ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION 2023

S4

MATHEMATICS **4052/01**
Paper 1 **23 August 2023**
Candidates answer on the Question Paper. **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST
Write your name, index number and class in the space at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiners' Use

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Marks									
Question	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
Marks									
Question	**19**	**20**	**21**	**22**	**23**	**24**			
Marks									

Table of Penalties	**Units**			**90**
	Clarity/ Logic			
	Accuracy/ Precision			

Parent's Name and Signature:	
Date:	

This document consists of **20** printed pages.

Answer **all** the questions.

1 Expand and simplify $x(4x-9y)(4y+9x)$.

2 Given $2^x+2^{3+x}=9$, find x.

3 Simplify $\dfrac{6x^2-2xy+15x-5y}{6x^2+13xy-5y^2}$.

4 Two integers, A and B, can be written as products of prime factors.

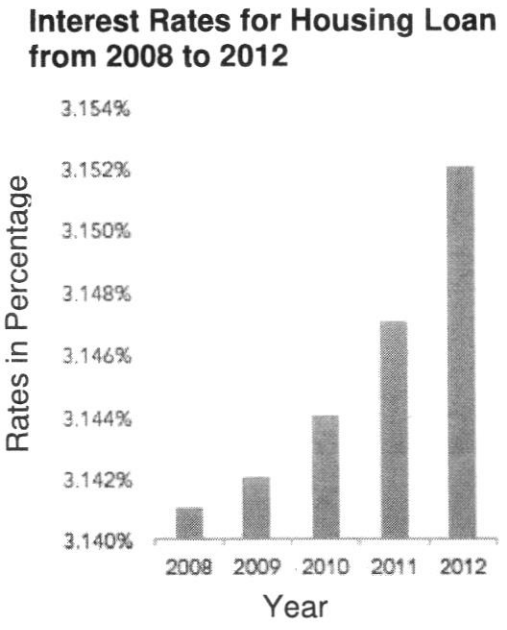
$A=5\times7$ $\quad$ $B=3^r\times5^{p-r}\times7$

The lowest common multiple of A and B is $3\times5^2\times7$.

(a) Write down the values of p and of r.

(b) Find the highest common factor of A and B.

5 Mr Lee drew this graph to show the interest rates for the housing loan from 2008 to 2012.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

6 **(a)** Simplify $\left(\frac{x^{12}}{64y^6}\right)^{-\frac{5}{6}}$, leaving your answer in positive index.

(b) $\dfrac{5^b}{25^{a-1}} = 125^c$

Find an expression for a in terms of b and c.

7 Factorise the following expressions completely.

(i) $9a^8 + 24a^4b + 16b^2$

(ii) Hence, factorise $144a^8 - \left(9a^8 + 24a^4b + 16b^2\right)$ completely.

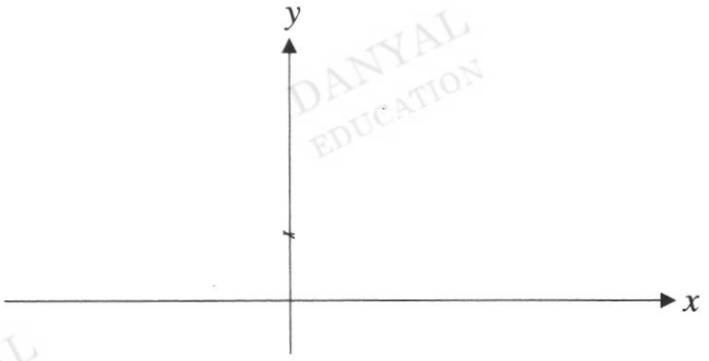
8 The gravitational force between two particles is inversely proportional to the square of the distance between them.

When two particles are r metres apart, the gravitational force between them is f newtons.

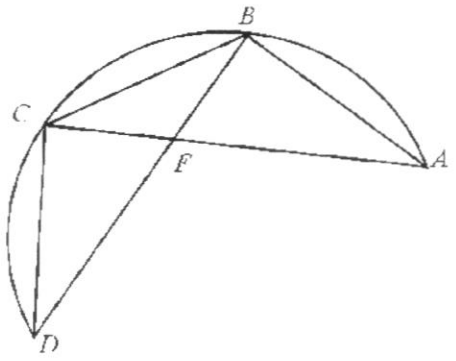
(a) Express the new gravitational force, in terms of f, between two particles when the distance between them is decreased by 60%.

(b) Calculate the percentage change in the gravitational force.

9 Sketch the graph of $y = 2^{-x}$ on the axes below.
Indicate clearly the values where the graph crosses the x- and y- axes.



10 A, B, C and D are vertices of a regular pentagon, and they lie on a circle.
AFC and BFD are straight lines.



Prove that triangle ABC is congruent to triangle BCD.
Give a reason for each statement you made.

11 By writing the expression $x^2 - 8x + 39$ in the form $(x-h)^2 + k$, where h and k are constants, explain why the expression is always positive.

12 $\xi = \{(x, y) : x \text{ and } y \text{ are integers, } -2 \le x < 3 \text{ and } -2 \le y \le 1\}$.

$A = \{(x, y) : x^2 + y^2 = 5, \quad x > y\}$.

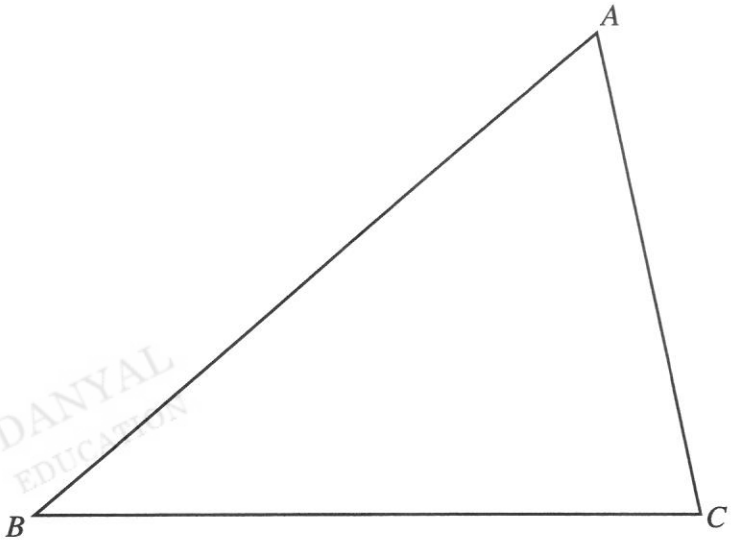
$B = \{(x, y) : xy < 0\}$.

List the elements in

(i) A,

(ii) B.

13 The diagram shows a plot of land, ABC.



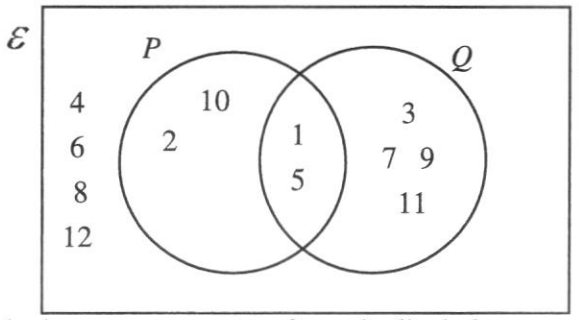
Scale = 1cm : 2m

(a) Construct the perpendicular bisector of AB.

(b) Construct the bisector of angle ACB.

(c) Mr Tan bought a plot of land, nearer to B than A and nearer to BC than to AC. Find the area that was bought in m^2.

14 The Venn diagram shows the elements of $\xi = \{\text{intergers } x : 1 \le x \le 12\}$



(i) Underline the incorrect statement from the list below.

$n(P) = 4$ $\quad 3 \in Q$ $\quad Q \cap Q' = \varnothing$

$\{1, 2, 5, 10\} \subset P$ $\quad n(P \cup Q)' = 4$

(ii) Find the value of

(a) $n(P \cup Q')$,

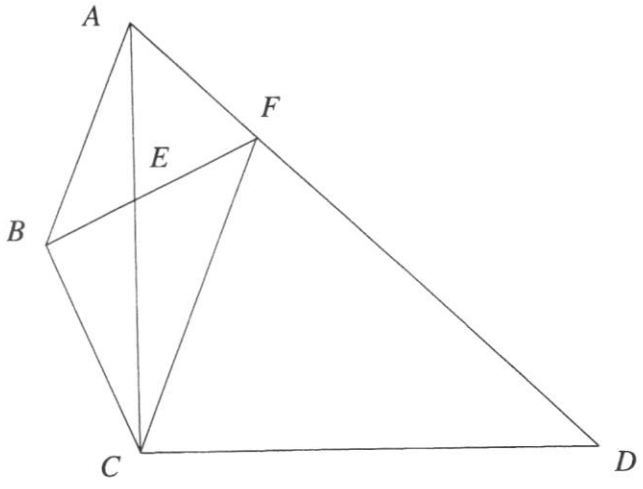
(b) $n[(P \cap Q)' \cap (P \cup Q)]$

15 **(a)** Solve the inequality $\frac{2x+2}{7} \le \frac{x+6}{4} < \frac{x-5}{2}$.

(b) Write down all the perfect squares that satisfy $\frac{2x+2}{7} \le \frac{x+6}{4} < \frac{x-5}{2}$.

16 In the figure below, $ABCF$ is a trapezium with $AB \parallel CF$.

The diagonals of the trapezium intersect at E. AFD and CD are straight lines.

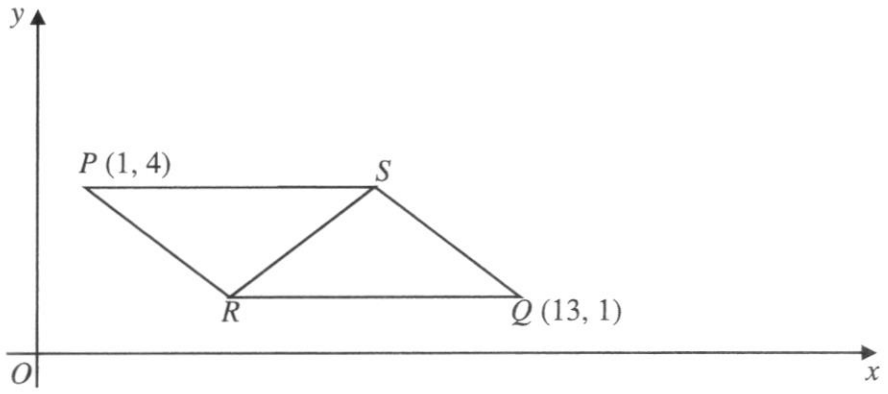


(a) Name two triangles that are similar.

(b) Explain why the two triangles in **(a)** are similar.

17 Glaciers are melting at an average rate of 13300 tonnes of water per second in 2022. Assuming there are 365 days in the year 2022, calculate the average amount of ice melts in the whole year. Give your answer in standard form, correct to 2 significant figures.

18 The diagram shows two congruent isosceles triangles.
The bases of the triangles are parallel to the x-axis.



(a) Write down the coordinates of R.

(b) Find the equation of the line PQ.

(c) Find the coordinates of the midpoint of PQ.

(d) Find the perimeter of $PSOR$.

19 A shop sells chocolate cookies (A) and chocolate cakes (B). These items are sold in boxes at three different outlets - Outlet 1, Outlet 2 and Outlet 3.
The table shows the number of boxes of A and of B sold at the three outlets.

	A	B
Outlet 1	45	40
Outlet 2	30	35
Outlet 3	40	x

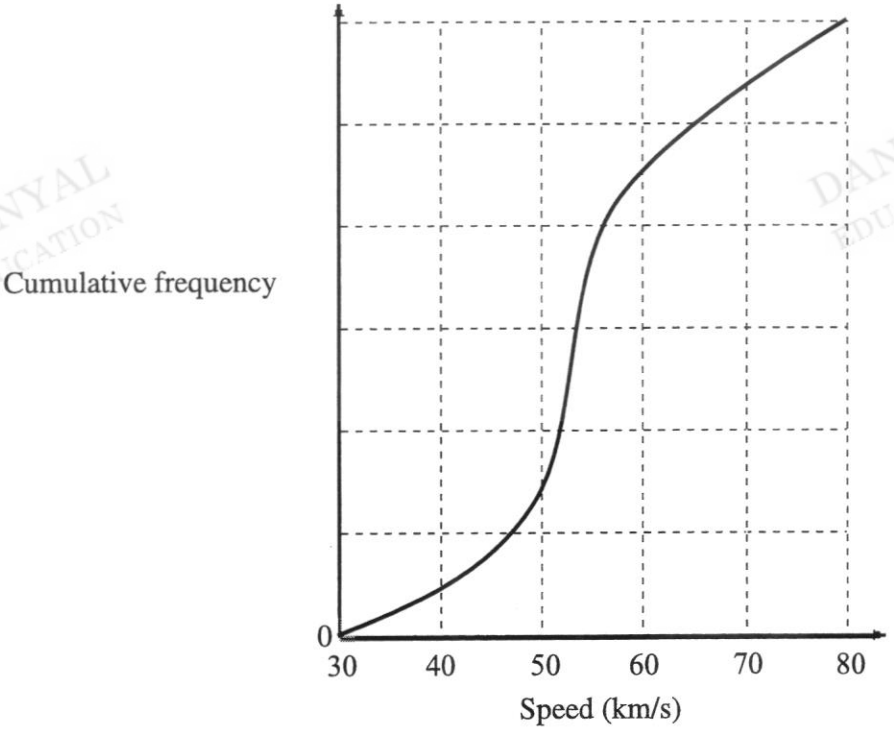
(a) The production costs of each box of A, and of B are \$10.00, and \$13.40 respectively.
Write down two matrices such that their product under matrix multiplication gives the selling prices that result in a 20% profit for a box of A and 25% profit for a box of B.

Evaluate the product to find the selling prices, in \$, of each box of A and of B.

(b) Using matrix multiplication twice, find the total amount of money collected for selling A and B at all three outlets in terms of x.

(c) The total amount of money collected at the three outlets for selling A and B is \$3189. Calculate the value of x.

20 The speeds of cars passing a checkpoint were recorded. The cumulative frequency curve below shows the distribution of the speeds. The speed limit on the road is 60km/h.



(a) Estimate the percentage of cars that exceeded the speed limit.

(b) Explain why it is not possible to find the number of cars that exceeded the speed limit.

21 A lake, measuring 48 cm^2 on a map has an actual area of 108 000 000 m^2.
Find the value of n if the scale of the map is in the form $1 : n$.

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22 Below are the first three lines of a sequence.
Line 1: $2^2 + 3 = 7$
Line 2: $3^2 + 5 = 14$
Line 3: $4^2 + 7 = 23$

(a) Write down the numbers for Line 6.

(b) Write down and simplify an expression for the nth term of the sequence.

(c) The difference, D, of the two consecutive terms is given by $T_{n+1} - T_n$.

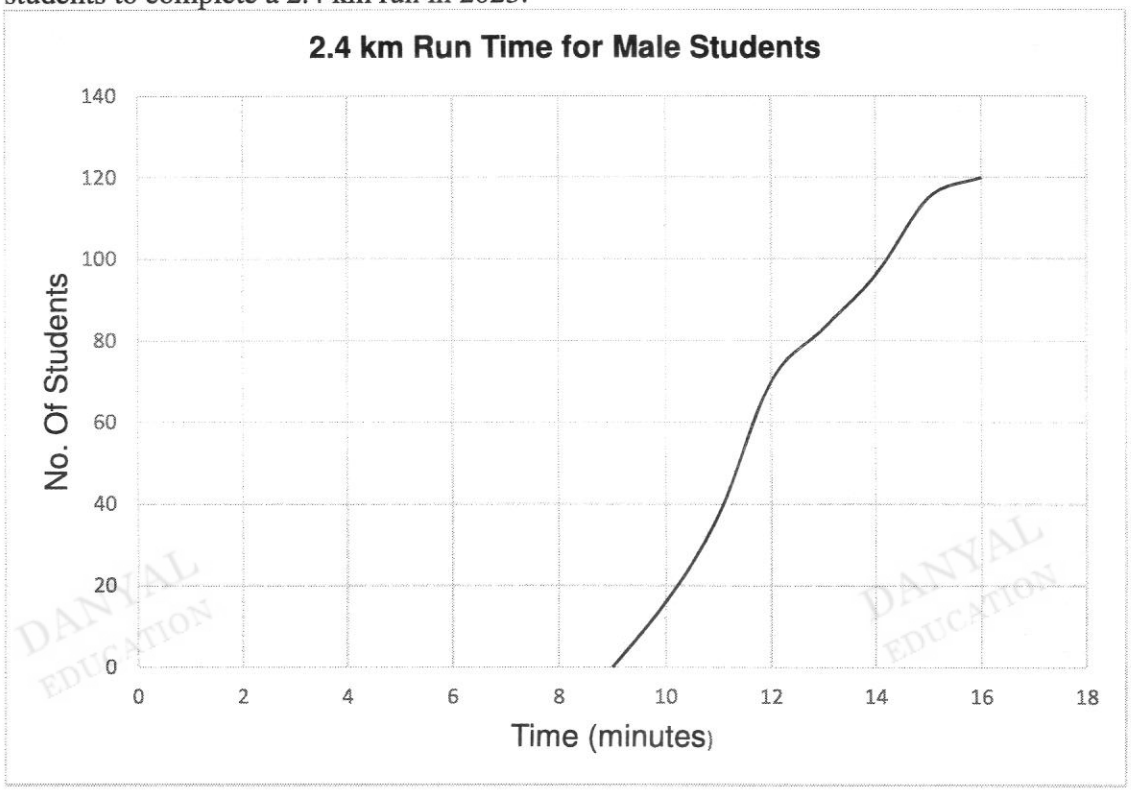
Find an expression for D.

(d) Hence, explain why the difference will always be an odd integer.

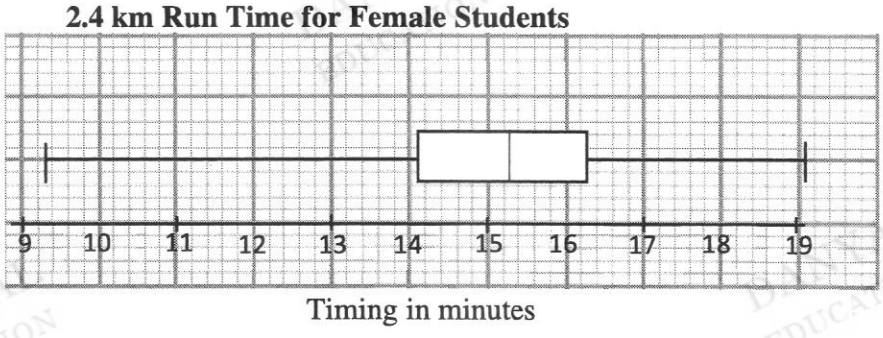
23 The table below shows the languages studied by 30 students.
There are seven students who only study Spanish but not French.
One of the 30 students is selected at random. The probability that the student studies French and not Spanish is $\frac{4}{15}$.
Two of the 30 students are selected at random. The probability that both students study neither French nor Spanish is $\frac{1}{145}$.
Complete the table of information about the 30 students.

	French	Not French
Spanish		
Not Spanish		

24 The cumulative frequency graph shows the distribution of the times of the first 120 male students to complete a 2.4 km run in 2023.



The box-and-whisker plot represents shows the distribution of the times of the first 120 female students to complete a 2.4 km run in 2023.



(a) 25% of the female students failed the 2.4 km run. State the time to pass the 2.4 km run.

(b) Using the two diagrams given, complete the table below.

	Lower Quartile	Median	Upper Quartile	Interquartile Range	
Male					
Female					

(c) Below are two statements comparing the times of the male and female students.
For each one, state you agree or disagree, giving a reason for each answer.

Statement	Agree/Disagree	Reason
The boys, on average, run faster than the girls.		
The timing for the girls run is more consistent as compared to the boys.		

25 18 painters can paint 3 buildings in 20 days.
After painting for 8 days, 8 painters left.
How many more days does the remaining painters need to complete the painting of the buildings? Give your answer to the nearest whole number.

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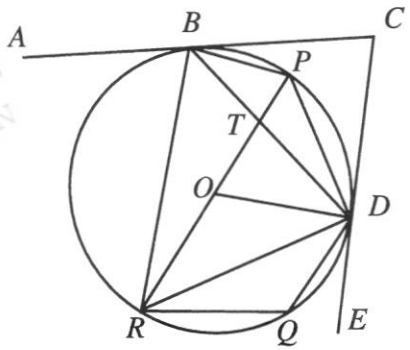
26 The lines ABC and CDE are tangents to the circle $BPDQR$ at B and at D respectively.
The centre of the circle is O. The straight lines POR and BTD intersect at T.
It is given that angle $ACD = 42°$ and angle $PBC = 31°$.



Giving all the relevant properties, find

(a) angle DBP,

(b) angle RBD,

(c) angle DQR.

27 A vase that is 17 cm tall can be modelled by a frustrum, A, on top of an egg-shaped container, B. A sketch of the vase is shown in Figure 1.

Figure 2 shows the cone that the frustum A is a part of. The diameter QRS of the top of the frustrum, is 8 cm and the diameter PVT of the bottom is 7 cm. The height of the frustrum, RV is 8 cm. R and V are the midpoints of QS and PT respectively.

Figure 1

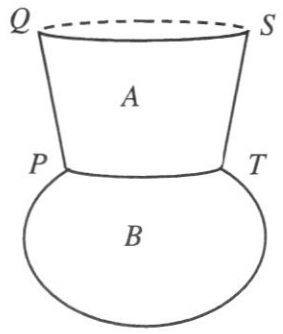
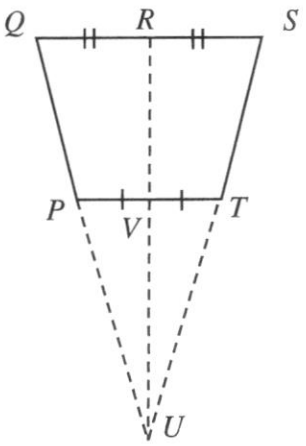


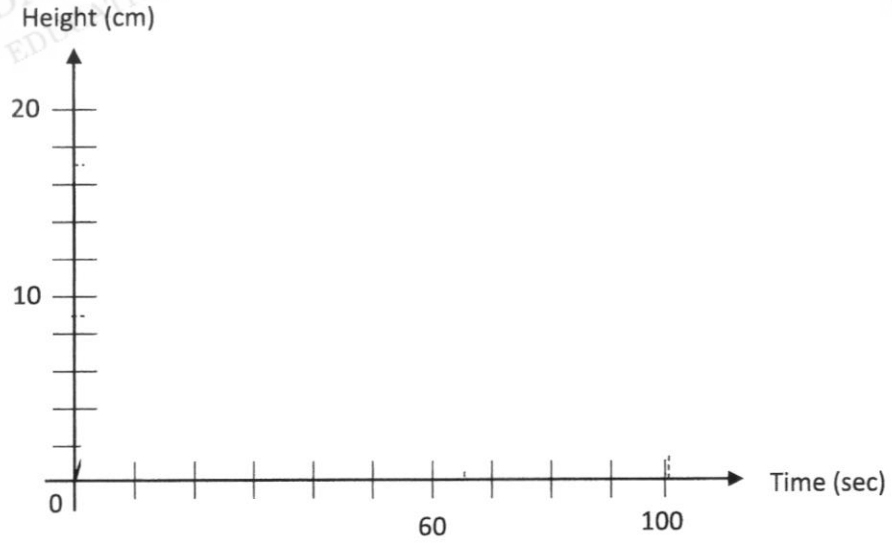
Figure 2



(a) Calculate the volume of the frustrum, A.

The volume of B is about 650 cm^3 and its height is 9 cm.
Water is poured into the vase at the rate of 10 cm^3 per second until it is full.

(b) Sketch the height of the water in the vase with respect to time.



End of paper

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2023

S4

MATHEMATICS **4052/02**

Paper 2 **28 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper and Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
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The total number of marks for this paper is 90.

For Examiners' Use

Questions	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Marks							
Questions	**8**	**9**	**10**	**11**			
Marks							
Table of Penalties	**Units**					**90**	
	Clarity/Logic						
	Accuracy/Precision						

Parent's Name and Signature:

Date:

This document consists of **21** printed pages and **1** blank page.

1 **(a)** Rearrange the formula $a = \dfrac{b^2+5}{3-b^2} + 2$ to make b the subject.

(b) Express as a single fraction in its simplest form $\dfrac{4}{2x-1} + \dfrac{2}{2x^2-3x+1}$.

(c) Solve $\dfrac{10x+7}{4} - \dfrac{2x^2-9}{x+2} = 7.$

2 **(a)** Myron puts his $30000 bonus into a fixed deposit at 3.35% per annum. The interest is compounded daily. There are 365 days in a year.

Calculate the amount of interest he will have at the end of two years.

(b) The price of a car is $160000.
Myron buys this car on hire purchase.
He makes a 30% down payment and takes up a 7-year loan tenure at a simple interest rate of 2.68% per annum.

Calculate his monthly instalment for the car.

(c) Myron pays monthly rent of $3600.
This is a 25% increment from his monthly rent last year.
Calculate his monthly rent last year.

(d) The exchange rate between Singapore dollars (S$) and Australian dollars (A$) is S$1 = A$1.12.

Myron spends A$980 in Sydney with his credit card.

All foreign currency transactions are subjected to a 1% conversion fee imposed by the respective card associations.
This foreign transaction amount, converted to S$, is further subjected to a 2.25% bank administrative fee.

Calculate the total amount on Myron's credit card bill, inclusive of the fees.
Give vour answer in S$. correct to the nearest cent.

3 The test marks of a group of 30 students were recorded. The marks are shown in the stem-and-leaf diagram.

Stem	Leaf
0	4 4 4
1	8 8 9 9 9 9 9 9
2	2 2 2 3 x 4 4 4 5 5 7 7 7 7 7 7 7 7 7

Key 1 | 9 means 19 marks

(i) Find the percentage of students who score less than 20 marks.

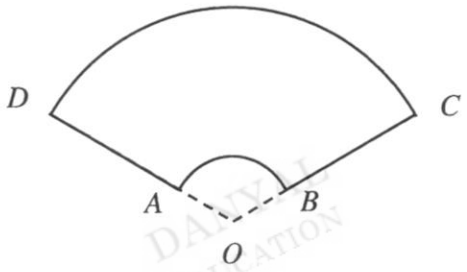
(ii) Given that the median is 23.5 marks, find the value of x.

(iii) Find the standard deviation of the marks.

(iv) It was found that the marks were recorded wrongly. Every student was short of two marks. Write down the correct values for the median and standard deviation.

(v) Explain whether mean or median is a better measure of central tendency.

4

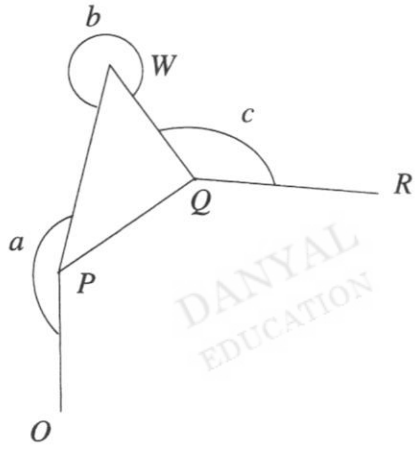


The diagram shows a mirror $ABCD$.
AB and DC are arcs of circles centre O with radii 15 cm and 45 cm respectively.
The perimeter of the mirror is 286 cm.

(a) Calculate the exact value of angle AOB in radians.

(b) Calculate the area of the mirror.

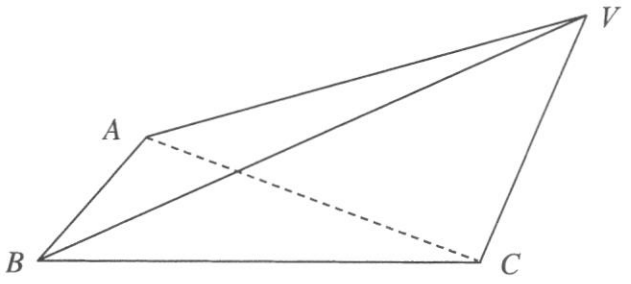
5 The diagram shows part of a regular polygon $OPQR$ attached to a triangle WPQ.
The exterior angle of the regular polygon is $40°$.



Calculate the sum of the angles a, b and c.

6 The figure shows a pyramid with a triangular base ABC with vertex V.

$AB = 5$ cm, $AC = 8$ cm, angle $BAC = 87°$, and the perpendicular height is 17 cm.



(a) Find the volume of the pyramid.

(b) The pyramid is melted to form a solid right cone with height 20 cm.

(i) Find the radius of the cone.

(ii) Find the total surface area of the cone.

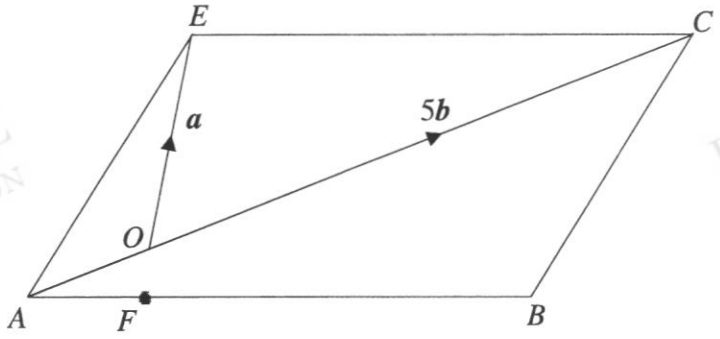
(iii) A frustrum is cut away to make a conical paperweight with a volume of 36 cm^3. What is the height of the frustrum?

7 **(a)** The position vector of point Y is $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$. X is the point (3, 1).

(i) Find the magnitude of vector XY.

(ii) Z is the point on XY produced with coordinates $(m, 9)$.

(b) $ABCE$ is a parallelogram. AOC is the diagonal of the parallelogram. The position vectors of C and E, relative to O, are $5\boldsymbol{b}$ and $\boldsymbol{a}$ respectively.



Given $\overrightarrow{AC} = 6\overrightarrow{AO}$ and $AF : FB = 1 : 4$.

(i) Express each of the following in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(a) $\overrightarrow{AO}$,

(b) $\overrightarrow{EC}$.

(c) $\overrightarrow{AF}$.

(d) position vector of F.

(ii) Write down one fact about O, E and F.

(iii) Find the value of $\dfrac{\text{Area of triangle } OFA}{\text{Area of triangle } CBA}$.

8 A, B, C and D are four locations on level ground.

A is 1.2 km due north of C. B is at a bearing of $046°$ from D and 1.5 km away from D.

Two straight pathways AC and BD intersect at T,
where T is the mid-point of AC and $BT : TD = 1 : 2$.

(i) Calculate the distance between A and B.

(ii) Calculate the bearing of A from B.

(iii) Ian walked from B to D. Calculate the shortest distance between Ian and C.

(iv) A vertical tower of height 70 m stands at T.
Calculate the angle of depression measured from the top of the tower to D.

9 Mr Lim and Mr Chan go running at MacRitchie Reservoir. Both men start from the same spot but Mr Chan being the slower runner, starts running at 0730 and Mr Lim starts at 0735. The route is 6.2 km long.

(i) If Mr Chan runs at an average speed of x km/h, write down an expression for the time taken, in hours, for the run.

(ii) Mr Lim runs the same route 3 km/h faster than Mr Chan and completes it 9 minutes before Mr Chan. Form an equation in terms of x and show that it reduces to $7x^2 + 21x - 558 = 0$.

(iii) Solve the equation $7x^2 + 21x - 558 = 0$.

(iv) Find the time taken by Mr Lim to overtake Mr Chan, giving your answer to the nearest minute.

10 **(a)** Complete the table of values for $y = -\frac{x^3}{10} + x + 1$.

x	–4	–3	–2	–1	0	1	2	3	4
y	3.4	0.7		0.1		1.9	2.2	1.3	–1.4

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of $y = -\frac{x^3}{10} + x + 1$ for $-4 \leq x \leq 4$ on a sheet of graph paper.

(c) Use your graph to determine the number of solutions for the equation $\frac{x^3}{10} = x + 1$ in the range $-4 \leq x \leq 4$.

(d) By drawing a tangent on your graph, find the gradient of the curve at the point $(-3, 0.7)$.

(e) By drawing a suitable straight line on your graph, solve the equation $x^3 - 9x = 0$ for $-4 \leq x \leq 4$.

11 Mr Chan owns a company that provides delivery services. He is reviewing the salaries of some of his staff and come up with a budget. Mr Chan finds the following information on the internet:

Table 1: Salaries for Drivers, Mechanics and Admin Executive

	Drivers	Vehicle Mechanic	Admin Executive
Monthly Salary Range	$2400 to $3000	$2500 to $4000	$3000 to $3500

Source: https://sg.indeed.com/career/salaries

In addition, both employer and employee must contribute a portion of the employee's monthly salary to the Central Provident Fund (CPF).

The following table summarises the current CPF contribution rates for Singaporeans and Singapore Permanent Residents (SPRs) across the different age groups.

Table 2: CPF Contribution Rates for Singaporeans and SPRs across different age groups

Employee's age (years)	Contribution Rates from 1 January 2023 (monthly wages > $750)		
	By employer (% of wage)	By employee (% of wage)	Total (% of wage)
55 and below	17	20	37
Above 55 to 60	14.5	15	29.5
Above 60 to 65	11	9.5	20.5
Above 65 to 70	8.5	7	15.5
Above 70	7.5	5	12.5

Source: https://www.cpf.gov.sg/employer/employer-obligations/how-much-cpf-contributions-to-pay

The following table gives the distribution of the different age groups according to the jobs.

Table 3: Distribution of Employees across age groups

Employee's age (years)	Drivers	Mechanics	Admin Executives
55 and below	13	4	1
Above 55 to 60	5	2	1
Above 60 to 65	0	0	0
Above 65 to 70	0	0	0
Above 70	0	0	0
Total	18	6	2

(a) A 35-year-old driver has a monthly salary of $2700.

Calculate the employee's CPF contribution, the employer's CPF contribution and the nett wage. The nett wage is an employee's earnings after all deductions are taken out which is also known as take-home pay.

(b) Mr Chan estimates that the monthly budget for the wages and employer's CPF contributions of all the drivers, mechanics, and admin executives should not exceed $100,000.

Is this a good estimate?
Justify your decision with calculations and state your assumption(s) made, if any.

MARKING SCHEME

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION 2023

S4

MATHEMATICS **4052/01**
Paper 1 **23 August 2023**
Candidates answer on the Question Paper. **2 hours 15 minutes**

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Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiners' Use

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Marks									
Question	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
Marks									
Question	**19**	**20**	**21**	**22**	**23**	**24**			
Marks									

Table of Penalties	**Units**			**90**
	Clarity/ Logic			
	Accuracy/ Precision			

Parent's Name and Signature: **Date:**	

This document consists of **20** printed pages.

Answer **all** the questions.

1 Expand and simplify $x(4x-9y)(4y+9x)$.

$$
\begin{aligned}
&x(4x-9y)(4y+9x)\\
&=\left(4x^2-9xy\right)(4y+9x)\\
&=16x^2y+36x^3-36xy^2-81x^2y\\
&=36x^3-36xy^2-65x^2y
\end{aligned}
$$

2 Given $2^x+2^{3+x}=9$, find x.

$$
\begin{aligned}
2^x+2^3\times 2^x&=9\\
9\left(2^x\right)&=9\\
2^x&=1\\
x&=0
\end{aligned}
$$

3 Simplify $\dfrac{6x^2-2xy+15x-5y}{6x^2+13xy-5y^2}$.

$$
\begin{aligned}
&\frac{6x^2-2xy+15x-5y}{6x^2+13xy-5y^2}\\
&=\frac{2x(3x-y)+5(3x-y)}{(3x-y)(2x+5y)}\\
&=\frac{(3x-y)(2x+5)}{(3x-y)(2x+5y)}\\
&=\frac{2x+5}{2x+5y}
\end{aligned}
$$

4 Two integers, A and B, can be written as products of prime factors.

$A=5\times 7 \qquad B=3^r\times 5^{p-r}\times 7$

The lowest common multiple of A and B is $3\times 5^2\times 7$.

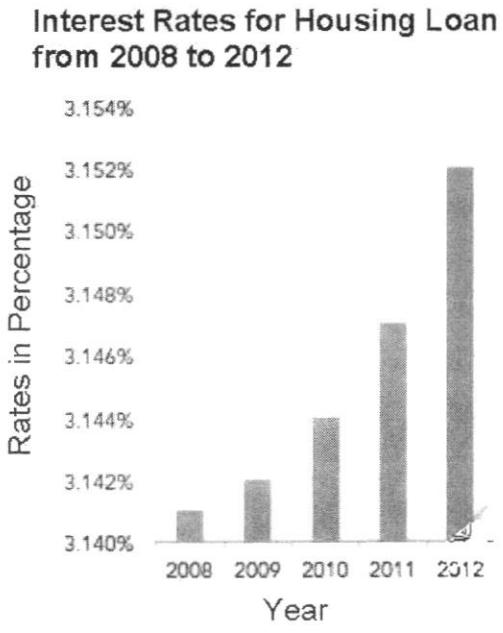
(a) Write down the values of p and of r.

$p=3$
$r=1$

(b) Find the highest common factor of A and B.

HCF = 35

5 Mr Lee drew this graph to show the interest rates for the housing loan from 2008 to 2012.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

The vertical axis does not start from zero.

It exaggerated the difference between the bars although their actual difference is less than 0.01%.

6 **(a)** Simplify $\left(\frac{x^{12}}{64y^6}\right)^{-\frac{5}{6}}$, leaving your answer in positive index.

$$\left(\frac{x^{12}}{64y^6}\right)^{-\frac{5}{6}} = \frac{x^{-10}}{2^{-5}y^{-5}}$$

$$= \frac{32y^5}{x^{10}}$$

OR

$$\left(\frac{x^{12}}{64y^6}\right)^{-\frac{5}{6}} = \left(\frac{64y^6}{x^{12}}\right)^{\frac{5}{6}}$$

$$= \frac{32y^5}{x^{10}}$$

(b) $\dfrac{5^b}{25^{a-1}} = 125^c$

Find an expression for a in terms of b and c.

$$\frac{5^b}{5^{2a-2}} = 5^{3c}$$
$$5^{b-2a+2} = 5^{3c}$$
$$a = \frac{b+2-3c}{2}$$

OR

$$\frac{5^b}{5^{2a-2}} = 5^{3c}$$
$$5^b = 5^{3c} \times 5^{2a-2}$$
$$5^b = 5^{3c+2a-2}$$
$$b = 3c + 2a - 2$$
$$a = \frac{b+2-3c}{2}$$

7 Factorise the following expressions completely.

(i) $9a^8 + 24a^4b + 16b^2$

$$9a^8 + 24a^4b + 16b^4 = \left(3a^4 + 4b\right)^2$$

(ii) Hence, factorise $144a^8 - \left(9a^8 + 24a^4b + 16b^2\right)$ completely.

$$144a^8 - \left(9a^8 + 24a^4b + 16b^2\right)$$
$$= 144a^8 - \left(3a^4 + 4b\right)^2$$
$$= \left(12a^4 - \left(3a^4 + 4b\right)\right)\left(12a^4 + \left(3a^4 + 4b\right)\right)$$
$$= \left(9a^4 - 4b\right)\left(15a^4 + 4b\right)$$

8 The gravitational force between two particles is inversely proportional to the square of the distance between them.

When two particles are r metres apart, the gravitational force between them is f newtons.

(a) Express the new gravitational force, in terms of f, between two particles when the distance between them is decreased by 60%.

(a) $f = \dfrac{k}{r^2}$, k is a constant

$k = fr^2$

Let f_n be new f

$f_n = \dfrac{fr^2}{(0.4r)^2}$

$\therefore f_n = 6.25f$

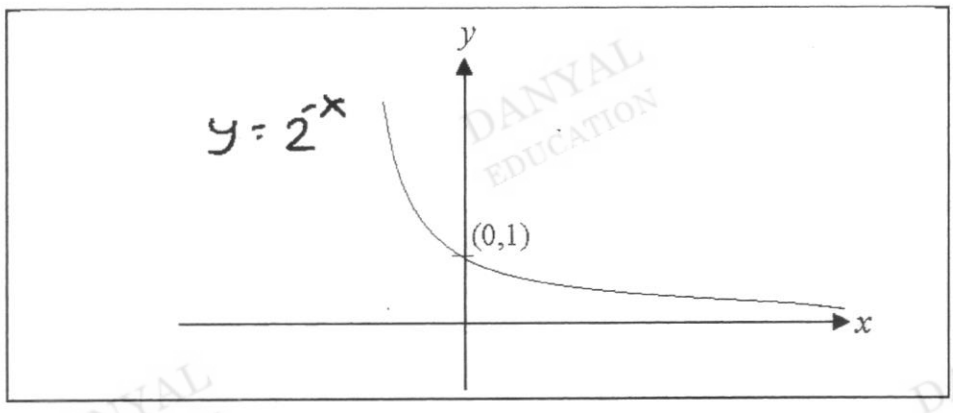
(b) Calculate the percentage change in the gravitational force.

(b) Percentage change $= \dfrac{6.25f - f}{f} \times 100\%$

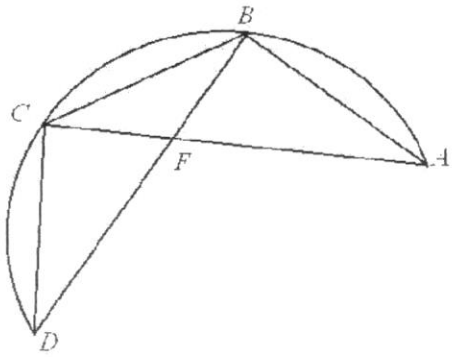
$= 525\%$

9 Sketch the graph of $y = 2^{-x}$ on the axes below.

Indicate clearly the values where the graph crosses the x- and y- axes.



10 A, B, C and D are vertices of a regular pentagon, and they lie on a circle.
AFC and BFD are straight lines.



Prove that triangle ABC is congruent to triangle BCD.
Give a reason for each statement you made.

In triangle ABC and triangle BCD,

$AB = BC$ (sides of regular polygon)
BC = CD (sides of regular polygon)
Angle ABC = angle BCD (int angles of a regular polygon)

Therefore, triangle $ABC \equiv$ triangle BCD (SAS)

OR

In triangle ABC and triangle BCD,
Angle ABC = angle BCD (int angles of a regular polygon)
Angle BAC = angle BDC (angles in the same segment)
$AB = BC$ (sides of regular polygon)

Therefore, triangle $ABC \equiv$ triangle BCD (AAS)

11 By writing the expression $x^2 - 8x + 39$ in the form $(x-h)^2 + k$, where h and k are constants, explain why the expression is always positive.

$x^2 - 8x + 39 = (x-4)^2 + 23$

$(x-4)^2 \geq 0$

$(x-4)^2 + 23 \geq 23 > 0$, hence expression is always positive.

12 $\xi = \{(x, y) : x \text{ and } y \text{ are integers, } -2 \leq x < 3 \text{ and } -2 \leq y \leq 1\}$.

$A = \{(x, y) : x^2 + y^2 = 5, \quad x > y\}$.

$B = \{(x, y) : xy < 0\}$.

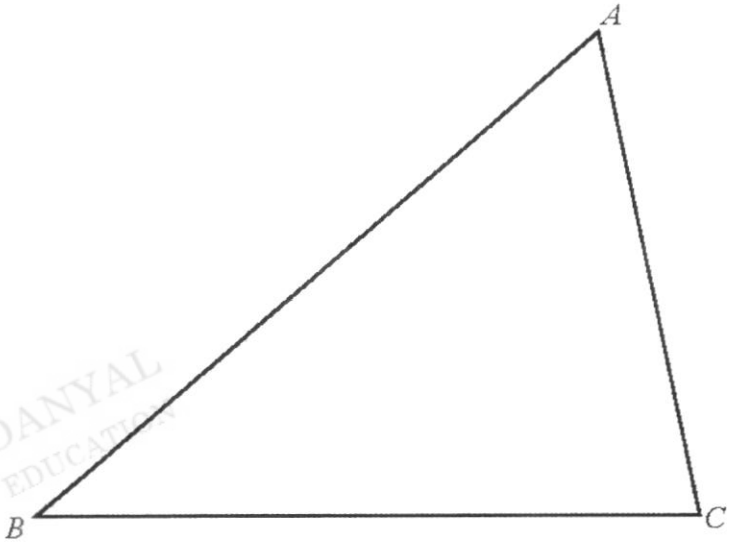
List the elements in
(i) A,

$(2,1), (2,-1), (-1,-2), (1,-2)$

(ii) B.

$(-2,1),(-1,1),(1,-2),(1,-1),(2,-2),(2,-1)$	B1

13 The diagram shows a plot of land, ABC.

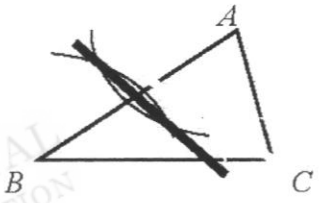


Scale = 1cm : 2m

(a) Construct the perpendicular bisector of AB.

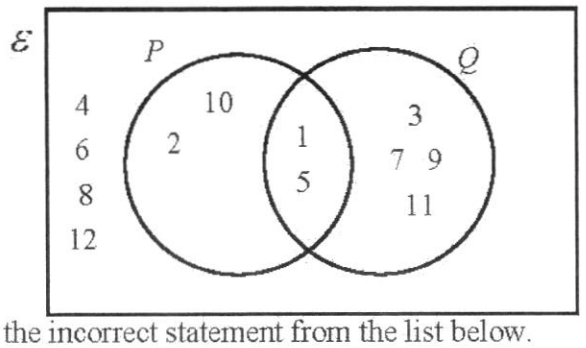
(b) Construct the bisector of angle ACB.

(c) Mr Tan bought a plot of land, nearer to B than A and nearer to BC than to AC. Find the area that was bought in m^2.



(c) Accept 74.2 to 76.7 m^2.

14 The Venn diagram shows the elements of $\xi = \{\text{intergers } x : 1 \le x \le 12\}$



(i) Underline the incorrect statement from the list below.

$n(P) = 4$ $\quad 3 \in Q$ $\quad Q \cap Q' = \varnothing$

$\{1, 2, 5, 10\} \subset P$ $\quad n(P \cup Q)' = 4$

(ii) Find the value of

(a) $n(P \cup Q')$,

8

(b) $n\left[(P \cap Q)' \cap (P \cup Q)\right]$

6

15 **(a)** Solve the inequality $\frac{2x+2}{7} \le \frac{x+6}{4} < \frac{x-5}{2}$.

$\frac{2x+2}{7} \le \frac{x+6}{4}$ and $\frac{x+6}{4} < \frac{x-5}{2}$

$8x + 8 \le 7x + 42$ $\quad 2x + 12 < 4x - 20$

$x \le 34$ $\quad 16 < x$

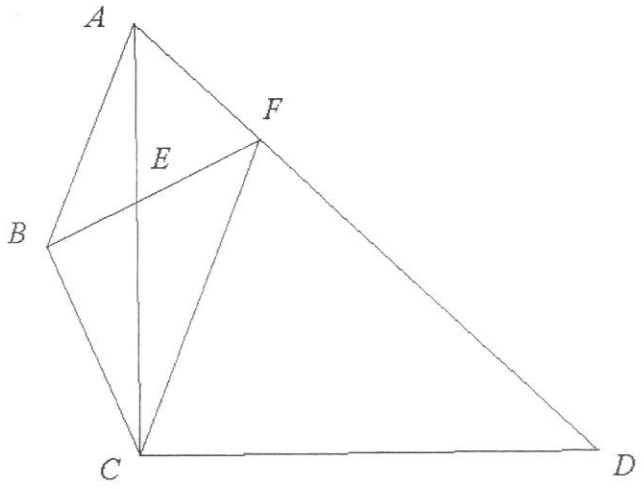
Hence, $16 < x \le 34$

(b) Write down all the perfect squares that satisfy $\frac{2x+2}{7} \le \frac{x+6}{4} < \frac{x-5}{2}$.

$x = 25$

16 In the figure below, $ABCF$ is a trapezium with $AB \parallel CF$.

The diagonals of the trapezium intersect at E. AFD and CD are straight lines.



(a) Name two triangles that are similar.

Triangle ABE is similar to triangle CFE.

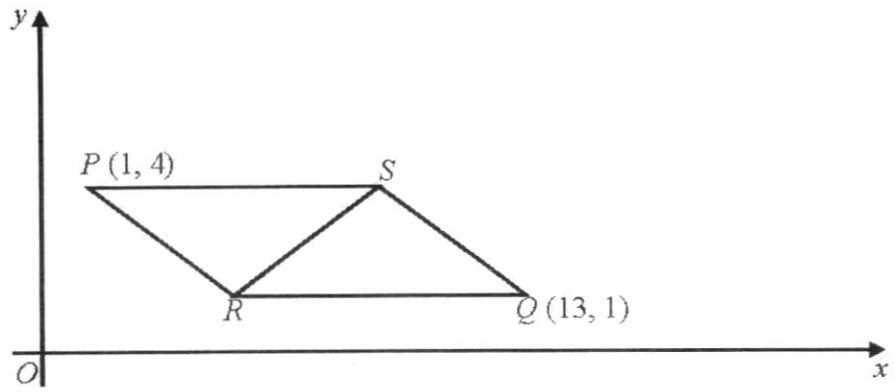
(b) Explain why the two triangles in **(a)** are similar.

In triangle ADE and triangle CFE, Angle AEB = angle FEC (vert. opp angles) Angle ABE = angle CFE (alternate angles, $AB \parallel CF$) Hence, by the test of AA, Triangle ADE is similar to triangle CFE.

17 Glaciers are melting at an average rate of 13300 tonnes of water per second in 2022. Assuming there are 365 days in the year 2022, calculate the average amount of ice melts in the whole year. Give your answer in standard form, correct to 2 significant figures.

Average amt. of ice melts $= 13300 \times 60 \times 60 \times 24 \times 365$ $= 4.2 \times 10^{11}$ tonnes

18 The diagram shows two congruent isosceles triangles.
The bases of the triangles are parallel to the x-axis.



(a) Write down the coordinates of R.

R has coordinates (5,1)

(b) Find the equation of the line PQ.

Method 1:

$$\frac{4-1}{1-13}=\frac{y-4}{x-1}$$
$$3x-3=-12y+48$$
$$12y=51-3x$$

Method 2:

$$\text{Gradient}=\frac{4-1}{1-13}=\frac{1}{-4}$$

Using pt (13,1):

$$y=-\frac{1}{4}x+c$$
$$1=-\frac{1}{4}(13)+c$$
$$c=\frac{17}{4}$$
$$\therefore y=-\frac{1}{4}x+\frac{17}{4}$$

(c) Find the coordinates of the midpoint of PQ.

$$\text{Midpoint of } PQ=\left(\frac{1+13}{2},\frac{4+1}{2}\right)$$
$$=(7,2.5)$$

(d) Find the perimeter of $PSQR$.

$$PR=\sqrt{(5-1)^2+(1-4)^2}$$
$$=5 \text{ units.}$$
$RQ = 8$ units.
Perimeter $= 8+8+5+5$
$= 26$ units.

19 A shop sells chocolate cookies (A) and chocolate cakes (B). These items are sold in boxes at three different outlets - Outlet 1, Outlet 2 and Outlet 3.
The table shows the number of boxes of A and of B sold at the three outlets.

	A	B
Outlet 1	45	40
Outlet 2	30	35
Outlet 3	40	x

(a) The production costs of each box of A, and of B are \$10.00, and \$13.40 respectively. Write down two matrices such that their product under matrix multiplication gives the selling prices that result in a 20% profit for a box of A and 25% profit for a box of B.

$$\begin{pmatrix} 1.2 & 0 \\ 0 & 1.25 \end{pmatrix} \quad \begin{pmatrix} 10.00 \\ 13.40 \end{pmatrix}$$

Or

$$\begin{pmatrix} 10.00 & 13.40 \end{pmatrix} \begin{pmatrix} 1.2 & 0 \\ 0 & 1.25 \end{pmatrix}$$

Or

$$\begin{pmatrix} 10 & 0 \\ 0 & 13.40 \end{pmatrix} \quad \begin{pmatrix} 1.2 \\ 1.25 \end{pmatrix}$$

Or

$$\begin{pmatrix} 1.2 & 1.25 \end{pmatrix} \begin{pmatrix} 10.00 & 0 \\ 0 & 13.40 \end{pmatrix}$$

Evaluate the product to find the selling prices, in \$, of each box of A and of B.

$$\begin{pmatrix} 1.2 & 0 \\ 0 & 1.25 \end{pmatrix} \begin{pmatrix} 10.00 \\ 13.40 \end{pmatrix} = \begin{pmatrix} 12.00 \\ 16.75 \end{pmatrix}$$

Or

$$\begin{pmatrix} 10.00 & 13.40 \end{pmatrix} \begin{pmatrix} 1.2 & 0 \\ 0 & 1.25 \end{pmatrix} = \begin{pmatrix} 12.00 & 16.75 \end{pmatrix}$$

Or

$$\begin{pmatrix} 10 & 0 \\ 0 & 13.40 \end{pmatrix} \begin{pmatrix} 1.2 \\ 1.25 \end{pmatrix} = \begin{pmatrix} 12.00 \\ 16.75 \end{pmatrix}$$

Or

$$\begin{pmatrix} 1.2 & 1.25 \end{pmatrix} \begin{pmatrix} 10.00 & 0 \\ 0 & 13.40 \end{pmatrix} = \begin{pmatrix} 12.00 & 16.75 \end{pmatrix}$$

$A = \$12.00$ and $B = \$16.75$

(b) Using matrix multiplication twice, find the total amount of money collected for selling A and B at all three outlets in terms of x.

$$\begin{pmatrix} 45 & 40 \\ 30 & 35 \\ 40 & x \end{pmatrix}\begin{pmatrix} 12.00 \\ 16.75 \end{pmatrix} = \begin{pmatrix} 45\times 12.00 + 40\times 16.75 \\ 30\times 12.00 + 35\times 16.75 \\ 40\times 12.00 + x\times 16.75 \end{pmatrix}$$

$$= \begin{pmatrix} 1210 \\ 946.25 \\ 480 + 16.75x \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1210 \\ 946.25 \\ 480 + 16.75x \end{pmatrix} = (1210 + 946.25 + 480 + 16.75x)$$

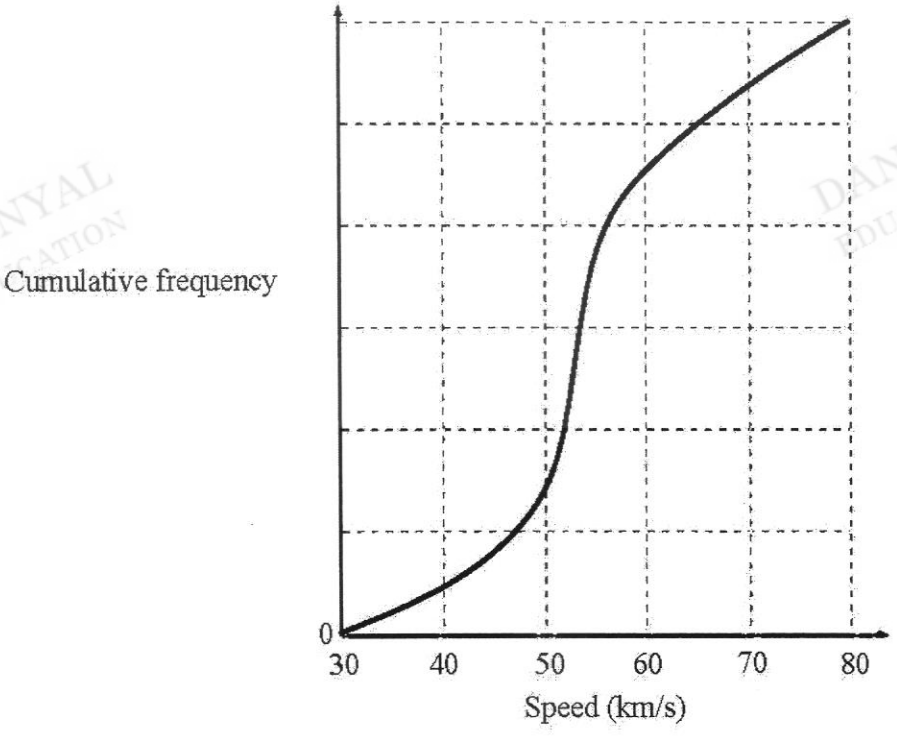
$$= (2636.25 + 16.75x)$$

The total amount of money collected is $\$2636.25 + 16.75x$

(c) The total amount of money collected at the three outlets for selling A and B is \$3189. Calculate the value of x.

$$2636.25 + 16.75x = 3189$$
$$16.75x = 552.75$$
$$x = 33$$

20 The speeds of cars passing a checkpoint were recorded. The cumulative frequency curve below shows the distribution of the speeds. The speed limit on the road is 60km/h.



(a) Estimate the percentage of cars that exceeded the speed limit.

$$\frac{1.5}{6}\times 100\% = 25\%$$

(b) Explain why it is not possible to find the number of cars that exceeded the speed limit.

The total number of cars is unknown.	B1

21 A lake, measuring 48 cm^2 on a map has an actual area of 108 000 000 m^2.
Find the value of n if the scale of the map is in the form $1 : n$.

48 cm^2 on map represents 108 000 000 m^2 on actual ground.
48 cm^2 on map represents 1080 000 000 000 cm^2 on actual ground.
1 cm^2 on map represents 22 500 000 000 cm^2 on actual gound.
$\therefore$ 1 cm on map represents 150 000 cm on actual ground.
Hence, scale of map = 1 : 150 000
$n = 150\,000$

22 Below are the first three lines of a sequence.
Line 1: $2^2 + 3 = 7$
Line 2: $3^2 + 5 = 14$
Line 3: $4^2 + 7 = 23$

(a) Write down the numbers for Line 6.

Line 6: $7^2 + 13 = 62$

(b) Write down and simplify an expression for the nth term of the sequence.

$$\begin{aligned} T_n &= (n+1)^2 + 2n + 1 \\ &= n^2 + 2n + 1 + 2n + 1 \\ &= n^2 + 4n + 2 \end{aligned}$$

(c) The difference, D, of the two consecutive terms is given by $T_{n+1} - T_n$.

Find an expression for D.

$$\begin{aligned} D &= (n+1)^2 + 4(n+1) + 2 - n^2 - 4n - 2 \\ &= n^2 + 2n + 1 + 4n + 4 - n^2 - 4n \\ &= 2n + 5 \end{aligned}$$

(d) Hence, explain why the difference will always be an odd integer.

Since 2n is always positive and when you add an odd number, 5, to it, the result will always be odd.

23 The table below shows the languages studied by 30 students.
There are seven students who only study Spanish but not French.
One of the 30 students is selected at random. The probability that the student studies French and not Spanish is $\frac{4}{15}$.
Two of the 30 students are selected at random. The probability that both students study neither French nor Spanish is $\frac{1}{145}$.
Complete the table of information about the 30 students.

	French	Not French
Spanish		
Not Spanish		

Number of students who speaks French and not Spanish

$= \frac{4}{15} \times 30$

$= 8$

Let the number of students who speaks neither French nor Spanish be n.

Probability that both students speaks neither French nor Spanish

$$\frac{(n)}{30}\left(\frac{n-1}{29}\right) = \frac{1}{145}$$

$$\frac{(n)(n-1)}{870} = \frac{1}{145}$$

$$145(n^2 - n) = 870$$

$$145n^2 - 145n - 870 = 0$$

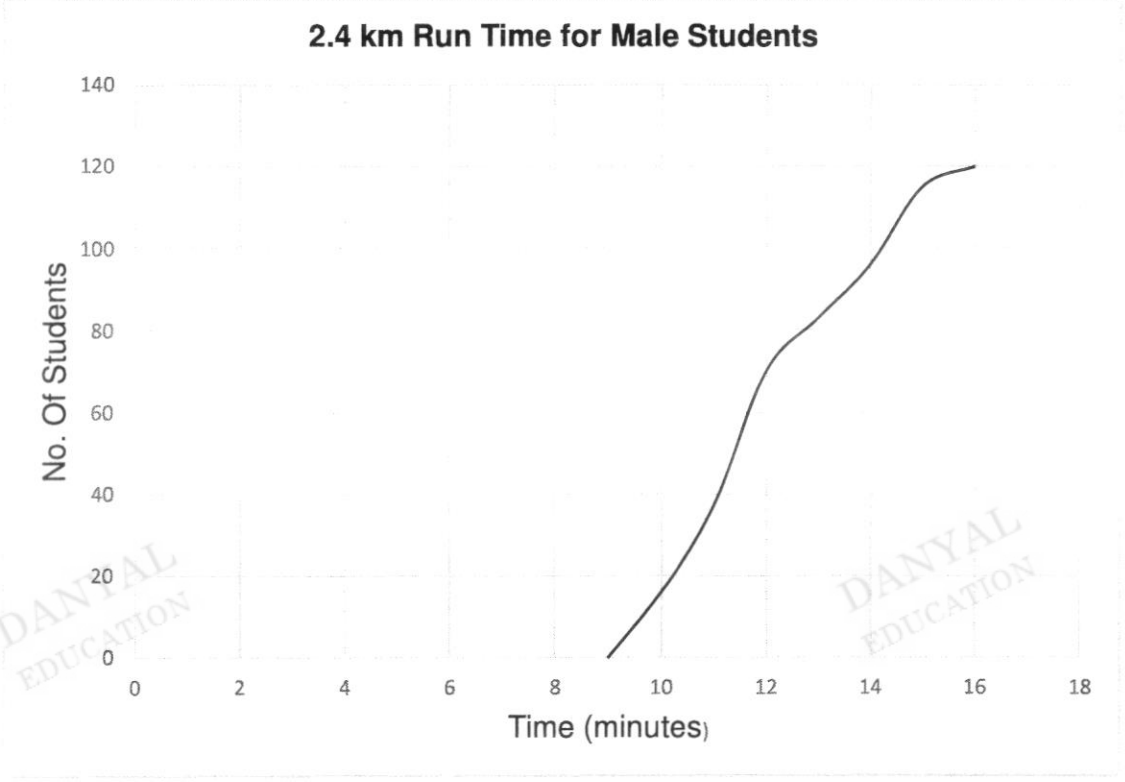
$$n^2 - n - 6 = 0$$

$$(n-3)(n+2) = 0$$

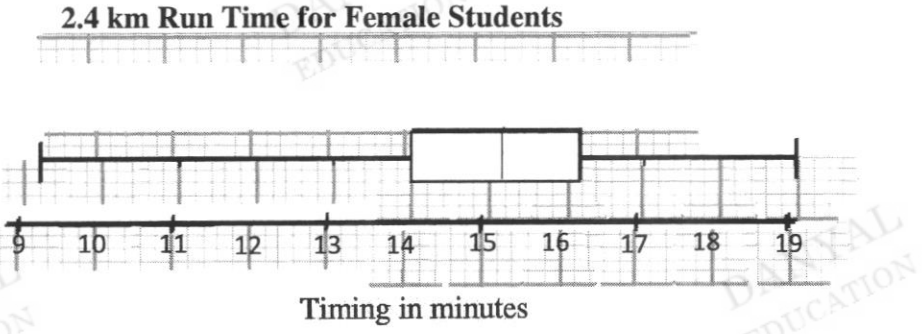
$n = -2$ (NA) or $n = 3$

	French	Not French
Spanish	12	7
Not Spanish	8	3

24 The cumulative frequency graph shows the distribution of the times of the first 120 male students to complete a 2.4 km run in 2023.



The box-and-whisker plot represents shows the distribution of the times of the first 120 female students to complete a 2.4 km run in 2023.



(a) 25% of the female students failed the 2.4 km run. State the time to pass the 2.4 km run.

16.3 minutes

(b) Using the two diagrams given, complete the table below.

	Lower Quartile	Median	Upper Quartile	Interquartile Range	
Male	10.6	11.6	13.6	3.0	
Female	14.1	15.3	16.3	2.2	

(c) Below are two statements comparing the times of the male and female students. For each one, state you agree or disagree, giving a reason for each answer.

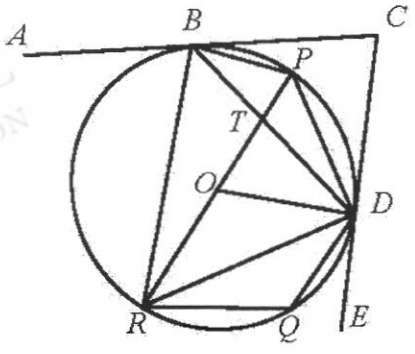
Statement	Agree/Disagree	Reason
The boys, on average, run faster than the girls.	Agree	The median time for male is 11.6 minutes which is lower than the median time for female which is 15.3 minutes.
The timing for the girls run is more consistent as compared to the boys.	Agree	The interquartile range is smaller for the girls.

25 18 painters can paint 3 buildings in 20 days.
After painting for 8 days, 8 painters left.
How many more days does the remaining painters need to complete the painting of the buildings? Give your answer to the nearest whole number.

18P : 3B : 20D
18P : 1.2B : 8D
10P : 1.2B : 14.4D
10P : 1.8B : 21.6D

The remaining painters will need **22** more days complete the painting of the buildings. (nearest whole number)

26 The lines ABC and CDE are tangents to the circle $BPDQR$ at B and at D respectively. The centre of the circle is O. The straight lines POR and BTD intersect at T.
It is given that angle $ACD = 42°$ and angle $PBC = 31°$.



Giving all the relevant properties, find

(a) angle DBP,

$CB = CD$ (Tangents from the same external point)
Triangle BCD is isosceles.

$\angle DBC = \angle CDB$ (Base angles of isosceles triangle)

$= \dfrac{180° - \angle BCD}{2}$

$= \dfrac{180° - 42°}{2}$

$= 69°$

$\angle DBP = \angle DBC - \angle PBC$

$= 69° - 31°$

$= 38°$

(b) angle RBD,

Method 1

$\angle DRP = \angle DBP$ (Angles in the same segment)

$= 38°$

$\angle PDR = 90°$ (Right angle in a semi-circle)

$\angle RPD = 180° - \angle PDR - \angle DRP$ (Angle sum of a triangle)

$= 180° - 90° - 38°$

$= 52°$

Method 2

$\angle PBR = 90°$ ($\angle$ in a semicircle)

$\angle RBD = 90° - \angle DBP$

$= 90° - 38°$

$= 52°$

(c) angle DQR.

$\angle DQR = 180° - \angle RPD$ (Angles in opposite segments)

$= 180° - 52°$

$= 128°$

27 A vase that is 17 cm tall can be modelled by a frustrum, A, on top of an egg-shaped container, B. A sketch of the vase is shown in Figure 1.

Figure 2 shows the cone that the frustum A is a part of. The diameter QRS of the top of the frustrum, is 8 cm and the diameter PVT of the bottom is 7 cm. The height of the frustrum, RV is 8 cm. R and V are the midpoints of QS and PT respectively.

Figure 1

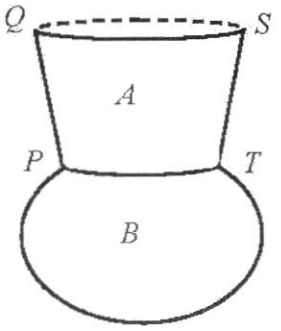
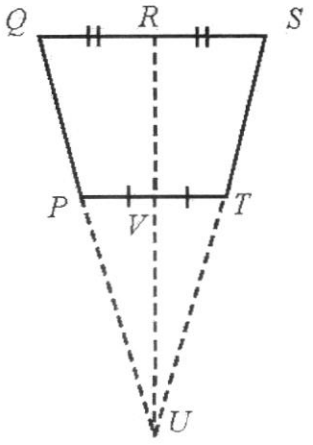


Figure 2



(a) Calculate the volume of the frustrum, A.

Let $UV = x$ cm.

Triangles UPT and UQS are similar.

$$\frac{x}{7} = \frac{x+8}{8}$$

$$8x = 7x + 56$$

$$x = 56$$

$UR = 56 + 8 = 64$ cm.

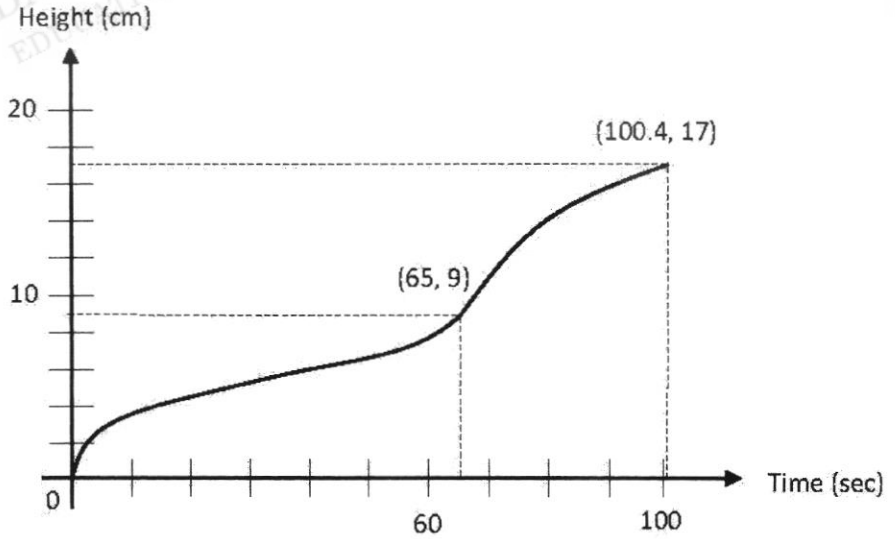
$$\text{Volume of A} = \frac{1}{3}\pi\left(\frac{8}{2}\right)^2(64) - \frac{1}{3}\pi\left(\frac{7}{2}\right)^2(56)$$

$$= 353.9528$$

$$= 354 \text{ cm}^3 (3s.f)$$

The volume of B is about 650 cm^3 and its height is 9 cm.
Water is poured into the vase at the rate of 10 cm^3 per second until it is full.

(b) Sketch the height of the water in the vase with respect to time.



End of paper

MARKING SCHEME

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2023

S4

MATHEMATICS **4052/02**

Paper 2 **28 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper and Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiners' Use

Questions	1	2	3	4	5	6	7
Marks							
Questions	8	9	10	11			
Marks							
Table of Penalties	**Units**					**90**	
	Clarity/Logic						
	Accuracy/Precision						

Parent's Name and Signature:

Date:

This document consists of **21** printed pages and **1** blank page.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** Rearrange the formula $a=\dfrac{b^2+5}{3-b^2}+2$ to make b the subject.

1(a)

$$a=\frac{b^2+5}{3-b^2}+2$$
$$3a-ab^2=b^2+5+6-2b^2$$
$$b^2-ab^2=11-3a$$
$$b^2(1-a)=11-3a$$
$$b^2=\frac{11-3a}{1-a}$$
$$b=\pm\sqrt{\frac{11-3a}{1-a}}$$

(b) Express as a single fraction in its simplest form $\dfrac{4}{2x-1}+\dfrac{2}{2x^2-3x+1}$.

1(b)

$$\frac{4}{2x-1}+\frac{2}{2x^2-3x+1}$$
$$=\frac{4}{2x-1}+\frac{2}{(2x-1)(x-1)}$$
$$=\frac{4(x-1)+2}{(2x-1)(x-1)}$$
$$=\frac{4x-4+2}{(2x-1)(x-1)}$$
$$=\frac{4x-2}{(2x-1)(x-1)}$$
$$=\frac{2(2x-1)}{(2x-1)(x-1)}$$
$$=\frac{2}{x-1}$$

(c) Solve $\dfrac{10x+7}{4}-\dfrac{2x^2-9}{x+2}=7$.

1(c)

$$\frac{10x+7}{4}-\frac{2x^2-9}{x+2}=7$$
$$(10x+7)(x+2)-4(2x^2-9)=28(x+2)$$
$$10x^2+27x+14-8x^2+36=28x+56$$
$$2x^2-x-6=0$$
$$(2x+3)(x-2)=0$$
$$x=-\frac{3}{2} \text{ or } 2$$

2 **(a)** Myron puts his \$30000 bonus into a fixed deposit at 3.35% per annum. The interest is compounded daily. There are 365 days in a year.

Calculate the amount of interest he will have at the end of two years.

2(a)

Total amount at the end of two years

$$= \$30000\left(1+\frac{3.35\%}{365}\right)^{365(2)}$$

$$= \$30000\left(1+\frac{335}{36500}\right)^{730}$$

$$\approx \$32078.7657$$

Amount of interest he will have at the end of two years

$$= \$32078.7657 - \$30000$$

$$= \$2078.77 \text{ (nearest cent)}$$

(b) The price of a car is \$160000.
Myron buys this car on hire purchase.
He makes a 30% down payment and takes up a 7-year loan tenure at a simple interest rate of 2.68% per annum.

Calculate his monthly instalment for the car.

2(b)

Remaining amount

$$= 0.7 \times \$160000$$

$$= \$112000$$

$$\text{Simple interest} = \frac{PRT}{100}$$

$$= \frac{\$112000 \times 2.68 \times 7}{100}$$

$$= \$21011.20$$

His monthly instalment for the car

$$= \frac{\$112000 + \$21011.20}{84}$$

$$= \$1583.47 \text{ (nearest cent)}$$

(c) Myron pays monthly rent of \$3600.
This is a 25% increment from his monthly rent last year.
Calculate his monthly rent last year.

2(c)

His monthly rent last year

$$= \frac{\$3600}{1.25}$$

$$= \$2880$$

(d) The exchange rate between Singapore dollars (S$) and Australian dollars (A$) is S$1 = A$1.12.

Myron spends A$980 in Sydney with his credit card.

All foreign currency transactions are subjected to a 1% conversion fee imposed by the respective card associations.
This foreign transaction amount, converted to S$, is further subjected to a 2.25% bank administrative fee.

Calculate the total amount on Myron's credit card bill, inclusive of the fees.
Give your answer in S$, correct to the nearest cent.

2(d)	Myron's spending in Singapore dollars $= \text{A\$980}\left(\frac{\text{S\$1}}{\text{A\$1.12}}\right)$ $= \text{S\$875}$ Myron's spending with conversion fee $= 1.01 \times \text{S\$875}$ $= \text{S\$883.75}$ Myron's total credit card bill $= \frac{102.25}{100} \times \text{S\$883.75}$ $= \text{S\$903.63}$ (nearest cent)

3 The test marks of a group of 30 students were recorded. The marks are shown in the stem-and-leaf diagram.

Stem	Leaf
0	4 4 4
1	8 8 9 9 9 9 9 9
2	2 2 2 3 *x* 4 4 4 5 5 7 7 7 7 7 7 7 7 7

Key 1 | 9 means 19 marks

(i) Find the percentage of students who score less than 20 marks.

3(i)	% of students who score < 20 marks $= \frac{11}{30}(100\%)$ $= 36\frac{2}{3}\%$

(ii) Given that the median is 23.5 marks, find the value of x.

3(ii)	$x = 4$

(iii) Find the standard deviation of the marks.

3(iii)	Standard Deviation ≈ 6.5946 marks ≈ 6.59 marks

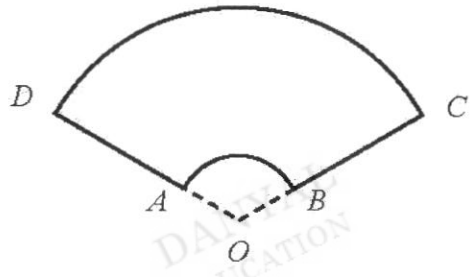
(iv) It was found that the marks were recorded wrongly. Every student was short of two marks. Write down the correct values for the median and standard deviation.

3(iv)	Median = 25.5 marks Standard deviation ≈ 6.59 marks

(v) Explain whether mean or median is a better measure of central tendency.

3(v)	The median is a better measure as the three test marks of 4 are **outliers / extreme values**.

4



The diagram shows a mirror $ABCD$.
AB and DC are arcs of circles centre O with radii 15 cm and 45 cm respectively.
The perimeter of the mirror is 286 cm.

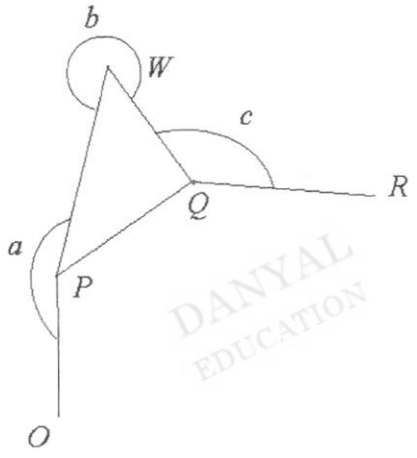
(a) Calculate the exact value of angle AOB in radians.

4(a)	Given Perimeter = 286 cm Arcs length = 296 – 2(30) = 226 cm $\frac{A\hat{O}B}{2\pi}\left[2\pi(15)+2\pi(45)\right]=226$ $A\hat{O}B(15+45)=226$ $A\hat{O}B=\frac{113}{30}=3\frac{23}{30}$ radians

(b) Calculate the area of the mirror.

4(b)	Area of the mirror $=\frac{A\hat{O}B}{2\pi}\left[\pi(45)^2-\pi(15)^2\right]$ $=\frac{A\hat{O}B}{2}\left(45^2-15^2\right)$ $=\frac{113}{2(30)}(30)(60)$ $=113(30)$ $=3390\ \text{cm}^2$

5 The diagram shows part of a regular polygon $OPQR$ attached to a triangle WPQ.
The exterior angle of the regular polygon is $40°$.

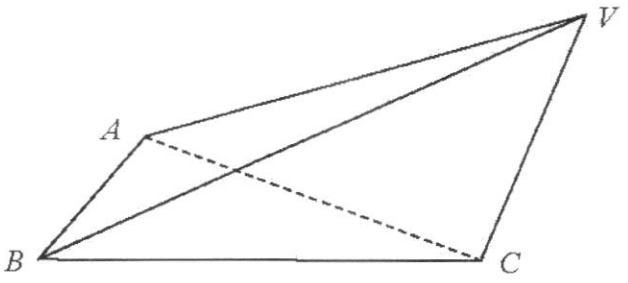


Calculate the sum of the angles a, b and c.

5	Int. angle + Ext. angle $= 180°$ (Adj. angles on a str. line) Int. angle $+ 40° = 180°$ Int. angle $= 180° - 40° = 140°$
	At P, Q and W, Sum of angles at a point $= 360° \times 3$ $= 1080°$ $\angle QPW + \angle PWQ + \angle WQP = 180°$ (Angle sum of a triangle) $a + b + c = 1080° - \angle OPQ - \angle PQR - (\angle QPW + \angle PWQ + \angle WQP)$ $= 1080° - 140° - 140° - 180°$ $= 620°$

6 The figure shows a pyramid with a triangular base ABC with vertex V.

$AB = 5$ cm, $AC = 8$ cm, angle $BAC = 87°$, and the perpendicular height is 17 cm.



(a) Find the volume of the pyramid.

6(a)	Volume of pyramid $=\frac{1}{3}\left[\frac{1}{2}(5)(8)\sin 87°\right](17)$ ≈ 113.178 $\approx 113 \text{ cm}^3$

(b) The pyramid is melted to form a solid right cone with height 20 cm.

(i) Find the radius of the cone.

6(b)(i)	Volume of cone = 113.178 $\frac{1}{3}\pi r^2 h = 113.178$ $r^2 = \frac{113.178\times 3}{20\pi}$ $r = \sqrt{\frac{113.178\times 3}{20\pi}} \left(\because r > 0\right)$ $r \approx 2.3246$ Radius of cone $\approx$ 2.32 cm

(ii) Find the total surface area of the cone.

6(b)(ii)	Total surface area of cone $= \pi r^2 + \pi r l$ $= \pi(2.3246)^2 + \pi(2.3246)\left(\sqrt{20^2 + 2.3236^2}\right)$ ≈ 164.02 $\approx 164 \text{ cm}^2$

(iii) A frustrum is cut away to make a conical paperweight with a volume of 36 cm^3. What is the height of the frustrum?

6(b)(iii)

$$\frac{\text{Vol. of conical paperweight}}{\text{Vol. of whole cone}} = \left(\frac{\text{Height of paperweight}}{\text{Height of whole cone}}\right)^3$$

$$\sqrt[3]{\frac{36}{113.178}} = \frac{\text{Height of paperweight}}{20}$$

$$\text{Height of paperweight} = 20 \times \sqrt[3]{\frac{36}{113.178}}$$

$$\approx 13.652 \text{ cm}$$

Therefore, height of the bottom to be removed

$= 20 - 13.652$

≈ 6.35 cm

OR

Let the height of the cone and paperweight be h_c cm and h_p cm respectively.

Let the radius of the cone and paperweight be r_c cm and r_p cm respectively.

$$\frac{\text{Height of paperweight}}{\text{height of cone}} = \frac{\text{radius of paperweight}}{\text{radius of cone}}$$

$$\frac{h_p}{h_c} = \frac{r_p}{r_c}$$

$$\frac{h_p}{20} = \frac{r_p}{2.3246}$$

$$r_p = \frac{2.3246 h_p}{20}$$

$$r_p = 0.11623 h_p$$

Volume of paperweight = 36 cm^2

$$\frac{1}{3}\pi r_p^2 h_p = 36$$

$$\frac{1}{3}\pi \left(0.11623 h_p\right)^2 h_p = 36$$

$$h_p^3 = \frac{36 \times 3}{\pi \times 0.11623^2}$$

$$h_p \approx 13.653 \text{ cm}$$

Height of the cone removed = 20 – 13.653

≈ 6.35 cm

7 **(a)** The position vector of point Y is $\begin{pmatrix}1\\6\end{pmatrix}$. X is the point (3, 1).

(i) Find the magnitude of vector XY.

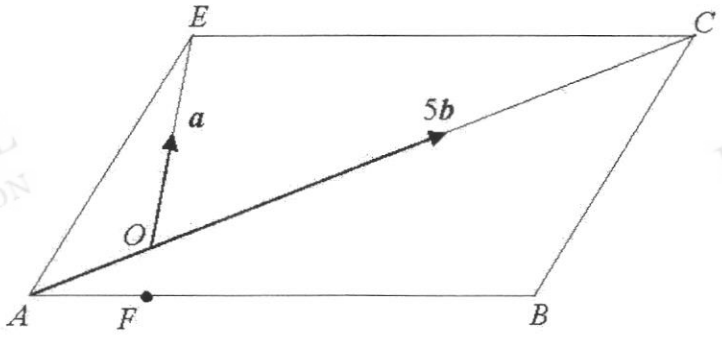
7(a)(i)	$\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY} = \begin{pmatrix}-3\\-1\end{pmatrix} + \begin{pmatrix}1\\6\end{pmatrix} = \begin{pmatrix}-2\\5\end{pmatrix}$ $\left\|\overrightarrow{XY}\right\| = \sqrt{(-2)^2 + 5^2} = \sqrt{29} = 5.39$ unit (3s.f)

(ii) Z is the point on XY produced with coordinates $(m, 9)$.

Find the value of m.

7(a)(ii)	$\overrightarrow{XZ} = k\overrightarrow{XY}$	$\overrightarrow{YZ} = k\overrightarrow{XY}$
	$\overrightarrow{XO} + \overrightarrow{OZ} = k\overrightarrow{XY}$	$\overrightarrow{YO} + \overrightarrow{OZ} = k\overrightarrow{XY}$
	$\begin{pmatrix}-3\\-1\end{pmatrix} + \begin{pmatrix}m\\9\end{pmatrix} = k\begin{pmatrix}-2\\5\end{pmatrix}$	$\begin{pmatrix}-1\\-6\end{pmatrix} + \begin{pmatrix}m\\9\end{pmatrix} = k\begin{pmatrix}-2\\5\end{pmatrix}$
	$\begin{pmatrix}m-3\\8\end{pmatrix} = \begin{pmatrix}-2k\\5k\end{pmatrix}$ or	$\begin{pmatrix}m-1\\3\end{pmatrix} = \begin{pmatrix}-2k\\5k\end{pmatrix}$
	$k = 1.6$	$k = 0.6$
	$m - 3 = -3.2$	$m - 1 = -1.2$
	$m = -0.2$	$m = -0.2$

(b) $ABCE$ is a parallelogram. AOC is the diagonal of the parallelogram. The position vectors of C and E, relative to O, are $5\boldsymbol{b}$ and $\boldsymbol{a}$ respectively.



Given $\overrightarrow{AC} = 6\overrightarrow{AO}$ and $AF : FB = 1 : 4$.

(i) Express each of the following in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(a) $\overrightarrow{AO}$,

7(b)(i)(a)	$\overrightarrow{AO} = \frac{1}{5}\overrightarrow{OC} = \boldsymbol{b}$

(b) $\overrightarrow{EC}$,

7(b)(i)(b)	$\overrightarrow{EC} = \overrightarrow{EO} + \overrightarrow{OC} = -\boldsymbol{a} + 5\boldsymbol{b} = 5\boldsymbol{b} - \boldsymbol{a}$

(c) $\overrightarrow{AF}$,

7(b)(i)(c)	$\overrightarrow{AF} = \frac{1}{5}\overrightarrow{AB} = \frac{1}{5}\overrightarrow{EC} = \frac{1}{5}(5\boldsymbol{b} - \boldsymbol{a})$

(d) position vector of F.

7(b)(i)(d)	$\overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{AF} = -\boldsymbol{b} + \frac{1}{5}(5\boldsymbol{b} - \boldsymbol{a}) = -\frac{1}{5}\boldsymbol{a}$

(ii) Write down one fact about O, E and F.

7(b)(ii)	Possible answer: a) The points are collinear. b) $OF : OE = 1 : 5$ c) $\overrightarrow{FO} = \frac{1}{5}\overrightarrow{OE}$ or $\overrightarrow{OF} = -\frac{1}{5}\overrightarrow{OE}$

(iii) Find the value of $\dfrac{\text{Area of triangle } OFA}{\text{Area of triangle } CBA}$.

7(b)(iii)	$\dfrac{\text{Area of } \Delta OFA}{\text{Area of } \Delta CBA} = \dfrac{\frac{1}{2}(AO)(AF)\sin\angle OAF}{\frac{1}{2}(AC)(AB)\sin\angle CAB}$ $= \dfrac{(AO)(AF)}{(AC)(AB)} \quad (\because \angle OAF = \angle CAB)$ $= \left(\frac{1}{6}\right)\left(\frac{1}{5}\right) = \frac{1}{30}$

8 A, B, C and D are four locations on level ground.

A is 1.2 km due north of C. B is at a bearing of 046° from D and 1.5 km away from D.

Two straight pathways AC and BD intersect at T,
where T is the mid-point of AC and $BT : TD = 1 : 2$.

(i) Calculate the distance between A and B.

8(i)

$AT = 0.6\text{km}$, $BT = 0.5\text{km}$

$\angle ATB = 46°$ (corr. $\angle$s betw. // lines)

Using Cosine Rule,

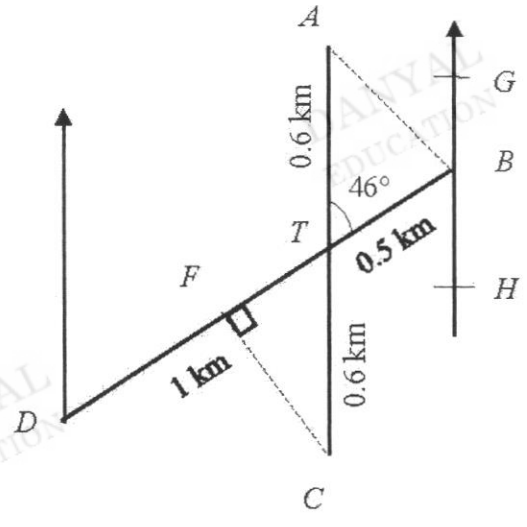
$$(AB)^2 = (0.6)^2 + (0.5)^2 - 2(0.6)(0.5)\cos 46°$$

$AB = 0.43955\text{ km}$ (5s.f.) $(\because AB > 0)$

$= 0.440\text{km}$ (3s.f)

(ii) Calculate the bearing of A from B.

8(ii)



	Method 1: Using Sine Rule, $\frac{0.43955}{\sin 46°} = \frac{0.5}{\sin \angle BAT}$ $\sin \angle BAT = \frac{0.5 \sin 46°}{0.43955}$ $\angle BAT = \sin^{-1}\left(\frac{0.5 \sin 46°}{0.43955}\right) = 54.9°$ (1 d.p) $\angle GBA = \angle BAT$ (alt $\angle$s, $AT // BG$) $\therefore$ Bearing of A from B $= 360° - 054.9°$ ($\angle s$ at a pt.) $= 305.1°$(1d.p)
	Method 2: Using Sine Rule, $\frac{0.6}{\sin \angle ABT} = \frac{0.43955}{\sin 46°}$ $\sin \angle ABT = \frac{0.6 \sin 46°}{0.43955}$ $\angle ABT = \sin^{-1}\left(\frac{0.6 \sin 46°}{0.43955}\right) = 79.1°$ (1 d.p) $\angle TBH = \angle ATB$ (alt $\angle$s, $AT // GH$) $\therefore$ Bearing of A from B $= 180° + 046° + 079.1° = 305.1°$ (1d.p)

(iii) Ian walked from B to D. Calculate the shortest distance between Ian and C.

8(iii)	Let the shortest distance be CF. $\sin 46° = \frac{CF}{0.6}$ $\therefore CF = 0.6 \times \sin 46°$ $= 0.432$km (3s.f)

(iv) A vertical tower of height 70 m stands at T.
Calculate the angle of depression measured from the top of the tower to D.

8(iv)	Let θ be the angle of depression. $\tan\theta = \dfrac{\text{Tower Height}}{DT} = \dfrac{70\text{ m}}{1000\text{ m}} = \dfrac{7}{100}$ $\therefore \theta = \tan^{-1}\left(\dfrac{7}{100}\right) = 4.0°$ (1d.p)
	Note: Penalise students if (1) no/inconsistent reasons given for angles [Clarity/Logic] (2) bearings not represented in 3 digits [Accuracy/Precision]

9 Mr Lim and Mr Chan go running at MacRitchie Reservoir. Both men start from the same spot but Mr Chan being the slower runner, starts running at 0730 and Mr Lim starts at 0735. The route is 6.2 km long.

(i) If Mr Chan runs at an average speed of x km/h, write down an expression for the time taken, in hours, for the run.

9(i)	Time taken by Mr Chan $= \dfrac{6.2}{x}$ hours

(ii) Mr Lim runs the same route 3 km/h faster than Mr Chan and completes it 9 minutes before Mr Chan. Form an equation in terms of x and show that it reduces to $7x^2 + 21x - 558 = 0$.

9(ii)	$\dfrac{6.2}{x} - \dfrac{6.2}{x+3} = \dfrac{5}{60} + \dfrac{9}{60}$ $\dfrac{6.2(x+3) - 6.2x}{x(x+3)} = \dfrac{14}{60}$ $\dfrac{18.6}{x(x+3)} = \dfrac{14}{60}$ $18.6 \times 60 = 14x(x+3)$ $14x^2 + 42x - 1116 = 0$ $7x^2 + 21x - 558 = 0$

(iii) Solve the equation $7x^2+21x-558=0$.

9(iii)	$7x^2+21x-558=0$ $x=\dfrac{-21\pm\sqrt{21^2-4(7)(-558)}}{2(7)}$ $=7.5534$ or -10.5534 (4d.p.) $=7.55$ or -10.6 (3s.f.)

(iv) Find the time taken by Mr Lim to overtake Mr Chan, giving your answer to the nearest minute.

9(iv)	Let t hours be the time taken for Mr Lim to catch up with Mr Chan. Mr Lim's average speed $=7.5534+3=10.5534$ km/h Distance travelled by Mr Lim $=10.5534t$ km Mr Chan's average speed $=7.5534$ km/h Distance travelled by Mr Chan $=7.5534\left(t+\dfrac{5}{60}\right)$ km $10.5534t=7.5534\left(t+\dfrac{5}{60}\right)$ $=7.5534t+0.62945$ $3t=0.62945$ $t=0.20982$ h $=0.20982\times60=12.589=13$minutes [FYI: The time when Mr Lim overtakes Mr Chan $=0735+0013=0748$ hours]

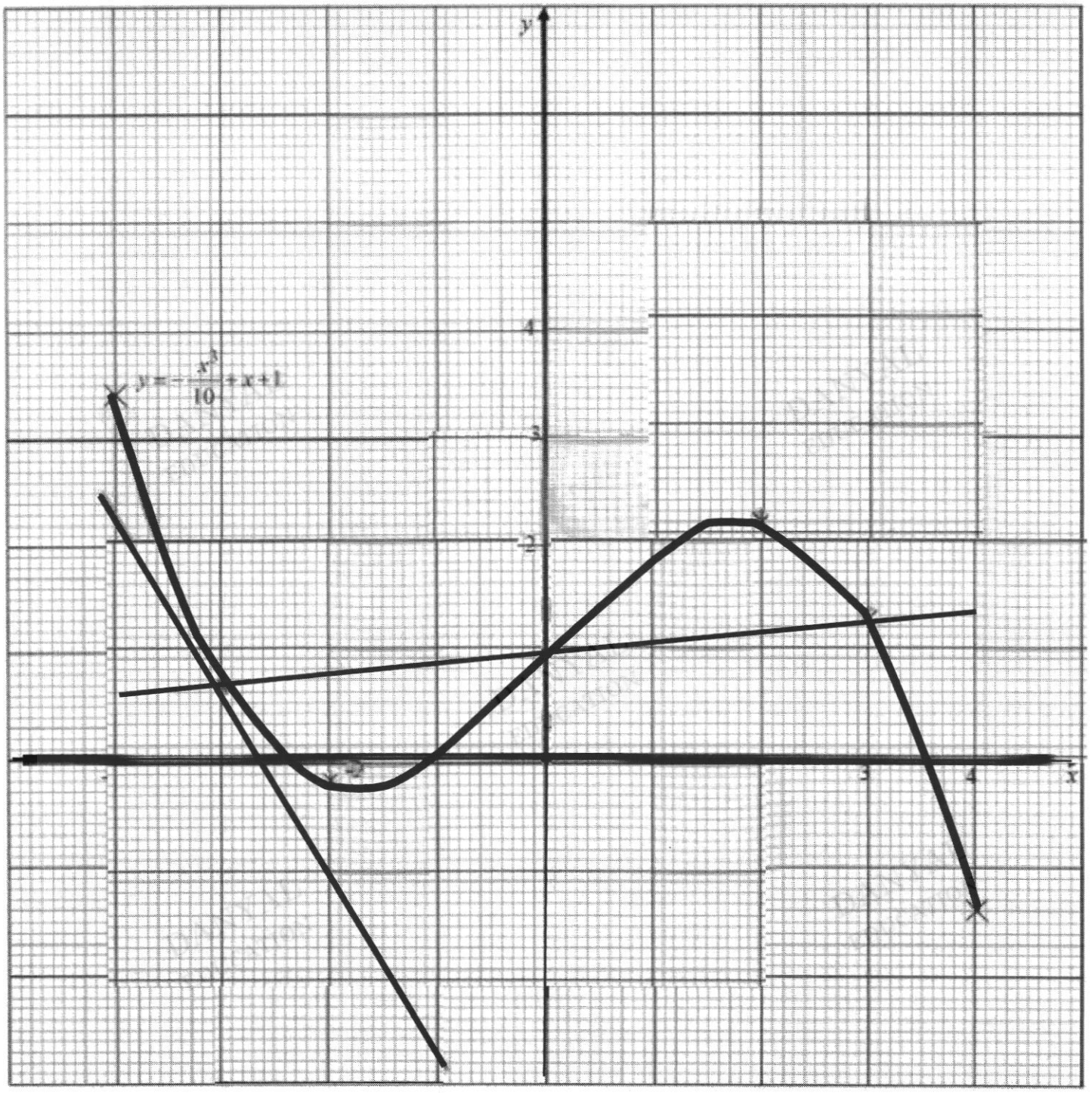
10 **(a)** Complete the table of values for $y=-\dfrac{x^3}{10}+x+1$.

x	–4	–3	–2	–1	0	1	2	3	4
y	3.4	0.7		0.1		1.9	2.2	1.3	–1.4

10(a)

x	–4	–3	–2	–1	0	1	2	3	4
y	3.4	0.7	**–0.2**	0.1	**1.0**	1.9	2.2	1.3	–1.4

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of $y=-\dfrac{x^3}{10}+x+1$ for $-4\le x\le 4$ on a sheet of graph paper.



(c) Use your graph to determine the number of solutions for the equation $\frac{x^3}{10} = x + 1$ in the range $-4 \leq x \leq 4$.

10(c)	**3 solutions**

(d) By drawing a tangent on your graph, find the gradient of the curve at the point $(-3, 0.7)$.

10(d)	From graph, Gradient $= \dfrac{2.3-(-2.8)}{-4-(-1)} = -1.7$

(e) By drawing a suitable straight line on your graph, solve the equation $x^3 - 9x = 0$ for $-4 \le x \le 4$.

10(e)	$x^3 - 9x = 0$ $-\dfrac{x^3}{10} + \dfrac{9x}{10} = 0$ $-\dfrac{x^3}{10} + x = \dfrac{x}{10}$ $-\dfrac{x^3}{10} + x + 1 = \dfrac{x}{10} + 1$ Draw $y = \dfrac{x}{10} + 1$ on the graph. The intersections of the line and the curve are the solutions. $x = -3.30$ or 0 or 3.30

11 Mr Chan owns a company that provides delivery services. He is reviewing the salaries of some of his staff and come up with a budget. Mr Chan finds the following information on the internet:

Table 1: Salaries for Drivers, Mechanics and Admin Executive

	Drivers	Vehicle Mechanic	Admin Executive
Monthly Salary Range	$2400 to $3000	$2500 to $4000	$3000 to $3500

Source: https://sg.indeed.com/career/salaries

In addition, both employer and employee must contribute a portion of the employee's monthly salary to the Central Provident Fund (CPF).

The following table summarises the current CPF contribution rates for Singaporeans and Singapore Permanent Residents (SPRs) across the different age groups.

Table 2: CPF Contribution Rates for Singaporeans and SPRs across different age groups

Employee's age (years)	Contribution Rates from 1 January 2023 (monthly wages > $750)		
	By employer (% of wage)	By employee (% of wage)	Total (% of wage)
55 and below	17	20	37
Above 55 to 60	14.5	15	29.5
Above 60 to 65	11	9.5	20.5
Above 65 to 70	8.5	7	15.5
Above 70	7.5	5	12.5

Source: https://www.cpf.gov.sg/employer/employer-obligations/how-much-cpf-contributions-to-pay

The following table gives the distribution of the different age groups according to the jobs.

Table 3: Distribution of Employees across age groups

Employee's age (years)	Drivers	Mechanics	Admin Executives
55 and below	13	4	1
Above 55 to 60	5	2	1
Above 60 to 65	0	0	0
Above 65 to 70	0	0	0
Above 70	0	0	0
Total	18	6	2

(a) A 35-year-old driver has a monthly salary of $2700.

Calculate the employee's CPF contribution, the employer's CPF contribution and the nett wage. The nett wage is an employee's earnings after all deductions are taken out which is also known as take-home pay.

11(a)	Employee's CPF contribution $= 2700 \times 0.2 = \$540$ Employer's CPF contribution $= 2700 \times 0.17 = \$459$ Employee's Nett Wage $= 2700 - 540 = \$2160$

(b) Mr Chan estimates that the monthly budget for the wages and employer's CPF contributions of all the drivers, mechanics, and admin executives should not exceed $100,000.

Is this a good estimate?
Justify your decision with calculations and state your assumption(s) made, if any.

11(b)

Using Maximum Salaries

Since we are checking the maximum budget, we can use the maximum salaries in each category of workers. We assume Mr Chan will use the maximum value and not go beyond.

Vehicle Mechanics

Employer's CPF Contribution for 55 and below

$= 4000 \times 0.17 \times 4 = \$2720$

Salary for Vehicle Mechanics 55 and below

$= 4000 \times 4 = \$16000$

Employer's CPF Contribution for 55 to 60

$= 4000 \times 0.145 \times 2 = \$1160$

Salary for Vehicle Mechanics 55 to 60

$= 4000 \times 2 = \$8000$

Total Employer's CPF $= 2720 + 1160$

$= \$3880$

Total Employer's Salary $= 16000 + 8000$

$= \$24000$

Total Employer's CPF and Salary $= 3880 + 24000$

$= \$27880$

Drivers

Employer's CPF Contribution for 55 and below

$= 3000 \times 0.17 \times 13 = \$6630$

Salary for Drivers 55 and below

$= 3000 \times 13 = \$39000$

Employer's CPF Contribution for 55 to 60

$= 3000 \times 0.145 \times 5 = \$2175$

Salary for Drivers 55 to 60

$= 3000 \times 5 = \$15000$

Total Employer's CPF $= 6630 + 2175$

$= \$8805$

Total Employer's Salary $= 39000 + 15000$

$= \$54000$

Total Employer's CPF and Salary $= 8805 + 54000$

$= \$62805$

	Admin Executives Employer's CPF Contribution for 55 and below $=3500\times0.17\times1=\$595$ Salary for Admin Execs 55 and below $=3500\times1=\$3500$ Employer's CPF Contribution for 55 to 60 $=3500\times0.145\times1=\$507.50$ Salary for Admin Execs 55 to 60 $=3500\times1=\$3500$ Total Employer's CPF = 595 – 507.50 = $1102.50 Total Employer's CPF and Salary = 3500 + 3500 = $7000 Total Employer's CPF and Salary = 1102.50 + 7000 = $8102.50 Total Employer's CPF and Salary for all employees = 27880 + 62805 + 8102.50 = $98787.50 Percentage difference of Mr Chan's estimate from the calculated maximum budget $=\frac{100000-98787.50}{98787.50}\times100$ =1.23% Mr Chan's estimate of maximum budget of $100000 is reasonable as it is only 1.23% from the calculated value using maximum salaries.
	<u>Using Mid-Value Salaries</u> Since we do not know the actual salaries, we will use the mid-value of each category of worker as an estimate. Vehicle Mechanics Employer's CPF Contribution for 55 and below $=3250\times0.17\times4=\$2210$ Salary for Vehicle Mechanics 55 and below $=3250\times4=\$13000$ Employer's CPF Contribution for 55 to 60 $=3250\times0.145\times2=\$942.50$ Salary for Vehicle Mechanics 55 to 60 $=3250\times2=\$6500$

Total Employer's CPF and Salary

$= 2210 + 13000 + 942.50 + 6500$

$= \$22652.50$

Drivers

Employer's CPF Contribution for 55 and below

$= 2700 \times 0.17 \times 13 = \$5967$

Salary for Drivers 55 and below

$= 2700 \times 13 = \$35100$

Employer's CPF Contribution for 55 to 60

$= 2700 \times 0.145 \times 5 = \$1957.50$

Salary for Drivers 55 to 60

$= 2700 \times 5 = \$13500$

Total Employer's CPF and Salary

$= 5967 + 35100 + 1957.50 + 13500$

$= \$56524.50$

Admin Executives

Employer's CPF Contribution for 55 and below

$= 3250 \times 0.17 \times 1 = \$552.50$

Salary for Admin Execs 55 and below

$= 3250 \times 1 = \$3250$

Employer's CPF Contribution for 55 to 60

$= 3250 \times 0.145 \times 1 = \$471.25$

Salary for Admin Execs 55 to 60

$= 3250 \times 1 = \$3250$

Total Employer's CPF and Salary

$= 552.50 + 3250 + 471.25 + 3250$

$= \$7523.75$

Total Employer's CPF and Salary for all employees

$= 22652.50 + 56524.50 + 7523.75$

$= \$86700.75$

Percentage difference of Mr Chan's estimate from the calculated maximum budget

$$= \frac{100000 - 86700.75}{86700.75} \times 100$$

$= 15.3\%$

The computed amount is 15.3% less than the \$100000. Mr Chan's estimate is reasonable as there is still some room for Mr Chan to increase wages.

Using Minimum Salaries

Vehicle Mechanics

Employer's CPF Contribution for 55 and below

$= 2500 \times 0.17 \times 4 = \$1700$

Salary for Vehicle Mechanics 55 and below

$= 2500 \times 4 = \$10000$

Employer's CPF Contribution for 55 to 60

$= 2500 \times 0.145 \times 2 = \$725$

Salary for Vehicle Mechanics 55 to 60

$= 2500 \times 2 = \$5000$

Total Employer's CPF and Salary

$= 1700 + 10000 + 725 + 5000$

$= \$17425$

Drivers

Employer's CPF Contribution for 55 and below

$= 2400 \times 0.17 \times 13 = \$5304$

Salary for Drivers 55 and below

$= 2400 \times 13 = \$31200$

Employer's CPF Contribution for 55 to 60

$= 2400 \times 0.145 \times 5 = \$1740$

Salary for Drivers 55 to 60

$= 2400 \times 5 = \$12000$

Total Employer's CPF and Salary

$= 5304 + 31200 + 1740 + 12000$

$= \$50244$

Admin Executives

Employer's CPF Contribution for 55 and below

$= 3000 \times 0.17 \times 1 = \$510$

Salary for Admin Execs 55 and below

$= 3000 \times 1 = \$3000$

Employer's CPF Contribution for 55 to 60

$= 3000 \times 0.145 \times 1 = \$435$

Salary for Admin Execs 55 to 60

$= 3000 \times 1 = \$3000$

Total Employer's CPF and Salary

$= 510 + 3000 + 435 + 3000$

$= \$6945$

	Total Employer's CPF and Salary for all employees $= 17425 + 50244 + 6945$ $= \$74614$ Percentage difference of Mr Chan's estimate from the calculated maximum budget $= \frac{100000 - 74614}{74614} \times 100$ $= 34.0\%$