Name:_____ ( )	Class:_____

SERVE·LEAD·EXCEL

80m

GREENDALE SECONDARY SCHOOL
End-Year Examination 2018

MATHEMATICS **4048/01**

Paper 1 4 October 2018

Secondary 3 Express **2 hours**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark or blue pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working may result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

Question	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13
Strand	A	A	N	N	S	G	G	S	A	G	A	N	N
Marks													

Question	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	Q22	Q23	Q24	Q25	
Strand	G	G	M	A	N	N	N	P	G	G	M	G	
Marks													

This document consists of 17 printed pages, including this cover page.

For Examiner's Use Only

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curve surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's Use Only

Answer **all** questions.

1 **(a)** Simplify $6x-5(5x-6)$.

Answer ..... [1]

(b) Factorise $16pq^2-12p^2q$.

Answer ..... [1]

---

2 Factorise $9b+6a-4ac-6bc$.

Answer ..... [2]

---

3 It is given that x is 40% lesser than m and y is 20% greater than n.
If $\frac{x}{y}$ is p% of $\frac{m}{n}$, find the value of p.

Answer p = ... [2]

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4 a and b are positive integers.
Show that $(a+b)^2-(a-b)^2$ is a multiple of 4 for all values of a and b.
Answer

[2]

---

5 The mean of five numbers is equal to the median of these numbers, which is 8.
Excluding the median, the mean of the other four numbers is x.
Find the value of x.

Answer $x =$ ..... [2]

---

6 The area of triangle ABC is 32 cm^2. Given that $AB = 8$ cm and $BC = 9$ cm, find the two possible values of $\angle ABC$.

Answer ..... ° or ..... ° [2]

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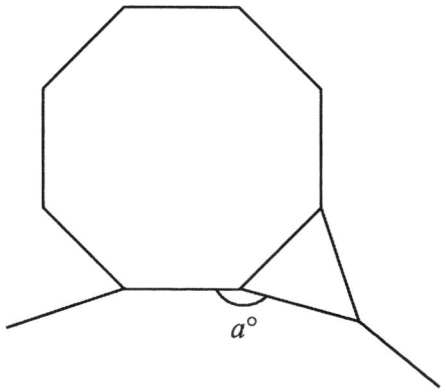
7 The diagram below is made up of a regular 8-sided polygon, an equilateral triangle and a regular n-sided polygon.



(a) An interior angle of the regular n-sided polygon is $a°$.
Find the value of a.

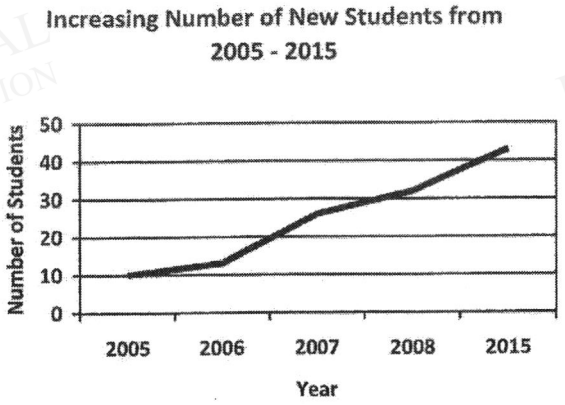
Answer $a =$ ..... [2]

(b) Find the value of n.

Answer $n =$ ..... [2]

---

8 The line graph below shows the number of new students per year in a tuition centre.



State and explain one way in which the graph above is misleading.

Answer .....

.....
[2]

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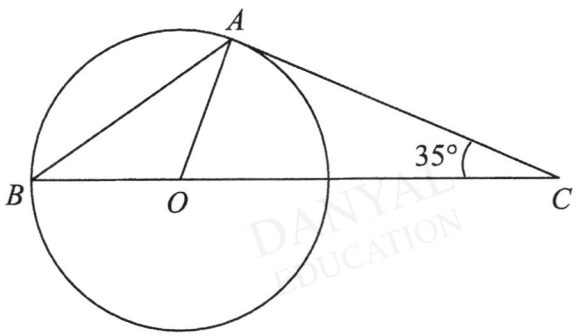
9 Write as a single fraction in its simplest form $\frac{x}{(x-3)^2} - \frac{3}{3-x}$.

Answer ..... [2]

---

10 A and B are points on the circle, centre O, and BOC is a straight line.
AC is a tangent at A and $\angle ACB = 35°$.



(a) Find

(i) $\angle OAC$,

Answer $\angle OAC =$ ..... ° [1]

(ii) $\angle OAB$.

Answer $\angle OAB =$ ..... ° [2]

(b) Given that D is a point along the major arc AB, find $\angle ADB$.

Answer $\angle ADB =$ ..... ° [1]

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11 **(a)** Express $x^2+10x+21$ in the form of $(x+p)^2+q$.

Answer ..... [1]

(b) Hence find the minimum value of $x^2+10x+21$.

Answer ..... [1]

(c) Find the equation of its line of symmetry of the graph of $y=x^2+10x+21$.

Answer ..... [1]

12 An Australian tourist exchanged some Australian dollars for Singapore dollars at the exchange rate of A$1 = S$1.055.

At the end of his visit, he changed his remaining S$837.50 back to Australian dollars at an exchange rate of A$1 = S$1.075.

Calculate the amount of money he lost in Australian dollars due to the difference in the exchange rates. Give your answer to the nearest cent.

Answer A$ = ... [3]

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13 **(a)** Khai deposited \$8000 in a bank at a compound interest of 2.4% per annum. Calculate the interest he would have at the end of 5 years.

Answer \$ ..... [3]

(b) At the end of 5 years, Khai withdrew his money from the bank and used 25% of it to buy a watch. How much did he pay for the watch?

Answer \$ ..... [1]

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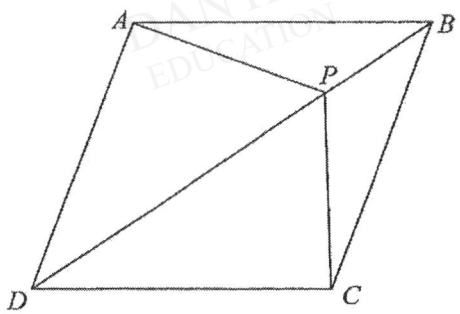
14 In the diagram below, $ABCD$ is a rhombus. P is a point on the diagonal.



Prove that ΔAPD is congruent to ΔCPD.

Answer

[2]

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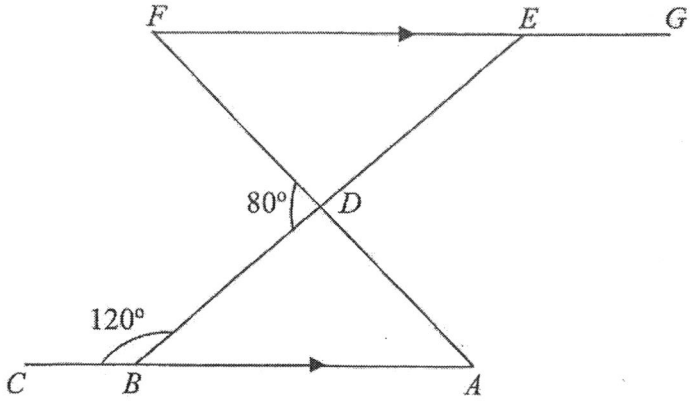
15 In the diagram below, *FEG* and *CBA* are a pair of parallel straight lines.



AF and *BE* are straight lines and they intersect at point *D*.
Given that $\angle BDF = 80^\circ$ and $\angle CBE = 120^\circ$,

find

(a) $\angle FED$,

Answer $\angle FED$ = ... ° [1]

(b) reflex $\angle DAB$.

Answer reflex $\angle DAB$ = ... ° [2]

(c) Explain, if the points *F*, *E*, *A* and *B* lie on the circumference of a circle with centre *D*.

Answer .....

.....

..... [2]

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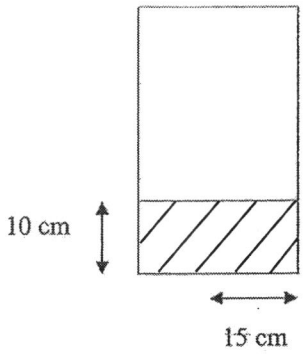
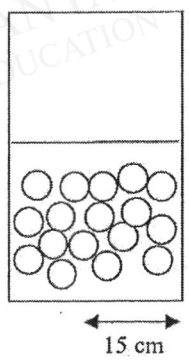
16 The cross section of a cylindrical water tank of base radius 15 cm is shown. The tank is filled with water to a depth of 10 cm.



(a) Find an expression for the volume of water in the tank in terms of π.

Answer ..... cm^3 [1]

(b) If 250 spherical marbles each of radius 0.7 cm are put into the tank, and are completely submerged in water, calculate new water level.



Answer ..... cm [3]

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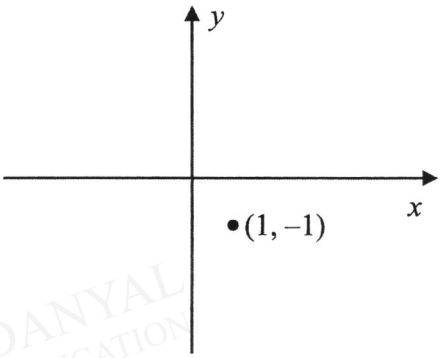
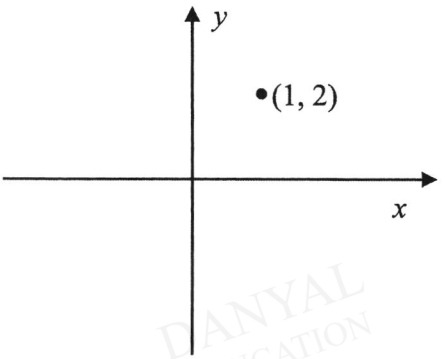
17 **(a)** On the axes given, sketch the following graphs, indicating the x and y intercepts where relevant.

(i) $y = -\frac{2}{x^2}$ **(ii)** $y = 2^x$ [2]





(b) Hence explain if $2^x + \frac{2}{x^2} = 0$ will have any solutions.

Answer .....

..... [1]

---

18 The number 2016 and 2160 are written as a product of its prime factors respectively as $2016 = 2^5 \times 3^2 \times 7$ and $2160 = 2^4 \times 3^3 \times 5$.

Find

(a) the highest common factor of 2016 and 2160,

Answer ..... [1]

(b) the smallest 3-digit number that is a factor of 2160,

Answer ..... [1]

(c) the minimum values of p and q, given that $2160 \times 5p = q^3$, where p and q are integers.

Answer $p =$ ..... [1]

$q =$ ..... [1]

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19 The table shows the cost of petrol and the fare per kilometre charged by a taxi driver for a customer's ride.

Cost of petrol per km	**Fare per km**
\$0.25	55 cents

(a) Find the ratio of the cost of petrol to the fare per km.

Answer ..... : ..... [1]

(b) On a particular day, the total cost of petrol for all customers' rides was \$88.50. Calculate

(i) the total distance travelled,

Answer ..... km [1]

(ii) the total fare collected by the taxi driver on that day.

Answer \$ ..... [1]

(c) The maintenance cost \$$A$, of the taxi is directly proportional to its total distance, d km, travelled. For every 5000 km travelled, the maintenance cost is \$150.
Find an equation to represent the relationship between A and d.

Answer ..... [1]

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20 A map of a city is drawn to a scale of $1:40\,000$.

(a) The distance between a post office and a police station is 4.2 km.
Find the distance, in centimetres, between them on the map.

Answer ..... cm [1]

(b) A school occupies an area of $0.68\ \text{cm}^2$ on the map.
Find the actual area, in square kilometres, of the school.

Answer ..... km^2 [2]

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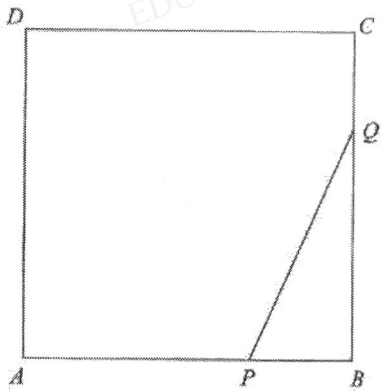
21 In the diagram, $ABCD$ is a square. P and Q are points on AB and BC respectively such that $AP = 2PB$ and $BQ = 2QC$.



A point is selected at random in the square $ABCD$.
Calculate the probability that the point selected lies outside triangle PBQ.

Answer ..... [2]

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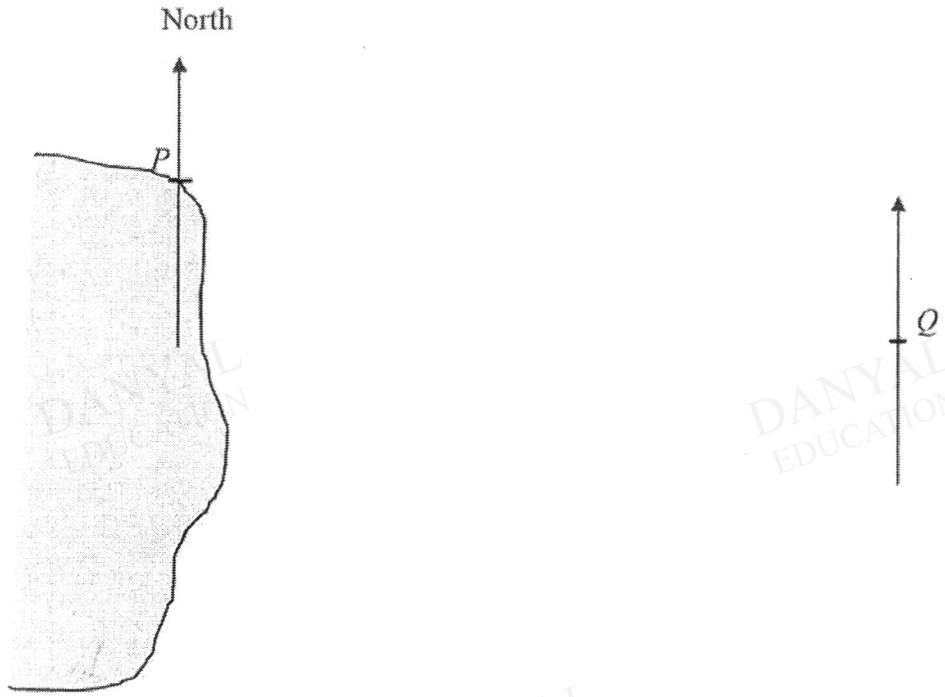
22 The scale drawing shows a lighthouse P and a police speedboat Q.
The scale is 1 cm to 4 km.



(a) A jet ski is 28 km from P on a bearing of 155^0.
Mark and label on the diagram the position J, of the ski. [1]

(b) The jet ski is out of petrol and sends out a distress call.
The police speed boat sets out from Q to travel to J.
The average speed of the police speed boat is 85 km/h.

(i) On what bearing does the police speed boat travel?

Answer ..... ^{0} [1]

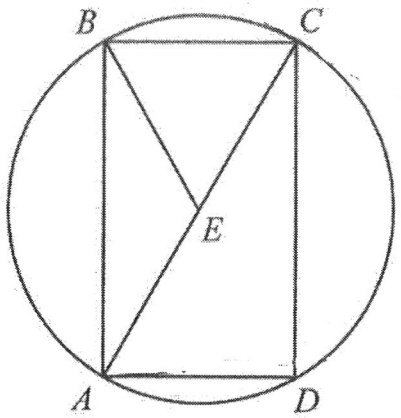
(ii) Calculate the travelling time of the police speed boat.

Give your answers in minutes and seconds, to the nearest second.

Answer ..... minutes ..... seconds [2]

For Examiner's Use Only

23 In the diagram below, a circle passes through the points A, B, C and D. $ABCD$ is a parallelogram and AEC is a straight line.



(a) Given that $AD = 3$ cm, $AB = 4$ cm and $AC = 5$ cm, explain why

(i) ΔABC is a right-angled triangle,

Answer .....

..... [1]

(ii) AC is the diameter of the circle.

Answer .....

..... [1]

(b) Given further that $\angle ECB = 70^\circ$ and $\angle ABE = 20^\circ$, explain why E is the centre of the circle.

Answer .....

..... [2]

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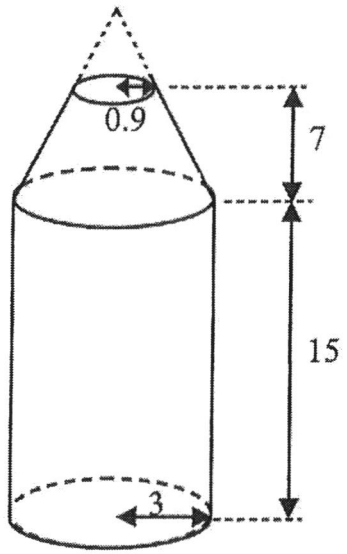
24 A shampoo bottle consists of a frustum of a right circular cone with vertical height 7 cm and top radius of 0.9 cm, attached to a cylinder of radius of 3 cm and height of 15 cm.



Assuming that the bottle's thickness is negligible, find the total capacity of the shampoo bottle, correct to 1 decimal place.

Answer ..... cm^3 [5]

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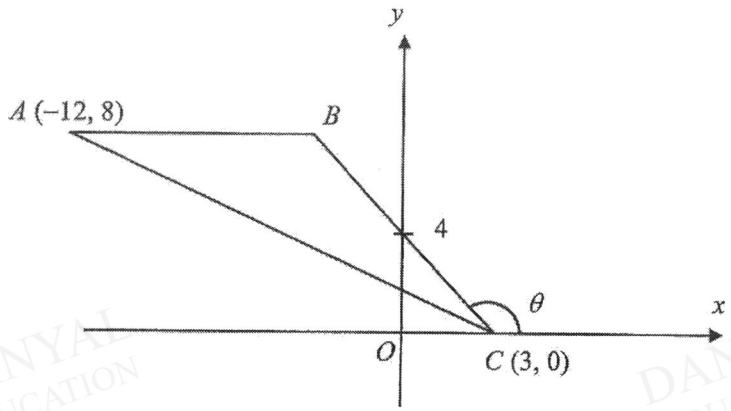
25 In the diagram below, the co-ordinates of points A and C are $(-12,8)$ and $(3,0)$ respectively. AB is a horizontal line segment. BC makes an angle of θ with the x-axis.



(a) Find the length of AC.

Answer ..... [2]

(b) State the equation of AB.

Answer ..... [1]

(c) Write down the value of $\cos\theta$.

Answer ..... [1]

(d) Find the coordinates of B.

Answer (....., ...) [2]

End-of-Paper

Name:_____ ( )	Class:_____

SERVE·LEAD·EXCEL

GREENDALE SECONDARY SCHOOL
End-of-Year Examination 2018

MATHEMATICS **4048/02**

Paper 2 1 October 2018

Secondary 3 Express **2 hours 30 minutes**

Candidates answer on Writing Paper.

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The total number of marks for this paper is **100.**

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$$\text{Curved surface area of a cone} = \pi rl$$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** questions.

1 (a) It is given that $p=\sqrt{\dfrac{1-t}{10-5t}}$.

(i) Find p when $t=-3$. [1]

(ii) Express t in terms of p. [3]

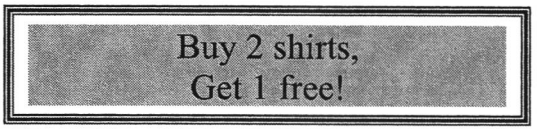
(b) Solve the equation $x-1=\dfrac{6}{2x-1}$. [2]

(c) Solve these simultaneous equations.

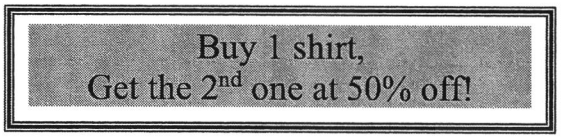
$$2x=y+6,$$
$$6x-2y=13.$$ [3]

(d) Simplify $\dfrac{27p^2-12}{9p^2-21p-18}$. [3]

2 (a) During the Great Singapore Sale, a retailer made the following offer ***A***:



After the Great Singapore Sale, he changed his offer to ***B***:



Determine whether offer ***A*** or ***B*** is the better deal for the customer.
Show your working clearly. [3]

(b) The cash price of a new car is $78 500.

(i) David buys the car on hire purchase.
He pays a deposit of one fifth of the cash price.
He then makes 36 monthly payments of $1900.

What is the total amount that David pays for the car? [2]

(ii) The original value of the car is its cash price of $78 500.
Each year the value of the car decreases by 10% of its value at the start of the year.
At the end of two years, David decides to sell the car.

Calculate the overall percentage reduction in the value of the car compared with its original value. [3]

3 The first four terms in a sequence of numbers, $u_1, u_2, u_3, u_4, \ldots$, are given below.

$$u_1 = 2^0 + 5 = 6$$
$$u_2 = 2^1 + 7 = 9$$
$$u_3 = 2^2 + 9 = 13$$
$$u_4 = 2^3 + 11 = 19$$

(a) Write down the expression for u_5 and show that $u_5 = 29$. [1]

(b) Write down the expression for u_6 and evaluate it. [1]

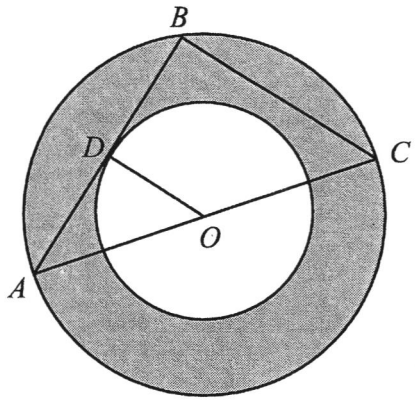
(c) Find an expression, in terms of n, for the nth term, u_n, of the sequence. [3]

(d) Evaluate u_{20}. [1]

(e) (i) Show that $2^{n-1} - 2^{n-2} = 2^{n-2}$. [1]

(ii) Find, and simplify, an expression, in terms of n, for $u_n - u_{n-1}$. [2]

4 The diagram shows two concentric circles with centre O.
A, B and C are points on the larger circle and D is a point on the smaller circle.
ADB is a tangent to the smaller circle.



(a) Show that triangles ABC and ADO are similar. [3]

(b) Given that radius of the smaller circle is 5 cm and angle $BCO = 60°$, show that the radius of the larger circle is 10 cm. [2]

(c) Find the ratio area of triangle ABC : area of quadrilateral $DOCB$. [2]

5 A cupcake shop sells cupcakes with assorted flavours.

(a) Chocolate, Cookies and Cream, and Salted Caramel cupcakes are baked in the ratio $3:7:5$ respectively. One week, 200 more Cookies and Cream cupcakes were baked than Chocolate cupcakes.

Work out the total number of cupcakes baked in the week. [2]

(b) Jamie and Pearlyn design the cupcakes in the shop.

(i) Jamie takes x seconds to design one cupcake.

Write an expression, in terms of x, for the number of cupcakes she designs in an hour. [1]

(ii) Pearlyn takes 50 seconds less than Jamie to design one cupcake.

Write an expression, in terms of x, for the number of cupcakes she designs in one hour. [1]

(iii) One morning, Jamie and Pearlyn each works for 4 hours. Altogether they design a total of 60 cupcakes.

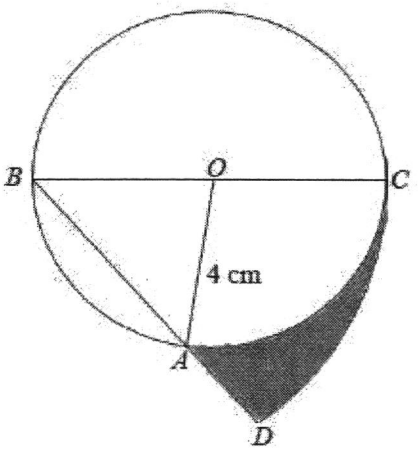
Write down an equation in x to represent this information and show that it reduces to

$$x^2 - 530x + 12000 = 0 .$$ [3]

(iv) Solve the equation $x^2 - 530x + 12000 = 0$. [3]

(v) Find the number of cupcakes Pearlyn designs in one hour. [2]

6 In the diagram, O is the centre of a circle of radius 4 cm and BC is the diameter.
CD is an arc of a circle with centre B and that the length of arc $AC = 7$cm.



(a) Show that $\angle AOC = \frac{7}{4}$ rad. [1]

(b) Calculate angle ABC in radians. [1]

(c) Show that $AD = 2.872$cm when rounded off to 3 decimal places. [3]

(d) Hence, calculate the perimeter of the shaded region. [2]

7 (a) The number of fish caught by a group of students in their fishing trip is recorded in the table below.

Number of fish	0	1	2	3	4	5
Number of students	10	12	7	x	3	2

(i) If the mode is 3, write down the minimum value of x. [1]

(ii) If the median is 2.5, write down the value of x. [1]

(iii) If 44% of the students caught at most 1 fish, find the value of x. [2]

(iv) If another student who caught 15 fish is added to the group of students, would the **mean** or the **median** be a better measure of average?
Give a reason for your answer. [1]

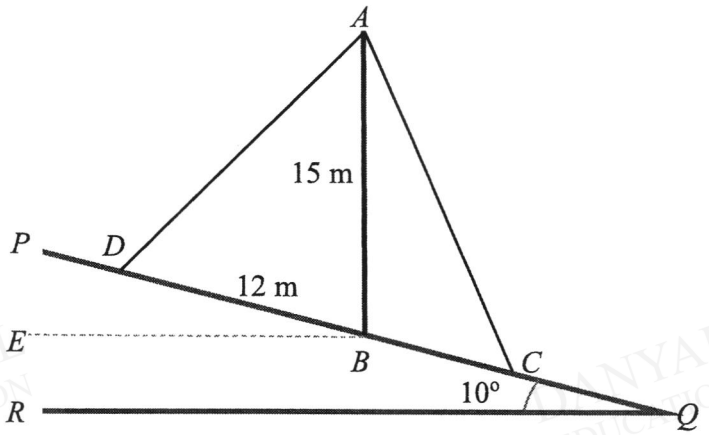
(b) Bag P contains 40 marbles of which x are green, 20 are yellow and the rest are blue.
Bag Q contains 60 marbles of which 15 are green, $2x$ are yellow and the rest are blue.

(i) If the probability of selecting a blue marble from bag P is $\frac{1}{8}$, show that the value of x is 15. [2]

(ii) Find the probability of selecting a blue marble from bag Q. [2]

(iii) If the marbles from bag P and bag Q are put into bag R, find the probability of selecting a yellow marble from bag R. [2]

8 A slope, PQ, is inclined at 10° to the level ground QR.
A pole, AB, stands on the slope such that AB is perpendicular to the line EB and EB is parallel to QR where BD and AB are 12 m and 15 m respectively.



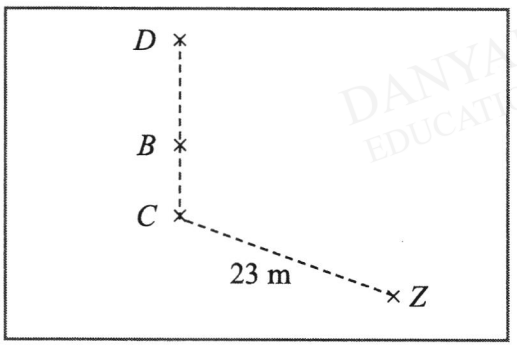
(a) Show that $AD = 17.5$ m, correct to 3 significant figures. [3]

(b) A bird stood at D and observed the top of the pole A.
Find the angle of elevation of A from the bird. [3]

(c) Given that the area of $\triangle ACD$ is 142 m^2, find the distance BC. [2]

(d) A piece of rope is used to attach the point A to a point along slope CD.
Find the shortest possible length of the rope. [2]

(e) The diagram below shows the top view of the surface of the slope. Z is a point on the same surface as B, C and D.
If CZ is 23 m and the bearing of Z from D is 155°, find the bearing of C from Z. [2]



9 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{x^3}{2} - 5x - 3.$$

Some corresponding values of x and y are given in the table below.

x	-3	-2	-1	0	1	2	3	4
y	-1.5	p	1.5	-3	-7.5	-9	-4.5	9

(a) Find the value of p. [1]

(b) Using a scale of 2cm to represent 1 unit, draw a horizontal x-axis for $-3 \le x \le 4$.
Using a scale of 1cm to represent 2 units, draw a vertical y-axis for $-10 \le y \le 10$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) The equation $\frac{x^3}{2} - 5x = 10$ has only one solution.

Explain how this can be seen from your graph. [2]

(d) By drawing a tangent, find the gradient of the curve at $(1, -7.5)$. [2]

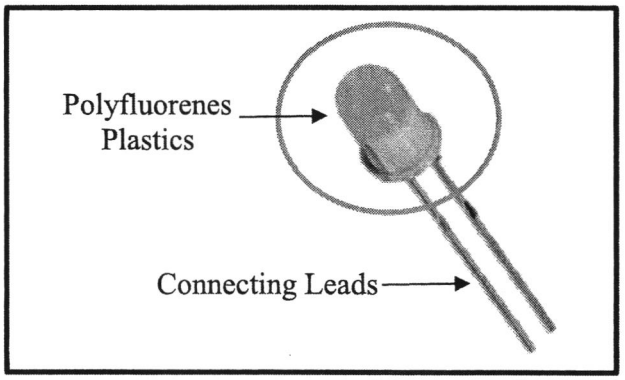
(e) (i) On the same axes, draw the line $y = 3 - 2x$ for $-3 \le x \le 4$. [1]

(ii) Write down the x-coordinate of the point where this line intersects the curve. [1]

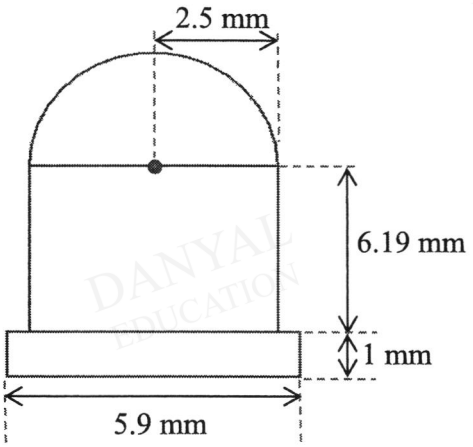
(iii) This value of x is a solution of the equation $x^3 + Ax + B = 0$.

Find the value of A and of B. [2]

10 Below are some information about LED light bulb.



In this question, the case of the LED bulb can be modelled as a hollow cylinder with a hollow hemispherical top and a solid cylindrical base.



(a) Calculate the surface area of the base of the LED light bulb. [1]

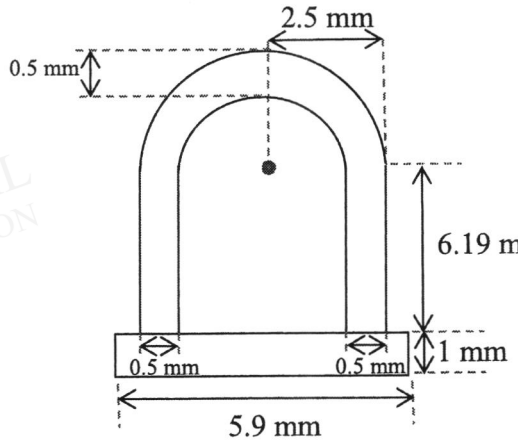
(b) Calculate the volume of the LED light bulb. [3]

Question 10 continues on next page

10 The case of the LED light bulbs is made from tinted plastics, Polyfluorenes. A manufacturer estimates that he can manufacture 16 000 LED light bulbs using 1 kg of Polyfluorenes plastics.

Useful information

Polyfluorenes plastics have a density of 0.00092 g/mm^3.
Thickness of the case of LED light bulb is 0.5 mm



(c) Explain if the manufacturer is accurate in his estimate, assuming the thickness of the connecting leads is negligible. [6]

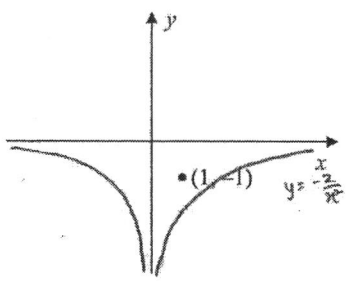
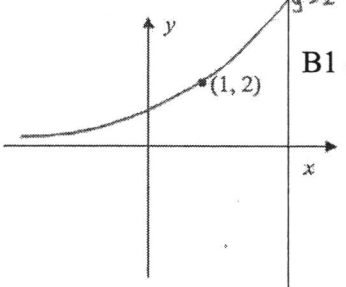
End - of - Paper

Greendale Secondary School
Secondary 3 Express
Mathematics End-of-Year Examination Paper 1/2018
Marking Scheme

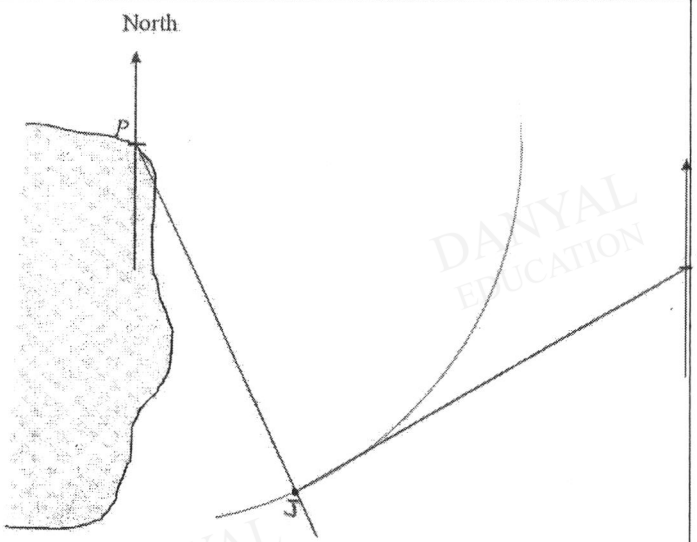
Q.		Solution	Marks	Remarks
1	(a)	$6x-5(5x-6)$ $=6x-25x+30$ $=-19x+30$	B1	Insert own bracket and did expansion
1	(b)	$16pq^2-12p^2q$ $=4pq(4q-3p)$	B1	Left out some factors like q.
2		$9b+6a-4ac-6bc$ $=3(3b+2a)-2c(2a+3b)$ $=(2a+3b)(3-2c)$	M1 A1	• Missed out operation (minus) in the second step. • Took out factor by stating: $(3b+2a)^2(3-2c)$
3		$x=0.6m$ $y=1.2n$ $\frac{x}{y}=\frac{p}{100}\left(\frac{m}{n}\right)$ $\frac{0.6m}{1.2n}=\frac{p}{100}\left(\frac{m}{n}\right)$ $p=50$	M1 A1	Poor understanding of the meaning of percentages.
4		$(a+b)^2-(a-b)^2$ $=a^2+2ab+b^2-(a^2-2ab+b^2)$ $=4ab$ 4ab is a multiple of 4, hence $(a+b)^2-(a-b)^2$ is a multiple of 4 for all values of a and b.	M1 A1	• A handful of students used guess and check and no marks were awarded as question states all values of a and b. • Final statement was missing. • Statement was wrongly stated. (eg: a and b are multiple of 4)
5		$mean=\frac{4x+8}{5}$ $8=\frac{4x+8}{5}$ $40=4x+8$ $4x=32$ $x=8$	M1 A1 (or B2)	Quite well done.

Q.		Solution	Marks	Remarks
6		$Area = 32$ $\frac{1}{2}(8)(9)\sin ABC = 32$ $\sin ABC = \frac{8}{9}$ $\angle ABC = \sin^{-1}\frac{8}{9}$ $\angle ABC \approx 62.7^0, 117.3^0$	B1 / B1	• Assume right angled triangle which is incorrect. • Rounding off error.
7	(a)	$\frac{(8-2)\times 180}{8} = 135^0$ $\angle a = 360 - 135 - 60$ $= 165^0 (\angle \text{ sum of })$	M1 A1	Quite well done.
	(b)	$(n-2)\times 180 = 165n$ $180n - 360 = 165n$ $15n = 360$ $n = 24$ $ext\angle = 180 - 165 = 15^0$ $n = \frac{360}{15} = 24 \text{ sides}$	M1 A1	
8		The new students from 2009 to 2014 are not shown. Student intake from 2009 to 2014 could be very low, hence it may not show an overall increasing trend from 2005 to 2015.	B1 B1	
9		$\frac{x}{(x-3)^2} - \frac{3}{3-x}$ $= \frac{x}{(x-3)^2} + \frac{3}{x-3}$ $= \frac{x+3(x-3)}{(x-3)^2}$ $= \frac{4x-9}{(x-3)^2}$	M1 A1	• Very badly done. • Poor understanding of algebra especially in changing the signs of the algebra. • Some students are still making mistakes like $(x-3)^2 = x^2 - 9$
10	(a)	$\angle OAC = 90^0 (rad \perp \tan)$	B1	
	(b)	$\angle AOB = 90 + 35 = 125 (ext \angle of \Delta)$ $\angle OAB = \frac{180-125}{2}$ $= 27.5^0 (base \angle s \ isos\Delta)$	M1 A1	• Most are able to find $\angle AOB$. • Those who skipped steps made mistake in find $\angle OAB$.
	(c)	$\angle ADB = \frac{125}{2}$ $= 62.5^0 (\angle s \text{ centre} = 2\angle s \text{ at circumference})$	B1	

Q.		Solution	Marks	Remarks
11	a	$x^2+10x+21=(x+5)^2-4$	B1	Problem with completing the square.
	b	Minimum value of -4.	B1 (ecf provided (a) is reasonable)	Most students do not understand the meaning of minimum value.
	c	Equation of line of symmetry $x=-5$	B1	A handful of students missed out "x" in the equation.
12		Before trip $A\$1=S\$1.055$ $A\$793.8388626=S\$837.50$ After trip $A\$1=S\$1.075$ $A\$779.0697674=S\$837.50$ Loss in Australian dollars $=A\$793.8388626-A\$779.0697674$ $=A\$14.77(\textit{nearest cents})$	M1 M1 A1	A small group of students found difference in exchange rate instead.
13	a	$8000\left(1+\frac{2.4}{100}\right)^5-8000$ $\approx \$1007.20\,(2dp)$	M1 $8000\left(1+\frac{2.4}{100}\right)^5$ M1 (8000) A1	Students forget to subtract \$8000.
	b	Total amount in bank $= \$8000+\$1007.20=\$9007.20$ Cost of watch $= \$9007.20\times 25\%=\$2251.80$	B1	
14		PD is the common length. (S) $\angle ADP=\angle CDP$ (diagonal of rhombus bisect angle) (A) $AD=CD$ (sides of a rhombus) (S) $\therefore \Delta APD\equiv\Delta CPD$ (SAS)	M1 (Any 2 proves) A1	• Most use SSS test. • Use the wrong angle to prove.

Q.		Solution	Marks	Remarks
15	(a)	$\angle FED = \angle ABD = 60^0 \ (alt \angle)$	B1	
	(b)	$\angle DAB = 180^0 - 60^0 - 100^0 = 20^0 \ (\angle s \ in \ a \ \Delta)$ $reflex \ \angle DAB = 360^0 - 20^0 = 340^0 \ (\angle s \ in \ a \ pt)$	M1 A1	Fail to know which is reflex angle.
	(c)	Not possible. $\angle FED$ is not equal to $\angle DAB$ and also not twice of $\angle FDB$. They do not satisfy the property of angle at centre is twice the angle at circumference. **OR** $\angle FED$ is not equal to $\angle DAB$ as it does not form angles in the same segment. **OR** $\angle DFE$ and $\angle DEF$ are not equal which means $DF \neq DE$ so DF and DE cannot be radius of the circle.	B1 B1	Use congruent triangle which is not enough to prove points are on circumference.
16	(a)	Volume of water in tank $= \pi(15)^2(10)$ $= 2250\pi$	B1	Use wrong formula.
	(b)	Volume of 250 spherical marbles $= 250 \times \frac{4}{3}\pi(0.7)^3$ $= 114\frac{1}{3}\pi$ $114\frac{1}{3}\pi + 2250\pi = \pi(15)^2(h)$ $2364\frac{1}{3} = 225h$ $h \approx 10.5 \ cm \ (3sf)$	M1 M1(ecf) A1	• Fail to recognise as a 3D solid. • Use area of circle to find volume of sphere.
17	(a)	(i) $y =$  (ii) $y = 2^x$ 	B1 ($y = -\frac{2}{x^2}$) B1 ($y = 2^x$)	Students ignored the points given or failed to consider the points.
	(b)	From the graphs above, there is no intersection of the curves, hence there will be NO solution for $2^x + \frac{2}{x^2} = 0$.	B1	• Students substituted values instead. • Determine if there is solution by checking if it cuts the x-axis or y-axis.

Q.		Solution	Marks	Remarks
18	(a)	$2^4 \times 3^2 = 144$	B1	
	(b)	$2^2 \times 3^3 = 108$	B1	Failed to get the smallest number
	(c)	$2^4 \times 3^3 \times 5 \times 5p = q^3$ $2^4 \times 3^3 \times 5 \times \left(2^2 \times 5^2\right) = q^3$ $\therefore 5p = 100$ $p = 20$ $\therefore q = 60$	 B1 B1	Most managed to get q but not p.
19	(a)	$25 : 55$ $5 : 11$	 B1	Failed to reduce to integer.
	(bi)	Distance $= \frac{88.50}{0.25} = 354\ km$	B1	
	(bii)	Total Fare Collected $=354 \times 0.55$ $=\$194.70$	 B1	Students added the amount spent on petrol as well.
	(c)	$A = kd$ $150 = 5000k$ $k = \frac{3}{100}$ $A = \frac{3}{100} d \,/\, d = 33\frac{1}{3} A \,/\, A = 0.03d$	 B1	
20	(a)	$1 : 40\,000$ $1\ cm : 0.4\ km$ $10.5\ cm : 4.2\ km$ Distance on map = 10.5 cm	 B1	Conversion of units are very weak.
	(b)	$1 : 40\,000$ $1\ cm : 0.4\ km$ $1\ cm^2 : 0.16\ km^2$ $0.68\ cm^2 : 0.1088\ km^2$ Actual area is $0.1088\ km^2$	 M1 A1	Those who got (a) wrong, will get (b) wrong.

Q.		Solution	Marks	Remarks
21		Let the length of square $ABCD$ be x Area square $ABCD = x^2$ Area of $\Delta PBQ = \frac{1}{2}\left(\frac{1}{3}x\right)\left(\frac{2}{3}x\right) = \frac{1}{9}x^2$ $P(\text{point outside } \Delta PBQ)$ $= \frac{x^2 - \frac{1}{9}x^2}{x^2}$ $= \frac{\frac{8}{9}x^2}{x^2}$ $= \frac{8}{9}$	M1 A1	Failed to understand that they should use area.
22	(a)		B1 (needs to show construction to get J)	Do not know how to construct bearing.
	(bi)	$240^\circ \pm 1^\circ$	B1 (ecf)	
	(bii)	$distance = 8.2 \times 4 = 32.8\ km$ $time = \frac{32.8}{85} \approx 0.385882352$ $= 23$ minutes 9 seconds	B1(ecf) B1(ecf)	Weak in conversion of time especially from minutes to seconds.

Q.		Solution	Marks	Remarks
23	(a)	$AC^2 = 5^2 = 25$ $AB^2 + BC^2 = 4^2 + 3^2 = 25$ By Converse of Pythagoras Theorem, ΔABC is a right-angled triangle.	B1	• Presentation is an issue, state Pythagoras Theorem straight from the beginning. • Students tend to assume is true first before proving.
	(b)	$\angle ABC = 90^0$ ($\angle$ in semi circle) Hence AC is the diameter of the circle	B1	
	(c)	$\angle BEC = 180 - 70 - 70$ $= 40^0$ $\angle BAC = 180 - 90 - 70$ $= 20^0$ Since $\angle BEC = 2\angle BAC$, the angle at the centre is twice the angle at the circumference, ***E*** is the centre of the circle. OR $\angle CBE = 90 - 70 = 20^0$ Base radius of isosceles triangle, $BE = EC$ (radius)	M1 A1	Prove diameter need not means E is the centre of the circle.
24		$\frac{0.9}{3} = \frac{h-7}{h}$ $0.9h = 3h - 21$ $2.1h = 21$ $h = 10$ Volume of cylinder $= \pi(3)^2(15)$ $= 135\pi\ cm^3$ Volume of frustum of cone $= \frac{1}{3}\pi(3)^2(10) - \frac{1}{3}\pi(0.9)^2(3)$ $= 29\frac{19}{100}\pi\ cm^3$ Total volume $= 135\pi + 29\frac{19}{100}\pi$ $\approx 515.8\ cm^3\ (1\ dp)$	M1 M1 M1 (ecf) M1 (ecf) A1	• Most students cannot get height.
25	(a)	Length $= \sqrt{(-12-3)^2 + (8-0)^2}$ $= 17\ units$	M1 A1	Made careless mistake in calculating length.
	(b)	$y = 8$	B1	
	(c)	$\cos\theta = -\frac{3}{5}$	B1	
	(d)	$B(-3, 8)$	B1 / B1	

Greendale Secondary School
Secondary 3 Express
Mathematics End-of-Year Examination Paper 2/2018
Marking Scheme

Q.		Solution	Marks	Remarks
1	(ai)	$p=\sqrt{\dfrac{1-t}{10-5t}}$ $p=\sqrt{\dfrac{1-(-3)}{10-5(-3)}}$ $p=\dfrac{2}{5}$ or 0.4	B1	
1	(aii)	$p=\sqrt{\dfrac{1-t}{10-5t}}$ $p^2=\dfrac{1-t}{10-5t}$ $p^2(10-5t)=1-t$ $10p^2-5p^2t=1-t$ $-5p^2t+t=1-10p^2$ $t(-5p^2+1)=1-10p^2$ $t=\dfrac{1-10p^2}{-5p^2+1}$ $=\dfrac{1-10p^2}{1-5p^2}$	M1 M1 A1	
1	(b)	$x-1=\dfrac{6}{2x-1}$ $(x-1)(2x-1)=6$ $2x^2-3x+1-6=0$ $2x^2-3x-5=0$ $(5x-2)(x+1)=0$ $x=2.5$ or $x=-1$	M1 A1	

1	(c)	$2x = y + 6$ $2x - y = 6 \rightarrow (1)$ $6x - 2y = 13 \rightarrow (2)$ $(1) \times 2 : 4x - 2y = 12 \rightarrow (3)$ $(2) - (3):$ $2x = 1$ $x = \frac{1}{2}$ Sub $x = \frac{1}{2}$ into (1) $2\left(\frac{1}{2}\right) - y = 6$ $y = -5$	M1 – Attempt to make one unknown the subject. A1 A1	
1	(d)	$\frac{27p^2 - 12}{9p^2 - 21p - 18}$ $= \frac{3(9p^2 - 4)}{3(3p^2 - 7p - 6)}$ $= \frac{(3p-2)(3p+2)}{(p-3)(3p+2)}$ $= \frac{3p-2}{p-3}$	M1 M1 A1	
		Total	**12 marks**	

Q.		Solution	Marks	Remarks
2	(a)	During GSS Offer ***A***: % of original price paid $= \frac{2}{3} \times 100\% = 66\frac{2}{3}\%$ After GSS Offer ***B***: % of original price paid $= \frac{1.5}{2} \times 100\% = 75\%$ Hence Offer ***A*** is better because the percentage of original price paid is lower.	M1 M1 A1	
2	(bi)	Deposit $= \frac{1}{5} \times \$78500$ $= \$15700$ Total instalments $= 36 \times \$1900$ $= \$68400$ Total amount paid $= \$15700 + \$68400$ $= \$84100$	M1 A1	
2	(bii)	Price after 1st year $= \frac{90}{100} \times \$78500$ $= \$70650$ Price after 2nd year $= \frac{90}{100} \times \$70650$ $= \$63585$ % reduction $= \frac{\$78500 - \$63585}{\$78500} \times 100\%$ $= 19\%$	M1 M1(ecf) A1	
		Total	**8 marks**	

Q.		Solution	Marks	Remarks
3	(a)	$u_5 = 2^4 + 13$ $= 29$ (Shown)	B1	
3	(b)	$u_6 = 2^5 + 15$ $= 47$	B1	
3	(c)	$u_n = 2^{n-1} + [2n+3]$ $= 2^{n-1} + 2n + 3$	M1, M1 A1	
3	(d)	$u_{20} = 2^{20-1} + 2(20) + 3$ $= 2^{19} + 40 + 3$ $= 524331$	B1	
3	(ei)	$2^{n-1} - 2^{n-2} = 2^{n-2} \times 2 - 2^{n-2} \times 1$ $= 2^{n-2} \times (2-1)$ $= 2^{n-2}$	B1	
3	(eii)	$u_n - u_{n-1}$ $= (2^{n-1} + 2n - 1) - [2^{n-1-1} + 2(n-1) - 1]$ $= 2^{n-1} + 2n - 1 - 2^{n-2} - 2n + 2 + 1$ $= 2^{n-1} - 2^{n-2} + 2$ $= 2^{n-2} + 2$	M1 A1	
		Total	**9 marks**	

Q.		Solution	Marks	Remarks
4	(a)	$\angle DAO = \angle BAC$ (shared angle) $\angle ADO = 90°$ (radius $\perp$ tangent) $\angle ABC = 90°$ ($\angle$ in semicircle) $\therefore \angle ADO = \angle ABC$ Since 2 corresponding angles are equal, the third angle is also equal, therefore ΔABC is similar to ΔADO.	M1 M1 A1	
4	(b)	$\angle DAO = 180° - 90° - 60°$ ($\angle$ sum of Δ) $= 30°$ $\sin 30° = \frac{5}{AO}$ $AO = \frac{5}{\sin 30°}$ $AO = 10$ cm (Shown)	 M1 A1	
4	(c)	$\frac{\text{Area of } \Delta ADO}{\text{Area of } \Delta ABC} = \left(\frac{10}{20}\right)^2$ $= \frac{1}{4}$ Area of ΔABC : Area of $DOCB$ 4 : 3	 M1 A1	
		Total	**7 marks**	

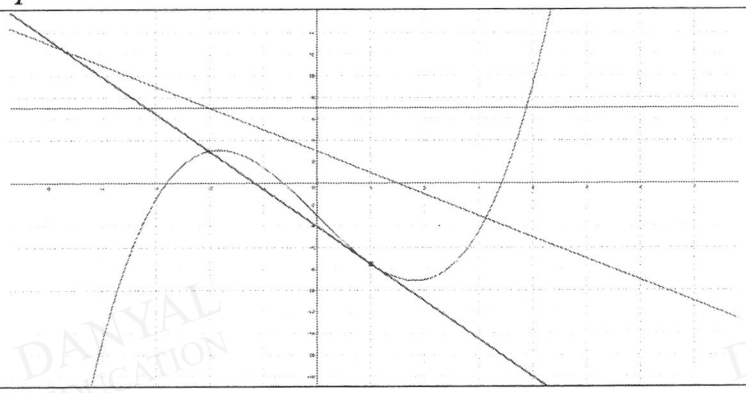
Q.		Solution	Marks	Remarks
5	(a)	$4u \rightarrow 200$ $15u \rightarrow \frac{15 \times 200}{4}$ $= 750$	M1 A1	
5	(bi)	$\frac{3600}{x}$	B1	
5	(bii)	$\frac{3600}{x-50}$	B1	
5	(biii)	$4\left(\frac{3600}{x}+\frac{3600}{x-50}\right)=60$ $\frac{3600}{x}+\frac{3600}{x-50}=15$ $3600(x-50)+3600(x)=15x(x-50)$ $3600x-180000+3600x=15x^2-750x$ $7200x-180000=15x^2-750x$ $15x^2-7950x+180000=0$ $x^2-530x+12000=0$	M1 M1 A1	
5	(biv)	$x^2-530x+12000=0$ $x=\frac{-(-530)\pm\sqrt{(-530)^2-4(1)(12000)}}{2(1)}$ $x=506.2985$ or $x=23.7014$ $x=506$ or $x=23.7$	M1 A1, A1	
5	(bv)	Number of cupcakes Pearlyn's bake $=\frac{3600}{506.2985-50}$ $=7.8895$ ≈ 7	M1 A1	
		Total	**12 marks**	

Q.		Solution	Marks	Remarks
6	(a)	$r\theta = 7$ $\theta = \frac{7}{4}$ rad or 1.75rad $\angle AOC = \frac{7}{4}$ rad or 1.75rad (Shown)	B1	
6	(b)	$\angle ABC = \frac{7}{4} \div 2 (\angle \text{ at centre} = 2\ \angle s \text{ at circumference})$ $= \frac{7}{8}$ rad	B1	
6	(c)	$\angle AOB$ $= \pi - \frac{7}{8} - \frac{7}{8}$ $= 1.3916$rad Using sine rule, $\frac{AB}{\sin(1.3916)} = \frac{4}{\sin\left(\frac{7}{8}\right)}$ $\therefore AB = 5.12798$cm Length of AD $= 8 - 5.12798$ $= 2.87201$cm $=2.872$cm	M1 M1 (ecf) A1	
6	(d)	Arc length of CD $=8\left(\frac{7}{8}\right)$ $= 7$cm Perimeter of shaded region $=7+7+2.872$ $=16.872$cm ≈ 16.9cm	M1 A1	
		Total	**7 marks**	

Q.		Solution	Marks	Remarks
7	(ai)	$x=13$	B1	
7	(aii)	$x-1+3+2=28$ $x=24$	B1	
7	(aiii)	$44\% \rightarrow 22$ $56\% \rightarrow \frac{22\times 56}{44}$ $=28$ $7+x+3+2=28$ $x=16$	M1 A1	
7	(aiv)	Median The student who caught 15 fish is an outlier to the set of data. This outlier data will cause the mean to inflate/increase a lot and not reflect the average accurately.	B1	
7	(bi)	P(blue from bag P) $=\frac{40-20-x}{40}$ $=\frac{20-x}{40}$ $\frac{20-x}{40}=\frac{1}{8}$ $20-x=5$ $x=15$ (Shown)	M1 A1	
7	(bii)	P(blue from bag Q) $=\frac{60-15-2(15)}{60}$ $=\frac{1}{4}$	M1 A1	
7	(biii)	P(select a yellow marble) $=\frac{20+2(15)}{60+40}$ $=\frac{1}{2}$	M1 A1	
		Total	**11 marks**	

Q.		Solution	Marks	Remarks
8	(a)	$\angle ABD = 90° - 10° = 80°$ (alternate angles) $AD^2 = 12^2 + 15^2 - 2(12)(15)\cos 80°$ $AD = \sqrt{306.487}$ $= 17.50677$ $= 17.5m$	M1 M1 A1	
8	(b)	Let F be a point on AB such that DF is perpendicular to AB. $DF = 12\sin 80°$ $= 11.8177\ m$ $\cos\angle ADF = \dfrac{DF}{AD}$ $\cos\angle ADF = \dfrac{11.8177}{\sqrt{306.487}}$ $\angle ADF = 47.543°$ $= 47.5°$ **OR**	 M1 M1 (ecf) A1	
8	(b)	$\dfrac{\sin\angle ADB}{15} = \dfrac{\sin 80°}{17.5}$ $\angle ADB = \sin^{-1}\left(\dfrac{15\sin 80°}{17.5}\right)$ $= 57.5778$ $\angle ADF = 57.5778 - 10$ $= 47.5778$ $= 47.6°$	$\dfrac{\sin\angle ADB}{15} = \dfrac{\sin 80°}{17.5067}$ $\angle ADB = \sin^{-1}\left(\dfrac{15\sin 80°}{17.5067}\right)$ $= 57.5433$ $\angle ADF = 57.5433 - 10$ $= 47.5433$ $= 47.5°$	M1 M1 (ecf) A1	
8	(c)	$\dfrac{1}{2}(AD)(DC)\sin\angle ADC = 142$ $\dfrac{1}{2}(17.5)(12 + BC)\sin(47.5778° + 10°) = 142$ $BC = 7.22541$ $= 7.23m$	 M1 (ecf) A1	

8	(d)	Let G be a point on CD such that AG is perpendicular to CD. $\sin\angle ADC = \frac{AG}{\sqrt{306.487}}$ $AG = \sqrt{306.487}\sin(47.543° + 10°)$ $= 14.7721$ $= 14.8m$	M1 A1	or use Area of Triangle
8	(e)	$\frac{\sin\angle DZC}{DC} = \frac{\sin\angle ZDC}{23}$ $\frac{\sin\angle DZC}{19.22542} = \frac{\sin 25°}{23}$ $\angle DZC = 20.687°$ Bearing of C from Z $= 360° - 25° - 20.687°$ $= 314.3°$	M1 A1	
		Total	**12 marks**	

Q.		Solution	Marks	Remarks
9	(a)	$p=3$	B1	
9	(b)		Correct Scale – B1 Correct Plot – B1 Correct Curve – B1	
9	(c)	$\frac{x^3}{2}-5x=10$ $\frac{x^3}{2}-5x-3=7$ The line $y=7$ cuts the curve $y=\frac{x^3}{2}-5x-3$ at only one point.	B1 B1	
9	(d)	Gradient $=-3.5$	Draw tangent – B1 Gradient –B1	
9	(ei)	Draw the line correctly. Refer to graph at (b)	B1	
9	(eii)	From the graph, $x=3.1$ *to* 3.3	B1	
9	(eii)	$\frac{x^3}{2}-5x-3=3-2x$ $x^3-10x-6=6-4x$ $x^3-6x-12=0$ $\therefore A=-6, B=-12$	B1, B1	
		Total	**12 marks**	

Q.		Solution	Marks	Remarks
10	(a)	Base Area $=\pi\left(\frac{5.9}{2}\right)^2$ $=27.3397..mm^2$	 B1	
10	(b)	Vol of cylinder and hemisphere $=\pi\left(\frac{5.9}{2}\right)^2(1)+\pi(2.5)^2(6.19)+\frac{1}{2}\times\frac{4}{3}\pi(2.5)^3$ $=27.3397..+121.54...+32.72...$ $=181.604..mm^3$	M1 – Award if any one of the substitution is correct. M2 – All substitutions are correct A1	
10	(c)	Vol of space in LED $=\pi(2)^2(6.19)+\frac{1}{2}\times\frac{4}{3}\pi(2)^3$ $=77.78...+16.755...$ $=94.540.....mm^3$ Vol of material for 1 LED case $=181.604...-\ 94.540...$ $=87.0631...mm^3$ Mass of 16000 LED $=16000\times87.0631\times0.00092$ $=1282.112g$ $=1.28kg$ Manufacturer is not accurate. He needs more than 1kg of Polyfluorenes	 M1, M1 M1(ecf) M1(ecf) M1(ecf) A1 (award based on their calculation)	
		Total	**10 marks**	