CANDIDATE NAME	( )	CLASS	

Anglo-Chinese School (Barker Road)

END-OF-YEAR EXAMINATION
SECONDARY THREE EXPRESS

ADDITIONAL MATHEMATICS 4049

2 HOURS 15 MINUTES

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For Examiner's Use

This question paper consists of **19** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

Answer **all** the questions.

1 **(i)** On the same axes, sketch the curves $y = e^{-x}$ and $y = \ln x$, showing the intercepts clearly. [2]

(ii) Hence state the number of solution(s) for the equation $e^{-x} = \ln x$. [1]

2 Find the coordinates of the points of intersection of the line $y = 2x + 3$ and the curve $y^2 = x + 3$. [4]

3 In the NBA finals, Chris Paul and Giannis are facing off against each other. The path, which the basketball travels from Chris Paul's hand to the hoops, is modelled by the equation $h = -2t^2 + 3t + c$, such that the height of the ball at time t seconds is h metres. It is given that Giannis can reach a blocking height of 3 metres.

Find the range of values of c such that Chris Paul's shot will not be blocked. [3]

4 The triangle ABC is such that its area is $\left(24+15\sqrt{3}\right)$ cm^2, the length of AB is $\left(4+8\sqrt{3}\right)$ cm and angle BAC is $60°$. **Without using a calculator**, find the length, in cm, of AC in the form $a+b\sqrt{3}$, where a and b are integers. [5]

5 Express $\dfrac{8x^2-3x-6}{(x-3)(2x^2+1)}$ in partial fractions. [5]

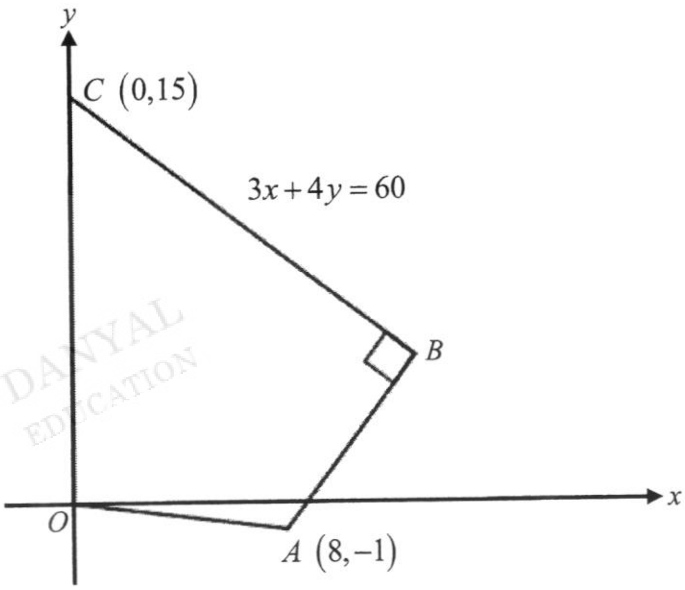
6 The total number of COVID-19 cases, C, in Singapore is given by $C = Ae^{kt}$, where A and k are constants and t is measured in months. The initial reported COVID-19 cases was 13 in January 2020 and there were 65 686 cases in July 2021.

(i) Find the value of A and of k. [4]

(ii) Calculate, to the nearest whole number, the minimum number of months for the total number of COVID-19 cases to exceed one hundred thousand in Singapore if there are no further measures to restrict the spread of COVID-19. [2]

7 **Solutions to this question by accurate drawing will not be accepted.**

In the diagram, the equation of BC is $3x+4y=60$, angle $ABC = 90°$, and the coordinates of A is $(8,-1)$ and C is $(0,15)$.



(i) Find the coordinates of B. [4]

(ii) Find the area of the quadrilateral $OABC$. [2]

8 (a) **Without using a calculator**, evaluate $\log_a 5a^2 + \log_a 2a^3 - \log_a 10$. [2]

(b) Solve the equation $\log_3 \frac{3}{p} + 2\log_{\frac{1}{9}} p = 3$. [5]

9 Given that the coefficient of x^3 in the expansion of $\left(x^2 - \frac{m}{x}\right)^9$ is $-\frac{14}{27}$,

(i) show that $m = \frac{1}{3}$, [3]

(ii) explain why there is no term independent of x in the expansion of $\left(243 + \frac{54}{x^3}\right)\left(x^2 - \frac{m}{x}\right)^9$. [4]

10 Given that $f(x) = 2x^3 - 5x^2 - 9$,

(i) find the remainder when $f(x)$ is divided by $x + 2$, [2]

(ii) show that $x - 3$ is a factor of $f(x)$ and hence explain why there is only one solution for the equation $f(x) = 0$. [5]

11 A circle passes through the points $A(3,3)$ and $B(7,-5)$. Its centre lies on the line $6y = x - 15$.

(i) Show that the coordinates of the centre of the circle is $(3,-2)$. [4]

(ii) Find the equation of the circle. [2]

Another circle has the equation $x^2+y^2-10x-12y+36=0$.

(iii) Will the two circles intersect?
Justify your answer with clear working. [3]

12 **(a)** Given that $\sin A=\dfrac{5}{13}$ and $\cos B=-\dfrac{7}{25}$, where A and B are in the same quadrant. **Without using a calculator**, find the exact value of

(i) $\cos A$, [1]

(ii) $\cos(A+B)$, [2]

(iii) $\sin\frac{A}{2}$, [4]

(iv) $\tan C$, given that $\tan(A+C) = -\frac{3}{41}$ and C is an acute angle. [2]

(b) The equation of a curve is $y = 3\sin\frac{1}{2}x - 2$.

(i) State the amplitude of y. [1]

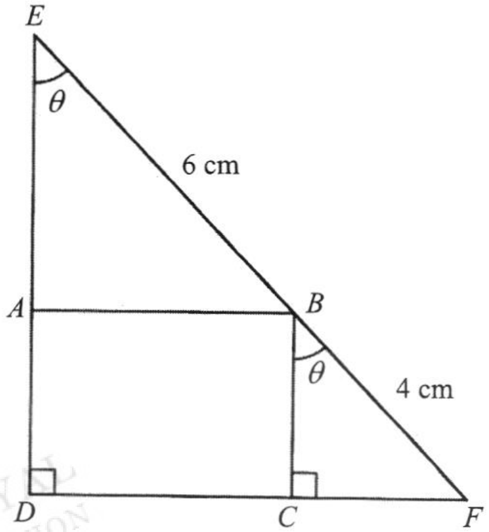
(ii) State the period of y. [1]

(iii) Sketch the graph of $y = 3\sin\frac{1}{2}x - 2$ for $0 \le x \le 2\pi$. [3]

13 **(a)** **(i)** Prove that $\dfrac{\cos 2\theta + 1}{2 - 2\sin\theta} = \sin\theta + 1$. [3]

(ii) Hence solve the equation $\dfrac{\cos 2\theta + 1}{2 - 2\sin\theta} = \dfrac{2}{\sin\theta}$ for $0° \le \theta \le 360°$. [4]

(b) In the triangle *DEF*, angle *EDF* = angle *BCF* = $\frac{\pi}{2}$, *EB* = 6 cm and *BF* = 4 cm.

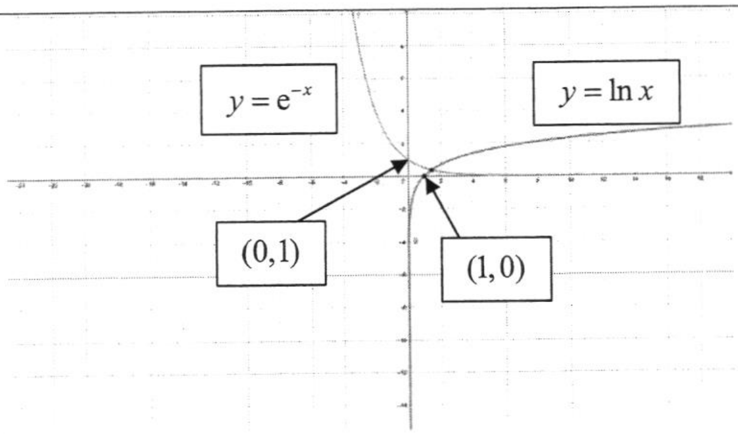


(i) Given that angle *DEF* = angle *CBF* = θ, where $0 < \theta < \frac{\pi}{2}$, show that *P*, the perimeter of the rectangle *ABCD* is given by $P = 8\cos\theta + 12\sin\theta$. [2]

(ii) Express P in the form $R\cos(\theta-\alpha)$ and hence find the value of θ for which $P = 10$ cm. [5]

Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 3 Express
Additional Mathematics
End of Year Examination 2021

1	(i)				
	(ii)	1 solution			
2		Sub $y=2x+3$ into $y^2=x+3$ $4x^2+11x+6=0$ Alternatively Sub $x=\frac{y-3}{2}$ into $y^2=x+3$			
		$(x+2)(4x+3)=0$ alternatively $(2y-3)(y+1)=0$			
		$x=-2$ or $x=-\frac{3}{4}$ alternative $y=-1$ or $y=\frac{3}{2}$			
		$(-2,-1)$ and $\left(-\frac{3}{4},\frac{3}{2}\right)$			
3		$h=-2\left(t^2-\frac{3}{2}t\right)+c$ $=-2\left(t-\frac{3}{4}\right)^2+\frac{9}{8}+c$ $\therefore \frac{9}{8}+c>3$ $c>\frac{15}{8}$			
4		$\frac{1}{2}\times AC\times\left(4+8\sqrt{3}\right)\times\sin 60°=24+15\sqrt{3}$			

Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 3 Express
Additional Mathematics
End of Year Examination 2021

		$\frac{1}{2}\times AC\times\left(4+8\sqrt{3}\right)\times\left(\frac{\sqrt{3}}{2}\right)=24+15\sqrt{3}$ $AC=\frac{24+15\sqrt{3}}{6+\sqrt{3}}\times\frac{6-\sqrt{3}}{6-\sqrt{3}}$ $=\frac{144-24\sqrt{3}+90\sqrt{3}-45}{33}$ $=\left(3+2\sqrt{3}\right)$ cm			
5		$\frac{8x^2-3x-6}{(x-3)(2x^2+1)}=\frac{A}{x-3}+\frac{Bx+C}{2x^2+1}$ $A=3,\ B=2,\ C=3$ $\frac{8x^2-3x-6}{(x-3)(2x^2+1)}=\frac{3}{x-3}+\frac{2x+3}{2x^2+1}$			
6	(i)	Sub $t=0$ and $C=13$, $A=13$ Sub $t=18$ and $C=65\ 686$, $13e^{18k}=65686$			

Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 3 Express
Additional Mathematics
End of Year Examination 2021

		$18k = \ln\left(\frac{65686}{13}\right)$ $k = \frac{1}{18}\ln\left(\frac{65686}{13}\right)$ $= 0.4737352729$ $= 0.474$ (3sf)			
	(ii)	$13e^{kt} > 100000$ $t > \ln\left(\frac{100000}{13}\right) \div \frac{1}{18}\ln\left(\frac{65686}{13}\right)$ $t > 18.9$ (3sf) Minimum number of months = 19 months			
7	(i)	$y = -\frac{3}{4}x + 15$ $m_{AB} = \frac{4}{3}$			

		$y+1=\frac{4}{3}(x-8)$ $y=\frac{4}{3}x-\frac{35}{3}$ Sub $y=-\frac{3}{4}x+15$ into $y=\frac{4}{3}x-\frac{35}{3}$, $x=12.8\left(\text{or }\frac{64}{5}\right)$ Sub $x=12.8$ into $y=-\frac{3}{4}x+15$, $y=5.4\left(\text{or }\frac{27}{5}\right)$ $\therefore B(12.8,5.4)\ \left(\text{or } B\left(\frac{64}{5},\frac{27}{5}\right)\right)$			
	(ii)	Area $=\frac{1}{2}\begin{vmatrix} 0 & 8 & 12.8 & 0 & 0 \\ 0 & -1 & 5.4 & 15 & 0 \end{vmatrix}$ $=\frac{1}{2}\left[(43.2+192)-(-12.8)\right]$ or $=\frac{1}{2}\left[\left(\frac{216}{5}+192\right)-\left(-\frac{64}{5}\right)\right]$ $= 124$ unit^2			
8	(a)	$\log_a a^5$ $=5$			

Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 3 Express
Additional Mathematics
End of Year Examination 2021

	(b)	$\log_3 \frac{3}{p} + \frac{2\log_3 p}{\log_3 3^{-2}} = 3$ $\log_3 \frac{3}{p} - \log_3 p = 3$ $\log_3 \frac{3}{p^2} = 3$ $\frac{3}{p^2} = 3^3$ $p = \frac{1}{3}$ or $-\frac{1}{3}$ (rejected)			
9	(i)	$T_{r+1} = \binom{9}{r}\left(x^2\right)^{9-r}\left(-\frac{m}{x}\right)^r$ $= \binom{9}{r}(-m)^r x^{18-3r}$ $18 - 3r = 3$ $r = 5$			

Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 3 Express
Additional Mathematics
End of Year Examination 2021

		$\binom{9}{5}(-m)^5=-\frac{14}{27}$ $m^5=\frac{1}{243}$ $m=\sqrt[5]{\frac{1}{243}}$ $=\frac{1}{3}$ (Shown)			
	(ii)	$18-3r=0$ $r=6$ $T_7=\binom{9}{6}\left(-\frac{1}{3}\right)^6$ $=\frac{28}{243}$ $\left(243+\frac{54}{x^3}\right)\left(x^2-\frac{m}{x}\right)^9=\left(243+\frac{54}{x^3}\right)\left(\ldots-\frac{14}{27}x^3+\frac{28}{243}+\ldots\right)$ $=\ldots243\left(\frac{28}{243}\right)+\left(\frac{54}{x^3}\right)\left(-\frac{14}{27}x^3\right)+\ldots$ $=\ldots+28-28+\ldots$ $=\ldots+0+\ldots$ Since the expansion gave a constant value of 0, there is no term independent in the expansion.			
10	(i)	$\mathrm{f}(-2)=2(-2)^3-5(-2)^2-9$			
		$=-45$			
	(ii)	$\mathrm{f}(3)=2(3)^3-5(3)^2-9$ $=0$ By Factor Theorem, $(x-3)$ is a factor. (Shown)			

Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 3 Express
Additional Mathematics
End of Year Examination 2021

		$\begin{array}{r} 2x^2+x+3 \\ x-3 \overline{) \; 2x^3-5x^2+0x-9} \\ -(2x^3-6x^2)+0x+0 \\ \hline x^2-0x-9 \\ -(x^2-3x)+0 \\ \hline 3x-9 \\ -(3x-9) \\ \hline 0 \end{array}$			
		$(x-3)(2x^2+x+3)=0$			
		$x=3$			
		Since discriminant of $2x^2+x+3$ is $-23<0$,			
		there are no real solutions for the quadratic equation, hence there is only 1 solution for the cubic equation.			
11	(i)	$m_{chord}=-2$ $m_{normal}=\frac{1}{2}$			
		Midpoint of $AB=(5,-1)$			
		Equation of perpendicular bisector: $y=\frac{1}{2}x-\frac{7}{2}$			

Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 3 Express
Additional Mathematics
End of Year Examination 2021

		Solve $y=\frac{1}{2}x-\frac{7}{2}$ and $6y=x-15$ simultaneously, $C(3,-2)$			
	(ii)	radius $=5$			
		$(x-3)^2+(y+2)^2=25$			
	(iii)	$(x-5)^2+(y-6)^2=25$			
		Distance between the 2 centres = $\sqrt{68}<5+5$			
		Since the distance between the 2 centres is less than the sum of radii, the two circles will intersect.			
12	(a) (i)	$-\frac{12}{13}$			
	(ii)	$\left(-\frac{12}{13}\right)\left(-\frac{7}{25}\right)-\left(\frac{5}{13}\right)\left(\frac{24}{25}\right)$			
		$=-\frac{36}{325}$			
	(iii)	$2\cos^2\frac{A}{2}-1=-\frac{12}{13}$			

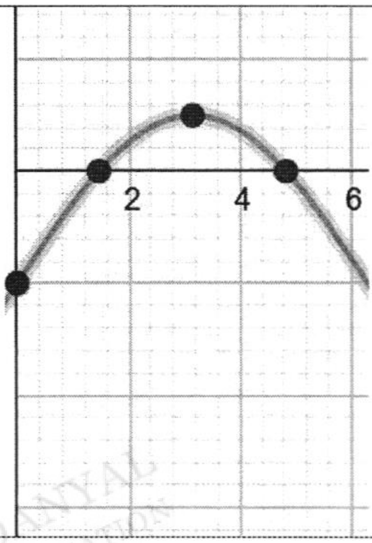
Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 3 Express
Additional Mathematics
End of Year Examination 2021

		$\cos\frac{A}{2}=\sqrt{\frac{1}{26}}$ or $-\sqrt{\frac{1}{26}}$ (rejected)			
		$2\left(\sin\frac{A}{2}\right)\left(\sqrt{\frac{1}{26}}\right)=\frac{5}{13}$ $\sin\frac{A}{2}=\frac{5}{\sqrt{26}}$ $\left(\text{or } \frac{5\sqrt{26}}{26}\right)$			
	(iv)	$\frac{-\frac{5}{12}+\tan C}{1-\left(-\frac{5}{12}\right)\tan C}=-\frac{3}{41}$			
		$\tan C=\frac{1}{3}$			
	(b) (i)	3			
	(ii)	720° or 4π			

Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 3 Express
Additional Mathematics
End of Year Examination 2021

	(iii)				
13	(a) (i)	$LHS=\dfrac{1-2\sin^2\theta+1}{2(1-\sin\theta)}$			
		$=\dfrac{2(1-\sin^2\theta)}{2(1-\sin\theta)}$			
		$=\dfrac{(1-\sin\theta)(1+\sin\theta)}{(1-\sin\theta)}$ $=\sin\theta+1$ $=RHS$ (proven)			
	(ii)	$\dfrac{\cos 2\theta+1}{2-2\sin\theta}=\dfrac{2}{\sin\theta}$ $\sin\theta+1=\dfrac{2}{\sin\theta}$ $\sin^2\theta+\sin\theta-2=0$			
		$(\sin\theta+2)(\sin\theta-1)=0$			
		$\sin\theta=1$ or $\sin\theta=-2$ (rejected)			
		$\therefore\theta=90°$			

Anglo-Chinese School
(Barker Road)

Marking Scheme
Secondary 3 Express
Additional Mathematics
End of Year Examination 2021

	(b) (i)	$\frac{AB}{6}=\sin\theta$ and $\frac{BC}{4}=\cos\theta$ $AB=6\sin\theta$ and $BC=4\cos\theta$ $\therefore P=2(6\sin\theta+4\cos\theta)$ $=8\cos\theta+12\sin\theta$ (Shown)			
	(ii)	$R=\sqrt{8^2+12^2}$ $=\sqrt{208}=4\sqrt{13}$			
		$\alpha=\tan^{-1}\left(\frac{12}{8}\right)$ $=0.9827937232$ $=0.983$ (3sf)			
		$\therefore 8\cos\theta+12\sin\theta=4\sqrt{13}\cos(\theta-0.983)$			
		$4\sqrt{13}\cos(\theta-0.983)=10$ $\cos(\theta-0.983)=\frac{10}{4\sqrt{13}}$ $\alpha=\cos^{-1}\left(\frac{10}{4\sqrt{13}}\right)$ $=0.8046336771$			
		$\theta=0.1781600461$ or 1.7874274 (rejected) $=0.178$ (3sf)			