Name:	Index Number:	Class:

YIO CHU KANG SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2018
SECONDARY THREE EXPRESS

MATHEMATICS
Paper 1

4048/01
2 hours
3 October 2018 (Wednesday)

READ THESE INSTRUCTIONS FIRST

Candidates answer on the Question Paper.

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 80.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142.

For Examiner's Use
80

Setter: Mdm Ng Lee Kiang

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1+\frac{r}{100})^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** Calculate $\dfrac{33.17+55.2^2}{0.3141+\sqrt[3]{647.9}}$ showing all the figures on your calculator display.

Answer ..... [1]

(b) Give your answer in **(a)** correct to 3 significant figures.

Answer ..... [1]

---

2 The area of triangle ABC is 60.5 cm^2. AB = 10.2 cm and BC = 12.6 cm.

Find the two possible sizes of the angle ABC.

Answer .....°*or*.....° [2]

---

3 A man buys y watermelons at \$1.20 each and $(2y + 1)$ apples at 80 cents each. If he wishes not to spend more than \$25 for his purchases,

(a) without simplifying, form an inequality involving y,

Answer ..... [1]

(b) solve the inequality and hence state the largest possible number of watermelons he can buy.

Answer ..... [2]

---

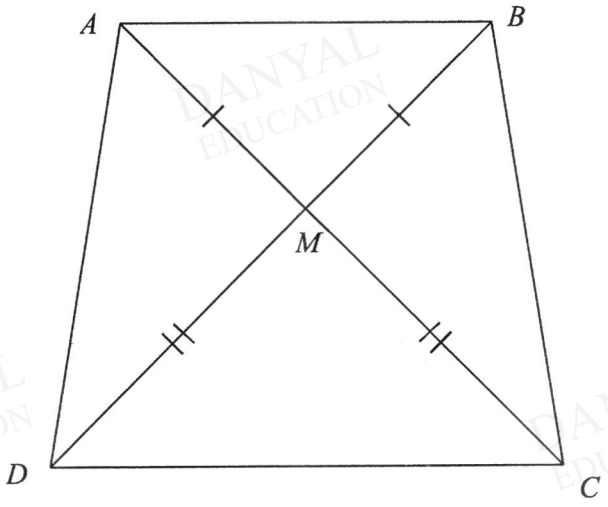
4 Anne invested a sum of money in an account paying compound interest at 2.5% per year. After 4 years, the money had earned total interest of \$519.07.

Calculate the sum of money Anne invested in the account.

Answer \$..... [3]

---

5 In the quadrilateral $ABCD$ shown below, the diagonals AC and BD intersect at M. $AM = BM$ and $DM = CM$.



(a) Show that triangle ACD is congruent to triangle BDC.

Answer

[2]

(b) Name a triangle that is congruent to triangle AMD.

Answer Triangle ..... [1]

---

6 Item A has a mass of 8×10^{-6} g.

(a) If item B weighs 2×10^{-7} g, how many item B are needed to weigh as much as one item A?

Answer .....[1]

(b) There are 1 million pieces of item A. Calculate their total mass in kg, giving your answer in standard form.

Answer .....kg [2]

---

7 **(a)** Express $x^2+11x-15$ in the form $(x+a)^2+b$.

Answer ..... [2]

(b) Hence solve the equation $x^2+11x-15=0$, giving your answers correct to two decimal places.

Answer x = ... *or*..... [2]

---

8 **(a)** Simplify $(7x^2y)^0 \div (4x^3)^{-1}$.

Answer ..... [2]

(b) Given that $27^k = 3^{25} \div 81$, find the value of k.

Answer ..... [2]

---

9 **(a)** Express $\frac{2}{3x-1} - \frac{x}{2x+1}$ as a single fraction.

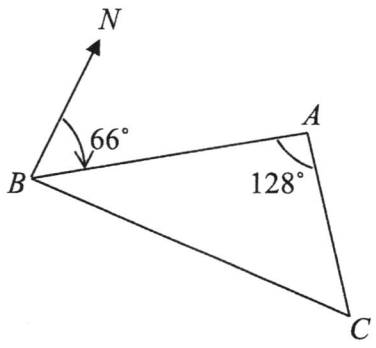
Answer ..... [2]

(b) Hence or otherwise, solve the equation $\frac{2}{3x-1} - \frac{x}{2x+1} = 0$.

Answer $x =$ ..... *or*..... [2]

---

10 In the diagram, A, B and C are three points on level ground. The bearing of A from B is 066°and angle $BAC = 128°$.



Calculate

(a) the bearing of B from A,

Answer ..... ° [2]

(b) the bearing of A from C.

Answer ..... ° [2]

---

11 In a sequence, the same number is subtracted each time to obtain the next term.
The first five terms of the sequence are

99 x y z 67

(a) Find the values of x, y and z.

Answer $x =$ .....

$y =$ .....

$z =$ ..... [2]

(b) Write down an expression for the nth term of this sequence.

Answer ..... [1]

(c) Explain why -234 is not a term of this sequence.

Answer .....

..... [2]

---

12 The costs of two geometrically similar cylindrical cup of mixed fruit juice are \$1.08 and \$5 respectively. It is assumed that the cost of the fruit juice is proportional to the volume of the fruit juice.

(a) Expressing your answer as a fraction in its lowest term, write down

(i) $\dfrac{\text{mass of smaller cup}}{\text{mss of larger cup}}$,

Answer .....[1]

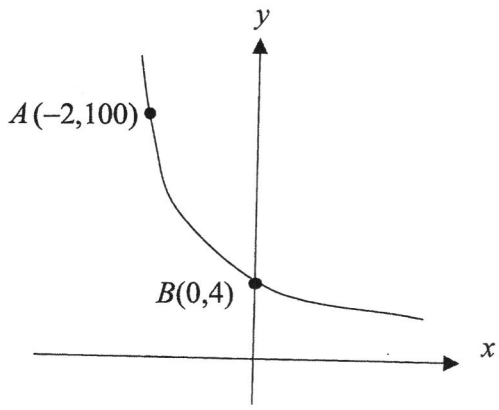
(ii) $\dfrac{\text{circumference of smaller cup}}{\text{circumference of larger cup}}$.

Answer .....[1]

(b) The height of the larger cup is 20 cm, calculate the height of the smaller cup.

Answer .....cm [2]

13 The sketch shows the graph of $y = ka^{-x}$.
The points $A(-2,100)$ and $B(0, 4)$ lie on the graph.



(a) Find the values of a and k.

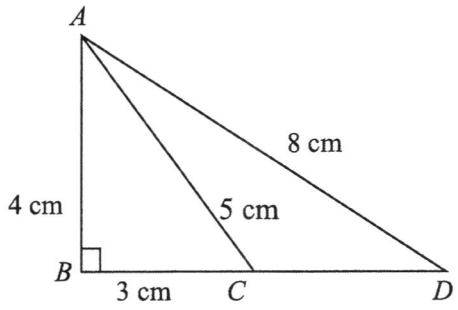
Answer $a =$ .....

$k =$ ..... [2]

(b) A line passes through AB. Find the equation of line AB.

Answer ..... [2]

14 In the triangle ABC, angle $ABC = 90^\circ$, $AB = 4$ cm, $BC = 3$ cm, $AC = 5$ cm and $AD = 8$ cm.



Calculate

(a) $\cos A\hat{C}D$,

Answer .....[1]

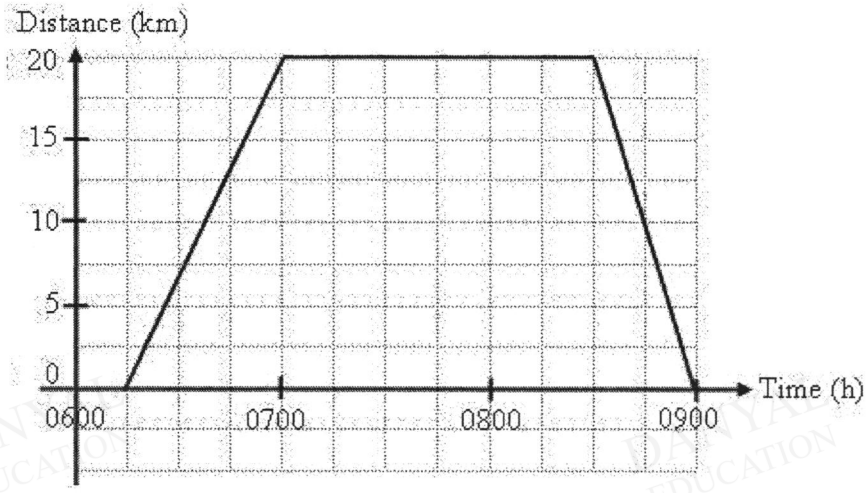
(b) angle BAC,

Answer .....° [2]

(c) the length of CD.

Answer .....cm [2]

15 The distance-time graph below shows the journey of a student, John Lim, travelling from his home to his school on a Saturday morning.
He returned home immediately after attending a remedial lesson.



(a) How long did John stay in his school for the remedial?

Answer .....minutes [1]

(b) Calculate John's speed for the journey from home to the school in km/h?

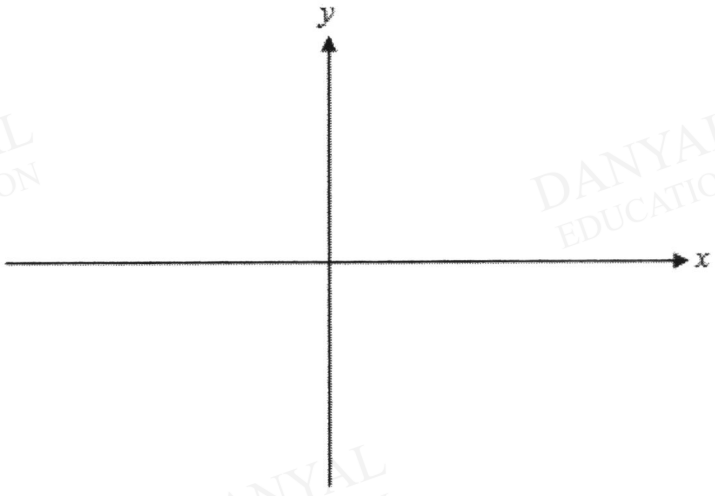
Answer .....km/h [2]

(c) Calculate John's average speed for the whole journey in m/s.

Answer .....m/s [2]

16 (a) On the axes below, sketch the graph of $y = x(x-2)$.
Indicate clearly the values where the curve cuts the x- and y- axes.

Answer [2]



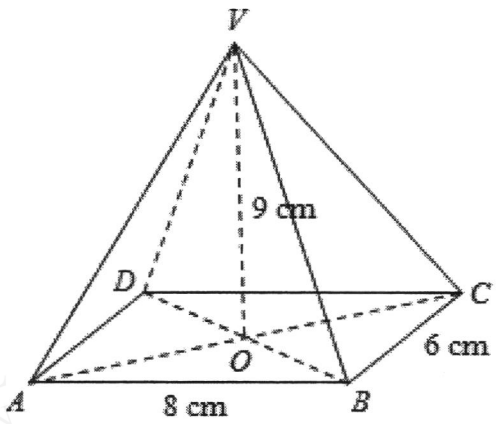
(b) State the minimum value of the curve $y = x(x-2)$.

Answer .....[1]

(c) Using the diagram in part **(a)**, find the range of values of p for which the line $y = p$ would intersect $y = x(x-2)$ exactly two times.

Answer .....[1]

17 The figure shows a pyramid $VABCD$ with a rectangular base. $AB = 8$ cm, $BC = 6$ cm and the height of the pyramid, VO, is 9 cm.



(a) Find

(i) the base area of the pyramid,

Answer .....cm^2 [1]

(ii) the volume of the pyramid.

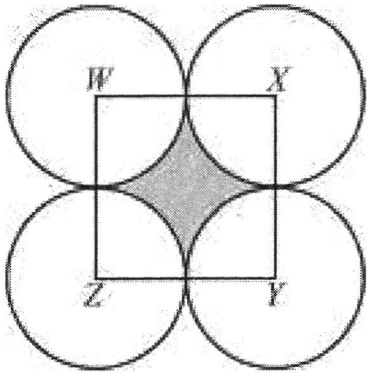
Answer .....cm^3 [1]

Harry filled the pyramid container completely with sand. The sand in the pyramid container was then poured into a cylindrical container with a base radius of 4 cm.

(b) Calculate the height of the sand in the cylinder. Leave your answer correct to 1 decimal place

Answer .....cm [3]

18 The figure shows a company logo which consists of 4 identical circles with centres W, X, Y and Z and a shaded central portion. The centre of each circle is joined to form the quadrilateral $WXYZ$.



If the radius of each circle is 5 cm, find, in terms of π,

(a) the perimeter of the shaded region,

Answer .....cm [2]

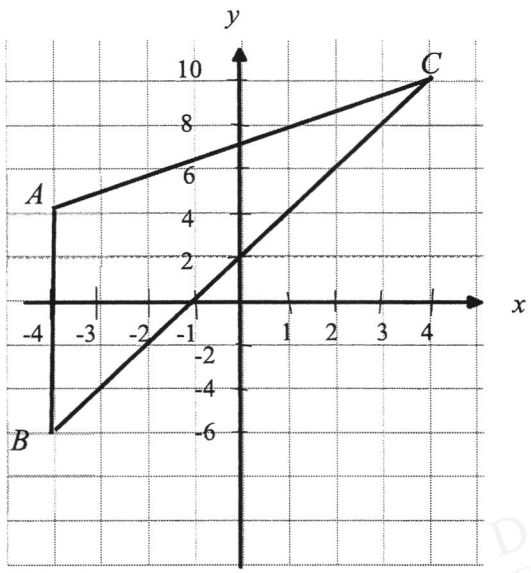
(b) the area of the shaded region,

Answer .....cm^2 [2]

(c) the total area of the logo.

Answer .....cm^2 [2]

19 The diagram shows a triangle with vertices $A(-4, 4)$, $B(-4, -6)$ and $C(4, 10)$.



(a) Find the length of line BC.

Answer .....units [1]

(b) Find the coordinates of the point D such that $ACDB$ is a parallelogram.

Answer D (..... , ...) [1]

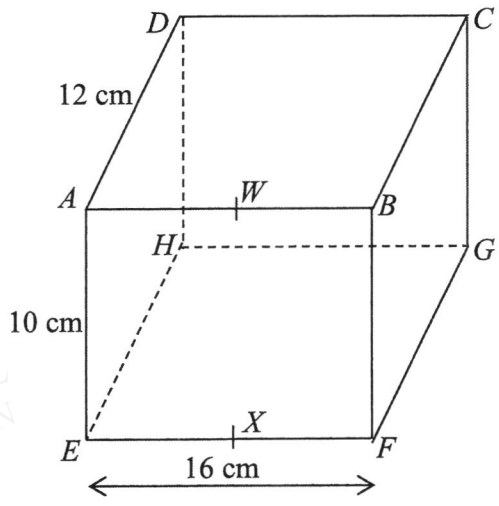
(c) Find the area of triangle ABC.

Answer .....square units [1]

(d) Find the perpendicular distance from A to BC.

Answer .....units [1]

20 The diagram shows a rectangular cuboid. W and X are the midpoints of AB and EF respectively. Given that $AD = 12$ cm, $AE = 10$ cm and $EF = 16$ cm.



(a) Find angle ADE.

Answer .....° [2]

(b) Calculate the length of WG.

Answer .....cm [2]

(c) Find angle WGX.

Answer .....° [2]

Name:	Index Number:	Class:

YIO CHU KANG SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2018
SECONDARY THREE EXPRESS

MATHEMATICS
Paper 2

4048/02
2 hours 30 minutes

Additional materials: Answer Paper
Graph paper (1 sheet)

5 October 2018 (Friday)

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use
100

Setter: Mrs Seck SH

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 The table below shows the population and land area of Singapore, Malaysia, United Kingdom and Australia in 2016.

Country	Population	Area (km^2)
Singapore	5.5×10^6	700
Malaysia	2.7×10^7	3.3×10^5
United Kingdom	6.0×10^7	2.4×10^5
Australia	2.2×10^7	7.7×10^6

(a) Find the ratio of the population of Singapore to the population of Australia. Give your answer in the form $1 : n$. [1]

(b) How many more people live in Malaysia than in Australia? Leave your answer in standard form. [2]

(c) Calculate the average number of people per km^2 (population density) in the United Kingdom. [1]

(d) Which country has the highest population density? Justify your answer with workings. [2]

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2 Alvin, a young adventurer, is planning a cycling expedition. He explores two possible routes.

(a) If he travels on route A, which is 120 km long, he expects to cover x km per hour. Route B, which is 5 km shorter than route A, has a more challenging terrain and he would only expect to cover $(x-2)$ km per hour.

Write down an expression, in terms of x, for the time he expects to take on

(i) route A, [1]

(ii) route B. [1]

(b) He estimates that route A will take 40 minutes less than route B.
Form an equation in x and show that it reduces to

$$2x^2+11x-720=0.$$ [3]

(c) Solve the equation $2x^2+11x-720=0$, giving both answers correct to one decimal place. [3]

(d) Calculate the time, in hours and minutes, that he expects to take on route B. [2]

---

3 **(a)** Express $\dfrac{7}{2(3p-1)}-\dfrac{3}{(1-3p)}$ as a single fraction in its simplest form. [3]

(b) Make a the subject of the formula $\dfrac{1}{3b}=\dfrac{2}{3a}+\dfrac{1}{5c}$. [3]

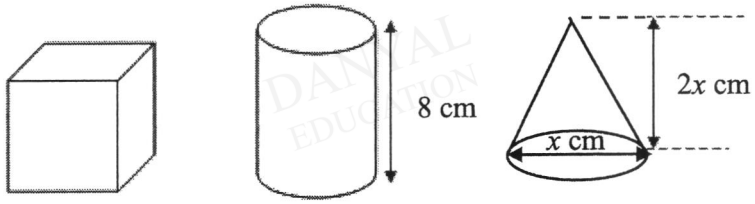
(c) Solve the simultaneous equations.

$$x-2y=8$$
$$3x=19+4y$$ [3]

(d) Simplify $2x^3\times\left(\dfrac{25x^2}{4y^4}\right)^{\frac{1}{2}}$. [3]

---

4 Mrs Wang sells scented candle wax in 3 shapes. The first shape is a cube, the second is a cylinder of height 8 cm, and the third is a cone of diameter x cm and height $2x$ cm. Each shape contains 343 cm^3 of candle wax.



(a) Taking $\pi=3.142$, calculate

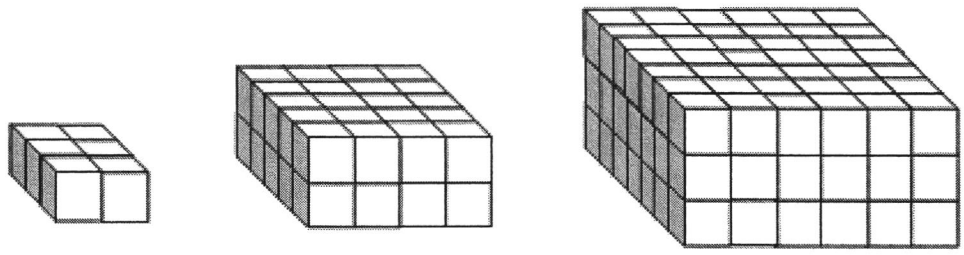
(i) the length of the cube, [1]

(ii) the radius of the base of the cylinder, [2]

(iii) the value of x. [3]

(b) Mrs Wang packed the cube candles into a carton box with dimensions 75 cm by 64 cm by 90 cm. Find the maximum number of cube candles that can be fitted into the carton box. [2]

5



Day 1 Day 2 Day 3

Andy learned to stack plastic cubes in his play centre. On Day 1, he formed a rectangular block with 6 plastic cubes. On Day 2, he enlarged his rectangular block by adding a plastic cube each to surround the previous day's block as show in the diagram above, and likewise for the number of plastic cubes in the block after Day 3.

Day	Total no. of plastic cubes used, N	No. of plastic cubes added to previous day's block, A
1	$1\times2\times3=6$	6
2	$2\times4\times5=40$	34
3	$3\times6\times7=126$	86
⋮	⋮	⋮
n	x	y
⋮	⋮	⋮

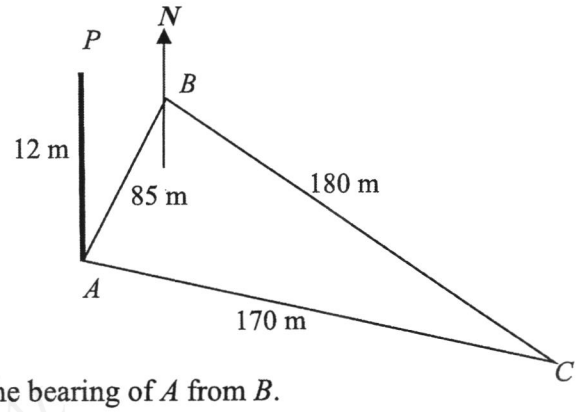
(a) Find the total number of plastic cubes in the rectangular block on Day 6. [2]

(b) Give a single reason why 2555 could not appear in column A. [1]

(c) Find x in terms of n. [2]

(d) What is the number of plastic cubes that Andy will need to add on from Day 30 to Day 31? [3]

6 In the diagram, ABC represents a horizontal triangular field and AP represents a vertical flagpole. B is 85 m from A on a bearing of 25° and C is 170 m from A. Length of BC is 180 m and the height of the flagpole is 12 m.



(a) Calculate the bearing of A from B. [1]

(b) Calculate angle ABC. [3]

(c) Find the angle of depression of C from P. [2]

(d) If the cost of the plot of land is \$50 per m^2, find the cost of the land ABC. [3]

(e) A man walks along BC. Calculate

(i) the shortest distance the man is from A as he walks along BC, [2]

(ii) the greatest angle of elevation of the top of the flagpole when viewed by the man as he travels from B to C. [2]

---

7 The utilities bill of a household consists of 3 components: water, gas and electricity. In a certain month, Angela's household used 17.5 m^3 of water, 68 kWh of gas and 610 kWh of electricity. The tariff rates for water, gas and electricity are \$1.17 per m^3, \$0.21 per kWh and \$0.26 per kWh respectively.

(a) Find the total amount payable by Angela for the month, excluding GST. [2]

(b) Given that the rate of GST on the utilities bill is 7%, find the GST that Angela has to pay. [1]

(c) In the following month, the consumption of gas decreased to 65 kWh, the electricity usage increased by 15%, and the consumption of water remained unchanged. Calculate

(i) the percentage decrease in the consumption of gas, [2]

(ii) the total utilities bill, inclusive of GST, [3]

(iii) the percentage change in Angela's utilities bill, inclusive of GST, as compared to the first month, stating whether it is an increase or decrease. [3]

---

8 **Answer the whole of this question on a sheet of graph paper.**
The table below shows some values of x and y, where $y = -x^2 + 4x + 1$.

x	-1	0	1	2	3	4	5
y	k	1	4	n	4	1	-4

(a) Find the value of k and of n. [2]

(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 1 unit on the y-axis, draw the graph of $y = -x^2 + 4x + 1$ for $-1 \leq x \leq 5$. [3]

(c) Use your graph to estimate the solutions of the equation $-x^2 + 4x + 1 = 0$. [2]

(d) By drawing a suitable straight line on the graph, solve the equation $-x^2 + 4x + 1 = 3 - x$. [3]

(e) By drawing a tangent, find the gradient of the curve $y = -x^2 + 4x + 1$ at the point where $x = 1$. [2]

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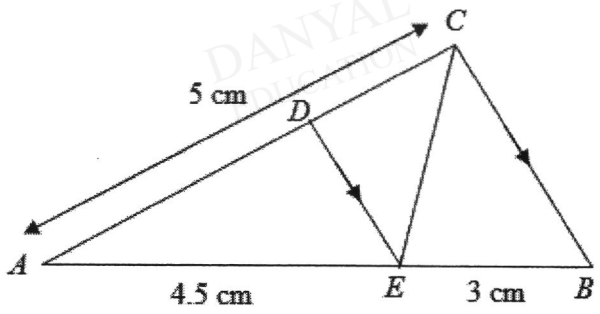
9 In the figure below, CB is parallel to DE, $AC = 5$ cm, $AE = 4.5$ cm and $EB = 3$ cm.



(a) Name a triangle that is similar to triangle ADE. State the reasons clearly. [2]

(b) Calculate the length of AD. [2]

(c) Find the ratio of

(i) area of triangle ADE : area of triangle ACB, [1]

(ii) area of triangle ADE : area of triangle ACE. [1]

(d) Given the area of triangle ABC is 18.75 cm², calculate the area of the trapezium $BCDE$. [2]

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10 Company X produces a mushroom-shaped table lamp which consists of a solid base and a lampshade as shown in Diagram 1.

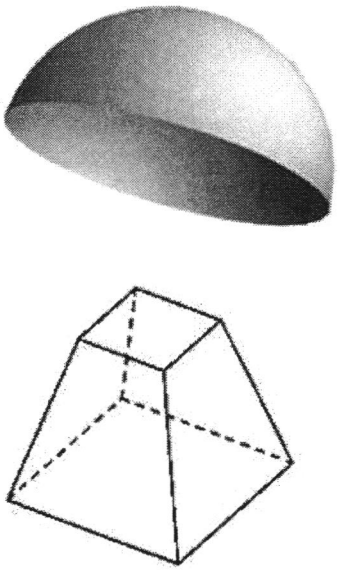

Diagram 1

The lampshade can be modelled by a hemisphere and the solid base has the shape of a frustum with cross section as shown in Diagram 2.

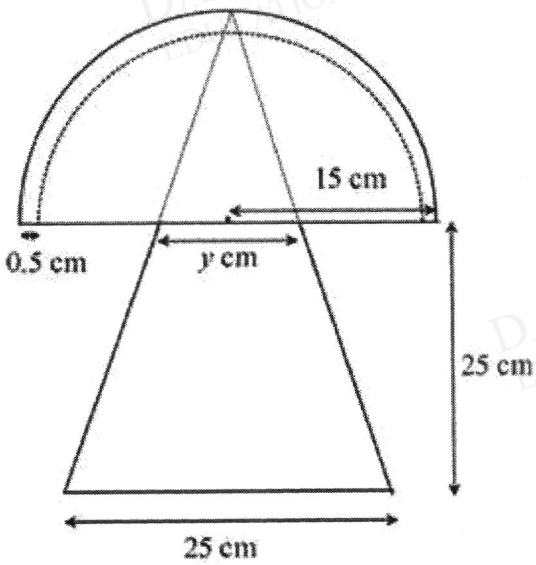

Diagram 2

Here are some information about the lamp :

The height of the base is 25 cm, and the lampshade has an external radius of 15cm.
The base of the frustum is a square of side 25 cm.
The lampshade is made of glass of thickness 0.5 cm.

(a) Show that $y = 9.375$. [1]

(b) Calculate the volume of glass used to make one such lampshade. [3]

(c) Find the total surface area of the glass lampshade. [3]

Heat resistant paints protect surfaces from cracks and peel offs. A protective layer of paint is applied on the entire lampshade and Company X produces 5000 of these mushroom-shaped table lamps. It has a choice of three brands of paint. The table below shows the information about the three brands of paint.

Brand	**A**	**B**	**C**
Usage	Wood, PVC, metal	Wood, glass, PVC	Wood, glass, metal
Application method	Spray	Spray	Spray
Coverage per can	2 m^2	3 m^2	2 m^2
Volume	400 ml	500 ml	400 ml
Dry time	30-60 mins	10 mins	20-60 mins
Price per can	\$ 8.10	\$12.50	\$ 10

(d) Which brand of paint is the best choice for Company X? Justify your answer and show your calculations clearly. [5]

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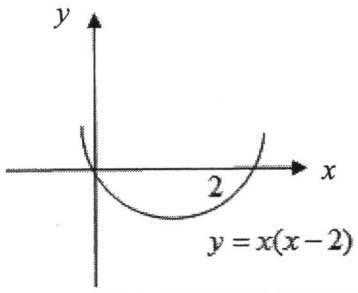
1a	343.4992415	B1	
1b	343	B1	
2	$\frac{1}{2}\times 10.2\times 12.6\times \sin A\hat{B}C = 60.5$ $\sin A\hat{B}C = 0.941487706$ $A\hat{B}C = 70.3°\ or\ 180° - 70.3° = 109.7°$	B2	
3a	$1.2y + 0.8(2y+1) \le 25$	B1	
3b	$1.2y + 0.8(2y+1) \le 25$ $1.2y + 1.6y + 0.8 \le 25$ $2.8y \le 24.2$ $y \le 8.64$ Maximum amount of watermelon = 8	M1 A1	
4	Let the principal be P $P(1+0.025)^4 - P = \$519.07$ $1.103812891P - P = \$519.07$ $0.103812891P = \$519.07$ $P = \$5000.05$	M1 M1 A1	
5a	*CD* is a common side. (base angles of isosceles triangle) Therefore triangle *ACD* is congruent to triangle *BDC* (SAS)	M1 A1	
5b	ΔAMD is congruent to ΔBMC (SAS)	B1	
6a	$\frac{8\times 10^{-6}}{2\times 10^{-7}}$ $= \frac{80\times 10^{-7}}{2\times 10^{-7}}$ $= 40$	B1	
6b	$8\times 10^{-6}\times 10^{6}$ $= 8$ g $= 0.008$ kg $= 8\times 10^{-3}$ kg	M1 A1	
7a		M1 A1	

7b	$x^2+11x-15=0$ $\left(x+\frac{11}{2}\right)^2-45\frac{1}{4}=0$ $\left(x+\frac{11}{2}\right)^2=45\frac{1}{4}$ $x+\frac{11}{2}=\pm 6.7268$ $x=1.23$ or $x=-12.23$	M1 A1	
8a	$(7x^2y)^0\div(4x^3)^{-1}=1\div\frac{1}{4x^3}$ $=4x^3$	M1 A1	
8b	$27^k=3^{25}\div 81$ $3^{3k}=3^{25}\div 3^4$ $3k=25-4$ $3k=21$ $k=7$	M1 A1	
9a	$\frac{2}{3x-1}-\frac{x}{2x+1}=\frac{2(2x+1)-x(3x-1)}{(3x-1)(2x+1)}$ $=\frac{4x+2-3x^2+x}{(3x-1)(2x+1)}$ $=\frac{-3x^2+5x+2}{(3x-1)(2x+1)}$ $=\frac{3x^2-5x-2}{(1-3x)(2x+1)}$	M1 A1	
9b	$3x^2-5x-2=0$ $(3x+1)(x-2)=0$ $x=-\frac{1}{3}$ or $x=2$	M1 A1	Accept any other method
10(a)	$\angle NAB=180°-66°$ (int. $\angle$s, // lines) $=114°$ Bearing $=360°-114°=246°$ ($\angle$s at a point)	M1 A1	
10(b)	$\angle NAC=360°-114°-128°$ ($\angle$s at a point) $=118°$ Bearing $=180°+118°$ (alt. $\angle$s, // lines) $=298°$	M1 A1	
11a	$x=91$ $y=83$ $z=75$	B2	B1 for any 2
11b	$107-8n$	B1	
11c	$107-8n=-234$ $-8n=-341$		

	n = 42.625 since *n* is not an integer, -234 is not a term in the sequence	B2	
12a(i)	$\frac{27}{125}$	B1	
12a(ii)	$\frac{3}{5}$	B1	
12b	$\frac{3}{5}=\frac{h}{20}$ $h = 12\text{cm}$	M1 A1	
13a	Substitute (0,4) into the equation $4 = ka^{-0}$ $k = 4$ Substitute (-2,100) into the equation $100 = 4a^2$ $a^2 = 100 \div 4$ $a^2 = 25$ $a = 5$	B1 B1	
13b	Gradient $= \frac{100-4}{-2-0} = -48$ $y = -48x + 4$	M1 A1	
14a	$\cos\angle ACD = -\cos\angle ACB = -\frac{3}{5}$	B1	
14b	$\tan\angle BAC = \frac{3}{4}$ $\angle BAC = 36.9°$	M1 A1	
14c	$(3 + CD)^2 = 8^2 - 4^2$ $3 + CD = \sqrt{64-16}$ $CD = 3.928 \approx 3.93$ cm	M1 A1	
15a	90 minutes	B1	
15b	Speed $= 20 \div \frac{3}{4}$ $= 26.7$ km/h	M1 A1	
15c	Or Average speed $= \frac{40\times1000}{2\frac{3}{4}\times60\times60}$ $= 4\frac{4}{99}$ m/s or 4.04 m/s	M1 A1	

16a		1 m intercepts at (0,0) and (2,0) 1m correct shape	
16b	Minimum value = -1	B1	
16c	For values of ***p* below -1 or p < -1**, the horizontal line will not intersect the curve as the **minimum value of the curve is -1**. For values greater than -1, the line will intersect the curve twice. P > -1	B1	
17a	Base area of pyramid $= 6\times 8$ $= 48\,\text{cm}^2$	B1	
17b	volume of pyramid $= \frac{1}{3}\times 48\times 9$ $= 144\,\text{cm}^3$	B1	
17c	Base area of cylinder $= \pi\times 4^2$ $= 16\pi\,\text{cm}^2$ Let the height of the sand in the cylinder be h cm. Since the sand in the pyramid is poured into the cylinder, volume of cylinder $= 144\,\text{cm}^3$ $16\pi\times h = 144$ $h = \frac{144}{16\pi}$ $h = 2.8648$ $h = 2.9$ (Correct to 1 decimal place)	M1 M1 A1	
18a	Perimeter of shaded region $= 2\pi r$ $= 2\pi(5)$ $= 10\pi\,\text{cm}$	M1 A1	
18b	Area of shaded region $= 10^2 - \pi r^2$ $= 100 - \pi(5)^2$ $= (100 - 25\pi)\,\text{cm}^2$	M1 A1	
18c	Total area = Area of 4 circles + Area of shaded region $= 4\times\pi r^2 + 100 - 25\pi$ $= 4\times 25\pi + 100 - 25\pi$ $= 100\pi + 100 - 25\pi$ $= (75\pi + 100)\,\text{cm}^2$	M1 A1	
19a	$\sqrt{(-4-4)^2 + (-6-10)^2} = 17.8885 = 17.9$ units (3 sig fig)	B1	
19b	D(4 , 0)	B1	

19c	$\frac{1}{2}\times 8\times 10 = 40$ square units	B1	
19d	Let the perpendicular distance be d $\frac{1}{2}\times d\times 17.8885 = 40$ $d = \frac{40\times 2}{17.8885} = 4.47$	 B1	
20a	$\tan\angle ADE = \frac{10}{12}$ $\angle ADE = \tan^{-1}\left(\frac{10}{12}\right)$ $\angle ADE = 39.80557109°$ $= 39.8°$ (1 *dec. pl.*)	M1 A1	
20b	$XG = \sqrt{8^2+12^2}$ $= \sqrt{208}$ *or* 14.4222051 $WG = \sqrt{10^2+208}$ *or* $\sqrt{10^2+14.4222051^2}$ $= 17.54992877$ $= 17.5$ (3 *s.f.*)	M1 A1	
20c	$\sin\angle WGX = \frac{10}{17.54992877}$ $\angle WGX = 34.73648129°$ $= 34.7°$ (1 *dec. pl*) OR $\tan WGX = \frac{10}{14.4222051}$ $= 34.73648128$ $= 34.7°$	M1 A1	

Solutions to 2018 3E Emath paper 2

Qn		Marks	Remarks
1a	Population of Singapore : Population of Australia		Easy qn,many students got full marks.
	$5.5\times10^6 : 2.2\times10^7$		
	$1 : \frac{2.2\times10^7}{5.5\times10^6}$		
	$1 : \frac{2.2}{5.5}\times10^{7-6}$		
	$1 : 0.4\times10$		
	$1 : 4$	B1	
	Hence, the ratio of population of Singapore to that of Australia is 1 : 4.		
b	$\text{Difference} = \text{Population of Malaysia} - \text{Population of Australia}$		
	$= 2.7\times10^7 - 2.2\times10^7$		
	$= (2.7-2.2)\times10^7$	M1	
	$= 0.5\times10^7$		
	$= 0.5\times10\times10^6$		
	$= 5\times10^5$ people	A1	
	Hence, there are more people live in Malaysia than in Australia.		
c	$\text{Average no. of people} = \frac{\text{Population of United Kingdom}}{\text{Area of United Kingdom}}$		
	$= \frac{6.0\times10^7}{2.4\times10^5}$		
	$= \frac{6.0}{2.4}\times10^{7-2}$		
	$= 2.5\times10^2$	B1	
	$= 250$ people/ km^2		
	Hence, the average number of people per km^2 in the United Kingdom is 250.		
d	Population density = population ÷ land area		Some did not work out pop
	Population density of Singapore = 7857.14		

Population density of Malaysia = 81.81
Population density of Australia = 2.86 M1

Population density of Singapore is the highest. A1

density of all the countries, so cannot conclude that SG is highest
6 marks

2ai

$$\text{Time taken for route } A = \frac{\text{Distance travelled}}{\text{Speed}}$$

$$= \frac{120}{x} \text{ h}$$ B1

Hence, the time that he expects to take on route A is $\frac{120}{x}$ h.

Many did not give units or gave wrong units

aii Since route B is 5 km shorter than route A,

$$\text{Distance covered in route } B = 120 - 5$$
$$= 115 \text{ km}$$

Since he expects to cover $(x-2)$ km/h in route B,

$$\text{Time taken by for route } B = \frac{\text{Distance travelled}}{\text{Speed}}$$

$$= \left(\frac{115}{x-2}\right) \text{h}$$ B1

Hence, the time he expects to take on route B is

Some used 120 instead of 115 km

b Since he estimates that route A will take 40 minutes or $\frac{2}{3}$ h less than route B,

Weaker students use 40 mins instead of $\frac{2}{3}$ hours

$$\text{Time taken for route } B - \text{Time taken for route } A = \frac{2}{3}$$

$$\frac{115}{x-2} - \frac{120}{x} = \frac{2}{3}$$

$$\frac{115x - 120(x-2)}{x(x-2)} = \frac{2}{3}$$ M1

$$\frac{115x - 120x + 240}{x^2 - 2x} = \frac{2}{3}$$

$$\frac{-5x + 240}{x^2 - 2x} = \frac{2}{3}$$

$$3(-5x + 240) = 2(x^2 - 2x)$$ M1

$$-15x + 720 = 2x^2 - 4x$$

$$2x^2 - 4x + 15x - 720 = 0$$

$$2x^2 + 11x - 720 = 0 \quad \text{(Shown)}$$ A1

Some mixed up the order of subtraction, some could not handle the algebraic manipulation to get the required quad equation.

c

Comparing with $ax^2 + bx + c = 0$, we have $a = 2$, $b = 11$ and .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(11) \pm \sqrt{(11)^2 - 4(2)(-720)}}{2(2)}$$ M1

$$= \frac{-11 \pm \sqrt{5881}}{4}$$

$$= 16.422 \text{ or } -21.922 \quad \text{(Correct to 3 decimal places)}$$

$$= 16.4 \text{ or } -21.9 \quad \text{(Correct to 1 decimal place)}$$ A2

Weaker students cannot give the correct formula,

Many rejected negative answer at this part.

Some did not read qn to give to 1 dp.

d

Since the speed is a positive value, $x = 16.422$.

$$\text{Time taken in route } B = \frac{115}{(16.422) - 2}$$ M1

$$= 7.9739 \text{ h}$$

$$= 7 \text{ h } 58 \text{ min}$$ A1

Some did not know how to change 0.9739 h to 58 mins.

Hence, the time he expects to take in route B is 7 h 58 min.

10 marks

3a

$$\frac{7}{2(3p-1)}-\frac{3}{(1-3p)}=\frac{7}{2(3p-1)}-\frac{3}{-(3p-1)}$$

$$=\frac{7}{2(3p-1)}+\frac{3}{3p-1}$$ M1

$$=\frac{7}{2(3p-1)}+\frac{3(2)}{2(3p-1)}$$

$$=\frac{7+6}{2(3p-1)}$$ M1

$$=\frac{13}{2(3p-1)}$$ A1

Ok

b

$$\frac{1}{3b}=\frac{2}{3a}+\frac{1}{5c}$$

$$\frac{1}{3b}-\frac{1}{5c}=\frac{2}{3a}$$

$$\frac{2}{3a}=\frac{1}{3b}-\frac{1}{5c}$$

$$\frac{2}{3a}=\frac{5c}{15bc}-\frac{3b}{15bc}$$ M1

$$\frac{2}{3a}=\frac{5c-3b}{15bc}$$

$$\frac{3a}{2}=\frac{15bc}{5c-3b}$$ M1

$$a=\frac{2}{3}\left(\frac{15bc}{5c-3b}\right)$$

$$=\frac{10bc}{5c-3b}$$ A1

Weaker students cannot handle fractional algebraic manipulations, having many misconceptions.

Some left answer without dividing by 3 or have fractions /decimals within the fraction.

c

$x-2y=8$ ----------(1)

$3x=19+4y$ ----------(2)

Rewrite (1) $x=8+2y$ ----------(3) M1

Put (3) in (2)

$3(8+2y)=19+4y$

$24+6y=19+4y$

Generally ok except for careless mistakes by weaker students.

$2y = -5$

$y = -2\frac{1}{2}$ A1

Put $y = -2\frac{1}{2}$ in (3)

$x = 8 + 2(-\frac{5}{2}) = 3$ A1

d

$$2x^3 \times \left(\frac{25x^2}{4y^4}\right)^{\frac{1}{2}} = 2x^3 \times \frac{5x}{2y^2} \qquad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (a^m)^n = a^{mn}$$ M1

$$= \frac{2 \times 5}{2} x^{3+1}\left(\frac{1}{y^2}\right) \qquad a^m \times a^n = a^{m+n}$$ M1

$$= \frac{5x^4}{y^2}$$ A1

Weaker students did not take square root of 25 and 4, some brought y^2 up.
12 marks

4ai Let the length of the cube be y cm.

Most can do.

$$\text{Volume of cube} = 343\ \text{cm}^3$$
$$y^3 = 343$$
$$y = \sqrt[3]{343}$$
$$y = 7$$ B1

Hence, the length of the cube is 7 cm.

ii Let the radius of the base of the cylinder be r cm.

Generally ok except for weaker students who mixed up the formula.

$$\text{Volume of cylinder} = 343\ \text{cm}^3$$
$$3.142 \times r^2 \times 8 = 343$$ M1
$$25.136r^2 = 343$$
$$r^2 = \frac{343}{25.136}$$
$$r = \pm\sqrt{\frac{343}{25.136}}$$
$$r = \pm 3.6940 \quad \text{(Correct to 5 sig. fig.)}$$
$$r = \pm 3.69 \quad \text{(Correct to 3 sig. fig.)}$$

Since $r > 0$, $r = 3.69$.

A1

Hence, the radius of the cylinder is 3.69 cm.

iii Volume of cone = 343 cm^3.

Many used *x* as radius instead of diameter.

$$\frac{1}{3}\pi\left(\frac{x}{2}\right)^2(2x) = 343$$

M1

$$\frac{1}{3}\times 3.142\times\left(\frac{x^2}{4}\right)(2x) = 343$$

Many got this step but cannot handle the manipulati on after that.

$$x^3 = \frac{343\times 2}{\frac{1}{3}\times 3.142}$$

M1

$$x = \sqrt[3]{654.9968}$$

$= 8.68$ cm

A1

b

A lot of students used vol of box divide by volume of cube.

$$\frac{75}{7} = 10.7$$

$$\frac{64}{7} = 9.14$$

M1

$$\frac{90}{7} = 12.86$$

Common misconcept in across classes.

Hence, maximum number of cube candles that can fit in the carton box = 10 x 9 x 12

= 1080

A1

8 marks

5a Continuing the sequence,

Many can see the pattern and give correct answer without much workings

Day	No. of plastic cubes used, *N*	No. of plastic cubes added,
1	$1\times 2\times 3 = 6$	6
2	$2\times 4\times 5 = 40$	34
3	$3\times 6\times 7 = 126$	86
4	$4\times 8\times 9 = 288$	$288 - 126 = 162$
5	$5\times 10\times 11 = 550$	$550 - 288 = 262$

M1

6	$6\times12\times13=936$	$936-550=386$	A1

Or

Total number of plastic cubes in the rectangular block in Day 6 is **936** . B2

b Observe that all the values in *A* are even numbers.
Since 2555 is not an even number, it could not appear in column *A*. B1

Some did not mention about odd nor even numbers, but showed a half page working to justify a 1 mark answer,

c No.of plastic cubes on day $1=1\times2\times3$

$1x\ 2\ (1)\ x\ [2(1)+1]$

No.of plastic cubes on day $2=2x4x5$

$=2\ x\ 2(2)\ X[2(2)+1]$

No.of plastic cubes on day $3=3\ x\ 6\ x\ 7$

$3=2(3)\ x\ [2(3)+1]$ M1

$\therefore\ x=n\times2(n)\times[2(n)+1]$

$=n(2n)(2n+1)$ Hence, $x = n(2n)(2n + 1)$. A1

Or

B2

Many can see the pattern and give correct answer without much workings

d No.of plastic cubes on day $31=31[2(31)][2(31)+1]$

$=121086$ M1

No. of plastic cubes on day 30 = 30[2(30)][2(30) + 1] M1

= 109800

Hence, no. of plastic cubes added on day 31 is 121 086 – 109800 A1

= 11286.

Many can see the pattern and give correct answer without much workings. Marks deducted for omission of essential workings for this part only 8 marks

6a	Bearing of A from B = 360° - 155° = 205°	B1	ok
b	Using Cosine Rule, $\cos\angle ABC = \frac{85^2+180^2-170^2}{2(85)(180)}$	M1	Generally ok except for weaker students.
	$= \frac{10725}{30600}$		
	$= \frac{143}{408}$	M1	Some didn't give to 1 dp.
	$\angle ABC = 69.482° = 69.5°$	A1	
c	$\tan\angle PAC = \frac{12}{170}$	M1	Some didn't give to 1 dp.
	$\angle PAC = \tan^{-1}\left(\frac{12}{170}\right)$	A1	
	$= 4.0°$		
d	Cost of land $ABC = 50 \times$ Area of ABC	M1	Due to rounding of angle ABC to different accuracy, many possible answers are acceptable as long as final answer is given to 2 dp.
	= 50x $\frac{1}{2}$ x 85 x 180 x sin 69.4826	M1	
	$ 358236.4156		
	=$358236.42	A1	
ei	$\sin\angle 69.4826 = \frac{\text{dist}}{85}$	M1	ok
	$\text{dist} \approx 79.6\text{ m}$	A1	
eii	$\tan\theta = \frac{12}{79.60809236}$	M1	Badly done in weaker classes . Many left blank.
	$\theta \approx 8.572139°$		
	$\approx 8.57°$		
	Greatest angle of elevation is 8.6°.	A1	
			13 marks

7a Amount payable $= 17.5(\$1.17) + 68(\$0.21) + 610(\$0.26)$ — M1

$= \$193.355$

$= \$193.36$ (2 decimal places) — A1

Accuracy is a problem here.

b $\text{GST} = \frac{7}{100} \times \$193.355$

$= \$13.53$ (2 decimal places) — B1

Accuracy is a problem here. Some did not read qn as asking for GST but gave price with GST.

ci $\%\text{ decrease} = \frac{68 - 65}{68} \times 100\%$ — M1

$= 4.41\%$ (3 sig. fig.) — A1

ok

ii $\text{Bill without GST} = 17.5\,(\$1.17) + 65\,(\$0.21) + \frac{115}{100}(610)(\$0.26)$ — M1

$= \$\,216.515$ — M1

Accuracy is also a problem here.

$\text{Bill with GST} = \frac{107}{100}(\$216.515)$

$= \$231.67$ (2 decimal places) — A1

iii $\text{Change in bill} = \$231.67 - (\$193.36 + \$13.53)$

$= \$24.78$ — M1

Some gave wrong denominated.

$\%\text{ change} = \frac{24.78}{193.36 + 13.53} \times 100\%$ — M1

$= 12.0\%$ (3 sig. fig.)

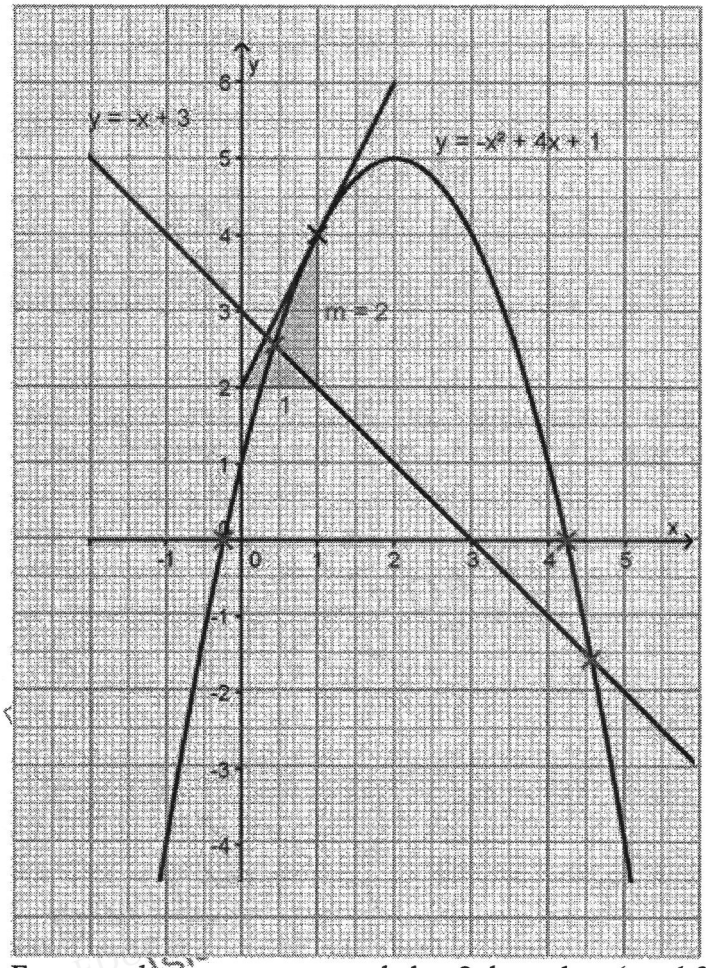
There is a 12.0% **increase**. — A1

11 marks

8a $x = -1,\ k = -4$ $x = 2,\ n = 5$ — B2

Weak students from 3E4 cannot even get these marks!

b



B1 correct scale

B1 correct points plotted & axes labelled

B1 smooth curve passing through all plotted points

Generally ok except for very weak students.

c From graph, x = answer rounded to 2 dp or less (read from individual graph, accept -0.336 to -0.136 ,4.16 to 4.36) — A2

d Draw the line $y = 3 - x$ — M1

From graph, x = answer rounded to 2 dp or less (read from individual graph, accept 0.338 to 0.538 ,4.46 to 4.66) — A2

e Tangent drawn at $x = 1$ with coordinates seen or triangle drawn. — B1

Gradient = 2 (read from individual graph, accept 1.5 to 2.4) — A1

12 marks

9a ΔACB with justifications below — A1 — No marks for just stating without reasons or details

$\angle CAB$ is common.
$\angle ACB = \angle ADE$ (corresponding angles, $CB \,//\, DE$) — M1
ΔACB is similar to ΔADE (AA similarity test)

b ok

$$\frac{AD}{AC} = \frac{AE}{AB}$$

$$\frac{AD}{5} = \frac{4.5}{7.5}$$ — M1

$$AD = \frac{4.5 \times 5}{7.5}$$

$$= 3 \text{ cm}$$ — A1

ci ok

$$\left(\frac{4.5}{7.5}\right)^2 = \left(\frac{3}{5}\right)^2$$

$$= \frac{9}{25}$$

Ratio = 9 : 25 — A1

cii Badly done

A1

Ratio = 3 : 5

d Ok. Ecf marks given

$$\frac{\text{Area of trapezium } BCDE}{\text{Area of } \Delta ABC} = \frac{25-9}{25}$$ — M1

$$\text{Area of trapezium } BCDE = \frac{16}{25} \times 18.75$$

$$= 12 \text{ cm}^2$$ — A1

8 marks

10a $\frac{y}{25} = \frac{15}{40}$

$y = 9.375$ (shown)

M1 Ratio must be seen or using area

b Volume of glass $= \frac{2}{3}\pi(15)^3 - \frac{2}{3}\pi(15-0.5)^3$

$= 683.558$

$= 684\,\text{cm}^3$ (3 sig. fig.)

M2 Weak students used

A1 wrong formula, some added frustrum volume.

c Total area of the lampshade to be painted

$= 2\pi(15)^2 + 2\pi(14.5)^2 + \pi(15)^2 - \pi(14.5)^2$

$= 2781.09\ \text{cm}^2$

$= 2780\ \text{cm}^2$.

M1M1 As in part b

A1

d

Total area to be painted $= 2781.09 \times 10^{-4} \times 5000\ \text{m}^2$

$= 1390.545\ \text{m}^2$

M1 Attempt to find area to be painted

Brand A cannot be used as it does not work on glass.

A1 Mention that Brand A cannot be used on glass

Brand B

$\frac{1390.545}{3} = 463.5$

464 cans needed. Cost = \$(464 x 12.50) = \$ 5800

M1 Attempt to find number of cans needed and cost for B and C

Brand C

$\frac{1390.545}{2} = 695.2$

696 cans needed. Cost = \$(696 x 10) = \$ 6960

Company X should choose brand B as it is cheaper.

A1 correct choice based on their **calculations**

12 marks