Name		Class				Index Number		

BROADRICK SECONDARY SCHOOL
SECONDARY 3 EXPRESS
END-OF-YEAR EXAMINATION 2022

ADDITIONAL MATHEMATICS 4049

Candidates answer on the Question Paper.
No Additional Materials are required.

October 2022
2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Write the question number attempted in the left column in the box provided.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use		
Reason	**Question Number**	**Marks Deducted**
No/Wrong Units		
Rounding-off		
Premature Rounding		
Others		

For Examiner's Use	
Question Number	**Marks Obtained**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
Total Marks	/90

This document consists of **17** printed pages.

Setter : Mr Yong JJ

1 Given that $\left[\left(\frac{7}{2}\right)^{-3} \times \sqrt{8}\right] \div \frac{1}{13} = 2^a \times 7^b \times 13^c$, find the value of each of a, b and c. [3]

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2 (a) Express $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ in the form $a + b\sqrt{15}$ where a and b are integers. [2]

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(b) If $k = \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$, show that $k^2 + \dfrac{1}{k^2} = 62$. [3]

3 Express $\dfrac{17x^2+8x-9}{\left(x^2+1\right)\left(3x-1\right)}$ in partial fractions. [5]

4 **(a)** Find the equation of the straight line that is parallel to $4y-2x-14=0$ and bisects the line joining the points $A(3, 1)$ and $B(1, -5)$. [3]

(b) Find the area of the triangle ABC, where C is the point $(0, -1)$. [2]

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5 **(a)** The graph $y = a\cos x + b$, where $a > 0$, has a maximum value of 7, and a minimum value of 3.

Find the value of a and of b. [2]

(b) Using these values of a and b found in **5(a),** sketch the graph $y = a\cos x + b$ in the space below for $0^{\circ} \leq x \leq 360^{\circ}$. [3]

6 The expression $\text{f}(x) = 2x^3 + ax^2 + bx - 9$, where a and b are constants, has a factor of $(x-3)$ and leaves a remainder of -24 when divided by $(x-1)$.

(a) Find the value of a and of b. [4]

(b) Using these values of a and b found in **6(a)**, find the roots of the equation $f(x) = 0$, showing all working clearly. [3]

7 (a) Explain why the coordinates $A(0,6)$, $B(2,1)$ and $C(7,3)$ are the 3 possible vertices of a square $ABCD$. [4]

(b) Find the coordinates of D, the fourth vertex of the square. [3]

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8 **(a)** Find the exact value of $\tan 150^\circ$. [2]

(b) Given that $270^\circ < \theta < 360^\circ$, and that $\sin\theta = -\frac{5}{13}$, find the values of $\cos\theta$ and $\tan\theta$. [3]

(c) Solve $\sin 2x = -\dfrac{\sqrt{3}}{2}$ for $0^\circ < x < 360^\circ$. [3]

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9 The height, h metres of a ball above the ground at time t seconds after it is thrown from a rooftop is given by $h = 80 + 64t - 16t^2$.

(a) Find the height reached by the ball 3 seconds after the ball is thrown. [1]

(b) Find the time when the ball hits the ground. [2]

(c) Use two different methods to explain why the ball cannot reach a height of 150 m. [6]

10 **(a)** Solve the simultaneous equations

$$\log_2 2x = 3 - \log_2(y+1)$$
$$\log_5(2x+1) = 1$$

[5]

(b) Given that $\log_3 x = a$ and $\log_3 y = b$, express $\log_3 \sqrt{\dfrac{x^4 y}{27}}$ in terms of a and b. [4]

11 (a) Find the coefficient of the term of x^7y^2 in the expansion of $\left(x+\frac{y}{3}\right)^9$. [2]

(b) In the expansion of $\left(x-\frac{k}{x}\right)^{14}$ for $k>0$, the coefficient of x^6 is 99 times the coefficient of x^{10}. Show that $k=3$. [6]

(c) Explain why the expansion of $\left(x-\frac{3}{x}\right)^{14}$ does not contain a term with x^7. [2]

12 A circle C has the equation $x^2-22x+y^2-16y+160=0$.

(a) Find the radius of C and the coordinates of the centre of C. [3]

(b) Explain why the point (8, 4) lies on circle C. [2]

(c) Find the equation of the line that is tangent to the circle at (8, 4). [4]

(d) A new circle is formed when C is reflected in the y-axis.
Find the equation of this new circle. [1]

Do the following question on a sheet of graph paper

13 It is known that x and y are related by an equation of the form $y = ax + bx^2$, where a and b are constants.

The table below shows recorded values of two related variables, x and y.

x	2	4	6	8	10
y	23.99	88.01	269.55	336.01	520.02

However, one of the values of y in the table above was recorded incorrectly.

(a) By choosing a suitable scale for both axes, plot $\dfrac{y}{x}$ against x using the graph grid on the next page and determine which value of y above is incorrectly recorded. [2]

(b) Draw the straight-line graph and use it to estimate the value of y to replace the incorrect recording you found in part **13(a)**. [2]

(c) Use your graph to estimate the value of a and of b. [3]

End of Paper

Ans for 2022 3E Add Math EYE

1	$a = 4.5$, $b = -3$, $c = 1$
2a	$4+\sqrt{15}$
2b	$k^2+\frac{1}{k^2}=31+8\sqrt{15}+31-8\sqrt{15}=62$
3	$\frac{17x^2+8x-9}{(x^2+1)(3x-1)}=\frac{7x+5}{(x^2+1)}+\frac{-4}{(3x-1)}$
4a	$y=\frac{1}{2}x-3$
4b	*Ans 7 units*
5a	$a = 2$ $b = 5$
6a	$a=1, b=-18$
6b	$x=-3, -0.5$ *or* 3
7a	*Grad of AB* = $-\frac{5}{2}$ *Grad of BC* = $\frac{2}{5}$ *Since product of the gradients of AB and BC are -1 → They are perpendicular* *Length of AB* = $\sqrt{29}$ *Length of BC* = $\sqrt{29}$ *Since AB and BC are perpendicular and are the same length, A,B, and C are the 3 possible vertices of a square*
7b	Point D is $(5,8)$

8a	$-\frac{1}{\sqrt{3}} or -\frac{\sqrt{3}}{3}$
8b	$\cos\theta = \frac{12}{13}$ $\tan\theta = -\frac{5}{12}$
8c	x = 120, 150, 300, 330
9a	Sub t =3→ h= 128 m
9b	It hits the ground after 5 seconds
9c	$h=144-16(t-2)^2$ From the above equation, we see the maximum height possible ball can reach is only 144m (since the lowest possible value of $16(t-2)^2$ is zero Method 2 Assume the height can reach h, then $-16t^2+64t+80=150$ Since the discriminant is negative, there is solution to the equation, meaning that at no time will the height of the ball reach 150m.
10a	x = 2, y =1
10b	$2a+\frac{1}{2}b-1.5$
11a	Coefficient of the term is 4
11c	Let 14-2r =7 → r =3.5 For the term x^7 to exist, the corresponding r has to be a positive integer. Since it is not, the x^7 does not appear in the expansion
12a	Centre is (11, 8), radius is 5
12c	$y=-\frac{3}{4}x+10$
12d	$(x+11)^2+(y-8)^2=25$
13a	The y-value 269.55 is incorrect
13b	y = 192 (accept 186 to 198)
13c	b =5 a = 2

Mark Scheme for 2022 3E Add Math EYE

1	$\left[\left(\frac{7}{2}\right)^{-3} \times \sqrt{8}\right] \div \frac{1}{13} = 2^a \times 7^b \times 13^c$ $\left[\left(\frac{2}{7}\right)^{3} \times \sqrt{2^3}\right] \div \frac{1}{13} = 2^a \times 7^b \times 13^c$ $\left[2^3 \times 7^{-3} \times 2^{1.5}\right] \times 13^1 = 2^a \times 7^b \times 13^c$ $\left[2^{4.5} \times 7^{-3}\right] \times 13^{-1} = 2^a \times 7^b \times 13^c$ $a = 4.5$, $b = -3$, $c = 1$	[B1] for each

2a	$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ $4+\sqrt{15}$	[M1] [A1]
2b	$k = 4+\sqrt{15} \to k^2 = 31+8\sqrt{15}$ $\frac{1}{k^2} = \frac{1}{31+8\sqrt{15}} = \frac{31-8\sqrt{15}}{31^2-64\cdot 15} = 31-8\sqrt{15}$ $k^2 + \frac{1}{k^2} = 31+8\sqrt{15}+31-8\sqrt{15} = 62$	[M1] [M1] [A1]

3	$\frac{17x^2+8x-9}{(x^2+1)(3x-1)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(3x-1)}$ $17x^2+8x-9 \equiv (3x-1)(Ax+B)+C(x^2+1)$ Sub $x = \frac{1}{3} \to C = -4$ Sub $x = 0 \to B = 5$ Sub $x = 1$ (or any suitable value) $\to A = 7$ $\frac{17x^2+8x-9}{(x^2+1)(3x-1)} = \frac{7x+5}{(x^2+1)} + \frac{-4}{(3x-1)}$	[M1] [M1] [M1] [M1] [A1]

[Turn Over

For Examiner Use

4a	$4y-2x-14=0 \rightarrow y=\frac{1}{2}x+3.5$ Gradient of line is 0.5 Midpoint of $A(3,1)$ and $B(1,-5)$ is $(2,-2)$ Sub $(2,-2)$ into $y=\frac{1}{2}x+C$ → C = -3 $y=\frac{1}{2}x-3$	[M1] [M1] [A1]
4b	Shoelace method on points $A(3,1)$, $B(1,-5)$ and C$(0,-1)$ Ans 7 units	[M1] **ECF for this method mark** [A1]

5a	a =2 b =5	[B1] [B1]
5b	**Award full ECF for this question Max 3** Max and Min clearly shown General shape of cosine curve – starts and ends at max value Axes labelled and every 90 deg labelled	 [B1] [B1] [B1]

6a	f(3)=0 → $9a+3b=-45$ f(1)= -24 → $a+b=-17$ solving using any method $a=1, b=-18$	[M1] [M1] [M1] [A1]
6b	Show long division working with $f(x)=2x^3+ax^2+bx-9$ divided by x -3 To get $2x^2+7x+3$	[M1]

	$2x^2+7x+3=(x+3)(2x+1)$ $x=-3,-0.5\ or\ 3$	[M1] [A1]

7a	Grad of AB = $-\frac{5}{2}$ Grad of BC = $\frac{2}{5}$ Since product of the gradients of AB and BC are -1 → They are perpendicular Length of AB = $\sqrt{29}$ Length of BC = $\sqrt{29}$ Since AB and BC are perpendicular and are the same length, A,B, and C are the 3 possible vertices of a square	[M1] if ss found at least 2 gradients [A1] [M1] [A1] conclusion
7b	Midpoint of AC =(3.5, 4.5) (3.5, 4.5) is also the midpoint of BD $\frac{x+2}{2}=3.5 \rightarrow x=5$ $\frac{y+2}{2}=4.5 \rightarrow y=8$ Point D is (5, 8) OR Student can use vector method (full credit if method suitable and final answer the same)	[M1] [M1] written or implied [A1]

8a	$\tan 150=-\tan 30$ $=-\frac{1}{\sqrt{3}}\ or -\frac{\sqrt{3}}{3}$	[M1] [A1]
8b	Applying PT to find that adjacent side is 12	[M1]

	$\cos\theta = \frac{12}{13}$	[B1]
	$\tan\theta = -\frac{5}{12}$	[B1]
8c	Base angle = 60 deg 2x= 240, 300, 600, 660 x = 120, 150, 300, 330	[A1] [B2] -1 mark if answer wrong or missing

9a	Sub t =3→ h= 128 m	[B1]
9b	Sub h =0→ $-16t^2+64t+80=0$ $t=-1,(rej)=5$ It hits the ground after 5 seconds	[M1] [A1]
9c	**Method 1** $-16t^2+64t+80=-16\left[(t-2)^2-9\right]$ $h=144-16(t-2)^2$ From the above equation, we see the maximum height possible ball can reach is only 144m (since the lowest possible value of $16(t-2)^2$ is zero **Method 2** Assume the height can reach h, then $-16t^2+64t+80=150$ Consider b^2-4ac for the equation $-16t^2+64t-70=0$ $b^2-4ac=(64)^2-4(-16)(-70)=-384$ Since the discriminant is negative, there is solution to the equation, meaning that at no time will the height of the ball reach 150m.	 [M1] attempt to complete the square [A1] [A1] conclusion [M1] discriminant [A1] [A1] conclusion

10a	$\log_2 2x = 3\log_2 2 - \log_2(y+1)$	[M1]

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	$\log_2 2x = \log_2\left(\frac{8}{y+1}\right)$ $2x = \frac{8}{y+1}$ Other eqn: $\log_5(2x+1) = 1 \rightarrow 2x+1 = 5$ Student solves (via any method) $2x = \frac{8}{y+1}$ and $2x+1 = 5$ x = 2, y =1	[A1] [A1] [A1][A1]
10b	$\frac{1}{2}\log_3\left(\frac{x^4 y}{27}\right)$ $\frac{1}{2}\left(\log_3 x^4 y - \log_3 27\right)$ $\frac{1}{2}\left(4\log_3 x + \log_3 y - 3\right)$ $2a + \frac{1}{2}b - 1.5$	[M1] [M1] [M1] [A1]

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11a	It is the 3rd term in the expansion $\binom{9}{2}x^7\left(\frac{y}{3}\right)=4x^7y^2$ Coefficient of the term is 4	[M1] [A1] or [B2]
11b	General Term $\binom{14}{4}x^{14-r}\left(\frac{-k}{x}\right)^r$ $\binom{14}{4}x^{14-2r}(-k)^r$ --(1) Let 14 -2r =6 ➔r =4 Subbing $r=4$ into (1), the coefficient of x^6 is $1001k^4$ Let 14 -2r =10 ➔r =2 Subbing $r=2$ into (1), the coefficient of x^{10} is $91k^2$ $\frac{1001k^4}{99k^2}=99$ $k^2=9\rightarrow k=3$ (Don't award ans mark if ss concludes k = -3)	[M1] [M1] [M1] [M1] [M1] [A1]
11c	Let 14 -2r =7 ➔r =3.5 For the term x^7 to exist, the corresponding r has to be a positive integer. Since it is not, the x^7 does not appear in the expansion	[M1] [A1]

12a	$(x-11)^2+(y-8)^2=25$ Centre is $(11,8)$, radius is 5	[M1]completing the square [A1] [A1]
12b	Sub $x=8, y=4$ $LHS=(8-11)^2+(4-8)^2$ =25 = RHS Since the equation holds when the coordinates are	[M1]

	substituted, the point is on the circle .	[A1]
12c	Gradient of line segment from the point to radius $m=\frac{8-4}{11-8}=\frac{4}{3}$ Gradient of the tangent is -0.75 (tan perpendicular to radius) Sub in (8,4) into $y=-\frac{3}{4}x+c$ $4=-\frac{3}{4}(8)+c \Rightarrow c=10$ $y=-\frac{3}{4}x+10$	[M1] attempt to calculate gradient [M1] [M1] [A1]
12d	$(x+11)^2+(y-8)^2=25$	[B1]

13a	Points plotted in a line (except one) The y-value 269.55 is incorrect	[M1] [A1]
13b	*Line on graph MUST be drawn in order to get get any marks for this question* $\frac{y}{x}=32$ (accept 31 to 33) y = 192 (accept 186 to 198)	 [M1] [A1]
13c	$\frac{42-22}{8-4}$ $b=5$ $a=2$	[M1] calc of gradient [A1] [B1]

END OF MARK SCHEME