



KRANJI SECONDARY SCHOOL

**3EXP
3R1**

END-OF-YEAR EXAMINATION 2018

**MATHEMATICS 4048
PAPER 1**

Level : Secondary Three

Date : 3 Oct 2018

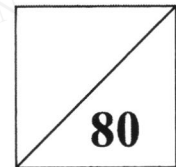
Stream : Express

Duration : 2 hours

Name : _____ ()

Marks :

Class : Secondary _____



READ THESE INSTRUCTIONS FIRST:

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 80.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give your answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

Set By: Ms Afnie

This question paper consists of 16 printed pages including the cover page.

[Turn over]

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

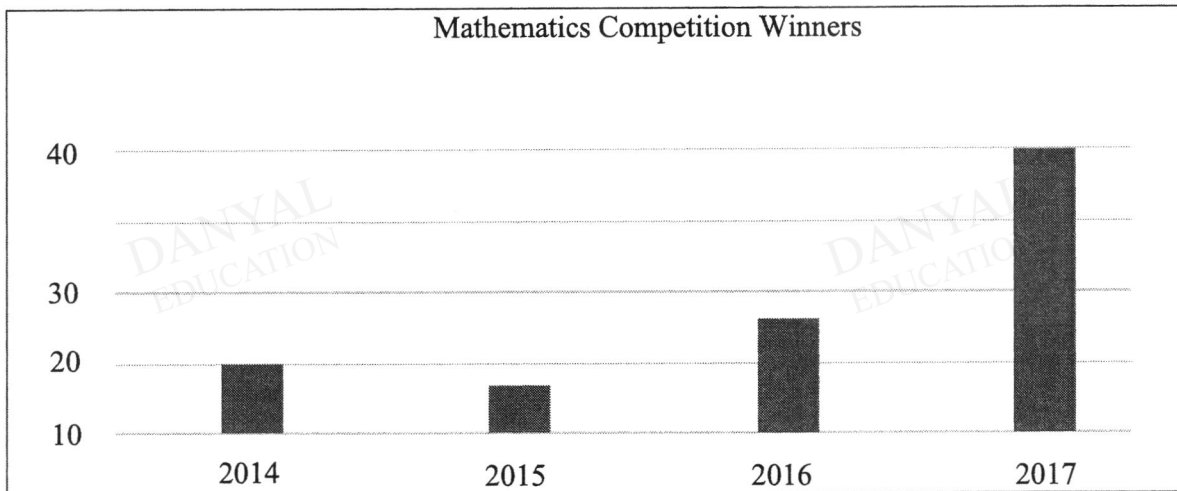
- 1 (a) Express $\frac{3}{16}$ as a percentage.

Answer % [1]

- (b) Express 108.2% as a fraction in its simplest form.

Answer [1]

- 2 The graph shows the number of winners for a mathematics competition over a number of years.



Explain one way in which the graph is misleading.

Answer

.....

.....

.....

[1]

3 Given that $x + 5y = 6$ and $x^2 - 25y^2 = 42$, find the value of $x - 5y$.

Answer [2]

4 A factory employs 12 workers to assemble laptops. Typically, the 12 workers can assemble 210 laptops in 3 hours. Assuming that all workers work at the same rate, how many more workers are needed in order to assemble 280 laptops in 30 minutes?

Answer workers [2]

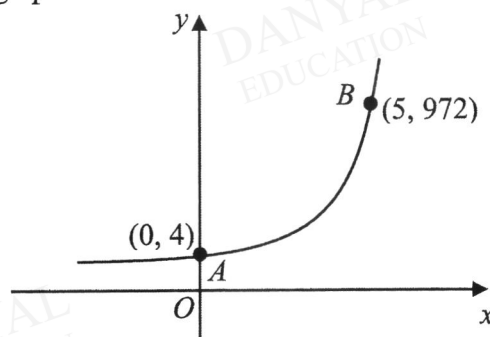
5 Factorise completely $2ab - 5c^2 + ac - 10bc$.

Answer [2]

- 6 The sine of an angle is 0.624.
Give two possible values for the angle in degrees.

Answer ° or ° [2]

- 7 The sketch shows the graph of $y = ka^x$.
The points $A(0, 4)$ and $B(5, 972)$ lies on the graph.



Find the values of k and a .

Answer $k =$ [1]

$a =$ [2]

- 8 Three bus services, A , B and C leave from the same bus interchange at different intervals. Services A , B and C leaves the bus interchange once every 6 minutes, 15 minutes and 18 minutes respectively. If the buses first leave together at 8.12 am, when will the buses leave together again?

Answer [3]

- 9 Solve $2^{3x} \times 16 = 4^5$.

Answer $x =$ [3]

- 10 Given that $\frac{2a-b}{7a+3b} = \frac{3}{5}$, find the exact value of $\frac{b}{a}$.

Answer [3]

11 The first four terms of a sequence is 1 , 6 , 11 , 16.

(a) Write down and simplify an expression for the n^{th} term.

Answer $T_n = \dots\dots\dots$ [1]

(b) Explain why 198 is not a term of this sequence.

Answer
.....
..... [2]

12 The table below shows the number of Lower Secondary students who have represented the school in a basketball competition.

	Boys	Girls
Secondary 1	5	3
Secondary 2	7	9

(a) A student is chosen at random. Find, as a fraction in its lowest terms, the probability that the student is a Secondary 1 boy,

Answer [1]

(b) Two students are chosen at random. Find, as a fraction in its lowest terms, the probability that

(i) they are both from Secondary 1,

Answer [1]

(ii) neither of them is a Secondary 1 boy.

Answer [2]

13 A line l is represented by the equation $3x - 4y = 18$. It crosses the x -axis at the point A and the y -axis at the point B .

(a) Find the coordinates of A and B .

Answer $A = (\dots\dots\dots, \dots\dots\dots)$ [1]

$B = (\dots\dots\dots, \dots\dots\dots)$ [1]

(b) Calculate the length of the line joining A and B .

Answer units [2]

14 Simplify the following expressions.

(a) $2x^3 \div \left(\frac{3}{4x}\right)^{-1}$.

Answer [2]

(b) $\sqrt[5]{y^{20}} \div y$.

Answer [2]

- 15 (a) Jenny solves the equation $(2x-1)^2 = (5x+2)^2$ as shown below.

$$\begin{aligned}(2x-1)^2 &= (5x+2)^2 \\ 2x-1 &= 5x+2 && \text{[Take square roots on both sides]} \\ x &= -1\end{aligned}$$

Do you agree with Jenny's solution? Explain your answer.

Answer

.....

.....

..... [2]

- (b) It is given that $x=2$ and $x=-7$ are the roots of a quadratic equation $ax^2 + bx + c = 0$, where a, b and c are integers. Find the quadratic equation.

Answer [2]

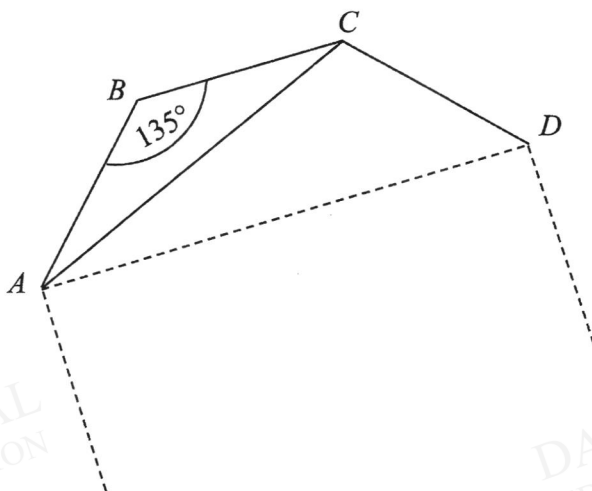
-
- 16 Consider the following numbers:

$$1\frac{1}{5}, \pi, \sqrt{3}, 0, -2, 1.\dot{3}$$

- (a) Which of the above number(s) is/are
- (i) integers, Answer [1]
- (ii) rational numbers, Answer [1]
- (iii) irrational numbers. Answer [1]
- (b) Arrange the above numbers in descending order.

Answer , , , , [2]

- 17 The diagram below shows a part of a regular polygon with n sides. Each interior angle of this polygon is 135° .



Find

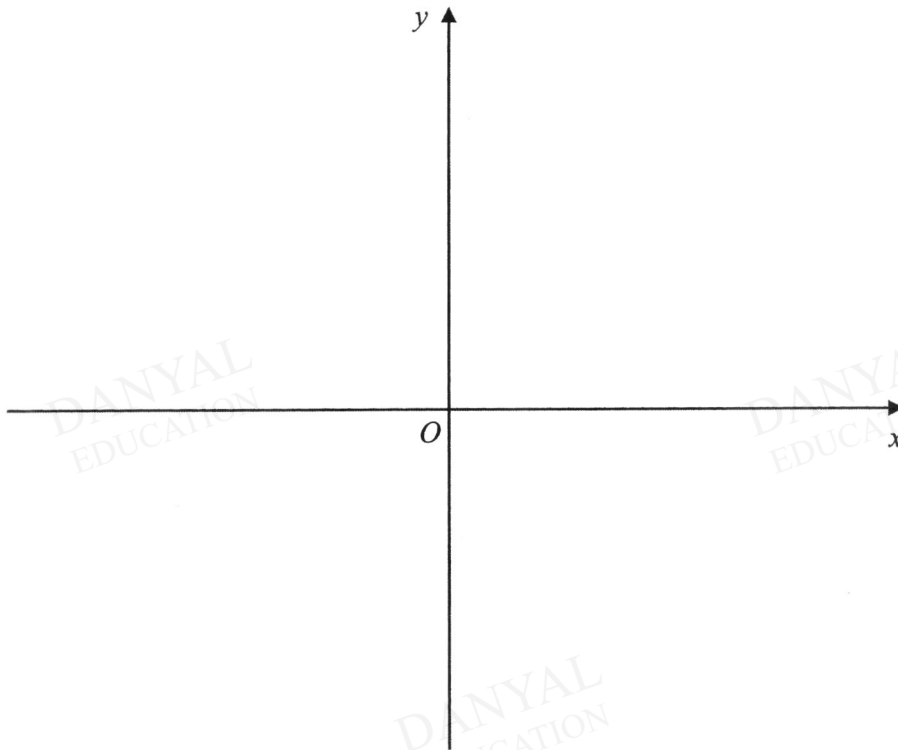
- (a) the value of n ,

Answer $n = \dots\dots\dots$ [2]

- (b) angle ACD .

Answer $\dots\dots\dots^\circ$ [2]

- 18 (a) Sketch the graph of $y = (x-2)(x+1)$ on the axes below. Indicate clearly the values where the graph crosses the x - and y - axes.



[2]

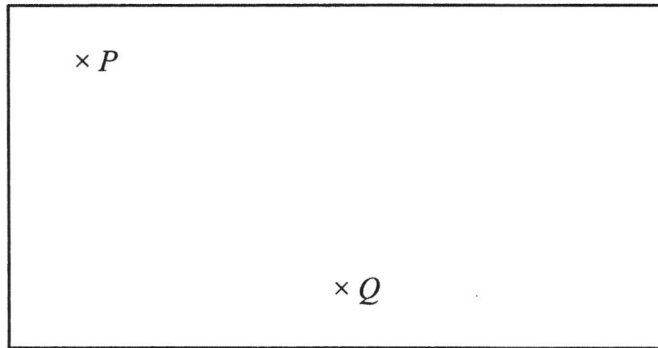
- (b) Write down the equation of the line of symmetry of the graph.

Answer [1]

- (c) Find the coordinates of the turning point.

Answer (..... ,) [1]

- 19 The map shows the actual position of two bus stops P and Q .
The scale of the map is 1 : 40 000.



- (a) Measure the distance PQ on the map.

Answer cm [1]

- (b) Hence, find the actual distance between P and Q , giving your answer in kilometres.

Answer km [1]

- (c) A new build-to-order estate is to be drawn on the map of the same scale. The area covered by this estate is 40 000 m². Calculate the area of the estate on the map, giving your answer in cm².

Answer cm² [2]

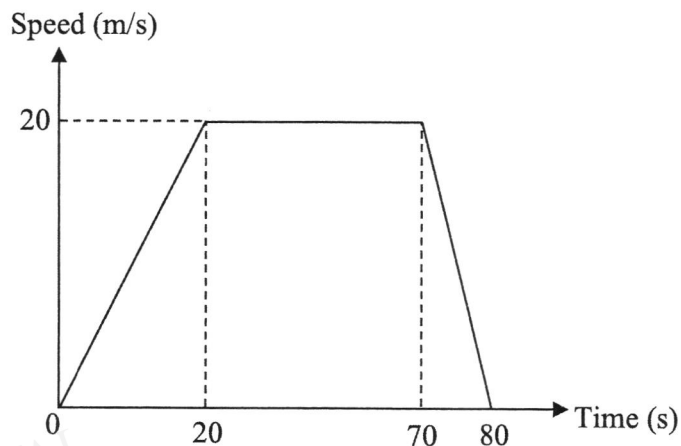
20 (a) Express $x^2 - 7x + 2$ in the form $(x+a)^2 + b$.

Answer [2]

(b) **Hence**, solve the equation $x^2 - 7x + 2 = 0$, giving your answers correct to two decimal places.

Answer $x =$ or $x =$ [3]

21 The diagram shows the speed-time graph for a train journey



(a) Calculate the deceleration of the train for the last 10 seconds of the journey.

Answer m/s^2 [1]

(b) Calculate the total distance travelled on the journey.

Answer m [2]

(c) The maximum speed of the train was 20 m/s.
Change 20 m/s into km/h.

Answer km/h [2]

- 22 The ages of 12 employees in the technical department of a company are shown in the stem-and-leaf diagram below.

2	1 3 6 7
3	0 2 3 5
4	1 9
5	3
6	
7	
8	1

Key: 2 | 1 means 21 years

- (a) Calculate the mean age.

Answer [1]

- (b) Calculate the median age.

Answer [1]

- (c) Why would the median be a better measure of central tendency of the ages of these employees?

Answer [1]

.....

- (d) Calculate the standard deviation of the ages.

Answer [1]

- (e) The standard deviation of the ages of employees in the sales department of the company is 5.74 years. What does this tell you about the ages of the employees in the sales department?

Answer [1]

.....

- 23 Points A , B and C are three checkpoints on flat ground. Points A and B are shown in the diagram below. Point C is located 18 km away from Point A , at a bearing of 052° .



- (a) Using a scale of 1 cm to represent 3 km, construct triangle ABC and label the position of checkpoint C clearly. [2]
- (b) Construct the perpendicular bisector of line AB . [1]
- (c) Construct the bisector of angle ABC . [1]
- (d) Another checkpoint D is equidistant from points A and B and equidistant from lines AB and BC . Indicate the position of checkpoint D and find the distance between checkpoints B and D .

Answer km [2]

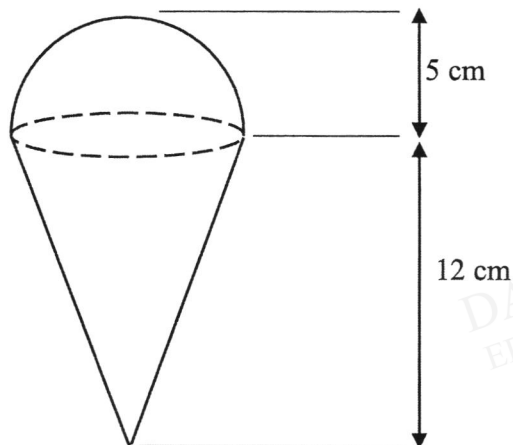
Answer all the questions.

- 1** Mr Gan plans to go on a holiday in Australia and intends to exchange 2000 Singapore Dollars (SGD) for Australian Dollars (AUD) at a money changer. Some of the exchange rates offered by the money changer are shown below.

	Selling	Buying
Australian Dollar (AUD)	1.025	1.011
Euro (EUR)	1.571	1.558

- (a) Calculate the amount of Australian Dollars he gets, correct to the nearest dollar. [1]
- (b) While packing, his wife finds 500 Euros (EUR) from a previous trip. Mr Gan decides to go back to the same money changer to exchange this 500 EUR for Australian Dollars on the same day. Calculate the amount of Australian Dollars he gets. [2]
- (c) After returning from Australia, Mr Gan finds that he has 400 AUD that has not been spent. He intends to exchange it for Singapore Dollars at the same money changer. Based on the table above, he calculates that he will receive 404.40 SGD. Why is his calculation probably inaccurate? [1]

- 2 An ice-cream vendor sells ice-cream in right circular cones. The vendor fills the entire cone with ice-cream and tops it off with ice-cream forming a hemisphere on top of the cone. The radius of the hemisphere is 5 cm and the perpendicular height of the cone is 12 cm as shown in the diagram below.



- (a) Calculate
- (i) the volume of ice-cream sold with each cone, [2]
 - (ii) the surface area of the cone in contact with the ice-cream. [3]
- (b) The vendor decides to offer an “upsized” option to customers where ice-cream is sold in a geometrically similar cone that has a perpendicular height of 18 cm. Calculate the percentage increase in volume of ice-cream sold in each “upsized” cone. [3]

- 3** A helicopter makes a trip to a destination 500 km away. The average speed of the helicopter is x km/h.
- (a) Write down an expression, in terms of x , for the time taken for the helicopter to reach its destination in hours. [1]
- (b) On the return trip, the helicopter reduced its average speed by 25 km/h. The return trip took 15 minutes longer than the outbound trip.
- (i) Write down an expression, in terms of x , for the time taken by the helicopter to make the return trip in hours. [1]
- (ii) Form an equation in x and show that it can be reduced to $x^2 - 25x - 50000 = 0$. [3]
- (iii) Solve the equation $x^2 - 25x - 50000 = 0$, giving both answers correct to 2 decimal places. [3]
- (iv) Calculate the time taken, in hours and minutes, for the helicopter to complete only the return trip. Give your answer correct to the nearest minute. [2]
- 4** (a) James needs a loan of \$90 000 to buy a new car.
Bank P charges a simple interest rate of 2.45% per annum.
Bank Q charges an interest rate of 2.22% per annum compounded monthly.
Which bank should he borrow from if he takes a 5-year loan?
Justify your answer. [4]
- (b) The sum of the digits of a two-digit number is 11.
The ten's digit is x and the one's digit is y .
If the digits are reversed, the new number is 5 more than three times the original number.
By forming two equations, solve the equations and find the original number. [5]

5 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = x^3 - 2x^2 - 3x$.

Some corresponding values of x and y are given in the following table.

x	0	0.5	1	1.5	2	2.5	3
y	0	-1.87	-4	-5.62	p	-4.37	0

(a) Find the value of p . [1]

(b) Using a scale of 4 cm to represent 1 unit, draw a horizontal x -axis for $0 \leq x \leq 3$.

Using a scale of 2 cm to represent 1 unit, draw a vertical axis for $0 \leq y \leq -6$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Find the values of x when $y = -3$. [1]

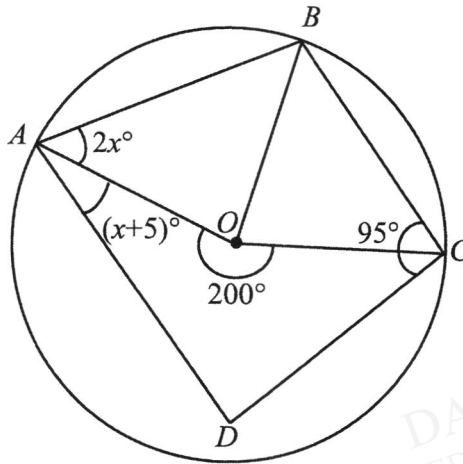
(d) By drawing a tangent, estimate the gradient of the curve at $x = 2.5$. [2]

(e) (i) On the same axes, draw the line $y = x - 5$ for $0 \leq x \leq 3$. [1]

(ii) Write down the x -coordinates of the points where the line intersects the curve. [1]

- 6 (a) It is given that $T = 2\pi\sqrt{\frac{L}{g}}$.
- (i) Find T when $L = 3.12$ and $g = 9.81$. [1]
- (ii) Express L in terms of T and g . [2]
- (b) (i) Solve the inequalities $2x - 5 < x + 2 \leq 3x$. [2]
- (ii) Hence, write down all the possible odd integers of x . [1]
- (c) Simplify fully
- (i) $\frac{x^2 - 49}{x + 7}$, [2]
- (ii) $\frac{5x}{3} - \frac{2(7-x)}{5}$. [2]

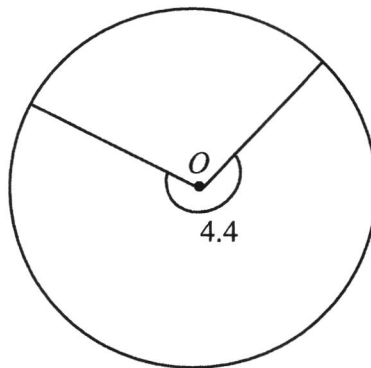
7 (a)



A , B , and C are points on the circle centre O .
 Angle $BAO = 2x^\circ$, angle $OAD = (x + 5)^\circ$, angle $BCD = 95^\circ$ and
 reflex angle $AOC = 200^\circ$. Calculate

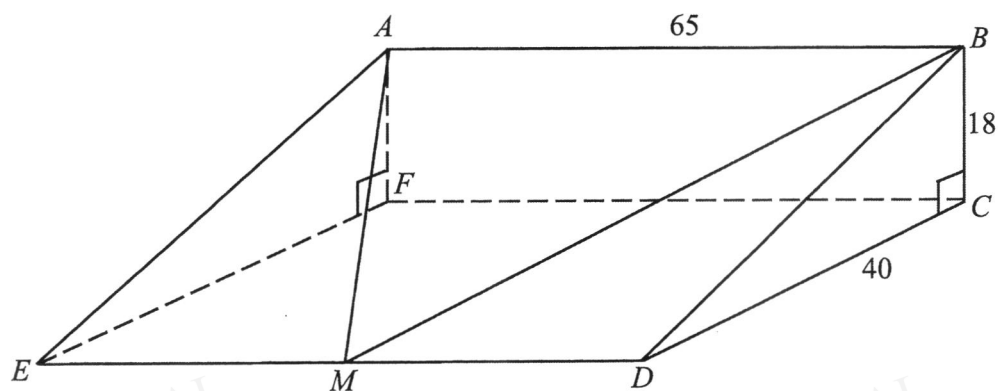
- (i) the value of x , [3]
- (ii) the angle ADC . [2]

(b)



A wheel of a car is represented by the circle above, centre O . For the car to move a distance of 100 cm, the wheel must rotate through an angle of 4.4 radians. Calculate

- (i) the radius of the wheel, [2]
- (ii) the number of revolutions for the car to travel 10 km, giving your answer correct to the nearest whole number. [2]



The diagram shows a triangular prism $ABCDEF$.

Angle $AFE = \text{angle } BCD = 90^\circ$.

$AB = 65$ cm, $BC = 18$ cm and $DC = 40$ cm.

The point M is the mid-point of ED .

- (a) Calculate the length MB . [3]
- (b) Show that angle $AMB = 73.1^\circ$, correct to 1 decimal place. [3]
- (c) Calculate the area of triangle AMB . [2]
- (d) Calculate the volume of the prism. [2]
- (e) Calculate the angle of elevation of B from M . [2]

- 9 The escape velocity is the minimum velocity or speed required to leave a planet. This is important because if the velocity of a spacecraft is less than the escape velocity, it will eventually fall back into the planet and crash. The escape velocity, in m/s, is calculated using the formula

$$\text{escape velocity} = \sqrt{\frac{2GM}{r}}$$

where

$$G = 6.672 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2},$$

M is the mass of the planet in kg and

r is the radius of the planet in m.

Some information on Earth and Mars is given in Table A.

Table A

Property	Earth	Mars
Mass	5.972×10^{24} kg	6.417×10^{23} kg
Radius	6378 km	3390 km
Distance apart (farthest)	378 million km	
Distance apart (nearest)	56 million km	

The Earth rotates from west to east. The velocity of the surface of the Earth varies according to where the velocity is measured. The surface velocity of the Earth, in m/s, is calculated using the formula

$$\text{surface velocity} = \omega r$$

where

$$\omega = 7.292 \times 10^{-5} \text{ rad/s and}$$

r is the effective radius in m.

The effective radius at various locations on Earth are given in Table B.

Table B

Location	Effective radius
Guiana Space Centre	6354 km
Cape Canaveral	5160 km
Washington, D.C	4964 km

[This question continues on the next page]

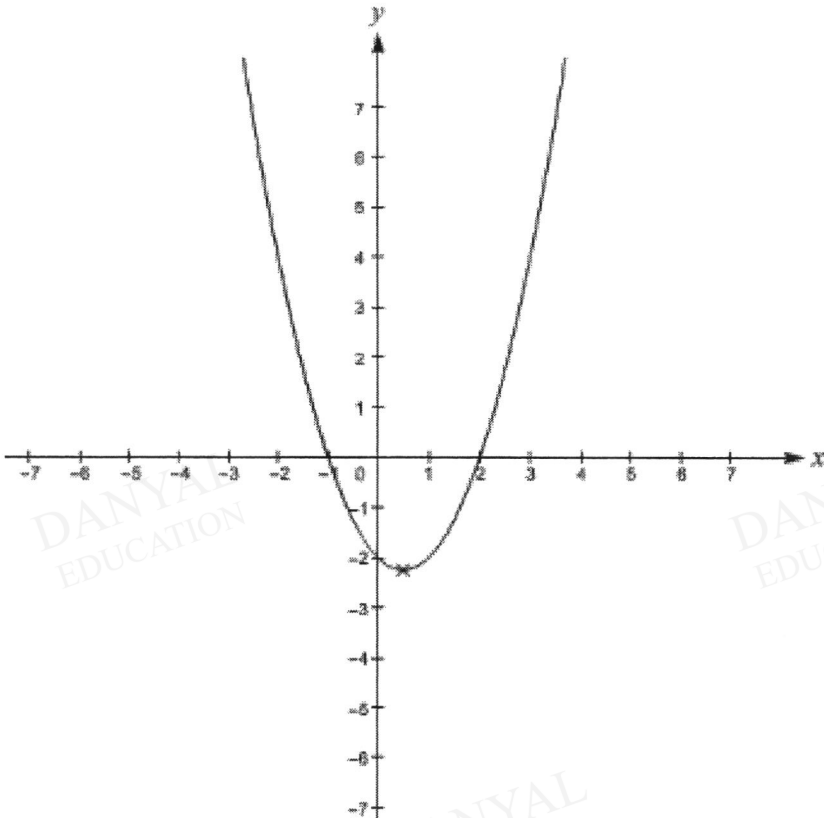
- (a) Calculate the escape velocity from Earth in m/s. [2]
- (b) A spacecraft launched in the direction of the Earth's rotation takes advantage of the surface velocity of the Earth.
- (i) From Table *B*, which is the best location to launch a spacecraft from? Give a reason for your answer based on information from Table *B*. [1]
- (ii) Calculate the surface velocity, in m/s, of the location you have chosen, to the nearest whole number. [2]
- (iii) Hence, calculate the velocity the spacecraft requires in order to achieve escape velocity from Earth. [1]
- (c) In the quest to send humans to Mars, one of the challenges that will arise is the time delay in radio communications. Radio signals travel at the speed of light. However, with the enormous distance between Earth and Mars, an astronaut on Mars contacting the space centre on Earth for help will have to wait a long while before getting a response.

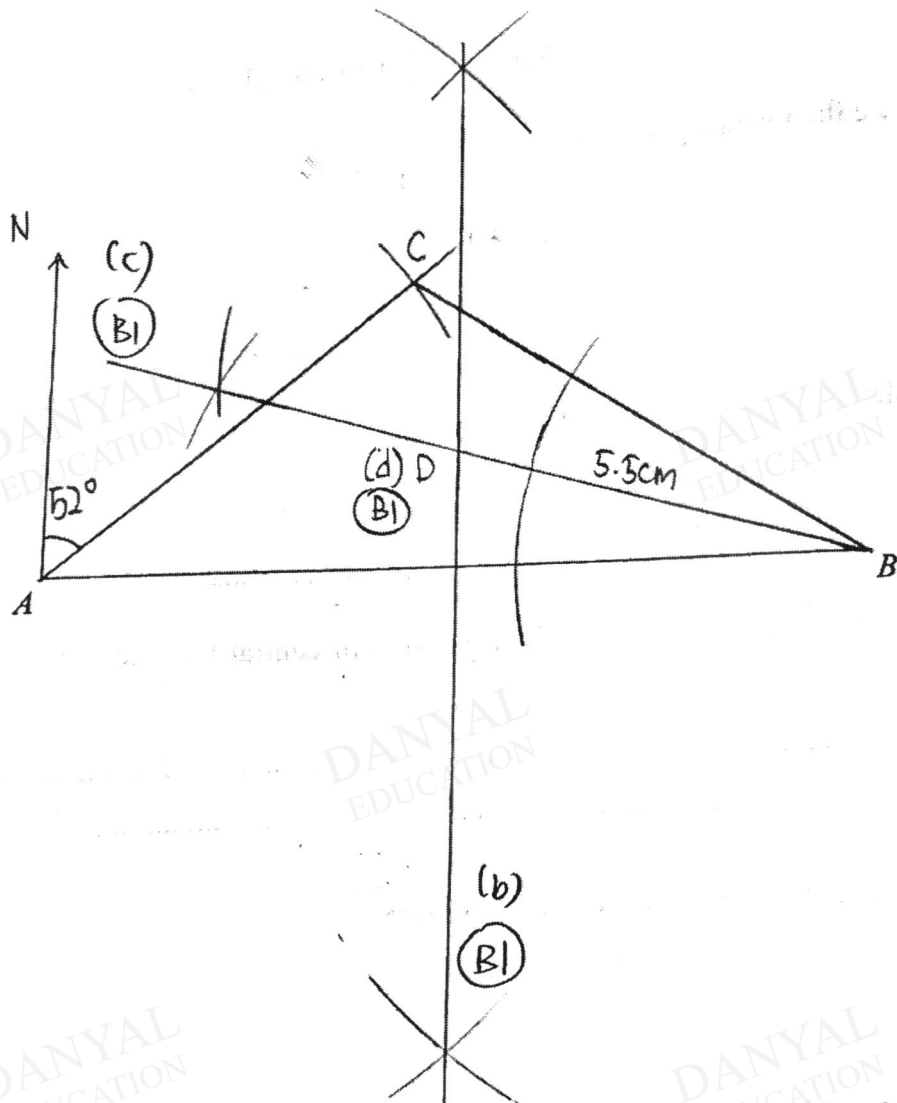
Calculate the shortest possible time that the astronaut on Mars will receive the response from the space centre on Earth after he has sent his signal to Earth. Give your answer in minutes. The speed of light is 2.9979×10^8 m/s. [3]

End of Paper

3EXP EOY P1 2018 Answers

1a	18.75%
1b	$1\frac{41}{500}$
2	The intervals on the vertical axis is not equally spaced leading to the conclusion that in 2017 there is a huge increase in number of winners.
3	7
4	84
5	$(2b+c)(a-5c)$
6	38.6° or 141.4°
7	$k=4, a=3$
8	9.42 am
9	2
10	$\frac{11}{14}$
11a	$5n-4$
11b	$n=40.4, n$ is not an integer
12a	$\frac{5}{24}$
12bi	$\frac{7}{69}$
12bii	$\frac{57}{92}$
13a	A (6, 0) B (0, -4.5)
13b	7.5
14a	$\frac{3x^2}{2}$
14b	y^3
15a	No. When square roots on both side, should have two possible solutions. Jenny missed out 1 solution.
15b	$x^2+5x-14=0$
16ai	0, -2
16aaii	$1\frac{1}{5}, 0, -2, 1.\dot{3}$
16aiii	$\pi, \sqrt{3}$
16b	$\pi, \sqrt{3}, 1.\dot{3}, 1\frac{1}{5}, 0, -2$
17a	8
17b	112.5°

18a	
18b	$x = 0.5$
18c	$(0.5, -2.25)$
19a	4.8 cm
19b	1.92 km
19c	0.25 cm^2
20a	$\left(x - 3\frac{1}{2}\right)^2 - 10\frac{1}{4}$
20b	0.30 or 6.70
21a	2 m/s^2
21b	1300 m
21c	72 km/h
22a	37.6
22b	32.5
22c	There is an outlier of age 81 years, which is 18 years older than the next oldest person, which causes the mean age to be raised significantly.
22d	16.1
22e	The difference in ages in the sales department is smaller due to the smaller spread in the age distribution of the department.



- (a) Using a scale of 1 cm to represent 3 km, construct triangle ABC and label the position of checkpoint C clearly. B2 for correct construction [2]
- (b) Construct the perpendicular bisector of line AB. [1]
- (c) Construct the bisector of angle ABC. [1]
- (d) Another checkpoint D is equidistant from points A and B and equidistant from lines AB and BC. Indicate the position of checkpoint D and find the distance between checkpoints B and D. Answer 6.5 (B1) km [2]

1 (a)

$$1.025 \text{ SGD} = 1 \text{ AUD}$$

$$2000 \text{ SGD} = \frac{1}{1.025} \times 2000 \text{ AUD}$$

$$2000 \text{ SGD} = 1951.219512 \text{ AUD}$$

He gets 1951 AUD (nearest dollar)

(b)

$$1 \text{ EUR} = 1.558 \text{ SGD}$$

$$500 \text{ EUR} = 1.558 \times 500 \text{ SGD}$$

$$500 \text{ EUR} = 779 \text{ SGD}$$

$$1.025 \text{ SGD} = 1 \text{ AUD}$$

$$779 \text{ SGD} = \frac{1}{1.025} \times 779 \text{ AUD}$$

$$779 \text{ SGD} = 760 \text{ AUD}$$

He gets 760 AUD

(c) The rate would have been different after he returned from Australia
Because exchange rates change on a daily basis.

2 (a) (i) Total volume of ice-cream

$$\begin{aligned} &= \left(\frac{1}{2} \times \frac{4}{3} \pi (5)^3 \right) + \left(\frac{1}{3} \pi (5)^2 (12) \right) \\ &= 575.9586532 = 576 \text{ cm}^3 \text{ (3sf)} \end{aligned}$$

(ii) slant length of cone = $\sqrt{5^2 + 12^2} = 13 \text{ cm}$
surface area of cone in contact with ice-cream
= $\pi(5)(13) = 204.2035225 = 204 \text{ cm}^2 \text{ (3sf)}$

(b) ratio of lengths = $\frac{18}{12} = \frac{3}{2}$

ratio of volumes = $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$

volume of ice-cream in one "upsized" cone = $\frac{27}{8} \times \frac{550}{3} \pi = \frac{2475}{4} \pi \text{ cm}^3$

percentage increase in volume

$$= \frac{\left(\frac{2475}{4} \pi - \frac{550}{3} \pi \right)}{\left(\frac{550}{3} \pi \right)} \times 100 = \frac{19}{8} \times 100 = 237.5\%$$

3

(a)

$$\frac{500}{x} \text{ hours}$$

(b) (i)

$$\frac{500}{x-25} \text{ hours}$$

(ii)

$$\frac{500}{x-25} - \frac{500}{x} = \frac{15}{60}$$

$$500(x)(60) - 500(x-25)(60) = 15(x)(x-25)$$

$$30000x - 30000(x-25) = 15x^2 - 375x$$

$$15x^2 - 375x = 30000x - 30000x + 750000$$

$$15x^2 - 375x - 750000 = 0$$

$$x^2 - 25x - 50000 = 0$$

(iii)

$$x^2 - 25x - 50000 = 0$$

$$x = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(1)(-50000)}}{2(1)}$$

$$x = \frac{25 \pm \sqrt{200625}}{2}$$

$$x = 236.4559108 \text{ or } -211.4559108$$

$$x = 236.46 \text{ or } -211.46$$

(iv) time taken for return trip

$$= \frac{500}{x-25} \text{ h}$$

$$= \frac{500}{236.4559108 - 25} \text{ h}$$

$$= 2.364559109 \text{ h}$$

$$= 2 \text{ h } (0.364559109 \times 60) \text{ min}$$

$$= 2 \text{ h } 21.87354654 \text{ min}$$

$$= 2 \text{ h } 22 \text{ min (nearest min)}$$

4

(a)

$$\text{Bank } P = \frac{90000 \times 2.45 \times 5}{100}$$

$$= \$11\,025$$

$$\text{Bank } Q = 90000 \left[1 + \frac{\frac{2.22}{100}}{12} \right]^{12 \times 5} - 90000$$

$$= \$10\,555.2293$$

He should borrow from Bank Q because Bank Q incur lesser interest.

(b)

$$x + y = 11 \text{ -----Eqn 1}$$

$$10y + x = 3(10x + y) + 5 \text{ ----- Eqn 2}$$

$$\text{Eqn 1: } x = 11 - y$$

$$\text{Eqn 2: } 7y - 29x = 5$$

Solve for y :

$$7y - 29(11 - y) = 5$$

$$7y - 319 + 29y = 5$$

$$36y = 324$$

$$y = 9, x = 2$$

Original number = 29

5

(a)

$$p = (2)^3 - 2(2)^2 - 3(2)$$

$$p = -6$$

(c)

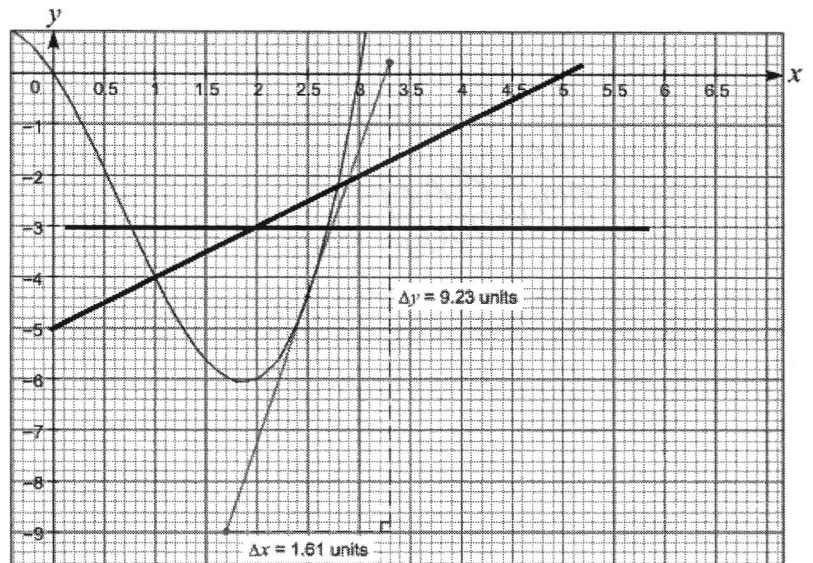
$$x = -0.76, x = 2.70$$

(d)

$$\text{Gradient} = 5.75$$

(e)

$$(ii) \quad x = 1, x = 2.79$$



6 (a) (i)

$$T = 2\pi\sqrt{\frac{3.12}{9.81}}$$

$$T = 3.543420241$$

$$T = 3.54 \text{ (3sf)}$$

(ii)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\sqrt{\frac{L}{g}} = \frac{T}{2\pi}$$

$$\frac{L}{g} = \left(\frac{T}{2\pi}\right)^2$$

$$L = g\left(\frac{T}{2\pi}\right)^2$$

(b) (i)

$$2x - 5 < x + 2 \leq 3x$$

$$2x - 5 < x + 2 \quad \text{and} \quad x + 2 \leq 3x \quad M1(\text{split})$$

$$2x - x < 2 + 5 \quad 2 \leq 2x$$

$$x < 7 \quad 1 \leq x$$

$$\therefore 1 \leq x < 7$$

(ii) Possible odd integers are 1, 3, 5

(c) (i)

$$\frac{x^2 - 49}{x + 7}$$

$$= \frac{(x-7)(x+7)}{x+7}$$

$$= x - 7$$

(ii)

$$\frac{5x}{3} - \frac{2(7-x)}{5}$$

$$= \frac{(5x)(5) - (3)(2)(7-x)}{15}$$

$$= \frac{25x - 42 + 6x}{15}$$

$$= \frac{31x - 42}{15}$$

$$\begin{aligned}\angle AOC &= 360^\circ - 200^\circ \\ &= 160^\circ\end{aligned}$$

$$\angle AOB = \angle BOC = \frac{160^\circ}{2} = 80^\circ$$

$$\angle OAB = \angle OBA = \frac{180^\circ - 80^\circ}{2} = 50^\circ$$

$$2x = 50$$

$$x = 25$$

(ii)

$$\angle OCB = 2x = 50^\circ$$

$$\angle OCD = 95^\circ - 50^\circ = 45^\circ$$

$$\angle OAD = x + 5 = 25 + 5 = 30^\circ$$

$$\angle ADC = 360^\circ - 45^\circ - 30^\circ - 200^\circ$$

$$\angle ADC = 85^\circ$$

(b) (i)

$$r(4.4) = 100$$

$$r = \frac{100}{4.4}$$

$$r = 22.72727273$$

$$r = 22.7 \text{ cm (3sf)}$$

$$\therefore \text{radius is } 22.7 \text{ cm}$$

(ii) total angle rotated

$$= \frac{10 \times 1000 \times 100}{\left(\frac{100}{4.4}\right)}$$

$$= 44000 \text{ rad}$$

No. of revolutions

$$= 44000 \times \frac{1}{2\pi}$$

$$= 7002.817496$$

$$= 7002 \text{ (nearest whole number)}$$

(a)

$$DB^2 = 40^2 + 18^2$$

$$DB^2 = 1924$$

$$MB^2 = MD^2 + DB^2$$

$$MB^2 = \left(\frac{65}{2}\right)^2 + 1924$$

$$MB = \sqrt{2980.25}$$

$$MB = 54.6 \text{ cm (3sf)}$$

(b)

$$65^2 = (\sqrt{2980.25})^2 + (\sqrt{2980.25})^2 - 2(\sqrt{2980.25})(\sqrt{2980.25}) \cos \angle AMB$$

$$65^2 = 2(2980.25) - 2(2980.25) \cos \angle AMB$$

$$\cos \angle AMB = \frac{2(2980.25) - 65^2}{2(2980.25)}$$

$$\cos \angle AMB = \frac{267}{917}$$

$$\angle AMB = 73.07217355^\circ$$

$$\angle AMB = 73.1^\circ (1dp)$$

(c) area of triangle AMB

$$= \frac{1}{2}(\sqrt{2980.25})(\sqrt{2980.25}) \sin 73.07217355^\circ$$

$$= 1425.561293$$

$$= 1430 \text{ cm}^2 (3sf)$$

(d) volume of prism

$$= \frac{1}{2}(40)(18) \times 65$$

$$= 23400 \text{ cm}^3$$

(e)

$$MC^2 = 40^2 + \left(\frac{65}{2}\right)^2$$

$$MC = \sqrt{2656.25} \text{ cm}$$

$$\tan \angle BMC = \frac{18}{\sqrt{2656.25}}$$

$$\angle BMC = 19.2518214^\circ$$

$$\angle BMC = 19.3^\circ (1dp)$$

Angle of elevation of B from M is 19.3°

9

(a) escape velocity from Earth

$$= \sqrt{\frac{2(6.672 \times 10^{-11})(5.972 \times 10^{24})}{6378 \times 1000}}$$

$$= 11177.91129$$

$$= 11200 \text{ m/s (3sf)}$$

(b) (i) Guiana Space Centre. It will have the greatest surface velocity.

Correct location with reason

Accept any logical reason such as:

- The effective radius is the largest
- It will allow the spacecraft to achieve escape velocity with less energy

(ii) surface velocity

$$= 7.292 \times 10^{-5} \times 6354 \times 1000$$

$$= 463.33368$$

$$= 463 \text{ m/s (nearest whole number)}$$

(iii) spacecraft velocity required

$$= 11177.91129 - 463.33368$$

$$= 10714.57449$$

$$= 10700 \text{ m/s (3sf)}$$

(c) Shortest possible distance

$$= 56 \times 10^6 \times 1000$$

$$= 5.6 \times 10^{10} \text{ m}$$

Time taken for signal to reach Earth from Mars

$$= \frac{5.6 \times 10^{10}}{2.9979 \times 10^8}$$

$$= 186.7974249 \text{ s}$$

Total time taken for astronaut to receive response

$$= 2 \times 186.7974249 \text{ s}$$

$$= \frac{2 \times 186.7974249}{60} \text{ min}$$

$$= 6.226580829 \text{ min}$$

$$= 6.23 \text{ min (3sf)}$$