BEATTY SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2021

SUBJECT : Mathematics

PAPER : 4048/1

LEVEL : Sec 3 Express

DURATION : 1 hour 30 minutes

DATE : 7 October 2021

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **60**.

For Examiner's Use
60

This paper consists of **14** printed pages (including this cover page)

[Turn over

2
Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 Solve the following equations simultaneously.

$$6x + y = 1$$

$$x + \frac{y}{3} = 1$$

Answer: $x =$ ..... , $y =$ ..... [3]

---

2 **(a)** Solve the inequality $2x - 11 \leq 15$.

Answer: ..... [1]

(b) Randy says there are a total of 7 prime numbers which satisfy the inequality $2x - 11 \leq 15$. Do you agree? Support your answer with clear workings.

Answer: Yes/No, because .....

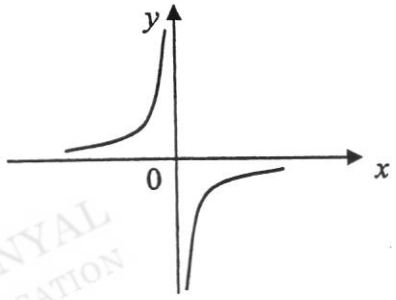
..... [2]

[Turn over

3 For each of the following graphs below, write down a possible equation.
In each case, select one of the equations from the box below.

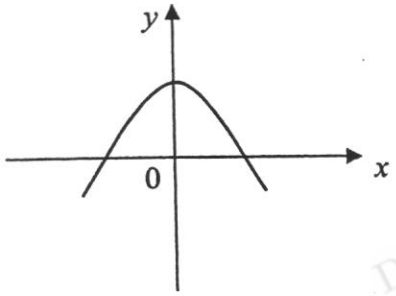
$y=-\frac{1}{x}$	$y=x^3$	$y=\frac{2}{x^2}$	$y=3x$
$y=x^2+1$	$y=\frac{2}{x}$	$y=1-x^2$	$y=10^x$

(a)



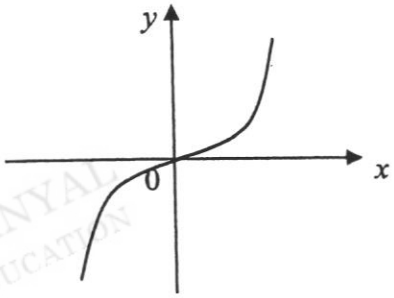
Answer: ..... [1]

(b)



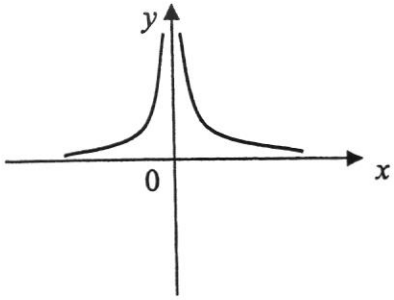
Answer: ..... [1]

(c)



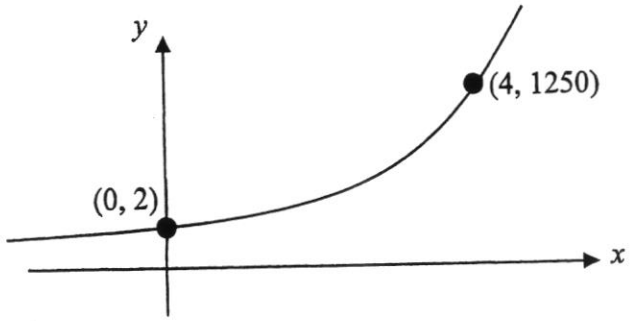
Answer: ..... [1]

(d)



Answer: ..... [1]

4 A graph passes through (0, 2) and (4, 1250).



(a) Is the graph for $y = ka^x$ or $y = kx^a$, where $k > 0$? Explain your answer.

Answer: .....

..... [1]

(b) Find the value of k and of a.

Answer: k = ... , a = ... [2]

5 **(a)** Simplify $\left(\frac{x^2}{5}\right)^3 \div \frac{50}{3x^0}$.

Answer: ..... [2]

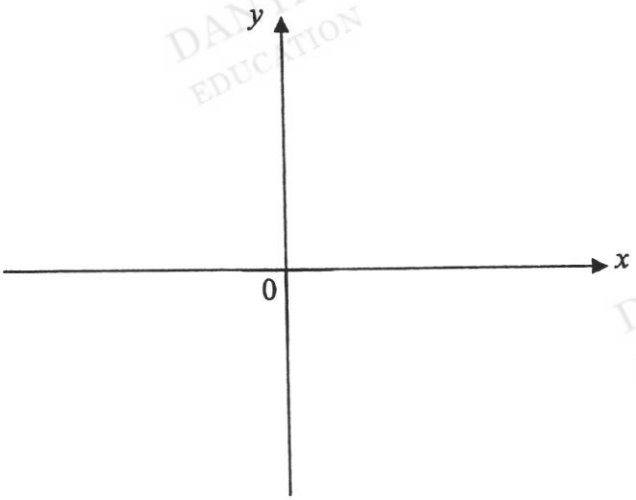
(b) Simplify $\left(\frac{(3a)^2 b}{20c^5}\right)^{-1} \times \frac{a^3 b^{-2}}{4c}$.

Answer: ..... [2]

---

6 **(a)** On the axes below, sketch the graph of $y = (x-1)^2 + 2$.

Answer: [2]



(b) Explain, by drawing a suitable line on your sketch, why the equation $(x-1)^2 = -1$ has no real solution.

Answer: .....

..... [2]

---

7 **(a)** y is inversely proportional to the square of x. When $x = 3$, $y = 10$.
Find the positive value of x when $y = 4.5$.

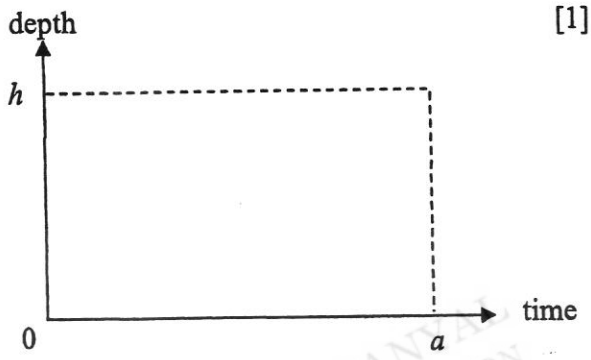
Answer: $x =$ ..... [2]

(b) y is varies directly with the cube root of x.
Find the percentage increase in y when x is doubled.

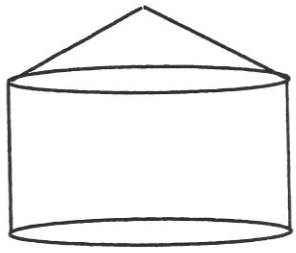
Answer: ..... % [2]

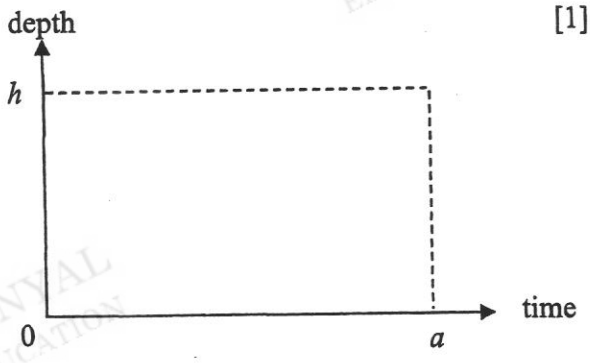
8 Each of the containers below has the same height h cm.
Each of them is filled with water at a constant rate. Each container takes the same amount of time, a minutes, to fill completely.
On the given axes, sketch the graph of the depth of water in the container over time.

(a) [1]

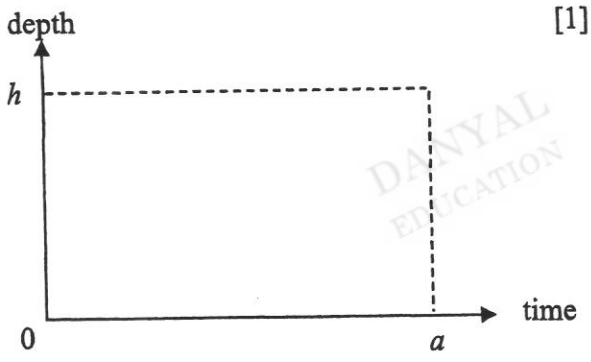


(b) [1]

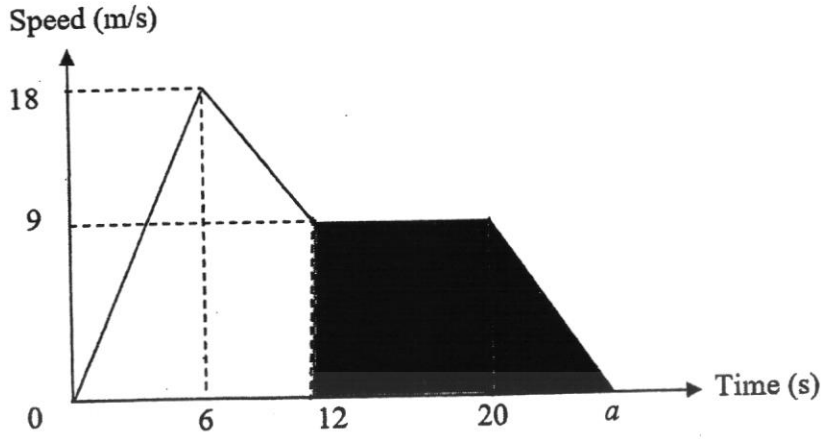




(c) [1]



9 The speed-time graph of a particle is shown below.



(a) Find the acceleration in the first 6 seconds.

Answer: ..... m/s^2 [1]

(b) Find the speed at $t = 10$ seconds.

Answer: ..... m/s [1]

The shaded area gives the distance travelled by the particle from $t = 12$ s to $t = a$ s.

(c) Given that the distance travelled by the particle from $t = 12$ s to $t = a$ s is 108 meters, find the value of a.

Answer: $a =$ ..... [1]

[Turn over

10 **(a)** Expand and simplify $5x+2(1-3x)$.

Answer: ..... [2]

(b) Factorise $4ay-2by+6a-3b$.

Answer: ..... [2]

---

11 **(a)** The heights of two geometrically similar containers are 10 cm and 12 cm.
If the total surface area of the smaller container is 650 cm^2, find the total surface area of the larger container.

Answer: ..... cm^2 [2]

(b) The volumes of another two geometrically similar containers are 400 cm^3 and 686 cm^3.
If the total surface area of the larger container is 300 cm^2, find the total surface area of the smaller container.

Answer: ..... cm^2 [2]

12 **(a)** Factorise $p^2 - \frac{1}{16}$.

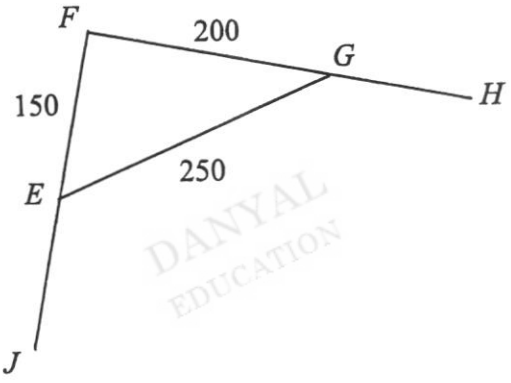
Answer: ..... [1]

(b) Factorise $6x^2 + 12x - 18$ completely.

Answer: ..... [2]

---

13 H is on FG produced and $FG = 200$ cm. J is on FE produced and $FE = 150$ cm. $EG = 250$ cm



(a) Is triangle EFG a right-angled triangle? Justify your answer.

Answer: .....

..... [2]

(b) Find the exact value of $\sin \angle EGF$.

Answer: ..... [1]

(c) Find the exact value of $\cos \angle JEG$.

Answer: ..... [1]

---

14 A road 800 m long measures 10 cm on a map.

(a) Find the length, in km, of another road that measures 20.5 cm on the map.

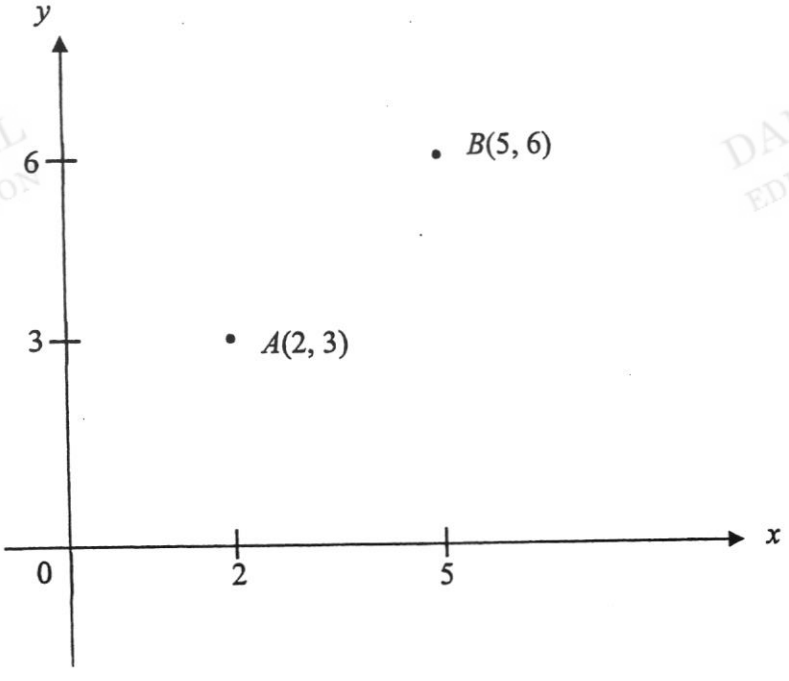
Answer: ..... km [2]

(b) A house has actual area of 725 m^2. Find the area of the house on the map.

Answer: ..... cm^2 [2]

---

15 A and B have coordinates (2, 3) and (5, 6) respectively.



15 **(a)** Find the equation of the line passing through A and B.

Answer: ..... [2]

(b) C is a point on the x-axis such that $BC = \sqrt{\frac{169}{4}}$.

By letting the coordinates of C be $(k, 0)$ or otherwise, find the two possible coordinates of C.

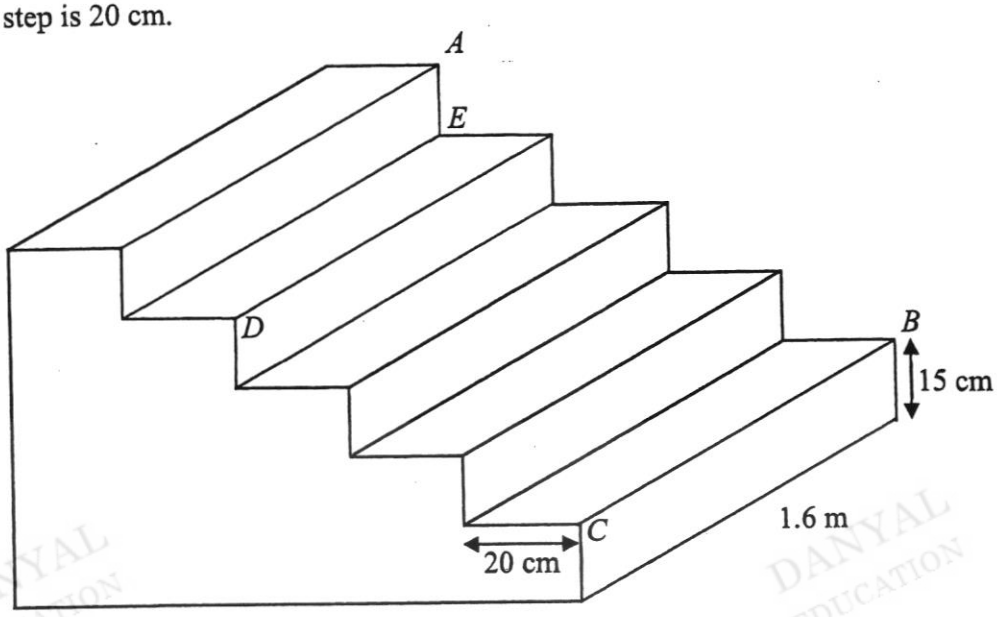
Answer: C (..... , ...) or C (..... , ...) [2]

(c) D is a point on the line $y = 6$ such that the area of triangle ABD is 12 unit^2.
Find the two possible coordinates of D.

Answer: D (..... , ...) and D (..... , ...) [2]

[Turn over

16 The width of a staircase is 1.6 m. Each step has a height of 15 cm, while the breadth of each step is 20 cm.



(a) Find the length of AC.

Answer: ..... cm [2]

(b) Find angle ADE.

Answer: ..... [2]

End of Paper

Answer Key

1. $x=-\frac{2}{3}, y=5$

2(a) $x \leq 13$ (b) No, because there are 6 prime numbers less than or equal to 13 (2, 3, 5, 7, 11 and 13).

3(a) $y=-\frac{1}{x}$

3(b) $y=1-x^2$

3(c) $y=x^3$

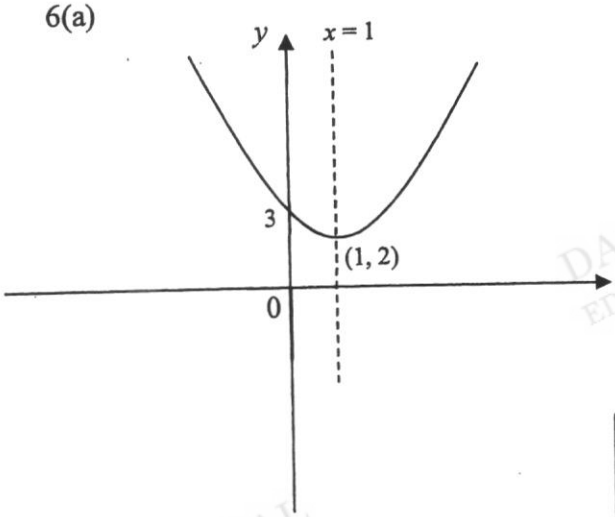
3(d) $y=\frac{2}{x^2}$

4(a) It is for $y=ka^x$ because when $x=0, y=k \neq 0$.

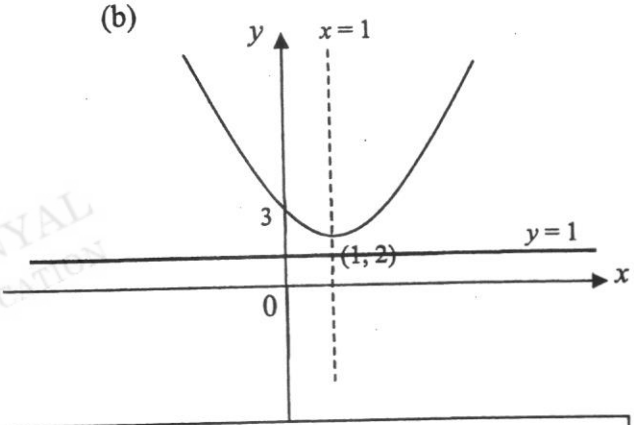
4(b) $a=5$, $k=2$

5(a) $\frac{3x^6}{6250}$ (b) $\frac{5ac^4}{9b^3}$

6(a)



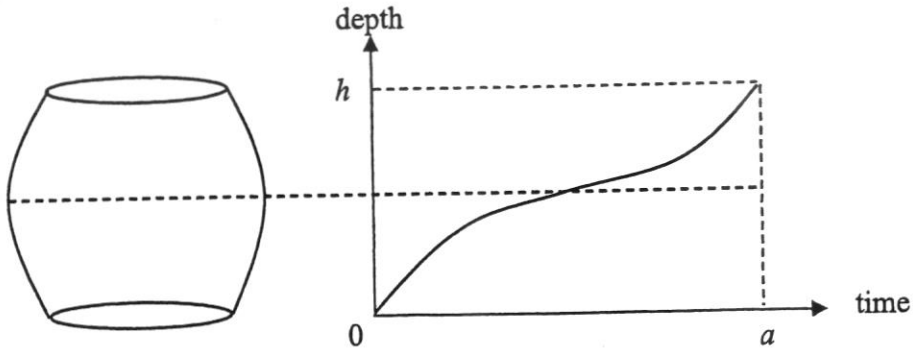
(b)



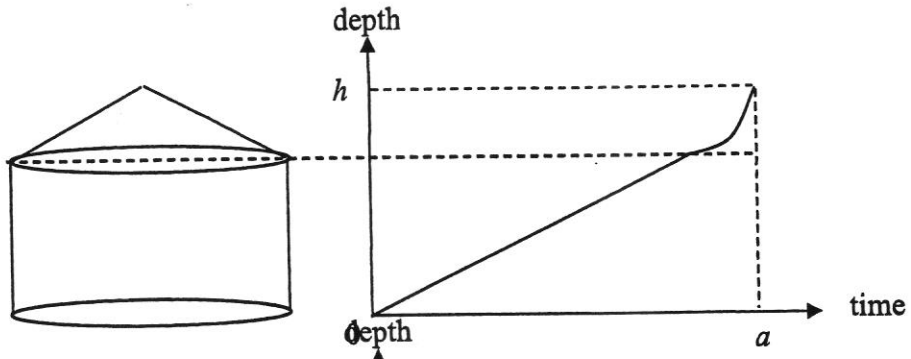
Since the line $y=1$ does not intersect the graph of $y=(x-1)^2+2$, the equation has no real solution.

7(a) $x=+\sqrt{20}=4.47$ (b) 26.0%

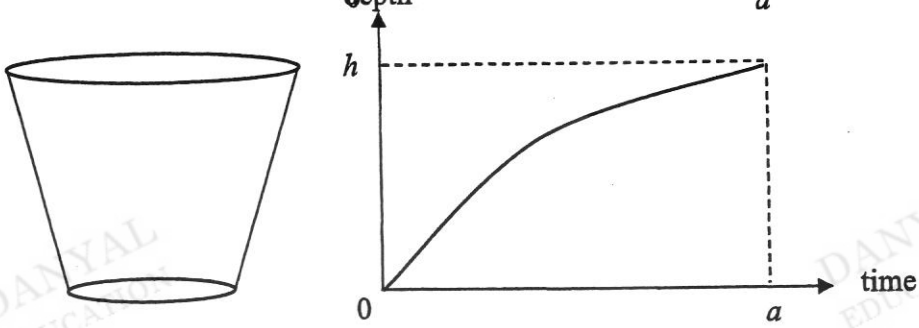
8(a)



8(b)



8(c)



9(a) 3 m/s^2 (b) 12 m/s (c) $a = 28$

10(a) $2-x$ (b) $(2a-b)(2y+3)$

11(a) 936 cm^2 (b) 209 cm^2

12(a) $\left(p+\frac{1}{4}\right)\left(p-\frac{1}{4}\right)$ (b) $6(x+3)(x-1)$

13(b) $\frac{3}{5}$ (c) $-\frac{3}{5}$

14(a) 1.64 km (b) $\frac{29}{256}$ cm^2

15(a) $y = x+1$ (b) C (7.5, 0) or C (2.5, 0) (c) D(–3, 6) and D(13, 6)

16(a) 189 cm (b) 5.3°

BEATTY SECONDARY SCHOOL

END-OF-YEAR EXAMINATION 2021

SUBJECT : Mathematics **LEVEL : Sec 3 Express**

PAPER : 4048/2 **DURATION : 2 hours**

DATE : 12 October 2021

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

This paper consists of 19 printed pages (including this cover page)

[Turn over

2

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1+\frac{r}{100})^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 (a) (i) Solve the simultaneous linear inequalities $\frac{x-2}{3} < \frac{2x+1}{5} \leq 3$.

Answer ..... [3]

(ii) Hence, state the smallest integer that satisfy the inequality

Answer ..... [1]

(b) Solve the fractional equation $\frac{x-9}{2} = 2 + \frac{17}{x+2}$.

Answer ..... [4]

(c) Given $L = \frac{1}{3}m\left(n + p^2\right)$.

(i) Evaluate L when $m = 4$, $n = -2$ and $p = \frac{1}{2}$.

Answer $L =$ ..... [1]

(ii) Express p as the subject of the formula.

Answer $p =$ ..... [2]

(d) Solve $2^{3-6x} = 32^{3-x}$.

Answer $x =$ ..... [2]

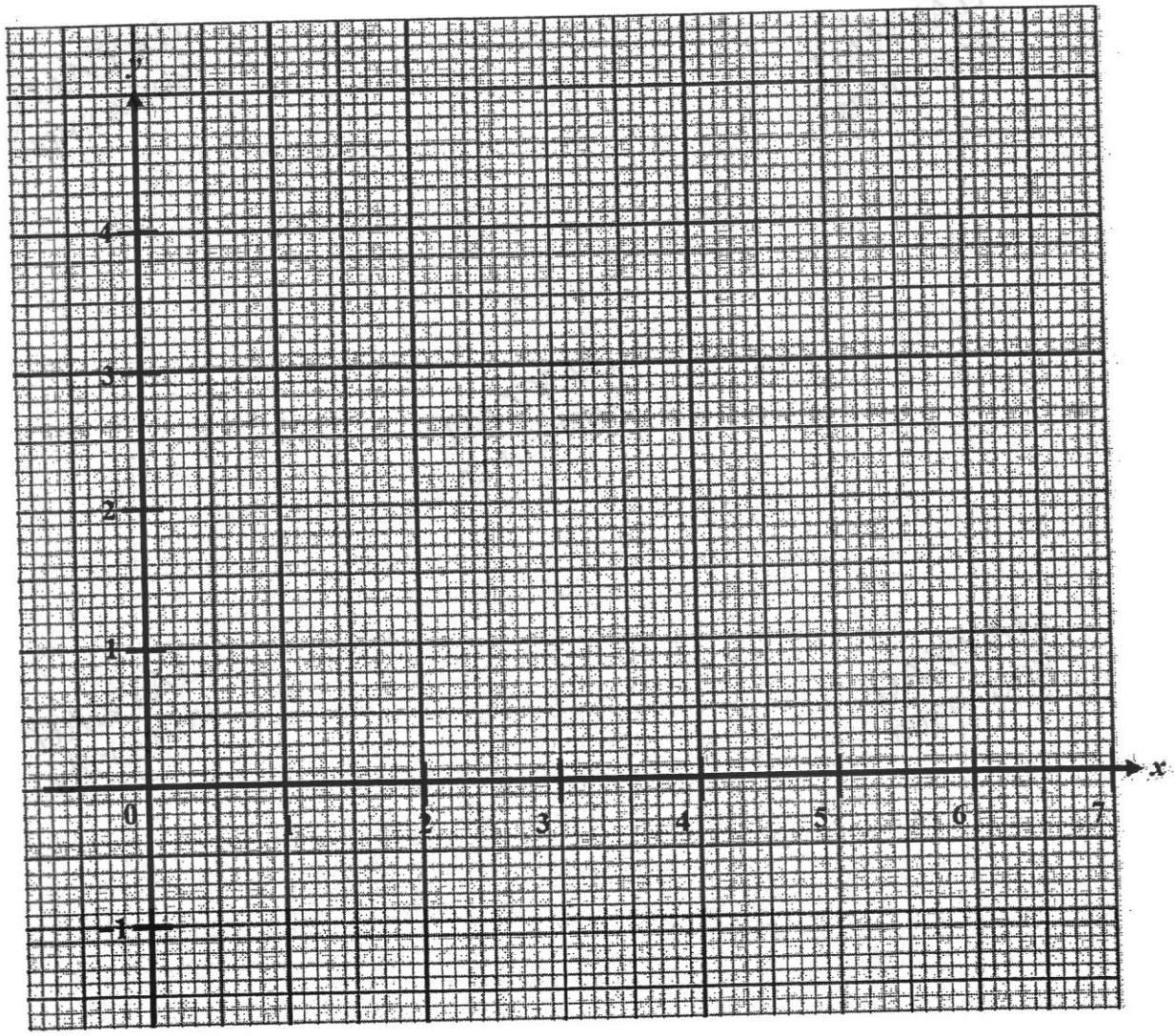
2 **(a)** Complete the table of values for $y = x + \frac{4}{x} - 5$.

Give your answer to 1 decimal place.

x	0.5	1	1.5	2	3	4	5	6	7
y	3.5	0	−0.8	−1	−0.7	0	0.8		2.6

[1]

(b) On the grid, draw the graph of $y = x + \frac{4}{x} - 5$ for $0.5 \leq x \leq 7$.



[3]

(c) Use your graph to find the solutions of the equation $x+\frac{4}{x}-6=0$ in the range $0.5 \leq x \leq 7$.

Answer $x =$ ..... or ..... [2]

(d) By drawing a tangent, find the gradient of the curve at (3, –0.7).

Answer $x =$ ..... [2]

(e) **(i)** On the same grid, draw the graph of $y=-\frac{1}{2}x+3$. [1]

(ii) Show that the points of intersection of the line and the curve gives the solutions of the equation $3x^2-16x+8=0$.

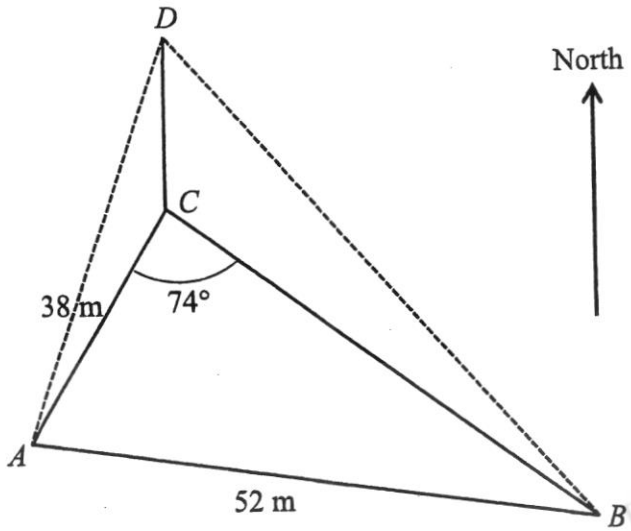
Answers:

[2]

(iii) Use your graph to solve the equation $3x^2-16x+8=0$.

Answer $x =$ ..... or ..... [1]

3



In the diagram, A, B and C are three points on a level school field.

It is given that $AC = 38$ m, $AB = 52$ m and angle $ACB = 74°$.

The bearing of A from B is 285°.

(a) Calculate the angle ABC.

Answer .....° [2]

(b) Find the bearing of C from B.

Answer .....° [1]

[Turn over

(c) Calculate the area of triangle ABC.

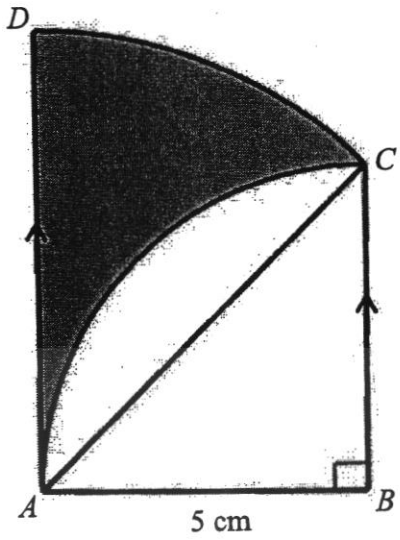
Answer ..... m^2 [2]

(d) A vertical metal beam was mounted on C, with its top at D. The angle of elevation of the top of the beam from A is 25°.

A man was walking along AB. Find the maximum angle of elevation of the top of the beam from the man along AB.

Answer .....° [5]

4



In the diagram, triangle ABC is a right-angled triangle. AC is an arc of a circle with centre B and CD is an arc of another circle with centre A. It is given that $AB = 5$ cm and AD is parallel to BC.

(a) Show that angle CAD is $\frac{\pi}{4}$ radians.

Answer:

[1]

(b) Find the perimeter of the shaded region.

Answer ..... cm [3]

(c) Find the area of the shaded region.

Answer ..... cm^2 [4]

5 Clinton and Harry took part in a city race where they each ran 21 km.

(a) Clinton ran at an average speed of x km/h.

Write down an expression, in terms of x, for the time he took to complete the race.

Answer ..... h [1]

(b) Harry ran at an average speed which was 3 km/h slower than Clinton's.

Write down an expression, in terms of x, the time he took to complete the race.

Answer ..... h [1]

(c) The difference between their time was 18 minutes.

Write down an expression x to represent this information and show that it reduces to $x^2 - 3x - 210 = 0$.

Answer:

[3]

(d) Solve the equation $x^2 - 3x - 210 = 0$, giving your answers to 2 decimal places.

Answer $x =$ ..... or ..... [3]

(e) Find the time that Harry took to complete the race, giving your answer in hours and minutes, correct to the nearest minutes.

Answer ..... h ..... min [2]

6 The employees of a company are offered a wage increase calculated according to one of the following schemes:

Scheme A: An increase of 5% of their present wages.

Scheme B: An increase of $16 per week plus 3% of their present wages.

(a) Mr Tay earns $480 per week. Which scheme should he choose? Support your answer with necessary working.

Answer Scheme ..... because.....

..... [2]

(b) Mr Kannan finds that either scheme will give him the same wage increase. How much is he earning presently?

Answer $..... [2]

[Turn over

(c) Benny divides his monthly income between food, transport and savings in the ratio 5 : 4 : 6 respectively.

(i) He sets aside \$6000 as savings. Find his monthly income.

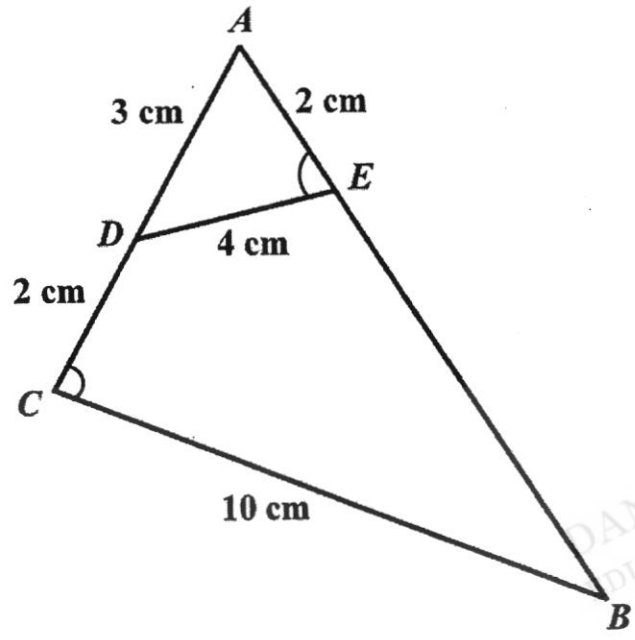
Answer \$..... [2]

(ii) He puts the \$6000 into an account paying compound interest of 3.8% per annum compounded half-yearly.

Calculate the interest he earns after 4 years, correct to the nearest cents.

Answer \$..... [3]

7



In the diagram, ADC and AEB are straight lines. It is given that $AD = 3$ cm, $CD = 2$ cm, $AE = 2$ cm, $DE = 4$ cm, $BC = 10$ cm and $\angle AED = \angle ACB$.

(a) Show that triangle ABC and triangle ADE are similar.

Answer:

[3]

[Turn over

(b) Find the length BE.

Answer ..... cm [2]

(c) Given that the area of triangle ADE is 6 cm^2, find the area of triangle ABC.

Answer ..... cm^2 [2]

(d) State the ratio of the area of triangle ADE to the area of quadrilateral $BCDE$.

Answer ..... [1]

8 Home renovation costs in Singapore vary depending on a few factors such as the size and type of your home. The average renovation cost of a resale 4-room HDB is around $67,000, while a resale 4-room condominium will cost around $82,000. A resale home is one where the new owner is taking over the property from the previous owner.

Meanwhile, a new 4-room HDB will cost around $44,000 to renovate, while a new 4-room condominium will cost around $39,000.

(a) Calculate the percentage difference between the average cost of renovating a resale 4-room HDB and a new 4-room HDB, using the new 4-room HDB renovation cost as the base

Answer ..... % [1]

(b) The following are other factors that will affect the renovation cost.

- Hacking means tearing down walls, rebuilding walls, or touching up walls.
- Flooring installation – either using ceramic tiles, laminate or marble
- You can built-in wardrobes, kitchen cabinets, desks and other fixtures instead of buying from furniture shops. This is known as carpentry.
- Decoration of walls – either painting or applying wallpaper.

The following table shows the breakdown of the cost of renovation based on a 3-room HDB.

Flooring installation	Ceramic Tiles $2.50/square feet	Laminate $4/square feet	Marble $7.50/square feet
Wall – Decoration	Paint $650	Wallpaper $845	
Wall – Hacking	HDB wall $500	Condo wall $800	

Carpentry (built-in items)	**Cost per metre**
Bottom kitchen cabinet	$345
Full height kitchen cabinet	$885
Full height wardrobe (swing door)	$755
Full height wardrobe (sliding door)	$837
Study table with drawers	$493
Study table without drawers	$427
Display cabinet	$886
Half height shoe cabinet	$525
Full height shoe cabinet	$804

(i) Given that 1 square feet is approximately 0.0929 square metre, convert the cost of laminate flooring from the cost per square feet to the cost per square metre, correcting your answer to the nearest dollar.

Answer $ ...../ square metre [2]

19

Ben and Jenny planned to get married and they bought a new 3-room HDB of size 70 square metres. They set aside a budget of $12 000 for renovating their new home. The following is a list of things they would like to have.

Laminate flooring for the whole house
Painting for the wall
Hack the wall between master bedroom and guest room
5-metre full height wardrobe (sliding door)
5-metre full height kitchen cabinet
2-metre full height shoe cabinet

(ii) Is their budget sufficient for their renovation plan? Support your answer with the necessary working.

Answer:

[7]

End of Paper

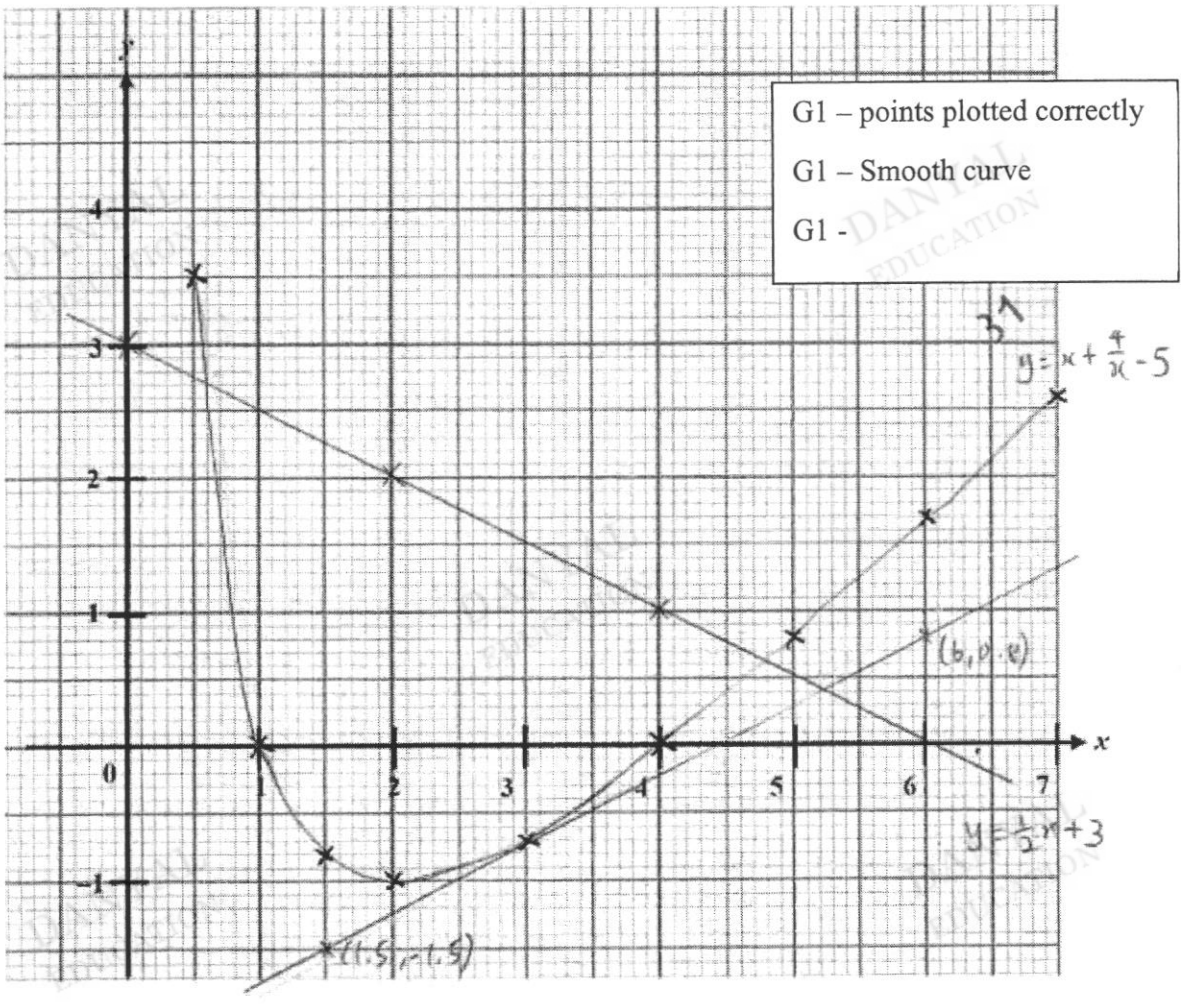
Answer Key

1(a)(i) $-13 < x \leq 7$ 1(a)(ii) -12 1(b) $x = 15$ or $x = -4$

1(c)(i) $-2\frac{1}{3}$ 1(c)(ii) $p = \pm\sqrt{\frac{}{m} - n}$ 1(d) $x = -12$

2(a) 1.7

2(b)



2(c) 0.8 or 5.2 2(d) 0.511 2(e)(iii) 0.6 or 4.8

3(a) 44.6° 3(b) 329.6° 3(c) 867 m^2 3(d) 28.0°

4(b) 20.5 cm 4(c) $12\frac{1}{2}$ cm^2

21

5(a) $\frac{21}{x}$ 5(b) $\frac{21}{x-3}$ 5(d) 16.07 or –13.07 (2dp)

5(e) 1 h 36 mins

6(a) Scheme B because the new wage is more than Scheme A.

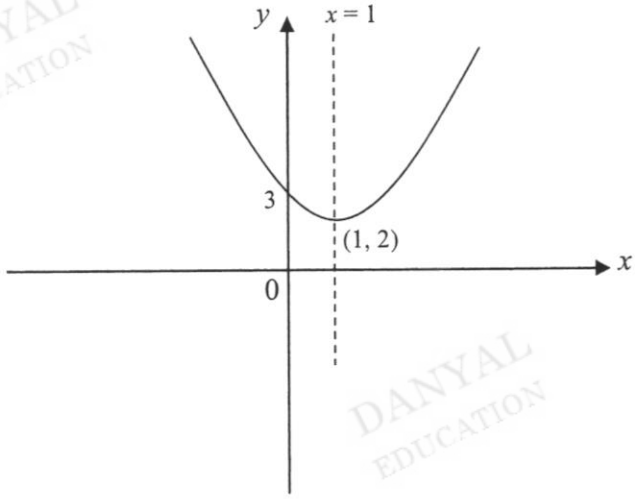
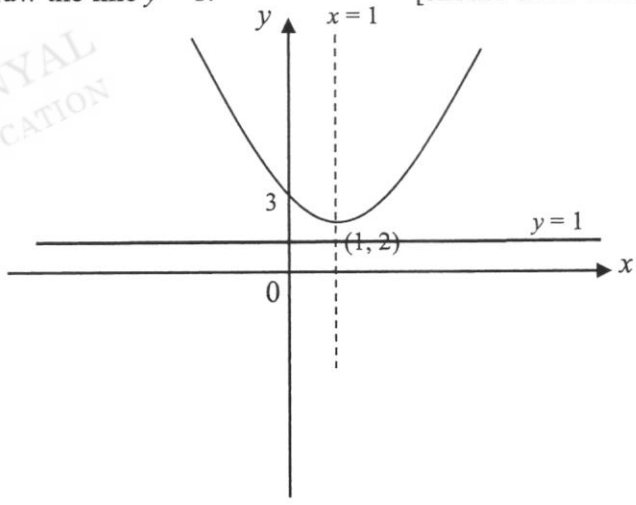
6(b) \$800 6(c)(i) \$15000 6(c)(ii) \$975.01

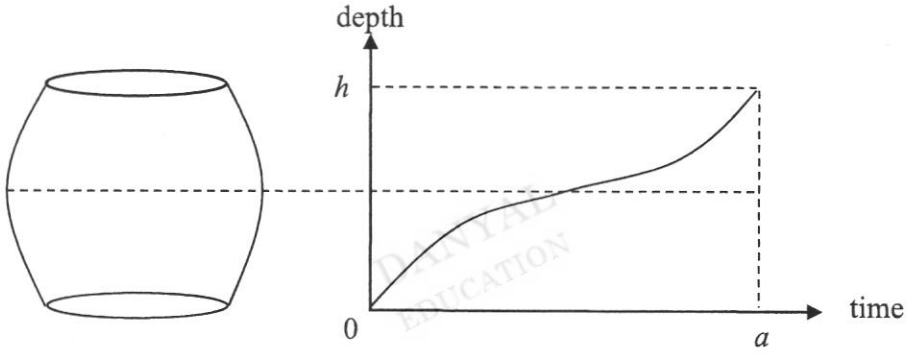
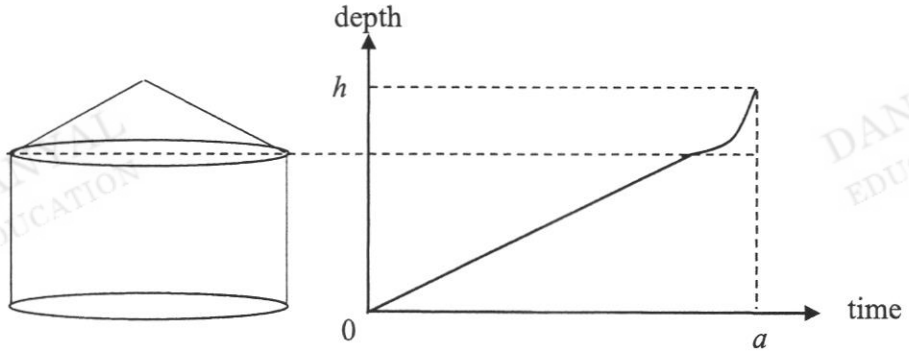
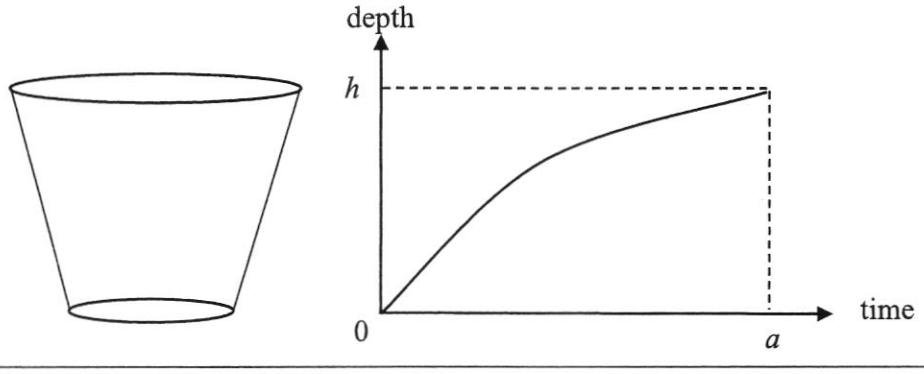
7(b) 5.5 cm 7(c) 37.5 cm^2 7(d) 4 : 21

8(a) 52.3% 8(b)(i) \$43

Mark Scheme for 3E paper 1

1	$6x+y=1$ ..... (1) $x+\frac{y}{3}=1 \rightarrow 3x+y=3$ .....(2) (1) – (2): $3x=-2$ $x=-\frac{2}{3}$ Then, $y=1-6\left(-\frac{2}{3}\right)=5$ Also accept answers obtained through substitution method: From (2): Sub $x=1-\frac{y}{3}$ into (1): $6\left(1-\frac{y}{3}\right)+y=1$ $6-y=1$ $y=5$ Then $x=-\frac{2}{3}$	M1 A1 A1
2(a)	$2x-11\le 15$ $2x\le 26$ $x\le 13$	B1
2(b)	No, because there are 6 prime numbers less than or equal to 13 (2, 3, 5, 7, 11 and 13). Clear workings: students must list all the 6 prime numbers.	B1
3(a)	$y=-\frac{1}{x}$	B1
3(b)	$y=1-x^2$	B1
3(c)	$y=x^3$	B1
3(d)	$y=\frac{2}{x^2}$	B1
4(a)	It is for $y=ka^x$ because when $x=0, y=k\neq 0$. It is for $y=ka^x$ because it is an exponential graph, not a power graph. Accept either of the above.	B1
4(b)	Sub (0, 2): $2=ka^0 \rightarrow k=2$ Sub (4, 1250): $1250=2a^4$ $a^4=625=5^4$ $a=5$ Deduct one mark if students obtain $a=\pm 5$	B1 B1

5(a)	$\left(\frac{x^2}{5}\right)^3 \div \frac{50}{3x^0} = \frac{x^6}{125} \times \frac{3}{50}$ (method mark for either $\frac{x^6}{125}$ or $x^0 = 1$) $= \frac{3x^6}{6250}$	M1 A1
5(b)	$\left(\frac{(3a)^2 b}{20c^5}\right)^{-1} \times \frac{a^3 b^{-2}}{4c} = \left(\frac{9a^2 b}{20c^5}\right)^{-1} \times \frac{a^3}{4b^2 c}$ (M1 for $\frac{20c^5}{9a^2 b}$ or $\frac{a^3}{4b^2 c}$) $= \frac{20c^5}{9a^2 b} \times \frac{a^3}{4b^2 c} = \frac{20a^3 c^5}{36a^2 b^3 c}$ $= \frac{5ac^4}{9b^3}$	M1 A1
6(a)		G1 – correct shape and y-intercept G1 – correct turning point
6(b)	$(x-1)^2 = -1 \rightarrow (x-1)^2 + 2 = 1$ Draw the line $y = 1$. [should show attempt to obtain $y = 1$]  Since the line $y = 1$ does not intersect the graph of $y = (x-1)^2 + 2$, the equation has no real solution.	B1 B1

7(a)	$y = \frac{k}{x^2}$ Sub $x = 3, y = 10$: $10 = \frac{k}{3^2} \rightarrow k = 90$ Hence $y = \frac{90}{x^2}$ (M1 for getting $k = 90$ or for $y = \frac{90}{x^2}$) Sub $y = 4.5$: $4.5 = \frac{90}{x^2}$ $x^2 = 20$ $x = +\sqrt{20} = 4.47$	M1 A1
7(b)	$y = k\sqrt[3]{x}$ When x is doubled, $y = k\sqrt[3]{2x}$ (no marks for writing this only) $\frac{\sqrt[3]{x} - \sqrt[3]{\ }}{k\sqrt[3]{x}} \times 100\% = \frac{\sqrt[3]{2} - 1}{1} \times 100\%$ $= 26.0\%$ (3 sf)	M1 A1
8(a)		B1
8(b)		B1
		B1

9(a)	$\frac{18-0}{6-0} = 3$ m/s^2	B1
9(b)	$m = \frac{9-18}{12-6} = -\frac{3}{2}$ Sub (6, 18) into $y = -\frac{3}{2}x + c$ $18 = -\frac{3}{2}(6) + c = -9 + c \rightarrow c = 27$ Henec $y = -\frac{3}{2}x + 27$ Sub $x = 10$, $y = -\frac{3}{2}(10) + 27 = 12$ m/s Accept other methods: eg. Similar triangles.	B1
9(c)	Area under graph = distance travelled $8 \times 9 + \frac{1}{2}(a-20)(9) = 108$ $\frac{1}{2}(a-20)(9) = 36$ $a - 20 = 8$ $a = 28$	B1
10(a)	$5x + 2(1-3x) = 5x + 2 - 6x$ $= 2 - x$ Accept $-x + 2$	M1 A1
10(b)	$4ay - 2by + 6a - 3b = 2y(2a-b) + 3(2a-b)$ $= (2a-b)(2y+3)$	M1 A1
11(a)	Length ratio = 10 : 12 = 5 : 6 Area ratio = $5^2 : 6^2 = 25 : 36$ (method mark for correct area ratio) Total s.a. of larger container = $\frac{650}{25} \times 36 = 936$ cm^2	M1 A1
11(b)	Volume ratio = 400 : 686 = 200 : 343 Length ratio = $\sqrt[3]{200} : \sqrt[3]{343} = \sqrt[3]{200} : 7$ (M1 for length or area ratio) Area ratio = $\left(\sqrt{\quad}\right)^2 : 7^2 = \left(\sqrt[3]{200}\right)^2 : 49$ Total s.a of smaller container = $\frac{300}{49} \times \left(\sqrt{200}\right)^2 = 209$ cm^2 (3 sf)	M1 A1

12(a)	$p^2-\frac{1}{16}=\left(p+\frac{1}{4}\right)\left(p-\frac{1}{4}\right)$	B1
12(b)	$6x^2+12x-18 = 6\left(x^2+2x-3\right)$ $= 6(x+3)(x-1)$	M1 A1
13(a)	$EG^2 = 250^2 = 62500$ $EF^2+FG^2 = 150^2+200^2 = 62500$ Since, $EF^2+FG^2 = EG^2$, by the converse of Pythagora's Theorem, triangle EFG is a right angle triangle, and $\angle EFG = 90°$. Or, by cosine rule, $\angle EFG = \cos^{-1}\left(\frac{150^2+200^2-250^2}{2(150)(200)}\right) = 90°$	M1 A1
13(b)	$\frac{150}{250}=\frac{3}{5}$	B1
13(c)	$-\frac{150}{250}=-\frac{3}{5}$	B1
14(a)	Map actual 10 cm 800 m 1 cm 80 m 20.5 cm 1640 m 1.64 km	M1 A1
14(b)	Map actual 1 cm 80 m $(1\text{ cm})^2$ $(80\text{ m})^2$ 1 cm^2 6400 m^2 0.113 cm^2 725 m^2 Accept $\frac{29}{256}\text{ cm}^2$	M1 A1

15(a)	Gradient = $m = \frac{6-3}{5-2} = 1$ Sub (2, 3) into $y = x + c$ $3 = 2 + c \quad \rightarrow c = 1$ Hence $y = x + 1$	M1 A1
15(b)	$\sqrt{\qquad\qquad -5)^2} = \sqrt{\frac{169}{4}}$ $36 + (k-5)^2 = \frac{169}{4}$ $(k-5)^2 = \frac{25}{4}$ $k - 5 = \pm\sqrt{\frac{25}{4}}$ $k = 5 + \sqrt{\frac{25}{4}}$ or $k = 5 - \sqrt{\frac{25}{4}}$ $= 7.5 \qquad\qquad = 2.5$ Hence C (7.5, 0) or C (2.5, 0) (answer mark is for both correct)	M1 A1
15(c)	Set $\frac{1}{2}(\text{base})(3) = 12$ base = 8 Hence $D(-3, 6)$ and $D(13, 6)$ (answer mark is for both correct)	M1 A1
16(a)	$AB = \sqrt{60^2 + 80^2} = \sqrt{10000} = 100$ cm $AC = \sqrt{100^2 + 160^2} = 189$ cm (3 sf)	M1 A1
16(b)	$ED = \sqrt{20^2 + 160^2} = 161.245$ Angle $ADE = \tan^{-1}\left(\frac{15}{161.245}\right) = 5.3°$ (1 dp)	M1 A1

BEATTY SECONDARY SCHOOL

END-OF-YEAR EXAMINATION 2021

SUBJECT : Mathematics **LEVEL : Sec 3 Express**

PAPER : 4048/2 **DURATION : 2 hours**

SETTER : Mr Lai Chee Kit **DATE : 12 October 2021**

CLASS :	NAME : MARK SCHEME	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

This paper consists of 19 printed pages (including this cover page)

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1+\frac{r}{100})^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** **(i)** Solve the simultaneous linear inequalities $\frac{x-2}{3} < \frac{2x+1}{5} \leq 3$.

$\frac{x-2}{3} < \frac{2x+1}{5}$	or $\frac{2x+1}{5} \leq 3$	
$5x-10 < 6x+3$	$2x+1 \leq 15$	
$x > -13$	$x \leq 7$	[M1. M1]
Hence $-13 < x \leq 7$ [A1]		

Answer ..... [3]

(ii) Hence, state the smallest integer that satisfy the inequality.

Answer ..... -12 [B1] [1]

(b) Solve the fractional equation $\frac{x-9}{2} = 2 + \frac{17}{x+2}$.

$\frac{x-9}{2} = 2 + \frac{17}{x+2}$	
$\frac{x-9}{2} = \frac{2(x+2)+17}{x+2}$	
$(x-9)(x+2) = 2(2x+21)$	[M1 for non-fractional equation]
$x^2 - 7x - 18 = 4x + 42$	
$x^2 - 11x - 60 = 0$	[M1]
$(x-15)(x+4) = 0$	[M1]
$x = 15$ or $x = -4$	[A1]

Answer ..... [4]

[Turn over

(c) Given $L=\frac{1}{3}m\left(n+p^2\right)$.

(i) Evaluate L when $m = 4$, $n = -2$ and $p=\frac{1}{2}$.

$-2\frac{1}{3}$, or $-\frac{7}{3}$ [B1]

Answer $L =$ ..... [1]

(ii) Express p as the subject of the formula.

$L=\frac{1}{3}m\left(n+p^2\right)$

$p^2=\frac{3L}{m}-n$ [M1]

$p=\pm\sqrt{\frac{3L}{m}-n}$ [A1]

Answer $p =$ ..... [2]

(d) Solve $2^{3-6x}=32^{3-x}$.

$2^{3-6x}=32^{3-x}$

$2^{3-6x}=2^{15-5x}$ [M1]

$3-6x=15-5x$

$x=-12$ [A1]

Answer $x =$ ..... [2]

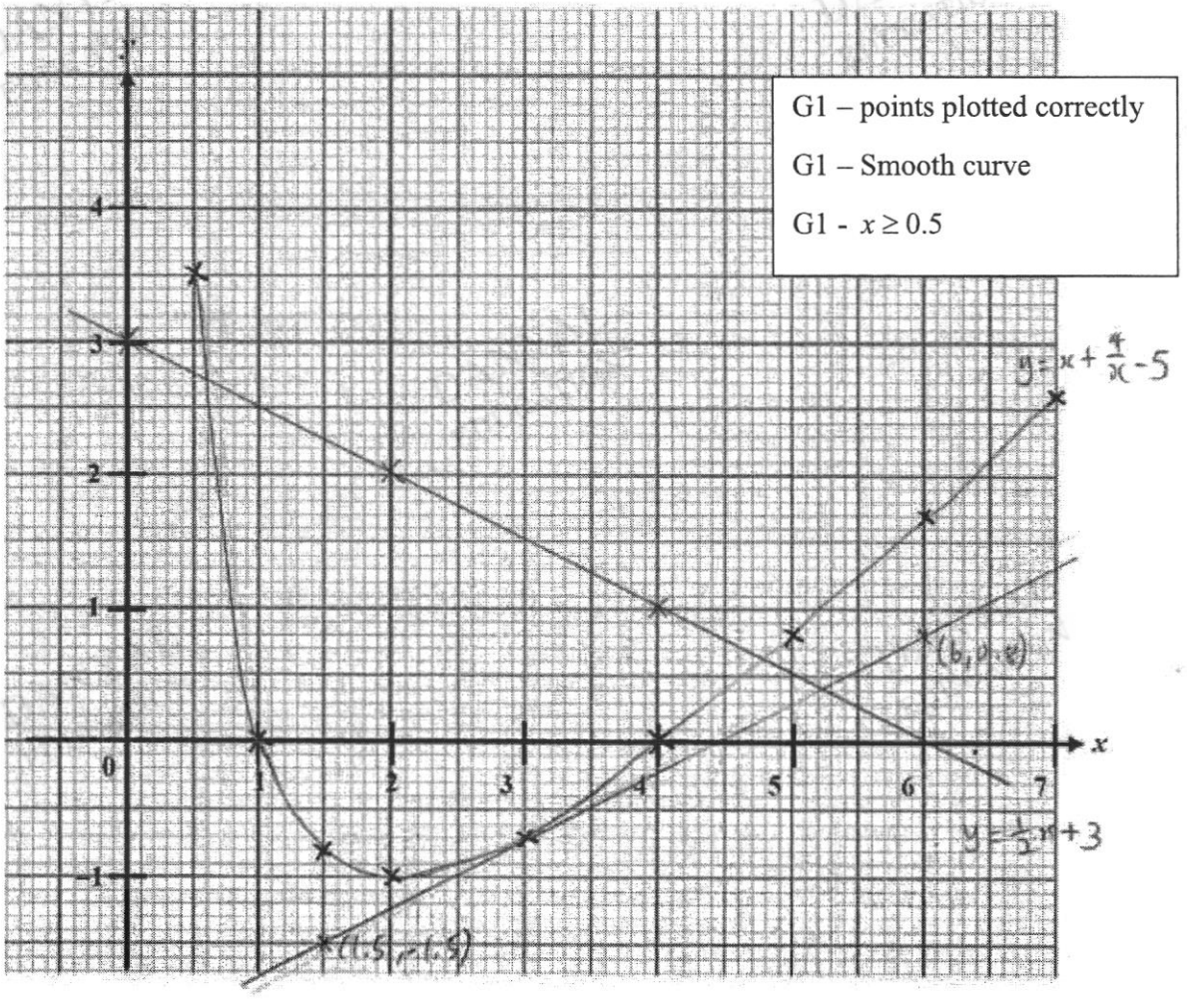
2 **(a)** Complete the table of values for $y = x + \frac{4}{x} - 5$.

Give your answer to 1 decimal place.

x	0.5	1	1.5	2	3	4	5	6	7
y	3.5	0	−0.8	−1	−0.7	0	0.8	***1.7*** ***[B1]***	2.6

[1]

(b) On the grid, draw the graph of $y = x + \frac{4}{x} - 5$ for $0.5 \leq x \leq 7$.



[3]

(c) Use your graph to find the solutions of the equation $x+\frac{4}{x}-6=0$ in the range $0.5 \le x \le 7$.

$x+\frac{4}{x}-6=0$

$x+\frac{4}{x}-5=1$

$y=1$ [B1]

0.8 (accept 0.7, 0.75, 0.85, 0.9) or

5.2 (accept 5.1, 5.15, 5.25, 5.3) [B1]

Answer $x =$ ..... or ..... [2]

(d) By drawing a tangent, find the gradient of the curve at (3, –0.7).

B1 for tangent drawn at correct point

$\text{Gradient} = \frac{-1.5-0.8}{1.5-6}$

$= 0.511$ (3sf) [B1]

(accept 0.43 to 0.62)

Answer ..... [2]

(e) **(i)** On the same grid, draw the graph of $y=-\frac{1}{2}x+3$. [1]

B1 for line drawn correctly

(ii) Show that the points of intersection of the line and the curve gives the solutions of the equation $3x^2-16x+8=0$.

$x+\frac{4}{x}-5=-\frac{1}{2}x+3$

$\frac{3}{2}x+\frac{4}{x}-8=0$

$\frac{3x^2+8-16x}{2x}=0$ [M1 for correct LCM]

$3x^2-16x+8=0$ (shown) [A1]

[2]

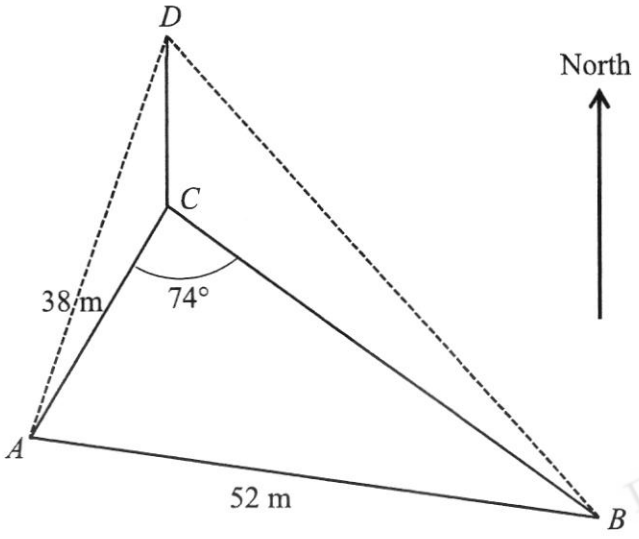
(iii) Use your graph to solve the equation $3x^2-16x+8=0$.

0.6 (accept 0.5, 0.55, 0.65, 0.7) or

4.8 (accept 4.7, 4.75, 4.85, 4.9) [B1]

Answer $x =$ ..... or ..... [1]

3



In the diagram, A, B and C are three points on a level school field.

It is given that $AC = 38$ m, $AB = 52$ m and angle $ACB = 74°$.

The bearing of A from B is 285°.

(a) Calculate the angle ABC.

$$\frac{\sin \angle ABC}{38} = \frac{\sin 74°}{52} \quad \text{[M1]}$$

$$\angle ABC = 44.6247°$$

$$= 44.6° \quad \text{[A1]}$$

Answer .....° [2]

(b) Find the bearing of C from B.

329.6° [B1]

Answer .....° [1]

(c) Calculate the area of triangle ABC.

$\angle CAB = 61.375°$

Area of $\triangle ABC = \frac{1}{2}(38)(52)\sin 61.375°$ [M1]

$= 867.243$

$= 867 \text{ m}^2$ [A1]

Answer ..... m^2 [2]

(d) A vertical metal beam was mounted on C, with its top at D. The angle of elevation of the top of the beam from A is 25°.

A man was walking along AB. Find the maximum angle of elevation of the top of the beam from the man along AB.

$\tan 25° = \frac{CD}{38}$

$CD = 17.71969$ m [M1]

Let E be the point on AB where the maximum angle of elevation occurs.

$\frac{1}{2} \times 52 \times CE = 867.243$ [M1]

$CE = 33.3555$ m [A1]

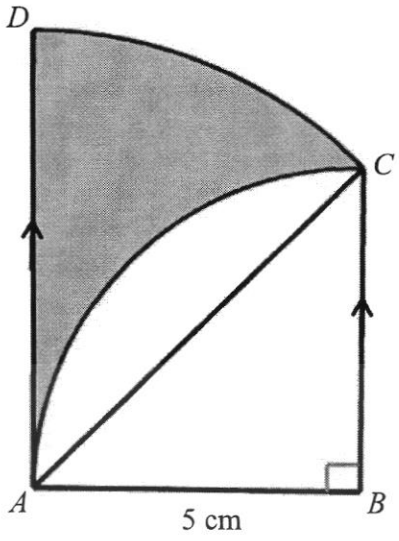
$\tan \angle CED = \frac{17.71969}{33.3555}$ [M1]

$\angle CED = 28.0°$ [A1]

Hence, maximum angle of elevations is 28.0°.

Answer .....° [5]

4



In the diagram, triangle ABC is a right-angled triangle. AC is an arc of a circle with centre B and CD is an arc of another circle with centre A. It is given that $AB = 5$ cm and AD is parallel to BC.

(a) Show that angle CAD is $\frac{\pi}{4}$ radians.

Answer:

$\angle BCA = \angle DAC$ (alternate angles, AD // BC)

$= \dfrac{\pi - \frac{\pi}{2}}{2}$ [B1]

$= \frac{\pi}{4}$ radians (shown)

[1]

(b) Find the perimeter of the shaded region.

$AD = AC = \sqrt{\quad 5^2}$

$= \sqrt{\quad}$

$= 7.07107$ cm [M1]

Arc $AC = \frac{5\pi}{2} = 7.85398$ cm

Arc $CD = \sqrt{\quad} \times \frac{\pi}{4} = 5.5536$ cm

Perimeter of shaded part $= \sqrt{50} + \sqrt{50} \times \frac{\pi}{4} + \frac{5\pi}{2}$ [M1]

$= 20.5$ cm (3sf) [A1]

Answer ..... cm [3]

(c) Find the area of the shaded region.

Area of segment $CD = \frac{1}{2} \times 5^2 \times \frac{\pi}{2} - \frac{1}{2} \times 5 \times 5$

$= \frac{25}{4}\pi - \frac{25}{2} = 7.13495$ cm^2 [M1]

[M1 for area either area of sector BAC or area of sector ACD]

Area of shaded region $= \frac{1}{2} \times \left(\sqrt{50}\right)^2 \times \frac{\pi}{4} - \left(\frac{25}{4}\pi - \frac{25}{2}\right)$ [M1]

$= 12\frac{1}{2}$ cm^2 or 12.5 cm^2 [A1]

Answer ..... cm^2 [4]

5 Clinton and Harry took part in a city race where they each ran 21 km.

(a) Clinton ran at an average speed of x km/h.

Write down an expression, in terms of x, for the time he took to complete the race.

Answer $\frac{21}{x}$ [B1] h [1]

(b) Harry ran at an average speed which was 3 km/h slower than Clinton's.

Write down an expression, in terms of x, the time he took to complete the race.

Answer $\frac{21}{x-3}$ [B1] h [1]

(c) The difference between their time was 18 minutes.

Write down an expression x to represent this information and show that it reduces to $x^2-3x-210=0$.

Answer:

$\frac{21}{x-3}-\frac{21}{x}=\frac{18}{60}$ [M1]

$\frac{21x-21(x-3)}{x(x-3)}=\frac{3}{10}$

$\frac{63}{x(x-3)}=\frac{3}{10}$

$630=3x^2-9x$

$3x^2-9x-630=0$ [M1]

$x^2-3x-210=0$ (shown) [A1]

[3]

(d) Solve the equation $x^2 - 3x - 210 = 0$, giving your answers to 2 decimal places.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(210)}}{2(1)} \quad [M1]$$

$= 16.07$ or -13.07 (2dp) [A1, A1]

Answer $x =$ ..... or ..... [3]

(e) Find the time that Harry took to complete the race, giving your answer in hours and minutes, correct to the nearest minutes.

$$\text{Time} = \frac{21}{16.07 - 3} \quad [M1]$$

$= 1.6067$ h

$= 1$ h 36 mins [A1]

Answer ..... h ..... min [2]

6 The employees of a company are offered a wage increase calculated according to one of the following schemes:

Scheme A: An increase of 5% of their present wages.

Scheme B: An increase of \$16 per week plus 3% of their present wages.

(a) Mr Tay earns \$480 per week. Which scheme should he choose? Support your answer with necessary working.

Scheme A: New wage = $1.05 \times \$480 = \$504$

Scheme B: New wage = $\$16 + 1.03 \times 480 = \$510.40$ [M1 for both new wages]

He should choose Scheme B because the new wage is more than Scheme A. [A1]

Answer Scheme ..... because.....

..... [2]

(b) Mr Kannan finds that either scheme will give him the same wage increase. How much is he earning presently?

Let x be his present wage.

$1.05x = 16 + 1.03x$ [M1]

$0.02x = 16$

$x = 800$

His present wage is \$800. [A1]

Answer \$..... [2]

(c) Benny divides his monthly income between food, transport and savings in the ratio 5 : 4 : 6 respectively.

(i) He sets aside \$6000 as savings. Find his monthly income.

$$\text{New monthly income} = \frac{6000}{6} \times 15 \quad [M1]$$

$$= \$15000 \quad [A1]$$

Answer \$..... [2]

(ii) He puts the \$6000 into an account paying compound interest of 3.8% per annum compounded half-yearly.

Calculate the interest he earns after 4 years, correct to the nearest cents.

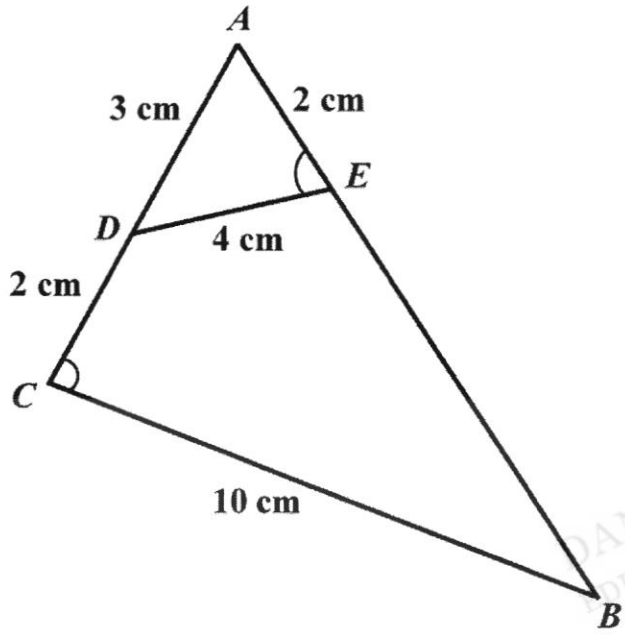
$$\text{Amount} = 6000\left(1+\frac{1.9}{100}\right)^{8} \quad [M1]$$

$$= \$6975.01 \quad [M1]$$

$$\text{Interest} = \$975.01 \text{ (nearest cent)} \quad [A1]$$

Answer \$..... [3]

7



In the diagram, ADC and AEB are straight lines. It is given that $AD = 3$ cm, $CD = 2$ cm, $AE = 2$ cm, $DE = 4$ cm, $BC = 10$ cm and $\angle AED = \angle ACB$.

(a) Show that triangle ABC and triangle ADE are similar.

Answer:

$\frac{AE}{AC} = \frac{2}{5}$

$\angle AED = \angle ACB$ (given)

$\frac{DE}{BC} = \frac{4}{10} = \frac{2}{5}$

[M2 for 3 statements, M1 for 2 statements]

Hence, triangle ABC is similar to triangle ADE (SAS-similarity). [A1, no need to write SAS]

OR

$\angle AED = \angle ACB$ (given)

$\angle EAD = \angle CAB$ (common angle)

Hence, triangle ABC is similar to triangle ADE (AA-similarity).

[3]

(b) Find the length BE.

$\frac{AB}{3} = \frac{5}{2}$

$AB = 7.5$ cm [M1]

$BE = 7.5 - 2$ `

$= 5.5$ cm [A1]

Answer ..... cm [2]

(c) Given that the area of triangle ADE is 6 cm^2, find the area of triangle ABC.

$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \left(\frac{5}{2}\right)^2$ [M1]

$\frac{\text{Area of } \Delta ABC}{6} = \frac{25}{4}$

Area of $\Delta ABC = 37.5$ cm^2 [A1]

Answer ..... cm^2 [2]

(d) State the ratio of the area of triangle ADE to the area of quadrilateral $BCDE$.

$4 : 21$ or $\frac{4}{21}$ [B1]

Answer ..... [1]

8 Home renovation costs in Singapore vary depending on a few factors such as the size and type of your home. The average renovation cost of a resale 4-room HDB is around $67,000, while a resale 4-room condominium will cost around $82,000. A resale home is one where the new owner is taking over the property from the previous owner.

Meanwhile, a new 4-room HDB will cost around $44,000 to renovate, while a new 4-room condominium will cost around $39,000.

(a) Calculate the percentage difference between the average cost of renovating a resale 4-room HDB and a new 4-room HDB, using the new 4-room HDB renovation cost as the base.

$$\% \text{ difference} = \frac{67000 - 44000}{44000} \times 100\%$$

$$= 52.3\% \text{ (3sf)} \quad [B1]$$

Answer ..... % [1]

(b) The following are other factors that will affect the renovation cost.

- Hacking means tearing down walls, rebuilding walls, or touching up walls.
- Flooring installation – either using ceramic tiles, laminate or marble
- You can built-in wardrobes, kitchen cabinets, desks and other fixtures instead of buying from furniture shops. This is known as carpentry.
- Decoration of walls – either painting or applying wallpaper.

The following table shows the breakdown of the cost of renovation based on a 3-room HDB.

Flooring installation	Ceramic Tiles $2.50/square feet	Laminate $4/square feet	Marble $7.50/square feet
Wall – Decoration	Paint $650	Wallpaper $845	
Wall – Hacking	HDB wall $500	Condo wall $800	

Carpentry (built-in items)	**Cost per metre**
Bottom kitchen cabinet	$345
Full height kitchen cabinet	$885
Full height wardrobe (swing door)	$755
Full height wardrobe (sliding door)	$837
Study table with drawers	$493
Study table without drawers	$427
Display cabinet	$886
Half height shoe cabinet	$525
Full height shoe cabinet	$804

(i) Given that 1 square feet is approximately 0.0929 square metre, convert the cost of laminate flooring from the cost per square feet to the cost per square metre, correcting your answer to the nearest dollar.

$$\frac{\$4}{1\,\text{square feet}} = \frac{\$4}{0.0929\,\text{sqm}} \quad [\text{M1}]$$

$$= \$43 \text{ (nearest dollar)} \quad [\text{A1}]$$

Answer $ ...../ square metre [2]

19

Ben and Jenny planned to get married and they bought a new 3-room HDB of size 70 square metre. They set aside a budget of $12 000 for renovating their new home. The following is a list of things they would like to have.

Laminate flooring for the whole house
Painting for the wall
Hack the wall between master bedroom and guest room
5-metre full height wardrobe (sliding door)
5-metre full height kitchen cabinet
2-metre full height shoe cabinet

(ii) Is their budget sufficient for their renovation plan? Support your answer with the necessary working.

Answer:

Cost of laminate flooring = $43 × 70 sqm = $3010 [Accept $3014] [C1]

Cost of painting = $650

Cost of hacking = $500

Cost of full height wardrobe (sliding door) = $837 × 5 m = $4185 [C1]

Cost of full height kitchen cabinet = $885 × 5 m = $4425 [C1]

Cost of full height shoe cabinet = $804 × 2 m = $1608 [C1]

Total cost of renovation = $3010 + $650 + $500 + $4185 + $4425 + $1608

[C1 for adding 6 individual costs]

= $14378 [Accept $14382] [A1]

Since $14378 > $12000, their budget is insufficient.

[A1 for correct conclusion based on calculated total cost]

[7]

End of Paper