



**AHMAD IBRAHIM SECONDARY SCHOOL**  
**GCE O-LEVEL PRELIMINARY EXAMINATION 2021**

**SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC**

Name:	Class:	Register No.:
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**MATHEMATICS**

Paper 1

**4048/01**

**23 August 2021**

**2 hours**

Candidates answer on the Question Paper.

**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any questions it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

**For Examiner's Use**

**/80**

**Mathematical Formulae***Compound Interest*

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

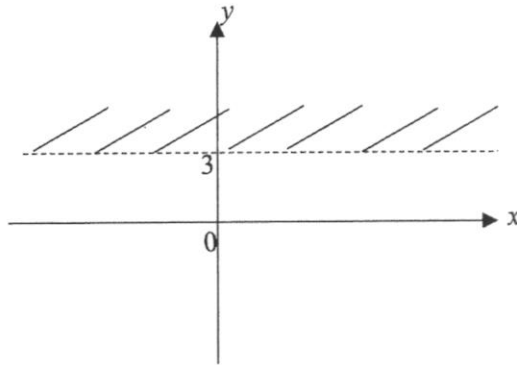
Answer **all** the questions.

- 1 Insert one pair of brackets to make the statement correct.

$$5 - 2 + 3 \times 2 = -5$$

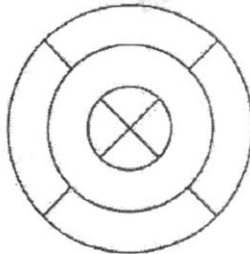
[1]

- 2 Write down an inequality that defines the points in the shaded region.



Answer ..... [1]

3



For the diagram above, write down

- (a) the number of lines of symmetry

Answer ..... [1]

- (a) the order of rotational symmetry.

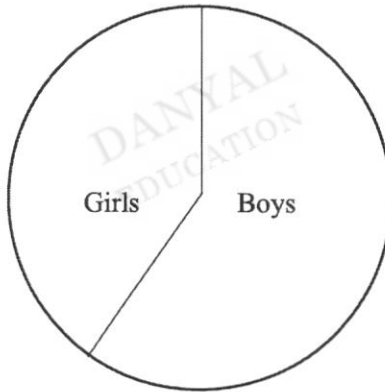
Answer ..... [1]

- 4 The bearing of a lighthouse from a coastguard station is  $115^\circ$ .  
What is the bearing of the coastguard station from the lighthouse?

Answer ..... [2]

- 5 The pie chart shows the number of girls and boys who scored full marks in a Mathematics test. Based on the chart, June concluded that on average, boys did better than girls in Mathematics. Explain why June made that conclusion and why she may be wrong.

**Students who scored full marks in Mathematics**



Answer .....

.....

.....

.....

..... [2]

- 6  $A$  is the point  $(3, 6)$  and  $B$  is the point  $(m, n)$  where  $m$  and  $n$  are both positive integers less than 10.

The length of the line segment is  $\sqrt{34}$  units.

Find the coordinates of one of the possible positions for  $B$ .

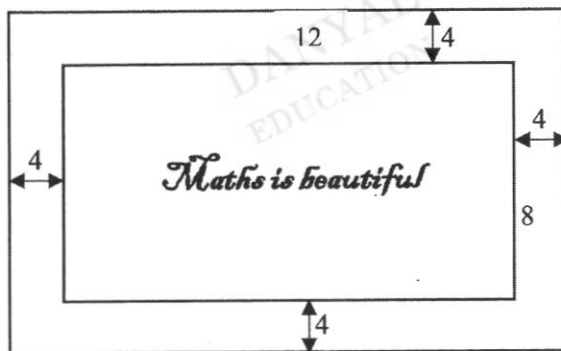
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Answer

(....., .....) [2]

7



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A rectangular poster with the words “Maths is beautiful”, 12 cm long and 8 cm wide, fits into a rectangular frame so that there is a border of 4 cm wide all the way round it. Use calculations to determine whether the two rectangles are similar.

Answer

[2]

- 8 A boy who was asked to find  $3\frac{1}{2}\%$  of a sum of money misread the question and found  $5\frac{1}{2}\%$  of it.  
His answer was \$247.10.  
Calculate what the answer should be.

*Answer* \$..... [2]

- 9 Mr Heng goes on a holiday for  $x$  days and takes \$ $f$  with him to spend.  
He spends \$ $g$  each day for the first  $y$  days.  
Write down expressions for

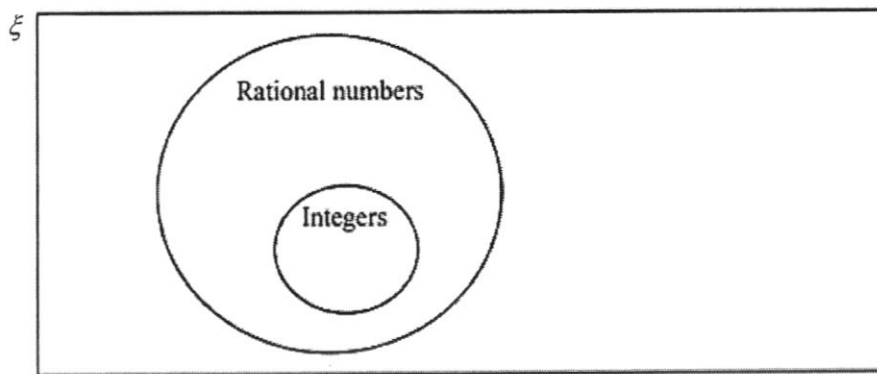
- (a) the number of dollars he has left after  $y$  days,

*Answer* \$..... [1]

- (b) the number of dollars he may spend each day for the rest of his holiday, without running out of money.

*Answer* \$..... [1]

10 (a)



On the Venn Diagram above, write each of these numbers in the appropriate subsets.

$$2.6, \frac{4}{17}, \sqrt{12}, \sqrt{\frac{112}{7}}$$

[2]

- (b) Three sets  $A$ ,  $B$  and  $C$  are such that  $A \subset C$ ,  $B \subset C$  and  $A \cap B = \emptyset$ . Draw a Venn diagram to show this information.

[1]

- 11 (a)  $2^8 = 4^x + 4^x + 4^x + 4^x$  .  
Find the value of  $x$ .

*Answer*  $x = \dots\dots\dots$  [2]

- (b) Write down the positive square root of  $\frac{4x^2 - 12x + 9}{25x^{16}}$  .

*Answer*  $\dots\dots\dots$  [1]

- 12 (a) Find the prime factors of 4410, giving your answer in index form.

*Answer*  $\dots\dots\dots$  [2]

- (b) The highest common factor of two numbers is 245.  
The lowest common multiple of these two numbers is 4410.  
Both numbers are greater than their highest common factor.  
Find the two numbers.

*Answer*  $\dots\dots\dots$  and  $\dots\dots\dots$  [2]



- 13 The mean and median of four numbers are 54 and 56 respectively.  
Find the mean of the largest and the smallest numbers.

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EDUCATION

*Answer* .....[3]

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14 Consider the number pattern below.

$$\frac{1}{11} = 0.090909\dots = 0.\dot{0}9$$

$$\frac{2}{11} = 0.181818\dots = 0.\dot{1}8$$

$$\frac{3}{11} = 0.272727\dots = 0.\dot{2}7$$

$$\frac{4}{11} = 0.363636\dots = 0.\dot{3}6$$

•

•

•

$$\frac{x}{11} = 636.3636\dots = 636.\dot{3}6$$

(i) Write down the 5<sup>th</sup> line of the pattern.

Answer ..... [1]

(ii) Find the value of  $x$ .

Answer  $x = \dots$  [2]

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15 (a) Factorise fully  $(a-b+c)^2 - (b-c+a)^2$ .

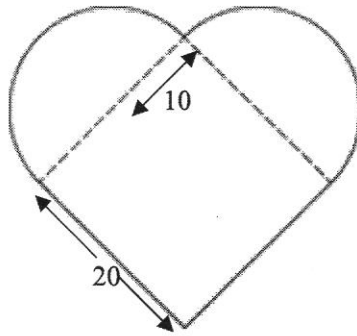
*Answer* ..... [2]

(b) Calculate the value of  $k$  if  $x^2 - 6x + k$  is a perfect square.

*Answer*  $k =$  ..... [2]

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16



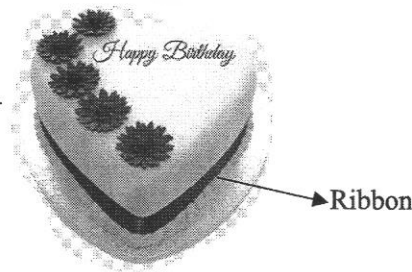
Instructions for baking a heart-shaped cake.

- Bake a square cake of side 20 cm.
- Bake two semicircle cakes of radius 10 cm.
- Join the two halves to the square cake, as shown in the diagram.

(a) Find the area of the top view of the cake.

Answer ..... cm<sup>2</sup> [1]

(b)



A red ribbon is fixed around the sides for the heart-shaped cake with the sides overlapping by 3 cm.  
Find the length of ribbon required.

Answer ..... cm [2]

- 17 The safe speed,  $v$  m/s, at which a train can go round a curve of radius  $r$  m is directly proportional to the square root of  $r$ .  
It is given that the safe speed at which a train can go round a curve of radius 121m is 22 m/s.

(i) Find the safe speed if the curve has a radius of 81m.

*Answer* ..... m/s [2]

- (i) A change in the safe speed produces a change in the radius.  
When the safe speed is halved, the value of radius changes by factor  $m$ .  
Find  $m$ .

*Answer*  $m =$ ..... [2]

18 In this question, the weather is either wet or dry.

When the weather is dry, the probability that Sarah will go walking is  $\frac{3}{5}$ .

When the weather is wet, the probability that Sarah will go walking is  $\frac{1}{10}$ .

The probability of a dry day is  $\frac{2}{3}$ .

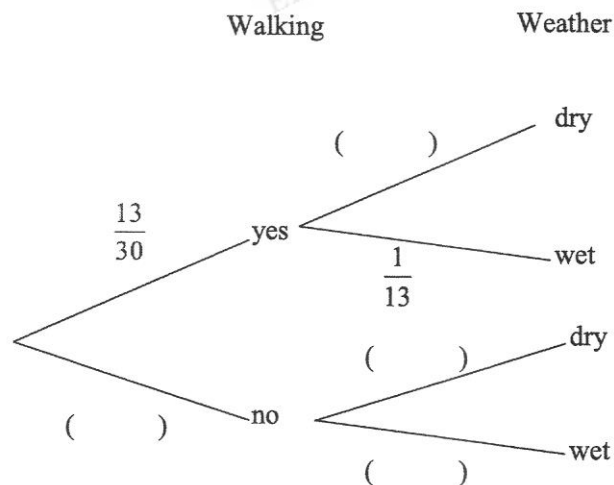
(a) Show that the probability that Sarah goes walking is  $\frac{13}{30}$ .

*Answer*

[2]

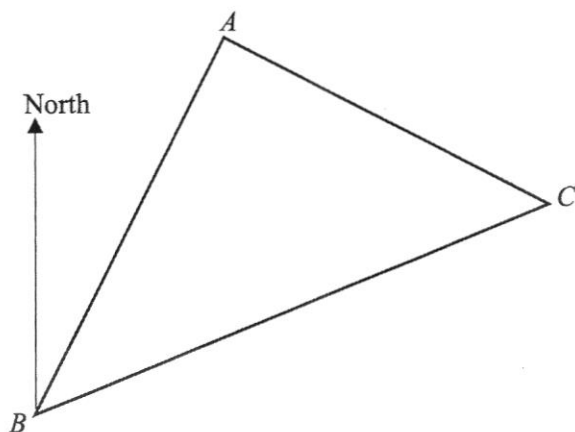
(b) The probability that Sarah does not go walking when the weather is wet is  $\frac{9}{30}$ .

Complete the tree diagram to show this information.



[2]

19

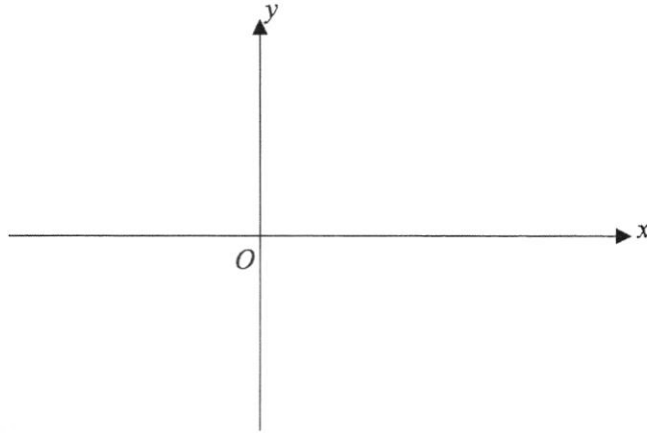


The diagram is a plan of a triangular field  $ABC$ , drawn to a scale of 1 cm to 50 m.  
 A tree,  $T$ , in the field is 150 m from  $A$  and is equidistant from  $BA$  and  $BC$ .

- (a) By making appropriate constructions on the diagram, indicate clearly the position of  $T$ . [2]
- (b) Use the diagram to find
- (i) the distance, in metres, from  $T$  to  $B$ , [1]
  - (ii) the bearing of  $T$  from  $B$ . [1]

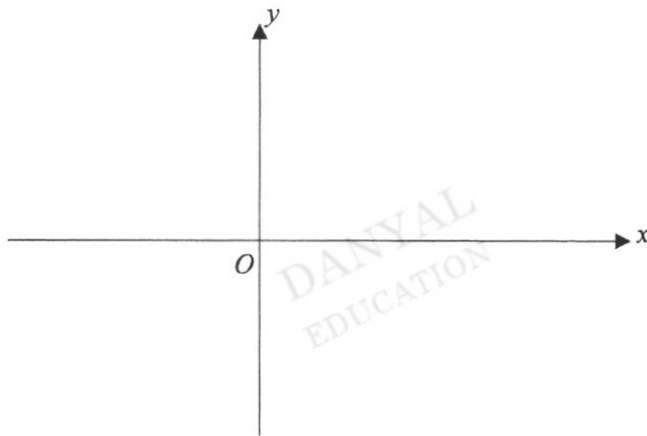
20 (a) On the axes below, sketch the graphs, indicating clearly the  $x$  and  $y$  intercepts if any.

(i)  $y = 3^x$



[1]

(ii)  $y = -3x^2$



[1]

(b) Hence, state the number of solution(s) to the equation  $3^x + 3x^2 = 0$ .  
Explain your answer.

Answer .....

.....

.....

..... [2]



- 21 (a) The solution of the inequality  $\frac{ax-1}{7} \leq \frac{3a+x}{4}$  is  $x \leq 46$ .

Show that  $a = 2$ .

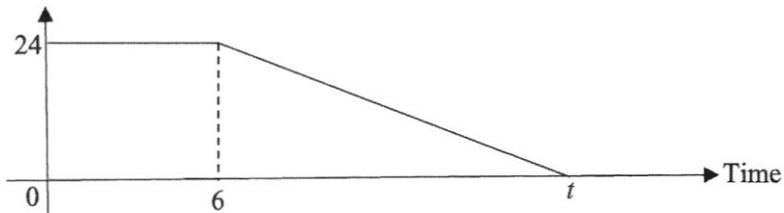
[3]

- (b) Hence, find a possible value of  $x$  if  $x$  is a perfect square and satisfies the simultaneous inequalities  $\frac{ax-1}{7} \leq \frac{3a+x}{4} < \frac{x-5}{2}$ .

Answer  $x = \dots\dots\dots$  [2]

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22 Speed (m/s)



The diagram shows the speed-time graph of an object which travels at a constant speed of 24 m/s for 6 seconds before slowing down at a rate of  $2 \text{ m/s}^2$  and coming to a rest after  $t$  seconds.

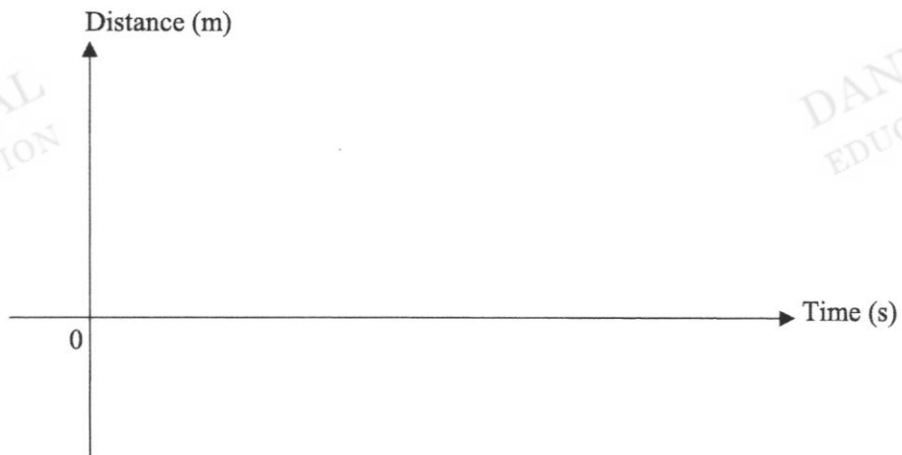
(a) Find the value of  $t$ .

Answer  $t = \dots\dots\dots$  [2]

(b) Calculate the average speed for the whole journey.

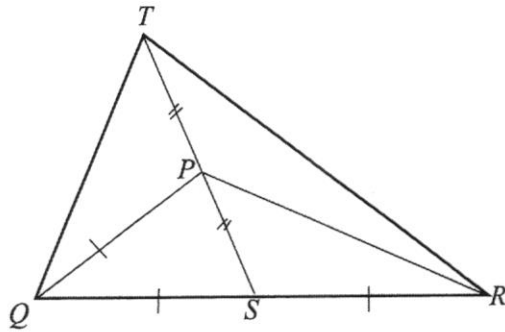
Answer  $\dots\dots\dots \text{m/s}$  [2]

(c) Sketch the distance-time graph of the object for the whole journey on the axes below.



[2]

23



$TQR$  is a triangle with  $SP = PT$  and  $QP = QS = SR$ .

- (a) Explain why angle  $TPQ =$  angle  $PSR$ .

*Answer*

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[2]

- (b) Show that  $TQ = PR$ .  
Give a reason for each statement you made.

*Answer*

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DANYAL  
EDUCATION

[3]

- (c) Show that triangle  $QPS$  and triangle  $PSR$  are equal in area.

*Answer*

[2]

- 24 Before the Chinese New Year (CNY) celebration, Mr Lee puts a combination of \$2 and \$10 notes into four types of red packets. Friends and relatives who visited Mr Lee during the CNY period are then given the appropriate red packets.

Design A contains  $x$  \$2 notes and no \$10 notes.

Design B contains equal number of \$2 notes and \$10 notes. Its number of \$2 notes is fewer than that of Design A by 2.

Design C contains twice as many \$10 notes as Design B. There are no \$2 notes in Design C.

Design D contains three times as many \$2 notes as Design A. The number of \$10 notes in Design D is equal to the number of \$10 notes in Design B.

- (a) (i) Complete the following table using the given information.

	Design			
	A	B	C	D
Number of \$2 notes				
Number of \$10 notes				

[1]

- (ii) Represent the data in the above table by a  $2 \times 4$  matrix  $N$ .

Answer  $N =$

[1]

- (b) The number of designs  $A$ ,  $B$ ,  $C$  and  $D$  red packets that were given away during the CNY

period can be represented by  $M = \begin{pmatrix} 20 \\ 10 \\ 6 \\ 5 \end{pmatrix}$ .

- (i) Find, in terms of  $x$ , the matrix  $NM$ .

Answer  $NM =$

[2]

- (iii) State what each element in matrix  $\mathbf{NM}$  represents.

*Answer* .....

.....

.....

..... [1]

- (c) The matrix  $\mathbf{D} = \begin{pmatrix} 2 & 10 \end{pmatrix}$ .  
Find, in terms of  $x$ , the matrix  $\mathbf{DNM}$ .

*Answer*  $\mathbf{DNM} =$  [1]

- (d) Find the value of  $x$  if the total amount of red packet money given away was \$500.

*Answer*  $x =$  ..... [1]

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**End of Paper**

Setter: Mrs Foo SG



**AHMAD IBRAHIM SECONDARY SCHOOL  
PRELIMINARY EXAMINATION 2021**

**SECONDARY 4 EXPRESS / 5 Normal (Academic)**

Name:	Class:	Register No.:
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**MATHEMATICS**

Paper 2

4048/02

25 Aug 2021

2 hours 30 minutes

Candidates answer on the Question Paper.

**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any questions it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

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For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

**For Examiner's Use**

**/100**

**Mathematical Formulae***Compound Interest*

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

- 1 (a) Write as a single fraction in its simplest form  $\frac{5}{5-2y} - \frac{2}{y+5}$ .

*Answer* ..... [2]

- (b) It is given that  $\frac{a}{h} = b\left(1 + \frac{k}{h}\right)$ , express  $h$  in terms of  $a$ ,  $b$  and  $k$ .

*Answer*  $h =$  ..... [3]



(c) Solve the equation  $\frac{6}{x-6} = 2 - \frac{1}{x-8}$ .

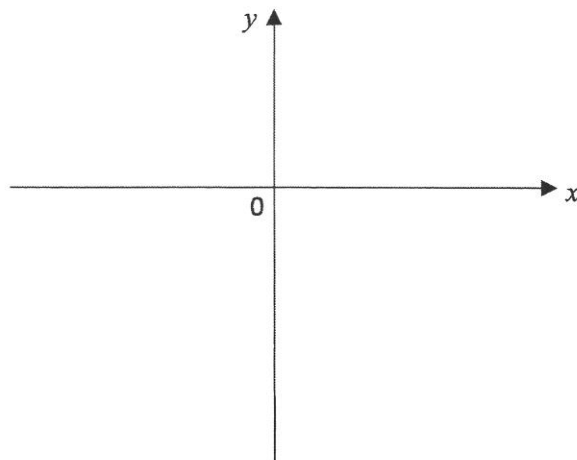
*Answer*  $x = \dots\dots\dots$  or  $\dots\dots\dots$  [3]

(d) (i) Express  $-25 + 3x + x^2$  in the form  $p + (q + x)^2$ .

*Answer*  $\dots\dots\dots$  [1]

(ii) Hence sketch the graph of  $y = -25 + 3x + x^2$  on the axes below.

Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point on the curve. [2]



- 2 (a) Mr Tan owns several cake shops in Singapore and Malaysia.  
 The price of a one-kilogram cake is \$35 in Singapore.  
 The price of the same cake in Malaysia is MYR 96.  
 The exchange rate between Singapore dollars (\$) and Malaysian Ringgit (MYR) is  
 \$1 = MYR 3.06.

Calculate how much more expensive the cake is in Singapore than in Malaysia.

*Answer* \$ ..... [1]

- (b) Mr Tan is planning to expand his cake shop business in Singapore.  
 His business manager has provided the visitor arrival data from the Singapore Tourism Board.

In 2019, the total visitor arrivals to Singapore was  $185.3 \times 10^5$ .

- (i) Calculate the mean number of visitor arrivals to Singapore each day in 2019.  
 Give your answer in standard form, correct to 3 significant figures.

*Answer* ..... [1]

- (ii) From 2018 to 2019, the number of visitor arrivals to Singapore decreased by 21.2%.  
 Calculate the number of visitor arrivals to Singapore in 2018.  
 Give your answer in millions, correct to 3 significant figures.

*Answer* ..... millions [2]

- (c) In January 2020, Mr Tan invested a sum of money in a bank paying a compound interest at 2.8% per annum.  
After one year, he received \$95776.75 from his investment.

Calculate the total interest he received in January 2021, giving your answer to the nearest dollar.

*Answer* \$ ..... [3]

- (d) Mr Tan needs to pay his Malaysia supplier MYR 2000 for the cake ingredients.  
The transportation fees for the ingredients were MYR 60.  
A 7% Goods and Services Tax (GST) is payable on the cost of goods and transportation that exceeds \$400.

Calculate the amount of money, in Singapore dollars, that Mr Tan must pay to import the ingredients.

*Answer* \$ ..... [2]

- 3 (a) Complete the table of values for  $y = 3x^2 + \frac{5}{x} - 4$ .

Find the value of  $p$  correct to 2 decimal places.

$x$	0.4	0.6	0.8	1.0	1.2	1.4	1.6
$y$	8.98	5.41	4.17	4.00	4.49	$p$	6.81

Answer  $p = \dots\dots\dots$  [1]

- (b) On the grid opposite, draw the graph of  $y = 3x^2 + \frac{5}{x} - 4$  for  $0.4 \leq x \leq 1.6$ . [3]

- (c) Explain why the equation  $3x^2 + \frac{5}{x} - 4 = k$  does not have solutions for some values of  $k$ . [2]

Answer

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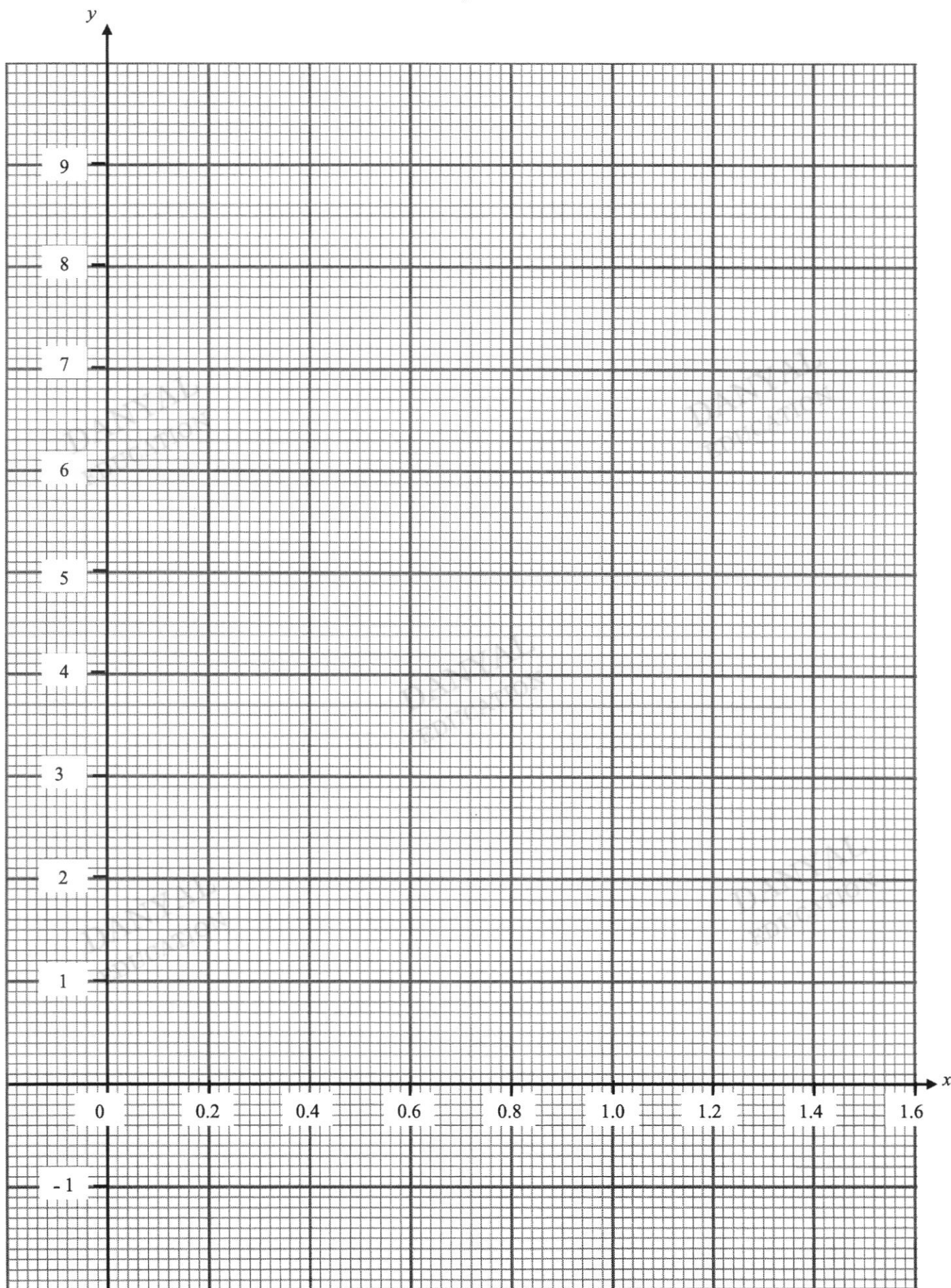
- (d) The equation  $3x^3 - 2x^2 - 9x + 5 = 0$  can be solved by drawing a suitable straight line on the grid.

- (i) Find the equation of the straight line.

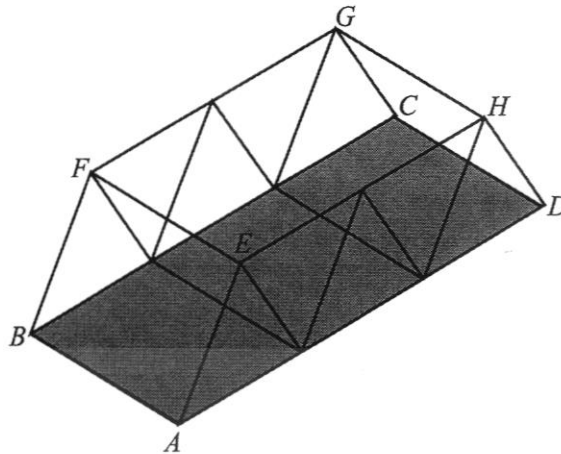
Answer ..... [1]

- (ii) By drawing this straight line, solve the equation  $3x^3 - 2x^2 - 9x + 5 = 0$  for  $0.4 \leq x \leq 1.6$ .

Answer  $x = \dots\dots\dots$  [2]



4



The figure shows a drawing of a bridge.

Each side of the bridge is made up of 5 equilateral triangles.

The shaded rectangular area  $ABCD$  is the base of the bridge.

$HD = 5.5$  m.

- (a) Calculate  $DE$ .

Answer ..... m [3]

A point,  $X$ , lies on  $BC$  and is vertically below point  $G$ .

- (b) (i) Angle  $AXD = 79.5^\circ$  and angle  $ADX = 65.0^\circ$ .  
Show that  $AX = 15.2$  m, correct to 3 significant figures.

Answer

[2]

- (ii) Calculate the angle of elevation of  $G$  from  $A$ .

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*Answer* ..... [3]

- (c) The greatest angle of elevation of  $G$  from a point along  $AD$  occurs at  $Y$ .  
Describe the relationship between  $XY$  and  $AD$ .

*Answer*

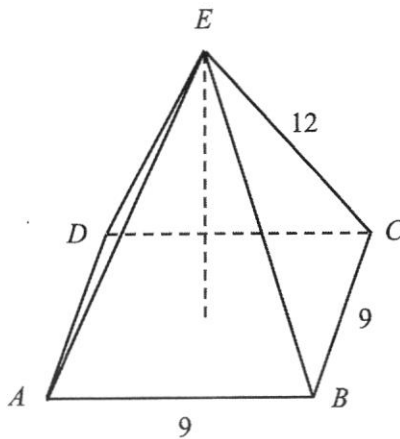
.....  
..... [1]

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5



The diagram shows a pyramid  $ABCDE$ .  
 The base of the pyramid is a square of side 9 cm.  
 $E$  is vertically above the centre of the square base.  
 $AE = BE = CE = DE = 12$  cm.

- (a) Find the total surface area of the pyramid  $ABCDE$ .

Answer .....  $\text{cm}^2$  [3]

The volume of the pyramid is  $275 \text{ cm}^3$ .  
 This metal solid is melted and molded into two identical spherical metal balls.

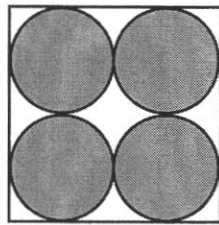
- (b) Find the radius of each metal ball.

Answer ..... cm [2]

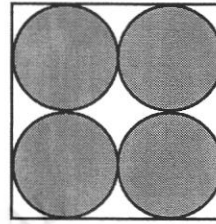


- (c) Given that one bucket of paint is able to paint an area of  $1200 \text{ cm}^2$ , how many buckets of paint are needed to paint 1500 of such metal balls?

Answer ..... [3]



Top view



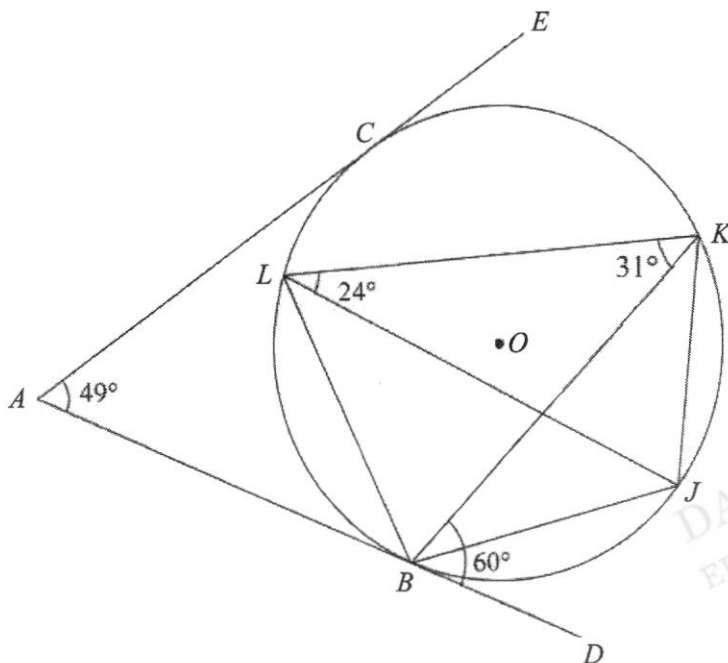
Side view

The diagram shows the top and side view of a box holding some spherical balls. The box is in the shape of a cube and the spherical balls just fit into the box.

- (d) Calculate the volume of empty space in the box.

Answer .....  $\text{cm}^3$  [3]

6 (a)



$B, C, J, K$  and  $L$  are points on the circle, centre  $O$ .  
 $AE$  and  $AD$  are tangents to the circle.  
 Angle  $JLB$  is equal to angle  $JBD$ .  
 Angle  $BAC = 49^\circ$ , angle  $JLK = 24^\circ$ , angle  $KBD = 60^\circ$  and angle  $BKL = 31^\circ$ .

Find, giving reasons for each answer,

(i) angle  $JBK$ ,

Answer ..... [1]

(ii) angle  $KBL$ ,

Answer ..... [2]

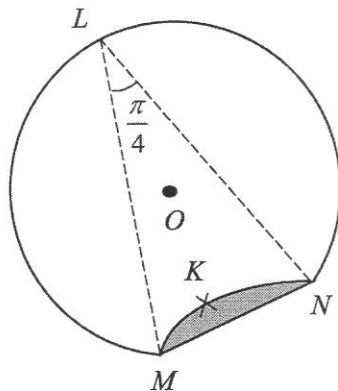
(iii) reflex angle  $KOL$ ,

Answer ..... [2]

(iv) angle  $COB$ .

Answer ..... [2]

(b)



The diagram shows a circle made of paper with centre  $O$ .

A segment of the circle is folded inwards.

Angle  $MLN = \frac{\pi}{4}$  radians and arc length  $MKN = 6$  cm

Calculate the length of the major arc  $MLN$ .

Answer ..... cm [4]

- 7 Two groups of students were surveyed on their weekly screen time in hours. The results are shown in the table.

14	32	55	24	56
23	48	17	36	59
<b>Group 1</b>				

42	23	29	25	19
23	90	45	35	29
<b>Group 2</b>				

- (a) 20% of students in Group 2 recorded a screen time of at least  $x$  hours. Find  $x$ .

*Answer* ..... hours [1]

- (b) Complete the table below. [2]

	Group 1	Group 2
Median	34	
Interquartile Range		19

- (c) Make a comparison between the screen times of the students in these two groups.

*Answer*

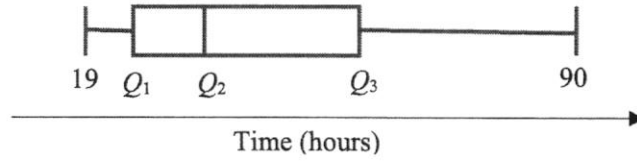
..... [1]

- (d) Explain why median is a better measure for average screen time for the two groups of students compared to the mean.

*Answer*

..... [1]

- (e) (i) The box-and-whisker plot for the distribution of screen time for students in Group 2 is drawn in the grid below. Write down the value for  $Q_1$ ,  $Q_2$  and  $Q_3$ .



Answer  $Q_1 = \dots\dots\dots$  hours

$Q_2 = \dots\dots\dots$  hours

$Q_3 = \dots\dots\dots$  hours

[2]

- (ii) Due to a recording error, the recorded amount of screen time for Group 1 is 1 hour more than the actual screen time.

State how the median and standard deviation of the screen times have been affected by this error.

Answer

.....

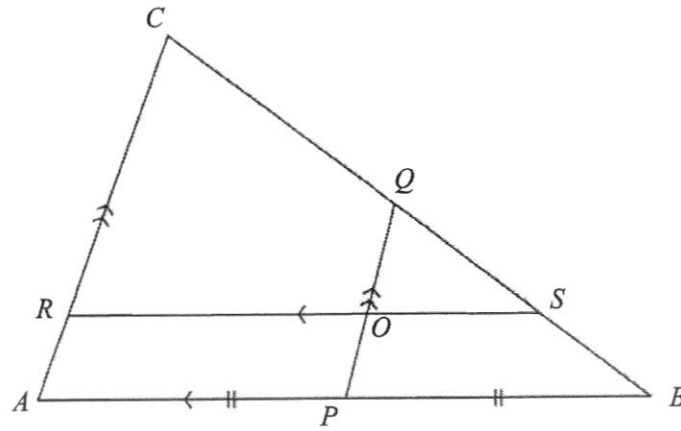
.....

.....

.....

[2]

8 (a)



In the diagram,  $SR$  is parallel to  $BA$  and  $PQ$  is parallel to  $AC$ .  
 $P$  is the midpoint of  $AB$  and  $BS : SC = 1 : 4$ .

(i) Prove that triangles  $CRS$  and  $CAB$  are similar.

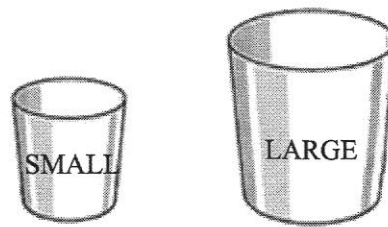
[2]

*Answer*

(ii) If the area of triangle  $ABC = 120 \text{ cm}^2$ , find the area of  $RSBA$ .

*Answer* .....  $\text{cm}^2$  [2]

(b)



The diagram shows two geometrically similar can.  
A soup manufacturer wants to fill the cans with soup.  
The ratio of the heights of two geometrically similar cans is 4 : 9.  
The surface area of the small can is  $240 \text{ cm}^2$ .

By considering the materials and volume of the soup the cans can hold, determine which can is more cost effective for the soup manufacturer.  
Use calculations to justify your answer.

[4]

*Answer*

DANYAL  
EDUCATION

DANYAL  
EDUCATION

DANYAL  
EDUCATION

- 9 (a) The first three terms in a sequence of numbers are given below.

$$Q_1 = 2^3 + 3 = 11$$

$$Q_2 = 2^4 + 5 = 21$$

$$Q_3 = 2^5 + 7 = 39$$

- (i) Find an expression, in terms of  $n$ , for  $Q_n$ .

Answer ..... [2]

- (ii) Explain why the value of  $Q_n$  must be odd for all values of  $n$ .

Answer

.....  
 ..... [1]

- (iii) Show that  $2^{n-2} - 2^{n-3} = 2^{n-3}$

Answer ..... [2]

- (b) In a sequence, the same number is subtracted each time to obtain the next term.  
 The first five terms of the sequence are

$$p, q, 2, r, -16$$

- (i) Write down an expression for the  $n$ th term of this sequence.

Answer ..... [2]



- (ii) The sum of the  $m$ th term and its previous term is  $-167$ .  
Find the value of  $m$ .

Answer ..... [2]

- (c) The table gives information about the ages of the members in the *Fitness First* club in Singapore.

	Members aged under 40	Members aged 40 or over
Male	40	24
Female	32	34

- (i) One of these members is selected at random.  
Find, as a fraction in its lowest terms, the probability that he or she is under 40.

Answer ..... [1]

- (ii) Two of the members are selected at random.  
Find the probability that

- (a) both members are female,

Answer ..... [2]

- (b) they are both aged 40 or over, but only one is a male member.

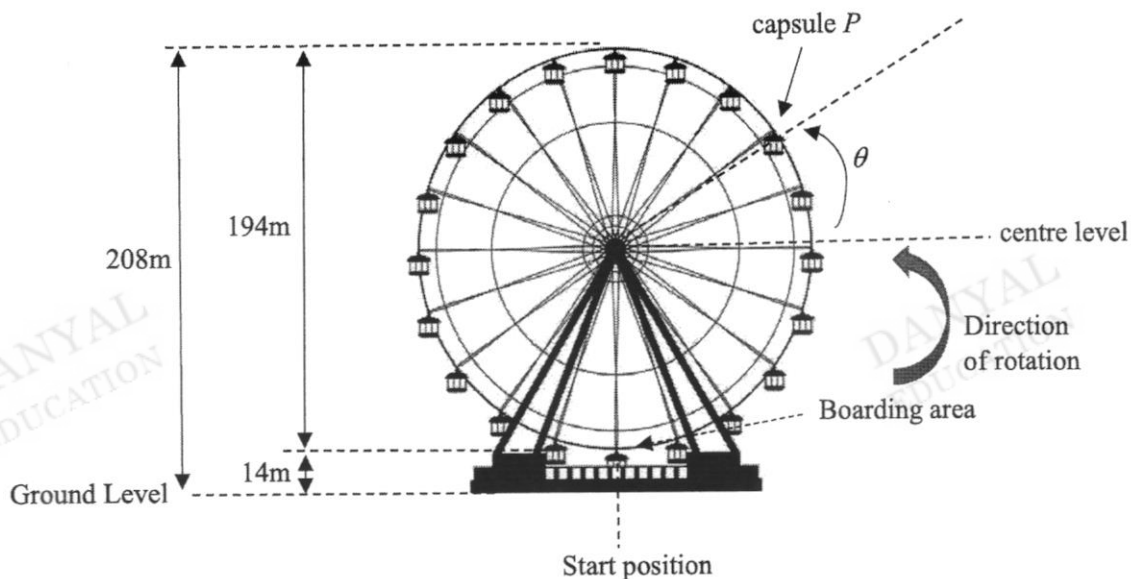
Answer ..... [2]

- 10 The Beijing Great Wheel is the world's largest Giant Observation Wheel and one of East Asia's biggest tourist attractions.

The Giant Observation Wheel has a height of 208 m and a diameter of 194 metres.

It rotates at the speed of 0.3 m/s.

Angle of rotation is measured from the start position in an anti-clockwise direction.



- (a) Find the time taken by a capsule to complete 1 revolution.  
Express your answer in minutes and seconds, correct to the nearest second.  
Take  $\pi$  as 3.142.

Answer ..... minutes ..... seconds [2]

- (b) The state square garden is 1km away from the Great Observation Wheel.  
There are fireworks displays at the garden from 8.45 pm to 8.50 pm and the best view for the fireworks occurs when the capsule P is at least 170 metres above the ground.

- (i) Find angle  $\theta$  when capsule P is exactly 170 metres above the ground level.

Answer  $\theta =$  ..... [2]

- (ii) Find the angle from the start position to the best viewing point where capsule  $P$  is.

*Answer* ..... [1]

- (c) A tourist family wants to get the best view of the **FULL** length of the fireworks display. They board the capsule at 8.32 pm and claims that they can enjoy the full length of fireworks display.

Do you agree with the claim?

Show working to support your answer and state one assumption that you made. [4]

---

**End of Paper**

Setter: Mr Tee Hong Heng



**AHMAD IBRAHIM SECONDARY SCHOOL**  
**GCE O-LEVEL PRELIMINARY EXAMINATION 2021**

**SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC**

Name: <b>MARKING SCHEME</b>	Class:	Register No.:
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**MATHEMATICS**

Paper 1

**4048/01**  
**23 August 2021**  
**2 hours**

Candidates answer on the Question Paper.

**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number on all the work you hand in.  
Write in dark blue or black pen.  
You may use an HB pencil for any diagrams or graphs.  
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any questions it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

**For Examiner's Use**

**/80**

**Mathematical Formulae***Compound Interest*

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

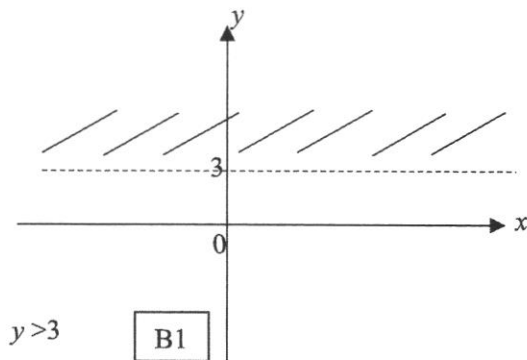
- 1 Insert one pair of brackets to make the statement correct.

$$5 - (2 + 3) \times 2 = -5$$

B1

[1]

- 2 Write down an inequality that defines the points in the shaded region.



Answer ..... [1]

3

For the diagram above, write down

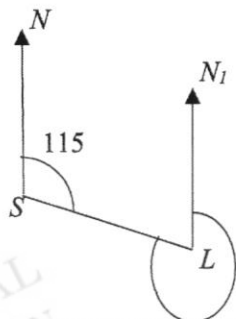
- (a) the number of lines of symmetry 4 B1

Answer ..... [1]

- (a) the order of rotational symmetry. 4 B1

Answer ..... [1]

- 4 The bearing of a lighthouse from a coastguard station is  $115^\circ$ .  
What is the bearing of the coastguard station from the lighthouse?



$$\angle SLN_1 = 180^\circ - 115^\circ \text{ (int angles, } SN \parallel N_1L) \\ = 65^\circ$$

M1

$$\text{Required bearing} = 360^\circ - 65^\circ \text{ (angles at a point)} \\ = 295^\circ$$

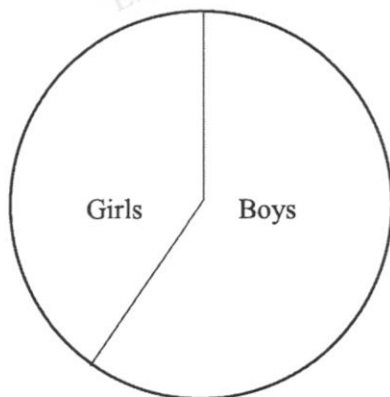
A1

NOTE: no reasons or not stating what they are finding – minus 1 mark

Answer ..... [2]

- 5 The pie chart shows the number of girls and boys who scored full marks in a Mathematics test. Based on the chart, June concluded that on average, boys did better than girls in Mathematics. Explain why June made that conclusion and why she may be wrong.

Students who scored full marks in Mathematics



Reason: From the pie – chart, angle/ area of the sector/ portion of circle representing boys who scored full marks is bigger than the angle of the sector representing girls who scored full marks.

B1

Why wrong: It only shows that there are more boys than girls who scored full marks but we cannot infer the performance of the boys as compared to the girls with just this information.

B1

- 6  $A$  is the point  $(3, 6)$  and  $B$  is the point  $(m, n)$  where  $m$  and  $n$  are both positive integers less than 10.

The length of the line segment is  $\sqrt{34}$  units.

Find the coordinates of one of the possible positions for  $B$ .

$$\sqrt{(m-3)^2 + (n-6)^2} = \sqrt{34}$$

$$(m-3)^2 + (n-6)^2 = 34$$

M1

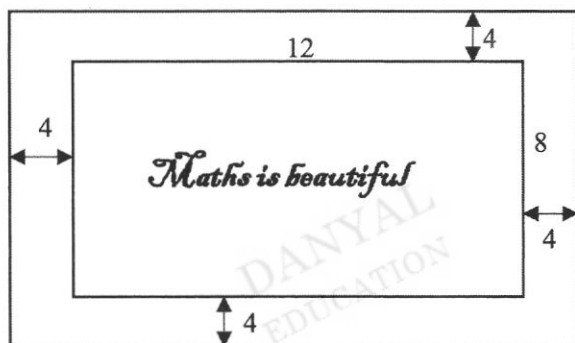
$$(8-3)^2 + (9-6)^2 = 34$$

$B$   $(8, 9)$  or  $(8, 3)$  or  $(6, 1)$

A1

Answer (.....,.....) [2]

7



A rectangular poster with the words "Maths is beautiful", 12 cm long and 8 cm wide, fits into a rectangular frame so that there is a border of 4 cm wide all the way round it.

Use calculations to determine whether the two rectangles are similar.

Answer

$$\frac{20}{16} \neq \frac{12}{8} \text{ or } \frac{20}{12} \neq \frac{16}{8}$$

M1

The two rectangles are not similar because the ratio of their corresponding sides are not equal. (need to state the highlighted very clearly)

A1



- 8 A boy who was asked to find  $3\frac{1}{2}\%$  of a sum of money misread the question and found  $5\frac{1}{2}\%$  of it.  
His answer was \$247.10.  
Calculate what the answer should be.

Let sum of money =  $x$

$$\frac{5.5}{100}x = 247.10$$

M1

$$x = 4492.727273$$

$$\text{Answer} = \frac{3.5}{100} \times \$4492.727273$$

A1

$$=\$157.25 \text{ (2 d.p)}$$

NOTE: if leave answer to 3 s.f, minus 1 mark from whole paper

Answer \$..... [2]

- 9 Mr Heng goes on a holiday for  $x$  days and takes \$ $f$  with him to spend.  
He spends \$ $g$  each day for the first  $y$  days.  
Write down expressions for

- (a) the number of dollars he has left after  $y$  days,

$$\text{Spent} = \$gy$$

B1

$$\text{Left} = \$(f - gy)$$

Answer \$..... [1]

- (b) the number of dollars he may spend each day for the rest of his holiday, without running out of money.

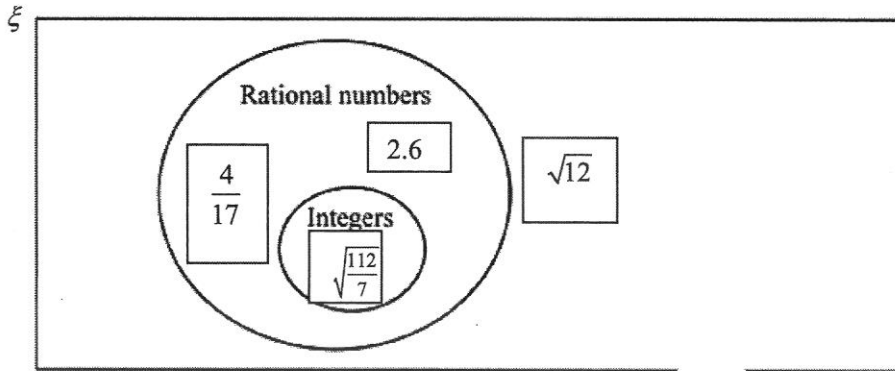
Number of days left =  $x - y$

$$\text{Amount he can spend each day} = \$ \left( \frac{f - gy}{x - y} \right)$$

B1

Answer \$..... [1]

10 (a)



On the Venn Diagram above, write each of these numbers in the appropriate subsets.

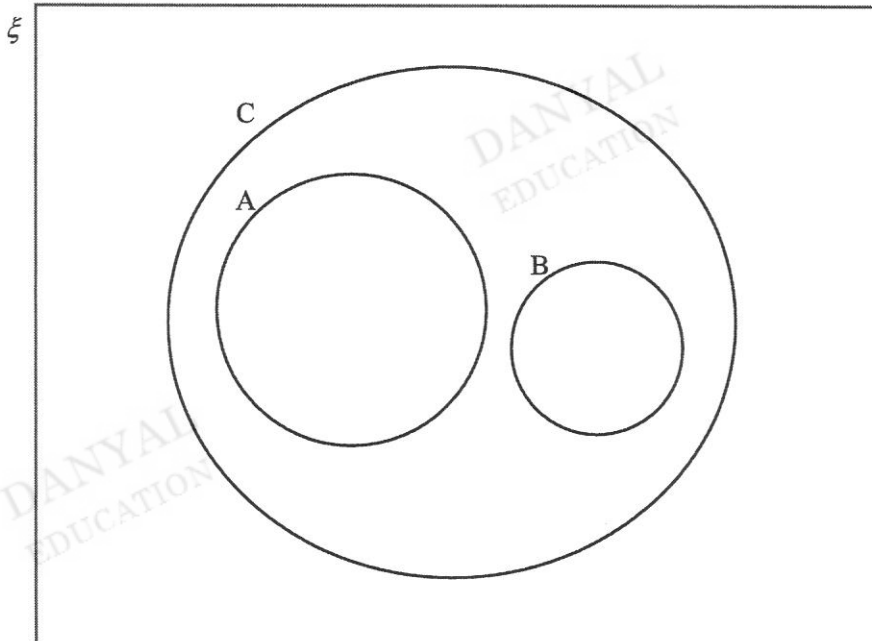
$$2.6, \frac{4}{17}, \sqrt{12}, \sqrt{\frac{112}{7}}$$

B2, all correct

B1, 2 correct

[2]

- (b) Three sets  $A$ ,  $B$  and  $C$  are such that  $A \subset C$ ,  $B \subset C$  and  $A \cap B = \emptyset$ .  
Draw a Venn diagram to show this information.



B1

[1]

- 11 (a)  $2^8 = 4^x + 4^x + 4^x + 4^x$ .  
Find the value of  $x$ .

$$2^8 = 4(4^x)$$

$$2^8 = 2^2(2^{2x})$$

$$2^8 = 2^{2+2x}$$

$$8 = 2 + 2x$$

$$2x = 6$$

$$x = 3$$

A1
----

Answer  $x = \dots\dots\dots$  [2]

- (b) Write down the positive square root of  $\frac{4x^2 - 12x + 9}{25x^{16}}$ .

$$\frac{2x - 3}{5x^8}$$

B1, B1
--------

Answer  $\dots\dots\dots$  [1]

- 12 (a) Find the prime factors of 4410, giving your answer in index form.

$$4410 = 2 \times 3^2 \times 5 \times 7^2$$

A1
----

M1 (ladder method or other method)
------------------------------------

Answer  $\dots\dots\dots$  [2]

- (b) The highest common factor of two numbers is 245.  
The lowest common multiple of these two numbers is 4410.  
Both numbers are greater than their highest common factor.  
Find the two numbers.

$$4410 = 2 \times 3^2 \times 5 \times 7^2$$

$$245 = 5 \times 7^2$$

$$x = 5 \times 7^2 \times 2 = 490$$

$$y = 5 \times 7^2 \times 3^2 = 2205$$

B1
----

B1
----

Answer  $\dots\dots\dots$  and  $\dots\dots\dots$  [2]

- 13 The mean and median of four numbers are 54 and 56 respectively.  
Find the mean of the largest and the smallest numbers.

$a \quad b \quad c \quad d$

$$\text{mean} = 54$$

$$\text{total} = 216$$

$$\text{median} = 56$$

$$\frac{b+c}{2} = 56$$

$$b+c = 112$$

$$a+d = 216 - 112$$

$$= 104$$

$$\text{Mean of } a+d = \frac{104}{2}$$

$$= 52$$

M1

M1

A1

Answer .....[3]

14 Consider the number pattern below.

$$\frac{1}{11} = 0.090909\dots = 0.\dot{0}9$$

$$\frac{2}{11} = 0.181818\dots = 0.\dot{1}8$$

$$\frac{3}{11} = 0.272727\dots = 0.\dot{2}7$$

$$\frac{4}{11} = 0.363636\dots = 0.\dot{3}6$$

•

•

•

$$\frac{x}{11} = 636.3636\dots = 636.\dot{3}6$$

(i) Write down the 5<sup>th</sup> line of the pattern.

$$\frac{5}{11} = 0.454545\dots = 0.\dot{4}5$$

B1
----

Answer ..... [1]

(ii) Find the value of  $x$ .

$$\frac{x}{11} = 636.3636 = 636 + 0.3636$$

$$\frac{x}{11} = 636 + \frac{4}{11}$$

$$x = 11(636) + 4$$

$$x = 7000$$

M1
----

A1
----

Answer  $x =$  ..... [2]

- 15 (a) Factorise fully  $(a-b+c)^2 - (b-c+a)^2$ .

$$\begin{aligned} & (a-b+c)^2 - (b-c+a)^2 \\ &= (a-b+c+b-c+a)(a-b+c-b+c-a) \\ &= (2a)[(2)(c-b)] \\ &= 4a(c-b) \end{aligned}$$

M1

A1

Answer ..... [2]

- (b) Calculate the value of  $k$  if  $x^2 - 6x + k$  is a perfect square.

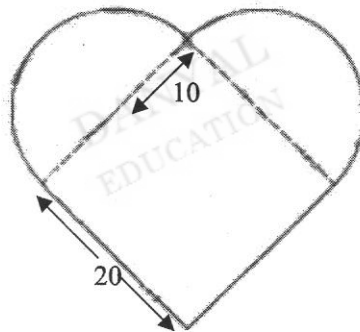
$$\begin{aligned} & x^2 - 6x + k \\ &= (x-3)^2 \\ &= x^2 - 6x + 9 \\ & k = 9 \end{aligned}$$

M1

A1

Answer  $k =$  ..... [2]

16



Instructions for baking a heart-shaped cake.

- Bake a square cake of side 20 cm.
- Bake two semicircle cakes of radius 10 cm.
- Join the two halves to the square cake, as shown in the diagram.

- (a) Find the area of the top view of the cake.

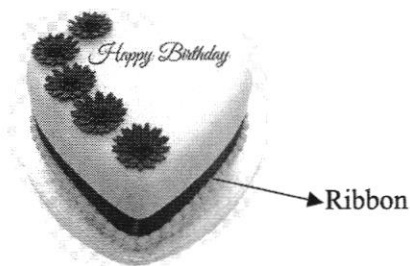
$$\begin{aligned} \text{Area} &= 20^2 + \pi(10)^2 \\ &= 400 + 100\pi \\ &= 714.16 \\ &= 714 \text{ cm}^2 \end{aligned}$$

B1

NOTE: If answer not rounded off to 3 s.f. - minus 1 from overall marks

Answer ..... cm<sup>2</sup> [1]

(b)



A red ribbon is fixed around the sides for the heart-shaped cake with the sides overlapping by 3 cm.

Find the length of ribbon required.

$$\begin{aligned} \text{Perimeter of cake} &= 2(20) + 2\pi(10) \\ &= (40 + 20\pi) \text{ cm} \end{aligned}$$

M1

$$\begin{aligned} \text{Length of ribbon required} &= 40 + 20\pi + 3 \\ &= 106 \text{ cm} \end{aligned}$$

A1

**NOTE: If answer not rounded off to 3 s.f. - minus 1 from overall marks**

Answer ..... cm [2]

- 17 The safe speed,  $v$  m/s, at which a train can go round a curve of radius  $r$  m is directly proportional to the square root of  $r$ .  
It is given that the safe speed at which a train can go round a curve of radius 121 m is 22 m/s.

(i) Find the safe speed if the curve has a radius of 81 m,

$$\begin{aligned} v &= k\sqrt{r} \\ 22 &= k\sqrt{121} \\ k &= \frac{22}{\sqrt{121}} \\ k &= \frac{22}{11} \\ k &= 2 \\ v &= 2\sqrt{r} \\ v &= 2\sqrt{81} \\ &= 18 \text{ m/s} \end{aligned}$$

Answer ..... m/s [2]

- (i) A change in the safe speed produces a change in the radius.  
When the safe speed is halved, the value of radius changes by factor  $m$ .  
Find  $m$ .

$$v = 2\sqrt{r}$$

$$\sqrt{r} = \frac{v}{2}$$

$$r = \frac{v^2}{4}$$

when speed is halved,

$$\begin{aligned} \text{New radius} &= \frac{\left(\frac{1}{2}v\right)^2}{4} \\ &= \frac{1}{4}\left(\frac{v^2}{4}\right) \end{aligned}$$

$$m = \frac{1}{4}$$

Answer  $m = \dots\dots\dots$  [2]

---



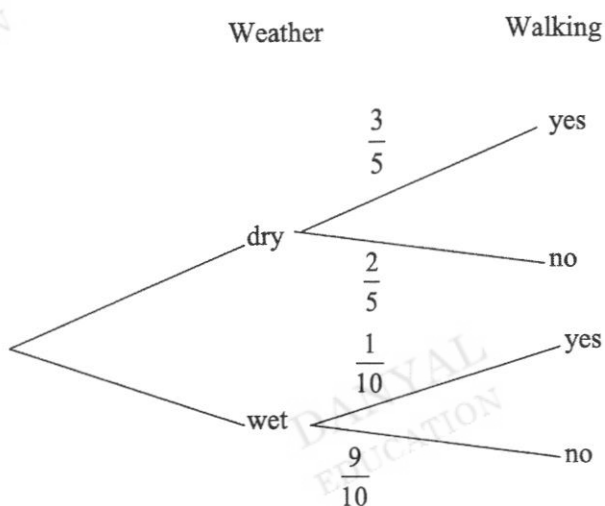
**8 In this question, the weather is either wet or dry.**

When the weather is dry, the probability that Sarah will go walking is  $\frac{3}{5}$ .

When the weather is wet, the probability that Sarah will go walking is  $\frac{1}{10}$ .

The probability of a dry day is  $\frac{2}{3}$ .

(a) Show that the probability that Sarah goes walking is  $\frac{13}{30}$ .



$$P(\text{walking}) = \left(\frac{2}{3} \times \frac{3}{5}\right) + \left(\frac{1}{3} \times \frac{1}{10}\right)$$

M1

$$= \frac{2}{5} + \frac{1}{30}$$

$$= \frac{12}{30} + \frac{1}{30}$$

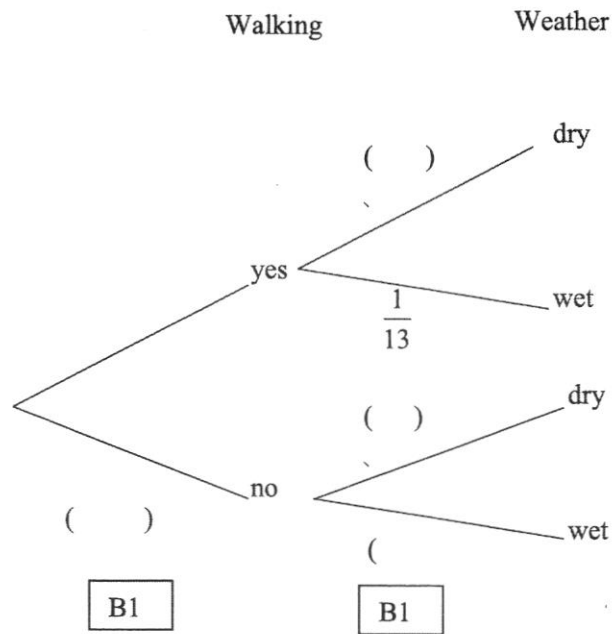
$$= \frac{13}{30} \text{ (shown)}$$

A1

The probability that Sarah does not go walking when the weather is wet is  $\frac{9}{30}$ .

Complete the tree diagram to show this information.

(b)



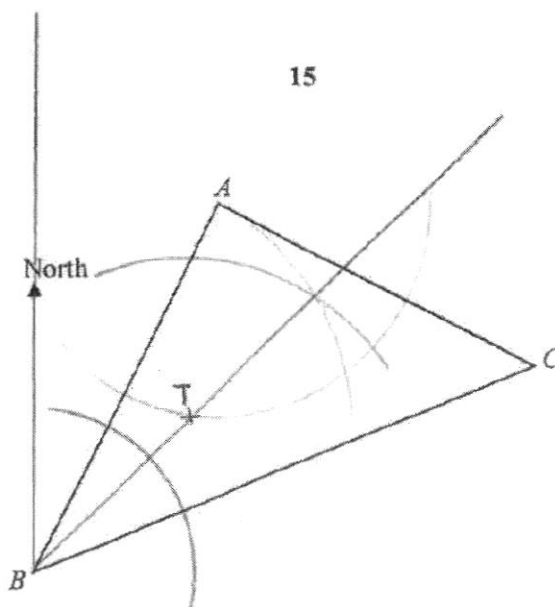
[2]

$$P(\text{does not go walking}) = \frac{17x}{30} = \frac{9}{30}$$

$$x = \frac{9}{30} \times \frac{30}{17}$$

$$= \frac{9}{17}$$

19



The diagram is a plan of a triangular field  $ABC$ , drawn to a scale of 1cm to 50 m. A tree,  $T$ , in the field is 150 m from  $A$  and is equidistant from  $BA$  and  $BC$ .

- (a) By making appropriate constructions on the diagram, indicate clearly the position of  $T$ .

B1 correct angle bisector of angle  $B$

B1 correct position  $T$

[2]

- (b) Use the diagram to find

- (i) the distance, in metres, from  $T$  to  $B$ ,

$$155\text{m} \leq TB \leq 160\text{m}$$

[1]

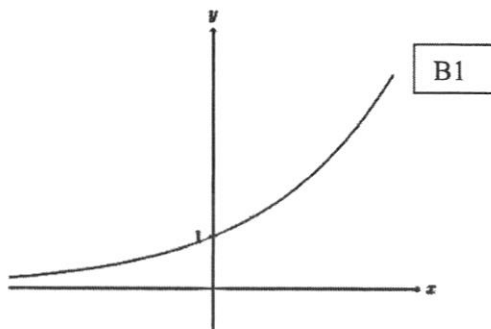
- (ii) the bearing of  $T$  from  $B$ .

$$46^\circ \pm 2$$

[1]

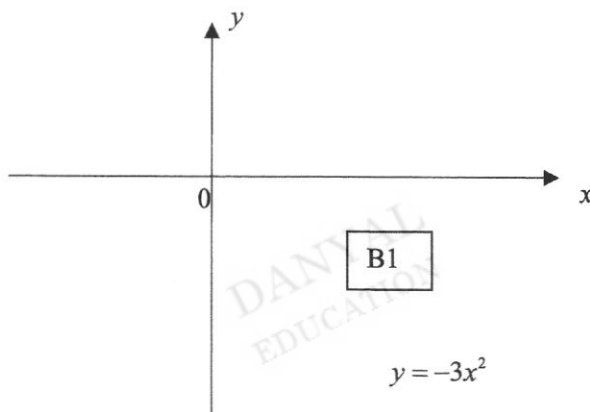
20 (a) On the axes below, sketch the graphs, indicating clearly the  $x$  and  $y$  intercepts if any.

(i)  $y = 3^x$



[1]

(ii)  $y = -3x^2$



[1]

(b) Hence, state the number of solution(s) to the equation  $3^x + 3x^2 = 0$ .  
Explain your answer.

*Answer*  $3^x = -3x^2$

B1

Since the graph of  $y = 3^x$  is always positive and the graph of  $y = -3x^2$  lies below the  $x$ -axis, the two graphs will not intersect and hence there will be no solutions to the given equation.

B1

[2]

- 21 (a) The solution of the inequality  $\frac{ax-1}{7} \leq \frac{3a+x}{4}$  is  $x \leq 46$ .

Show that  $a = 2$ .

$$\frac{ax-1}{7} \leq \frac{3a+x}{4}$$

$$4ax-4 \leq 21a+7x$$

$$4ax-7x \leq 21a+4$$

$$x(4a-7) \leq 21a+4$$

$$x \leq \frac{21a+4}{4a-7}$$

M1

$$\frac{21a+4}{4a-7} = 46$$

M1

$$21a+4 = 184a-322$$

$$326 = 163a$$

$$a = 2 \text{ (shown)}$$

A1

[3]

- (b) Hence, find a possible value of  $x$  if  $x$  is a perfect square and satisfies the simultaneous inequalities  $\frac{ax-1}{7} \leq \frac{3a+x}{4} < \frac{x-5}{2}$ .

$$\frac{6+x}{4} < \frac{x-5}{2}$$

$$12+2x < 4x-20$$

$$32 < 2x$$

$$2x > 32$$

$$x > 16$$

$$16 < x \leq 46$$

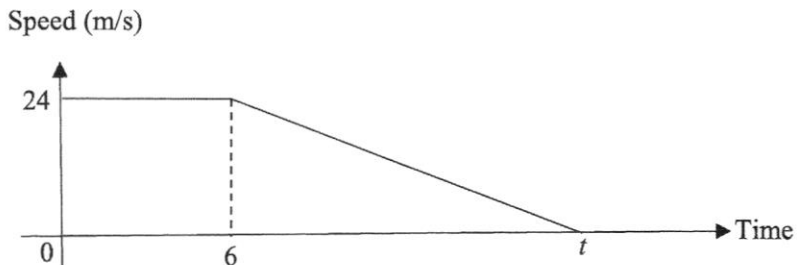
$$x = 25 \text{ or } 36$$

M1

A1

Answer  $x = \dots\dots\dots$  [2]

22



The diagram shows the speed-time graph of an object which travels at a constant speed of 24 m/s for 6 seconds before slowing down at a rate of 2 m/s<sup>2</sup> and coming to a rest after  $t$  seconds.

(a) Find the value of  $t$ .

$$\frac{24 - 0}{6 - t} = -2$$

M1

$$24 = -12 + 2t$$

$$2t = 36$$

$$t = 18$$

A1

Answer  $t = \dots\dots\dots$  [2]

(b) Calculate the average speed for the whole journey.

$$\text{Total distance} = (6 \times 24) + \left(\frac{1}{2} \times 12 \times 24\right)$$

$$= 144 + 144$$

$$= 288 \text{ m}$$

M1

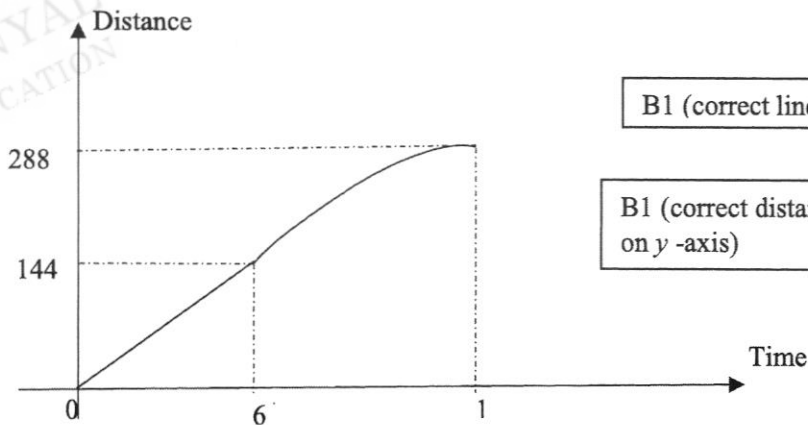
$$\text{Average speed} = \frac{288}{18}$$

$$= 16 \text{ m/s}$$

A1

Answer  $\dots\dots\dots$  m/s [2]

(c) Sketch the distance-time graph of the object for the whole journey on the axes below.

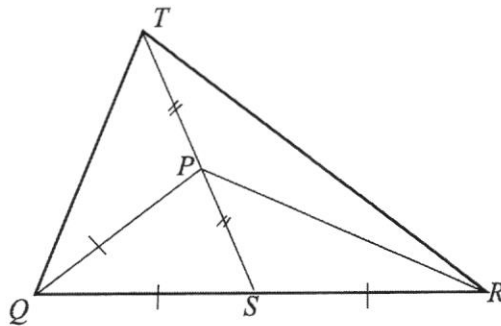


B1 (correct line and curve)

B1 (correct distances indicated on y-axis)

[2]

23



$TQR$  is a triangle with  $SP = PT$  and  $QP = QS = SR$ .

- (a) Explain why angle  $TPQ =$  angle  $PSR$ .

*Answer*

$$\angle QPS = \angle QSP \text{ (base angle of isos triangle)}$$

$$\angle TPQ = 180^\circ - \angle QPS \text{ (adjacent angles on a straight line)}$$

$$= 180^\circ - \angle QSP$$

$$= \angle PSR$$

M1

A1

- (b) Show that  $TQ = PR$ .

Give a reason for each statement you made.

*Answer*

$$\angle TPQ = \angle PSR \text{ (from (a))}$$

$$ST = PT \text{ (given)}$$

$$QP = RS \text{ (given)}$$

$$\text{Hence, } \triangle TPQ \cong \triangle PSR \text{ (SAS)}$$

$$TQ = PR \text{ (corresponding side of congruent triangles)}$$

M1

M1

A1

- (c) Show that triangles  $QPS$  and triangle  $PSR$  are equal in area.

Give a reason for each statement you made.

$$\begin{aligned} \frac{\text{area of } \triangle QPS}{\text{area of } \triangle PSR} &= \frac{\frac{1}{2} \times QS \times h}{\frac{1}{2} \times SR \times h} \quad \text{M1} \\ &= \frac{QS}{SR} \\ &= \frac{1}{1} \end{aligned}$$

Since the ratio of the area of  $\triangle QPS$  to area of  $\triangle PSR$  is 1:1, the two triangles are equal in area.

A1

[2]

[3]

- 24 Before the Chinese New Year (CNY) celebration Mr Lee put a combination of \$2 and \$10 notes into four types of red packets. Friends and relatives who visited Mr Lee during the CNY period are then given the appropriate red packets.

Design A contains  $x$  \$2 notes and no \$10 notes.

Design B contains equal number of \$2 notes and \$10 notes. Its number of \$2 notes is fewer than that of Design A by 2.

Design C contains twice as many \$10 notes as Design B. There are no \$2 notes in Design C.

Design D contains three times as many \$2 notes as Design A. The number of \$10 notes in Design D is equal to the number of \$10 notes in Design B.

- (a) (i) Complete the following table using the given information.

	Design			
	A	B	C	D
Number of \$2 notes	$x$	$x-2$	0	$3x$
Number of \$10 notes	0	$x-2$	$2x-4$	$x-2$

[1]

B1

- (ii) Represent the data in the above table by a  $2 \times 4$  matrix  $N$ .

$$N = \begin{pmatrix} x & x-2 & 0 & 3x \\ 0 & x-2 & 2x-4 & x-2 \end{pmatrix} \quad \text{B1}$$

Answer  $N =$ 

[1]

- (b) The number of designs  $A$ ,  $B$ ,  $C$  and  $D$  red packets that were given away during the CNY

period can be represented by  $M = \begin{pmatrix} 20 \\ 10 \\ 6 \\ 5 \end{pmatrix}$ .

- (i) Find, in terms of  $x$ , the matrix  $NM$ .

$$NM = \begin{pmatrix} x & x-2 & 0 & 3x \\ 0 & x-2 & 2x-4 & x-2 \end{pmatrix} \begin{pmatrix} 20 \\ 10 \\ 6 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} 20x+10x-20+15x \\ 10x-20+12x-24+5x-10 \end{pmatrix} \quad \text{M1}$$

$$= \begin{pmatrix} 45x-20 \\ 27x-54 \end{pmatrix} \quad \text{A1}$$

Answer  $NM =$ 

[2]



(iii) State what each element in matrix **NM** represents.

$45x - 20$  represents the total number of \$2 notes that Mr Lee gave away. [1]  
 $27x - 45$  represents the total number of \$10 notes that Mr Lee gave away.

B1

(c) The matrix **D** =  $\begin{pmatrix} 2 & 10 \end{pmatrix}$ .

Find, in terms of  $x$ , the matrix **DNM**.

$$\mathbf{DNM} = \begin{pmatrix} 2 & 10 \end{pmatrix} \begin{pmatrix} 45x - 20 \\ 27x - 54 \end{pmatrix}$$

$$= (360x - 580)$$

B1

*Answer* **DNM** = [1]

(d) The total amount of red packet money given away was \$500.  
 Find the value of  $x$ .

$$360x - 580 = 500$$

$$360x = 1080$$

$$x = 3$$

A1

*Answer*  $x = \dots\dots\dots$  [1]

**End of Paper**

Mrs Foo SG

# MARKING SCHEME



## AHMAD IBRAHIM SECONDARY SCHOOL PRELIMINARY EXAMINATION 2021

### SECONDARY 4 EXPRESS / 5 Normal (Academic)

Name:	Class:	Register No.:
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#### MATHEMATICS

Paper 2

4048/02

25 Aug 2021

2 hours 30 minutes

Candidates answer on the Question Paper.

#### READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any questions it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use

**/100**

**Mathematical Formulae***Compound Interest*

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

- 1 (a) Write as a single fraction in its simplest form  $\frac{5}{5-2y} - \frac{2}{y+5}$ .

$$\begin{aligned} & \frac{5}{5-2y} - \frac{2}{y+5} \\ &= \frac{5(y+5) - 2(5-2y)}{(5-2y)(y+5)} \\ &= \frac{5y+25-10+4y}{(5-2y)(y+5)} && \text{M1} \\ &= \frac{9y+15}{(5-2y)(y+5)} = \frac{3(3y+5)}{(5-2y)(y+5)} && \text{A1} \end{aligned}$$

Answer ..... [2]

- (b) It is given that  $\frac{a}{h} = b\left(1 + \frac{k}{h}\right)$ , express  $h$  in terms of  $a$ ,  $b$  and  $k$ .

$$\begin{aligned} \frac{a}{h} &= b\left(1 + \frac{k}{h}\right) && \text{Expand M1} \\ \frac{a}{h} &= b + \frac{bk}{h} \\ \frac{a}{h} - \frac{bk}{h} &= b \\ \frac{a-bk}{h} &= b && \text{Isolate h M1} \\ h &= \frac{a-bk}{b} \text{ or } \frac{a}{b} - k && \text{A1} \end{aligned}$$

$$\begin{aligned} \frac{a}{h} &= b\left(1 + \frac{k}{h}\right) \\ \frac{a}{b} &= h\left(1 + \frac{k}{h}\right) && \text{swop b and h M1} \\ \frac{a}{b} &= h + k && \text{expand M1} \\ h &= \frac{a-bk}{b} \text{ or } \frac{a}{b} - k \end{aligned}$$

Answer  $h =$  ..... [3]

- (c) Solve the equation  $\frac{6}{x-6} = 2 - \frac{1}{x-8}$ .

$$6(x-8) = 2(x-6)(x-8) - (x-6)$$

$$6x - 48 = 2(x^2 - 14x + 48) - x + 6$$

$$6x - 48 = 2x^2 - 29x + 102$$

$$2x^2 - 35x + 150 = 0 \quad \text{M1}$$

$$(2x-15)(x-10) = 0$$

$$x = 7.5 \text{ or } 10 \quad \text{A2}$$

Answer  $x = \dots\dots\dots$  or  $\dots\dots\dots$  [3]

- (d) (i) Express  $-25 + 3x + x^2$  in the form  $p + (q + x)^2$ .

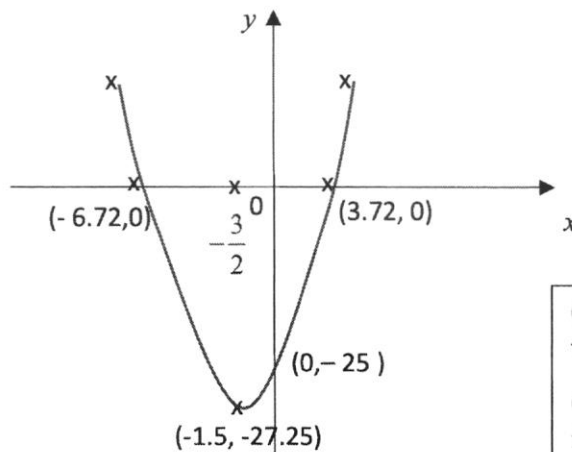
$$= x^2 + 3x + \left(\frac{+3}{2}\right)^2 - \left(\frac{+3}{2}\right)^2 - 25 = \left(x + \frac{3}{2}\right)^2 - \frac{109}{4}$$

$$= -\frac{109}{4} + \left(\frac{3}{2} + x\right)^2 \quad \text{A1 OR } -27.25 + (1.5 + x)^2$$

Answer  $\dots\dots\dots$  [1]

- (ii) Hence sketch the graph of  $y = -25 + 3x + x^2$  on the axes below.

Indicate **clearly the coordinates** of the points where the graph crosses the axes and the turning point on the curve. [2]



Correct Shape thru  
turning point – B1

Correct x and y-  
intercept – B1

- 2 (a) Mr Tan owns several cake shops in Singapore and Malaysia.  
 The price of a one-kilogram cake is \$35 in Singapore.  
 The price of the same cake in Malaysia is MYR 96.  
 The exchange rate between Singapore dollars (\$) and Malaysian Ringgit (MYR) is  
 \$1 = MYR 3.06.

Calculate how much more expensive the cake is in Singapore than in Malaysia.

$\text{Price of cake in Malaysia in S\$} = \frac{96}{3.06} = \$31.37 \text{ (2 d.p.)}$ $\text{Price difference} = \$35 - \$31.37 = \$3.63 \text{ (nearest cent)} \quad \text{B1}$
---

*Answer* \$ ..... [1]

- (b) Mr Tan is planning to expand his cake shop business in Singapore.  
 His business manager has provided the visitor arrival data from the Singapore Tourism Board.

In 2019, the total visitor arrivals to Singapore was  $185.3 \times 10^5$ .

- (i) Calculate the mean number of visitor arrivals to Singapore each day in 2019.  
 Give your answer in standard form, correct to 3 significant figures.

$\begin{aligned} &\text{Number of visitors} \\ &= \frac{185.3 \times 10^5}{365} = 5.08 \times 10^4 \quad \text{B1} \end{aligned}$
---

*Answer* ..... [1]

- (ii) From 2018 to 2019, the number of visitor arrivals to Singapore decreased by 21.2%.  
 Calculate the number of visitor arrivals to Singapore in 2018.  
 Give your answer in millions, correct to 3 significant figures.

$\begin{aligned} &78.8\% - 185.3 \times 10^5 \\ &100\% - 23515228 = 185.3 \times 10^5 \times (100/78.8) \text{ M1} \\ &= 23.5 \text{ millions (3 s.f.)} \quad \text{A1} \end{aligned}$
--

*Answer* ..... millions [2]

- (c) In January 2020, Mr Tan invested a sum of money in a bank paying a compound interest at 2.8% per annum.  
After one year, he received \$95776.75 from his investment.

Calculate the total interest he received in January 2021, giving your answer to the nearest dollar.

$$95\,776.75 = P \left( 1 + \frac{2.8}{100} \right)^1 \quad \text{M1}$$

$$P = \frac{95\,776.75}{\left( 1 + \frac{2.8}{100} \right)} \quad \text{M1}$$

$$P = \$ 93\,168.00475$$

$$\text{Interest} = 95\,776.75 - 93\,168.00475$$

$$= \$ 2\,608.705253 = \$ 2\,609 \text{ (nearest dollar)} \quad \text{A1}$$

Answer \$ ..... [3]

- (d) Mr Tan needs to pay his Malaysia supplier MYR 2000 for the cake ingredients.  
The transportation fees for the ingredients were MYR 60.  
A 7% Goods and Services Tax (GST) is payable on the cost of goods and transportation that exceeds \$400.

Calculate the amount of money, in Singapore dollars, that Mr Tan must pay to import the ingredients.

$$\begin{aligned} &\text{Amount of money} \\ &= \left( \frac{2000 + 60}{3.06} \right) \times 1.07 \quad \text{M1} \\ &= 673 \frac{31}{153} \times 1.07 \\ &= 720.3267974 = \$ 720.33 \text{ (2 d.p.)} \quad \text{A1} \end{aligned}$$

Answer \$ ..... [2]

- 3 (a) Complete the table of values for  $y = 3x^2 + \frac{5}{x} - 4$ .

Find the value of  $p$  correct to 2 decimal places.

$x$	0.4	0.6	0.8	1.0	1.2	1.4	1.6
$y$	8.98	5.41	4.17	4.00	4.49	$p$	6.81

$p = 5.45$	B1
------------	----

Answer  $p = \dots\dots\dots$  [1]

- (b) On the grid opposite, draw the graph of  $y = 3x^2 + \frac{5}{x} - 4$  for  $0.4 \leq x \leq 1.6$ . [3]

- (c) Explain why the equation  $3x^2 + \frac{5}{x} - 4 = k$  does not have solutions for some values of  $k$ . [2]

Answer

.... The minimum value of $y$ is 4. B1	.....
.... For $k < 4$ , the horizontal line $y = k$ does not intersect with the curve. B1	.....
....	.....

- (d) The equation  $3x^3 - 2x^2 - 9x + 5 = 0$  can be solved by drawing a suitable straight line on the grid.

- (i) Find the equation of the straight line.

$3x^3 - 2x^2 - 9x + 5 = 0$	
$3x^2 - 2x - 9 + \frac{5}{x} = 0$	
$3x^2 + \frac{5}{x} - 9 = 2x$	
$3x^2 + \frac{5}{x} - 9 + 5 = 2x + 5$	
$y = 2x + 5$	A1

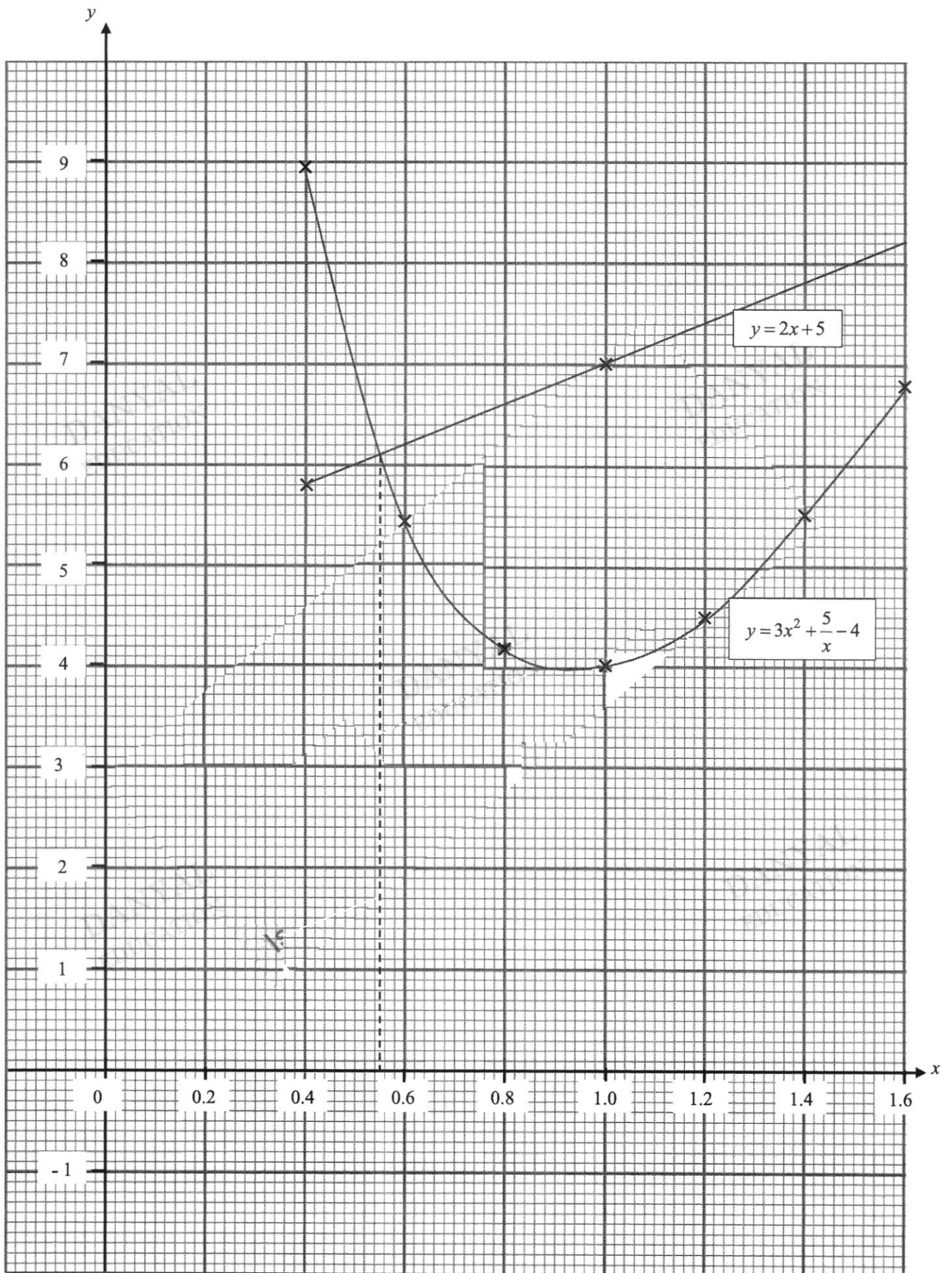
Answer [1]

- (ii) By drawing this straight line, solve the equation  $3x^3 - 2x^2 - 9x + 5 = 0$  for  $0.4 \leq x \leq 1.6$ .

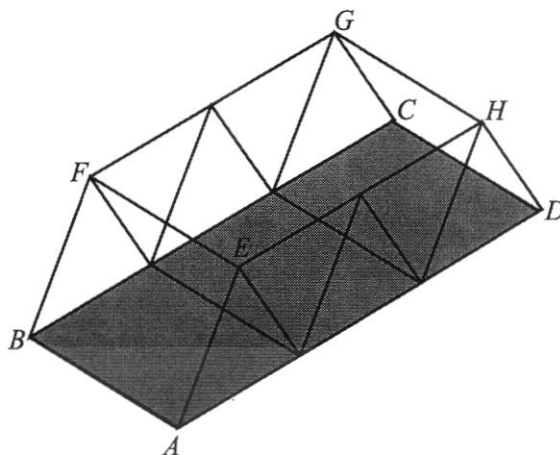
Draw the line	B1
$x = 0.54 \pm 0.02$	B1

Answer  $x = \dots\dots\dots$  [2]





4



The figure shows a drawing of a bridge.

Each side of the bridge is made up of 5 equilateral triangles.

The shaded rectangular area  $ABCD$  is the base of the bridge.

$HD = 5.5$  m.

(a) Calculate  $DE$ .

$$\angle DHE = 120^\circ \quad (\text{angles in an equilateral triangle})$$

$$EH = 2 \times 5.5 = 11 \text{ m} \quad (\text{angle or length}) \text{ M1}$$

$$DE^2 = HD^2 + EH^2 - 2(HD)(EH)\cos 120^\circ$$

$$DE = \sqrt{5.5^2 + 11^2 - 2(5.5)(11)\cos 120^\circ} \quad \text{M1}$$

$$DE = \sqrt{211\frac{3}{4}} = 14.55163221 = 14.6 \text{ m (3 s.f.)} \quad \text{A1}$$

Alternative solution:

Let  $M$  be vertically below  $E$ .

$$EM = \sqrt{5.5^2 - 2.75^2} = 4.7631 \quad \text{M1}$$

$$MD = 2 \times 5.5 + 2.75 = 13.75$$

$$DE = \sqrt{13.75^2 + 4.7631^2} = 14.6 \text{ m} \quad \text{M1, A1}$$

Alternative solution:

In triangle  $EAD$ ,

$$DE = \sqrt{5.5^2 + 16.5^2 - 2(5.5)(16.5)\cos 60^\circ} \quad \text{M1, M1, A1}$$

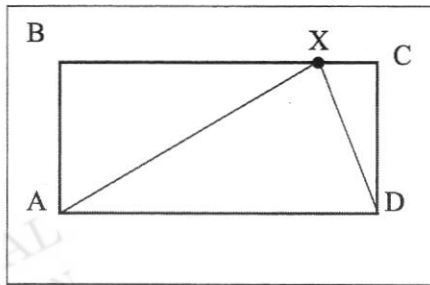
$$= 14.6$$

Answer ..... m [3]

A point,  $X$ , lies on  $BC$  and is vertically below point  $G$ .

- (b) (i) Angle  $AXD = 79.5^\circ$  and angle  $ADX = 65.0^\circ$ .  
Show that  $AX = 15.2$  m, correct to 3 significant figures.

Answer



$$\frac{AX}{\sin 65^\circ} = \frac{AD}{\sin 79.5^\circ} \quad \text{M1}$$

$$AX = \frac{3 \times 5.5}{\sin 79.5^\circ} \times \sin 65^\circ$$

$$= 15.20875042$$

$$= 15.2 \text{ m (3 s.f.)} \quad \text{A1}$$

[2]

Alternative solution

$$\sin 54.5 = \frac{BX}{AX} \quad \text{M1}$$

$$AX = \frac{5.5 + 5.5 + 2.75}{\sin 54.5} \quad \text{A1}$$

$$= 16.9 \text{ m}$$

- (ii) Calculate the angle of elevation of  $G$  from  $A$ .

$$GC^2 = XG^2 + XC^2$$

$$XG = \sqrt{5.5^2 - 2.75^2} = 4.763139721 \text{ m} \quad \text{M1}$$

$$\tan \angle GAX = \frac{GX}{AX} = \frac{4.76314}{15.2088} \quad \text{M1}$$

Angle of Elevation =  $\angle GAX$

$$= \tan^{-1}\left(\frac{4.76314}{15.2088}\right) = 17.4^\circ \text{ (1 d.p.)} \quad \text{A1}$$

Answer ..... [3]

- (c) The greatest angle of elevation of  $G$  from a point along  $AD$  occurs at  $Y$ .  
Describe the relationship between  $XY$  and  $AD$ .

*Answer*

$Y$  is a point on  $AD$  such that  $XY$  is perpendicular to  $AD$ .

OR  $XY$  is the shortest distance from  $X$  to  $AD$ .

B1

[1]

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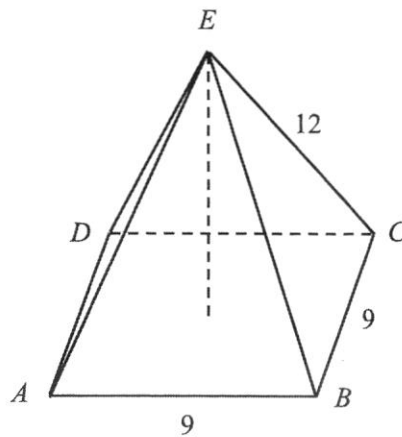
DANYAL  
EDUCATION

DANYAL  
EDUCATION

DANYAL  
EDUCATION

DANYAL  
EDUCATION

5



The diagram shows a pyramid  $ABCDE$ .  
 The base of the pyramid is a square of side 9 cm.  
 $E$  is vertically above the centre of the square base.  
 $AE = BE = CE = DE = 12$  cm.

- (a) Find the total surface area of the pyramid  $ABCDE$ .

Let  $X$  be the mid-point of  $BC$ .  $BX = 4.5$  cm

$$\text{Slant Height } EX = \sqrt{12^2 - 4.5^2} = 11.12428773 \text{ cm} \quad \text{M1}$$

$$\text{Surface Area} = 9 \times 9 + 4 \times \frac{1}{2} \times 9 \times 11.1243 \quad \text{M1}$$

$$= 281.2371791 \text{ cm}^2$$

$$= 281 \text{ cm}^2 \text{ (3 s.f.)} \quad \text{A1}$$

Answer .....  $\text{cm}^2$  [3]

The volume of the pyramid is  $275 \text{ cm}^3$ .

This metal solid is melted and molded into two identical spherical metal balls.

- (b) Find the radius of each metal ball.

$$\text{Volume of 1 spherical metal balls} = \frac{275}{2} = 137.5 \text{ cm}^3$$

$$\frac{4}{3} \times \pi \times r^3 = 137.5 \quad \text{M1}$$

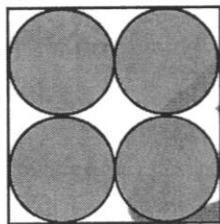
$$r = 3.201877382 = 3.20 \text{ cm (3 s.f.)} \quad \text{A1}$$

Answer ..... cm [2]

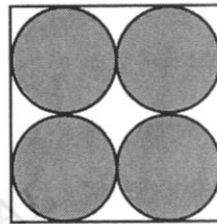
- (c) Given that one bucket of paint is able to paint an area of  $1200 \text{ cm}^2$ , how many buckets of paint are needed to paint 1500 of such metal balls?

Surface Area of 1 spherical ball	
$= 4 \times \pi \times 3.20188^2 = 128.8306674 \text{ cm}^2$	M1
Total Surface Area of 1500 metal balls	
$= 1500 \times 128.8306674 = 193246.0011 \text{ cm}^2$	M1
No. of buckets of paint required	
$= \frac{193246.0011}{1200} = 161.0383342$	
$= 162$ (nearest bucket)	A1

Answer ..... [3]



Top view



Side view

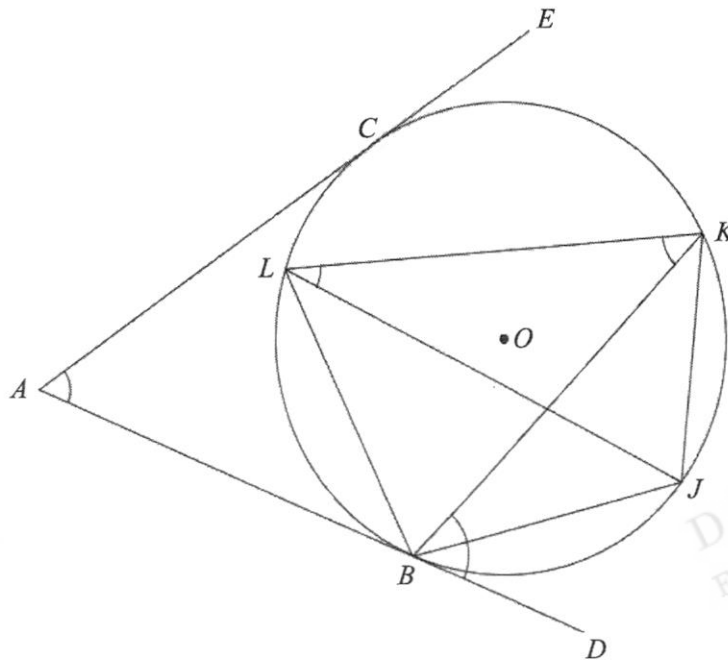
The diagram shows the top and side view of a box holding some spherical balls. The box is in the shape of a cube and the spherical balls just fit into the box.

- (d) Calculate the volume of empty space in the box.

Length of one side of cube $= 3.201877382 \times 4 = 12.8075 \text{ cm}$	
Volume of the cube $= 12.8075^3 = 2100.84056 \text{ cm}^3$	M1
Volume of 8 spherical balls $= 8 \times 137.5$	
$= 1100 \text{ cm}^3$	M1
Volume of empty space $= 2100.84056 - 1100$	
$= 1000.84056 = 1000 \text{ cm}^3$ (3 s.f.)	A1

Answer .....  $\text{cm}^3$  [3]

6 (a)



$B, C, J, K$  and  $L$  are points on the circle, centre  $O$ .  
 $AE$  and  $AD$  are tangents to the circle.  
 Angle  $JLB$  is equal to angle  $JBD$ .  
 Angle  $BAC = 49^\circ$ , angle  $JLK = 24^\circ$ , angle  $KBD = 60^\circ$  and angle  $BKL = 31^\circ$ .

Find, giving reasons for each answer,

(i) angle  $JBK$ ,

$\angle JBK = 24^\circ$ (angles in same segment) B1
---

Answer ..... [1]

(ii) angle  $KBL$ ,

$\angle JBD = 60^\circ - 24^\circ = 36^\circ$ $\angle JLB = \angle JBD$ (Given) $\angle KBL = 180^\circ - 31^\circ - 24^\circ - 36^\circ$ ( $\angle$ sum of $\triangle KBL$ ) $= 89^\circ$	M1  A1
---	--------------

Answer ..... [2]

(iii) reflex angle  $KOL$ ,

$\angle KOL = 360^\circ - 2 \times 89^\circ$ (angle at centre = $2 \times$ angle at circumference) $= 182^\circ$	M1, A1
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Answer ..... [2]

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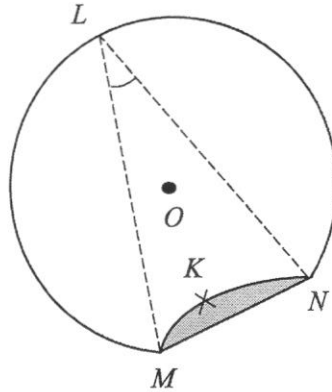
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(iv) angle  $COB$ .

$$\begin{aligned} \angle ACO = \angle ABO = 90^\circ & \text{ (tangent perpendicular to radius) M1} \\ \angle COB = 360^\circ - 90^\circ - 90^\circ - 49^\circ & \text{ (sum of angles in quadrilateral)} \\ & = 131^\circ \qquad \qquad \qquad \text{A1} \end{aligned}$$

Answer ..... [2]

(b)



The diagram shows a circle made of paper with centre  $O$ .  
A segment of the circle is folded inwards.

Angle  $MLN = \frac{\pi}{4}$  radians and arc length  $MKN = 6$  cm

Calculate the length of the major arc  $MLN$ .

$$\begin{aligned} \angle MON &= 2 \times \frac{\pi}{4} = \frac{\pi}{2} && \text{M1} \\ \text{Reflex } \angle MON &= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2} && \text{M1} \\ \text{Major Arc length } MLN & && \\ &= \frac{3\pi}{2} \times 3.819 && \text{M1} \\ &= 17.998596 \text{ cm} && \\ &= 18.0 \text{ cm (3 s.f.)} && \text{A1} \end{aligned}$$

$$\begin{aligned} \text{Arc length } MKN &= 6 && \\ r \times 2 \times \frac{\pi}{4} &= 6 && \text{M1} \\ r &= 3.819 \text{ cm (correct to 4 d.p.)} && \text{A1} \\ \text{Major Arc length } MLN & && \\ &= 2\pi \times 3.819 - 6 && \text{M1} \\ &= 17.998596 \text{ cm} && \\ &= 18.0 \text{ cm (3 s.f.)} && \text{A1} \end{aligned}$$

Answer ..... cm [4]



- 7 Two groups of students were surveyed on their weekly screen time in hours. The results are shown in the table.

14	32	55	24	56
23	48	17	36	59
<b>Group 1</b>				

42	23	29	25	19
23	90	45	35	29
<b>Group 2</b>				

- (a) 20% of students in Group 2 recorded a screen time of at least  $x$  hours. Find  $x$ .

$\frac{20}{100} \times 10 = 2 \text{ students}$ $x = 45 \text{ hours}$ <p style="text-align: right;">A1</p>
---

Answer ..... hours [1]

- (b) Complete the table below. [2]

	Group 1	Group 2
Median	34	
Interquartile Range		19

$\text{Median} = \frac{29 + 29}{2} = 29 \quad \text{B1}$ $\text{IQR} = 55 - 23 = 32 \quad \text{B1}$
--

- (c) Make a comparison between the screen times of the students in these two groups.

Answer

<p>Group 2 students has shorter screen time since they have a smaller median of 29h compared to 34h for median of Group 1.</p> <p>Or Group 2 students has less spread out screen time than Group 1 students as their interquartile range (19h) is less than Group 1's 32h.</p> <p style="text-align: right;">B1</p>
---

..... [1]

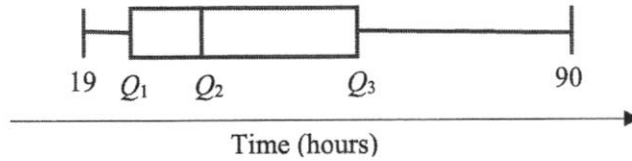
- (d) Explain why median is a better measure for average screen time for the two groups of students compared to the mean.

Answer

<p>There are extreme values, such as 90, present in Group 2, which will affect the calculation of the mean but not the median.</p> <p style="text-align: right;">B1</p>
---

..... [1]

- (e) (i) The box-and-whisker plot for the distribution of screen time for students in Group 2 is drawn in the grid below. Write down the value for  $Q_1$ ,  $Q_2$  and  $Q_3$ .



1-2 mistakes (deduct 1 mark)  
 3 mistakes (deduct 2 marks)  
 $Q_1 = 23, Q_2 = 29,$   
 $Q_3 = 42$

Answer  $Q_1 = \dots\dots\dots$  hours

$Q_2 = \dots\dots\dots$  hours

$Q_3 = \dots\dots\dots$  hours

[2]

- (ii) Due to a recording error, the recorded amount of screen time for Group 1 is 1 hour more than the actual screen time.

State how the median and standard deviation of the screen times have been affected by this error.

Answer

..... The actual median will decrease by 1 hour from 34h to 33h. B1 ...

..... The standard deviation remains the same. B1 ...

[2]

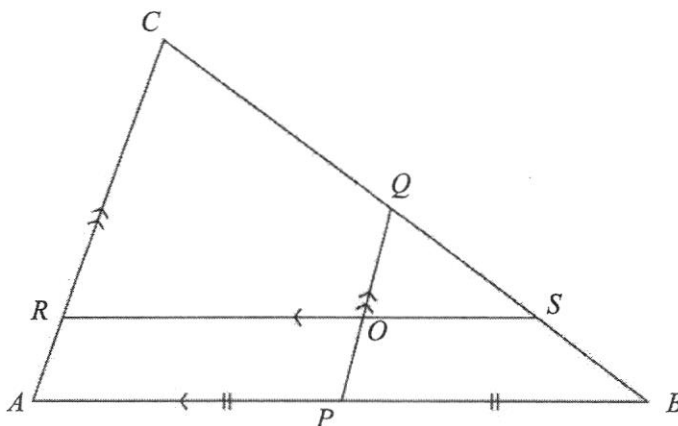
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8 (a)



In the diagram,  $SR$  is parallel to  $BA$  and  $PQ$  is parallel to  $AC$ .  
 $P$  is the midpoint of  $AB$  and  $BS : SC = 1 : 4$ .

(i) Prove that triangles  $CRS$  and  $CAB$  are similar.

[2]

Answer

B1 (either statement), B1 (property)

(ii) If the area of triangle  $ABC = 120 \text{ cm}^2$ , find the area of  $RSBA$ .

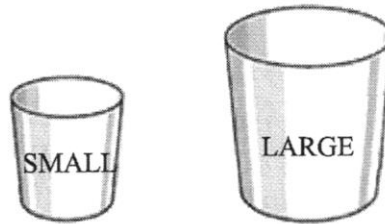
$$\frac{\text{Area of } \triangle CRS}{\text{Area of } \triangle CAB} = \left(\frac{4}{5}\right)^2$$

$$\text{Area of } \triangle CRS = \left(\frac{4}{5}\right)^2 \times 120 = 76.8 \text{ cm}^2 \quad \text{M1}$$

$$\text{Area of } RSBA = 120 - 76.8 = 43.2 \text{ cm}^2 \quad \text{A1}$$

Answer .....  $\text{cm}^2$  [2]

(b)



The diagram shows two geometrically similar can.  
 A soup manufacturer wants to fill the cans with soup.  
 The ratio of the heights of two geometrically similar cans is 4 : 9.  
 The surface area of the small can is 240 cm<sup>2</sup>.

By considering the materials and volume of the soup the cans can hold, determine which can is more cost effective for the soup manufacturer.  
 Use calculations to justify your answer.

[4]

*Answer*

$$\frac{240}{\text{Area of Large Can}} = \left(\frac{4}{9}\right)^2$$

$$\text{Area of Large Can} = 1215 \text{ cm}^2 \quad \text{M1}$$

$$\frac{V_{\text{small}}}{V_{\text{large}}} = \left(\frac{4}{9}\right)^3 = \frac{64}{729} \quad \text{M1}$$

Ratio of volume of soup per cm<sup>2</sup> of material for small can

$$= 64 \text{ parts} \div 240 \text{ cm}^2 \cong 0.267$$

Ratio of volume of soup per cm<sup>2</sup> of material for large can

$$= 729 \text{ parts} \div 1215 \text{ cm}^2 \cong 0.6 \quad \text{M1 (either ratio)}$$

Hence the large can is more cost effective since it has a larger ratio of volume of soup to area of material needed.

A1

- 9 (a) The first three terms in a sequence of numbers are given below.

$$Q_1 = 2^3 + 3 = 11$$

$$Q_2 = 2^4 + 5 = 21$$

$$Q_3 = 2^5 + 7 = 39$$

- (i) Find an expression, in terms of  $n$ , for  $Q_n$ .

$$2^3, 2^4, 2^5, \dots \quad T_n = 2^{n+2} \quad \text{B1 (either series)}$$

$$3, 5, 7, \dots \quad T_n = 3 + (n-1)2 = 2n + 1$$

$$Q_n = 2^{n+2} + 2n + 1 \quad \text{A1}$$

Answer ..... [2]

- (ii) Explain why the value of  $Q_n$  must be odd for all values of  $n$ .

Answer

Since  $2^{n+2} + 2n$  is even for all values of  $n$  and 1 is odd, the sum of an even and odd number is odd. B1

[1]

- (iii) Show that  $2^{n-2} - 2^{n-3} = 2^{n-3}$ .

$$2^{n-2} - 2^{n-3} = 2^{n-3}(2-1) = 2^{n-3} \quad \text{M1, A1}$$

$$\begin{aligned} 2^{n-2} - 2^{n-3} &= \frac{2^n}{4} - \frac{2^n}{8} = \frac{2^{n+1} - 2^n}{8} \\ \text{Or} \\ &= \frac{2^n(2-1)}{2^3} = 2^{n-3} \quad \text{M1, A1} \end{aligned}$$

Answer ..... [2]

- (b) In a sequence, the same number is subtracted each time to obtain the next term. The first five terms of the sequence are

$$p, q, 2, r, -16$$

- (i) Write down an expression for the  $n$ th term of this sequence.

Let the same number be 'd'.

$$2 - d = r$$

$$r - d = -16$$

$$d = 9 \quad \text{or common difference} = -9 \quad \text{M1}$$

$$T_n = 29 - 9n \quad \text{A1}$$

Answer ..... [2]

- (ii) The sum of the  $m$ th term and its previous term is  $-167$ .  
Find the value of  $m$ .

$$T_m = 29 - 9m$$

$$T_{m-1} = 29 - 9(m-1)$$

$$29 - 9m + [29 - 9(m-1)] = -167 \quad \text{M1}$$

$$m = 13 \quad \text{A1}$$

Answer ..... [2]

- (c) The table gives information about the ages of the members in the *Fitness First* club in Singapore.

	Members aged under 40	Members aged 40 or over
Male	40	24
Female	32	34

- (i) One of these members is selected at random.  
Find, as a fraction in its lowest terms, the probability that he or she is under 40.

$$P(< 40 \text{ yrs})$$

$$= \frac{40 + 32}{40 + 32 + 24 + 34} = \frac{72}{130} = \frac{36}{65} \quad \text{B1}$$

Answer ..... [1]

- (ii) Two of the members are selected at random.  
Find the probability that

- (a) both members are female,

$$P(\text{both are female})$$

$$= \frac{32 + 34}{130} \times \frac{32 + 33}{129}$$

$$= \frac{66}{130} \times \frac{65}{129} \quad \text{M1}$$

$$= \frac{11}{43} \quad \text{A1}$$

Answer ..... [2]

- (b) they are both aged 40 or over, but only one is a male member.

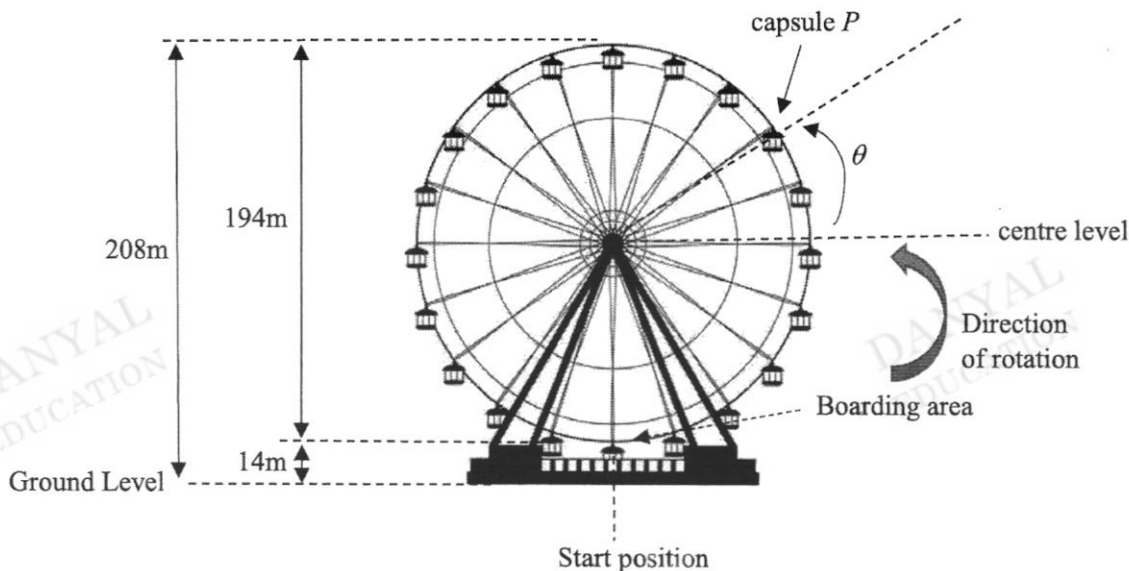
$$P(> 40 \text{ yrs, only 1 male})$$

$$= \frac{24}{130} \times \frac{34}{129} + \frac{34}{130} \times \frac{24}{129} \quad \text{M1}$$

$$= \frac{272}{2795} \quad \text{A1}$$

Answer ..... [2]

- 10 The Beijing Great Wheel is the world's largest Giant Observation Wheel and one of East Asia's biggest tourist attractions.  
 The Giant Observation Wheel has a height of 208 m and a diameter of 194 metres.  
 It rotates at the speed of 0.3 m/s.  
 Angle of rotation is measured from the start position in an anti-clockwise direction.



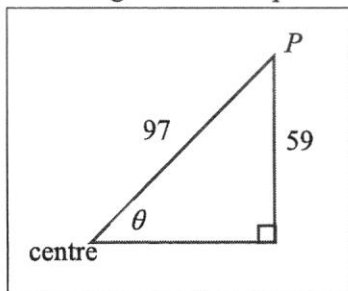
- (a) Find the time taken by a capsule to complete 1 revolution.  
 Express your answer in minutes and seconds, correct to the nearest second.  
 Take  $\pi$  as 3.142.

$\text{Time Taken} = \frac{\pi \times 194}{0.3} \text{ secs}$	M1
$= 2031.826667 \text{ secs}$	
$= 33.86377778 \text{ min}$	
$= 33 \text{ min } 52 \text{ secs}$	A1

Answer ..... minutes ..... seconds [2]

- (b) The state square garden is 1km away from the Great Observation Wheel. There are fireworks displays at the garden from 8.45 pm to 8.50 pm and the best view for the fireworks occurs when the capsule P is at least 170 metres above the ground.

- (i) Find angle  $\theta$  when capsule P is exactly 170 meters above the ground level.



Let angle from the centre level of the wheel to a height of 170m from the ground be  $\theta$ .

$\sin \theta = \frac{59}{97}$	M1
$\theta = 37.46289^\circ$	A1

Answer  $\theta =$  ..... [2]

- (ii) Find the angle from the start position to the best viewing point where capsule  $P$  is.

$$\begin{aligned} &\text{Angle from start to best viewing point at 170 m} \\ &\text{above ground} \\ &= 90^\circ + 37.46289^\circ = 127.4629^\circ = 127.5^\circ \end{aligned}$$

Answer ..... [1]

- (c) A tourist family wants to get the best view of the **FULL** length of the fireworks display. They board the capsule at 8.32 pm and claims that they can enjoy the full length of fireworks display.

Do you agree with the claim?

Show working to support your answer and state one assumption that you made.

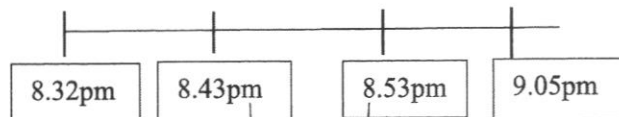
[4]

$$\begin{aligned} \text{Time to reach 170 m from start position} &= \frac{127.4629}{360} \times 2031.826667 \\ &= 719.3958181 \text{ secs} \\ &= 11.9899303 \text{ min} = 11 \text{ min } 49 \text{ sec} \end{aligned}$$

M1

Or Time for the fireworks to be in view

$$= \frac{105.07422^\circ}{360^\circ} \times 33.86377778 = 9.88 \text{ min} = 9 \text{ min } 53 \text{ s}$$



Conclusion: Yes, the family can get the best fireworks full length of the fireworks display since the ride over 170m lasted till 8.52pm.

B1

Assumption: The capsule moves at constant speed throughout

B1

**End of Paper**

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