BEATTY SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2022
SECONDARY THREE EXPRESS

CANDIDATE NAME			
CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS **4049**

Paper Choose an item. **10 October 2022**

Setter: Ms Joanna Chong HY **2 hours 15 minutes**

Candidates answer on the Question Paper
Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Given non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiner's Use

This document consists of **18** printed pages and **0** blank pages.

1 Find the range of values of p for which $y = x^2 + p(x-1) + 4 - x$ is positive for all real values of x. [4]

2 Given that $\sqrt{7}(k+2) = 3(k-2)$, express k in the form $a + b\sqrt{7}$. [3]

3 **(a)** By completing the square, prove that the minimum value of $y = 6x^2 + 4x - 7$ is $-7\frac{2}{3}$. [3]

(b) Two statements are made about a quadratic expression $f(x)$.

Statement A : The graph of $f(x)$ has a turning point at $(-3, 5)$.
Statement B : $f(x)$ cannot be expressed as $f(x) = a(x-p)(x-q)$ where a, p and q are real numbers.

State whether each of the following is true, false, or if there is insufficient information to conclude. Justify your answer with reasons.

(i) $f(x) = 2(x+3)^2 + 5$. [2]

(ii) $(-3, 5)$ is a minimum point. [2]

4 Solve each of the following equations for $0^o \leq \theta \leq 360^o$.

(a) $\sin(\theta - 25^o) = \frac{1}{2}$. [3]

(b) $2\tan^2\theta - 1 = 5\sec\theta$. [5]

5 A student claimed that $\frac{5x^2-3x+8}{(x-1)(x^2+4)} = \frac{2}{x-1}+\frac{3x+1}{x^2+4}$, $x \neq 1$.

(i) Without finding the partial fractions of $\frac{5x^2-3x+8}{(x-1)(x^2+4)}$ and without substituting any values of x, show that the student's answer is wrong. [2]

(ii) Express $\frac{5x^2-3x+8}{(x-1)(x^2+4)}$ as a sum of two partial fractions. [5]

6 **(a)** Solve $3^{2x}+18=3^{x+2}$. [3]

(b) Solve the following equation $\log_{25}(5x-6)-\dfrac{1}{\log_2 5}=\log_5(x-1)$. [4]

7 Given that $f(x) = 2x^3 + (a-4)x^2 + ax - 6a$,

(i) show that $x - 2$ is a factor of f(x), [1]

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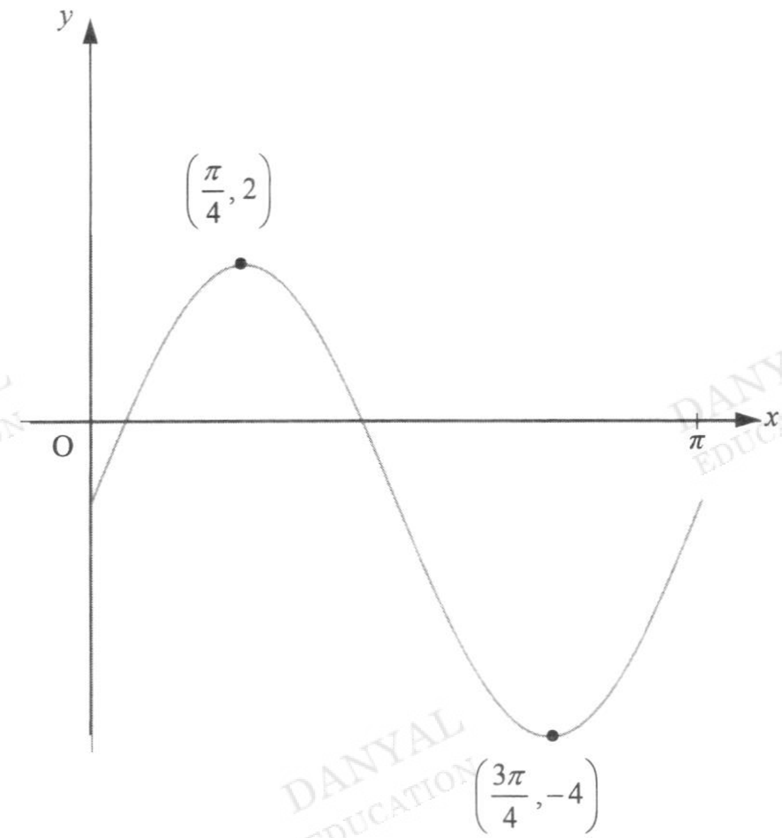
(iii) If f(x) has three real roots, find the range of values of a. [2]

8 The diagram shows the curve $y = a\sin bx + c$ for $0 \le x \le \pi$ radians. The curve has a maximum point at $\left(\frac{\pi}{4}, 2\right)$ and a minimum point at $\left(\frac{3\pi}{4}, -4\right)$.



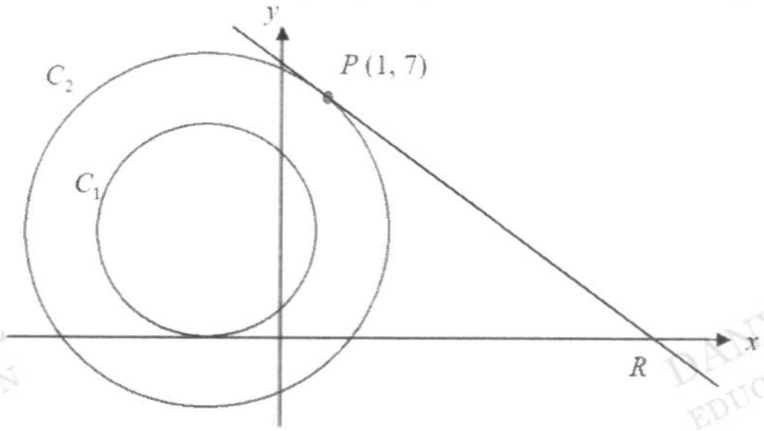
(i) Explain why $b = 2$. [2]

(ii) Find the equation of the curve. [2]

(iii) On the same axes, draw the graph of $y = 4\cos 2x + 2$. Label the y-intercept and the coordinates of minimum point(s) on the graph clearly. [3]

9 Concentric circles are circles which have the same centre.
The diagram shows two concentric circles C_1 and C_2.
PR is a tangent to the circle C_2 at *P* (1, 7).
The equation of C_1 is $x^2 + y^2 + 4x - 6y + 4 = 0$.

(i) Show that the centre of C_1 is $(-2, 3)$ and find the radius of C_1. [3]



(ii) Find the equation of C_2. [2]

(iii) Given that PR cuts the x-axis at point R. Find the co-ordinates of R. [4]

(iv) A third circle, C_3 is tangent to both the x and y axes. What can you deduce about the centre of the circle? [1]

10 **(a)(i)** Write down the first three terms of the expansion $\left(2-\frac{1}{4x}\right)^5$ in descending powers of x. [2]

(ii) Given that the coefficient of $\frac{1}{x}$ in the expansion of $\left(2-\frac{1}{4x}\right)^5(3x-a)$ is 255, find the value of a. [3]

(b) The expansion of $\left(2x^2 - \frac{1}{\sqrt{x}}\right)^n$, where n is a positive integer, has a term that is independent of x. Find the smallest value of n. [3]

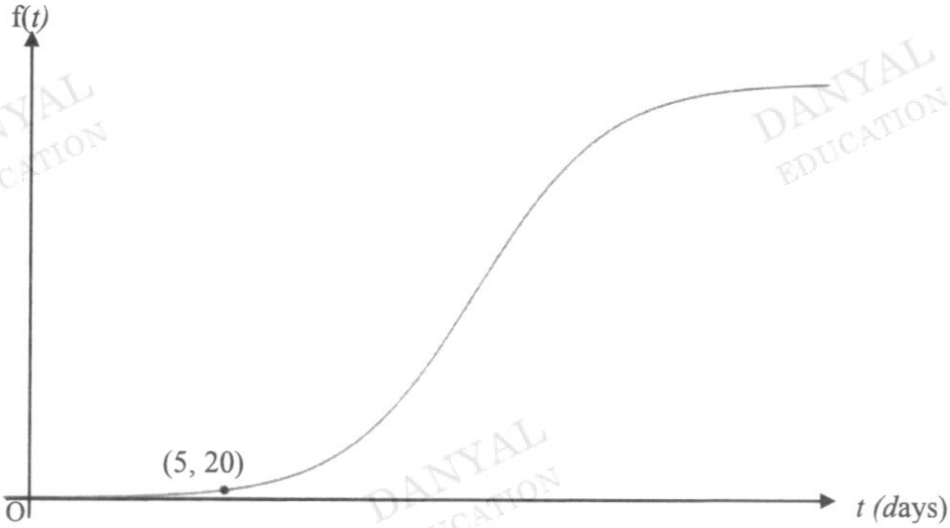
11 An influenza epidemic spreads through a population rapidly, at a rate that depends on two factors:

- The more people who have the flu, the more rapidly it spreads, and
- the more uninfected people there are, the more rapidly it spreads.

For a particular community of 1000 people, researchers found that they could model the spread of the flu using the logistic model,

$$f(t) = \frac{1000}{1+999e^{kt}},$$

where f(*t*) is the number of people infected, *t* is time measured in days after the first infection and *k* is a constant. The following graph shows how the model fits the data. The point (5, 20) lies on the curve.



(i) Show that the value of the constant, *k*, is approximately –0.603. [3]

(ii) Predict the number of people infected after 10 days. [2]

(iii) How many days will it take for 750 people to be infected? Give your answer correct to 1 decimal place. [2]

(iv) Student A suggested that an exponential curve of the form

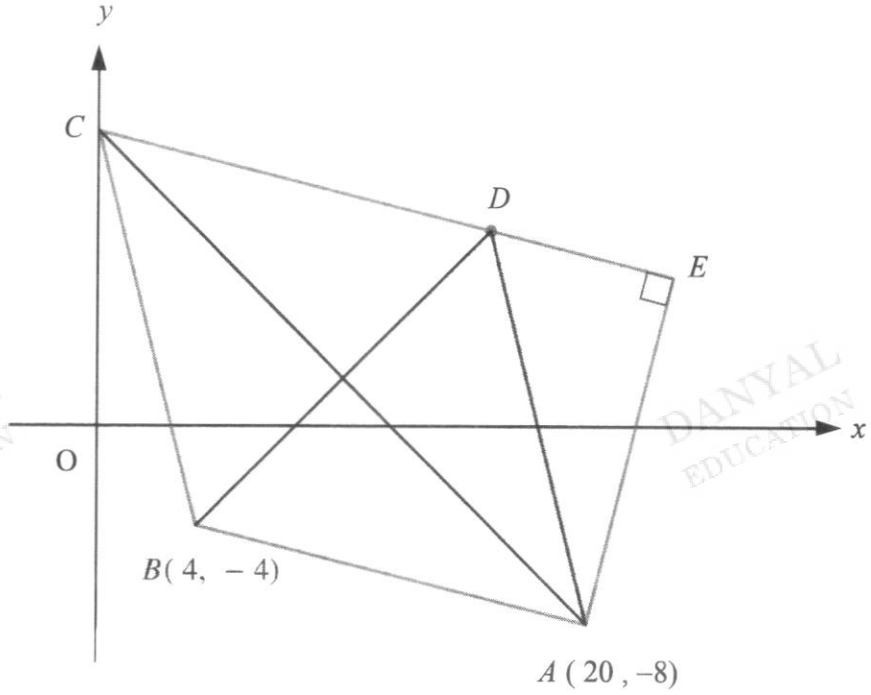
$$g(t) = e^{kt}, \text{ where } k = \frac{\ln 20}{5}$$

can be used to fit the data set just as well.

Student B claimed that the logistic curve shown in the diagram is a more accurate model.

Who do you agree with? Explain your answer. [2]

12 In the diagram, $ABCD$ is a rhombus. The coordinates of A and B are $(20,-8)$ and $(4,-4)$ respectively.
C is a point on the y-axis and E is a point on CD produced such that $\angle BAE = \angle CEA = 90^o$.
The length of AB is $\sqrt{272}$ units.



(i) Show that the point C is (0, 12). [3]

(ii) Find the coordinates of D. [3]

(iii) Find the equation of AE. [3]

(iv) Find the area of the rhombus $ABCD$. [2]

(v) Show that $\cos\angle OCB = \frac{4}{\sqrt{17}}$. [2]

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[Turn over

1 Find the range of values of p for which $y = x^2 + p(x-1) + 4 - x$ is positive for all real values of p. [4]

$y = x^2 + px - x + 4 - p$

Discriminant $= b^2 - 4ac < 0$ --- M1

$(p-1)^2 - 4(1)(4-p) < 0$

$p^2 - 2p + 1 - 16 + 4p < 0$ --- M1

$p^2 + 2p - 15 < 0$

$(p-3)(p+5) < 0$ --- M1

$-5 < p < 3$ --- A1

2 Given that $\sqrt{7}(k+2) = 3(k-2)$. Express k in the form $a + b\sqrt{7}$ where a and b are integers. [3]

$k(3-\sqrt{7}) = 2\sqrt{7} + 6$

$k = \dfrac{2\sqrt{7}+6}{3-\sqrt{7}} \times \dfrac{3+\sqrt{7}}{3+\sqrt{7}}$ --- M1(conjugate surd)

$= \dfrac{6\sqrt{7}+14+18+6\sqrt{7}}{9-7}$ --- M1(expansion), M1(denominator)

$= \dfrac{12\sqrt{7}+32}{2}$

$= 16 + 6\sqrt{7}$ --- A1

[Turn over

3 **(a)** By completing the square, prove that the minimum value of $y = 6x^2 + 4x - 7$ is $-7\frac{2}{3}$. [3]

$$6x^2 + 4x - 7$$
$$= 6\left(x^2 + \frac{2}{3}x\right) - 7$$
$$= 6\left[\left(x + \frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right] - 7 --- M1$$
$$= 6\left[\left(x + \frac{1}{3}\right)^2 - \frac{1}{9}\right] - 7$$
$$= 6\left(x + \frac{1}{3}\right)^2 - \frac{2}{3} - 7$$
$$= 6\left(x + \frac{1}{3}\right)^2 - 7\frac{2}{3} --- M1$$

Since $\left(x + \frac{1}{3}\right)^2 \geq 0$ (perfect square), $6\left(x + \frac{1}{3}\right)^2 - 7\frac{2}{3} \geq -7\frac{2}{3}$, $---A1$

hence, minimum value is $-7\frac{2}{3}$.

(b) Two statements are made about a quadratic expression $f(x)$.

Statement A : The graph of $f(x)$ has a turning point at $(-3, 5)$.
Statement B : $f(x)$ cannot be expressed as $f(x) = a(x-p)(x-q)$ where a, p and q are real numbers.

State whether each of the following is true, false, or if there is insufficient information to conclude. Justify your answer with reasons.

(i) $f(x) = 2(x+3)^2 + 5$. [2]

(ii) $(-3, 5)$ is a minimum point. [2]

(i) $f(x)$ is of the form $y = a(x+3)^2+5$ since $(-3, 5)$ is the turning point but the constant, a can be any real number and may not be '2'. --- $B1$
Hence, there is insufficient information to conclude that $f(x) = 2(x+3)^2+5$--$B1$

(ii) Since $f(x)$ cannot be factorised into factors that are real, it means that **$f(x) = 0$ has no real roots**.---$B1$
As the **turning point lies above the x-axis**, the graph of $f(x)$ must open upwards to avoid cutting the x-axis, hence $(-3, 5)$ is a minimum point. True. $B1$

4 Solve each of the following equations for $0^o \le \theta \le 360^o$.

(a) $\sin^2(\theta - 25^o) = \frac{1}{2}$. [3]

(b) $2\tan^2\theta - 1 = 5\sec\theta$. [5]

(a)

$\sin(\theta - 25^o) = \frac{1}{2}$

$basic\ angle, \alpha = 30^o$ ---M1

$\theta - 25^o = 30^o, 150^o$ ----M1

$\theta = 75^o, 175^o$ ----A1

(b)

$2\tan^2\theta - 1 = 5\sec\theta$

$2(\sec^2\theta - 1) - 1 = 5\sec\theta$ ---M1

$2\sec^2\theta - 3 = 5\sec\theta$

$2\sec^2\theta - 5\sec\theta - 3 = 0$

$(2\sec\theta + 1)(\sec\theta - 3) = 0$---M1

$\sec\theta = -\frac{1}{2}$ or $\sec\theta = 3$

$\cos\theta = -2$ (NA) $\quad \cos\theta = \frac{1}{3}$

M1 $\quad$ basic angle, $\alpha = 70.528^o$ ---M1

$\theta = 70.5^o, 289.5^o$---A1

5 A student claimed that $\dfrac{5x^2-3x+8}{(x-1)(x^2+4)} = \dfrac{2}{x-1}+\dfrac{3x+1}{x^2+4}$, $x \neq -1$.

(i) Without finding the partial fractions of $\dfrac{5x^2-3x+8}{(x-1)(x^2+4)}$ and without substituting values of x, show that the student's answer is wrong. [2]

(ii) Express $\dfrac{5x^2-3x+8}{(x-1)(x^2+4)}$ as a sum of two partial fractions. [5]

(i) $\dfrac{2}{x-1}+\dfrac{3x+1}{x^2+4}$

$$\frac{2(x^2+4)+(3x+1)(x-1)}{(x-1)(x^2+4)} - - - M1$$

$$= \frac{2x^2+8+3x^2-3x+x-1}{(x-1)(x^2+4)}$$

$$= \frac{5x^2-2x+7}{(x-1)(x^2+4)} - - - A1$$

(ii)

$$\frac{5x^2-3x+8}{(x-1)(x^2+4)} = \frac{A}{(x-1)}+\frac{Bx+C}{(x^2+4)} - - - M1$$

$5x^2-3x+8 = A(x^2+4)+(Bx+C)(x-1)$

Let $x=1$

$10 = A(5)$

$A = 2 - - - - - M1$

Let $x = 0 - - - - M1$

$8 = 4A - C$

$C = 0$

Comparing coefficients of x^2:

$5=A+B$

$B = 5-2 = 3 - - - - M1$

$$\frac{5x^2-3x+8}{(x-1)(x^2+4)} = \frac{2}{(x-1)}+\frac{3x}{(x^2+4)} - - - B1$$

6 **(a)** Solve $3^{2x}+18=3^{x+2}$. [3]

$3^{2x}+18=3^{x+2}$

Let $u=3^{x}$ ---M1

$u^{2}-9u+18=0$

$(u-3)(u-6)=0$ ---M1

$u=3 \quad or \quad u=6$

$3^{x}=3 \qquad 3^{x}=6$

$x=1 \qquad x=\dfrac{\lg 6}{\lg 3}=1.63 \quad (3sf)$ --- A1(for both answers)

(b) Solve the following equation $\log_{25}(5x-6)-\dfrac{1}{\log_{2}5}=\log_{5}(x-1)$. [4]

$\log_{25}(5x-6)-\dfrac{1}{\log_{2}5}=\log_{5}(x-1)$

$\dfrac{\log_{5}(5x-6)}{\log_{5}25}-\log_{5}2=\log_{5}(x-1)$ --- M1 [*change of base*]

$\log_{5}(5x-6)-2\log_{5}2=2\log_{5}(x-1)$ --- M1

$\log_{5}\left(\dfrac{5x-6}{4}\right)=\log_{5}(x-1)^{2}$ --- M1 [*quotient law*]

$\left(\dfrac{5x-6}{4}\right)=(x-1)^{2}$

$4x^{2}-13x+10=0$

$(x-2)(4x-5)=0$

$x=2 \quad or \quad x=1\dfrac{1}{4}$ --- A1 [*for both answers*]

[Turn over

7 Given that $f(x) = 2x^3 + (a-4)x^2 + ax - 6a$,

(i) show that $x-2$ is a factor of $f(x)$, [1]

(ii) find the other quadratic factor in terms of a and x. [4]

(iii) If $f(x)$ has three real roots, find the range of values of a. [2]

(i) $f(2) = 2(2)^3 + (a-4)(2)^2 + a(2) - 6a$

$= 16 + 4a - 16 + 2a - 6a$

$= 0 \text{ --- } B1$

(ii)

$$\begin{array}{r l}
 & 2x^2 + ax + 3a \\
x-2 \,) & 2x^3 + (a-4)x^2 + ax - 6a \\
 & \underline{2x^3 - 4x^2} \quad \text{---}\ M1 \\
 & ax^2 + ax \\
 & \underline{ax^2 - 2ax} \\
 & 3ax - 6a \\
 & \underline{3ax - 6a} \quad \text{---}\ M1 \\
 & 0 \quad \text{---}\ M1
\end{array}$$

The other quadratic factor is $2x^2 + ax + 3a$. --- $A1$

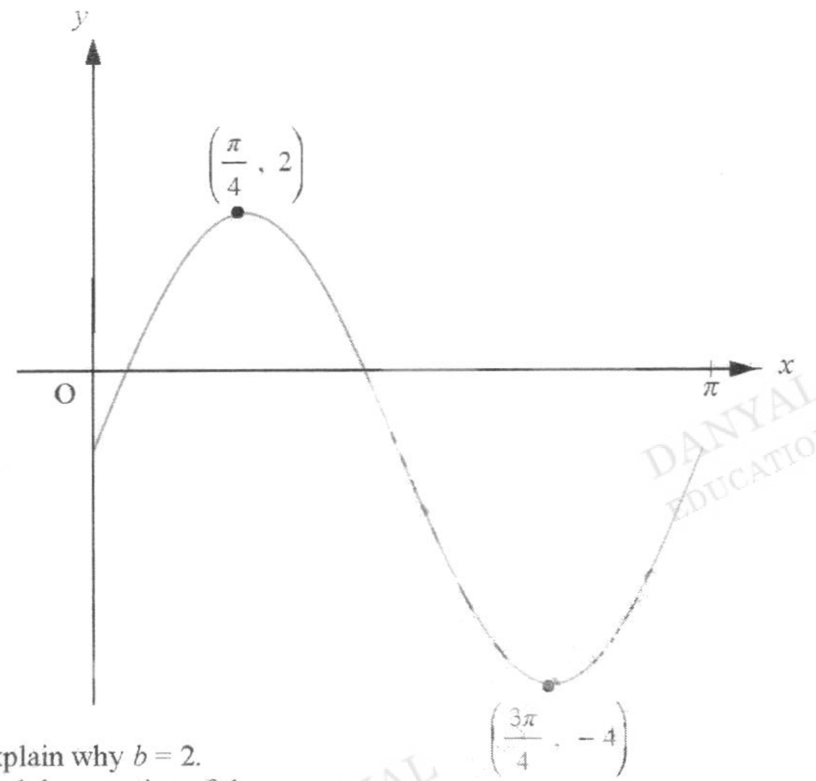
(iii) $a^2 - 4(2)(3a) \geq 0 \text{ ---- } M1$

$a^2 - 24a \geq 0$

$a(a-24) \geq 0$

$a \leq 0 \quad, a \geq 24 \text{ ---- } A1$

8 The diagram shows the curve $y = a\sin bx + c$ for $0 \leq x \leq \pi$ radians. The curve has a maximum point t $\left(\frac{\pi}{4}, 2\right)$ and a minimum point at $\left(\frac{3\pi}{4}, -4\right)$.



(i) Explain why $b = 2$. [2]

(ii) Find the equation of the curve. [2]

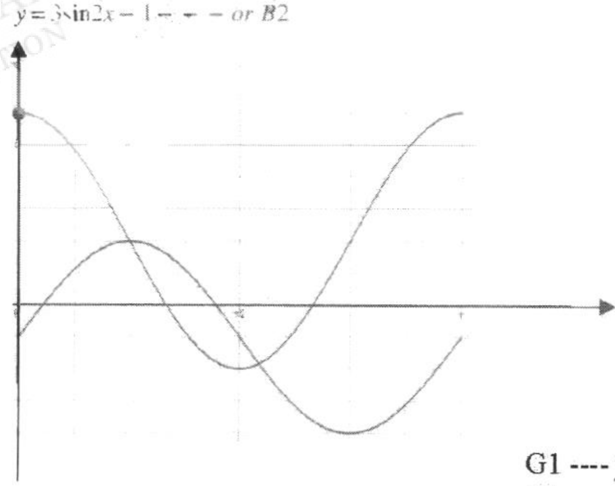
(iii) On the same axes, draw the graph of $y = 4\cos 2x + 2$. Label the y-intercept and the coordinates of minimum point(s) on the graph clearly. [3]

(i) Since there is one complete cycle in π shown in the diagram, $b = \frac{2\pi}{\pi} = 2$.---B2/M1A1

(ii) $a = \frac{2-(-4)}{2} = 3$---B1

$c = 2 - 3 = -1$---B1

$y = 3\sin 2x - 1$---or B2



G1 ---- y-intercept ($y = 6$)

G1 ---- 1 cycle

G1 ---- min point ($\frac{\pi}{2}$, -2)

9 The diagram shows two concentric circles C_1 and C_2.
The equation of C_1 is $x^2 + y^2 + 4x - 6y + 4 = 0$.

(i) Find the radius and centre of C_1. [3]

(ii) Find the equation of C_2. [2]

(iii) Given that PR is a tangent to the circle C_2 at $P(1, 7)$, find the co-ordinates of R. [4]

(iv) A third circle, C_3 is tangent to both the x and y axes. What can you deduce about the centre of the circle? [1]

(i) $x^2 + y^2 + 4x - 6y + 4 = 0$

$(x+2)^2 - (2)^2 + (y-3)^2 - (-3)^2 + 4 = 0 - - - M1$

$(x+2)^2 + (y-3)^2 = 9$

$centre\ (-2, 3)\ ,\ radius = 3 - - - A1, A1$

(ii) Subst (1, 7) into equation

$(1+2)^2 + (7-3)^2 = r^2 - - - M1$

$r^2 = 25 - - - M1$

$C_2 : (x+2)^2 + (y-3)^2 = 25 - - - A1$

(iii) gradient of radius $= \frac{4}{3} - - - M1$

Gradient of $PR = -\frac{3}{4}$------$M1$

Subst (1, 7) - - - $M1$

Equation of PR :

$$y = -\frac{3}{4}x + 7\frac{3}{4} - - - A1$$

(iv) The centre of the circle lies on the line $y = x$ or the line $y = -x$. ---------$B1$
Or coordinates (k, k) or (k, $-k$) ---- B1

10 **(a)(i)** Write down the first three terms of the expansion $\left(2-\frac{1}{4x}\right)^5$ in descending powers of x. [2]

(ii) Given that the coefficient of $\frac{1}{x}$ in the expansion of $\left(2-\frac{1}{4x}\right)^2(3x-a)$ is 255, find the value of a. [3]

(b) The expansion of $\left(2x^2-\frac{1}{\sqrt{x}}\right)^n$, where n is a positive integer, has a term that is independent of x. Find the smallest value of n. [3]

(a)(i) $\left(2-\frac{1}{4x}\right)^5=(2)^5+\binom{5}{1}(2)^4\left(-\frac{1}{4x}\right)+\binom{5}{2}(2)^3\left(-\frac{1}{4x}\right)^2+\ldots---M1$

$=32-\frac{20}{x}+\frac{5}{x^2}-\ldots---A1$

(ii)

$$\left(2-\frac{1}{4x}\right)^5(3x-a)=\left(32-\frac{20}{x}+\frac{5}{x^2}-\ldots\right)(3x-a)$$

$$=\frac{20a}{x}+\frac{15}{x}+\ldots---M1$$

Coefficient of $\frac{1}{x}=255$

$20a+15=255---M1$

$a=12---A1$

(b) $\left(2x^2-\frac{1}{\sqrt{x}}\right)^n$

$T_{r+1}=\binom{n}{r}(2x^2)^{n-r}\left(-\frac{1}{\sqrt{x}}\right)^r---M1$

$=\binom{n}{r}2^{n-r}(-1)^r x^{2n-2\frac{1}{2}r}$

Power of x :

$2n-2\frac{1}{2}r=0---M1$

$n=\frac{5}{4}r$

Since n and r must be positive integers, *smallest* $n=\frac{5}{4}\times4=5---A1$

[Turn over

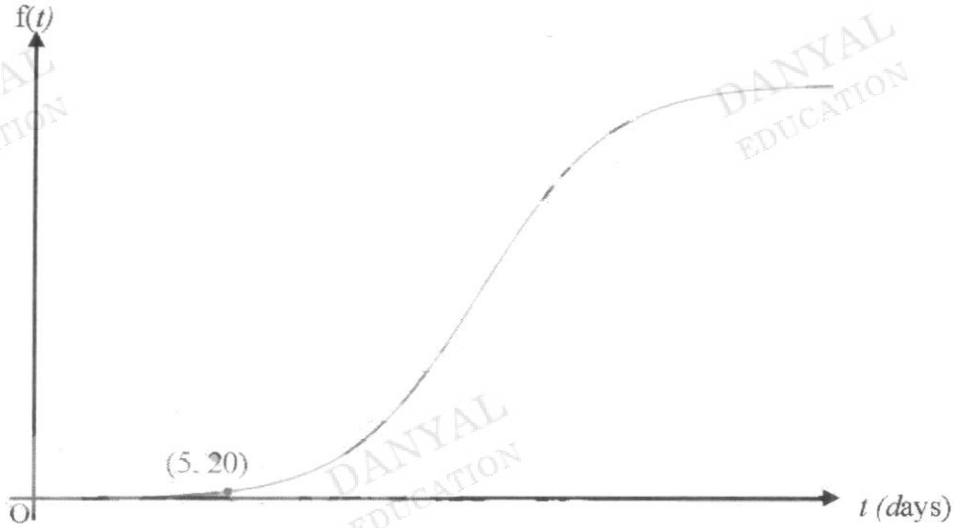
11 An influenza epidemic spreads through a population rapidly, at a rate that depends on two factors:

- The more people who have the flu, the more rapidly it spreads, and
- the more uninfected people there are, the more rapidly it spreads.

For a particular community of 1000 people, researchers found that they could model the spread of the flu using the logistic model,

$$f(t) = \frac{1000}{1+999e^{kt}},$$

where f(t) is the number of people infected, t is time measured in days after the first infection and k is a constant. The following graph shows how the model fits the data.



(i) Show that the value of the constant, k, is approximately -0.603. [3]

(ii) Predict the number of people infected after 10 days. [2]

(iii) How many days will it take for 750 of the population to be infected? Give your answer correct to 1 decimal place. [2]

(i)

$$20 = \frac{1000}{1+999e^{5k}} - - - M1$$

$$1+999e^{5k} = 50$$

$$e^{5k} = \frac{49}{999}$$

$$k = \frac{1}{5}\ln\left(\frac{49}{999}\right) - - - M1$$

$$= -0.602986\ldots$$

$$= -0.603(3sf) - - - A1$$

(ii)

$$f(t) = \frac{1000}{1+999e^{10k}} - - - M1$$

$$= 293.8\ldots$$

$$f(10) = 294 - - - A1$$

(iii)

$$750 = \frac{1000}{1+999e^{kt}} - - - M1$$

$$1+999e^{kt} = \frac{4}{3}$$

$$e^{kt} = \frac{\frac{1}{3}}{999} = \frac{1}{2997}$$

$$t = \frac{\ln\left(\frac{1}{2997}\right)}{k} = 13.3\ (3sf)\ days - - - M1$$

(vi) Student A suggested that an exponential curve of the form

$$g(t) = e^{kt}, \text{ where } k = \frac{\ln 20}{5}$$

can be used to fit the data set just as well.
Student B claimed that the logistic curve shown in the diagram is a more accurate model.

Who do you agree with? Explain your decision. [2]

Student B.
The exponential curve $g(t)$ may pass through the data point (5, 20), it concaves as t increases, suggesting an increasing rate of infection. - - - $B1$
The logistic curve, however, tapers off towards the end, reflecting the decreasing rate of infection as more and more people are infected in the community, hence it is a more accurate model. - - - $B1$

12 In the diagram, $ABCD$ is a rhombus. The coordinates of A and B are $(20,-8)$ and $(4,-4)$ respectively.

C is a point on the y-axis and E is a point on CD produced such that $\angle BAE = \angle CEA = 90°$.

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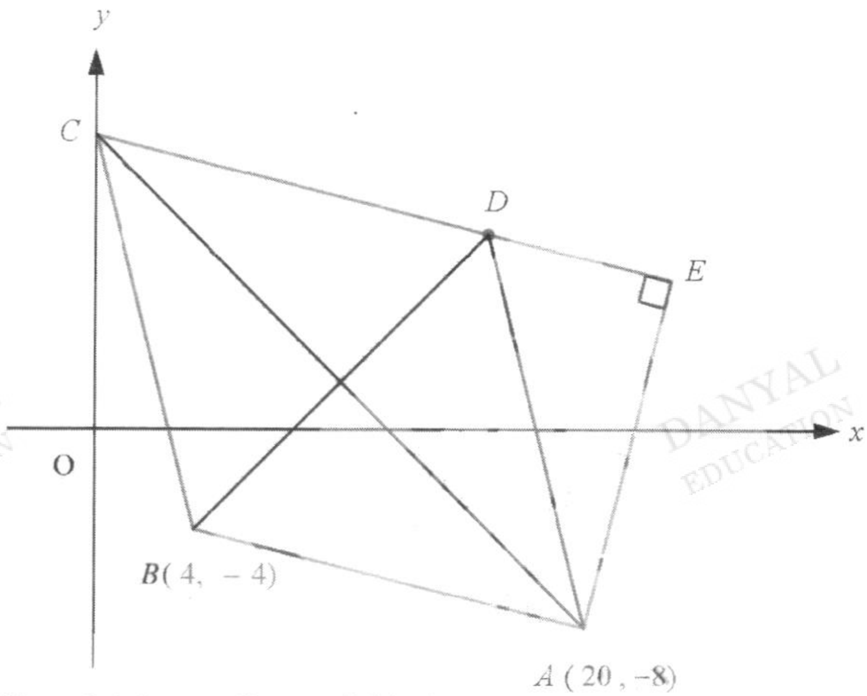

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(iii) Find the equation of AE. [3]

(iv) Find the area of the rhombus $ABCD$. [2]

(v) Show that $\cos\angle OCB = \dfrac{4}{\sqrt{17}}$. [2]

(i)

$$\sqrt{(4-0)^2+(-4-y)^2}=\sqrt{272}\ ---M1$$

$$16+16+8y+y^2=272$$

$$y^2+8y-240=0$$

$$(y-12)(y+20)=0\ ---M1$$

$$y=12 \text{ or } y=-20 \ (rej)$$

$$C(0,12)\ ---A1 \text{ with } y=-20 \text{ rejected}$$

(ii)

$$Midpoint\ of\ AC,\ M=\left(\frac{20+0}{2},\frac{-8+12}{2}\right)$$
$$=(10,2)---M1$$
$$\left(\frac{4+x_D}{2},\frac{-4+y_D}{2}\right)=(10,2)---M1$$
$$x_D=16,\ y_D=8$$
$$D(16,8)---A1$$

(iii)

$$\text{Gradient of } AB=-\frac{1}{4}$$
$$\text{Gradient of } AE=4---M1$$

Equation of AE:

$$-8=4(20)+c$$
$$c=-88---M1$$
$$y=4x-88---A1$$

(iv)

$$\frac{1}{2}\begin{vmatrix}0 & 4 & 20 & 16 & 0\\ 12 & -4 & -8 & 8 & 12\end{vmatrix}---M1$$
$$=\frac{1}{2}[320-(-160)]$$
$$=240\text{unit}^2---A1$$

(v)

$$\cos\angle OCB=\frac{16}{\sqrt{272}}---M1$$
$$=\sqrt{\frac{256}{272}}=\sqrt{\frac{16}{17}}$$
$$=\frac{4}{\sqrt{17}}---A1$$

[Turn over