ANG MO KIO SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

MATHEMATICS
Paper 1

4048/01

Monday   **30 August 2021**   **2 hours**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

For Examiner's Use
80

This document consists of **22** printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curve surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector Area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $\left(\dfrac{3}{x}\right)^{-2}$.

Answer .... [1]

(b) Solve $3^y \times 4^y = 12^{5-y}$.

Answer $y =$ .... [2]

2 Given that $2x - 3 < \dfrac{2}{3}(9x - 6)$, solve the inequality and hence find the smallest possible value of x if x is a prime number.

Answer $x =$ .... [3]

3   The coordinates of points A and B are (5, –3) and (7, 2) respectively.

(a) Find the length of the line AB.

Answer .... [2]

(b) The line $hx - 2y = k$ is parallel to AB and passes through the point (3, 8).
Find the value of h and of k.

Answer $h =$ .... $k =$ .... [3]

4   Express as a single fraction in its simplest form

$$2 - \frac{3a-b}{a+b}.$$

Answer .... [2]

5   The number of people living in a city in 2010 was given as 279 400, correct to the nearest hundred.
The number of people living in the same city in 2020 was given as 531 000, correct to the nearest thousand.
Find the maximum increase in the number of people living in the city between the year 2010 and the year 2020.
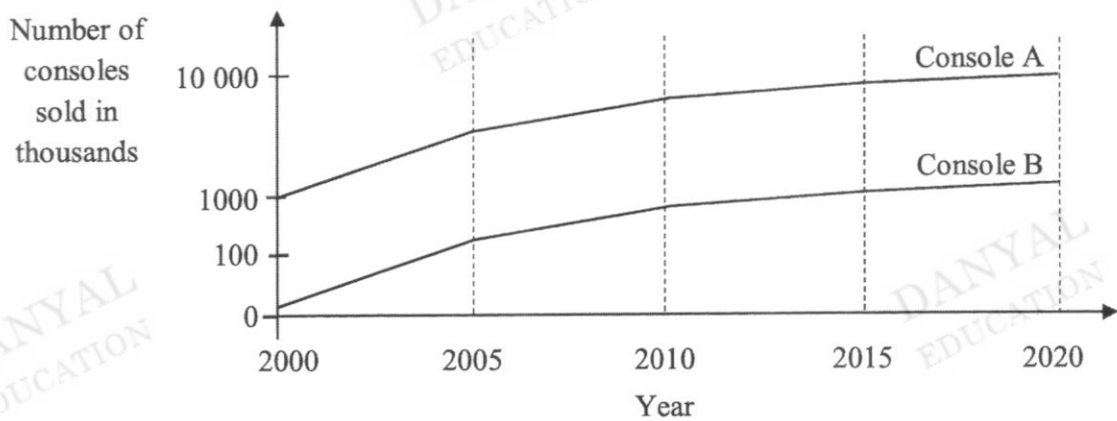
Answer .... [2]

6 Solve the simultaneous equations

$$x - 4y = 17,$$
$$2x + 3y = 1.$$

Answer $x =$ .... $y =$ .... [3]

7 The graph shows the sales figures of 2 gaming consoles manufactured by a company.

A sales executive from the company claimed that the chart showed comparable growth in the sales of the 2 gaming consoles. Do the chart support his claim? Justify your answer with reference to the chart.

Answer ....

....

.... [1]

8   Two coloured chips are taken from a box at random with replacement.
    The box contains 3 green chips and 8 yellow chips.

(a) Emily said that the probability that both chips are yellow is $\dfrac{28}{55}$.

    Explain what she has done wrong.

    Answer ....

    ....

    ....

    ....

    .... [1]

(b) Find the probability that at least one of the chips is yellow.

    Answer .... [2]

9   x is inversely proportional to the cube root of y.

It is given that $x = 6$ for a particular value of y.

Find the new value of x when this value of y decreases by 87.5%.

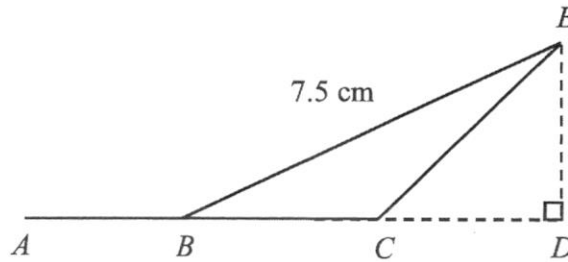*Answer* $x =$ .... [2]

---

10   An online sales platform offers x % cash rebate capped at $ y dollars for each order.
Find the minimum amount in dollars, in terms of x and y, one should purchase in each
order to maximise the rebate.

Answer $ .... [2]

11 In the diagram, $ABCD$ is a straight line. $BE = 7.5$ cm, angle $CDE = 90°$ and $\cos \angle ABE = -\dfrac{4}{5}$.

Find the length of ED.

Answer .... cm   [2]

12 Patrick made a fruit cordial drink by mixing water and syrup in the ratio of 18 : 7. After finding the drink was too sweet, he added 1.3 litres of water such that the ratio of water to syrup became 17 : 3. Find the amount of the syrup used.

Answer .... litres   [3]

13 (a) Express 180 as a product of its prime factors.

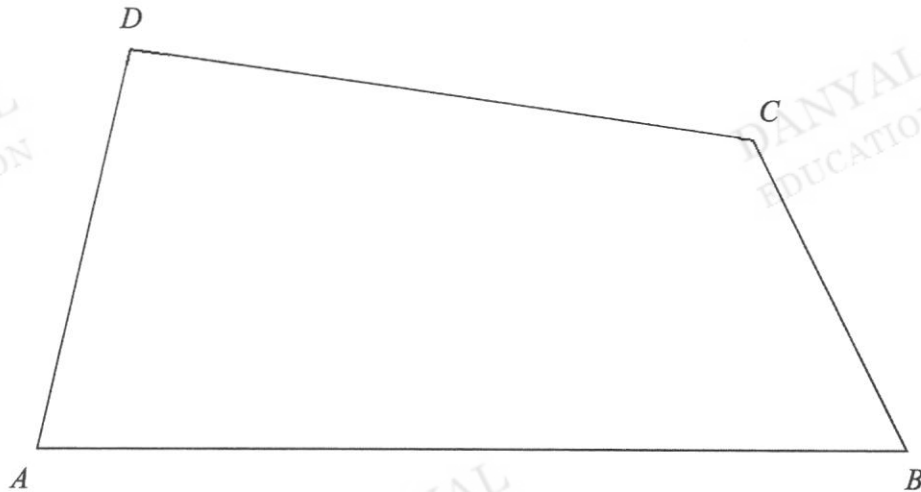*Answer* .... [1]

(b) The number $180pq$ is a perfect square.

p and q are composite numbers that are larger than 10 and p is smaller than q.

Find the smallest possible value of p and the value of q.

*Answer* $p =$ .... $q =$ .... [2]

14 The diagram shows a quadrilateral $ABCD$.

On the diagram,

(a) construct the perpendicular bisector of BC, [1]

(b) construct the angle bisector of angle ABC, [1]

(c) A point E, inside the quadrilateral $ABCD$, is equidistant from B and C and closer to AB than BC. Mark and label a possible location of point E. [1]

15 In the diagram, A and B are due south of C. The lines PAE and QBD are parallel.
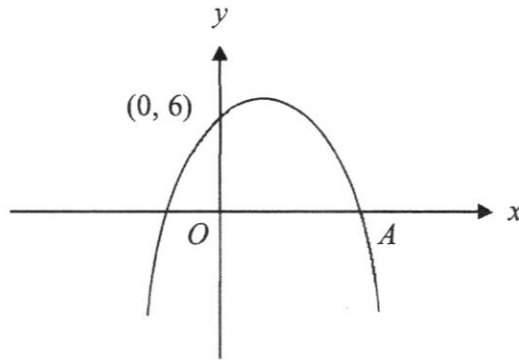Angle BAE = 50° and the bearing of D from C is 137°

North

C

Q   B   D

50°
P   A   E
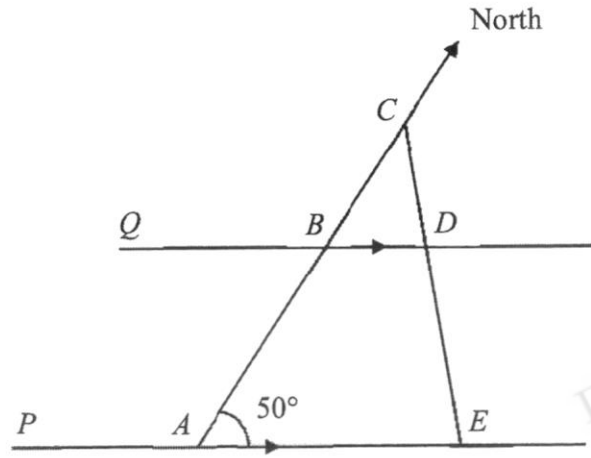
Find

(a) the bearing of Q from B,

Answer .... ° [1]

(b) reflex angle DEA.

Answer .... ° [2]

16 The curve $y = -(x-3)(2x+b)$ cuts the x-axis at the point A and the y-axis at the point $(0, 6)$.

(a) Find the value of b.

Answer $b =$ .... [1]

(b) State the coordinates of point A.

Answer (...................... , ...) [1]

(c) Find the maximum value of y.

Answer $y =$ .... [2]

17 A restaurant charges different delivery fees for customers staying in different zones. The matrix **P** shows the number of orders from zone A, B and C on a Saturday and Sunday respectively.

$$\begin{array}{cccc} & A & B & C & \\ P = \begin{pmatrix} 85 & 42 & 16 \\ 90 & 65 & 28 \end{pmatrix} & & & \begin{array}{l} \text{Saturday} \\ \text{Sunday} \end{array} \end{array}$$

(a) The restaurant charges \$2, \$x and \$8 for each delivery to zone A, B and C respectively. Represent this information in a 3 × 1 matrix **Q**.

Answer **Q** =   [1]

(b) Find, in terms of x, the matrix **R** = **PQ**.

Answer **R** =   [2]

(c) Given that on Sunday the restaurant collected \$232.50 more in delivery fees compared to Saturday, find the value of x.

Answer x = ... [1]

18 The stem-and-leaf diagram shows the amount of time spent on exercising in a gym by 20 members on a particular day.

Stem	Leaf
2	5
3	0 2 6 9
4	3 5 5
5	1 6 8
6	0 0 0 3 4 8
7	0 5
8	0

Key: 2 | 5 represents 25 minutes

(a) Find the modal time spent on exercising.

Answer .... min   [1]

(b) Find the mean time spent on exercising.

Answer .... min   [1]

(c) Given that 70% of the gym members exercised for more than x minutes, state the smallest possible integer value of x.

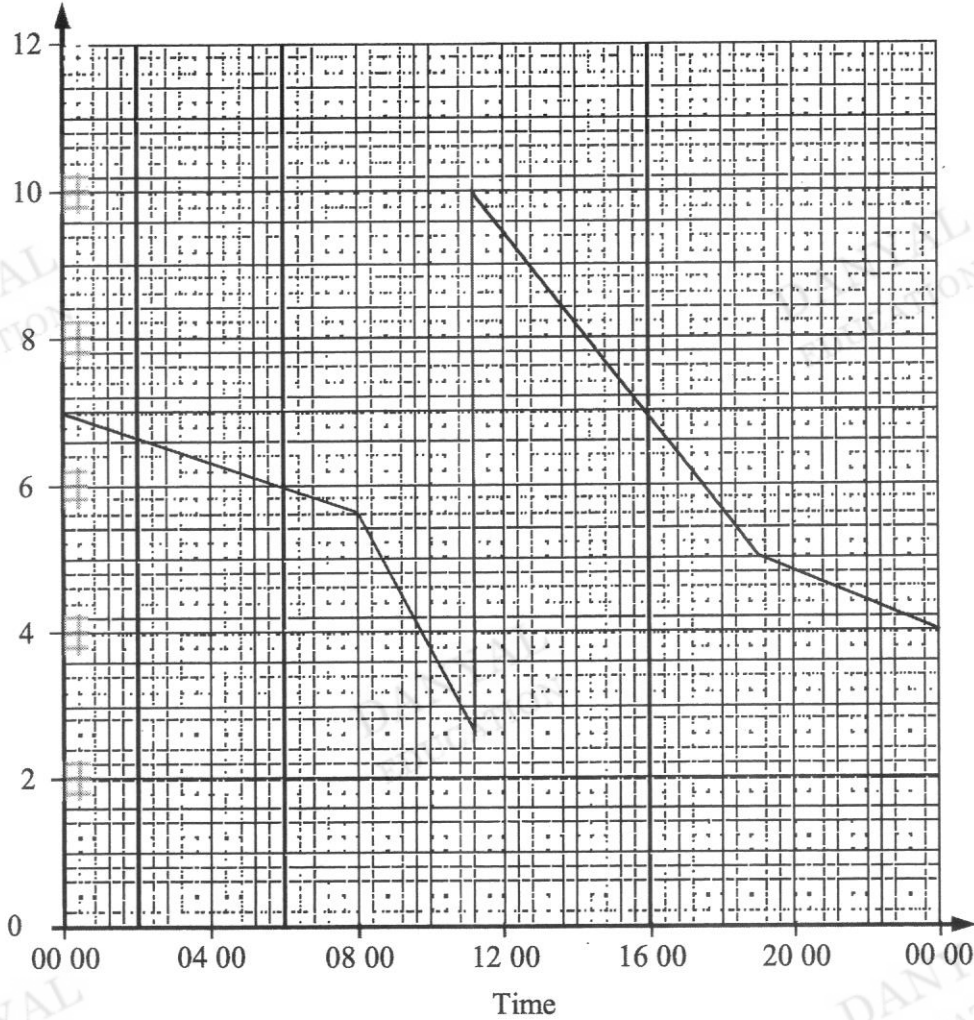*Answer* $x =$ .... [1]

(d) It is discovered that 2 of the values have been recorded wrongly.
The number 32 should have been 40 and the number 51 should have been 43.
Explain how the mean will be affected.

Answer ....

.... [1]

19   The graph shows the concentration of chlorine, in milligrams per litre, of water in a swimming
     pool throughout a particular day. A device replenishes chlorine automatically when the
     concentration falls below a certain level. $\left(1 \text{ milligram} = 10^{-3} \text{ gram}\right)$

Concentration of Chlorine (mg/L)

(a)   At what time was the chlorine replenished?

                                        Answer .... [1]

(b)   The concentration of chlorine drops slowly as time passes but rapidly when the pool is
      being used. Given that the pool is not open for 24 hours, write down the possible
      opening hours of the pool.

                            Answer .... to .... [1]

(c) Given that there are 2 million litres of water in the pool, find the amount of chlorine, in kilograms, added to the pool.

Answer .... kg   [2]

---

20 In a hexagonal tile, 2 interior angles are $x°$ each and remaining interior angles are $y°$ each. Explain if it's possible to place 3 of these tiles adjacent to one another so that there is no gap in between. Justify your answer with working.

Answer ....

....

....

....

.... [2]

---

21   **(a)** $\varepsilon = \{x : x \text{ is a positive integer and } x \leq 15\}$

$A = \{x : x \text{ is an even number}\}$

$B = \{x : x \text{ is an integer whose last digit is } 5\}$

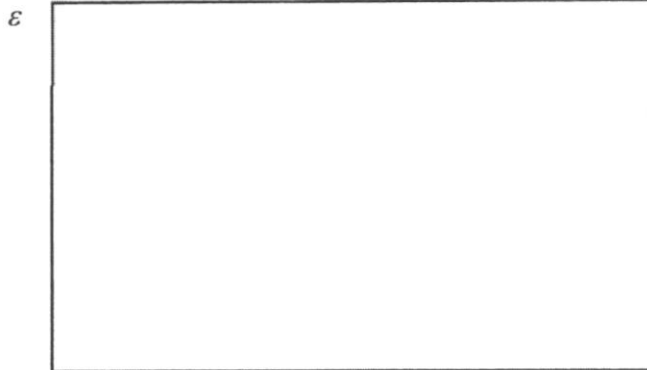   **(i)**   List the elements in set A'.

                 Answer .... [1]

       (ii)   State the number of elements in the set $A \cap B$.

                 Answer .... [1]

(b)   The universal set, ε, contains three sets, C, D and E. The three sets satisfy the following conditions:
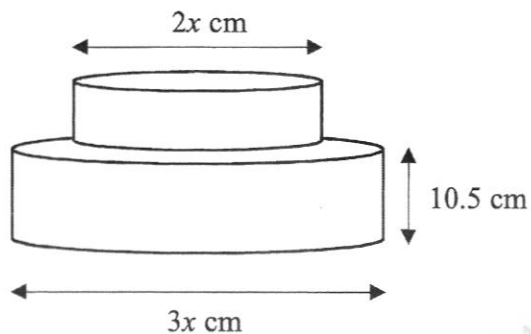
$$D \subset C, D \cap E = \varnothing \text{ and } C' \cap E \neq \varnothing.$$

Complete the Venn diagram below to illustrate these sets.

[2]

22 The diagram shows a birthday cake made up of 2 geometrically similar cylindrical tiers. The diameters of the upper tier and lower tier are $2x$ cm and $3x$ cm respectively. The height of the lower tier is 10.5 cm.
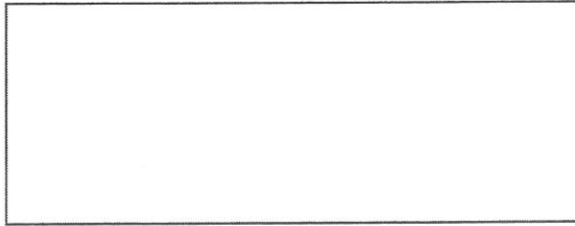

(a) Find the total height of the cake.

Answer .... cm [2]

(b) Given that the cake weighs a total of 1.4 kg, find the weight of the lower tier.

Answer .... kg [2]

23 The diagram shows the floor plan of each level of a 6-storey shopping mall drawn to the scale of 1 cm represent 15 metres.

The Gross Floor Area (GFA) of a building is the sum of the floor areas of all the spaces within the building.

(a) Using the plan, find the total gross floor area of the shopping mall.

Answer .... m² [2]

(b) Occupancy limit of the mall was changed from one person per 10 square metres of GFA to one person per 16 square metres of GFA. Find the decrease in number of people allowed in the mall.

Answer .... [2]

24 **(a)** Factorise completely $12xy - 5 + 20x - 3y$.

Answer .... [2]

(b) The total surface area of a solid cube is $\left(6a^2 - 48ab + 96b^2\right)$ cm^2.

    (i) Find, in terms of a and b, the length of each side of the cube.

Answer .... [2]

    (ii) Hence find the volume of the cube if $a = 7$ and $b = 1$.

Answer .... cm^3 [1]

25   The diagram shows a trapezium $ABCD$ with AB parallel to DC. $AB = 6.75$ cm, $BD = 9$ cm, $DC = 12$ cm and $AD = 13.5$ cm.
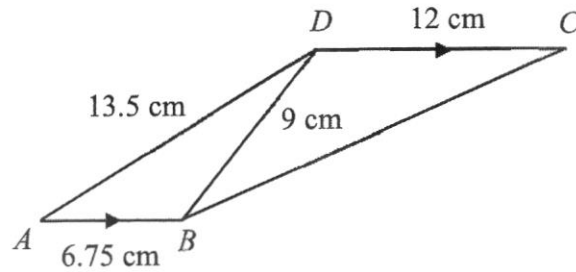
(a)   Show that triangle ABD is similar to triangle BDC.
      Give a reason for each statement you make

Answer ....

....

....

....

.... [3]

(b)   Find the perimeter of trapezium $ABCD$.

Answer .... cm   [2]

END OF PAPER

Class	Index Number	Name

ANG MO KIO SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

MATHEMATICS
Paper 2

4048/02

Tuesday   **31 August 2021**   **2 hours 30 minutes**

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
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Answer **all** questions.
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Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

For Examiner's Use

100

This document consists of **24** printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 (a) Simplify $\dfrac{(3ab)^2}{3} \div \dfrac{2a}{b^0}$

Answer .... [2]

(b) $a = \sqrt{\dfrac{b+2c}{3-b}}$

(i) Evaluate a when $b = -4$ and $c = 3$.

Answer .... [1]

(ii) Express b in terms of a and c.

Answer .... [3]

(c) **(i)** Express $9 - 5x + x^2$ in the form $p + (q + x)^2$.

Answer .... [2]

(ii) Sketch the graph of $y = 9 - 5x + x^2$ on the axes below.
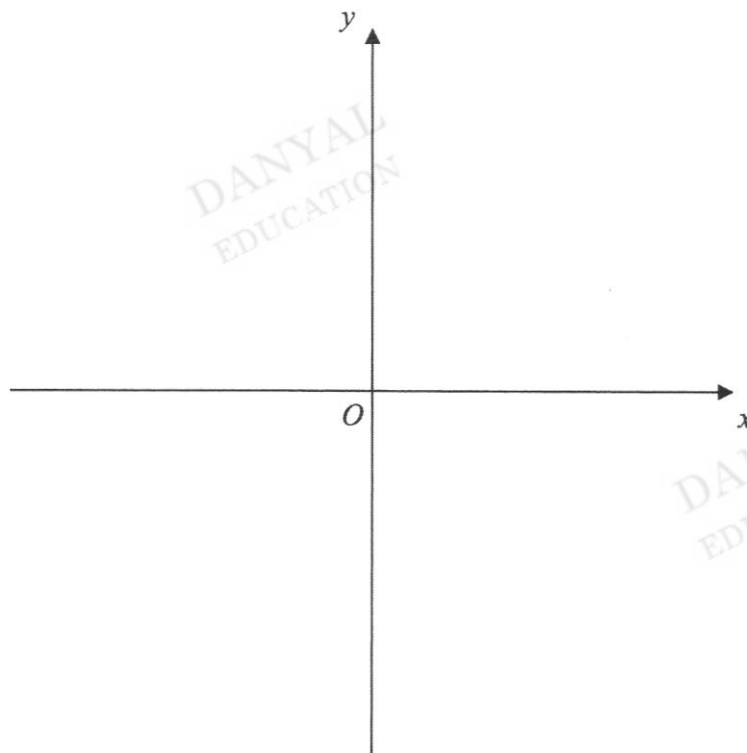
[2]

(iii) Write down the equation of the line of symmetry for $y = 9 - 5x + x^2$.

Answer .... [1]

2   The diagram represents a field *PQRS* on horizontal ground in which
     $PS = 43$ m, $PQ = 130$ m, $QR = 58$ m, $PR = 97$ m, $\angle PRS = 26.3°$ and $\angle RPS = 63°$.
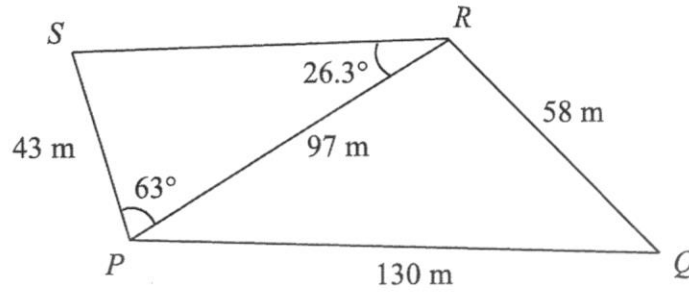
(a)   Find angle *PRQ*.

Answer .... °   [3]

(b)   Calculate the area of the field *PQRS*.

Answer .... m²   [3]

(c) Calculate the shortest distance from *S* to *PR*.

Answer .... m   [2]

(d) A pole is erected at *S*. *T* is the top of the pole. Given that the angle of elevation of *T* from *P* is 3.9°, find the range of the angles of elevation of *T* along the path *PR*.

Answer ....° to ....°   [5]

3 **(a)** The price of an iPad Air is $1 299 in Singapore. The price of the same iPad Air in Japan is ¥104 280. The exchange rate between Singapore dollars ($) and Japanese Yen (¥) is $1 = ¥81.49.
Where should you buy the iPad Air from?

I should buy the iPad Air from .... because

....

.... [3]

(b) The price of a MacBook Air increased from $999 in 2012 to $1299 in 2020. The price increased by x % every year. Find the value of x.

Answer $x =$ .... [2]

(c)   The table shows the sales of Apple's iPhone and the total revenue for 2019 and 2020.

Year	2019	2020
Number of iPhone	187.2 million	196.9 million
Revenue from sale of iPhones	$142 billion	$138 billion
Total revenue from all Apple's products	$260 billion	$274 billion

(i)   Calculate how many more iPhones were sold in 2020 than in 2019. Give your answer in standard form.

Answer .... [1]

(ii)   Calculate the percentage decrease in iPhone's revenue from 2019 to 2020.

Answer .... % [2]

(iii)   Calculate the percentage of Apple's total revenue that comes from the sales of iPhone in 2020.

Answer .... % [2]

4   The variables x and y are connected by the equation

$$y = \frac{x^3}{2} + 3x - 5.$$

Some corresponding values of x and y are given in the table below.

x	-4	-3	-2	-1	0	1	2	3	4
y	-49	-27.5	-15	-8.5	-5	-1.5	5	17.5	p

(a)   Find the value of p.

Answer $p =$ .... [1]

(b)   On the grid opposite, draw the graph of $y = \frac{x^3}{2} + 3x - 5$ for $-4 \le x \le 4$ [3]

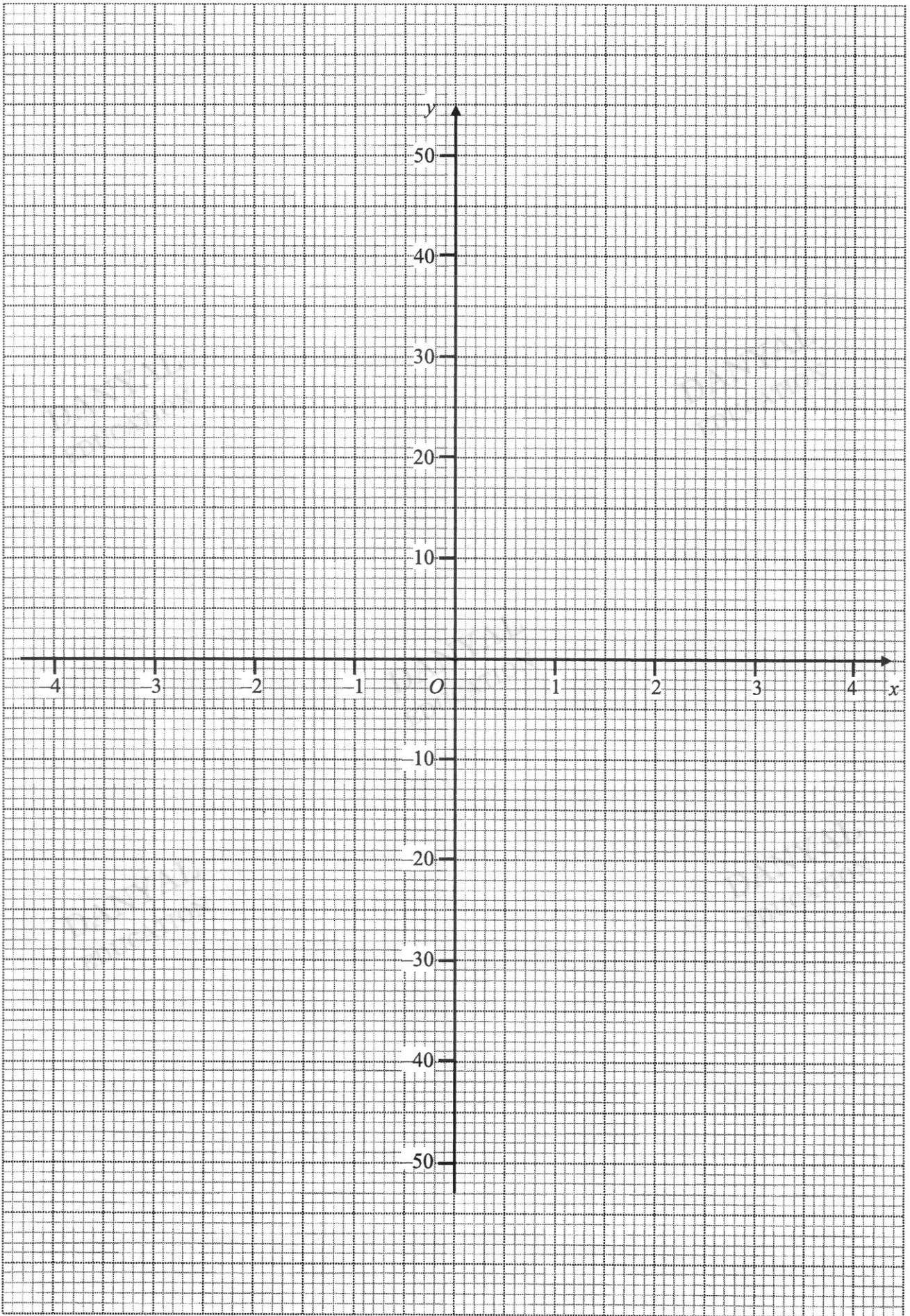(c)   Use your graph to write down an inequality in x to describe the range of values of where $y > 10$.

Answer   .... [1]

(d)   (i)   On the same grid, draw the graph of $3y - 29x + 5 = 0$ for $-4 \le x \le 4$.   [2]

(ii)   Show that the points of intersection of the line and the curve give the solution of the equation $3x^3 - 40x - 20 = 0$.

[2]

(iii)   Use your graph to solve equation $3x^3 - 40x - 20 = 0$.

Answer $x =$ .... or .... or .... [2]

5   The first three terms in a sequence of numbers T_1, T_2, T_3, ... are given below.

$$T_1 = 2 \times 1^2 + 2 = 4$$
$$T_2 = 2 \times 2^2 + 3 = 11$$
$$T_3 = 2 \times 3^2 + 4 = 22$$

(a)   **(i)**   Write down the fourth line of the sequence.

Answer .... [1]

(ii)   Find an expression, in terms of n, for T_n.

Answer $T_n =$ .... [1]

(iii)   Determine if 1175 can be one of the numbers in the sequence.

Answer ....

....

....

....

....

....

....

.... [2]
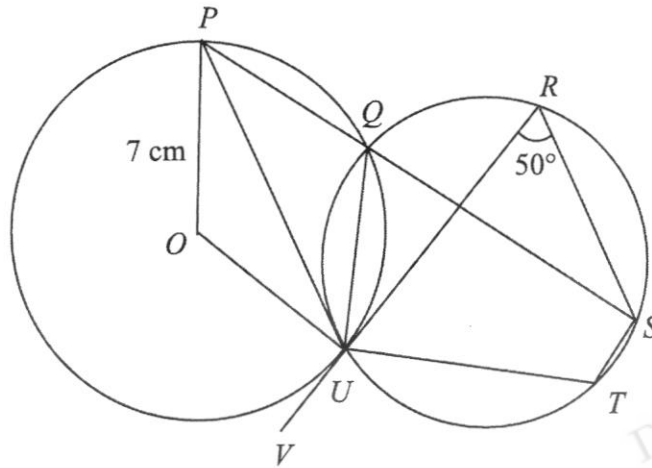
(b) The first four terms in another sequence are 2, 5, 8, 11.

    (i) Find an expression, in terms of n, for the nth term, S_n, of this sequence.

Answer $S_n =$ .... [1]

    (ii) Find the 53rd term of this sequence.

Answer .... [1]

6 (a) In the diagram below, RUV is a tangent to the circle with centre O, angle $SRU = 50°$ and PQS is a straight line.

(i) Stating your reasons clearly, find

(a) angle STU,

Answer .... ° [1]

(b) reflex angle POU,

Answer .... ° [2]

(c) angle PUO,

Answer .... ° [1]

(d) angle PUR.

Answer .... ° [1]

(ii) What can you say about the lines *RS* and *PU*? Explain.

Answer ....

....

.... [1]

(b) In the diagram below, *OPQR* is a sector of a circle with centre *O*. The sector is folded such that the centre *O* touches the arc *PR* at point *Q*.
Angle *TQS* = 1.31 radians, *PT* = 1.8 cm, *QT* = 5.2 cm and *RS* = 3.1 cm.
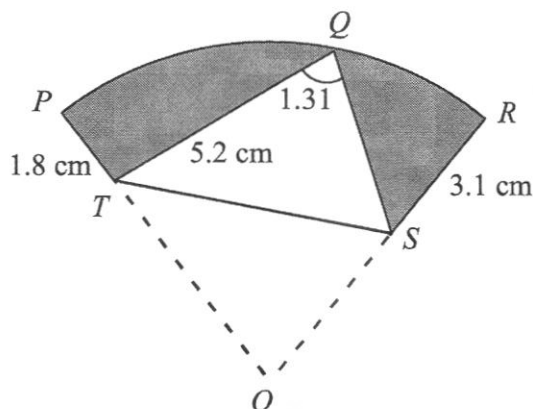
(i) Find the length of arc *PQR*.

Answer .... cm [1]

(ii) Find the area of the shaded region.

Answer .... cm² [3]

7   A pond in an ecogarden contains 2500 litres of water.

(a)   A large pump have a water supply of x litres per minute. Write down an expression, in terms of x, for the number of minutes the pump would take to fill up the pond.

Answer .... min   [1]

(b)   A small pump have a water supply of $(x - 10)$ litres per minute. Write down an expression, in terms of x, for the number of minutes the pump would take to fill up the pond.

Answer .... min   [1]

(c)   It takes 4 hours longer to fill up the pool using the small pump than it does using the large pump.
Write down an equation in x to represent this information, and show that it reduces to

$$6x^2 - 60x - 625 = 0.$$

Answer

[3]

(d)   Solve the equation $6x^2 - 60x - 625 = 0$, giving your solutions correct to one decimal place.

Answer $x =$ ....   or   ....   [3]

(e)   Find the time taken for the 2 pumps to operate together to fill up the pond. Give your answer in hours and minutes correct to the nearest minute.

Answer .... hr .... min   [3]

8   The diagram shows a building structure in the shape of a hemisphere on top of a cuboid.

The circumference of the base of the hemisphere touches the four edges of the top of the cuboid.

Point X is the highest point of the structure at the top of the hemisphere.

$AB = 20$ m and $AE = 15$ m.
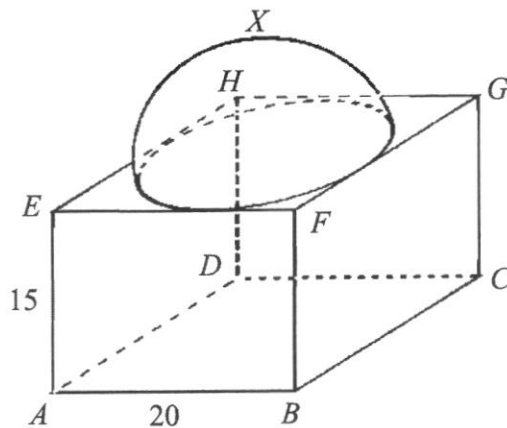
(a)   Find the height of the whole structure.

Answer .... m   [1]

(b)   Calculate the volume of the building structure.

Answer .... m^3 [3]

(c) Calculate the surface area of the building structure.

Answer .... m^2 [3]

(d) Find angle XAC.

Answer .... °   [3]

9 **(a)** The pie chart below shows the distribution of score for 40 students in a quiz. A student can score a minimum of 0 and a maximum of 5 in the quiz.

Quiz Score
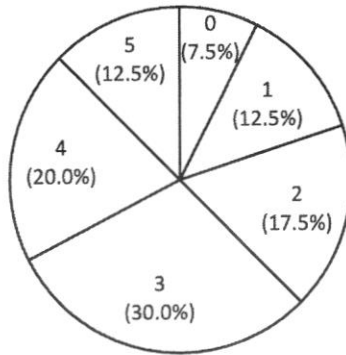
(i) Find the number of students getting the score of 3.

Answer .... [1]

(ii) Find the median score for the quiz.

Answer .... [1]

(iii) Find the range of score for the quiz.

Answer .... [1]

(iv) Find the interquartile range of the score for the quiz.

Answer .... [1]

(b) A box contains some green and some red cards. Each card comes with a picture of a living thing or a non-living thing.
The table below shows the probabilities of drawing a card at random from the box

	Green	Red
Living thing	$\dfrac{1}{12}$	$\dfrac{1}{3}$
Non-living thing	$\dfrac{1}{4}$	m

(i) Find the value of m.

Answer $m =$ .... [1]

(ii) Given that there are 60 cards in the box, how many cards with a picture of a non-living thing should be removed so that there is an equal probability of drawing a card with a living or non-living thing from the box.

Answer .... [2]

10   A disposable plastic cup can be represented by a right frustum which is a parallel
truncation of a right cone as shown in the diagram below. The cup has a diameter of
84 mm at the top, a diameter of 60 mm at the bottom and a height of 165 mm.

84 mm

165 mm

60 mm
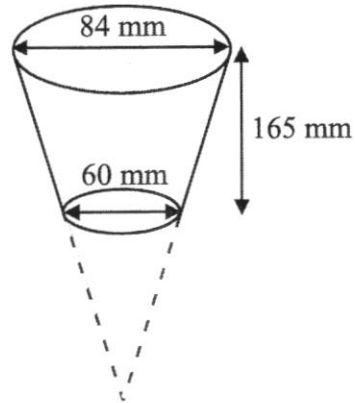
(a)   Show that the plastic cup has a volume of 678019 mm^3.

Answer

[3]

(b) Bubbles Bubble Tea sells beverages using disposable cup of two sizes. The dimensions of the cups are given in the table below.

The material cost for manufacturing the cups are given in the table below.
The minimum thickness of a plastic cup is 0.25 mm for the manufacturing of the whole cup.
The minimum thickness of a paper cup is 0.35 mm for the side of the cup and 0.70 mm for the bottom of the cup.

Size	Top diameter	Bottom diameter	Height of cup	Selling Price of beverage
Regular	84 mm	70 mm	130 mm	$5.00
Large	84 mm	60 mm	165 mm	$6.50

Material	Thickness (mm)	Cost per m² ($)
Paper	0.35	$0.62
Plastic	0.25	$0.65

The shop needs 1500 large cups each week. Given that the cost of material for 1500 large plastic cups is $39.24, which material should the shop use for the manufacturing the cups?

Answer   The shop should use .... for manufacturing

the cups because ....

....

.... [8]

END OF PAPER
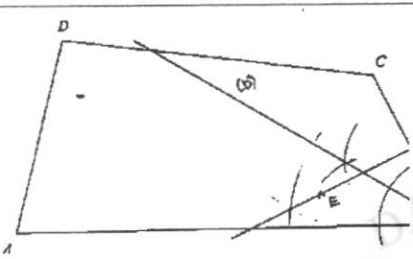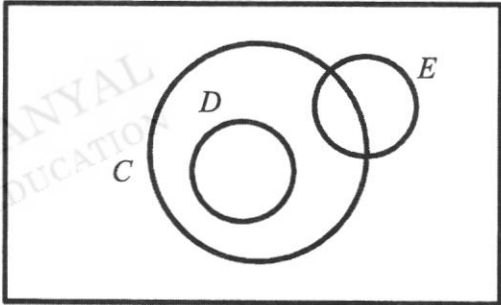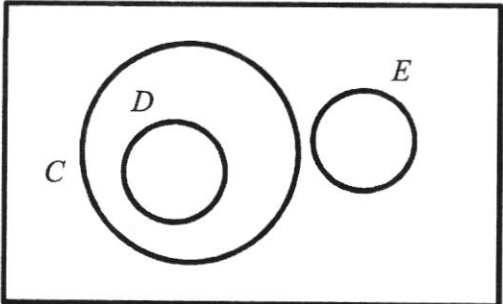
AMKSS 2021 Prelim 4E5N EM P1 Answer Scheme

	Answer	Marks
1 (a)	$\left(\dfrac{3}{x}\right)^{-2} = \dfrac{x^2}{9}$	B1
1(b)	$3^y \times 4^y = 12^{5-y}$ $12^y = 12^{5-y}$ $y = 5 - y$ $2y = 5$ $y = 2.5$	M1 (change to same base) A1
2	$2x - 3 < \dfrac{2}{3}(9x - 6)$ $2x - 3 < 6x - 4$ $-4x < -1$ $x > \dfrac{1}{4}$ Smallest possible value of $x = 2$.	M1 A1 B1 (no mark if inequality is wrong)
3(a)	$AB = \sqrt{(5-7)^2 + (-3-2)^2}$ $= 5.385164807$ $= 5.39$ units (3sf)	M1 A1
3(b)	Gradient of AB $\dfrac{2-(-3)}{7-5} = \dfrac{5}{2}$ $2y = hx - k$ $y = \dfrac{h}{2}x - \dfrac{k}{2}$ $h = 5$ $k = 5(3) - 2(8) = -1$	M1 A1 A1
4	$2 - \dfrac{3a-b}{a+b}$ $= \dfrac{2(a+b) - (3a-b)}{a+b}$ $= \dfrac{2a + 2b - 3a + b}{a+b}$ $= \dfrac{-a + 3b}{a+b}$	 M1 (combine into single fraction) A1
5	Maximum increase $= 531499 - 279350$ $= 252149$	M1 (for either 531 499 or 279 350) A1

	Answer	Marks
6	$x - 4y = 17 \ldots (1)$ $2x + 3y = 1 \ldots (2)$ From (1): $x = 4y + 17 \ldots (3)$ Subs. (3) into (2): $2(4y + 17) + 3y = 1$ $\quad 8y + 34 + 3y = 1$ $\qquad\quad 11y = -33$ $\qquad\qquad y = -3$ $\qquad x = 4(-3) + 17 = 5$	M1 (substitution or elimination) A1 A1
7	**No** because **the scale for the vertical axis is inconsistent**. It gives the impression that there is comparable growth in sales of the 2 gaming consoles.	B1
8(a)	Emily calculated the probability **without replacement**. It should be $\dfrac{8}{11} \times \dfrac{8}{11} = \dfrac{64}{121}$.	B1 (need to mention **without replacement** or explain Emily should not reduce 1 from the total since there is replacement)
8(b)	P(at least 1 yellow) $= 1 - \left(\dfrac{3}{11} \times \dfrac{3}{11}\right)$ $\qquad$ accept $2\left(\dfrac{3}{11} \times \dfrac{8}{11}\right) + \dfrac{8}{11} \times \dfrac{8}{11}$ $=$ $= \dfrac{112}{121}$	M1 A1
9	$x = \dfrac{k}{\sqrt[3]{y}}$ When $x = 6$, $y = a$ $6 = \dfrac{k}{\sqrt[3]{a}}$ $k = 6\sqrt[3]{a}$ When $y = \dfrac{12.5}{100}a = \dfrac{1}{8}a$ $x = \dfrac{6\sqrt[3]{a}}{\sqrt[3]{\dfrac{1}{8}a}}$ $= \dfrac{6\sqrt[3]{a}}{\dfrac{1}{2}\sqrt[3]{a}}$ $= 12$	 M1 A1
10	$x\% \rightarrow \$y$ $1\% \rightarrow \$\dfrac{y}{x}$ $100\% \rightarrow \$\dfrac{100y}{x}$	 M1 A1 (or B2)

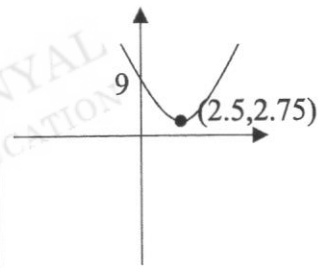|	Answer	Marks
11	$\dfrac{BD}{7.5}=\dfrac{4}{5}$ $BD = 6$ cm $ED = \sqrt{7.5^2 - 6^2} = 4.5$ cm	M1 A1
12	Before: $\quad 18:7 = 54:21$ After: $\quad 17:3 = 119:21$ 65 units $\to$ 1.3 litres 21 litres $\to \dfrac{1.3}{65}\times 21 = 0.42$ litres	M1 (change to equivalent ratios) M1 A1
13(a)	$2^2 \times 3^2 \times 5$	B1
13(b)	Smallest perfect square $= 2^4 \times 3^4 \times 5^2$ $pq = 2^2 \times 3^2 \times 5$ $\quad = (2^2 \times 3)\times(3 \times 5)$ $\quad = 12 \times 15$ $p = 12$ $q = 15$	 B1 B1
14(a)		B1
14(b)		B1
14(c)		B1
15(a)	bearing of Q from B $= 180° + 50° \quad$ (alternate $\angle$s) $= 230°$	 B1
15(b)	$\angle ACE = 180° - 137° = 43° \quad$ (adj $\angle$s on str. line) $\angle DEA = 360° - (180° - 50° - 43°)$ $\quad = 273° \;(\angle$s at a point$)$	M1 A1
16(a)	$6 = -(0-3)(2(0)+b)$ $6 = 3b$ $b = 2$	 B1
16(b)	$(3, 0)$	B1
16(c)	$-(x-3)(2x+2) = 0$ $x = 3 \quad$ or $\quad x = -1$ Line of symmetry is $x = \dfrac{3+(-1)}{2} = 1$ $y = -(1-3)(2(1)+2) = 8$	 M1 A1

	Answer	Marks
17(a)	$Q = \begin{pmatrix} 2 \\ x \\ 8 \end{pmatrix}$	B1
17(b)	$R = \begin{pmatrix} 85 & 42 & 16 \\ 90 & 65 & 28 \end{pmatrix} \begin{pmatrix} 2 \\ x \\ 8 \end{pmatrix}$ $= \begin{pmatrix} 298 + 42x \\ 404 + 65x \end{pmatrix}$	 B2 (or M1, A1)
17(c)	$298 + 42x + 232.50 = 404 + 65x$ $23x = 126.50$ $x = 5.50$	 B1
18(a)	60 min	B1
18(b)	53 min	B1
18(c)	43	B1
18(d)	Mean **remains the same / No change** to the mean	B1
19(a)	1112	B1
19(b)	0800 to 1900	B1
19(c)	$= (10 - 2.6) \times 2000000$ $= 14800000$ mg $= 14.8$ kg	M1 A1
20	Sum of interior angles of hexagon $(6-2) \times 180 = 720°$ $2x + 4y = 720°$ $x + 2y = 360°$ Yes, it is possible to put 3 sides adjacent to one another.	M1 (correct sum of int. angles of hexagon) A1
21(a)(i)	$\{1, 3, 5, 7, 11, 13, 15\}$	B1
21(a)(ii)	0	B1
21(b)	 OR 	B1 (for $D \subset C$) B1 (for correct E) No mark if not labelled.

	Answer	Marks
22(a)	Let h be height of the upper tier. $\dfrac{h}{10.5} = \dfrac{2x}{3x}$ $h = 7$ Total height $= 7 + 10.5 = 17.5$ cm.	M1 A1
22(b)	$\left(\dfrac{2}{3}\right)^3 = \dfrac{8}{27}$ 35 units $\rightarrow$ 1.4 kg 27 units $\rightarrow \dfrac{27}{35} \times 1.4 = 1.08$ kg	M1 (find ratio of volume) A1
23(a)	GFA of 1 level $= (3 \times 15) \times (8 \times 15) = 5400$ m^2 Total GFA $= 5400 \times 6 = 32400$ m^2 **OR** GFA of 1 level $= (3 \times 15) \times (8.1 \times 15) = 5467.5$ m^2 Total GFA $= 5467.5 \times 6 = 32805$ m^2	M1 A1
23(b)	Decrease in number $= \dfrac{32400}{10} - \dfrac{32400}{16}$ $= 1215$ **OR** Decrease in number $= \dfrac{32805}{10} - \dfrac{32805}{16}$ $= 1230.1875$ (accept 1230 or 1231)	M1 A1
24(a)	$12xy - 5 + 20x - 3y$ $= 12xy + 20x - 3y - 5$ $= 4x(3y + 5) - (3y + 5)$ $= (3y + 5)(4x - 1)$	M1 A1
24(b)(i)	Area of each side of cube $= \dfrac{6a^2 - 48ab + 96b^2}{6}$ $= a^2 - 8ab + 16b^2$ $= (a - 4b)^2$ Length of each side $= (a - 4b)$ cm.	M1 A1
24(b)(ii)	$(a - 4b)^3 = ((7) - 4(1))^3$ $= 27$ cm^3	B1 (must use (i))
25(a)	$\dfrac{AB}{BD} = \dfrac{6.75}{9} = \dfrac{3}{4}$ $\dfrac{BD}{DC} = \dfrac{9}{12} = \dfrac{3}{4}$ $\dfrac{AB}{BD} = \dfrac{BD}{DC}$ $\angle ABD = \angle BDC$ (alternate angles) Triangle ABD is similar to Triangle BDC (Ratios of 2 pairs of corresponding side and included angle equal)	M1 (show equal ratio) M1 (for angle) A1

	Answer	Marks
25(b)	$\dfrac{BC}{AD} = \dfrac{4}{3}$ $\dfrac{BC}{13.5} = \dfrac{4}{3}$ $BC = 18$ cm Perimeter $= 6.75 + 18 + 12 + 13.5 = 50.25$ cm	M1 (correct length of BC) A1

Qn	Solutions	Marks	Remarks
1a	$\dfrac{(3ab)^2}{3} \div \dfrac{2a}{b^0}$		
	$\dfrac{9a^2b^2}{3} \times \dfrac{1}{2a}$	M1	
	$\dfrac{3ab^2}{2}$	A1	
1bi	$0.5345224838 = 0.535$	B1	
1bii	$a = \sqrt{\dfrac{b+2c}{3-b}}$		Do not except if students did not state $b =$
	$a^2 = \dfrac{b+2c}{3-b}$	M1	
	$3a^2 - a^2b = b + 2c$		
	$3a^2 - 2c = b + a^2b$	M1	
	$b(1+a^2) = 3a^2 - 2c$		
	$b = \dfrac{3a^2 - 2c}{1+a^2}$ or $b = \dfrac{2c - 3a^2}{-1-a^2}$	A1	
1ci	$x^2 - 5x + 9$		
	$= x^2 - 5x + \left(\dfrac{-5}{2}\right)^2 + 9 - \left(\dfrac{-5}{2}\right)^2$	M1	
	$= \left(x - \dfrac{5}{2}\right)^2 + \dfrac{11}{4}$		
	$= \dfrac{11}{4} + \left(-\dfrac{5}{2} + x\right)^2$	A1 or B2	
1cii		min pt and shape B1	
		y-intercept B1	
1ciii	$x = 2.5$	B1	
			11

2a	$130^2 = 97^2 + 58^2 - 2(97)(58)\cos \angle PRQ$	M1	
	$-11252 \cos \angle PRQ = 4127$		
	$\cos \angle PRQ = -\dfrac{4127}{11252}$	M1	
	$\angle PRQ = 111.5171212°$		
	$\angle PRQ = 111.5°$	A1	
	OR		
	$\cos \angle PRQ = \dfrac{97^2 + 58^2 - 130^2}{2(97)(58)}$	M1	
	$\cos \angle PRQ = -\dfrac{4127}{11252}$		
	$\angle PRQ = 111.5171212°$	M1	
	$\angle PRQ = 111.5°$	A1	
2b	Area		
	$= 0.5(97)(58) \sin 111.5 + 0.5(97)(43) \sin 63$	M1 + M1	
	$= 2616.956426 + 1858.194106$		
	$= 4475.150533$ (based on 111.5171212)		
	$= 4475.458725$ (based on 111.5)		
	$= 4480$	A1	
2c	$0.5 \times 97 \times h = 1858.194106$	M1	
	$h = 38.31328054 = 38.3$	A1	
	OR		
	$\sin 63° = h \div 43$	M1	
	$h = 38.31328054 = 38.3$	A1	
2d	$\tan 3.9° = \dfrac{ST}{43}$		
	$ST = 2.93144591$	M1	
	$\dfrac{SR}{\sin 63} = \dfrac{43}{\sin 26.3}$		
	$SR = 86.47206439$	M1	
	$SR = 86.43385477$ (using cosine rule)		
	$\tan \theta_{min} = \dfrac{2.93144591}{86.47206439}$	M1	
	$\theta_{min} = 1.941611802°$ $(1.942469468°)$		
	$\tan \theta_{max} = \dfrac{2.93144591}{38.31328054}$	M1	
	$\theta_{max} = 4.375320211$		
	$1.9°$ to $4.4°$	A1	
			13

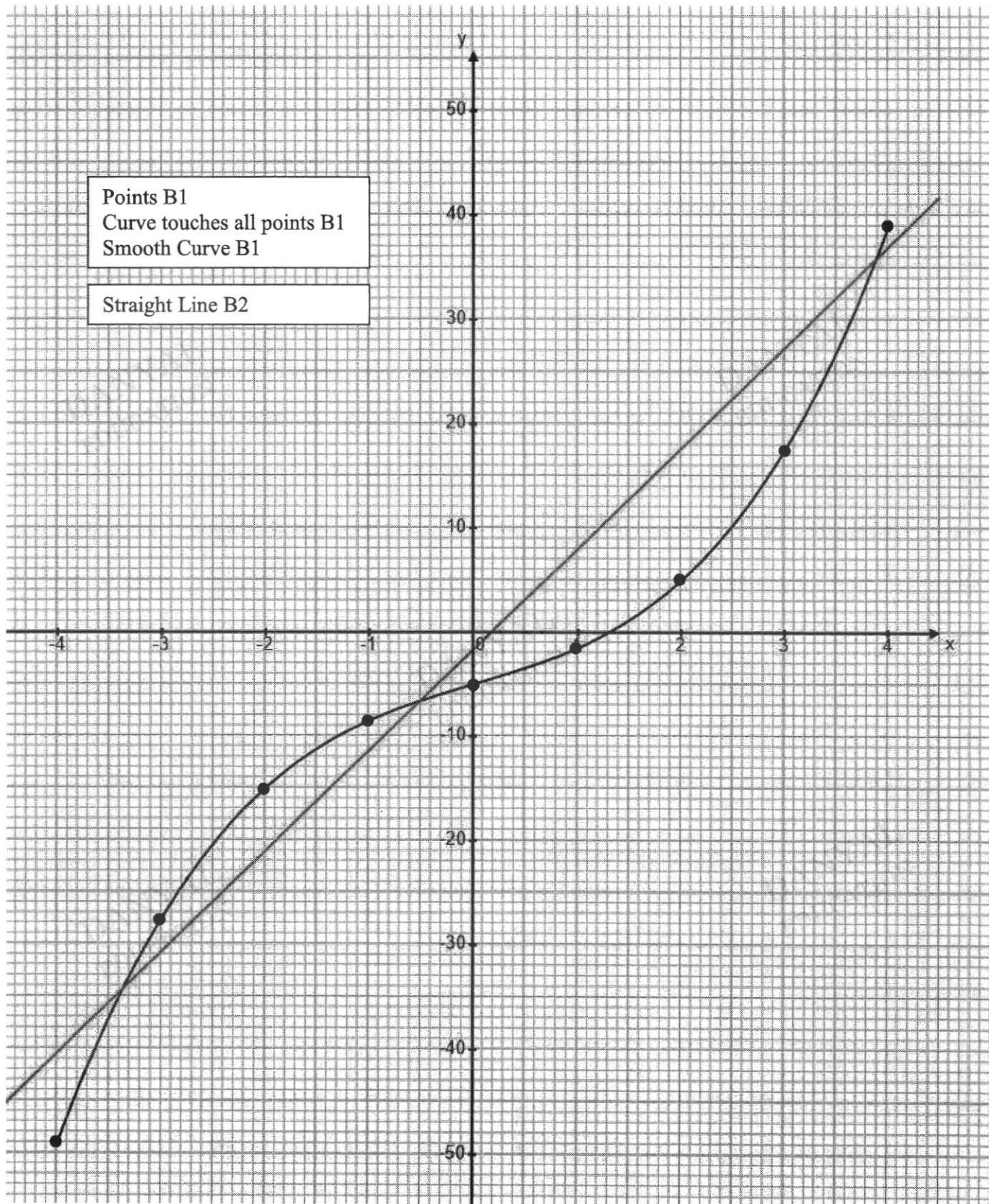|			
3a	Japan ¥: $1 = ¥81.49$ $1299 = 81.49 \times 1299 = ¥105855.51$ Singapore $: ¥81.49 = 1 ¥104280 = 1 ÷ 81.49 × 104280 = 1279.67 Buy from <u>Japan</u> Cheaper by ¥1575.51 $19.33	M1 A1 A1	Must state cheaper by how much
3b	$999\left(1+\dfrac{x}{100}\right)^8 = 1299$ $\left(1+\dfrac{x}{100}\right)^8 = \dfrac{1299}{999}$ $1+\dfrac{x}{100} = \sqrt[8]{\dfrac{1299}{999}}$ $1+\dfrac{x}{100} = 1.033369069$ $x = 3.336906861 = 3.34$	M1 A1	
3ci	$196.9 \times 10^6 - 187.2 \times 10^6$ $= 9.7 \times 10^6$	B1	
3cii	$\dfrac{142-138}{142} \times 100$ $= 2.816901408 = 2.82\%$	M1 A1	
3ciii	$\dfrac{138}{274} \times 100$ $= 50.36496358 = 50.4\%$	M1 A1	
			10
4a	$p = 39$	B1	
4b	Points, Curve passes through all points, Smooth curve	B1,B1,B1	See graph
4c	$x > 2.5$ (2.4 to 2.6)	B1	
4di	Draw straight line	B2	See graph
4dii	$3y - 29x + 5 = 0$ $3y = 29x - 5$ $y = \dfrac{29}{3}x - \dfrac{5}{3}$ $\dfrac{29}{3}x - \dfrac{5}{3} = \dfrac{x^3}{2} + 3x - 5$ $58x - 10 = 3x^3 + 18x - 30$ $3x^3 + 18x - 58x - 30 + 10 = 0$ $3x^3 - 40x - 20 = 0$ (shown)	M1 A1	
4diii	3.75 to 3.95 (3.879652106) −0.6 to −0.4 (−0.5099456443) −3.45 to −3.25 (−3.369706462)	2 correct B1 B2	
			11

5ai	$T_4 = 2 \times 4^2 + 5 = 37$ OR $2 \times 4^2 + 5 = 37$	B1 B1	
5aii	$T_n = 2 \times n^2 + (n + 1)$ or $2n^2 + n + 1$	B1	
5aiii	$2n^2 + n + 1 = 1175$ $2n^2 + n - 1174 = 0$ $n = 23.97937267$ Since n needs to be a positive integer, 1175 cannot be one of the numbers in the sequence	 M1 A1	
5b	$S_n = 3n - 1$	B1	
5c	158	B1	
			6
6aia	$\angle STU = 180° - 50° = 130°$ ($\angle$s in opps seg)	B1	Minus 1 mark from whole question if no reason or wrong reason
6aib	$\angle SQU = \angle SRU = 50°$ ($\angle$s in same seg) $\angle PQU = 180 - 50 = 130°$ ($\angle$s on a str line) $\angle POU = 130° \times 2 = 260°$ ($\angle$ at ctr $= 2\angle$s at circumference)	M1 A1	
6aic	$\angle POU = 360 - 260 = 100°$ $\angle PUO = (180 - 100) \div 2 = 40°$ (isos $\triangle PUO$)	 B1	
6aid	$\angle PUR = 90 - 40 = 50°$ (tangent perpendicular to radius)	 B1	
6aii	$\angle SRU = \angle PUR$ (alternate $\angle$s of parallel lines) RS and PU are parallel	 B1	
6bi	Arc length $PQR = 1.31 \times 7$ Arc length $PQR = 9.17$	 B1	
6bii	Area of sector $= 0.5 \times 7^2 \times 1.31 = 32.095$ Area of $\triangle QST = 0.5 \times 5.2 \times (7 - 3.1) \times \sin 1.31$ Area of $\triangle QST = 9.797115409$ Area of shaded region $= 32.095 - 2(9.797115409)$ $= 12.50076918 = 12.5$ cm^2	M1 M1 A1	
			10

7a	$\dfrac{2500}{x}$	B1	
7b	$\dfrac{2500}{x-10}$	B1	
7c	$\dfrac{2500}{x-10} - \dfrac{2500}{x} = 4 \times 60$	M1	
	$\dfrac{2500}{x-10} - \dfrac{2500}{x} = 240$		
	$2500(x) - 2500(x-10) = 240(x)(x-10)$	M1	
	$2500x - 2500x + 25000 = 240x^2 - 2400x$		
	$240x^2 - 2400x - 25000 = 0$	M1	
	$6x^2 - 60x - 625 = 0$		
7d	$x = \dfrac{-(-60) \pm \sqrt{(-60)^2 - 4(6)(-625)}}{2(6)}$		
	$x = \dfrac{60 \pm \sqrt{18600}}{12}$	M1	
	$x = 16.36515141, -6.365151414$	M1	
	$x = 16.4, -6.4$ (1 dp)	A1	
7e	L+S 1 min ➔ $16.36515141 + 6.36515141\ l$		
	L+S 1 min ➔ $22.73030283\ l$	M1	
	$1\ l$ ➔ $1 \div 22.73030283$		
	$2500\ l$ ➔ $1 \div 22.73030283 \times 2500$	M1	
	Time taken ➔ 109.9853363 min		
	Time taken ➔ 1 hour and 50 mins	A1	
			11

8a	Radius of the hemisphere = 10 m Height of structure = 10 + 15 = 25 m	B1	
8b	Volume of hemisphere $= \dfrac{1}{2} \times \dfrac{4}{3}\pi 10^3 = \dfrac{2000}{3}\pi = 2094.395102$	M1	
	Volume of cuboid $= 20 \times 20 \times 15 = 6000$	M1	
	Volume = 2094.395102 + 6000 Volume = 8094.395102 = 8090 m^3	A1	
8c	SA of cuboid $= 20 \times 20 + 4(20 \times 15) + [20 \times 20 - (\pi \times 10^2)]$ $= 2000 - 100\pi$ $= 1685.840735$	M1	
	SA of hemisphere $= 0.5 \times 4 \times \pi \times 10^2$ $= 200\pi$ $= 628.3185307$	M1	
	Total SA $= 1685.840735 + 628.3185307$ $= 2314.159265$ $= 2310$ m^2	A1	
8d	$AC^2 = 20^2 + 20^2$ $AC = \sqrt{800} = 28.28427125$	M1	
	$\tan \theta = \dfrac{25}{\sqrt{800} \div 2}$	M1	
	$\theta = 60.5037915 = 60.5°$	A1	
			10
9ai	$30 \div 100 \times 40 = 12$	B1	
9aii	Median = 3	B1	
9aiii	Range = 5 − 0 = 5	B1	
9aiv	Interquartile range = 4 − 2 = 2	B1	
9bi	$1 - \dfrac{1}{12} - \dfrac{1}{4} - \dfrac{1}{3} = \dfrac{1}{3}$	B1	
9bii	$\left(\dfrac{1}{12} + \dfrac{1}{3}\right) = \left(\dfrac{1}{4} + \dfrac{1}{3}\right) - x$ $x = \dfrac{1}{6}$	M1	
	$\dfrac{1}{6} \times 60 = 10$	A1	
	OR no. of living $= 60\left(\dfrac{1}{12} + \dfrac{1}{3}\right) = 25$	M1	
	no. of non-living $= 60\left(\dfrac{1}{4} + \dfrac{1}{3}\right) = 35$ $35 - 25 = 10$	A1	
			7

10a	Let h = height of small cone		
	$\dfrac{h}{h+165} = \dfrac{60}{84}$	Ha1	
	$84h = 60h + 9900$		
	$h = 412.5$		
	Total height $= 412.5 + 165 = 577.5$	Ha2	
	Volume of the cup		
	$= \left(\dfrac{1}{3}\pi \times 42^2 \times 577.5\right) - \left(\dfrac{1}{3}\pi \times 30^2 \times 412.5\right)$		
	$= 339570\pi - 123750\pi$		
	$= 1066790.617 - 388772.0909$		
	$= 678018.5265 = 678019$ (shown)	V1	
10a	Slant height$_{\text{Small}} = \sqrt{412.5^2 + 30^2}$		
	Slant height$_{\text{Small}} = 413.5894704$ mm	Ss1	
	Slant height$_{\text{Big}} = \sqrt{577.5^2 + 42^2}$		
		Sb1	
	Slant height$_{\text{Big}} = 579.0252585$ mm		
	SA of cone$_{\text{Small}} = \pi \times 30 \times 413.5894704$		
	SA of cone$_{\text{Small}} = 38979.88925$ mm^2	SAs1	
	SA of cone$_{\text{Big}} = \pi \times 42 \times 579.0252585$		
	SA of cone$_{\text{Big}} = 76400.58293$ mm^2	SAb1	
	SA of circle $= \pi \times 30^2 = 900\pi = 2827.433388$ mm^2		
	SA$_{\text{Paper}} = 76400.58293 - 38979.88925 + \left(2 \times 2827.433388\right)$		
	SA$_{\text{Paper}} = 37420.69368 + 5654.866776$	SAPa1	
	SA$_{\text{Paper}} = 43075.56046$ mm^2		
	Paper sheets needed $= 1500 \times \dfrac{43075.56046}{1000000}$	PaS1	
	Paper sheets needed $= 64.61334069$		
	Cost of Paper $= 64.61334069 \times \$0.62$		
	Cost of Paper $= 40.06027123 = \$40.06$	CPa1	
	OR		
	Cost of Paper $= 65 \times \$0.62 = \$40.30$		
	Use <u>Plastic</u> because it is $0.82 (\$1.06 if use 65 sheets of plastic) cheaper for the material cost.	A1	
			11

Points B1
Curve touches all points B1
Smooth Curve B1

Straight Line B2