,400	64,400	91,800			305
15/16	24	5 5/8	3 5/16	20 5/8	760	52,600	73,700	73,700	105,000			285
1	25	6	3 9/16	22	860	59,700	83,600	83,600	119,500	84,000	129,000	267
1 1/16	30	6 3/8	3 3/4	23 3/8	970	67,200	94,100	94,100	135,000			251
1 1/8	29	6 3/4	4	24 3/4	1,080	75,000	105,000	105,000	150,000	106,000	161,000	237
1 13/16	30	7 1/8	4 1/4	26 1/8	1,220	83,400	116,500	116,500	167,000			225
1 1/4	32	7 1/2	4 1/2	27 1/2	1,350	92,200	129,000	129,000	184,000	130,000	198,000	213
1 5/16	33	7 7/8	4 3/4	28 7/8	1,490	1,500	142,000	142,000	203,000			203
1 3/8	34	8 1/4	4 15/16	30 1/4	1,630	111,000	155,000	155,000	222,000	157,000	235,000	195
1 7/16	36	8 5/8	5 3/16	31 5/8	1,780	120,500	169,000	169,000	241,000			187
1 1/2	38	9	5 3/8	33	1,940	131,000	183,500	183,500	262,000	185,000	280,000	179
1 9/16	40	9 3/8	5 5/8	34 3/8	2,090	142,000	198,500	198,500	284,000			171
1 5/8	42	9 3/4	5 7/8	35 3/4	2,240	153,000	214,000	214,000	306,000	216,000	325,000	165
1 11/16	43	10 1/8	6 1/16	37 1/8	2,410	166,500	229,000	229,000	327,000			159
1 3/4	44	10 1/2	6 5/16	38 1/2	2,590	176,000	247,000	247,000	352,000	249,000	380,000	153
1 13/16	46	10 7/8	6 1/2	39 7/8	2,790	188,500	264,000	264,000	377,000			147
1 7/8	48	11 1/4	6 3/4	41 1/4	2,980	201,000	281,000	281,000	402,000	285,000	432,000	143
1 15/16	50	11 5/8	7	42 5/8	3,180	214,000	299,000	299,000	427,000			139
2	51	12	7 3/16	44	3,360	227,000	318,000	318,000	454,000	322,000	488,000	133

1 fathom is equivalent to 1.8 metres and 1 inch is 25.4 mm.
A single unit chain can be modelled by the figure below in which two cylindrical metal pieces are bent and joined together at the edges.



(a) Calculate the surface area of the single unit chain in the form of $k\pi$ inches^2, where k is an integer, if the diameter of each cylinder is 1 inch.
It is assumed that the surface area of the figure does not change.

Answer ..... inches^2 [1]

(b) Calculate the volume of the single unit chain, in mm^3, if the diameter of each cylinder is 1 inch.

Answer ..... mm^3 [2]

(c) Such anchor chains are sold in sets of 15 fathom shots.
The weight per 15 fathom shots in the table is given in kg.
A fishing boat goes out to sea with the intention of anchoring at a depth of 600 m.
The maximum allowable load for the chain is 45 tonnes.
If a 1.5 inches chain size is selected, determine, showing your calculation, whether the chain size choice is recommended. (1 tonne = 1000 kg)

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Answer ..... [7]

----- End of Paper -----

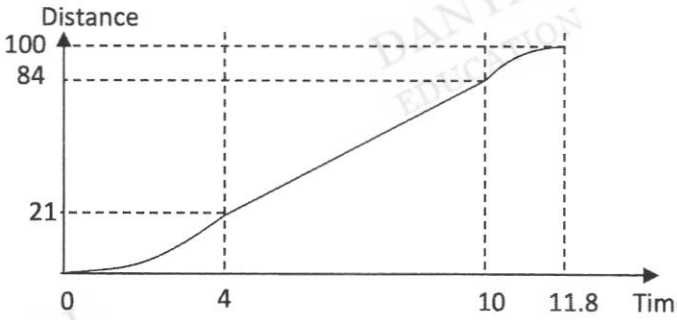
Have you communicated with Clarity & Precision?

Mark Scheme for 2021 4E5N MYE Mathematics Paper 1 (4048/1)

1	1731.97666 = 1730 (to 3 sig fig)	 B1
2	Diagram 2	B1
3	Angle $OAP = 90°$ (tan ⊥radius) Angle $CAP = 180° - 60° = 120°$ (interior angles in parallelogram) Angle $OAC = 120° - 90° = 30°$	M1 – either one of the reasons shown A1
4	(a) $2^{\frac{5}{2}}$	B1
	(b) $\frac{3x}{4y^2} \times \frac{16}{15x^2y}$ $= \frac{4}{5xy^3}$	M1 A1
5	(a) The heights of the bars were not drawn to any vertical scale.	B1
	(b) The distorted height difference made the viewers see that Argentina tests almost as many people as the USA where in fact the USA test is 20 times more than that of Argentina.	B1
6	45% = 9/20 (a) Area X : Area $Y = 20^2 : 9^2 = 400 : 81$ (b) Length of longer parallel side of $Y = \frac{9}{20} \times 18 = 8.1$ cm	M1 A1 B1
7	(a) 106 min (b) Any time between 82 min and 103 min inclusive	B1 B1
8	$\frac{2x+1}{7} - \frac{x-1}{3} = -1$ $3(2x+1) - 7(x-1) = (-1)(21)$ $6x + 3 - 7x + 7 = (-21)$ $x = 31$	 M1: removing the denom M1: expansion A1
9	(a) $x^2 + 5x - 5 = x^2 + 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 5$ $= \left(x + \frac{5}{2}\right)^2 - \frac{45}{4}$ (b) $x = -\frac{5}{2}$	M1 A1 B1
10	(a) P(a girl is chosen) = 1 – 0.42 = 0.58 (b) 0.58 – 0.42 = 0.16 Total number of children $= \frac{8}{0.16} = 50$	B1 M1, A1 (o.e.)
11	(a) $5n - 4$ (b) $5n - 4 = 616$ $n = 620 \div 5$ $= 124$ 124th term	B1 M1 A1 (Accept $n = 124$)
12	5 parts $= \frac{5}{8} \times 4880 = \$3050$ Remaining to be divided = (\$1830 – 90) ÷ 2 = \$1740 ÷ 2 = \$870 Chef receives \$3050, Kim receives = \$960, Yong receives = \$870	M1 M1 A1 for all 3 correct ans
13	(a) $y = \frac{5(-2)+3}{6-7(-2)} = -\frac{7}{20}$ (b) $y = \frac{5x+3}{6-7x}$ $y(6 - 7x) = 5x + 3$ $6y - 7xy = 5x + 3$ $5x + 7xy = 6y - 3$ $x(7y + 5) = 6y - 3$ $x = \frac{6y - 3}{7y + 5}$	B1 Accept −0.35 M1: collecting x terms to one side A1

DANYAL EDUCATION

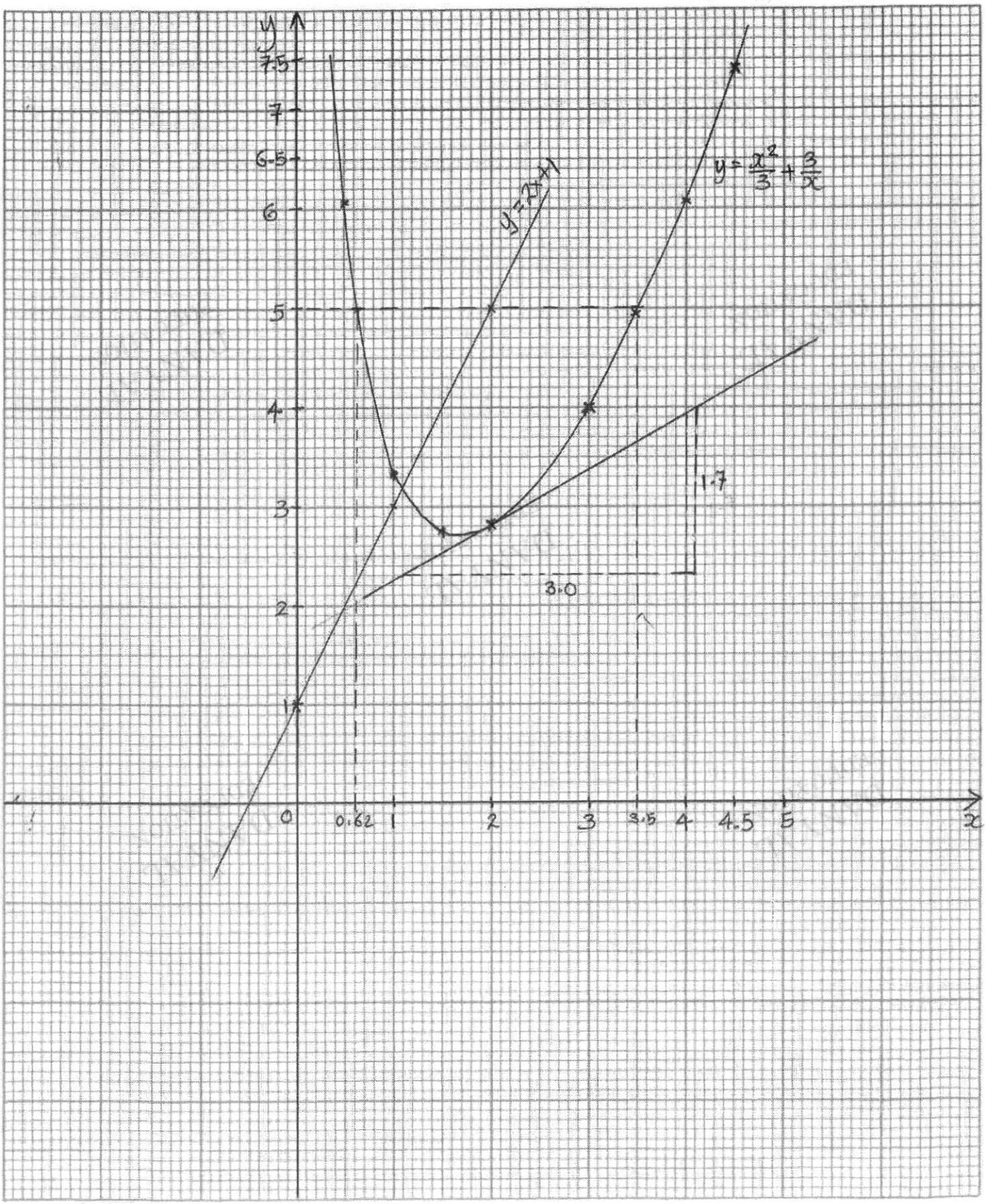
14	<table><tr><td></td><td>58</td><td>37</td></tr></table> 30% of new total = 37 100% of new total = $\frac{37}{0.30} = 123\frac{1}{3}$ Smallest total number = 124 ∴ Smallest number of dancers needed to join club = 29 Alternative method: Let x be the number of new dancers $\frac{x+58}{x+95} \geq 0.7$ $x + 58 \geq 0.7(x + 95)$ $0.3x \geq 66.5 - 58$ $x \geq \frac{8.5}{0.3}$ Smallest x =29	M1 A1 A1 M1 M1 A1
15	Initial investment amount = P $P(1 + \frac{3.5}{100})^{10} = P + 6159$ $[(1.035)^{10} - 1]P = 6159$ P $= \frac{6159}{(1.035)^{10}-1}$ = \$15 000 (nearest \$)	M1 M1 A1
16	(a) (i) {3} (ii) {1, 4, 8, 10} (b) $X \cup Y'$	B1 B1 B1
17	(a) $3(5x + 4y) - 2(x - 3y) = 13x + 18y$ (b) $3pq - 6pr + 2r - q = 3p(q - 2r) - (q - 2r)$ $= (q - 2r)(3p - 1)$	B1 M1 A1
18	(a) Selling price = $\$280 + 1.35 \times 280$ = \$658 (b) 100 yen = \$1.21 22 900 yen =\$ $\frac{1.21}{100} \times 22900$ = \$277.09 The watch costs less (\$277.09 compared with \$280) in Japan given the exchange rate. Alternatively, convert \$280 to yen = 23140 yen to make comparison	M1 A1 M1 A1
19	$tan\ 34° = \frac{8}{PX}$ $PX = \frac{8}{tan\ 34°}$ = 11.860 m Cos ($\angle XPZ$) = $\frac{11.860}{22}$ Angle between escalator PZ and the basement = 57.4°	M1 A1 M1 A1
20	(a) $504 = 2^3 \times 3^2 \times 7$ (b) $392 = 2^3 \times 7^2$ LCM = $2^3 \times 3^2 \times 7^2 = 3528$ (c) HCF = 56	B1 M1 B1 B1
21	(a) (i) angle QPT = 51° (corresponding angles, $PQ//TS$) (ii) angle TQR = angle PTQ (alternate angles, PTU//QR) = 180° – 68° – 51° (angle sum of triangle) = 61° (iii) angle QTS = 68° (alternate angles, $PQ//TS$) angle RST = 360° – 116° – 61° – 68° = 115° (sum of angles in a quadrilateral)	B1 B1: either one reason shown M1: either reason for QTS or angle of quad shown A1 B1

	(b) Since angle TQR + angle QRS do not add up to 180°, they are not interior angles between parallel lines, $\therefore$ QT is not parallel RS.	
22	(a) (i) Since angle GDF=90°(right angle in a square) Angle DFG = 45°(angle sum of triangle) $\therefore$ DG = DF (isosceles triangle) or $\tan 45° = \frac{DF}{DG} = 1$ (ii) In triangles BGA and BFC, (1) BA = BC (sides of a square) (2) Angle A = Angle C = 90°(angles in a square) (3) Since DG = DF and AD = DC GA = FC $\therefore$ triangles BGA and BFC are congruent (SAS) (b) From (a) GB = FB But GB = GF (sides of rhombus) $\therefore$ triangle GFB is an equilateral triangle and $\angle$BGF = 60° $\angle$AGB = 180° – 45° - 60° = 75° (adjacent angles on a straight line)	B1 - Isosceles triangle shown B2: All three conditions stated [B1: any two conditions stated] A1: test used M1: state reason why $\angle BGF = 60°$ A1
23	(a) acceleration in first 3 seconds = $10.5 \div 4 = 2.625$ m/s^2 (b) Let speed = v $\frac{1}{2}(6 + 10)10.5 + \frac{1}{2}(v + 10.5)1.8 = 100$ $84 + 0.9v + 9.45 = 100$ $v = \frac{6.55}{0.9}$ = 7.28 m/s (3 s.f.) or $7\frac{5}{18}$ m/s  	B1 cao (exact) M1 M1 A1 B1 Correctly shaped curve B1 Correct distance label
24	(a) (i) 38 years old (ii) 46 – 30 = 16 years (b) 5% of club members = 4 members 76 members less than minimum age Minimum age to qualify = 56 years old (c) Number of members in the 20-30 years old = 12 P(members between 20-30 years old) = $\frac{12}{80} = \frac{3}{20}$	B1 M1, A1 M1 Reading from cf 76 A1 M1 A1

Candidate Name .....

Subject ... M/E E.Math ... Paper .. 2 ...

Question No. .. 3(b) ...



BP 47

2021 BBSS 4E5N MYE Mathematics (4048/2) Paper 2 Marking Scheme

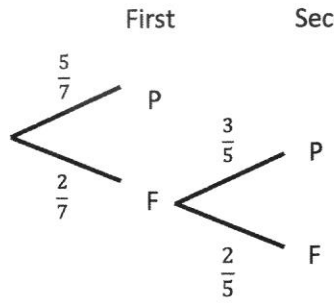
Question	Solution	Marks
1(a)	$\mathbf{T} = 10\mathbf{S}$ $\mathbf{T} = 10\begin{pmatrix}6 & 9\\3 & 5\end{pmatrix}$ $\mathbf{T} = \begin{pmatrix}60 & 90\\30 & 50\end{pmatrix}$	A1
1(b)	$\mathbf{F} = \begin{pmatrix}50\\80\end{pmatrix}$	B1
1(c)	$\mathbf{Q} = \begin{pmatrix}60 & 90\\30 & 50\end{pmatrix}\begin{pmatrix}50\\80\end{pmatrix}$ $\mathbf{Q} = \begin{pmatrix}3000+7200\\1500+4000\end{pmatrix}$ $\mathbf{Q} = \begin{pmatrix}10200\\5500\end{pmatrix}$	M1 A1
1(d)	The elements of **Q** represent the total amount of money Henry collected for 10-week block sessions on weekdays and at weekends respectively.	A1
1(e)	Total amount of money earned $= (10+6)(50)(10)(0.9) + (5+4)(80)(10)(0.9)$ $= \$\,13680$	$(10+6)(50)(10)(0.9)$: M1 $(5+4)(80)(10)(0.9)$: M1 A1
2(a)	$3 - 5x > 8$ $x < \frac{8-3}{-5}$ $x < -1$	M1 A1
2(b)	$x - 2y = 7 \quad ----(1)$ $3x = 11 + 4y ----(2)$ From (1), $x = 7 + 2y ----(3)$ Sub (3) into (2), $3(7+2y) = 11 + 4y$ $21 + 6y = 11 + 4y$ $y = -5$ $x = -3$	M1 A1 A1
2(c)	$\frac{6}{(3x-1)^2} - \frac{1}{(1-3x)}$ $= \frac{6}{(3x-1)^2} + \frac{1}{(3x-1)}$ $= \frac{6+3x-1}{(3x-1)^2}$ $= \frac{3x+5}{(3x-1)^2}$ $\frac{6}{(3x-1)^2} - \frac{1}{(1-3x)}$ $= \frac{6}{(3x-1)^2} - \frac{1-3x}{(1-3x)^2}$ $= \frac{6+3x-1}{(3x-1)^2}$ $= \frac{3x+5}{(3x-1)^2}$	M1 A1
2(d)	$\left(\frac{4x^{-1}}{xy^2}\right)^{\frac{1}{2}} = \left(\frac{2}{xy}\right)^{(2)\left(\frac{1}{2}\right)}$ $= \frac{2}{xy}$	M1 A1

2(e)	$\frac{49-p^2}{21-3p}$ $=\frac{(7-p)(7+p)}{3(7-p)}$ $=\frac{1}{3}(7+p)$	$(7-x)(7+x)$: M1 $3(7-x)$: M1 A1
3(a)	$y=\frac{3^2}{3}+\frac{3}{3}=4$	A1
3(b)	Refer to the attached graph.	Scale & interval & label – B1 Able to mark coordinates correctly – B1 Able to join all coordinates with a smooth curve – B1
3(c)	$x = 0.6$ and 3.5 (accept 0.5 to 3.6)	B1, B1
3(d)	Gradient $=\frac{4.0-2.3}{4.1-1.15}=\frac{1.7}{3.0}$ $=\frac{17}{30}=0.567$ (3s.f) (accept 0.55 to 0.60)	M1 A1
3(e)	$\frac{x^3+9}{3x}=\frac{6x^2+3x}{3x}$ $\frac{x^2}{3}+\frac{3}{x}=2x+1$ By drawing, $y=2x+1$, $x = 1.1$	M1 M1 A1
4(a)	$\frac{\frac{2}{3}(36)}{x}=\frac{24}{x}$	A1
4(b)	$\frac{\frac{1}{3}(36)}{x-4}=\frac{12}{x-4}$	A1
4(c)	$\frac{24}{x}+\frac{12}{x-4}=3$ $24(x-4)+12x=3x(x-4)$ $24x-96+12x=3x^2-12x$ $x^2-16x+32=0$ (shown)	M1 M1 A1
4(d)(i)	$x=\frac{-(-16)\pm\sqrt{(-16)^2-4(1)(32)}}{(2)(1)}$ $x=\frac{16\pm\sqrt{128}}{2}$ $x = 13.65685425 = 13.66$ (2d.p) or $x = 2.343145751 = 2.34$ (2d.p)	M1 A1 A1
4(d)(ii)	Since time taken cannot be negative, $\therefore\ x = 2.34$ km/h (2d.p) is rejected	A1

4(e)	Time taken $= \dfrac{12}{13.65685425 - 4}$ $= 1.24240687$ hrs $= 1.24$ hrs $= 74.55844122$ mins $= 75$ mins	M1 A1
5(a)(i)	$RQ^2 = 250^2 + 400^2 - 2(250)(400)\cos 60^\circ$ $RQ = \sqrt{250^2 + 400^2 - 2(250)(400)\cos 60^\circ}$ $RQ = \sqrt{122500} = 350$ metres	M2 A1
5(a)(ii)	Area of triangle RPQ $= \dfrac{1}{2}(250)(400)\sin 60^\circ$ $= 43301.27019 = 43300$ m^2 (3 s.f)	M1 A1
5(a)(iii)	$\dfrac{\sin \angle RQP}{250} = \dfrac{\sin 60^\circ}{\sqrt{122500}}$ $\angle RQP = \sin^{-1}\left(\dfrac{\sin 60^\circ}{\sqrt{122500}}\right)(250) = \sin^{-1}0.618589574$ $\angle RQP = 38.2132107^\circ = 38.2^\circ$ (1 d.p.)	M1 A1
5(a)(iv)	Bearing of Q from $R = 270^\circ - 38.2^\circ = 231.8^\circ$	A1
5(b)	Let the height of lighthouse be h m. Let the angle of elevation of the top of the lighthouse from Q be θ. $\dfrac{h}{250} = \tan 9^\circ$ $h = 250 \tan 9^\circ$ $\tan\theta = \dfrac{250\tan 9^\circ}{\sqrt{122500}}$ $\theta = \tan^{-1}\left(\dfrac{250\tan 9^\circ}{\sqrt{122500}}\right) = \tan^{-1}0.113131743$ $\theta = 6.54528^\circ = 6.5^\circ$ (1d.p.)	M1 M1 A1
6(a)	The upper quartile will lie in $60 < s \leq 70$	A1
6(b)		If students used calculators and obtained the right answers (without any working) we will award full marks.

Speed (s km/h)	Mid-Value (x)	x^2	f	fx	fx^2
$20 < s \leq 40$	30	900	65	1950	58500
$40 < s \leq 50$	45	2025	80	3600	162000
$50 < s \leq 60$	55	3025	115	6325	347875
$60 < s \leq 70$	65	4225	130	8450	549250
$70 < s \leq 90$	80	6400	10	800	64000
			$\sum f = 400$	$\sum fx = 21125$	$\sum fx^2 = 1181625$

$\sum f = 400$, $\sum fx = 21125$, $\sum fx^2 = 1181625$

	Mean $= \frac{21125}{400} = 52.8125$ km/h $\text{S.D} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$ $\text{S.D} = \sqrt{\frac{1181625}{400} - (52.8125)^2}$ $\text{S.D} = \sqrt{164.902344}$ $\text{S.D} = 12.84143075 = 12.8\ (3\text{s.f})$	M1 A1 M1 A1
6(c)	The mean speed (57.6 km/h) on Saturdays is faster than on weekdays (52.8125 km/h). The spread of the speed on weekday (S.D = 12.8) are wider on weekdays than on Saturdays (S.D = 7.5).	A1 A1
7(a)(i)	<table><tr><td></td><td>2</td><td>3</td><td>5</td><td>7</td><td>11</td><td>13</td></tr><tr><td>2</td><td>(2,2)</td><td>(2,3)</td><td>(2,5)</td><td>(2,7)</td><td>(2,11)</td><td>(2,13)</td></tr><tr><td>3</td><td>(3,2)</td><td>(3,3)</td><td>(3,5)</td><td>(3,7)</td><td>(3,11)</td><td>(3,13)</td></tr><tr><td>5</td><td>(5,2)</td><td>(5,3)</td><td>(5,5)</td><td>(5, 7)</td><td>(5, 11)</td><td>(5, 13)</td></tr><tr><td>7</td><td>(7,2)</td><td>(7,3)</td><td>(7,5)</td><td>(7,7)</td><td>(7,11)</td><td>(7,13)</td></tr><tr><td>11</td><td>(11, 2)</td><td>(11,3)</td><td>(11,5)</td><td>(11,7)</td><td>(11,11)</td><td>(11,13)</td></tr><tr><td>13</td><td>(13,2)</td><td>(13,3)</td><td>(13,5)</td><td>(13,7)</td><td>(13,11)</td><td>(13,13)</td></tr></table>	A2 We will accept if students represented it in Cross diagram or Dot diagram.
7(a)(ii)	$\text{P(even)} = \frac{1}{36}$	A1
7(a)(iii)	$\text{P(sum more than 16)} = \frac{10}{36} = \frac{5}{18}$	A1
7(a)(iv)	$\text{P(at least one of the numbers is a multiple of 5)} = \frac{11}{36}$	A1
		A1
7(b)(i)	P (participant gains employment)	M1

	$= \frac{5}{7} + \left(\frac{2}{7} \times \frac{3}{5}\right)$ $= \frac{31}{35}$	A1
7(b)(ii)	Expected number of participants $= 70 \times \frac{31}{35} = 62$	A1
8(a)	Given that midpoint of AB is M. $AO = BO =$ radius $= \frac{90}{2} = 45$ cm, $OM == 65 - 45 = 20$ cm $\cos\frac{1}{2}\angle AOB = \frac{20}{45}$ $\angle AOB = 2\cos^{-1}\frac{20}{45}$ $\angle AOB = 127.2244001^{o} = 127.2^{o}$ (1d.p.) Area of triangle AOB $= \frac{1}{2}(45^2)(\sin 127.2244001^{o})$ $= 806.2257746$ cm^2 Area of sector ACB $= \left(\frac{360^{o} - 127.2244001^{o}}{360^{o}}\right)(\pi)(45^2)$ $= 4113.48435$ cm^2 Area of major segment $AMBC$ = Area of triangle AOB + Area of sector ACB $= 806.2257746 + 4113.48435 = 4919.710125$ $= 4920$ cm^2 (3 s.f.)	M1 M1 M1 M1 A1
8(b)(i)	Angle $AED = 90^{o}$ (Angle in a semicircle)	B1
8(b)(ii)	Angle EAD $= 180^{o} - 90^{o} - 58^{o}$ (Angle sum of triangle) $= 32^{o}$	A1
8(b)(iiii)	Angle $EBD = 32^{o}$ (Angles in the same segment)	B1
8(b)(iv)	Angle $DEB = 180^{o} - 141^{oo} = 39^{o}$ (Angles in the opp. segment)	B1

8b(v)	Angle *EFA* $= 39^{o}+58^{o}$ (ext. angles of triangle) $= 97^{o}$	A1
9(a)	Surface area of the open cylinder $= (2\pi)(4)(8) = 201.06193\ \text{cm}^2$ Surface area of cone (without the base) $= (\pi)(4)\left(\sqrt{(4)^2+(6)^2}\right) = 90.61739\ \text{cm}^2$ Total external surface area of the container $= 201.06193 + 90.61739 = 291.67932 = 292\ \text{cm}^2$ (3 s. f.)	M1 M1 A1
9(b)	Volume of ball $= \frac{4}{3}(\pi)(4^3)$ $268.08257 = 268\ \text{cm}^2$ (3 s. f.)	M1 A1
9(c)	Volume of cone $= \frac{1}{3}(\pi)(4^2)(6) = 100.53097\ \text{cm}^2$ Remaining volume $= 268.08257 - 100.53097 = 167.5516\ \text{cm}^2$ Height in the cylinder, h $= 6 + \frac{167.5516}{(\pi)(4^2)}$ $= 9.33333 = 9.33\ \text{cm}$	M1 M1 M1 A1
9(d)	Time $= \frac{268.08257}{2}$ $= 134.04129 = 134$ seconds (3. s, f) = 2 minutes 14 seconds	M1 A1
10(a)	The surface area of the single unit chain $= (2)(\pi)(1)(7) = 14\pi\ \text{inches}^2$	A1
10(b)	The volume of the single unit chain $= (2)(\pi)(12.7^2)(7)(25.4)$ $= 180185.1796 = 180\,000\ \text{mm}^3$ (3. s, f)	M1 A1

10(c)	Convert 600 metres to fathom $= \frac{600}{1.8} = 333\frac{1}{3}$ fathom	M1
	Number of set needed $= \left(333\frac{1}{3}\right) \div (15)$ $= 22.222 = 22.2$ sets (3 s. f)	M1
	Therefore, number of sets to buy = 23 (round off)	M1
	Weight of chain $= (23 \times 1940)$ kg $= 44620$ kg	M1
	Convert weight from kg to tonnes $= \left(\frac{44620}{1000}\right)$ tonnes $= 44.62$ tonnes	M1
	Since the weight of chain (44.62 tonnes) **is less than** the maximum allowable load of 45 tonnes,	M1
	therefore, **1.5 inches chain size is recommended**.	A1

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