Name : $\qquad$ Reg No. $\qquad$ Class : $\qquad$

EXAMINATION : END-OF-YEAR EXAMINATION
LEVEL : SECONDARY 3 EXPRESS DATE: 02 Oct 2018
SUBJECT : MATHEMATICS O-LEVEL (4048/1) PAPER: 1

DURATION : 2 hours
SETTER(S) : Mr ONG CHEE LIM Parent's/Guardian's Signature:

## INSTRUCTIONS TO CANDIDATES

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen in the spaces provided on the Question Paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Answer all questions.
The number of marks is given in brackets [ ] at the end of each question or part question. If working is needed for any question, it must be shown with the answer. Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.

For Examiner's Use
/80

## Mathematical Formulae

## Compound interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

## Mensuration

Curved surface area of a cone $=\pi r l$
Surface area of a sphere $=4 \pi r^{2}$

$$
\text { Volume of a cone }=\frac{1}{3} \pi r^{2} h
$$

Volume of a sphere $=\frac{4}{3} \pi r^{3}$

$$
\text { Area of triangle } A B C=\frac{1}{2} a b \sin C
$$

Arc length $=r \theta$, where $\theta$ is in radians
Sector area $=\frac{1}{2} r^{2} \theta$, where $\theta$ is in radians

Trigonometry

$$
\begin{aligned}
& \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{aligned}
$$

## Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

## Answer all the questions.

1 Given that $\frac{1}{343}=7^{p}$, find $p$.

$$
\text { Answer } p=
$$

2 Represent $-2.5<x \leq 5$ on the number line below.


3 Simplify $18 x-3(2 x-5)^{2}$.

4 Show that, for all positive integer $k, 3^{k}+3^{k+1}$ is a multiple of 4 .

Answer

5 Water is poured into each empty container at a constant rate. After $t$ seconds, the depth of water in the container is $d \mathrm{~cm}$. sketch the graph of $d$ against $t$.
(a)


(b)



6 The stem-and-leaf diagram shows the height, in centimetres, of some students.

| 14 | 7 | 9 |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 15 | 1 | 4 | 8 |  |  |  |  |  |  |  |
| 16 | 0 | 1 | 3 | 5 | 7 | 7 | 7 | 8 | 9 | 9 |
| 17 | 0 | 2 | 4 | 5 | 7 | 8 | 9 |  |  |  |
| 18 | 0 | 2 |  |  |  |  |  |  |  |  |

Key: $14 \mid 7$ represents 147 cm
For these students, find
(a) the modal height,
$\qquad$
(b) the median height.

Answer
cm

7 An advertisement for milk powder shows the amount of calcium in the milk produced over a period of time after a cow gives birth.

Amount of calcium


State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

8 The diagram shows an irregular polygon $A B C D E$ in which $\angle A B Q=x^{\circ}, \angle B C R=2 x^{\circ}$, $\angle C D E=3 x^{\circ}-40^{\circ}, \angle T E A=x^{\circ}$ and $\angle B A E=160^{\circ}-x^{\circ}$.

Find the value of $x$.


$$
\text { Answer } x=
$$

9 The gravitational force, $F$, between two huge objects is inversely proportional to square of the distance between the two objects.

Given that the force is 40 N when the distance is $r \mathrm{~km}$, find the gravitational force if the distance is halved.

10 Sketch the graph of $y=(x+2)(x-4)$ on the axes below.
Indicate clearly the values where the graph crosses the $x$ - and $y$-axes.


11 A shopkeeper buys 1440 pencils at 25 cents each.
He sells 40 dozens at $\$ 3.60$ per dozen and another 30 dozens at $\$ 4.20$ per dozen.
Find the price he must sell each of the remaining pencils in order to make a profit of $16 \frac{2}{3} \%$.


In the diagram above, $Q$ is the point $(7,6)$ and $R$ is the point $(-2,0)$. $P Q$ is parallel to the $x$-axis and $P R$ is parallel to the $y$-axis.
(a) Write down the coordinates of $P$.

Answer $P=(\ldots \ldots \ldots, \ldots \ldots .$.
(b) If $S(1, b)$ is a point on $Q R$, find the value of $b$.

13 The drawing in the answer space below shows the positions of a port $P$, a ship $S$ and a lighthouse $L$.

(a) Construct the perpendicular bisector of $S L$.
(b) Construct the bisector of angle PSL.
(c) A motorboat has broken down. It is nearer to $S$ than $L$ and nearer to $S L$ than to $P S$. Shade the region where the motorboat is located.

14 Solve $\frac{a+1}{3}-\frac{a-2}{2}=1$.

15 Adam invested a sum of money in a bank account paying compound interest at $2.5 \%$ per year. After 4 years, Adam earned a total interest of \$674.79.

Calculate the sum of money Adam invested in the account.


In the diagram, $\angle P Q R=\angle S P T=90^{\circ}$.
$T$ is the midpoint of $P Q, P Q=2 Q R$ and $P Q=P S$.
(a) Show that the triangles $P Q R$ and $S P T$ are congruent.
Answer ............ shown
(b) Name another angle that is equal to $\angle T S P$.

(c) Hence, show that $\angle S T P=\angle R P S$.

17 (a) Factorise $2 x^{2}-11 x-21$.
(b) Hence factorise completely $2(2 y+1)^{2}-11(2 y+1)-21$.

18 Points $A, B, C$ and $D$ lie on a circle with centre $O$. $A B=5.0 \mathrm{~cm}, B C=7.14 \mathrm{~cm}$ and angle $B D A=x^{\circ}$

Find the value of $x$.
Show your working and state all properties clearly.


19 In the diagram, $A, B$, and $C$ are points on a circle, centre $O$. $O C$ is parallel to $A B$.
Angle $O A C=x^{\circ}$ and angle $O C B=5 x^{\circ}$.
Find the value of $x$.
Show your working and state all properties clearly.


Answer $x=$

20 (a) Use prime factorisation to explain why $15 \times 135$ is a perfect square.

Answer
(b) $\quad a$ and $b$ are both prime numbers.

Find the values of $a$ and $b$ such that $2025 \times \frac{a}{b}$ is a perfect cube.

$$
\text { Answer } \begin{aligned}
a & = \\
b & =
\end{aligned}
$$

21 The graphs of $y=x^{2}+3 x-4$ and $y=2 x+2$ are drawn on the grid.

(a) Explain why the equation $x^{2}+3 x-4=k$ does not have any solution for some values of $k$.

## Answer

(b) The points of intersection of the curve and the straight line give the solutions of a quadratic equation.

Find the quadratic equation, giving your answer in the form $a x^{2}+b x+c=0$.
(c) The equation $x^{2}+4 x-2=0$ can be solved by drawing a suitable straight line on the grid.
(i) Find the equation of the straight line.

## Answer

(ii) By drawing this straight line, solve the equation $x^{2}+4 x-2=0$.

$$
\text { Answer } x=\ldots \ldots \ldots \ldots \text { or } x=
$$

22 Triangle $A B C$ is a right-angled triangle and $D$ is a point on $B C$.
It is given that $A B=C D=12 \mathrm{~cm}$ and $\tan \angle A C B=\frac{4}{7}$.
Find
(a) the area of triangle $A B C$,


Answer .............................cm ${ }^{2}$
(b) the exact value of $\cos \angle A D C$.

23 China's high speed train can travel at an average speed of $300 \mathrm{~km} / \mathrm{h}$.
The distance between Beijing and Shanghai is 1318 km .
The distance between Beijing and Guangzhou is approximately 2300 km .
(a) Change $300 \mathrm{~km} / \mathrm{h}$ into $\mathrm{m} / \mathrm{s}$.
$\qquad$
(b) Calculate the time taken for the train to travel from Beijing to Shanghai, leaving your answers in hours, minutes and seconds.
$\qquad$
(c) Mr Tan took the train from Beijing to Guangzhou. He left Beijing at 7.15 am . What time did he arrive at Guangzhou?

24 Baseballs and other spherical objects are often packed in boxes that are cuboids.
The diagram shows 4 identical baseballs packed tightly inside a cubical box.
The radius of each baseball is 4 cm .


Find
(a) the volume of a baseball, leaving your answer in terms of $\pi$,

$$
\text { Answer ............................... cm }{ }^{3}
$$

(b) the volume of the box,
$\qquad$
Answer ..............................cm ${ }^{3}$
(c) the percentage of the volume of the box that is not occupied by the baseball, correcting your answer to 1 decimal place.

25 The diagram shows an inverted cone of height $h$ and radius $r$. It contains water to a depth of $\frac{1}{2} h$.

(a) Find ratio of area of surface $B$ : area of surface $A$.
(b) Find the volume of water if the cone can hold $480 \mathrm{~cm}^{3}$ of water when full.
$\qquad$
(c) The cone is now inverted again such that the liquid rests on the flat circular surface of the cone as shown below. Find, in terms of $h$, an expression for $d$, the distance of the liquid surface from the top of the cone.


## WOODLANDS RING SECONDARY SCHOOL

$\qquad$
Name :
Reg No. $\qquad$ Class : $\qquad$

EXAMINATION : END-OF-YEAR EXAMINATION
LEVEL : SECONDARY 3 EXPRESS DATE: 03 Oct 2018
SECONDARY 3 NORMAL ACADEMIC
SUBJECT : MATHEMATICS O-LEVEL (4048/2)
PAPER: 2
DURATION : 2 hours
SETTER(S) : Mr Soh Kian Hong Parent's/Guardian's Signature:

## INSTRUCTIONS TO CANDIDATES

Write your answers and working on the separate answer papers provided.
Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
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Answer all questions.
The number of marks is given in brackets [ ] at the end of each question or part question.
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The use of an approved scientific calculator is expected, where appropriate.
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For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.

At the end of the examination, fasten all your work securely together.

## Mathematical Formulae

Compound interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

## Mensuration

$$
\begin{gathered}
\text { Curved surface area of a cone }=\pi r l \\
\text { Surface area of a sphere }=4 \pi r^{2} \\
\text { Volume of a cone }=\frac{1}{3} \pi r^{2} h \\
\text { Volume of a sphere }=\frac{4}{3} \pi r^{3} \\
\text { Area of triangle } A B C=\frac{1}{2} a b \sin C
\end{gathered}
$$

Arc length $=r \theta$, where $\theta$ is in radians
Sector area $=\frac{1}{2} r^{2} \theta$, where $\theta$ is in radians

## Trigonometry

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A
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Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

Answer all the questions.

1 (a) Solve the inequality $\frac{x+9}{3} \geq \frac{5-x}{7}$.
(b) Simplify $\frac{15 a^{2}}{c^{2}} \div \frac{6 a}{14 b c^{3}}$.
(c) It is given that $s=u t+\frac{1}{2} a t^{2}$.
(i) Find $s$ when $u=1.5, a=2$ and $t=4$.
(ii) Express $a$ in terms of $s, u$ and $t$.


In the diagram, $A B C D$ is parallelogram. $M$ is the mid-point of $B C$ and $A E C$ and $D E M$ are straight lines. $E M=4 \mathrm{~cm}$.
(a) Show that triangle $A D E$ is similar to triangle $C M E$. Give a reason for each statement you make.
(b) Find the length of $D E$.
(c) Calculate, as a fraction, the numerical value of the ratio
(i) $\frac{\text { area of triangle } C M E}{\text { area of triangle } A D E}$,
(ii) $\frac{\text { area of triangle } A D E}{\text { area of triangle } A D C}$.


The diagram shows a circle $A B C D$, centre $O$.
$B C$ produced and $A D$ produced meet at point $F$.
$B E D$ is a straight line. Angle $O B E=11^{\circ}$ and angle $B C E=58^{\circ}$.
Find, giving reasons for each answer,
(a) angle $A B C$, [1]
(b) angle $B A C$,
(c) angle $B A D$,
(d) angle $A C D$.

4 The table shows a sequence of figures formed by drawing lines

|  |  |  |  |
| :--- | :---: | :---: | :---: |
|  | Figure 1 | Figure 2 | Figure 3 |
| Number of <br> non-vertical lines | 2 | 6 | 12 |

(a) Find the number of non-vertical lines in figure 4 and figure 5 .
(b) Find an expression, in terms of $n$, in the form $a n^{2}+b n$ for the number of non-vertical lines found in figure $n$.
(c) Find the number of non-vertical lines in figure 56.
(d) Hadi claims that one of the figures has 243 non-vertical lines. Is he correct? Explain your answer.

5 (a) Frank is a salesman who is paid $1.5 \%$ commission on his sales. His sales in January was \$26000.
(i) Find his commission in January.
(ii) His sales increase by $12 \%$ from January to February and decrease by $25 \%$ from February to March.
Find his sales in the month of March.
(b) The exchange rate between United States dollars (US\$) and Singapore dollars(S\$) was US $\$ 1.00=$ S $\$ 1.37$.
Paul bought a shirt from an online shop for US\$23.99.
Find the amount of money he paid in Singapore dollars.
$6 \quad x \mathrm{~kg}$ of aluminium metal costs $\$ 25$.
An alloy, which is 1 kg lighter, also costs $\$ 25$.
(a) Write down an expression, in terms of $x$, for the cost of 1 kg of aluminium.
(b) Write down an expression, in terms of $x$, for the cost of 1 kg of alloy.
(c) For an art sculpture, Nurul paid $\$ 55$ for 10 kg of aluminium and 5 kg of alloy. Write down an equation to represent this information and show that it simplifies to

$$
\begin{equation*}
11 x^{2}-86 x+50=0 \tag{3}
\end{equation*}
$$

(d) Solve the equation $11 x^{2}-86 x+50=0$.
(e) Explain why one of the solutions in part (d) must be rejected as the mass of the aluminium.
(f) Calculate the cost of 1 kg of aluminium.

7


Points $A, B$ and $C$ are on level ground.
$B$ is due east of $C$.
$A$ is 515 m from $C$ on a bearing of $055^{\circ}$.
$B$ is 584 m from $C$.
(a) Calculate $A B$.
(b) Calculate the bearing of $A$ from $B$.
(c) Calculate the shortest distance from $A$ to $B C$.
(d) $A T$ is a vertical tower at $A$.

The angle of elevation of the top of the tower from $C$ is $25^{\circ}$.
Calculate the height of the tower, $A T$.
$8 \quad A$ is the point $(5,11)$ and $B$ is the point $(-4,-2)$ respectively.
(a) Find the length of the line $A B$.
(b) Find the equation of the line $A B$.
(c) The equation of line $p$ is $9 y=13 x+9$.
(i) Show how you can tell that the line $p$ does not intersect the line $A B$.
(ii) Another line $q$ is $3 y-5 x=6$.

Find the coordinates of the point of intersection of the line $p$ and the line $q$.

9 Answer the whole of this question on a single sheet of graph paper.
The variables $x$ and $y$ are connected by the equation

$$
y=\frac{x^{3}}{30}+\frac{20}{x}-10 .
$$

Some corresponding values of $x$ and $y$, correct to 2 decimal places, are given in the table below.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $p$ | 0.27 | -2.43 | -2.87 | -1.83 | 0.53 | 4.29 |

(a) Calculate the value of $p$.
(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal $x$-axis for $0 \leq x \leq 7$. Using a scale of 1 cm to represent 1 unit, draw a vertical $y$-axis for $-3 \leq y \leq 11$.
On your axes, plot the points given in the table and join them with a smooth curve.
(c) By drawing a line, use your graph to find the solution to the equation

$$
\begin{equation*}
\frac{x^{3}}{30}+\frac{20}{x}=15 \tag{2}
\end{equation*}
$$

(d) By drawing a tangent, find the gradient of the curve at $x=2$.
(e) (i) On the same axes, draw the line $y+2 x=10$.
(ii) Write down the $x$-coordinates of the points where the line intersects the curve.

10 The figure below shows a conical paper cup used by a company water dispenser. The cup has diameter of 6 cm and a height of 8 cm .

(a) Calculate the full volume of water each cup can hold. Leave your answer to the nearest whole number.
(b) The water dispenser tank is cylindrical with diameter 50 cm and height 60 cm .
(i) Find the volume of water in a full dispenser tank.

Leave your answer in 2 significant figures.
(ii) Teresa, the company manager, wishes to purchase cups and water dispenser tanks for her company.

Information that Teresa needs is found below.

| Item | Description | Unit Cost |
| :--- | :--- | :--- |
| Conical cups | 1 pack of 50 | $\$ 2.50$ |
|  | 1 pack of 250 | $\$ 10.00$ |
| Cylindrical <br> water tanks | 1 drum ( $\leq 50$ drums) | $\$ 20.00$ |
|  | Bulk price ( $>50$ drums) | $\$ 15.00$ |

She estimates that 10000 litres of water is consumed per year.
Using your answers in (a) and (bi), work out a suitable amount of money Teresa needs to budget for her company's water dispenser per year.
Justify the decision you make and show your calculations clearly.


|  |  |  |
| :---: | :---: | :---: |
| 6(a) | Mode $=1167 \mathrm{~cm}$ | A1 |
| (b) | Median $=\frac{167+168}{2}=167.5 \mathrm{~cm}$ | B1 |
| 7 | Vertical axis does not start from zero <br> Hence this makes change seems like $25 \%$ of original, when the change is actually $60 \%$ of original. | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \end{aligned}$ |
| 8 | Method 1 (using exterior angles) $\begin{aligned} x+2 x+[180-(3 x-40)]+x+[180-(160-x)] & =360 \\ 240+2 x & =360 \\ x & =60 \end{aligned}$ <br> Method 2 (using interior angles) $\begin{aligned} (180-x)+(180-2 x)+(3 x-40)+(180-x)+(160-x) & =180(5-2) \\ 660-2 x & =540 \\ x & =60 \end{aligned}$ | M1 <br> A1 |
| 9 | $F=\frac{k}{d^{2}}$ $F=\frac{k}{d^{2}}$ <br> Sub $F=40, d=r$ <br> New $F=\frac{k}{\left(\frac{d}{2}\right)^{2}}$ <br> $40=\frac{k}{r^{2}}$ $k=40 r^{2}$ <br> $d^{2}$ <br> $=4(40)$   <br> $=160 \mathrm{~N}$  $\quad$$4 k=\frac{40 r^{2}}{d^{2}}$  <br>  Sub $d=\frac{1}{2} r$ <br>  $F=\frac{40 r^{2}}{\left(\frac{1}{2} r\right)^{2}}$ <br>  $F=\frac{40 r^{2}}{\frac{1}{4} r^{2}}$ <br>  $\therefore F=160 \mathrm{~N}$ | M1 |


| 10 |  |  |
| :--- | :--- | :--- | :--- |




|  |  |  |
| :---: | :---: | :---: |
| 19 | $\angle \mathrm{OCA}=x$  (base $\angle$ of isos triangle)  <br> obtuse $\angle \mathrm{AOC}$ $=180-2 x$  (angle sum of triangle) <br> reflex $\angle \mathrm{AOC}$ $=2(5 x)$  ( $\angle$ at centre $=2 \angle$ at circumference) <br>  $=10 x$  $\quad$$180-2 x+10 x=360$  (angles at a point) <br> $x$ $=22.5$  | M1 M1 M1 A1 |
| 20(a) | $\begin{aligned} 15 \times 135 & =(3 \times 5) \times\left(3^{3} \times 5\right) \\ & =\left(3^{2} \times 5\right)^{2}=45^{2} \end{aligned}$ <br> $45^{2}$ is a perfect square or the powers are even / powers are multiples of 2 | M1 <br> M1 <br> A1 |
| (b) | $a=5, b=3$ | B2 |
| 21(a) | When k is less than minimum point of the graph, there is not intersection of the graph of $y=x^{2}+3 x-4$ with graph of $\mathrm{y}=\mathrm{k}$, and thus no solution of the equation $x^{2}+3 x-4=k$. <br> Note: accept $(\mathrm{k}<-6$ or $\mathrm{k}<-7)$. | A1 |
| (b) | $x^{2}+3 x-4=2 x+2$ Shift all terms to LHS, $x^{2}+x-6=0$ | B1 |
| (c) | $\begin{aligned} & x^{2}+4 x-2=0 \\ & x^{2}+3 x-4=-x-2 \end{aligned}$ <br> suitable straight line : $y=-x-2$  | B1 |
| (d) | Draw line of $y=-x-2$ (e.c.f. for a straight line drawn) | M1 A2 |
|  |  |  |


| 22(a) | $\tan \angle A C B=\frac{4}{7}=\frac{12}{12+S Q}$ $\frac{4}{7}=\frac{12}{B C}$ <br> $4(12+S Q)=12 \times 7$  <br> $48+4 \mathrm{SQ}=84$  <br> $\mathrm{SQ}=9$ $\mathrm{BC}=21$ <br>   <br> Area $=\frac{1}{2} \times 12 \times(9+12)$  <br>  $=126 \mathrm{~cm}^{2}$ Area $=\frac{1}{2} \times 12 \times(21)$ <br>  $=126 \mathrm{~cm}^{2}$ | M1 <br> A1 |
| :---: | :---: | :---: |
| (b) | $\begin{aligned} & A D^{2}=12^{2}+9^{2} \quad \text { (Pythagoras' Theorem) } \\ & A D=15 \mathrm{~cm} \\ & \cos P S R=-\cos P S Q=-\frac{9}{15}=-\frac{3}{5} \end{aligned}$ | M1 <br> A1 |
| 23(a) | $83 \frac{1}{3} \mathrm{~m} / \mathrm{s}$ <br> (accept $83.3 \mathrm{~m} / \mathrm{s}),\left(\frac{250}{3} \mathrm{~m} / \mathrm{s}\right.$ not accepted) | A1 |
| (b) | $\begin{aligned} & \text { Time taken }=\frac{1318}{300} \\ &=\frac{659}{150} \mathrm{~h} \quad \text { or } 4.39 \mathrm{~h} \text { or } 4 \frac{59}{150} \mathrm{~h} \\ &=4 \mathrm{~h} 23 \mathrm{~m} 36 \mathrm{~s} \end{aligned}$ | M1 <br> A1 |
| (c) | $\begin{aligned} \text { Time taken } & =\frac{2300}{300} \\ & =7 \frac{2}{3}=7 \mathrm{~h} 40 \mathrm{mins} \end{aligned}$ <br> Reached at 2.55 pm or 1455 h | M1 A1 |
| 24(a) | $\begin{aligned} \text { Volume of a sphere } & =\frac{4}{3} \times \pi \times 4^{3} \\ & =85 \frac{1}{3} \pi \mathrm{~cm}^{3} \quad \text { accept } 85.3 \pi \mathrm{~cm}^{3}, \frac{256}{3} \pi \mathrm{~cm}^{3} \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { M1 } \\ \text { A1 } \end{array}$ |
| (b) |  | M1 |


|  | $\begin{aligned} \text { Volume of box } & =16 \times 16 \times 8 \\ & =2048 \mathrm{~cm}^{3} \end{aligned}$ | A1 |
| :---: | :---: | :---: |
| (c) | $\begin{aligned} \text { Volume of } 4 \text { spheres } & =4 \times 85 \frac{1}{3} \pi \\ & =341 \frac{1}{3} \pi \mathrm{~cm}^{3} \\ \text { or } & =1072.33 \mathrm{~cm}^{3} \end{aligned}$ <br> Percentage of box not occupied by spheres $\begin{aligned} & =\frac{2048-341 \frac{1}{3} \pi}{2048} \times 100 \% \\ & =47.6 \% \quad \text { (Correct to } 1 \text { decimal place }) \end{aligned}$ | M1 <br> A1 |
| 25(a) | Let $A_{A}$ be the area of surface A Let $A_{B}$ be the area of surface B $\begin{aligned} & \frac{A_{B}}{A_{A}}=\left(\frac{1}{2}\right)^{2}=\frac{1}{4} \\ & 1: 4 \quad \text { accept } 0.25: 1 \text { or } \frac{1}{4}: 1 \end{aligned}$ | A1 |
| (b) | Let $V_{B}$ be the vol of the water $V_{A}$ be the vol of the cone $\begin{aligned} & \frac{V_{B}}{V_{A}}=\left(\frac{1}{2}\right)^{3}=\frac{1}{8} \\ & V_{B}=\frac{480}{8}=60 \end{aligned}$ | M1 <br> A1 |
| (c) | Let $V_{C}$ be the vol of the empty cone $V_{A}$ be the vol of the empty cone $\frac{V_{C}}{V_{A}}=\frac{480-60}{480}=\frac{7}{8} \quad \text { (ratio of vol of similar solids) }$ <br> Let $h_{C}$ be height of the empty cone, $h_{A}$ be height of the empty cone $\begin{aligned} & \frac{h_{C}}{h_{A}}=\sqrt[3]{\frac{7}{8}} \quad \text { (ratio of length of similar solids) } \\ & h_{C}=\frac{\sqrt[3]{7}}{2} h \text { or } \sqrt[3]{\frac{7}{8}} \mathrm{~h} \end{aligned}$ | M1 <br> A1 |
|  |  |  |

## 2018 WRSS Sec 3E EOY P2 Solutions

| 1a | $\begin{aligned} & \frac{x+9}{3} \geq \frac{5-x}{7} \\ & 7(x+9) \square 3(5-x) \\ & 7 x+63 \square 15-3 x \\ & 10 x \square-48 \\ & x \geq-4.8 /-4 \frac{4}{5} /-\frac{24}{5} \end{aligned}$ | M1 A1 |
| :---: | :---: | :---: |
| b | $\begin{aligned} & =\frac{15 a^{2}}{c^{2}} \times \frac{14 b c^{3}}{6 a} \\ & =\mathbf{3 5 a b} \boldsymbol{c} \end{aligned}$ | M1 A1 |
| ci | $\begin{aligned} s & =u t+\frac{1}{2} a t^{2} \\ & =1.5(4)+0.5(2)(4)^{2}=\mathbf{2 2} \end{aligned}$ | B1 |
| 1cii | $\begin{aligned} & \frac{1}{2} a t^{2}=s-u t \\ & a t^{2}=2(s-u t) \\ & a=\frac{2(s-u t)}{t^{2}} / \frac{2 s-2 u t}{t^{2}} /-\frac{2(u t-s)}{t^{2}} /-\frac{2 u t-2 s}{t^{2}} / \frac{2 s}{t^{2}}-\frac{2 u}{t} \end{aligned}$ | M1 A1 |


| 2a | $\begin{aligned} & \angle D A E=\angle M C E(\text { alternate } \angle \mathrm{s}, \mathrm{AD} / / \mathrm{BC}) \\ & \angle A D E=\angle C M E(\text { alternate } \angle \mathrm{s}, \mathrm{AD} / / \mathrm{BC}) \\ & \angle A E D=\angle C E M(\text { vert. opp. } \angle \mathrm{s}) \end{aligned}$ <br> By AA similarity test, $\triangle A D E$ is similar to $\triangle C M E$. <br> OR <br> $\triangle A D E$ is similar to $\triangle C M E$ because 3 pairs of corresponding angles are equal. <br> Note <br> A1 will not be awarded if AA similarity test or 3 pairs of corresponding angles are equal not stated. | M1 A1 |
| :---: | :---: | :---: |
| b | Since both $\Delta s$ are similar, hence the ratios of the corresponding sides are the same. $\begin{aligned} \frac{D E}{M E} & =\frac{A D}{M C} \\ \frac{D E}{4 c m} & =\frac{2}{1} \\ D E & =\mathbf{8} \mathbf{~ c m} \end{aligned}$ <br> (Note: $M$ is the mid-point of $B C$ ) | M1 A1 |
| ci | $\frac{\text { area of triangle } C M E}{\text { area of triangle } A D E}=\left(\frac{l_{1}}{l_{2}}\right)^{2}=\left(\frac{1}{2}\right)^{2}=\frac{1}{4}$ <br> Note <br> Wrong working: No mark awarded (e.g. assume the side or height of triangles to be of certain length. | B1 |
| 2cii | $\frac{\text { area of triangle } A D E}{\text { area of triangle } A D C}=\frac{\frac{1}{2}(\text { Base } \mathrm{AE})(\text { common ht })}{\frac{1}{2}(\text { Base } \mathrm{AC})(\text { common ht })}=\frac{2 \text { parts }}{3 \text { parts }}=\frac{2}{3}$ <br> Note <br> Wrong working: No mark awarded (e.g. assume the side or height of triangles to be of certain length. | B1 |


| 3a | $\angle \mathrm{ABC}=90^{\circ}(\angle \mathrm{s}$ in the semi-circle $)$ | B1 |
| :---: | :---: | :---: |
| b | $\begin{aligned} \angle \mathrm{BAC} & =180^{\circ}-90^{\circ}-58^{\circ} \quad(\text { Sum of } \angle \mathrm{s}, \Delta) \text { or }(\angle \mathrm{s} \text { sum of } \Delta) \\ & =32^{\circ} \end{aligned}$ | B1 |
| c | $\begin{aligned} & \angle \mathrm{OAB}=\angle \mathrm{OBA}=32^{\circ}(\text { base } \angle \mathrm{s}, \text { isosceles } \triangle) \\ & \angle \mathrm{CBD}=90^{\circ}-11^{\circ}-32^{\circ}=47^{\circ} \end{aligned}$ <br> Hence, $\angle \mathrm{CAD}=47^{\circ}$ ( $\angle \mathrm{s}$ in same segment) $\therefore \angle \mathrm{BAD}=32^{\circ}+47^{\circ}=79^{\circ}$ <br> Note <br> Missing important working: 1 m deducted (in red) | M1 <br> A1 |
| 3d | $\angle \mathrm{BCD}=180^{\circ}-79^{\circ}=101^{\circ}$ ( $\angle$ in opposite segment) <br> Hence, $\angle \mathrm{ACD}=101-58=43^{\circ} \quad$ or <br> $\angle \mathrm{ADC}=90^{\circ}$ ( $\angle \mathrm{s}$ in the semi-circle) <br> Hence, $\angle \mathrm{ACD}=180-90-47=43^{\circ}$ <br> Note <br> Missing important working: 1 m deducted (in red) | M1 <br> A1 <br> M1 <br> A1 |


| 4na | Figure $4=4 \times 5=\mathbf{2 0}$ <br> Figure $5=5 \times 6=\mathbf{3 0}$ <br> Note <br> Did not write " 20 " is for Figure 4 or Figure 5: 1 m deducted | B1 B1 |
| :---: | :---: | :---: |
| b | Sequence pattern is: <br> $\mathrm{Fig}_{1}: \quad 1^{2}+1=2$ <br> $\mathrm{Fig}_{2}: \quad 2^{2}+2=6$ <br> $\mathrm{Fig}_{3}: \quad 3^{2}+3=12 \ldots$ <br> Expression for the number of non-vertical lines in Figure $n=\boldsymbol{n}^{2}+\boldsymbol{n}$ | B1 |
| c | $\text { Number of non-vertical lines in figure } \begin{aligned} 56 & =56^{2}+56 \\ & =\mathbf{3 1 9 2} \end{aligned}$ | B1 |
| 4d | Method 1 <br> According to Hadi's claim, $\begin{aligned} n^{2}+n & =243 \\ n^{2}+n-243 & =0 \\ n & =\frac{-1 \pm \sqrt{973}}{2}=15.0964 \ldots \end{aligned}$ <br> n represents the Figure Number and thus, it must be a positive integer <br> Since $n$ is not a positive integer, Hadi's claim is wrong as there is no figure with 243 non-vertical lines. <br> Note: <br> One colour one mark. Student can either solve or state to prove that $n$ is not a positive integer. <br> Method 2 <br> Number of non-vertical lines $\begin{aligned} & =n^{2}+n \\ & =n(n+1) \end{aligned}$ <br> 4 represents the Figure Number and thus, it must be a positive integer <br> Since $n(n+1)$ represents the product of two consecutive integers and it will always result in an even integen, Hadi's claim is wrong as there is no figure with 243 nonvertical lines as it is an odd integer. <br> Note: <br> One colour one mark. <br> If students only state that number of non-vertical line must be an even integer: Award 1m only <br> Method 3 <br> Fig $15=240$ <br> Fig 16=272 <br> Since $n$ represents the Figure Number and it must be a positive integer, there is nd figure with 243 non-vertical lines. |  |


| 5ai | $\frac{1.5 \%}{100 \%} \times \$ 26000, \quad \therefore \text { commission }=\$ 390$ | B1 |
| :---: | :---: | :---: |
| aii | $\begin{aligned} 100 \% & =\$ 26000 \\ 112 \% & =\frac{112}{100} \times 26000 \\ & =\$ 29120(\mathrm{Feb}) \\ 100 \% & =\$ 29120 \\ 75 \% & =\frac{75}{100} \times 29120 \\ & =\$ 21840(\text { in March }) \end{aligned}$ | M1 <br> A1 |
| 5b | $\begin{aligned} \text { S\$ paid } & =\frac{1.37}{1} \times \$ 23.99 \\ & =32.8663 \approx \mathbf{\$ 3 2 . 8 7} \end{aligned}$ <br> Note: <br> No d.p: -1m | M1 <br> A1 |


| 6a | $\$\left(\frac{25}{x}\right)=$ cost of 1 kg of Aluminium | B1 |
| :---: | :---: | :---: |
| b | \$( $\left.\frac{25}{x-1}\right)$ | B1 |
| c | $\begin{aligned} 10\left(\frac{25}{x}\right)+5\left(\frac{25}{x-1}\right) & =55 \\ 250(x-1)+125 x & =55 x(x-1) \\ 55 x^{2}-430 x+250 & =0 \\ 11 x^{2}-86 x+50 & =0 \text { (Shown) } \end{aligned}$ | M1 <br> M1 <br> A1 |
| d | $\begin{aligned} & a=11, b=-86, c=50 \\ & x=\frac{-(-86) \pm \sqrt{(-86)^{2}-4(11)(50)}}{2(11)} \\ & x=\frac{86+\sqrt{5196}}{22} \text { or } \frac{86-\sqrt{5196}}{22} \\ & x=7.1856 \ldots \quad \text { or } \quad x=0.63257 \ldots \\ & x=7.19 \quad \text { or } \quad x=0.633 \quad \ldots \ldots \ldots .(3 \text { sig. figs). } \end{aligned}$ | M1 $\mathrm{A} 1, \mathrm{~A} 1$ |
| 6e | $x=0.633$ is rejected as the term $\left(\frac{25}{x-1}\right)$ needs to be positive ( 2 conditions). Or, $\mathrm{x}>1$ for the term $\left(\frac{25}{x-1}\right)$. | B1 |
| 6f | Cost of 1 kg of $\mathrm{Al}=\left(\frac{25}{7.1856 \ldots}\right)=3.4791 \ldots \approx \$ 3.48$ ( 3 sig. figs) | B1 |


| 7 a | $\angle A C B=90^{\circ}-055^{\circ}=35^{\circ}$ <br> Cosine Rule: $\begin{aligned} (A B)^{2} & =515^{2}+584^{2}-2(515)(584) \cos 35^{\circ} \\ & =113544.6623 \ldots \\ A B & =336.9638 \ldots \approx \mathbf{3 3 7} \mathbf{~ m}(\mathbf{3} \text { sig. figs }) \end{aligned}$ | B1 <br> M1 <br> A1 |
| :---: | :---: | :---: |
| b | $\begin{aligned} & \frac{\sin \angle A B C}{515}=\frac{\sin 35}{336.9638} \quad:(\text { Sine Rule }) \\ & \angle A B C=\sin ^{-1}\left(\frac{515 \times \sin 35}{336.9638}\right)=\sin ^{-1}(0.87662 \ldots)=61.2382 \ldots \approx 61.2^{\circ} \end{aligned}$ $\text { Hence, bearing of } \begin{aligned} A \text { from } B & =180^{\circ}+90^{\circ}+61.2^{\circ} \\ & \approx \mathbf{3 3 1 . 2 ^ { \circ }}(1 \text { dec. place }) \end{aligned}$ | M1 <br> M1 <br> A1 |
| c | Let $x$ be the shortest distance from $A$ to $A C$. $\begin{aligned} \sin 35 & =\frac{x}{515} \quad \text { or } \quad \frac{1}{2}(584)(x)=\frac{1}{2}(515)(584) \sin 35 \\ x & =\sin 35^{\circ} \times 515=295.3918 \ldots \approx 295 \mathrm{~m}(3 \text { sig. figs }) \end{aligned}$ | M1 <br> A1 |
| 7d | $\begin{aligned} \tan 25^{\circ} & =\frac{A T}{515} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ A T & =\tan 25^{\circ} \times 515=240.1484 \ldots \approx 240 \mathrm{~m} \text { (3 sig. figs) } \end{aligned}$ | M1 <br> A1 |

\begin{tabular}{|c|c|c|}
\hline 8 a \& \[
\text { Length } \begin{aligned}
A B \& =\sqrt{(5-(-4))^{2}+\left(11-(-2)^{2}\right)} \\
\& =\sqrt{81+169}=\sqrt{250}=15.8113 \ldots \approx \mathbf{1 5 . 8} \text { unit (3 sig. figs) }
\end{aligned}
\] \& \begin{tabular}{l}
M1 \\
A1
\end{tabular} \\
\hline b \& \begin{tabular}{l}
Gradient \(A B=\frac{11-(-2)}{5-(-4)}=\frac{13}{9}\) or \(1 \frac{4}{9}\) \\
Substitute \((5,11)\) into: \(\quad(11)=\left(\frac{13}{9}\right)(5)+c \Rightarrow c=\frac{34}{9}\) \\
Equation \(A B\) is:
\[
y=\frac{13}{9} x+\frac{34}{9} \text { or } 9 y=13 x+\mathbf{3 4}
\]
\end{tabular} \& M1

A1 <br>

\hline 8ci \& | line $p: 9 y=13 x+9$ (Given) |
| :--- |
| Hence, $y=\frac{13}{9} x+1$ |
| Since line $p$ and line $A B$ shared the same gradient (13/9), both lines are parallel. And parallel lines do not intersect. (all 3 conditions stated) |
| Or substitute line $p$ into line $A B$, to get an equation of no real solutions, thus no intersection points between the 2 lines. | \& M1

A1 <br>
\hline
\end{tabular}

| 8cii | line $p: 9 y=13 x+9 \Rightarrow y=\frac{13}{9} x+1$, substitute into line $q: 3 y-5 x=6$ $\begin{aligned} 3\left(\frac{13}{9} x+1\right) & =5 x+6 \\ \frac{13 x-15 x}{3} & =3 \\ -2 x & =9 \quad \ldots \ldots \ldots \ldots \ldots \ldots \end{aligned}$ <br> Hence, $y$ coordinate $\quad \Rightarrow y=\frac{13}{9}(-4.5)+1=-5.5$ <br> Coordinate is: $(-4.5,-5.5)$ |
| :---: | :---: |


| 9a | $p=10.03(2 \mathrm{dec} . \mathrm{pls}) \text { or } 10.0(3 \mathrm{sig} . \text { figs }) \text { or } 10 \frac{1}{30}$ | B1 |
| :---: | :---: | :---: |
| b | Accurately plotted coordinates (at least 3 pairs) $\qquad$ see graph <br> Correct scales \& ranges for $x$ and $y$ axes + label of curve \& axes <br> Smoothness of curve | B1 <br> B1 <br> B1 |
| c | $\frac{x^{3}}{30}+\frac{20}{x}=15$ is the intersection points between the graph, $y=\frac{x^{3}}{30}+\frac{20}{x}-10$ and $y=5$. Intersection point are: $\boldsymbol{x}=\mathbf{1 . 3 0} \pm \mathbf{0 . 2}$ | B1:draw $y=5$ <br> B1 |
| d | Draw tangent at (2, 0.27). Suggest a new plotting point of $(1.5,3.4)$ Grad. $\mathrm{T}=\frac{0.27-5}{2-1}=-4.73$ <br> $-4.90 \leq$ accepted gradient range $\leq-4.30 \quad$ (Note: calculated gradient $=-4.60$ ) | B1 B1 |
| ei | Draw the line $y+2 x=10$ | B1 |
| 9eii | Intersection points are: $\boldsymbol{x}=\mathbf{1 . 1 5} \pm \mathbf{0 . 2}$ and $\boldsymbol{x}=\mathbf{5 . 4 5} \pm \mathbf{0 . 2}$ | B1,B1 |


| 10a | $\begin{aligned} \text { Vol. of conical cup } & =\frac{1}{3} \pi(3)^{2}(8) \\ & =75.3982 \ldots \\ & \approx \mathbf{7 5} \mathbf{c m}^{3} \quad \text { (nearest whole number) } \end{aligned}$ | M1 A1 |
| :---: | :---: | :---: |
| bi | $\begin{aligned} \text { Vol. of cylinder tank } & =\pi(25)^{2}(60) \\ & =117809.7245 \ldots \\ & \approx \mathbf{1 2 0} 000 \mathbf{c m}^{\mathbf{3}} \quad(\mathbf{2} \text { sig. figs }) \end{aligned}$ | M1 $\mathrm{A} 1$ |

Note: $1 \mathrm{~m}^{3}=1000000 \mathrm{~cm}^{3}, 0.00001 \mathrm{~m}^{3}=1 \mathrm{~cm}^{3}, \quad 1 \mathrm{ml}=1 \mathrm{~cm}^{3}$

| bii | No. of tanks needed $\begin{aligned} & =10000 \text { litre } \times 1000 \mathrm{~cm}^{3} \\ & =\frac{10^{7}}{120000}=83.333 \ldots \approx 84 \text { tanks } \end{aligned}$ <br> No. of cups needed $\frac{10^{7}}{75}=133333.333 \ldots \approx 133334 \text { cups }$ <br> No. of $\underline{250 \text { packs_required }=\frac{133334}{250}=533.336=+\quad \text { ren }}$ 533 or 534 packs <br> No. of 50 packs required $=2$ packs <br> (if 533 packs) $\begin{aligned} \text { Cost of water tanks } & =\$ 15 \times 84 \\ & =\$ 1260 \end{aligned}$ $\begin{aligned} \text { Cost of cups } & =(\$ 10 \times 533)+(\$ 2.50 \times 2) \\ & =\$ 5335 \end{aligned}$ $\begin{array}{ll} \text { Total budget } & =\$ 1260+5335 \\ (84 \text { tanks }) & =\$ 6595 \end{array}$ <br> or <br> (for 85 tanks) $\quad=\underline{\$ 610}$ <br> or $(\$ 10 \times 534$ packs $)+(\$ 15 \times 85)=\underline{\$ 6600}$ | or $\begin{aligned} & \frac{10^{7}}{117809.7245 \ldots}=84.8826 \ldots \\ & (\approx 85 \text { tanks }) \\ & \text { or } \frac{10^{7}}{75.3982 \ldots}=132629.1609 . . \\ & \begin{array}{c} (\approx 132630 \text { cups }) \\ \frac{132630}{250}=530.52 \\ \quad=(531 \text { packs }) \end{array} \end{aligned}$ <br> (3 packs if 530 packs) $\$ 15 \times 85=\$ 1275$ $\begin{aligned} & (\$ 10 \times 530)+(\$ 2.50 \times 3) \\ & =\$ 5307.50 \\ & =\$ 1260+5307.5 \\ & =\$ 6567.50 \end{aligned}$ $=\underline{\text { or } \$ 6582.50}$ | Find vol M1 <br> M1 <br> Find tanks <br> M1 <br> A1 |
| :---: | :---: | :---: | :---: |

## Namp (2018) 3E EOY MaM P2

Subreck Q9 (Grop)
Class
Date


