2023 Sec 4 AMath Prelim - Anglican High

1 Express $\dfrac{4}{x^2(x+2)}$ in partial fractions. [4]

2 (a) Solve the cubic equation $2x^3 - 17x^2 + 47x - 42 = 0$. [4]

(b) Hence solve the equation $16w^3 - 68w^2 + 94w - 42 = 0$. [2]

3 Given that $\frac{1}{1-\sqrt{2}} \times \left(1+3\sqrt{2}\right)^2 = a + b\sqrt{2}$, find the value of a and of b. [4]

4 The graph of a cubic polynomial $f(x) = x^3 + px^2 + \left(p^2 + p - 7\right)x - 2p^2 - 6p + 6$ meets the x-axis only at 2.

(a) Show that $x - 2$ is a factor of $f(x)$. [2]

(b) Given that $f(x)=(x-2)\left[x^2+(p+2)x+\left(p^2+3p-3\right)\right]$, find the range of values of p for f(x) to have only 1 solution. [3]

2) ( )

5 (a) Find the set of values of the constant k for which the line $y+k^2=2x$ meets the curve $y=x^2+4x-3$. [5]

(b) Hence, write down the value(s) of k for which the line $y+k^2=2x$ is a tangent to the graph $y=x^2+4x-3$. [1]

6 A polynomial has a remainder of 3 when divided by $x-1$ and a remainder of 5 when divided by $x-3$.

Find the remainder when the polynomial is divided by $(x-1)(x-3)$. [4]

7 **(i)** Given that $y=\dfrac{e^{x-3}}{2\left(e^{1-x}\right)}$, find $\dfrac{dy}{dx}$. [2]

(ii) Hence, calculate the exact rate of change of x at the instant when $x = 3$ and y is decreasing at the rate of e^3 units per second. [2]

8 It is given that $y = \dfrac{x^2}{\ln x}$ where $x > 1$. Find the range of values of x for which $y = \dfrac{x^2}{\ln x}$ is a decreasing function. [4]

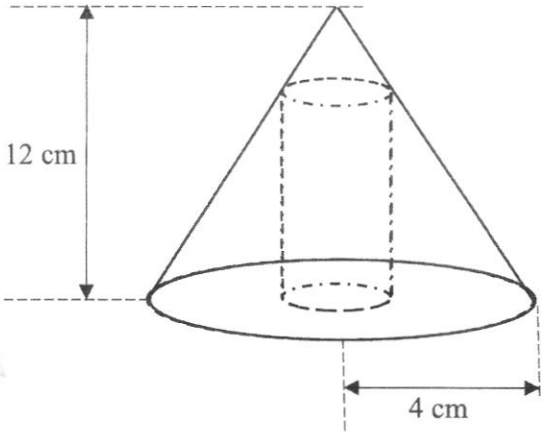
9 **(a)** Given that $y = \ln\left(\cos^2 x\right)$, show that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -2\sec^2 x$. [2]

(b) The equation of a curve is $y = \sqrt{x}\cos x$, find the gradient of the curve at $x = \pi$ radians in terms of π. [3]

10 Find the value of the constant k for which $y = \dfrac{x^3}{e^x}$ is a solution for the equation.

$$e^x\left(y + \frac{\mathrm{d}y}{\mathrm{d}x}\right) = 2kx^2.$$ [5]

11 The diagram shows a cylinder placed inside a circular cone of radius 4 cm and height 12 cm so that its base is level with the base of the cone.



(i) If the radius of the cylinder is r cm, show that its height h cm is given by $h = 12 - 3r$. [1]

(ii) Given that r can vary, find the maximum volume of the cylinder in terms of π.[4]

12 **(i)** Given that the coefficient of the third term in the binomial expansion of $\left(x-\frac{1}{3x^2}\right)^n$ is $\frac{22}{3}$, show that $n = 12$. [3]

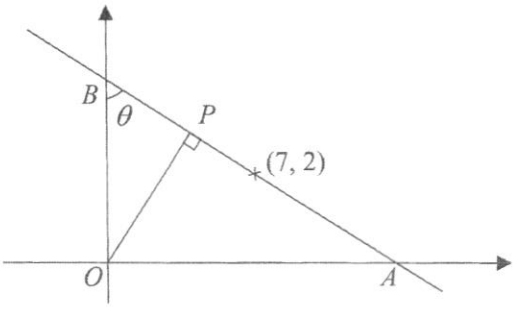
(ii) Hence, explain, showing your working clearly, why there are no constant terms in the expansion of $\left(x^3+\frac{8}{15}\right)\left(x-\frac{1}{3x^2}\right)^n$. [5]

13 **(i)** Express $\log_6 2^{3n+1} + \log_6 3^{4n+1} - \log_6 3^n$ in the form $an+b$,

a and b are integers. [3]

(ii) Solve the equation $\log_{\frac{1}{3}} \sqrt{2x-1} = \log_9 (x+5) + \frac{1}{2}$. [4]

14 In the diagram below, a line passes through the x-axis at point A and y-axis at point B. The point (7, 2) lies on the line.



(i) Given that OP is perpendicular to AB, and $\angle ABO = \theta$, find OA in the form $a + b\tan\theta$, where a and b are integers. [2]

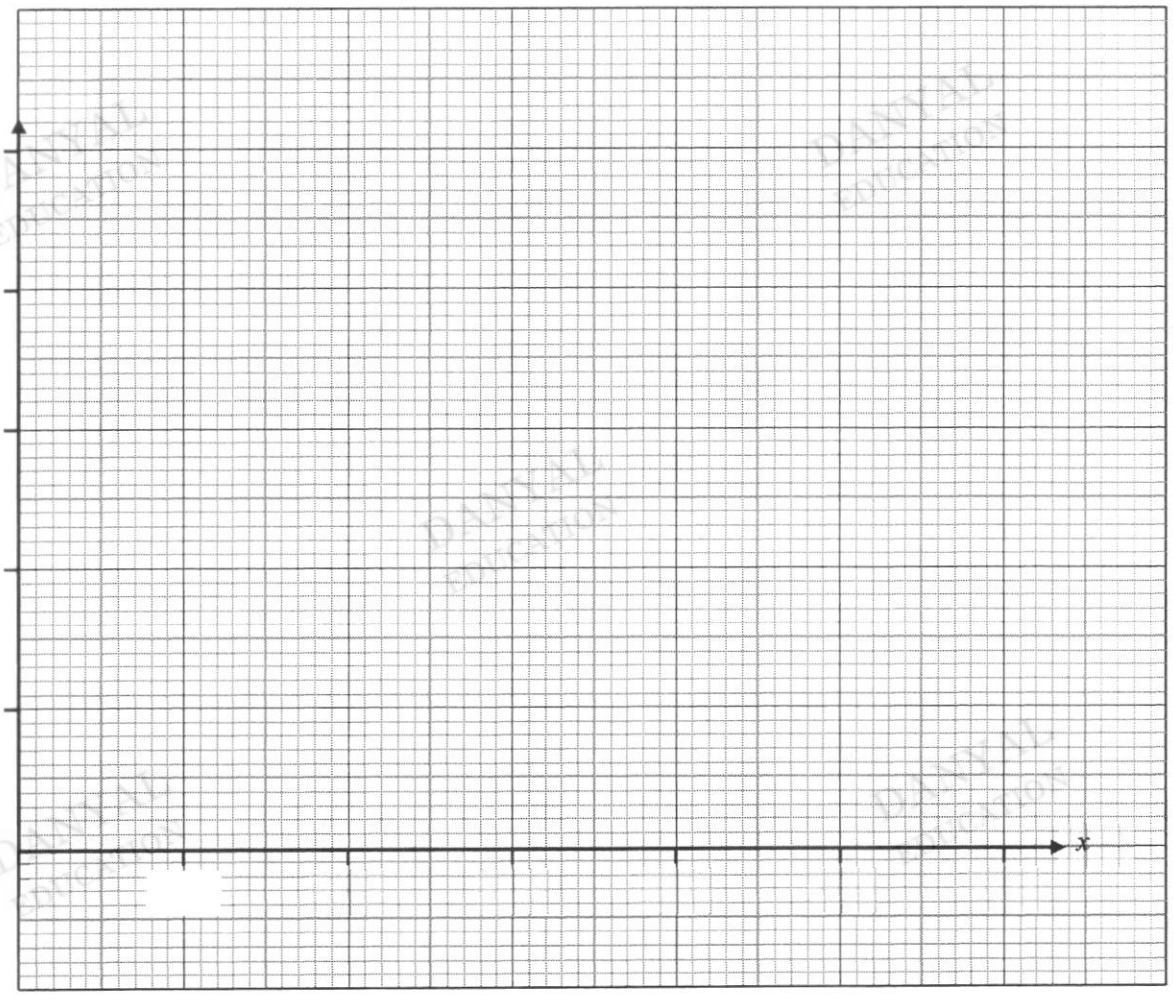
(ii) Show that $OP = 7\cos\theta + 2\sin\theta$. [2]

(iii) If $OP = 6$ units, determine the value of θ. [3]

15 The table below shows the experimental values of two variables x and y.

x	0.1	0.2	0.3	0.4	0.5	0.6
y	3.16	3.98	5.01	6.31	7.94	10.0
$ln\, y$	1.15	1.38	1.61	1.84	2.07	2.30

(i) On the grid below, draw a straight line graph of $\ln y$ against x [2]



(ii) Find the gradient of your straight line and hence express y in the form Ae^{bx}, where A and b are constants. [4]

(iii) By drawing a suitable straight line on your graph, solve the equation $Ae^{bx} = e^{2-3x}$. [2]

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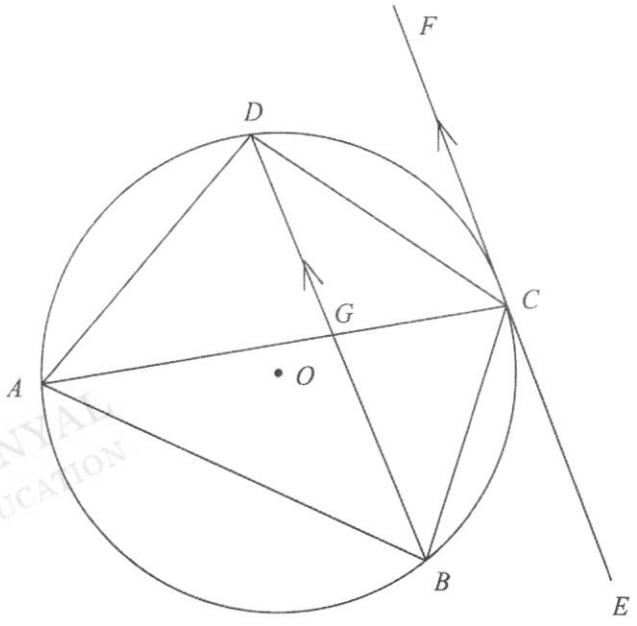


In the diagram, EF is tangent to the circle at C. O is the centre of the circle and DB is parallel to EC. Show that

(i) Triangle GBC is similar to triangle GAD. [2]

(ii) $GB \times AD = GA \times BC$. [1]

(iii) Triangle DBC is isosceles. [2]

(iv) $\angle DAB = 2$ times of $\angle DBC$. [3]

Solution for 2023 P1

1 Express $\dfrac{4}{x^2(x+2)}$ in partial fractions. [4]

$\dfrac{4}{x^2(x+2)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+2}$ $4 = Ax(x+2) + B(x+2) + C(x^2)$ *Subst.* $x = 0$, $4 = 2B$ $B = 2$ *Subst.* $x = -2$, $4 = 4C$ $C = 1$ *Subst.* $x = 1$, $4 = 3A + 6 + 1$ $A = -1$ $\dfrac{4}{x^2(x-1)(x-2)} = \dfrac{-1}{x} + \dfrac{2}{x^2} + \dfrac{1}{x+2}$	

2 (a) Solve the cubic equation $2x^3 - 17x^2 + 47x - 42 = 0$. [4]

Let f(x) $= 2x^3 - 17x^2 + 47x - 42$ f(2)=0 $(x-2)$ is a factor of f(x). $2x^3 - 17x^2 + 47x - 42 = (x-2)(ax^2 + bx + c)$ $2x^3 - 17x^2 + 47x - 42 = (x-2)(2x^2 - 13x + 21)$ $2x^3 - 17x^2 + 47x - 42 = 0$ $(x-2)(2x^2 - 13x + 21) = 0$ $(x-2)(2x-7)(x-3) = 0$ $x - 2 = 0$ or $2x - 7 = 0$ or $x - 3 = 0$ $x = 2$ or $x = \dfrac{7}{2}$ or $x = 3$	

(b) Hence solve the equation $16w^3 - 68w^2 + 94w - 42 = 0$. [2]

$16w^3 - 68w^2 + 94w - 42 = 0$ $2(2w)^3 - 17(2w)^2 + 47(2w) - 42 = 0$ From (a), $x = 2$ or $x = \frac{7}{2}$ or $x = 3$ $\therefore 2w = 2$ or $2w = \frac{7}{2}$ or $2w = 3$ $w = 1$ or $w = \frac{7}{4}$ or $w = \frac{3}{2}$	

3 Given that $\frac{1}{1-\sqrt{2}} \times \left(1+3\sqrt{2}\right)^2 = a + b\sqrt{2}$, find the value of a and of b. [4]

$\frac{1}{1-\sqrt{2}} \times \left(1+3\sqrt{2}\right)^2 = a + b\sqrt{2}$ LHS $= \left(\frac{1}{1-\sqrt{2}}\right)\left(19+6\sqrt{2}\right)$ $= \frac{19+6\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$ $= \frac{19+19\sqrt{2}+6\sqrt{2}+12}{1-2}$ $= -\left(31+25\sqrt{2}\right)$ $a = -31$, $b = -25$	

4 The graph of a cubic polynomial $f(x) = x^3 + px^2 + \left(p^2 + p - 7\right)x - 2p^2 - 6p + 6$ meets the x-axis only at 2.

(a) Show that $x - 2$ is a factor of $f(x)$. [2]

$f(x) = x^3 + px^2 + \left(p^2 + p - 7\right)x - 2p^2 - 6p + 6$ $f(2) = 2^3 + p(2)^2 + \left(p^2 + p - 7\right)(2) - 2p^2 - 6p + 6$ $f(2) = 8 + 4p + 2p^2 + 2p - 14 - 2p^2 - 6p + 6$ $f(2) = 0$ $\therefore (x-2)$ is a factor of $f(x)$	

(b) Given that $f(x)=(x-2)\left[x^2+(p+2)x+\left(p^2+3p-3\right)\right]$, find the range of values of p for f(x) to have only 1 solution. [3]

For $x^2+(p+2)x+\left(p^2+3p-3\right)=0$, $a=1, b=p+2, c=p^2+3p-3$, Discriminant <0 $(p+2)^2-4(1)\left(p^2+3p-3\right)<0$ $p^2+4p+4-4p^2-12p+12<0$ $-3p^2-8p+16<0$ $3p^2+8p-16>0$ $(3p-4)(p+4)>0$ $p<-4$ or $p>\frac{4}{3}$	

5 (a) Find the set of values of the constant k for which the line $y+k^2=2x$ meets the curve $y=x^2+4x-3$. [5]

$x^2+4x-3=2x-k^2$ $x^2+2x+k^2-3=0$ Discriminant ≥ 0 $(2)^2-4(1)\left(k^2-3\right)\geq 0$ $4-4k^2+12\geq 0$ $4k^2-16\leq 0$ $k^2-4\leq 0$ $(k-2)(k+2)\leq 0$ $-2\leq k\leq 2$	

(b) Hence, write down the value(s) of k for which the line $y+k^2=2x$ is a tangent to the graph $y=x^2+4x-3$. [1]

$k=-2$ or 2	

6 A polynomial has a remainder of 3 when divided by $x-1$ and a remainder of 5 when divided by $x-3$.

Find the remainder when the polynomial is divided by $(x-1)(x-3)$. [4]

Since degree of $(x-1)(x-3)$ is 2, Remainder of $f(x)$ is $R=ax+b$ f(x)=Q(x)$(x-1)(x-3)+ax+b$ *When* $x=1, R=3=f(1)$ f(1) $=a(1)+b$ $3=a+b$ $-Eqn1$ *When* $x=3, R=5=f(5)$ f(3)=$3a+b$ $5=3a+b-Eqn2$ $a=1, b=2$ Remainder $=x+2$	

7 **(i)** Given that $y=\dfrac{e^{x-3}}{2\left(e^{1-x}\right)}$, find $\dfrac{dy}{dx}$. [2]

$y=\dfrac{e^{x-3}}{2\left(e^{1-x}\right)}$ $=\dfrac{1}{2}e^{x-3-(1-x)}$ $=\dfrac{1}{2}e^{2x-4}$ $\dfrac{dy}{dx}=\dfrac{1}{2}e^{2x-4}(2)$ $=e^{2x-4}$	

(ii) Hence, calculate the exact rate of change of x at the instant when $x=3$ and y is decreasing at the rate of e^3 units per second. [2]

Given $\frac{dy}{dt}=-e^3$ units/s $\left.\frac{dy}{dx}\right\|_{x=3}=e^2$ $\frac{dy}{dt}=\frac{dy}{dx}\cdot\frac{dx}{dt}$ $\frac{dx}{dt}=-e^3\div\left(e^2\right)$ $=-e$ units/s	

8 It is given that $y=\frac{x^2}{\ln x}$ where $x>1$. Find the range of values of x for which $y=\frac{x^2}{\ln x}$ is a decreasing function. [4]

$\frac{dy}{dx}=\frac{(\ln x)(2x)-x^2\left(\frac{1}{x}\right)}{(\ln x)^2}$ $=\frac{2x\ln x-x}{(\ln x)^2}$ For decreasing function, $\frac{dy}{dx}<0$ $\frac{2x\ln x-x}{(\ln x)^2}<0$ $x(2\ln x-1)<0$ $\ln x<\frac{1}{2}$ $x<e^{\frac{1}{2}}$ Hence, range of values of x for $y=\frac{x^2}{\ln x}$ is decreasing is $1<x<\sqrt{e}$.	

9 **(a)** Given that $y = \ln\left(\cos^2 x\right)$, show that $\dfrac{d^2y}{dx^2} = -2\sec^2 x$. [2]

$\dfrac{dy}{dx} = \dfrac{1}{\cos^2 x}\left(-2\cos x \sin x\right)$ $= -2\tan x$ $\dfrac{d^2y}{dx^2} = -2\sec^2 x$	

(b) The equation of a curve is $y = \sqrt{x}\cos x$, find the gradient of the curve at $x = \pi$ radians in terms of π. [3]

$\dfrac{dy}{dx} = \sqrt{x}\left(-\sin x\right) + \left(\cos x\right)\left(\dfrac{1}{2}\right)x^{-\frac{1}{2}}$ $= \dfrac{1}{2\sqrt{x}}\cos x - \sqrt{x}\sin x$ At $x = \pi$ radians, $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{\pi}}\cos\pi - \sqrt{\pi}\sin\pi$ $= \dfrac{-1}{2\sqrt{\pi}}$ *or* $\dfrac{-\sqrt{\pi}}{2\pi}$	

10 Find the value of the constant k for which $y = \dfrac{x^3}{e^x}$ is a solution for the equation.

$$e^x\left(y + \frac{dy}{dx}\right) = 2kx^2.$$ [5]

$$\frac{dy}{dx} = \frac{e^x\left(3x^2\right) - x^3e^x}{e^{2x}}$$

$$= \frac{3x^2 - x^3}{e^x}$$

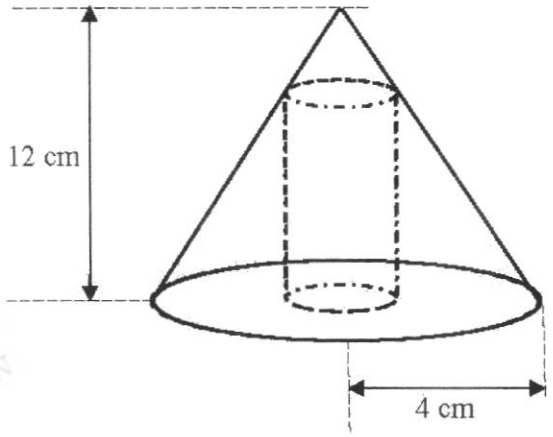
$$e^x\left(y + \frac{dy}{dx}\right) = 2kx$$

$$e^x\left[\frac{x^3}{e^x} + \left(\frac{3x^2 - x^3}{e^x}\right)\right] = 2kx^2$$

$$3x^2 = 2kx^2$$

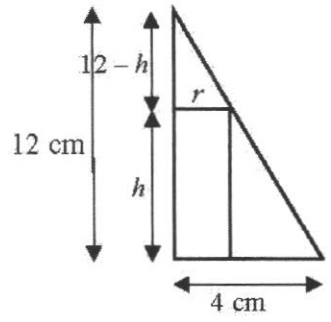
$$k = \frac{3}{2} \text{ o.e.}$$

11 The diagram shows a cylinder placed inside a circular cone of radius 4 cm and height 12 cm so that its base is level with the base of the cone.



(i) If the radius of the cylinder is r cm, show that its height h cm is given by $h = 12 - 3r$. [1]

(i)



Considering the similar triangles:

$$\frac{12-h}{r} = \frac{12}{4}$$

$$12 - h = 3r$$

$$h = 12 - 3r$$

(ii) Given that r can vary, find the maximum volume of the cylinder in terms of π.[4]

(ii)

Volume of cylinder, $V = \pi r^2 h$

$$= \pi r^2 (12 - 3r)$$

$$V = 12\pi r^2 - 3\pi r^3$$

$$\frac{dV}{dr} = 24\pi r - 9\pi r^2$$

At stationary point, $\frac{dV}{dr} = 0$

$$24\pi r - 9\pi r^2 = 0$$

$$3\pi r(8 - 3r) = 0$$

$r = 0$ (rejected) or $r = \frac{8}{3}$

$$\frac{d^2V}{dr^2} = 24\pi - 18\pi r$$

$$\left.\frac{d^2V}{dr^2}\right|_{r=\frac{8}{3}} = 24\pi - 18\pi\left(\frac{8}{3}\right) < 0$$

Maximum volume $= 12\pi\left(\frac{8}{3}\right)^2 - 3\pi\left(\frac{8}{3}\right)^3$

$$= \frac{256\pi}{3} - \frac{512\pi}{9}$$

$$= \frac{256\pi}{9} \text{ cm}^2$$

12 **(i)** Given that the coefficient of the third term in the binomial expansion of $\left(x-\frac{1}{3x^2}\right)^n$ is $\frac{22}{3}$, show that $n=12$. [3]

$T_3 = T_{2+1}$ $= \binom{n}{2}(x)^{n-2}\left(-\frac{1}{3x^2}\right)^2$ $= \frac{n(n-1)}{2}\left(-\frac{1}{3}\right)^2 (x)^{n-2-4}$ $= \frac{n(n-1)}{2}\left(-\frac{1}{3}\right)^2 (x)^{n-6}$ $\frac{n(n-1)}{2}\left(-\frac{1}{3}\right)^2 = \frac{22}{3}$ $\frac{n(n-1)}{18} = \frac{22}{3}$ $n^2 - n - 132 = 0$ $(n-12)(n+11) = 0$ $n = 12 \quad or \quad -11$ $n = 12\,(shown)$	

(ii) Hence, explain, showing your working clearly, why there are no constant terms in the expansion of $\left(x^3+\frac{8}{15}\right)\left(x-\frac{1}{3x^2}\right)^n$. [5]

(ii)

$$T_{r+1}=\binom{12}{r}(x)^{12-r}\left(-\frac{1}{3x^2}\right)^r$$

$$=\binom{12}{r}\left(-\frac{1}{3}\right)^r(x)^{12-3r}$$

For term in x^{-3},

$12-3r=-3$

$3r=15$

$r=5$

$$T_{5+1}=\binom{12}{5}\left(-\frac{1}{3}\right)^5(x)^{-3}$$

$$=-\frac{88}{27}x^{-3}$$

For term in x^0,

$12-3r=0$

$3r=12$

$r=4$

$$T_{4+1}=\binom{12}{4}\left(-\frac{1}{3}\right)^4(x)^0$$

$$=\frac{55}{9}$$

For independent term in $\left(x^3+\frac{8}{15}\right)\left(x-\frac{1}{3x^2}\right)^n$,

$$\left(x^3\right)\left(-\frac{88}{27}x^{-3}\right)+\left(\frac{8}{15}\right)\left(\frac{55}{9}\right)=-\frac{88}{27}+\frac{88}{27}$$

$$=0$$

Therefore, there are no constant terms.

13 **(i)** Express $\log_6 2^{3n+1} + \log_6 3^{4n+1} - \log_6 3^n$ in the form $an+b$,

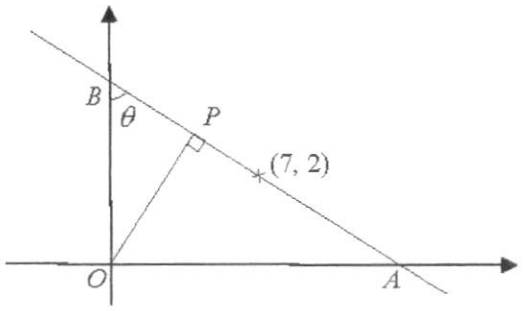
a and b are integers. [3]

$$\log_6 2^{3n+1} + \log_6 3^{4n+1} - \log_6 3^n = \log_6 \left(\frac{2^{3n+1} 3^{4n+1}}{3^n} \right)$$
$$= \log_6 \left(2^{3n+1} 3^{3n+1} \right)$$
$$= \log_6 6^{3n+1}$$
$$= 3n+1$$

(ii) Solve the equation $\log_{\frac{1}{3}} \sqrt{2x-1} = \log_9 (x+5) + \frac{1}{2}$. [4]

$$\log_{\frac{1}{3}} \sqrt{2x-1} = \log_9 (x+5) + \frac{1}{2}$$
$$\frac{1}{2} \left(\frac{\log_3 (2x-1)}{\log_3 3^{-1}} \right) = \frac{\log_3 (x+5)}{\log_3 3^2} + \frac{1}{2}$$
$$-\frac{1}{2} \log_3 (2x-1) = \frac{1}{2} \log_3 (x+5) + \frac{1}{2}$$
$$-\log_3 (2x-1) = \log_3 (x+5) + \log_3 3$$
$$\log_3 (2x-1)^{-1} = \log_3 3(x+5)$$
$$\frac{1}{2x-1} = 3(x+5)$$
$$1 = 3(x+5)(2x-1)$$
$$1 = 6x^2 + 27x - 15$$
$$6x^2 + 27x - 16 = 0$$
$$x = \frac{-27 \pm \sqrt{27^2 - 4(6)(-16)}}{12}$$
$$= \frac{-27 \pm \sqrt{1113}}{12}$$
$$= -5.03014 \quad or \quad 0.53014$$
$$= 0.530$$

14 In the diagram below, a line passes through the x-axis at point A and y-axis at point B. The point (7, 2) lies on the line.



(i) Given that OP is perpendicular to AB, and $\angle ABO = \theta$, find OA in the form $a + b\tan\theta$, where a and b are integers. [2]

(i)	$OA = 7 + \dfrac{2}{\tan(90° - \theta)}$ *or* $7 + 2\tan\theta$	

(ii) Show that $OP = 7\cos\theta + 2\sin\theta$. [2]

(ii)	$OP = OA\sin(90° - \theta)$ $= OA\cos\theta$ $OP = \left(7 + \dfrac{2}{\tan(90° - \theta)}\right)\cos\theta$ $= (7 + \dfrac{2\cos(90° - \theta)}{\sin(90° - \theta)})\cos\theta$ $= 7\cos\theta + \dfrac{2\sin\theta}{\cos\theta}(\cos\theta)$ $= 7\cos\theta + 2\sin\theta$	

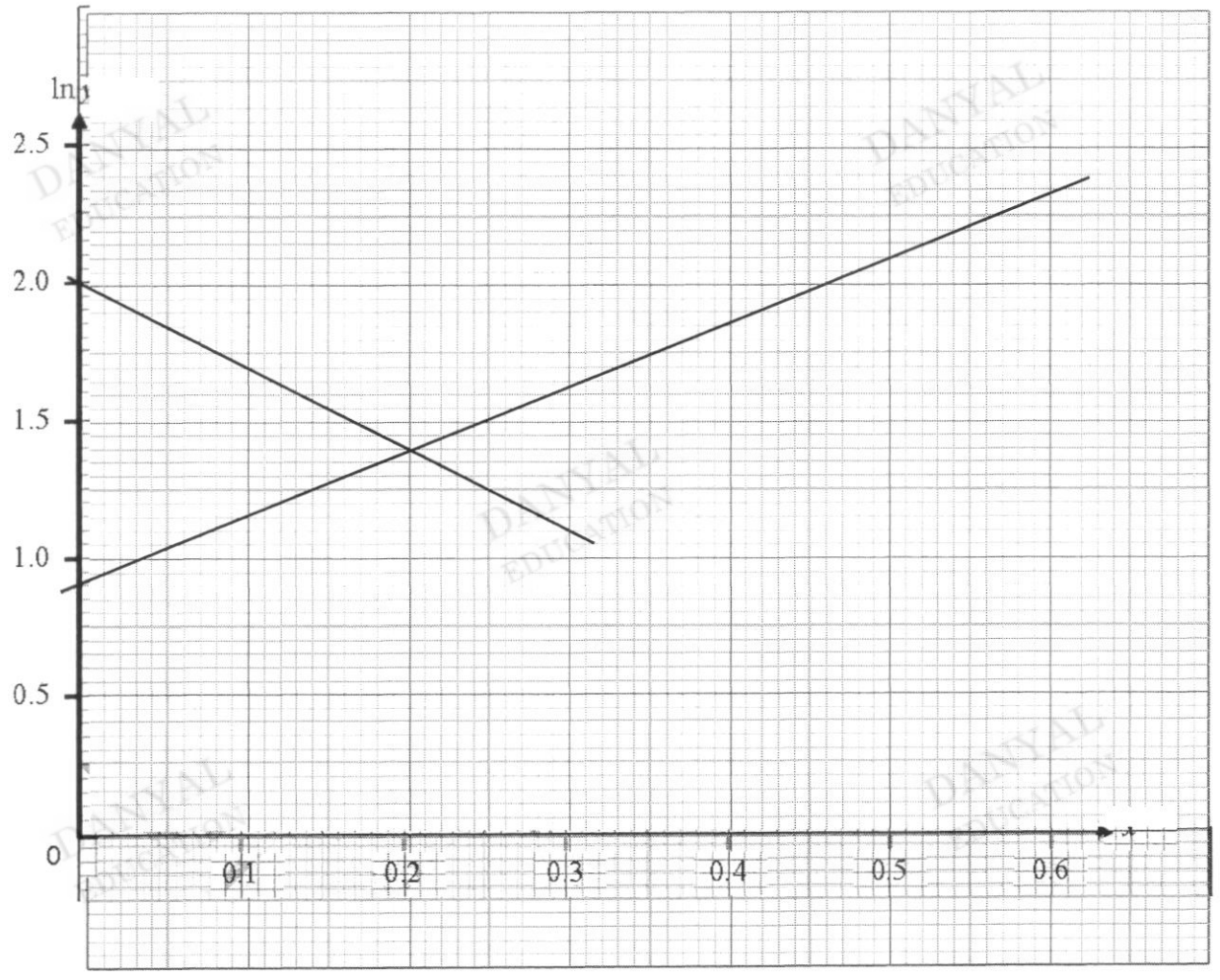
(iii) If $OP = 6$ units, determine the value of θ. [3]

(iii)	$R = \sqrt{49+4}$ $= \sqrt{53}$ $\alpha = \tan^{-1}(\frac{2}{7}) = 15.945°$ $7\cos\theta + 2\sin\theta = \sqrt{53}\cos(\theta - 15.945°)$ $\sqrt{53}\cos(\theta - 15.945°) = 6$ $\cos(\theta - 15.945°) = \frac{6}{\sqrt{53}}$ $\alpha = 34.496°$ $\theta - 15.945° = 34.496°$ $\theta = 50.441$ $= 50.4°$	

15 The table below shows the experimental values of two variables x and y.

x	0.1	0.2	0.3	0.4	0.5	0.6
y	3.16	3.98	5.01	6.31	7.94	10.0
$\ln y$	1.15	1.38	1.61	1.84	2.07	2.30

(i) On the grid below, draw a straight line graph of $\ln y$ against x [2]



(ii) Find the gradient of your straight line and hence express y in the form Ae^{bx}, where A and b are constants. [4]

Gradient $= \dfrac{2.0-1.25}{0.47-0.15}$ $= 2.34375$ $= 2.34$ $\ln y = 2.34375x + 0.9$ $y = e^{2.34375x+0.9}$ $= e^{0.9}e^{2.34375x}$ $= 2.4596e^{2.34375x}$ $y = 2.46e^{2.34x}$	

(iii) By drawing a suitable straight line on your graph, solve the equation $Ae^{bx} = e^{2-3x}$. [2]

(iii) $y = e^{2-3x}$ $\ln y = -3x + 2$ From the graph, $x = 0.205$	

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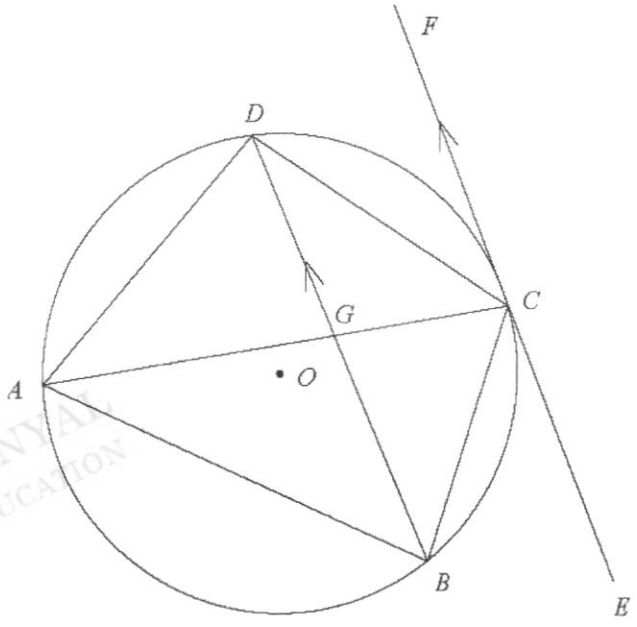


In the diagram, EF is tangent to the circle at C. O is the centre of the circle and DB is parallel to EC. Show that

(i) Triangle GBC is similar to triangle GAD. [2]

$\angle BGC = \angle DGA$ (vertically opposite angle) $\angle GCB = \angle DAG$ (angle in the same segment) $\angle GBC = \angle GAD$ (angle in the same segment) By AAA or AA property, Triangle GBC is similar to triangle GAD.	

(ii) $GB \times AD = GA \times BC$. [1]

$\frac{GB}{GA} = \frac{BC}{AD}$ $GB \times AD = GA \times BC$ (shown)	

(iii) Triangle DBC is isosceles. [2]

$\angle DBC = \angle BCE$ (alternate angle, DB is parallel to EF) $\angle BDC = \angle BCE$ (alternate segment theorem) $\therefore$ Triangle DBC is isoceles.	

(iv) $\angle DAB = 2$ times of $\angle DBC$. [3]

$\angle DAC = \angle DBC$ (angle in the same segment) $\angle CAB = \angle BDC$ (angle in the same segment) $= \angle DBC$ (isoceles triangle DBC) $\angle DAB = \angle DAC + \angle CAB$ $= \angle DBC + \angle DBC$ $= 2$ times of $\angle DBC$ *Alternatively* $\angle CAB = \angle BCE$ (alternate segment theorem) $= \angle DBC$ (alternate angle, BD is parallel to EF) $\angle DAB = \angle DAC + \angle CAB$ $= \angle DBC + \angle DBC$ $= 2$ times of $\angle DBC$	

MARKING SCHEME

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2023

S4

MATHEMATICS
Paper 2

4052/02
28 August 2023
2 hours 15 minutes

Candidates answer on the Question Paper and Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiners' Use

Questions	1	2	3	4	5	6	7
Marks							
Questions	8	9	10	11			
Marks							
Table of Penalties	**Units**					**90**	
	Clarity/Logic						
	Accuracy/Precision						

Parent's Name and Signature:

Date:

This document consists of **21** printed pages and **1** blank page.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** Rearrange the formula $a = \dfrac{b^2+5}{3-b^2}+2$ to make b the subject.

1(a)	$a = \dfrac{b^2+5}{3-b^2}+2$ $3a - ab^2 = b^2+5+6-2b^2$ $b^2 - ab^2 = 11-3a$ $b^2(1-a) = 11-3a$ $b^2 = \dfrac{11-3a}{1-a}$ $b = \pm\sqrt{\dfrac{11-3a}{1-a}}$	M1 – remove fraction M1 – grp terms with 'b^2' on LHS A1

(b) Express as a single fraction in its simplest form $\dfrac{4}{2x-1}+\dfrac{2}{2x^2-3x+1}$.

1(b)	$\dfrac{4}{2x-1}+\dfrac{2}{2x^2-3x+1}$ $=\dfrac{4}{2x-1}+\dfrac{2}{(2x-1)(x-1)}$ $=\dfrac{4(x-1)+2}{(2x-1)(x-1)}$ $=\dfrac{4x-4+2}{(2x-1)(x-1)}$ $=\dfrac{4x-2}{(2x-1)(x-1)}$ $=\dfrac{2(2x-1)}{(2x-1)(x-1)}$ $=\dfrac{2}{x-1}$	M1 – factorise denominator M1 – express as a single fraction A1

(c) Solve $\dfrac{10x+7}{4}-\dfrac{2x^2-9}{x+2}=7$.

1(c)	$\frac{10x+7}{4}-\frac{2x^2-9}{x+2}=7$ $(10x+7)(x+2)-4(2x^2-9)=28(x+2)$ $10x^2+27x+14-8x^2+36=28x+56$ $2x^2-x-6=0$ $(2x+3)(x-2)=0$ $x=-\frac{3}{2}$ or 2	M1 – remove/combine fraction to get quad. eqn. M1 – factorising A1

2 (a) Myron puts his \$30000 bonus into a fixed deposit at 3.35% per annum. The interest is compounded daily. There are 365 days in a year.

Calculate the amount of interest he will have at the end of two years.

2(a)	Total amount at the end of two years $=\$30000\left(1+\frac{3.35\%}{365}\right)^{365(2)}$ $=\$30000\left(1+\frac{335}{36500}\right)^{730}$ $\approx \$32078.7657$ Amount of interest he will have at the end of two years $=\$32078.7657-\$30000$ $=\$2078.77$ (nearest cent)	M1 for using compound interest formula A1 – [(penalize under A/P) if not rounded off to 2d.p.]

(b) The price of a car is \$160000.
Myron buys this car on hire purchase.
He makes a 30% down payment and takes up a 7-year loan tenure at a simple interest rate of 2.68% per annum.

Calculate his monthly instalment for the car.

2(b)	Remaining amount $=0.7\times\$160000$ $=\$112000$ Simple interest $=\frac{PRT}{100}$ $=\frac{\$112000\times 2.68\times 7}{100}$ $=\$21011.20$	A1 – correct interest M1

	His monthly instalment for the car $= \frac{\$112000 + \$21011.20}{84}$ $= \$1583.47$ (nearest cent)	A1 – [(A/P) if not rounded off to 2d.p.]

(c) Myron pays monthly rent of $3600.
This is a 25% increment from his monthly rent last year.
Calculate his monthly rent last year.

2(c)	His monthly rent last year $= \frac{S3600}{1.25}$ $= S2880$	B1

(d) The exchange rate between Singapore dollars (S$) and Australian dollars (A$) is S$1 = A$1.12.

Myron spends A$980 in Sydney with his credit card.

All foreign currency transactions are subjected to a 1% conversion fee imposed by the respective card associations.
This foreign transaction amount, converted to S$, is further subjected to a 2.25% bank administrative fee.

Calculate the total amount on Myron's credit card bill, inclusive of the fees.
Give your answer in S$, correct to the nearest cent.

2(d)	Myron's spending in Singapore dollars $= A\$980\left(\frac{S\$1}{A\$1.12}\right)$ $= S\$875$ Myron's spending with conversion fee $= 1.01 \times S\$875$ $= S\$883.75$ Myron's total credit card bill $= \frac{102.25}{100} \times S\$883.75$ $= S\$903.63$ (nearest cent)	M1 – correct conversion M1 A1 – [(penalizsd A/P) if not rounded off t 2d.p.]

3 The test marks of a group of 30 students were recorded. The marks are shown in the stem-and-leaf diagram.

Stem	Leaf
0	4 4 4
1	8 8 9 9 9 9 9 9
2	2 2 2 3 x 4 4 4 5 5 7 7 7 7 7 7 7 7

Key 1 | 9 means 19 marks

(i) Find the percentage of students who score less than 20 marks.

3(i)	% of students who score < 20 marks $=\frac{11}{30}(100\%)$ $=36\frac{2}{3}\%$	B1

(ii) Given that the median is 23.5 marks, find the value of x.

3(ii)	$x=4$	B1

(iii) Find the standard deviation of the marks.

3(iii)	Standard Deviation ≈ 6.5946 marks ≈ 6.59 marks	B1

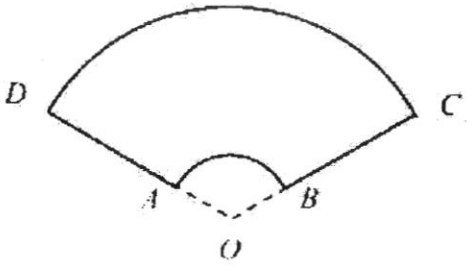
(iv) It was found that the marks were recorded wrongly. Every student was short of two marks.
Write down the correct values for the median and standard deviation.

3(iv)	Median = 25.5 marks Standard deviation ≈ 6.59 marks	B1 B1 - [or same wrong value as in (ii)]

(v) Explain whether mean or median is a better measure of central tendency.

3(v)	The median is a better measure as the three test marks of 4 are **outliers / extreme values**.	B1

4



The diagram shows a mirror $ABCD$.
AB and DC are arcs of circles centre O with radii 15 cm and 45 cm respectively.
The perimeter of the mirror is 286 cm.

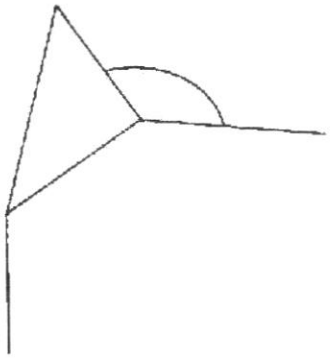
(a) Calculate the exact value of angle AOB in radians.

4(a)	Given Perimeter = 286 cm Arcs length = $296 - 2(30) = 226$ cm $\frac{A\hat{O}B}{2\pi}\left[2\pi(15)+2\pi(45)\right]=226$ $A\hat{O}B(15+45)=226$ $A\hat{O}B=\frac{113}{30}=3\frac{23}{30}$ radians	M1 A1

(b) Calculate the area of the mirror.

4(b)	Area of the mirror $=\frac{A\hat{O}B}{2\pi}\left[\pi(45)^2-\pi(15)^2\right]$ $=\frac{A\hat{O}B}{2}\left(45^2-15^2\right)$ $=\frac{113}{2(30)}(30)(60)$ $=113(30)$ $=3390\ \text{cm}^2$	M1 A1

5 The diagram shows part of a regular polygon $OPQR$ attached to a triangle WPQ.
The exterior angle of the regular polygon is $40°$.



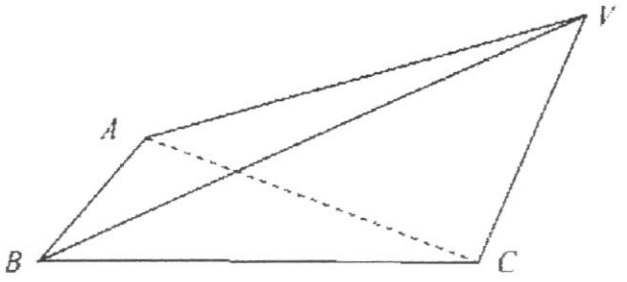
P
Q
R
W
O
a
b
c

Calculate the sum of the angles a, b and c.

5	Int. angle + Ext. angle = 180° (Adj. angles on a str. line) Int. angle + 40° = 180° Int. angle = 180° − 40° = 140°	B1
	At P, Q and W, Sum of angles at a point = 360° × 3 = 1080° $\angle QPW + \angle PWQ + \angle WQP = 180°$ (Angle sum of a triangle) $a + b + c = 1080° - \angle OPQ - \angle PQR - (\angle QPW + \angle PWQ + \angle WQP)$ = 1080° − 140° − 140° − 180° = 620°	M1 A1

6 The figure shows a pyramid with a triangular base ABC with vertex V.

AB = 5 cm, AC = 8 cm, angle BAC = 87°, and the perpendicular height is 17 cm.



(a) Find the volume of the pyramid.

6(a)	Volume of pyramid $=\frac{1}{3}\left[\frac{1}{2}(5)(8)\sin 87°\right](17)$ ≈ 113.178 $\approx 113 \text{ cm}^3$	 M1 A1

(b) The pyramid is melted to form a solid right cone with height 20 cm.

(i) Find the radius of the cone.

6(b)(i)	Volume of cone = 113.178 $\frac{1}{3}\pi r^2 h = 113.178$ $r^2 = \frac{113.178\times 3}{20\pi}$ $r = \sqrt{\frac{113.178\times 3}{20\pi}} \quad (\because r>0)$ $r \approx 2.3246$ Radius of cone ≈ 2.32 cm	 M1 A1

(ii) Find the total surface area of the cone.

6(b)(ii)	Total surface area of cone $= \pi r^2 + \pi r l$ $= \pi(2.3246)^2 + \pi(2.3246)\left(\sqrt{20^2 + 2.3236^2}\right)$ ≈ 164.02 $\approx 164 \text{ cm}^2$	 M1 A1

(iii) A frustrum is cut away to make a conical paperweight with a volume of 36 cm^3. What is the height of the frustrum?

6(b)(iii)	$\dfrac{\text{Vol. of conical paperweight}}{\text{Vol. of whole cone}} = \left(\dfrac{\text{Height of paperweight}}{\text{Height of whole cone}}\right)^3$ $\sqrt[3]{\dfrac{36}{113.178}} = \dfrac{\text{Height of paperweight}}{20}$ Height of paperweight $= 20 \times \sqrt[3]{\dfrac{36}{113.178}}$ ≈ 13.652 cm	M1
	Therefore, height of the bottom to be removed $= 20 - 13.652$ ≈ 6.35 cm	M1 A1
	OR Let the height of the cone and paperweight be h_c cm and h_p cm respectively. Let the radius of the cone and paperweight be r_c cm and r_p cm respectively. $\dfrac{\text{Height of paperweight}}{\text{height of cone}} = \dfrac{\text{radius of paperweight}}{\text{radius of cone}}$ $\dfrac{h_p}{h_c} = \dfrac{r_p}{r_c}$ $\dfrac{h_p}{20} = \dfrac{r_p}{2.3246}$ $r_p = \dfrac{2.3246 h_p}{20}$ $r_p = 0.11623 h_p$	M1
	Volume of paperweight $= 36$ cm^2 $\dfrac{1}{3}\pi r_p^2 h_p = 36$ $\dfrac{1}{3}\pi (0.11623 h_p)^2 h_p = 36$ $h_p^3 = \dfrac{36 \times 3}{\pi \times 0.11623^2}$ $h_p \approx 13.653$ cm	M1
	Height of the cone removed $= 20 - 13.653$	A1

	≈ 6.35 cm	

7 **(a)** The position vector of point Y is $\begin{pmatrix}1\\6\end{pmatrix}$. X is the point (3, 1).

(i) Find the magnitude of vector XY.

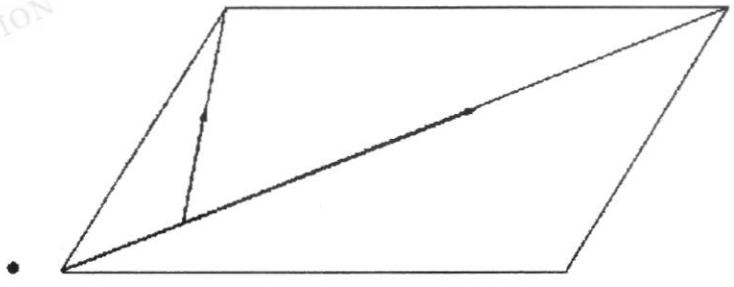
7(a)(i)	$\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY} = \begin{pmatrix}-3\\-1\end{pmatrix} + \begin{pmatrix}1\\6\end{pmatrix} = \begin{pmatrix}-2\\5\end{pmatrix}$ $\left\|\overrightarrow{XY}\right\| = \sqrt{(-2)^2 + 5^2} = \sqrt{29} = 5.39$ unit (3s.f)	M1 – find $\overrightarrow{XY}$ A1

(ii) Z is the point on XY produced with coordinates $(m, 9)$.

Find the value of m.

7(a)(ii)	$\overrightarrow{XZ} = k\overrightarrow{XY}$ $\overrightarrow{XO} + \overrightarrow{OZ} = k\overrightarrow{XY}$ $\begin{pmatrix}-3\\-1\end{pmatrix} + \begin{pmatrix}m\\9\end{pmatrix} = k\begin{pmatrix}-2\\5\end{pmatrix}$ $\begin{pmatrix}m-3\\8\end{pmatrix} = \begin{pmatrix}-2k\\5k\end{pmatrix}$ $k = 1.6$ $m - 3 = -3.2$ $m = -0.2$ or $\overrightarrow{YZ} = k\overrightarrow{XY}$ $\overrightarrow{YO} + \overrightarrow{OZ} = k\overrightarrow{XY}$ $\begin{pmatrix}-1\\-6\end{pmatrix} + \begin{pmatrix}m\\9\end{pmatrix} = k\begin{pmatrix}-2\\5\end{pmatrix}$ $\begin{pmatrix}m-1\\3\end{pmatrix} = \begin{pmatrix}-2k\\5k\end{pmatrix}$ $k = 0.6$ $m - 1 = -1.2$ $m = -0.2$	M1 – find $\overrightarrow{XZ}$ or $\overrightarrow{YZ}$ M1 – find k A1

(b) $ABCE$ is a parallelogram. AOC is the diagonal of the parallelogram. The position vectors of C and E, relative to O, are $5\boldsymbol{b}$ and $\boldsymbol{a}$ respectively.



C

E

F

Given $\overrightarrow{AC} = 6\overrightarrow{AO}$ and $AF : FB = 1 : 4$.

(i) Express each of the following in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(a) $\overrightarrow{AO}$,

7(b)(i)(a)	$\overrightarrow{AO} = \frac{1}{5}\overrightarrow{OC} = \boldsymbol{b}$	B1

(b) $\overrightarrow{EC}$,

7(b)(i)(b)	$\overrightarrow{EC} = \overrightarrow{EO} + \overrightarrow{OC} = -\boldsymbol{a} + 5\boldsymbol{b} = 5\boldsymbol{b} - \boldsymbol{a}$	B1

(c) $\overrightarrow{AF}$,

7(b)(i)(c)	$\overrightarrow{AF} = \frac{1}{5}\overrightarrow{AB} = \frac{1}{5}\overrightarrow{EC} = \frac{1}{5}(5\boldsymbol{b} - \boldsymbol{a})$	B1

(d) position vector of F.

7(b)(i)(d)	$\overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{AF} = -\boldsymbol{b} + \frac{1}{5}(5\boldsymbol{b} - \boldsymbol{a}) = -\frac{1}{5}\boldsymbol{a}$	B1

(ii) Write down one fact about O, E and F.

7(b)(ii)	Possible answer: a) The points are collinear. b) $OF : OE = 1 : 5$ c) $\overrightarrow{FO} = \frac{1}{5}\overrightarrow{OE}$ or $\overrightarrow{OF} = -\frac{1}{5}\overrightarrow{OE}$	B1

(iii) Find the value of $\dfrac{\text{Area of triangle } OFA}{\text{Area of triangle } CBA}$.

7(b)(iii)	$\dfrac{\text{Area of } \Delta OFA}{\text{Area of } ACBA} = \dfrac{\frac{1}{2}(AO)(AF)\sin\angle OAF}{\frac{1}{2}(AC)(AB)\sin\angle CAB}$ $= \dfrac{(AO)(AF)}{(AC)(AB)} \quad (\because \angle OAF = \angle CAB)$ $= \left(\dfrac{1}{6}\right)\left(\dfrac{1}{5}\right) = \dfrac{1}{30}$	M1 A1

8 A, B, C and D are four locations on level ground.

A is 1.2 km due north of C. B is at a bearing of $046°$ from D and 1.5 km away from D.

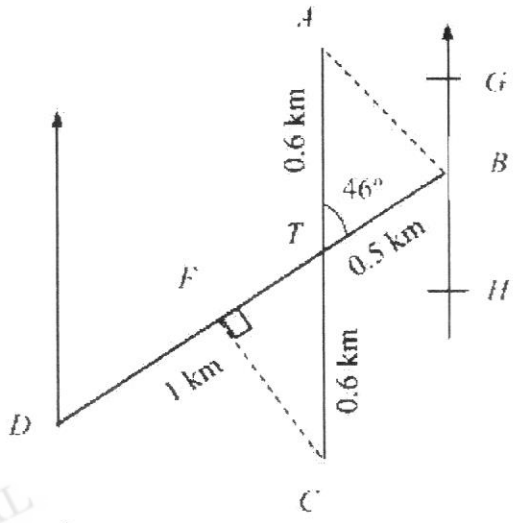
Two straight pathways AC and BD intersect at T,
where T is the mid-point of AC and $BT : TD = 1 : 2$.

(i) Calculate the distance between A and B.

8(i)	$AT = 0.6\text{km}$, $BT = 0.5\text{km}$ $\angle ATB = 46°$ (corr. $\angle$s betw. // lines) Using Cosine Rule, $(AB)^2 = (0.6)^2 + (0.5)^2 - 2(0.6)(0.5)\cos 46°$ $AB = 0.43955 \text{ km}$ (5s.f.) $(\because AB > 0)$ $= 0.440\text{km}$ (3s.f)	M1 – find $\angle ATB$ M1 – using Cosine Rule A1

(ii) Calculate the bearing of A from B.

8(ii)		

		
	Method 1: Using Sine Rule, $\frac{0.43955}{\sin 46°} = \frac{0.5}{\sin \angle BAT}$ $\sin \angle BAT = \frac{0.5 \sin 46°}{0.43955}$ $\angle BAT = \sin^{-1}\left(\frac{0.5 \sin 46°}{0.43955}\right) = 54.9°$ (1 d.p) $\angle GBA = \angle BAT$ (alt $\angle$s, $AT // BG$) $\therefore$ Bearing of A from B $= 360° - 054.9°$ ($\angle s$ at a pt.) $= 305.1°$(1d.p)	M1 – using Sine Rule M1A1
	Method 2: Using Sine Rule, $\frac{0.6}{\sin \angle ABT} = \frac{0.43955}{\sin 46°}$ $\sin \angle ABT = \frac{0.6 \sin 46°}{0.43955}$ $\angle ABT = \sin^{-1}\left(\frac{0.6 \sin 46°}{0.43955}\right) = 79.1°$ (1 d.p) $\angle TBH - \angle ATB$ (alt $\angle$s, $AT // GH$) $\therefore$ Bearing of A from B $= 180° + 046° + 079.1° = 305.1°$ (1d.p)	M1 – using Sine Rule M1A1

(iii) Ian walked from B to D. Calculate the shortest distance between Ian and C.

8(iii)	Let the shortest distance be CF. $\sin 46° = \frac{CF}{0.6}$ $\therefore CF = 0.6 \times \sin 46°$ $= 0.432\text{km}$ (3s.f)	M1 A1

(iv) A vertical tower of height 70 m stands at T.
Calculate the angle of depression measured from the top of the tower to D.

8(iv)	Let θ be the angle of depression. $\tan\theta = \frac{\text{Tower Height}}{DT} = \frac{70\text{ m}}{1000\text{ m}} = \frac{7}{100}$ $\therefore \theta - \tan^{-1}\left(\frac{7}{100}\right) = 4.0°$ (1d.p)	M1 A1
	Note: Penalise students if (1) no/inconsistent reasons given for angles [Clarity/Logic] (2) bearings not represented in 3 digits [Accuracy/Precision]	

9 Mr Lim and Mr Chan go running at MacRitchie Reservoir. Both men start from the same spot but Mr Chan being the slower runner, starts running at 0730 and Mr Lim starts at 0735. The route is 6.2 km long.

(i) If Mr Chan runs at an average speed of x km/h, write down an expression for the time taken, in hours, for the run.

9(i)	Time taken by Mr Chan $= \frac{6.2}{x}$ hours	B1

(ii) Mr Lim runs the same route 3 km/h faster than Mr Chan and completes it 9 minutes before Mr Chan. Form an equation in terms of x and show that it reduces to $7x^2 + 21x - 558 = 0$.

9(ii)	$\frac{6.2}{x}-\frac{6.2}{x+3}=\frac{5}{60}+\frac{9}{60}$ $\frac{6.2(x+3)-6.2x}{x(x+3)}=\frac{14}{60}$ $\frac{18.6}{x(x+3)}=\frac{14}{60}$ $18.6\times 60=14x(x+3)$ $14x^2+42x-1116=0$ $7x^2+21x-558=0$	M1 – combining the fractions / × by LCM of denominators M1 – form the quadratic equation AG

(iii) Solve the equation $7x^2+21x-558=0$.

9(iii)	$7x^2+21x-558=0$ $x=\frac{-21\pm\sqrt{21^2-4(7)(-558)}}{2(7)}$ $=7.5534$ or -10.5534 (4d.p.) $=7.55$ or -10.6 (3s.f.)	M1 A1

(iv) Find the time taken by Mr Lim to overtake Mr Chan, giving your answer to the nearest minute.

9(iv)	Let t hours be the time taken for Mr Lim to catch up with Mr Chan. Mr Lim's average speed $=7.5534+3=10.5534$ km/h Distance travelled by Mr Lim $=10.5534t$ km Mr Chan's average speed $=7.5534$ km/h Distance travelled by Mr Chan $=7.5543\left(t+\frac{5}{60}\right)$ km	M1 M1

	$10.5534t = 7.5543\left(t + \frac{5}{60}\right)$ $= 7.5543t + 0.62945$ $3t = 0.62945$ $t = 0.20982$ h $= 0.20982 \times 60 = 12.589 = 13$ minutes [FYI: The time when Mr Lim overtakes Mr Chan $= 0735 + 0013 = 0748$ hours]	A1

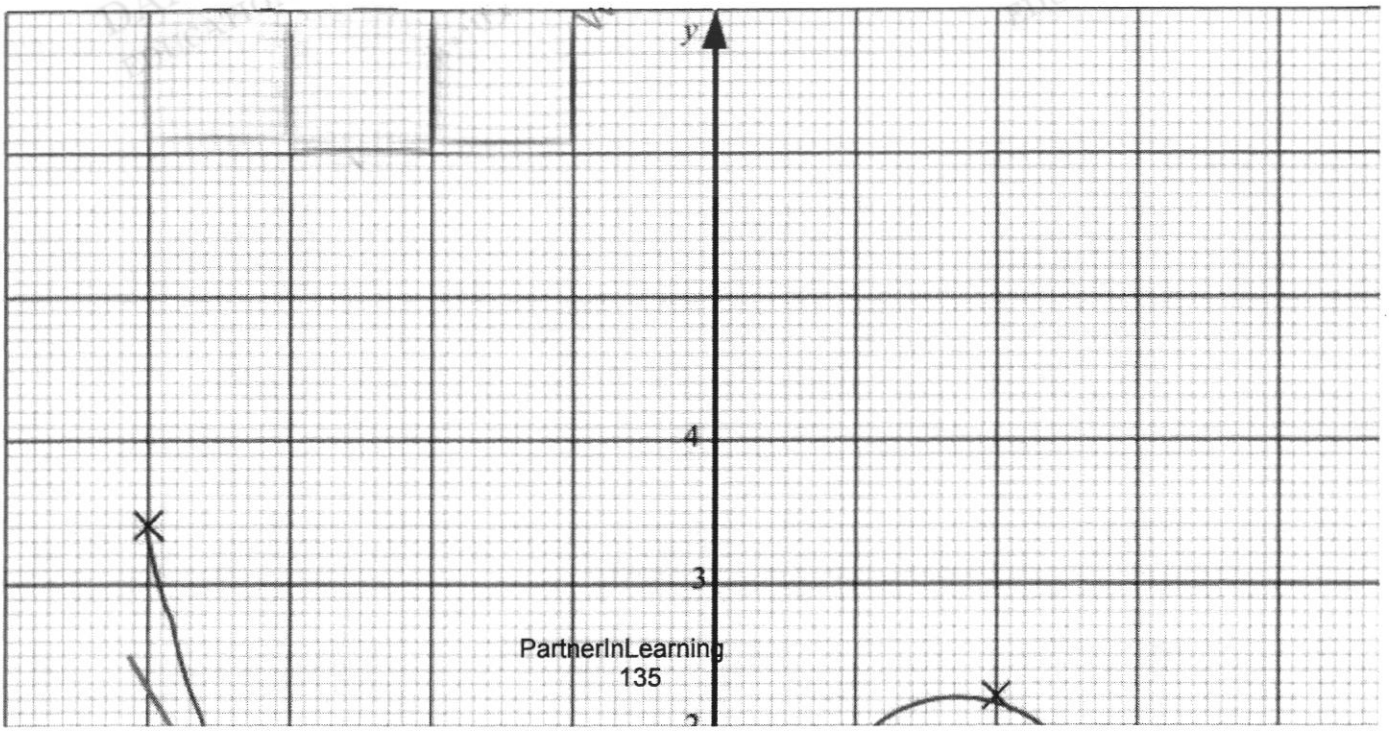
10 **(a)** Complete the table of values for $y = -\frac{x^3}{10} + x + 1$.

x	–4	–3	–2	–1	0	1	2	3	4
y	3.4	0.7		0.1		1.9	2.2	1.3	–1.4

10(a)		B1 – Both correct

x	–4	–3	–2	–1	0	1	2	3	4
y	3.4	0.7	**~~–0.2~~**	0.1	**~~1.0~~**	1.9	2.2	1.3	–1.4

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of $y = -\frac{x^3}{10} + x + 1$ for $-4 \le x \le 4$ on a sheet of graph paper.



(c) Use your graph to determine the number of solutions for the equation $\frac{x^3}{10} = x+1$ in the range $-4 \le x \le 4$.

10(c)	3 solutions	B1

(d) By drawing a tangent on your graph, find the gradient of the curve at the point $(-3, 0.7)$.

10(d)	From graph, Gradient $= \frac{2.3-(-2.8)}{-4-(-1)} = -1.7$	M1 – draw tangent A1 – accept ± 0.2

(e) By drawing a suitable straight line on your graph, solve the equation $x^3 - 9x = 0$ for $-4 \le x \le 4$.

10(e)	$x^3 - 9x = 0$ $-\frac{x^3}{10} + \frac{9x}{10} = 0$ $-\frac{x^3}{10} + x = \frac{x}{10}$ $-\frac{x^3}{10} + x + 1 = \frac{x}{10} + 1$ Draw $y = \frac{x}{10} + 1$ on the graph. The intersections of the line and the curve are the solutions. $x = -3.30$ or 0 or 3.30	M1A1 G1 – Str. line w eqn G1 - accept ± 0.2, except 0

11 Mr Chan owns a company that provides delivery services. He is reviewing the salaries of some of his staff and come up with a budget. Mr Chan finds the following information on the internet:

Table 1: Salaries for Drivers, Mechanics and Admin Executive

	Drivers	Vehicle Mechanic	Admin Executive
Monthly Salary Range	$2400 to $3000	$2500 to $4000	$3000 to $3500

Source: https://sg.indeed.com/career/salaries

In addition, both employer and employee must contribute a portion of the employee's monthly salary to the Central Provident Fund (CPF).

The following table summarises the current CPF contribution rates for Singaporeans and Singapore Permanent Residents (SPRs) across the different age groups.

Table 2: CPF Contribution Rates for Singaporeans and SPRs across different age groups

Employee's age (years)	Contribution Rates from 1 January 2023 (monthly wages > $750)		
	By employer (% of wage)	By employee (% of wage)	Total (% of wage)
55 and below	17	20	37
Above 55 to 60	14.5	15	29.5
Above 60 to 65	11	9.5	20.5
Above 65 to 70	8.5	7	15.5
Above 70	7.5	5	12.5

Source: https://www.cpf.gov.sg/employer/employer-obligations/how-much-cpf-contributions-to-pay

The following table gives the distribution of the different age groups according to the jobs.

Table 3: Distribution of Employees across age groups

Employee's age (years)	Drivers	Mechanics	Admin Executives
55 and below	13	4	1
Above 55 to 60	5	2	1
Above 60 to 65	0	0	0
Above 65 to 70	0	0	0
Above 70	0	0	0
Total	18	6	2

(a) A 35-year-old driver has a monthly salary of $2700.

Calculate the employee's CPF contribution, the employer's CPF contribution and the nett wage. The nett wage is an employee's earnings after all deductions are taken out which is also known as take-home pay.

11(a)	Employee's CPF contribution $= 2700 \times 0.2 = \$540$ Employer's CPF contribution $= 2700 \times 0.17 = \$459$ Employee's Nett Wage $= 2700 - 540 = \$2160$	B1 B1 B1

(b) Mr Chan estimates that the monthly budget for the wages and employer's CPF contributions of all the drivers, mechanics, and admin executives should not exceed $100,000.

Is this a good estimate?
Justify your decision with calculations and state your assumption(s) made, if any.

11(b)	<u>Using Maximum Salaries</u> Since we are checking the maximum budget, we can use the maximum salaries in each category of workers. We assume Mr Chan will use the maximum value and not go beyond. Vehicle Mechanics Employer's CPF Contribution for 55 and below $= 4000 \times 0.17 \times 4 = \$2720$ Salary for Vehicle Mechanics 55 and below $= 4000 \times 4 = \$16000$ Employer's CPF Contribution for 55 to 60 $= 4000 \times 0.145 \times 2 = \$1160$ Salary for Vehicle Mechanics 55 to 60 $= 4000 \times 2 = \$8000$ Total Employer's CPF $= 2720 + 1160$ $= \$3880$ Total Employer's Salary $= 16000 + 8000$ $= \$24000$ Total Employer's CPF and Salary $= 3880 + 24000$ $= \$27880$ Drivers	B1 M1 M1

	Employer's CPF Contribution for 55 and below $= 3000 \times 0.17 \times 13 = \$6630$ Salary for Drivers 55 and below $= 3000 \times 13 = \$39000$ Employer's CPF Contribution for 55 to 60 $= 3000 \times 0.145 \times 5 = \$2175$ Salary for Drivers 55 to 60 $= 3000 \times 5 = \$15000$ Total Employer's CPF $= 6630 + 2175$ $= \$8805$ Total Employer's Salary $= 39000 + 15000$ $= \$54000$ Total Employer's CPF and Salary $= 8805 + 54000$ $= \$62805$ Admin Executives Employer's CPF Contribution for 55 and below $= 3500 \times 0.17 \times 1 = \$595$ Salary for Admin Execs 55 and below $= 3500 \times 1 = \$3500$ Employer's CPF Contribution for 55 to 60 $= 3500 \times 0.145 \times 1 = \$507.50$ Salary for Admin Execs 55 to 60 $= 3500 \times 1 = \$3500$ Total Employer's CPF $= 595 + 507.50$ $= \$1102.50$ Total Employer's CPF and Salary $= 3500 + 3500$ $= \$7000$ Total Employer's CPF and Salary $= 1102.50 + 7000$ $= S8102.50$ Total Employer's CPF and Salary for all employees $= 27880 + 62805 + 8102.50$ $= S98787.50$	M1 M1 – comparison of amount of difference A1

	Percentage difference of Mr Chan's estimate from the calculated maximum budget $=\frac{100000-98787.50}{98787.50}\times 100$ $=1.23\%$ Mr Chan's estimate of maximum budget of \$100000 is reasonable as it is only 1.23% from the calculated value using maximum salaries.	
	<u>Using Mid-Value Salaries</u> Since we do not know the actual salaries, we will use the mid-value of each category of worker as an estimate. Vehicle Mechanics Employer's CPF Contribution for 55 and below $=3250\times 0.17\times 4=\$2210$ Salary for Vehicle Mechanics 55 and below $=3250\times 4=\$13000$ Employer's CPF Contribution for 55 to 60 $=3250\times 0.145\times 2=\$942.50$ Salary for Vehicle Mechanics 55 to 60 $=3250\times 2=\$6500$ Total Employer's CPF and Salary $=2210+13000+942.50+6500$ $=\$22652.50$ Drivers Employer's CPF Contribution for 55 and below $=2700\times 0.17\times 13=\$5967$ Salary for Drivers 55 and below $=2700\times 13=\$35100$ Employer's CPF Contribution for 55 to 60 $=2700\times 0.145\times 5=\$1957.50$ Salary for Drivers 55 to 60 $=2700\times 5=\$13500$ Total Employer's CPF and Salary $=5967+35100+1957.50+13500$ $=\$56524.50$	B1 M3 for computation on employer's CPF and worker's salaries.

	Admin Executives Employer's CPF Contribution for 55 and below $= 3250 \times 0.17 \times 1 = \$552.50$ Salary for Admin Execs 55 and below $= 3250 \times 1 = \$3250$ Employer's CPF Contribution for 55 to 60 $= 3250 \times 0.145 \times 1 = \$471.25$ Salary for Admin Execs 55 to 60 $= 3250 \times 1 = \$3250$ Total Employer's CPF and Salary $= 552.50 + 3250 + 471.25 + 3250$ $= \$7523.75$ Total Employer's CPF and Salary for all employees $= 22652.50 + 56524.50 + 7523.75$ $= S86700.75$	M1
	Percentage difference of Mr Chan's estimate from the calculated maximum budget $= \frac{100000 - 86700.75}{86700.75} \times 100$ $= 15.3\%$	A1
	The computed amount is 15.3% less than the \$100000. Mr Chan's estimate is reasonable as there is still some room for Mr Chan to increase wages.	

	Using Minimum Salaries Vehicle Mechanics Employer's CPF Contribution for 55 and below $= 2500 \times 0.17 \times 4 = \$1700$ Salary for Vehicle Mechanics 55 and below $= 2500 \times 4 = \$10000$ Employer's CPF Contribution for 55 to 60 $= 2500 \times 0.145 \times 2 = \$725$ Salary for Vehicle Mechanics 55 to 60 $= 2500 \times 2 = \$5000$ Total Employer's CPF and Salary $= 1700 + 10000 + 725 + 5000$ $= \$17425$ Drivers Employer's CPF Contribution for 55 and below $= 2400 \times 0.17 \times 13 = \$5304$ Salary for Drivers 55 and below $= 2400 \times 13 = \$31200$ Employer's CPF Contribution for 55 to 60 $= 2400 \times 0.145 \times 5 = \$1740$ Salary for Drivers 55 to 60 $= 2400 \times 5 = \$12000$	Using the minimum salaries would not be a reasonable way to test for the maximum budget. M3 for computing salaries and employer's contributions

	Total Employer's CPF and Salary $= 5304 + 31200 + 1740 + 12000$ $= \$50244$ Admin Executives Employer's CPF Contribution for 55 and below $= 3000 \times 0.17 \times 1 = \$510$ Salary for Admin Execs 55 and below $= 3000 \times 1 = \$3000$ Employer's CPF Contribution for 55 to 60 $= 3000 \times 0.145 \times 1 = \$435$ Salary for Admin Execs 55 to 60 $= 3000 \times 1 = \$3000$ Total Employer's CPF and Salary $= 510 + 3000 + 435 + 3000$ $= \$6945$ Total Employer's CPF and Salary for all employees $= 17425 + 50244 + 6945$ $= S74614$ Percentage difference of Mr Chan's estimate from the calculated maximum budget $= \frac{100000 - 74614}{74614} \times 100$ $= 34.0\%$	M1 for some comparison of the computed budget with the \$1000000