AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2023

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:

MATHEMATICS **4052/01**
Paper 1 **1 August 2023**
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any questions it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
/90

Answer **all** the questions.

1 Calculate $\dfrac{-10.5^3}{4.8-\sqrt{19}}$ and correct your answer to 2 significant figures.

Answer ..... [1]

---

2 Solve the equation $\dfrac{3x-5}{4}-\dfrac{x}{6}=3$.

Answer $x =$ ..... [2]

---

3 **(a)** Express 4680 as a product of its prime factors, giving your answers in index notation.

Answer ..... [1]

(b) The HCF and LCM of 120 and m are 12 and 4680 respectively.
Given that $12 = 2^2 \times 3$, find the value of m.

Answer $m =$ ..... [2]

---

4 Evaluate $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\times\ldots\times\left(1-\frac{1}{100}\right)$, leaving your answer as a fraction.

Answer .....[2]

---

5 **(a)** Express $-x^2+5x-7$ in the form $-(x+p)^2+q$.

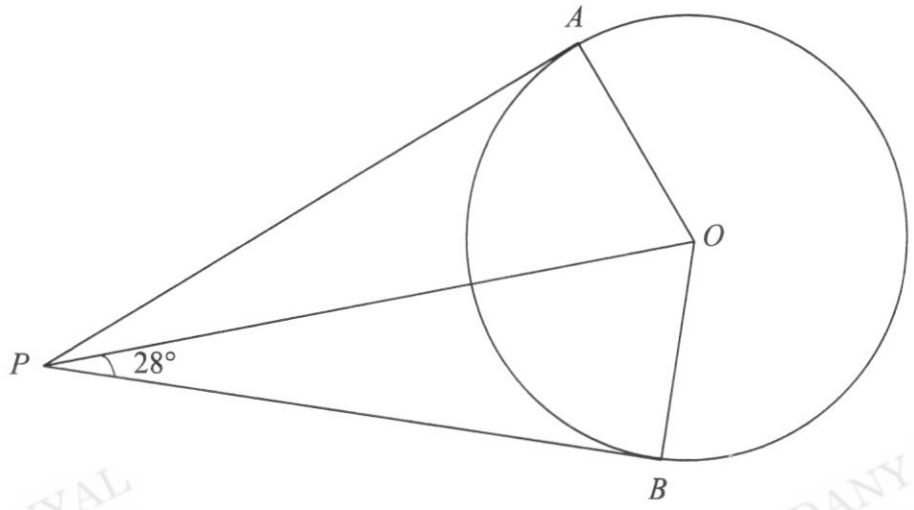
Answer .....[2]

(b) Hence explain why there is no real solution for $-x^2+5x-7=0$.

Answer .....

..... [1]

6



PA and *PB* are tangents to a circle, centre *O*.

Show that triangle *AOP* and triangle *BOP* are congruent.
Give a reason for each statement you make.

Answer

[2]

7 P is directly proportional to the cube of Q.
P is increased by 700%.
Calculate the percentage change in Q.

Answer .....% [2]

---

8 A tuft of grass grows approximately 70 picometre in 1 millisecond.
Estimate how much the same tuft of grass will grow in 1 second.
Express your answer in metres expressed in standard form. [pico = 10^{-12}]

Answer .....m [2]

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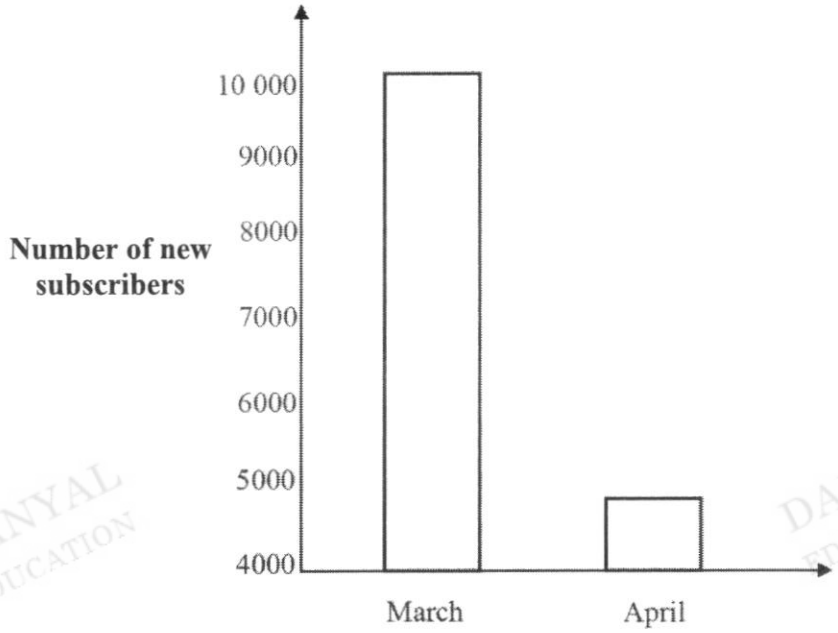
9 **Number of new BubbleTV subscribers in March and April 2023**



State one feature of the bar graph that may be misleading and explain how this may lead to a misinterpretation.

Answer .....

.....

..... [2]

---

10 Given that $x=9$ is a solution of the equation $7(x^2+3kx+5)=(1-k)x-1$, find the value of k.

Answer $k=$ ..... [2]

---

11 Simplify $\dfrac{15am-4n-5m+12an}{8n^2+6mn-5m^2}$.

Answer ..... [3]

---

12 For real numbers x, y and z, it is given that $\dfrac{x}{y}=\dfrac{y}{z}=\dfrac{z}{x}$, and $y \neq z$.

(a) Show that $x+y+z=0$.

Answer

[2]

(b) Find the value of x for $\dfrac{3}{x}-\dfrac{2}{y+z}=12$.

Answer $x =$ ..... [2]

---

13 **(a)** Solve the inequalities $5-4x<2x+1\leq x+8$.

Answer .....[2]

(b) Represent the solution on the number line.

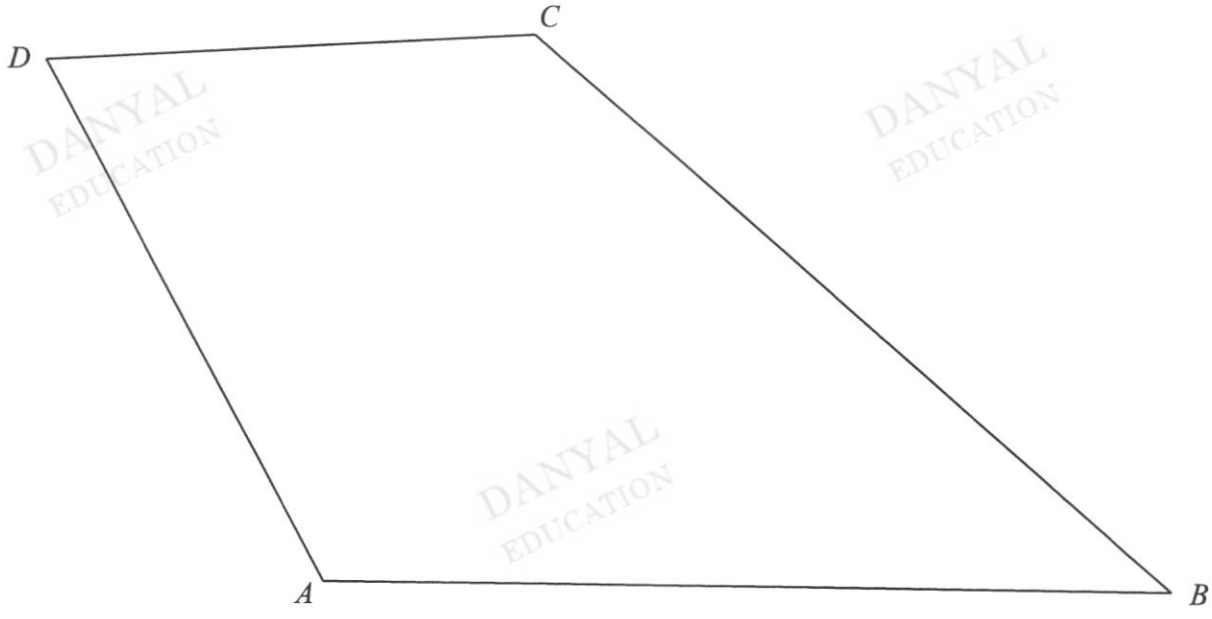
Answer

[1]

(c) Write down the smallest prime number that satisfies $5-4x<2x+1\leq x+8$.

Answer .....[1]

14 The diagram shows the plan of a neighbourhood play area $ABCD$ in the shape of a quadrilateral.
It is drawn to a scale of 1 centimetre represent 1 kilometre.



A sandy court is located in the play area, nearer to D than to A and nearer to CD than to AD.
Shade and label the region S, where the sandy court is to be located. [3]

15 Coffee beans are packed in bags of 250 g and 750 g.
The number of 250 g bags and 750 g bags are in the ratio 1 : 2.
When two of the 750 g bags are repacked into 250 g bags, the ratio becomes 5 : 3.

(a) Find the original number of 250 g bags.

Answer .....[3]

(b) Find the total mass of the coffee beans in kilograms.

Answer ..... kg [1]

---

16 P and Q are two end points of a diameter of a circle with centre C.
The coordinate of P is $(-1, 5)$.
The coordinate of C is $(3, 6)$.

(a) Find the length of the diameter of the circle.

Answer .....[2]

(b) Find the gradient of PQ.

Answer .....[2]

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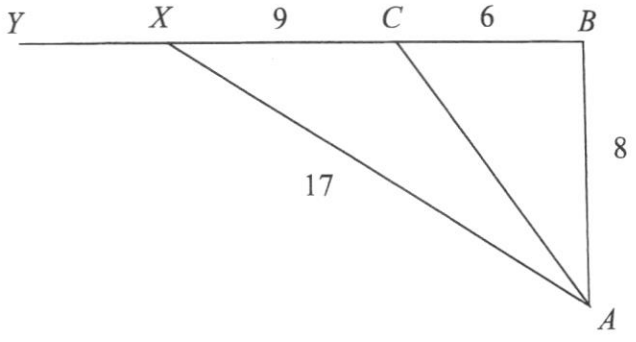
17 In the diagram below, $XC = 9$ cm, $CB = 6$ cm , $BA = 8$ cm and $XA = 17$ cm.



(a) Given that $YXCB$ is a straight line, show that angle $CBA = 90^\circ$.

Answer

[2]

(b) Given that $\sin \angle ACX = \frac{4}{5}$, find the exact value of $\sin \angle CAX$.

Answer .....[2]

18 **(a)** Factorise $y^3 - y$.

Answer .....[2]

(b) Hence explain why $y^3 - y$ is divisible by 2 for any positive integer value of y.

Answer .....

.....[2]

---

19 **(a)** **(i)** Simplify $4x^2 \times 3x^0 \div 12x^{-5}$.

Answer .....[1]

(ii) Simplify $\left(\frac{a^5}{256b^4}\right)^{-\frac{1}{4}}$, giving your answer in positive index form.

Answer .....[1]

(b) Solve $27 = \sqrt[n]{243}$.

Answer .....[2]

---

20 The first four terms in a sequence of numbers are given below.

$$T_1 = 3^2 - (-1)^2 = 8$$
$$T_2 = 4^2 - 0^2 = 16$$
$$T_3 = 5^2 - 1^2 = 24$$
$$T_4 = 6^2 - 2^2 = 32$$

(a) Write down an expression, in the same form, in terms of n, to represent T_n.

Answer T_n = ... =..... [1]

(b) Hence find the positive integer value of h and k such that $h^2 - k^2 = 600$.

Answer h = ...

k = ... [3]

21 A regular n-sided polygon is cut from a circular piece of paper of radius 5.4 cm such that all vertices of the polygon are on the circumference of the circle.

Given that one interior angle of the polygon is 144°, find

(a) the value of n,

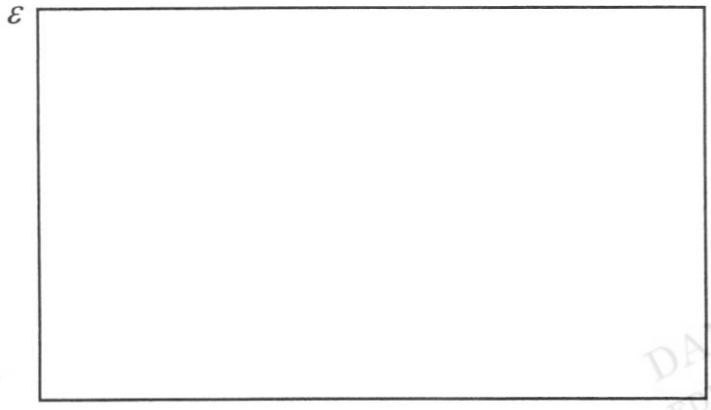
Answer $n =$ ..... [1]

(b) the area of paper discarded.

Answer ..... cm^2 [3]

22 **(a)** A, B and C are subsets of the universal set ε where $A \cap C = C$, $A \cap B \neq \varnothing$ and $B \cap C = \varnothing$.
Represent these sets on a clearly labelled Venn diagram.

Answer



[2]

(b) $\varepsilon = \{x : x \text{ is an integer}, -3 < x < 10\}$
$A = \{x : x \text{ is a prime number}\}$
$B = \{x : x > 0\}$
List the elements in A'.

Answer ..... [1]

(c) Find $n(A \cap B')$.

Answer ..... [1]

23 The back-to-back stem-and-leaf diagram shows the times, in minutes taken by two groups of students to complete a task.

Group A								Group B									
							1	*h*									
				8	7	6	4	1	2	9							
9	9	9	8	6	5	3	3	0	3	2	4	4	6				
			6	6	6	6	4	2	4	2	2	2	4	6	7	7	8	8	8
									5	0	*k*								

Key: 1 | 2 | 9 means a time of 21 minutes in Group A and a time of 29 minutes in Group B

(a) Find the modal time for group A.

Answer ..... min [1]

(b) Find the median time for group A.

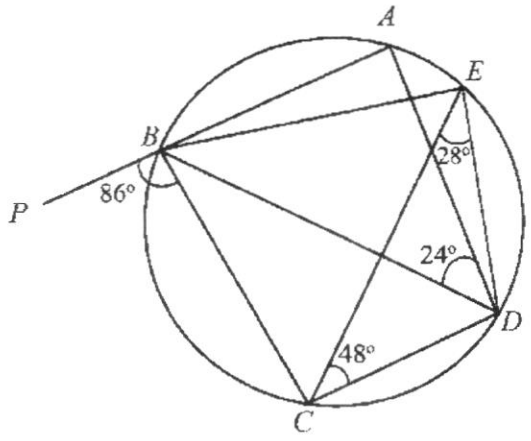
Answer ..... min [1]

(c) The range of time for group B is 45 minutes.
The mean time for group B is 41 minutes.
By forming two equations, find the value of h and of k.

Answer h =.....

k =..... [4]

24



In the diagram, A, B, C, D and E are points on a circle.
ABP is a straight line.
It is given that Angle $ADB = 24°$, Angle $DCE = 48°$, Angle $CED = 28°$ and Angle $CBP = 86°$.
Find, giving reasons for each answer,

(a) **(i)** angle CBD,

Answer ..... [1]

(ii) angle BDC,

Answer ..... [2]

(iii) angle ADE.

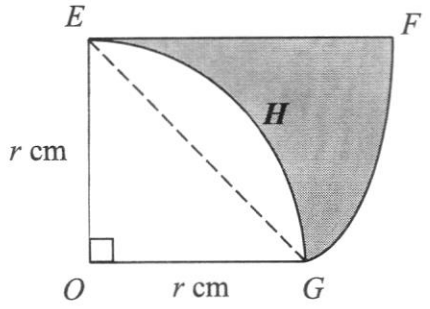
Answer ..... [1]

(b) Explain why BD is a diameter of the circle.

Answer

[2]

25 In the diagram below, $\angle EOG = 90°$, EF is parallel to OG and $OE = r$ cm.
EG is an arc of a circle centre O and FG is an arc of a circle centre E.



What fraction of the diagram is shaded? Leave your answer in terms of π.

Answer ..... [5]

26 Two travel companies, Owl Flight and GA Air sell air tickets to Thailand, Vietnam and India.
The matrix **B** shows the number of air tickets each company was allocated to sell in December 2022.

$$\mathbf{B} = \begin{matrix} \\ \textbf{Thailand} \\ \textbf{Vietnam} \\ \textbf{India} \end{matrix} \begin{matrix} \textbf{Owl} & \textbf{GA} \\ \begin{pmatrix} 125 & 80 \\ 207 & 135 \\ 96 & 75 \end{pmatrix} \end{matrix}$$

(a) The price of an air ticket to Thailand is \$450.
The price of an air ticket to Vietnam is \$600.
The price of an air ticket to India is \$800.
Represent this information in a 1×3 matrix **A**.

Answer **A** = [1]

(b) Evaluate the matrix **C** = **AB**.

Answer **C** = [2]

(c) State what each element of matrix **C** represents.

Answer .....

.....

..... [1]

(d) By the end of December 2022, GA Air did not manage to sell all the air tickets.
The number of air tickets to Vietnam sold was 100% more than the number of air tickets to Thailand sold.
The number of air tickets to India sold was 40% of the number of air tickets to Vietnam sold.
Given that 23 air tickets to India were not sold, find the number of air tickets to Thailand GA Air sold.

Answer ..... [2]

End of Paper

AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2023

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:

MATHEMATICS **4052/02**
Paper 2 **02 August 2023**
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
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For Examiner's Use
/ 90

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Factorise $25p^2 - 1$.

Answer ..... [1]

(b) $a = \dfrac{b^2 + 3}{b - c^2}$

(i) Find a when $b = -2$ and $c = \dfrac{1}{2}$.

Answer ..... [1]

(ii) Rearrange the formula to make c the subject.

Answer ..... [3]

(c) Write as a single fraction in its simplest form $\dfrac{3x}{(3x-2)^2}-\dfrac{2}{2-3x}$.

Answer .....[2]

(d) Solve the equation $\dfrac{3}{x-3}+\dfrac{2x-1}{x^2-5x+6}=3$

Answer .....[3]

2 The variables x and y are connected by the equation $y = \frac{1}{5}\left(50 - \frac{60}{x} - x^2\right)$.

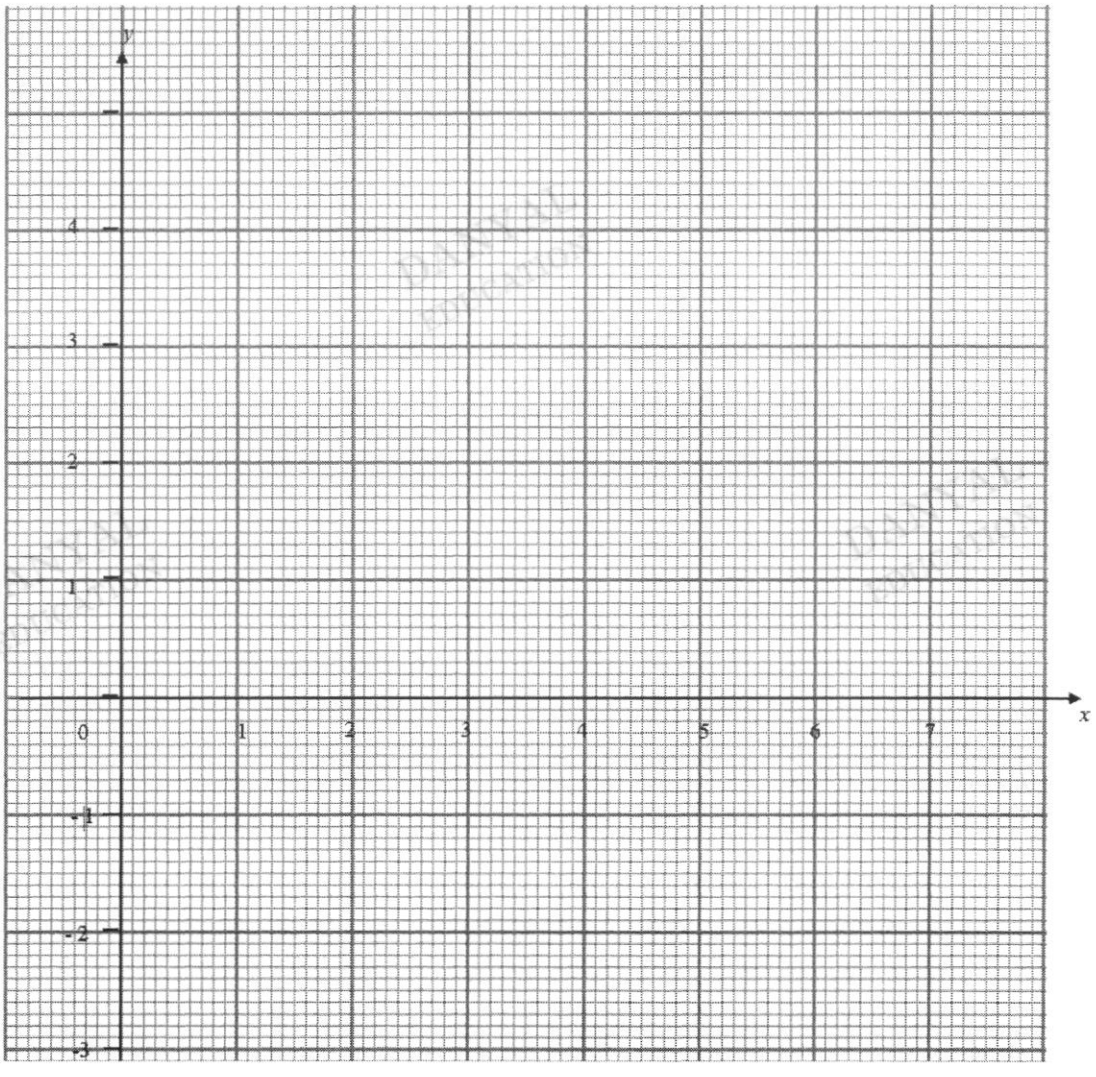
Some of the corresponding values of x and y are given in the following table.

x	1	2	2.5	3.5	4	5	6	7
y	–2.2	3.2	4.0	4.1	3.8	2.6	0.8	w

(a) Find the value of w.

..... [1]

(b) On the grid, draw the graph of $y = \frac{1}{5}\left(50 - \frac{60}{x} - x^2\right)$ for $1 \leq x \leq 7$. [3]



(c) Explain how your graph shows that there are more than one solution of the equation

$$\frac{1}{5}\left(50-\frac{60}{x}-x^2\right)=0.$$

Answer .....

.....[1]

(d) By drawing a tangent, find the value of x when the gradient of the curve is 4.

Answer .....[2]

(e) By drawing a suitable straight line on the same axes, solve the equation

$$2x^2+5x+\frac{120}{x}=102.$$

Answer x = ...or x = ...[3]

3 **(a)** Mr Xander, his wife and their 5 children aged 16 years old, 13 years old, 11 years old and a pair of twins who are 3 years old dined in a buffet restaurant.

The restaurant offered a promotion of 50% discount to children aged 10 to 12 years old and free dining for children below 10 years old.

The buffet restaurant charges S$25.50 per pax and the bill is subjected to 10% service charge and 8% GST.

Calculate the total bill paid by Mr Xander.

Answer $.....[2]

(b) A sofa is priced at $8600.

Salmiah bought the sofa on hire purchase with these terms:

A downpayment of $500 and the balance to be paid in monthly instalments over 3 years at a simple interest rate of 2.5% per annum.

Calculate her monthly instalment.

Answer $.....[2]

(c) John borrows \$10 000 from a bank for 3 years.

The total compound interest he has to pay the bank at the end of 3 years is \$768.

Given that interest is compounded yearly, calculate the interest rate per year.

Give your answer correct to 2 significant figures.

Answer .....% [2]

(d) The exchange rate on a particular day at Changi Airport between Japanese Yen (¥) and Singapore dollars (SGD) was 1SGD = ¥106.10

On the same day, the exchange rate between Thai Baht (฿) and Singapore dollars (SGD) was 1 SGD =฿26.

Zen wants to change ¥30 000 into Thai Baht.

He has a choice of either changing it in Changi Airport or when he is in Thailand.

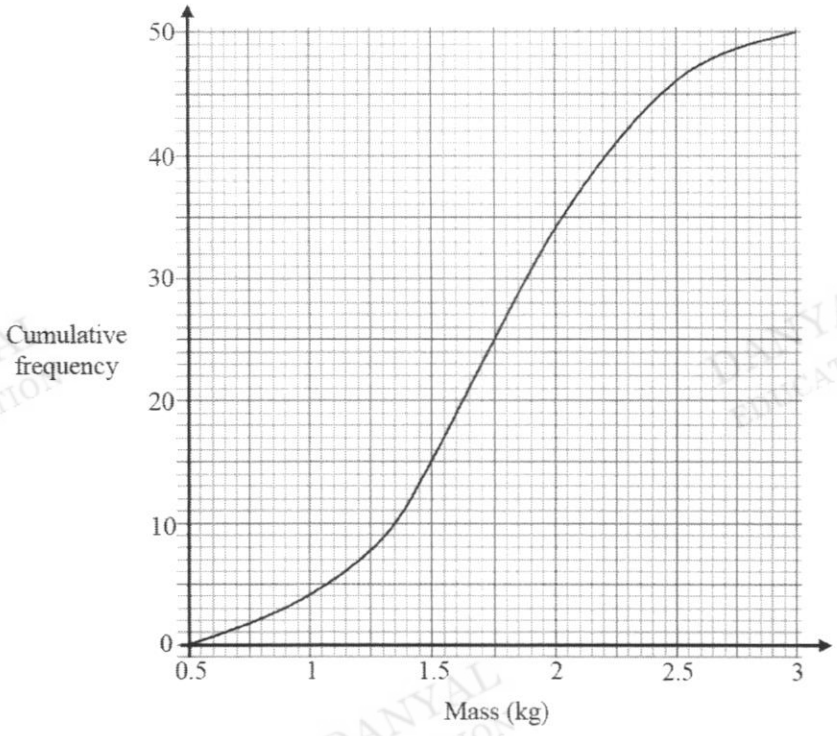
Assuming that the exchange rates do not change and that the exchange rate in Thailand will be ¥100 = ฿28, suggest where he should change his money.

Justify your decision with calculations.

Answer : He should change his money in ..... [3]

4 Jessie owns a duck farm and she conducts a study on the masses of 50 healthy ducks in her farm, Farm J.

(a) The cumulative frequency curve shows the distribution of the masses.



(i) Complete the grouped frequency table for the masses of the ducks.

Mass (m kg)	$0.5 \leq m < 1.0$	$1.0 \leq m < 1.5$	$1.5 \leq m < 2.0$	$2.0 \leq m < 2.5$	$2.5 \leq m < 3.0$
Frequency					

[1]

(ii) Calculate an estimate of the mean mass of each duck.

Answer .....[1]

(iii) Calculate an estimate of the standard deviation.

Answer .....[1]

(iv) Explain why the mean and standard deviation are estimates.

Answer .....

.....

.....

.....[1]

(v) A research article states that 92% of healthy ducks have a mass greater than 1.3 kg.

Comment on whether Jessie's data supports this claim.

Answer .....

.....

.....

.....

.....[2]

(b) Quinn also has 50 ducks in her farm, Farm Q.

Her ducks have a mean mass of 1.65 kg and standard deviation of 0.95.

Make two comments comparing the masses of ducks from both farms.

Answer .....

.....

.....

.....

.....

.....[2]

(c) Mary has three 50 cent coins and two 10 cent coins in her pocket.

She takes two coins out of her pocket, at random, one after another.

The coins are not replaced.

(i) Find the probability that both coins are of different denominations.

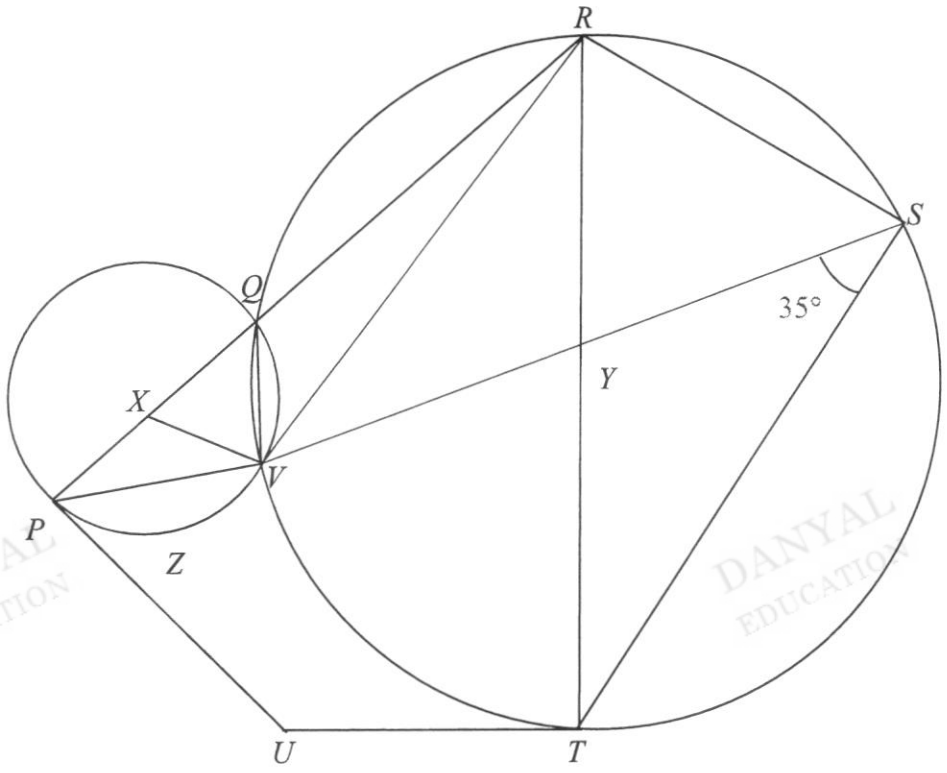
Answer .....[2]

(ii) Mary takes out a third coin from her pocket.

Find the probability that the total value of the three coins taken out is 70 cents.

Answer .....[2]

5



The diagram shows two circles with centre X and Y respectively.

The circles intersect each other at Q and V.

PU is tangent to the smaller circle at P and TU is tangent to the bigger circle at T.

(a) Show that triangle RVY is similar to triangle STY.

Give a reason for each statement you make.

.....

.....

.....

.....[2]

(b) Given radius $XV = 5$ cm and angle $VST = 35°$, find the area of segment PZV.

Answer ..... [5]

6 **(a)** **(i)** $\overrightarrow{AB} = \begin{pmatrix} 2p \\ 5p \end{pmatrix}$.

$|\overrightarrow{AB}| = \frac{5\sqrt{29}}{2}$.

Find the two possible values of p.

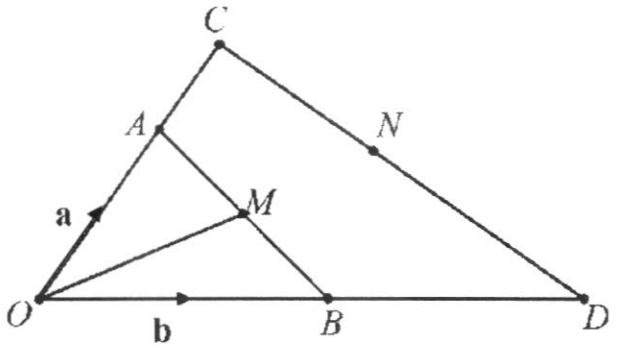
Answer .....[2]

(ii) A line joins the two points $X(-6, 2)$ and $Z(6, -2)$.

Use vectors to show whether or not the point $Y(3, 1)$ lies on this line. [3]

(b) In the diagram, M is the midpoint of AB and B is the midpoint of OD.

$OA = \frac{2}{3}OC$, $CN = \frac{3}{7}CD$, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.



(i) Express each of the following vectors in terms of **a** and **b**.

(a) $\overrightarrow{CD}$,

Answer .....[1]

(b) $\overrightarrow{ON}$.

Answer .....[2]

(ii) Write down the ratio of

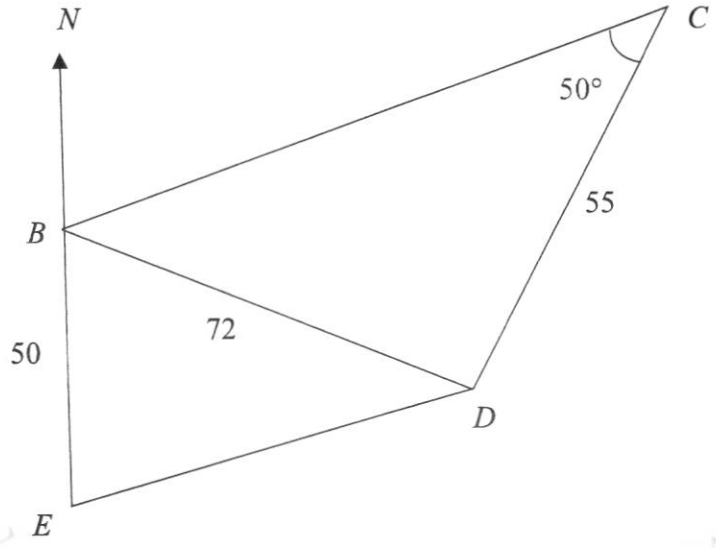
(a) area of ΔOAB : area of ΔOCB ,

Answer .....[1]

(b) area of ΔOAB : area of ΔOCD.

Answer .....[2]

7



The diagram shows a plot of land BCDE gazetted for development into a school.

$BE = 50$ m, $BD = 72$ m, $CD = 55$ m and angle $BCD = 50°$.

E is due south of B and the bearing of D from B is $113°$.

(a) Find the distance DE.

Answer .....[3]

(b) Find the bearing of B from C.

Answer .....[3]

(c) Calculate the area of $BCDE$.

Answer .....[2]

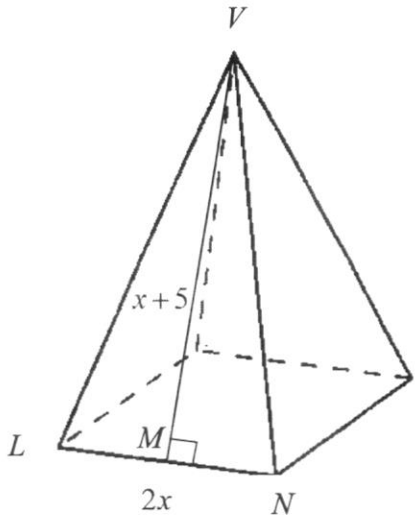
(d) An engineer planned to erect a flag pole at B such that the angle of elevation of the top of the flag pole from E is $13°$.

He estimated and told the builder to prepare wood of length 11 m for the pole.

Is his estimation correct?

Answer .....[3]

8



The diagram shows a model of a commemorative structure in the shape of a square-based pyramid.

$VM = (x+5)$ cm and $LN = 2x$ cm.

(a) Given that the total surface area of the pyramid is 50 cm^2, form an equation in x and show that it reduces to $4x^2 + 10x - 25 = 0$. [3]

(b) Solve the equation $4x^2 + 10x - 25 = 0$.

Answer $x =$ .....or $x =$ .....[3]

(c) Explain why one of the solutions in part **(b)** must be rejected.

Answer .....

.....

.....[1]

(d) Given that the height of the model is 6.36 cm and the scale of the model is 1: 500, find the actual volume of the structure, in m^3.

Answer .....m^3 [3]

9 John earns $1650 every month and this is sufficient to cover his family's expenses in 2022. The costs are given in the table below.

Living expenses in 2022

Groceries	$250 per week
Utilities	$200 per month
Transport	$300 per month

(a) Given that there are 52 weeks in a year, calculate his total expenditure for 2022.

Answer ..... [1]

(b) Inflation causes prices of goods and services to increase yearly.

The table below shows the percentage change in price from the previous year.

Consumer Price Index			
Categories	**Percentage Change (%)**		
	2021	2022	2023 (expected)
Groceries	3.5	8	20
Healthcare	10	13	23
Transport	6	−2	12.5
Communication	−0.9	−4.2	3
Recreation & Culture	2.8	1	1.5
Utilities	8	0	15
Miscellaneous Goods & Services	−1.1	0.3	0.5

Calculate John's expected total expenditure for 2023.

Answer ..... [3]

(c) Due to inflation, John realised that his current income is insufficient to cover his family's expected living expenses in 2023.

To make up for the shortfall, he decides to take up an additional part-time job during the weekends.

Below are the possible options for him.

Fast Food Service Crew	**Credit Card Sales Promoter**	**Food Delivery Rider**
$8 per hour	Basic pay of $4 per hour and an additional $10 commission per successful credit card sign up. Based on the company's statistics, on average, a person is able to get a successful sign up every 2 hours.	An average of $15 per hour Every food delivery rider is required to purchase a food delivery starter kit which consists of a long sleeve base, helmet, reflective jacket and insulated backpack at $50.

Suggest which part-time job he should take up and the number of hours he needs to work per week to make up for the shortfall.

Justify any decision you make and show your calculations clearly.

Answer.....

.....[6]

END OF PAPER

AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2023

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:

MATHEMATICS **4052/01**
Paper 1 **1 August 2023**
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any questions it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
/90

This document consists of **22** printed pages.

Answer **all** the questions.

1 Calculate $\dfrac{-10.5^3}{4.8-\sqrt{19}}$ and correct your answer to 2 significant figures.

$\dfrac{-10.5^3}{4.8-\sqrt{19}} = -2600$ (2 s.f.)

Answer ..... [1]

---

2 Solve the equation $\dfrac{3x-5}{4}-\dfrac{x}{6}=3$.

$\dfrac{3x-5}{4}-\dfrac{x}{6}=3$

$\dfrac{9x-15-2x}{12}=3$

$\dfrac{7x-15}{12}=3$

$7x-15=36$

$x=7\dfrac{2}{7}$

Answer $x =$ ..... [2]

---

3 **(a)** Express 4680 as a product of its prime factors, giving your answers in index notation.

$4680 = 2^3 \times 3^2 \times 5 \times 13$

Answer ..... [1]

(b) The HCF and LCM of 120 and m are 12 and 4680 respectively.
Given that $12 = 2^2 \times 3$, find the value of m.

120 =	$2^3 \times 3 \times 5$
m =	$2^2 \times 3^2 \times 13$
HCF =	$2^2 \times 3$
LCM =	$2^3 \times 3^2 \times 5 \times 13$

$m = 468$

Answer $m =$ ..... [2]

---

4 Evaluate $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\times...\times\left(1-\frac{1}{100}\right)$, leaving your answer as a fraction.

$$\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\times...\times\left(1-\frac{1}{100}\right)$$
$$=\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times...\frac{99}{100}$$
$$=\frac{2}{100}$$
$$=\frac{1}{50}$$

Answer .....[2]

---

5 **(a)** Express $-x^2+5x-7$ in the form $-(x+p)^2+q$.

$$-x^2+5x-7$$
$$=-(x^2-5x+7)$$
$$=-[(x-\frac{5}{2})^2-(-\frac{5}{2})^2+7]$$
$$=-(x-\frac{5}{2})^2-\frac{3}{4}$$

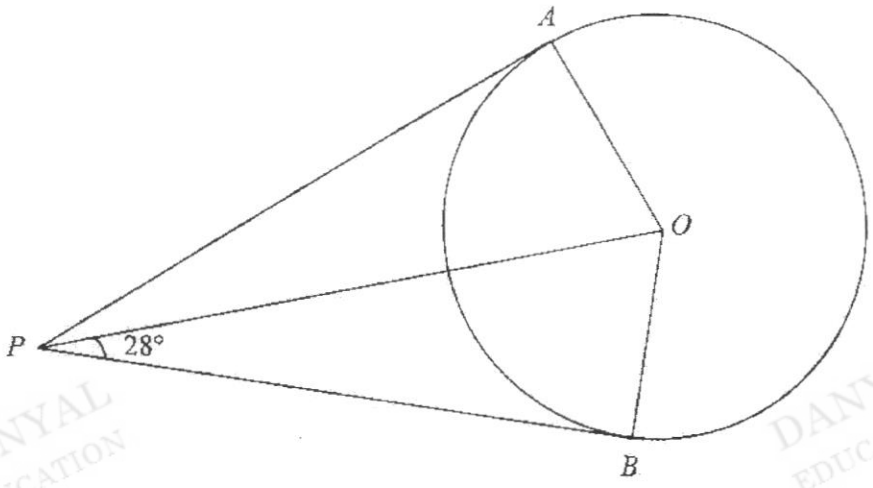
Answer .....[2]

(b) Hence explain why there is no real solution for $-x^2+5x-7=0$.

Th shape of the curve opens downwards indefinitely and since the maximum point of the curve is $(\frac{5}{2},-\frac{3}{4})$, *the curve lies entirely below the x-axis. There is no intersection with the x-axis and hence no solution.*

OR The maximum point is at $(\frac{5}{2},-\frac{3}{4})$, therefore the graph will not cut x axis and hence no solution.

6



In the figure, not drawn to scale, two tangents drawn from points A and B on a circle with centre O meet at point P.

Show that triangle AOP and triangle BOP are congruent.
Give a reason for each statement you make.

Answer

$\angle OAP = \angle OBP = 90°$ (radius $\perp$ tangent)
OP (common hypotenuse)
$OA = OB$ (radius of circle)
triangle $AOP \equiv$ triangle BOP (RHS Congruency Test)

OR

$AP = BP$ (tangent from external point)
OP common side
$OA = OB$ (radius of circle)
triangle $AOP \equiv$ triangle BOP (SSS Congruency test)

[2]

7 P is directly proportional to the cube of Q.
P is increased by 700%.
Calculate the percentage change in Q.

$P = kQ^3$

$k = \frac{P}{Q^3}$

$P_{new} = k(Q_{new})^3$

$8P = k(Q_{new})^3$

$8P = \frac{P}{Q^3}(Q_{new})^3$

$(Q_{new})^3 = 8Q^3$

$Q_{new} = \sqrt[3]{8Q^3}$

$Q_{new} = 2Q$

Percentage Increase

$= \frac{2Q - Q}{Q} \times 100\%$

$= 100\%$

OR

$P = kQ^3$

$Q^3 = \frac{P}{k}$

$Q_{new}^{\ 3}$

$= \frac{P_{new}}{k}$

$= \frac{8P}{k}$

Percentage Increase

$= \frac{\sqrt[3]{\frac{8P}{k}} - \sqrt[3]{\frac{P}{k}}}{\sqrt[3]{\frac{P}{k}}} \times 100\%$

$= \frac{\sqrt[3]{8} - 1}{1} \times 100\%$

$= 100\%$

Answer .....[2]

---

8 A tuft of grass grows approximately 70 picometre in 1 millisecond.
Estimate how much the same tuft of grass will grow in 1 second.
Express your answer in metres expressed in standard form. [pico = 10^{-12}]

1×10^{-3} s represent 70×10^{-12} m

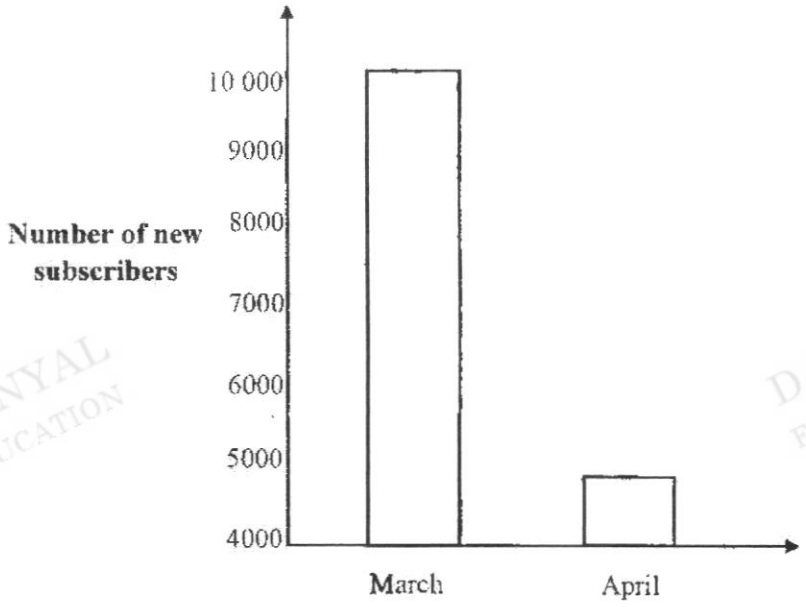
1 s represent $\frac{70 \times 10^{-12}}{10^{-3}}$ m

1 s represent 7×10^{-8} m

Answer .....m [2]

---

9 **Number of new BubbleTV subscribers in March and April 2023**



State one feature of the bar graph that may be misleading and explain how this may lead to a misinterpretation.

Feature: The vertical axis starts from 4000 and does not start from 0.

How it is misleading: The differences are exaggerated. The subscribers in March look 4-6 times more than that of April on the bar graph. However in reality, it is only about twice the number.

---

10 Given that $x=9$ is a solution of the equation $7(x^2+3kx+5)=(1-k)x-1$, find the value of k.

$7(81+27k+5)=9-9k-1$

$567+189k+35=9-9k-1$

$198k=-594$

$k=-3$

Answer $k=$ ..... [2]

---

11 Simplify $\dfrac{15am-4n-5m+12an}{8n^2+6mn-5m^2}$.

$$\frac{15am-4n-5m+12an}{8n^2+6mn-5m^2}$$
$$=\frac{5m(3a-1)+4n(3a-1)}{(4n+5m)(2n-m)}$$
$$=\frac{(3a-1)(5m+4n)}{(4n+5m)(2n-m)}$$
$$=\frac{3a-1}{2n-m}$$

12 For real numbers x, y and z, it is given that $\dfrac{x}{y}=\dfrac{y}{z}=\dfrac{z}{x}$, and $y\neq z$.

(a) Show that $x+y+z=0$.

$$\frac{x}{y}=\frac{y}{z}=\frac{z}{x}$$
$$xz=y^2 \quad \text{and} \quad xy=z^2$$
$$xz-xy=y^2-z^2$$
$$x(z-y)=(y-z)(y+z)$$
$$x(z-y)=-(z-y)(y+z)$$
$$x=-y-z$$
$$x+y+z=0 \text{ (shown)}$$

Answer ..... [2]

(b) Find the value of x for $\dfrac{3}{x}-\dfrac{2}{y+z}=12$.

since $x+y+z=0$,
$x=-(y+z)$.
$$\frac{3}{x}-\frac{2}{y+z}=12$$
$$\frac{3}{x}+\frac{2}{x}=12$$
$$\frac{5}{x}=12$$
$$x=\frac{5}{12}$$

Answer $x=$ ..... [2]

13 **(a)** Solve the inequalities $5-4x<2x+1\leq x+8$.

$5-4x<2x+1$

$-6x<-4$

$x>\frac{2}{3}$

AND

$2x+1\leq x+8$

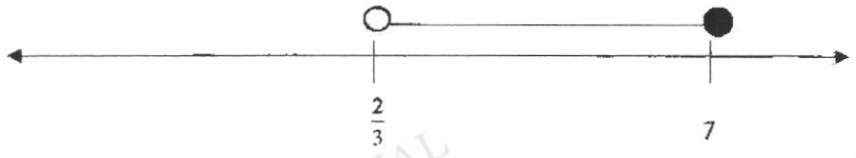
$x\leq 7$

$\therefore \frac{2}{3}<x\leq 7$

Answer .....[2]

(b) Represent the solution on the number line.

Answer



[1]

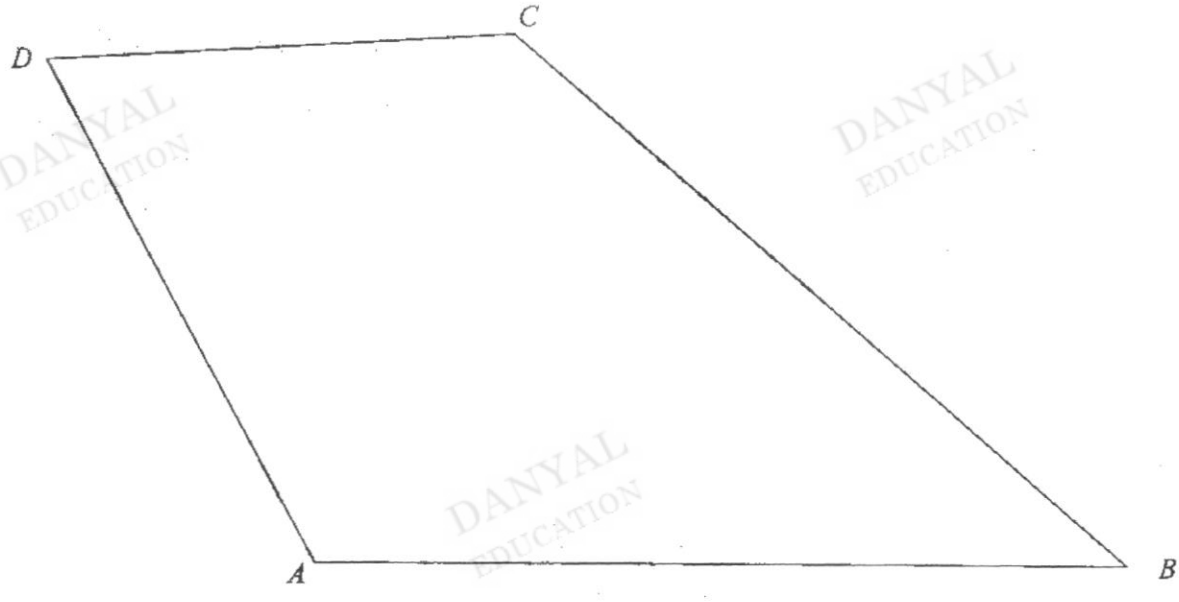
(c) Write down the smallest prime number that satisfies $5-4x<2x+1\leq x+8$.

Smallest Prime number = 2

Answer .....[1]

14 The diagram shows the plan of a neighbourhood play area $ABCD$ in the shape of a quadrilateral.
It is drawn to a scale of 1 centimetre represent 1 kilometre.

(ANSWER: see attachment at the back)



A sandy court is located in the play area, nearer to D than to A and nearer to CD than to AD.
Shade and label the region S, where the sandy court is to be located. [3]

15 Coffee beans are packed in bags of 250 g and 750 g.
The number of 250 g bags and 750 g bags are in the ratio 1 : 2.
When two of the 750 g bags are repacked into 250 g bags, the ratio becomes 5 : 3.

(a) Find the original number of 250 g packages,

let x be the original number of packages of 250 g then the number of packages for 750 g is $2x$.
$1500 \div 250 = 6$

$$\frac{x+6}{2x-2} = \frac{5}{3}$$

$3x + 18 = 10x - 10$

$7x = 28$

$x = 4$

Original no of 250 g packages = x = **4**

Answer .....[3]

(b) Find the total mass of the coffee beans in kilograms.

Mass of coffee beans
$250(4) + 750(8)$
$= 7kg$

Answer ..... kg [1]

16 P and Q are two end points of a diameter of a circle with centre C.
The coordinate of P is $(-1,\ 5)$.
The coordinate of C is $(3,\ 6)$.

(a) Find the length of the diameter of the circle.

Length
$= 2\left(\sqrt{(5-6)^2+(-1-3)^2}\right)$
$= = 8.24621$
$= 8.25$ units (3 s.f.)

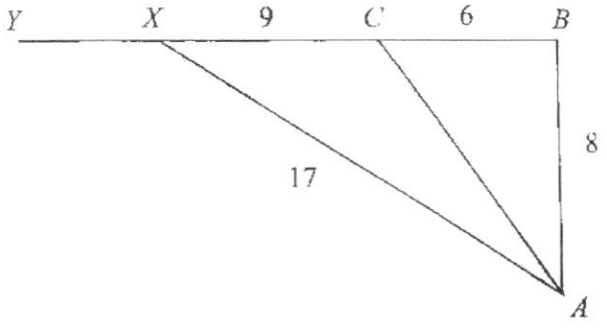
Answer ..... [2]

(b) Find the gradient of PQ.

Gradient
= gradient of PC
$= \dfrac{5-6}{-1-3}$
$= \dfrac{1}{4}$

Answer ..... [2]

17 In the diagram below, $XC = 9$ cm, $CB = 6$ cm , $BA = 8$ cm and $XA = 17$ cm.



(a) Given that $YXCB$ is a straight line, show that angle $CBA = 90°$.

Answer

$XB^2 + AB^2$

$= 15^2 + 8^2$

$= 289$

AX^2

$= 17^2$

$= 289$ [2]

Since $XB^2 + AB^2 = AX^2$, by the converse of Pythagoras' theorem, $\angle XBA = 90°$. Hence, $\angle CBA = 90°$

OR

$$\cos \angle CBA = \frac{15^2 + 8^2 - 17^2}{2(15)(8)}$$

$\angle CBA = 90°$

(b) Given that $\sin \angle ACX = \frac{4}{5}$, find the exact value of $\sin \angle CAX$.

$$\frac{\sin \angle CAX}{9} = \frac{\sin \angle ACX}{17}$$

$$\frac{\sin \angle CAX}{9} = \frac{\frac{4}{5}}{17}$$

$$\sin \angle CAX = \frac{\frac{4}{5}}{17} \times 9$$

$$\sin \angle CAX = \frac{36}{85}$$

Answer .....[2]

---

18 **(a)** Factorise $y^3 - y$.

$$y^3 - y$$
$$= y(y^2 - 1)$$
$$= y(y+1)(y-1)$$

Answer .....[2]

(b) Hence explain why $y^3 - y$ is divisible by 2 for any positive integer value of y.

Either y or $y-1$ is an even number as they are consecutive numbers.
Any number multiplies by an even number is divisible by 2.
OR
Since $y-1$, y and $y+1$ are consecutive integers, at least one of them must be even. Since there are even factor(s), $y^3 - y$ is divisible by 2.

---

19 **(a)** **(i)** Simplify $4x^2 \times 3x^0 \div 12x^{-5}$.

$4x^2 \times 3x^0 \div 12x^{-5}$
$= 12x^2 \div 12x^{-5}$
$= x^7$

Answer .....[1]

(ii) Simplify $\left(\frac{a^5}{256b^4}\right)^{-\frac{1}{4}}$, giving your answer in positive index form.

$\left(\frac{a^5}{256b^4}\right)^{-\frac{1}{4}}$
$= \left(\frac{256b^4}{a^5}\right)^{\frac{1}{4}}$
$= \frac{4b}{a^{\frac{5}{4}}}$

Answer .....[1]

(b) Solve $27 = \sqrt[n]{243}$.

$3^3 = (3^{\frac{5}{n}})$
$3 = \frac{5}{n}$
$3n = 5$
$n = \frac{5}{3}$

Answer .....[2]

20 The first four terms in a sequence of numbers are given below.

$$T_1 = 3^2 - (-1)^2 = 8$$
$$T_2 = 4^2 - 0^2 = 16$$
$$T_3 = 5^2 - 1^2 = 24$$
$$T_4 = 6^2 - 2^2 = 32$$

(a) Write down an expression, in the same form, in terms of n, to represent T_n.

$T_n = (n+2)^2 - (n-2)^2 = 8n$

Answer T_n = ... = ... [1]

(b) Hence find the positive integer value of h and k such that $h^2 - k^2 = 600$.

$600 \div 8 = 75$
Pattern number is 75.
$h = 75 + 2 = 77$
$k = 75 - 2 = 73$

Answer h = ...

k = ... [3]

21 A regular n-sided polygon is cut from a circular piece of paper of radius 5.4 cm such that all vertices of the polygon are on the circumference of the circle.

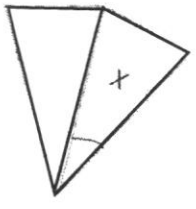
Given that one interior angle of the polygon is 144°, find

(a) the value of n,

$$n = \frac{360°}{180° - 144°}$$
$$= 10$$

Answer $n =$ ..... [1]

(b) the area of paper discarded.



$$x = \frac{360°}{10}$$
$$= 36°$$

Area of Polygon

$$= 10\left(\frac{1}{2} \times 5.4 \times 5.4 \times \sin 36°\right)$$
$$= 85.6991$$

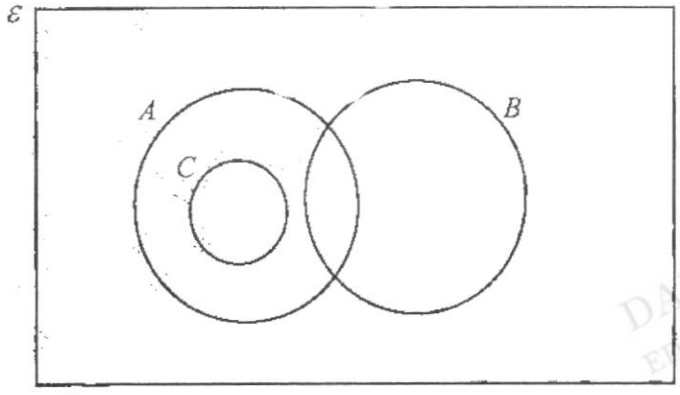
Amount of paper discarded

$$= \pi(5.4)^2 - 85.6991$$
$$= 5.9097$$
$$= 5.91 cm^2$$

Answer ..... cm^2 [3]

22 (a) A, B and C are subsets of the universal set ε where $A \cap C = C$, $A \cap B \neq \varnothing$ and $B \cap C = \varnothing$.
Represent these sets on a clearly labelled Venn diagram.

Answer



[2]

(b) $\varepsilon = \{x : x \text{ is an integer, } -3 < x < 10\}$
$A = \{x : x \text{ is a prime number}\}$
$B = \{x : x > 0\}$
List the elements in A'.

$-2, -1, 0, 1, 4, 6, 8, 9$

Answer ..... [1]

(c) Find $n(A \cap B')$.

$n(A \cap B') = 0$

Answer ..... [1]

23 The back-to-back stem-and-leaf diagram shows the times, in minutes taken by two groups of students to complete a task.

Group A											Group B									
									1	*h*										
				8	7	6	4	1	2	9										
9	9	9	8	6	5	3	3	0	3	2	4	4	6							
			6	6	6	6	4	2	4	2	2	2	4	6	7	7	8	8	8	
									5	0	*k*									

Key: 1 | 2 | 9 means a time of 21 minutes in Group A and a time of 29 minutes in Group B

(a) Find the modal time for group A.

46 minutes

Answer ..... min [1]

(b) Find the median time for group A.

37 minutes

Answer ..... min [1]

(c) The range of time for group B is 45 minutes.
The mean time for group B is 41 minutes.
By forming two equations, find the values of h and k.

Let x be the shortest time possible and y be the longest time possible for Group B,

$y - x = 45 - - - -(1)$

since mean is 43 minutes

$669 + x + y = (18)(41)$

$x + y = 69 - - - -(2)$

$(2) - (1)$:

$2x = 24$

$x = 12$

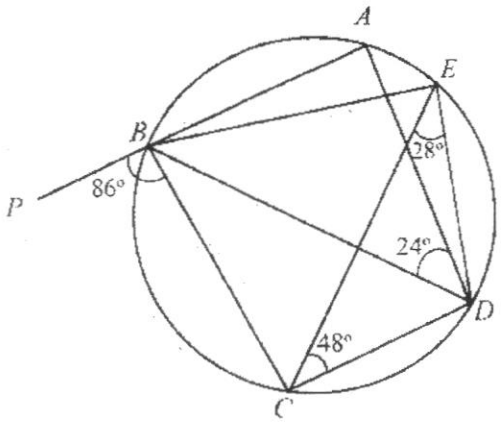
$y = 45 + 12 = 57$

$\therefore h = 2,\ k = 7$

Answer $h =$.....

$k =$..... [4]

24



In the diagram, A, B, C, D and E are points on a circle.
ABP is a straight line.
It is given that Angle $ADB = 24°$, Angle $DCE = 48°$, Angle $CED = 28°$ and Angle $CBP = 86°$.
Find, giving reasons for each answer,

(a) **(i)** angle CBD,

$\angle CBD = 28°$ (angles in same segment)

Answer ..... [1]

(ii) angle BDC,

$\angle ABC$

$= 180° - 86°$(adj $\angle$ on a straight line)

$= 94°$

$\angle BDC$

$= 180° - 94° - 24°$($\angle$ in opp segments)

$= 62°$

Answer ..... [2]

(iii) and angle ADE.

$\angle EBD$

$= 48°$ ($\angle$ in the same segment)

$\angle DBC$

$= 28°$($\angle$ in the same segment)

$\angle ADE$

$= \angle ABE$ ($\angle$ in the same segment)

$= 94° - 28° - 48°$

$= 18°$

Answer ..... [1]

(b) Explain why BD is a diameter of the circle.

Answer

$\angle BCD = 180° - \angle CBD - \angle CDB$ ($\angle$ sum of triangle)

$= 180° - 62° - 28°$

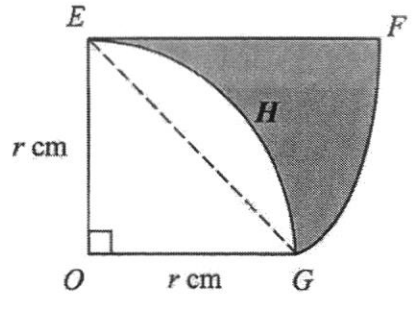
$= 90°$

Since $\angle BAD = 90°$, BD is the diameter by the property rt. $\angle$ in semicircle.

[2]

---

25 In the diagram below, $\angle EOG = 90°$, EF is parallel to OG and $OE = r$ cm. EG is an arc of a circle centre O and FG is an arc of a circle centre E.



What fraction of the diagram is shaded? Leave your answer in terms of π.

Length of EG

$=\sqrt{r^2+r^2}$

$=\sqrt{2}r$

Area of diagram *EOGF*

=triangle OEG + sector EGF

$=\frac{1}{2}\times r\times r+\frac{45}{360}\times\pi\times\left(\sqrt{2}r\right)^2$

$=\frac{1}{2}r^2+\frac{1}{8}\times\pi\times 2r^2$

$=\frac{1}{2}r^2+\frac{1}{4}\pi r^2$

Area of shaded region

$=\left(\frac{1}{2}r^2+\frac{1}{4}\pi r^2\right)-\frac{1}{4}\pi r^2$

$=\frac{1}{2}r^2$

Fraction shaded

$=\frac{\frac{1}{2}r^2}{\frac{1}{2}r^2+\frac{1}{4}\pi r^2}$

$=\frac{\frac{1}{2}r^2}{\frac{1}{2}r^2(1+\frac{1}{2}\pi)}$

$=\frac{1}{\frac{2+\pi}{2}}$

$=\frac{2}{2+\pi}$

Answer ..... [5]

---

26 Two travel companies, Owl Flight and GA Air sell air tickets to Thailand, Vietnam and India.

The matrix **B** shows the number of air tickets each company was allocated to sell in December 2022.

$$\mathbf{B}=\begin{matrix}\\ \text{Thailand}\\ \text{Vietnam}\\ \text{India}\end{matrix}\begin{matrix}\begin{matrix}\textbf{Owl} & \textbf{GA}\end{matrix}\\ \begin{pmatrix}125 & 80\\ 207 & 135\\ 96 & 75\end{pmatrix}\end{matrix}$$

(a) The price of an air ticket to Thailand is \$450.
The price of an air ticket to Vietnam is \$600.
The price of an air ticket to India is \$800.
Represent this information in a 1×3 matrix **A**
$A=\begin{pmatrix}450 & 600 & 800\end{pmatrix}$

Answer **A** = [1]

(b) Evaluate the matrix **C = AB**.

$$C=\begin{pmatrix}450 & 600 & 800\end{pmatrix}\begin{pmatrix}125 & 80\\ 207 & 135\\ 96 & 75\end{pmatrix}$$

$$=\begin{pmatrix}257250 & 177000\end{pmatrix}$$

Answer **C** = [2]

(c) State what each element of matrix **C** represents.

The total revenue for Owl flight is \$257250 and the total revenue for GA flight is \$177000 if all the allocated tickets are sold.

(d) By the end of December 2022, GA Air did not manage to sell all the air tickets.
The number of air tickets to Vietnam sold was 100% more than the number of air tickets to Thailand sold.
The number of air tickets to India sold was 40% of the number of air tickets to Vietnam sold.
Given that 23 air tickets to India were not sold, find the number of air tickets to Thailand GA Air sold.

Number of air tickets sold (India)

$=75-23$

$=52$

Number of air tickets sold (Vietnam)

$=\frac{52}{40}\times 100$

$=130$

Number of air tickets sold (Thailand)

$=\frac{130}{2}$

$=65$

Answer ..... [2]

End of Paper

AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2023

SECONDARY 4 EXPRESS

Name: MARKING SCHEME	Class:	Register No.:

MATHEMATICS **4052/02**
Paper 2 **02 August 2023**
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
/ 90

This document consists of **23** printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Factorise $25p^2 - 1$.

Answer ..... $(5p+1)(5p-1)$ [1]

(b) $a = \dfrac{b^2+3}{b-c^2}$

(i) Find a when $b = -2$ and $c = \dfrac{1}{2}$.

$$a = \frac{(-2)^2+3}{(-2)-\left(\frac{1}{2}\right)^2}$$

$$= -3\frac{1}{9}$$

Answer ..... [1]

(ii) Rearrange the formula to make c the subject.

$$a = \frac{b^2+3}{b-c^2}$$

$$ab - ac^2 = b^2 + 3$$

$$ac^2 = ab - b^2 - 3$$

$$c^2 = \frac{ab-b^2-3}{a}$$

$$c = \pm\sqrt{\frac{ab-b^2-3}{a}}$$

Answer ..... [3]

(b) Write as a single fraction in its simplest form $\frac{3x}{(3x-2)^2}-\frac{2}{2-3x}$.

$$\frac{3x}{(3x-2)^2}-\frac{2}{2-3x}=\frac{3x}{(3x-2)^2}+\frac{2}{3x-2}$$
$$=\frac{3x+2(3x-2)}{(3x-2)^2}$$
$$=\frac{9x-4}{(3x-2)^2}$$

Answer ..... [2]

(c) Solve the equation $\frac{3}{x-3}+\frac{2x-1}{x^2-5x+6}=3$

$$\frac{3}{x-3}+\frac{2x-1}{x^2-5x+6}=3$$
$$\frac{3}{x-3}+\frac{2x-1}{(x-2)(x-3)}=3$$
$$3(x-2)+2x-1=3(x-2)(x-3)$$
$$3x-6+2x-1=3x^2-15x+18$$
$$3x^2-20x+25=0$$
$$x=5 \quad \text{or} \quad x=1\frac{2}{3}$$

Answer ..... [3]

2 The variables x and y are connected by the equation $y=\frac{1}{5}\left(50-\frac{60}{x}-x^2\right)$.

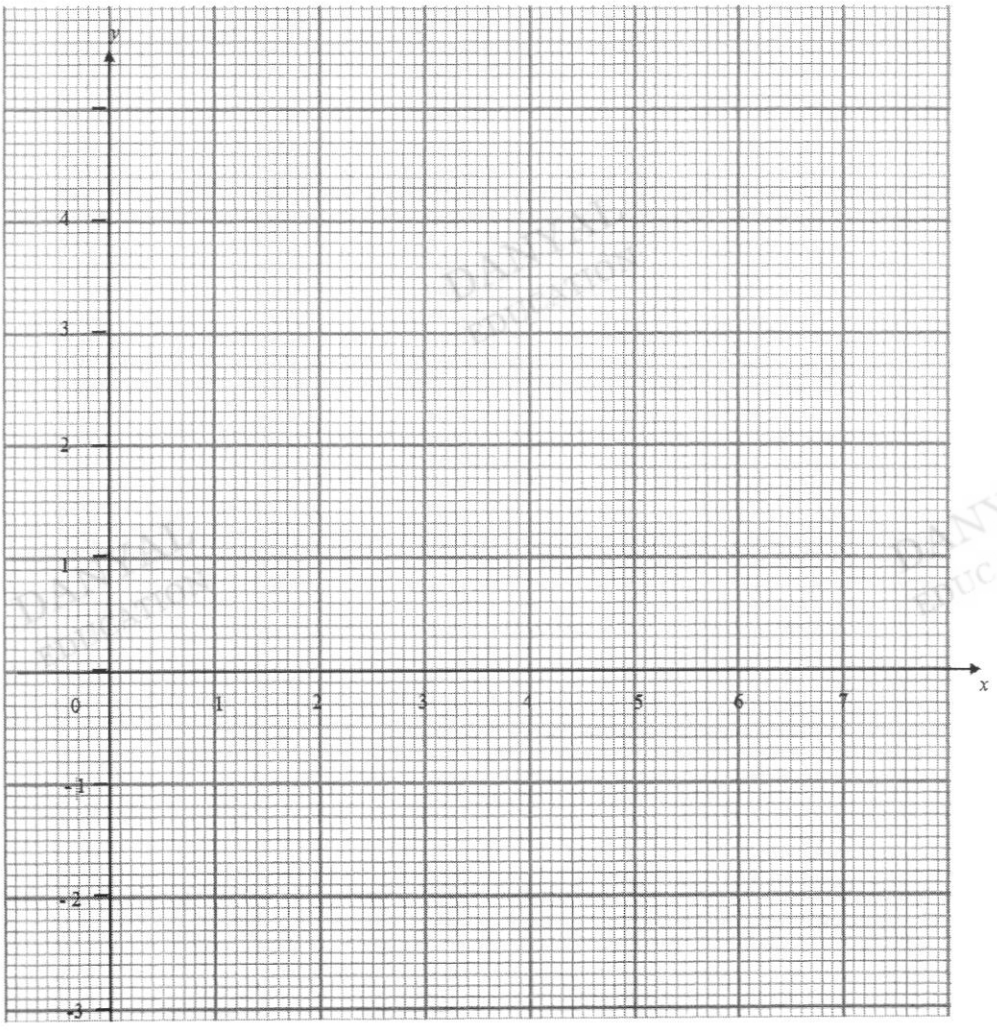
Some of the corresponding values of x and y are given in the following table.

x	1	2	2.5	3.5	4	5	6	7
y	−2.2	3.2	4.0	4.1	3.8	2.6	0.8	w

(a) Find the value of w.

$$w=\frac{1}{5}\left(50-\frac{60}{7}-7^2\right)$$
$$=-1.5$$

(b) On the grid, draw the graph of $y=\frac{1}{5}\left(50-\frac{60}{x}-x^2\right)$ for $1\le x\le 7$. [3]



(c) Explain how your graph shows that there are more than one solution of the equation

$\frac{1}{5}\left(50-\frac{60}{x}-x^2\right)=0$.

The curve intersects the x axis two times. Thus there are 2 solutions. [1]

(d) By drawing a tangent, find the value of x when the gradient of the curve is 4.

Drawing of correct tangent with gradient 4

$x = 1.6$ (Accept x = 1.5 to 1.8)

Answer .....[2]

(e) By drawing a suitable straight line on the same axes, solve the equation

$2x^2+5x+\frac{120}{x}=102$.

$2x^2+5x+\frac{120}{x}=102$

$x^2+\frac{5}{2}x+\frac{60}{x}=51$

$\frac{5}{2}x=51-\frac{60}{x}-x^2$

$\frac{5}{2}x-1=50-\frac{60}{x}-x^2$

$\frac{1}{5}\left(\frac{5}{2}x-1\right)=\frac{1}{5}\left(50-\frac{60}{x}-x^2\right)$

$\frac{1}{2}x-\frac{1}{5}=y$

Draw graph of $y=\frac{1}{2}x-\frac{1}{5}$

$x = 1.25$ or $x = 5.15$ (± 0.1)

Answer x = ...or x = ...[3]

3 **(a)** Mr Xander, his wife and their 5 children aged 16 years old, 13 years old, 11 years old and a pair of twins who are 3 years old dined in a buffet restaurant.

The restaurant offered a promotion of 50% discount to children aged 10 to 12 years old and free dining for children below 10 years old.

The buffet restaurant charges S$25.50 per pax and the bill is subjected to 10% service charge and 8% GST.

Calculate the total bill paid by Mr Xander.

$$\frac{108}{100}\times\left(\frac{110}{100}\times 4\frac{1}{2}\times \$25.50\right)$$
$$= \$136.32 \text{ (2 d.p)}$$

Answer $..... [2]

(b) A sofa is priced at $8600.

Salmiah bought the sofa on hire purchase with these terms:

A downpayment of $500 and the balance to be paid in monthly instalments over 3 years at a simple interest rate of 2.5% per annum.

Calculate her monthly instalment.

$$\text{Total payment to be made} = 8600 + \frac{8100\times 2.5\times 3}{100}$$
$$= \$9207.50$$
$$\text{Monthly instalment} = \$\frac{(9207.50-500)}{3\times 12}$$
$$= \$241.88 \text{ (2 d.p)}$$

Answer $..... [2]

(c) John borrows \$10 000 from a bank for 3 years.

The total compound interest he has to pay the bank at the end of 3 years is \$768.

Calculate the interest rate per year.

Give your answer correct to 2 significant figures.

$$10768 = 10000\left(1+\frac{r}{100}\right)^3$$

$$r = \left(\sqrt[3]{\frac{10768}{10000}} - 1\right) \times 100$$

$= 2.5$ (2 s.f.)

(d) The exchange rate at Changi Airport between Japanese Yen (¥) and Singapore dollars (SGD) was 1SGD = ¥106.10.

On the same day, the exchange rate between Thai Baht (฿) and Singapore dollars (SGD) was 1 SGD =฿26.

Zen wants to change ¥30 000 into Thai Baht.

He has a choice of either changing it in Changi Airport or when he is in Thailand.

Assuming that the exchange rates do not change and given that the exchange rate in Thailand will be ¥100 = ฿28, which will be a better choice?

In Changi,

To change to 30000 Yen, she needs $= \frac{30000}{106.10}$

$= S\$282.752$

She will get $= 282.752 \times 26$ Baht

$= 7351.552$ Baht

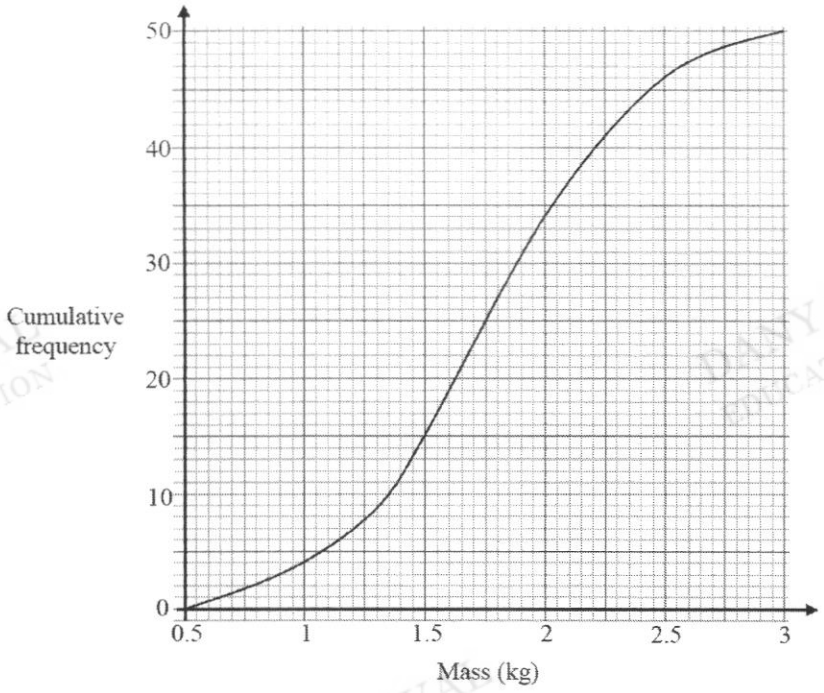
In Thailand $= 30000 \times \frac{28}{100}$

$= 8400$ Baht

Answer : Changing in……Thailand…… will be a better choice [3]

4 Jessie owns a duck farm and she conducts a study on the masses of 50 healthy ducks in her farm, Farm J.

(a) The cumulative frequency curve shows the distribution of the masses.



(i) Complete the grouped frequency table for the masses of the ducks.

Mass (m kg)	$0.5 \leq m < 1.0$	$1.0 \leq m < 1.5$	$1.5 \leq m < 2.0$	$2.0 \leq m < 2.5$	$2.5 \leq m < 3.0$
Frequency	4	11	19	12	4

[1]

(ii) Calculate an estimate of the mean mass of each duck.

Answer .....1.76.....[1]

(iii) Calculate an estimate of the standard deviation.

Answer ...0.525.....[1]

(iv) Explain why the mean and standard deviation are estimates.

Answer

The exact values of the masses of the ducks are unknown.

(v) A research article states that 92% of healthy ducks have a mass greater than 1.3 kg.

Comment on whether Jessie's data supports this claim.

Percentage of chicken with mass greater than 1.3 kg

$$= \frac{41}{50} \times 100\%$$

$$= 82\%$$

Answer

Jessie's data does not support the claim as the percentage of ducks with mass greater than 1.3 kg in his farm is 82% which is less than 92%.

(b) Quinn also has 50 ducks in her farm, Farm Q.

Her ducks have a mean mass of 1.65 kg and standard deviation of 0.95.

Make two comments comparing the masses of ducks from both farms.

Answer

The ducks in Farm J are heavier as their mean mass is 1.76 kg as compared to Farm Q's mean mass of 1.65 kg.

The mass of ducks in Farm Q is more consistent as the standard deviation is 1.65 as compared to Farm J's standard deviation of 0.525.

(c) Mary has three 50 cent coins and two 10 cent coins in her pocket.

She takes two coins out of her pocket, at random, one after another.

The coins are not replaced.

(i) Find the probability that both coins are of different denominations.

P(2 diff denominations)

$$= \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)+\left(\frac{2}{5}\right)\left(\frac{3}{4}\right)$$

$$= \frac{3}{5}$$

Answer .....[2]

(ii) Mary takes out a third coin from her pocket.

Find the probability that the total value of the three coins taken out is 70 cents.

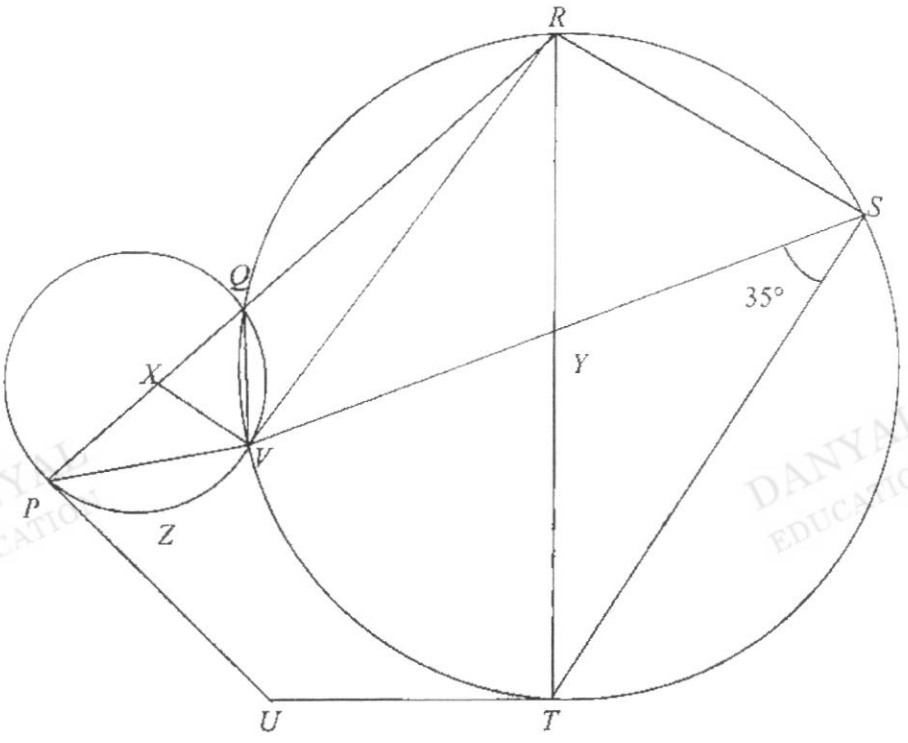
P(total is 70 cents)

$$= \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)+\left(\frac{2}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right)+\left(\frac{2}{5}\right)\left(\frac{1}{4}\right)\left(\frac{3}{3}\right)$$

$$= \frac{3}{10}$$

Answer .....[2]

5



The diagram above shows two circles with centre X and Y respectively.

The circles intersect each other at Q and V.

PU is tangent to the smaller circle at P and TU is tangent to the bigger circle at T.

(a) Show that triangle RVY is similar to triangle STY.

Give a reason for each statement you make.

$\angle VRY = \angle TSY$ (Angles in the same segment)

$\angle RVY = \angle SYT$ (Vert. opp angles)

$\angle RVY = \angle STY$ (Angle in the same segment)

$\therefore$ Triangle RVY is similar to triangle STY.

(2 corresponding angles are the same)

(b) Given radius $XV = 5$ cm and angle $VST = 35°$, find the area of segment PZV.

$\angle RST = 90°$ (rt angle in semicircle)

$\angle RSV = 90° - 35° = 55°$

$\angle VQR = 180° - 55° = 125°$ (angles in opposite segment)

$\angle XQR = 180° - 125° = 55°$ (adj. angles on a st. line)

$\angle PXV = 2 \times 55° = 110°$ (ext $\angle$ of triangle)

$$\text{Area of segment} = \left(\frac{110°}{360°} \times \pi \times 5^2\right) - \left[\frac{1}{2}(5)(5)\sin 110°\right]$$

$$= 12.3 \text{ cm}^2$$

Answer ..... [5]

6 (a) (i) $\overrightarrow{AB} = \begin{pmatrix} 2p \\ 5p \end{pmatrix}$.

$|\overrightarrow{AB}| = \frac{5\sqrt{29}}{2}$.

Find the two possible values of p.

$$\sqrt{(2p)^2 + (5p)^2} = \frac{5\sqrt{29}}{2}$$

$$\sqrt{29p^2} = \frac{5\sqrt{29}}{2}$$

$$29p^2 = \frac{25 \times 29}{4}$$

$$p = \pm\sqrt{\frac{25 \times 29}{4} \times \frac{1}{29}}$$

$$= \pm\frac{5}{2}$$

Answer .....[2]

(ii) A line joins the two points $X(-6, 2)$ and $Z(6, -2)$.

Use vectors to show whether or not the point $Y(3, 1)$ lies on this line. [3]

$$\overrightarrow{XZ} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 12 \\ -4 \end{pmatrix}$$

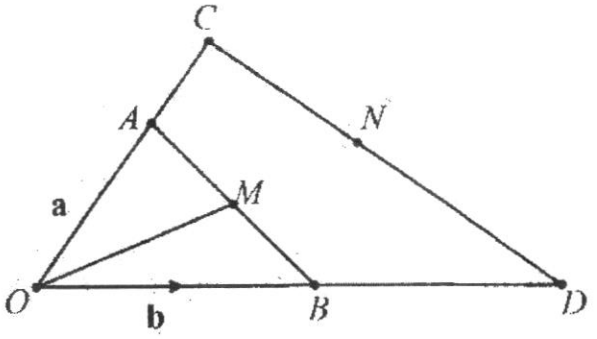
$$\overrightarrow{XY} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -6 \\ 2 \end{pmatrix} \text{ or } \overrightarrow{YZ} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ -1 \end{pmatrix} \qquad = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$

Since $\overrightarrow{XZ} \neq k\overrightarrow{XY}$ or $\overrightarrow{XZ} \neq k\overrightarrow{YZ}$,
or Since $\begin{pmatrix} 12 \\ -4 \end{pmatrix} \neq k\begin{pmatrix} 9 \\ -1 \end{pmatrix}$,
or since $\begin{pmatrix} 12 \\ -4 \end{pmatrix} \neq k\begin{pmatrix} 3 \\ -3 \end{pmatrix}$, $Y(3,1)$ does not lie on this line.

(b) In the diagram, M is the midpoint of AB and B is the midpoint of OD.

$OA = \frac{2}{3}OC, CN = \frac{3}{7}CD$, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.



(i) Express each of the following vectors in terms of **a** and **b**.

(a) $\overrightarrow{CD}$,

$$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$$
$$= 2b - \frac{3}{2}a$$

Answer .....[1]

(b) $\overrightarrow{ON}$.

$$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN}$$
$$= \frac{3}{2}a + \frac{3}{7}(2b - \frac{3}{2}a)$$
$$= \frac{6}{7}(a + b)$$

OR

$$\overrightarrow{ON} = \overrightarrow{OD} + \overrightarrow{DN}$$
$$= 2b + \frac{4}{7}(-2b + \frac{3}{2}a)$$
$$= \frac{6}{7}(a + b)$$

Answer .....[2]

(ii) Write down the ratio of

(a) area of ΔOAB : area of ΔOCB ,

Answer 2 : 3

(b) area of ΔOAB : area of ΔOCD.

$$\frac{\text{area of } \Delta OAB}{\text{area of } \Delta OCD} = \frac{\text{area of } \Delta OAB}{\text{area of } \Delta OCB} \times \frac{\text{area of } \Delta OCB}{\text{area of } \Delta OCD}$$

$$= \frac{2}{3} \times \frac{1}{2}$$

$$= \frac{1}{3}$$

Ratio = 1:3

OR

$$\frac{\text{area of } \Delta OAB}{\text{area of } \Delta OCD} = \frac{\frac{1}{2} \times OA \times OB \times \sin \angle COD}{\frac{1}{2} \times OC \times OD \times \sin \angle COD}$$

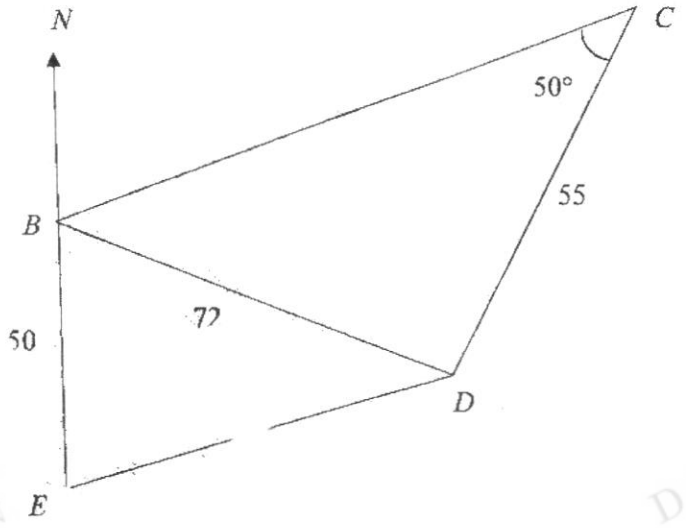
$$= \frac{\frac{1}{2} \times \frac{2}{3} OC \times \frac{1}{2} OD \times \sin \angle COD}{\frac{1}{2} \times OC \times OD \times \sin \angle COD}$$

$$= \frac{1}{3}$$

Ratio = 1 : 3

Answer 1 : 3 .....[2]

7



The diagram shows a plot of land BCDE gazetted for development into a school.

$BE = 50$ m, $BD = 72$ m, $CD = 55$ m and angle $BCD = 50°$.

E is due south of B and the bearing of D from B is 113°.

(a) Find the distance DE.

$$\angle EBD = 180° - 113°$$
$$= 67° \text{ (adj. } \angle \text{ on st. line)}$$
$$DE = \sqrt{50^2 + 72^2 - 2(50)(72)\cos 67°}$$
$$= 69.791$$
$$= 69.8 \text{ m (3 s.f.)}$$

(b) Find the bearing of B from C.

$$\frac{\sin \angle DBC}{55} = \frac{\sin 50°}{72}$$
$$\angle DBC = 35.815°$$
$$\text{Bearing of } B \text{ from } C = 180° + (113° - 35.815°)$$
$$= 257.2° \text{ (1 d.p.)}$$

OR

$$\frac{\sin \angle DBC}{55} = \frac{\sin 50°}{72}$$
$$\angle DBC = 35.815°$$
$$\text{Bearing of } B \text{ from } C = 360° - (180° - (113° - 35.815°))$$
$$= 257.2° \text{ (1 d.p.)}$$

(c) Calculate the area of $BCDE$.

$$\text{Area of } BCDE = \frac{1}{2}(50)(72(\sin 67°) + \frac{1}{2}(72)(55)\sin(180° - 50° - 35.815°)$$
$$= 3631.6$$
$$= 3630 \text{ m}^2 (3 \text{ s.f.})$$

(d) An engineer planned to erect a flag pole at B such that the angle of elevation of the top of the flag pole from E is $13°$.

He estimated and told the builder to prepare wood of length 11 m for the pole.

Is his estimation correct?

$$\tan 13° = \frac{\text{height}}{50}$$
$$\text{height} = 50 \tan 13°$$
$$= 11.5 \text{ m (3 s.f.)}$$

No, his estimation is not correct

OR

If 11 m pole is used,

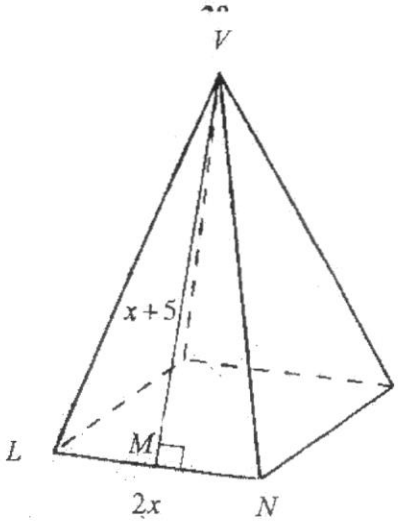
$$\tan \theta° = \frac{11}{50}$$

angle of elevation, $\theta = \tan^{-1} \frac{11}{50}$

$$= 12.4° \text{ (1 d.p.)}$$

No, his estimation is not correct

8



The diagram shows a model of a commemorative structure in the shape of a square-based pyramid.

$VM = (x+5)$ cm and $LN = 2x$ cm.

(a) Given that the total surface area of the pyramid is 50 cm^2, form an equation in x and show that it reduces to $4x^2 + 10x - 25 = 0$. [3]

$$\text{Area of } \Delta VLN = \left(\frac{1}{2} \times 2x \times (x+5)\right)$$
$$= (x^2 + 5x) \text{ cm}^2$$
$$\text{Area of square base} = (2x)^2$$
$$= 4x^2 \text{ cm}^2$$
$$4(x^2 + 5x) + 4x^2 = 50$$
$$8x^2 + 20x = 50$$
$$4x^2 + 10x - 25 = 0 \text{ (shown)}$$

(b) Solve the equation $4x^2 + 10x - 25 = 0$.

$$4x^2 + 10x - 25 = 0$$
$$x = \frac{-10 \pm \sqrt{10^2 - 4(4)(-25)}}{2(4)}$$
$$= 1.54508 \quad \text{or} \quad = -4.0450$$
$$= 1.55 (3 \text{ s.f.}) \quad = -4.05 (3 \text{ s.f.})$$

Answer $x =$ .....or $x =$ .....[3]

(c) Explain why one of the solutions in part (c) must be rejected.

Answer

The length of the triangle ($2x$ cm) must be more than 0 cm, hence -4.05 must be rejected.

OR,

When $x = -4.05$, length $2x$ becomes negative and length cannot be negative. [1]

(d) Given that the height of the model is 6.36 cm and the scale of the model is 1: 500, find the actual volume of the structure, in m^3.

Volume of pyramid

$= \frac{1}{3}(2\times1.54508)^2(6.36)$

$= 20.24406\ cm^3$

$= 20.24406 \times 125m$

$= 2530\ m^3$ (3 s.f.)

or

Actual height $= 6.36 \times 5 = 31.8$ m

Actual length/ beadth $= 2(1.54508)\times5 = 15.4508$ m

Volume of pyramid

$= \frac{1}{3}(15.4508)(15.408)(31.8)$

$= 2530.508539m^3$

$= 2530\ m^3$ (3 s.f.)

Answer .....m^2 [3]

9 John earns $1650 every month and this is sufficient to cover his family's expenses in 2022. The costs are given in the table below.

Living expenses in 2022

Groceries	$250 per week
Utilities	$200 per month
Transport	$300 per month

(a) Given that there are 52 weeks in a year, calculate his total expenditure for 2022.

Total expenditure in 2022
$= (\$250 \times 52) + (\$200 \times 12) + (\$300 \times 12)$
$= \$19000$

Answer ..... [1]

(b) Inflation causes prices of goods and services to increase yearly.

The table below shows the percentage change in price from the previous year.

Consumer Price Index			
Categories	**Percentage Change (%)**		
	2021	2022	2023 (expected)
Groceries	3.5	8	20
Healthcare	10	13	23
Transport	6	−2	12.5
Communication	−0.9	−4.2	3
Recreation & Culture	2.8	1	1.5
Utilities	8	0	15
Miscellaneous Goods & Services	−1.1	0.3	0.5

Calculate John's expected total expenditure for 2023.

Expected total expenditure in 2023
$= (\$250 \times 52 \times 1.2) + (\$200 \times 12 \times 1.15) + (\$300 \times 12 \times 1.125)$
or
$= (\$13000 \times \frac{120}{100}) + (\$2400 \times \frac{115}{100}) + (\$3600 \times \frac{112.5}{100})$
or
$= (52 \times \$300) + (12 \times \$230) + (12 \times \$337.50)$

$= \$22410$ [A1]

Answer ..... [3]

(c) Due to inflation, John realised that his current income is insufficient to cover his family's expected living expenses in 2023.

To make up for the shortfall, he decides to take up an additional part-time job during the weekends.

Below are the possible options for him.

Fast Food Service Crew	Credit Card Sales Promoter	Food Delivery Rider
\$8 per hour	Basic pay of \$4 per hour and an additional \$10 commission per successful credit card sign up. Based on the company's statistics, on average, a person is able to get a successful sign up every 2 hours.	An average of \$15 per hour Every food delivery rider is required to purchase a food delivery starter kit which consists of a long sleeve base, helmet, reflective jacket and insulated backpack at \$50.

Suggest which part-time job he should take up and the number of hours he needs to work per week to make up for the shortfall.

Justify any decision you make and show your calculations clearly.

Shortfall per year = their $\$22410-(\$1650\times12)$

$=\$2610$

Shortfall per week = their $\$2610\div52$

$=\$50.192$

Fast Food Service Crew:	Credit Card Sales Promoter:	Food Delivery Rider:
their $\$50.192\div8$ $=6.274$ hours	their $\$50.192\div(4+4+10)\times2$ or their $\$50.192\div(4+\frac{10}{2})$ $=5.577$ hours	(their $\$50.192+\$50)\div\$15$ $=6.679$ hours

Minimum 6 hours for credit card sales promoter and 7 hours for food delivery rider and fast food service crew.

Choice of part-time job should be backed up with sensible reasoning.

OR

Shortfall per year = their \$22410 − (\$1650 × 12)
= \$2610

Fast Food Service Crew:	Credit Card Sales Promoter:	Food Delivery Rider:
their \$2610 ÷ 8 = 326.25 hours yearly their \$326.25 ÷ 52 = 6.27 ≈ 7 hours	their \$2610 ÷ $(4+\frac{10}{2})$ = 290 hours yearly their 290 ÷ 52 = 5.577 ≈ 6 hours	(their \$2610 + \$50) ÷ \$15 = 177.33 hours yearly their 177.33 ÷ 52 = 3.41 ≈ 4 hours weekly

Minimum 7 hours for Fast Food Service, 6 hours for credit card sales promoter and 4 hours for food delivery rider and fast food service crew.

Choice of part-time job should be backed up with sensible reasoning.

OR

Shortfall per year = their $\frac{\$22410}{12}$ − \$1650
= \$217.50

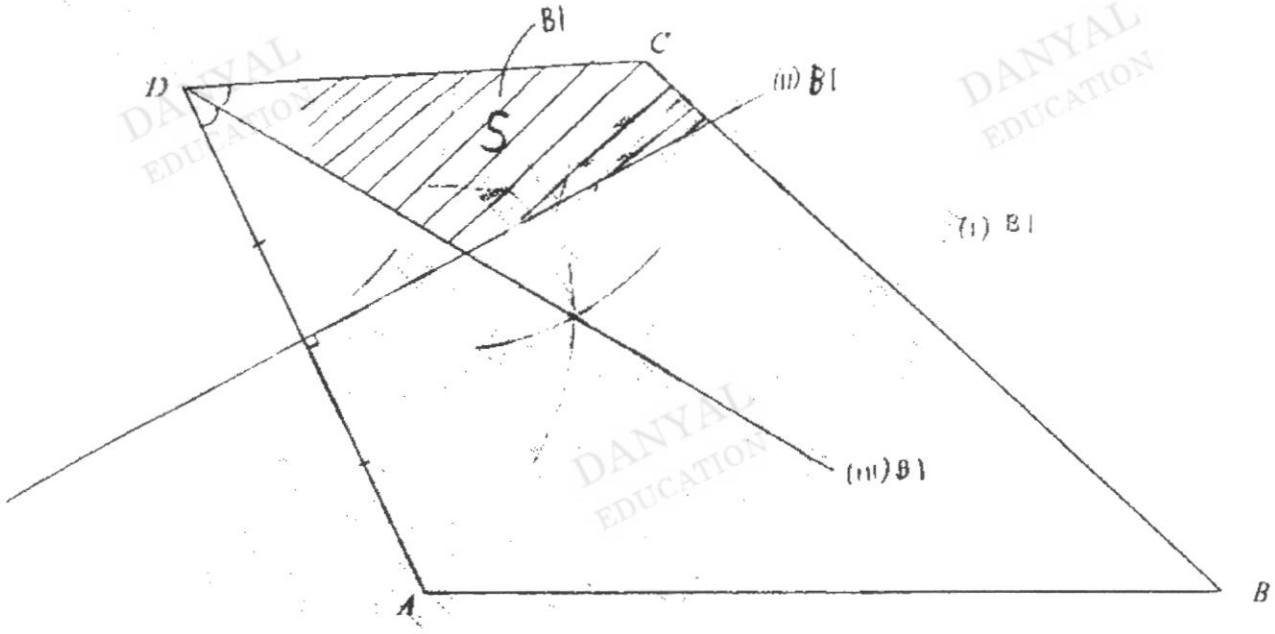
Fast Food Service Crew:	Credit Card Sales Promoter:	Food Delivery Rider:
their \$217.50 ÷ 8 = 27.1875 hours monthly their \$27.1875 ÷ 4 = 6.7968 ≈ 7 hours weekly	their \$217.50 ÷ $(4+\frac{10}{2})$ = 24.16 hours monthly their 24.16 ÷ 4 = 6.04 ≈ 7 hours weekly	(their \$217.50 + \$50) ÷ \$15 = 17.833 hours monthly their 17.833 ÷ 4 = 4.458 ≈ 5 hours weekly

Minimum 7 hours for Fast Food Service, 7 hours for credit card sales promoter and 5 hours for food delivery rider and fast food service crew.

Choice of part-time job should be backed up with sensible reasoning.

END OF PAPER

11 The diagram shows the plan of a neighbourhood play area $ABCD$ in the shape of a quadrilateral.
Use the diagram to answer the questions below.
1 centimetre represents 1 kilometre.



Inside the play area, a sandy play area is located in the region where it is

- less than 5 km from C, — B1 [see (i), arc of 5 cm around C]
- nearer to D then A and — B1 [see (ii), perpendicular bisector]
- nearer to CD ~~and BC~~ then DA. — B1 [see (iii), angle bisector]

Shade and label the region S. — B1 [4]

2 The variables x and y are connected by the equation $y=\frac{1}{5}\left(50-\frac{60}{x}-x^2\right)$.

Some of the corresponding values of x and y are given in the following table.

x	1	2	2.5	3.5	4	5	6	7
y	−2.2	3.2	4.0	4.1	3.8	2.6	0.8	w

(a) Find the value of w.

$w = -1\frac{18}{35}$

$= -15$ (id.p) - 1.5

(b) On the grid, draw the graph of $y=\frac{1}{5}\left(50-\frac{60}{x}-x^2\right)$ for $1\le x\le 7$. [3]