Candidate Name: ____________________ Class ____ Index No. ____

FUHUA SECONDARY SCHOOL

Secondary Three Express

Mid-Year Examination 2018

3E

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MATHEMATICS 4048/01

Paper 1

10 May 2018
0755 – 0925
1 hour 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to 3 significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **60**.

PARENT'S SIGNATURE	**FOR EXAMINER'S USE**		
	Units		**/ 60**
	Statements/Accuracy		
	Poor Presentation		

Setter: Mr Chen Hong Ming Vetter: Ms Winnifred Lim

This question paper consists of 9 printed pages including this page.

1 **(a)** Express 1485 as the product of its prime factors.

(b) Find the smallest possible integer value of k such that $1485k$ is a perfect square.

(c) The lowest common multiple of 1485 and the number X is 5940.
The highest common factor of 1485 and the number X is 45.
Find the value of X.

Answer (a).. [1]

(b) k = ... [1]

(c) X = ... [1]

---

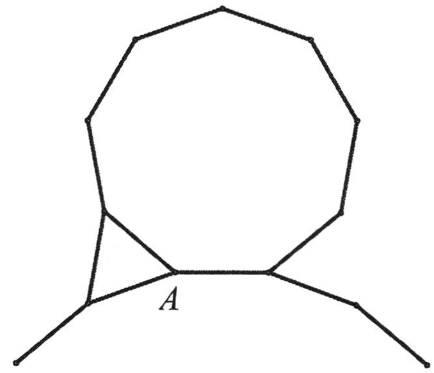
2 **(a)** Solve the inequality $x-1<5-x\leq 2x+17$.

Answer (a).. [2]

(b) Represent the solution on the number line below.

[1]

3 An equilateral triangle, a nonagon and an unknown regular polygon share a vertex *A*.
The 3 shapes share a side with one another without overlapping.
How many sides does the unknown regular polygon have?



Answer .... sides [3]

4 The Mercedes taxi in Singapore has a fare structure as shown in the table below.

Fixed Boarding Fare	\$3.90
Variable Fare	\$0.30 for every 400 metres thereafter **or part thereof**

Billy only has \$14 in his wallet.

(a) Form an inequality for distance (in kilometres) that Billy can afford to travel on the taxi.

(b) Hence, find the maximum distance that Billy can travel on the taxi.

Answer (a).. [1]

(b).....................................km [2]

5 A map of Singapore has a scale of 1 : 200 000.

(a) The length of the Singapore River on the map is 1.6 cm.
Calculate the actual length, in kilometres, of the Singapore River.

(b) The actual area of Gardens by the Bay is 1.01 km^2.
Calculate the area on the map, in square centimetres, of Gardens by the Bay.

Answer (a).......................................km [1]

(b).......................................cm^2 [2]

6 Without the use of a calculator and leaving your answers in standard form, evaluate

(a) $3.2\times10^{14}+7.9\times10^{13}$,

(b) $1.2\times10^{-17}-1.9\times10^{-18}$,

(c) $(3\times10^{20})^3$.

Answer (a)....................................... [1]

(b)....................................... [1]

(c)....................................... [1]

7 The first four terms of a sequence are 5, 11, 17 and 23.

(a) Write down the 8^{th} term in the sequence.

(b) Write down an expression for the general term of the sequence.

(c) Is the number 591 a term in the sequence? Justify your answer.

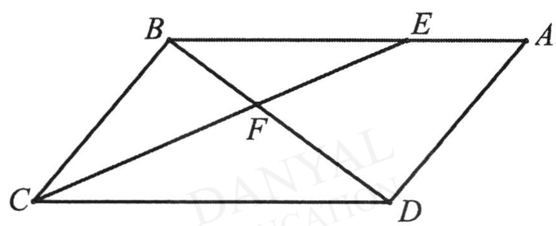
Answer (a).................................... [1]

(b).................................... [1]

(c)..

.... [1]

8 In the diagram below, $ABCD$ is a parallelogram such that $BE : CD = 2 : 3$.
The line BD and CE intersects at F.



(a) Show that ΔBFE is similar to ΔDFC. State clearly your reasons.

(b) Find

(i) $\dfrac{\text{Area of } \Delta BFE}{\text{Area of } \Delta DFC}$,

(ii) $\dfrac{\text{Area of } \Delta BFC}{\text{Area of } \Delta DFC}$.

Answer (a)..

....

.... [2]

(bi).................................... [1]

(bii).................................... [1]

9 The period of a pendulum, T seconds, is directly proportional to the square root of the pendulum's length, L metres.

(a) Given that $T = 1.2$s when $L = 0.36$m, form an equation connecting T and L.

(b) Find the percentage increase in T when L increases by 300%.

Answer (a).................................... [2]

(b).................................... [3]

10 There is roughly 250 billion stars in our galaxy, the milky way. The nearest planet that is a candidate for human habitation is in the star system of *Proxima Centauri*, approximately 12 light years away. Given that 1 light year is the distance that light travels in 1 year, and that the speed of light is 3.0×10^8 m/s,

(a) express 250 billion in standard form,

(b) calculate the distance from Earth to *Proxima Centauri*, giving your answer in standard form, correct to 2 significant figures.

(c) The fastest rocket can reach a speed of about 265 000 km/h. Calculate the number of years, correct to 2 significant figures, it would take to reach *Proxima Centauri*.

Answer (a).................................... [1]

(b)....................................m [2]

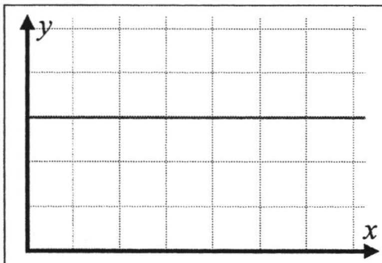
(c)....................................years [2]

11 The table below lists 5 of the E-Maths Paper 1 equations that Mr Chen wanted to test students to sketch for the Mid-Year Examinations.

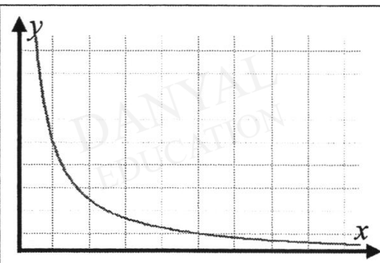
Equation	Equation
A	$y = 4\left(3^{-x}\right)$
B	$xy = 100$
C	$3x - 2y - 5 = 0$
D	$y = 10 - 3x^2$
E	$y = \pi(2 - x^2)^0$

On his way to print the questions, he dropped his answers and mixed up the 5 correct sketches with 4 more sketches that he prepared for the End-of-Year Examinations.

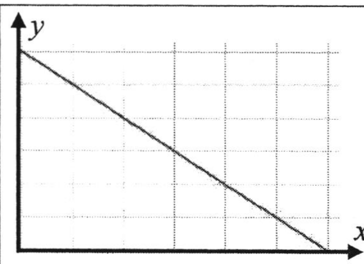
Examining the 9 sketches below, label the 5 sketches that match the equations above. [5]



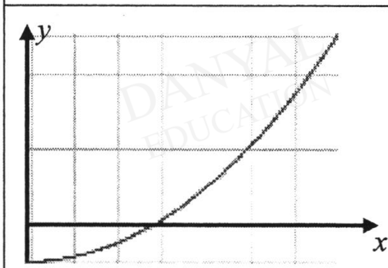
Equation ______



Equation ______



Equation ______



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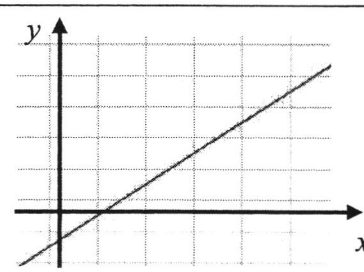
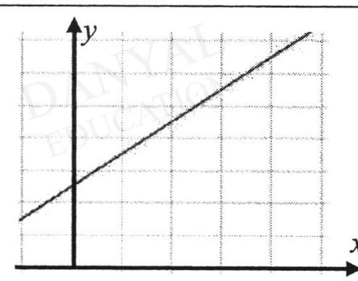
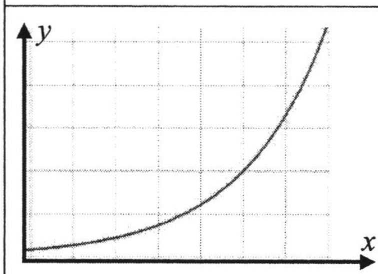


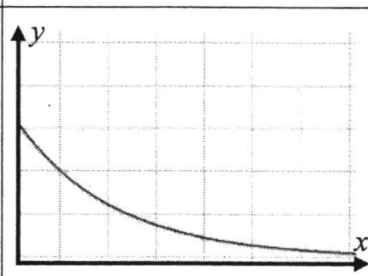
Equation ______



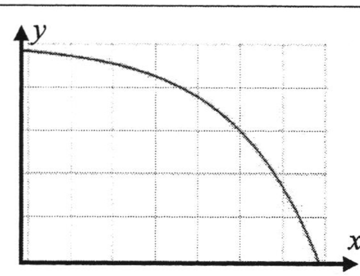
Equation ______



Equation ______



Equation ______



Equation ______

12 A, B and C are the points $(-1, 4)$, $(5, 7)$ and $(3, -4)$.

(a) Find the length of AB.

Answer (a)..............................units [2]

(b) Hence, show that ΔABC is a right-angled triangle.

Answer (b)..

....

....

....

.... [2]

(c) State the angle in ΔABC that is a right-angle.

Answer (c) Angle.............................. [1]

13 Given the line $2y+3x-4=0$ and the coordinate $P(-6, 7)$,

(a) Given also that the line intersects $y = 1$ at Q, find the gradient of PQ.

(b) Find the equation of a line that passes through P and $R(-1, 1)$.

(c) Find the area of triangle PQR.

Answer (a)...................................... [3]

(b)...................................... [3]

(c)..............................units^2 [1]

14 **(a)** Express $x^2 - 6x + 5$ in the form $(x - h)^2 + k$.

Answer (a).. [2]

(b) State the line of symmetry.

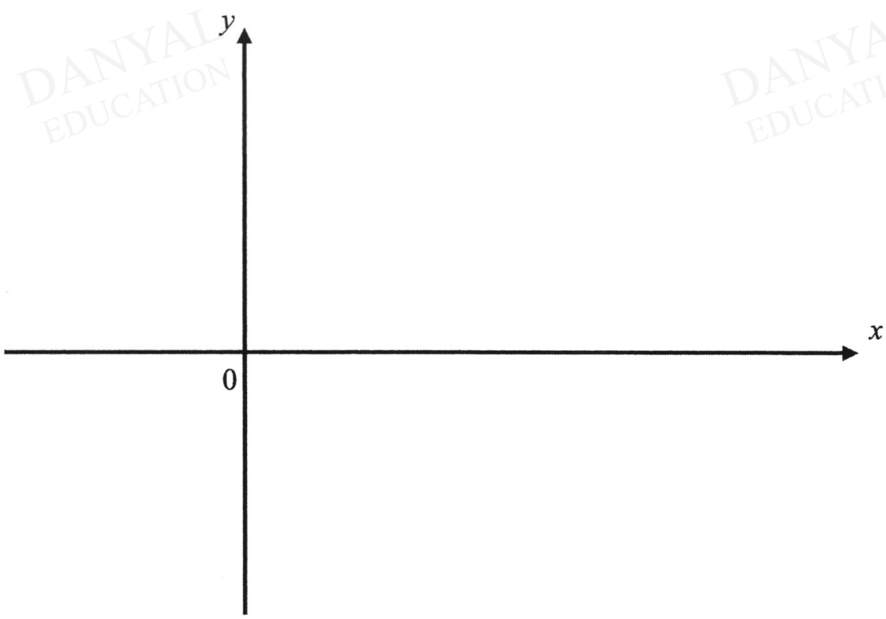
(b).. [1]

(c) **Using part (a)**, solve $x^2 - 6x + 5 = 0$

(c) $x =$ .... or .... [2]

(d) **Hence**, sketch $y = x^2 - 6x + 5$ on the axis below.
Label clearly, the intercepts, and turning point.

(d)



[3]

End of Paper

Class	Index No.

Candidate Name: ____________________

FUHUA SECONDARY SCHOOL

Secondary Three Express

Mid-Year Examination 2018

3E

MATHEMATICS **4048/02**
PAPER 2

Additional Materials:
Answer Paper (6)
Graph Paper (1)

DATE 8 May 2018
TIME 0755 – 0925
DURATION 1 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate writing paper provided.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **60**.

PARENT'S SIGNATURE	FOR EXAMINER'S USE		
	Units		/ 60
	Statements/Accuracy		
	Poor Presentation		

This question paper consists of 4 printed pages including this page.

Setter: *Mr Chia Chun Teck* *Vetter:* *Miss Winnifred Lim/Mr Chen HM*

[Turn Over

1 Given $-7 < 3x - 1 \leq x + 7$ and $-14 \leq 7y \leq 49$, and x and y are integers, find the smallest value of xy and the largest value of $(2x + y)(2x - y)$. [4]

2 (a) Simplify the fraction $\dfrac{2p-8p^3}{6p+3}$. [3]

(b) Given that $b = \sqrt{\dfrac{a(x^2-8)}{y}}$, express x in terms of a, b and y. [2]

(c) Given that $\dfrac{x+3y}{5x-4y} = \dfrac{2}{3}$, find the ratio $x : y$. [2]

3 (a) Factorise completely $a^2 + 16b^2 - 8ab - 2a + 8b$. [2]

(b) Solve the equation $\dfrac{1}{4x^2-4x+1} + \dfrac{3}{2x-1} = 4$. [3]

Hence, without solving, deduce the solutions for the equation

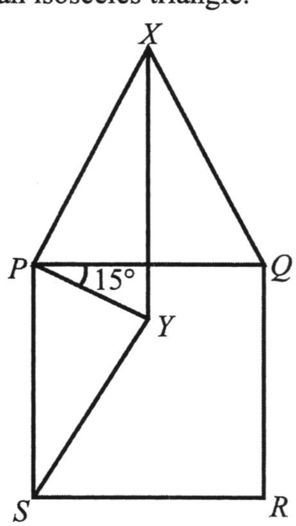
$$\frac{1}{(a-1)^2} + \frac{3}{a-1} = 4.$$ [2]

4 In the diagram below, $PQRS$ is a square and PQX is an equilateral triangle.
The line XY bisects $\angle PXQ$ and $\angle YPQ = 15°$.

(a) Prove that $\angle XPY = \angle SPY = 75°$. [2]

(b) Show that ΔXPY is congruent to ΔSPY. [3]

(c) Prove that ΔSPY is an isosceles triangle. [3]



5 **(a)** Simplify $\sqrt[3]{27a^9b^{-3}} \times \frac{1}{9}\left(a^{-\frac{1}{2}}b^{\frac{1}{4}}\right)^{-2} \div (ab^0)$, expressing your answer in positive indices. [3]

(b) Given that $7^a = 3$ and $7^b = 8$, find the value of 7^{2b-3a}. [2]

(c) Solve the following equations.

(i) $9^{x+5} = \frac{1}{729}$ [2]

(ii) $4^x \times 3^{2x} = 36$ [2]

6 Ken, Joshua and Muthu were running on a 400 m circular track. Ken started running from point O in an anti-clockwise direction with a speed of v m/s. At the same time, Joshua and Muthu started running from point O, but in a clockwise direction with speeds $(v + 3)$ m/s and $(v - 4)$ m/s respectively.

(i) Show that the time passed before Ken and Joshua meet each other on the track is $\frac{400}{2v+3}$ seconds. [1]

(ii) Find, in terms of v, the time passed before Ken and Muthu meet each other on the track. Leave your answer in simplest form. [1]

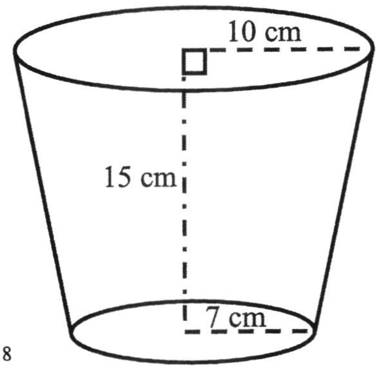
(iii) Given that Ken meets Muthu 24 seconds after passing Joshua, form an equation in terms of v and show that it simplifies to $6v^2 - 3v - 193 = 0$. [3]

(iv) Solve the equation $6v^2 - 3v - 193 = 0$. [3]

(v) Hence, find the time taken for Joshua to run one round around the track. [2]

7 Mr Chen wants to order some popcorn holder cups for the upcoming Annual Speech Day. The popcorn cups are in the form of truncated cones that are 15 cm high, with base and top radii 7 cm and 10 cm respectively. A sample of the cup is shown below.
Calculate the volume of each popcorn cup. [4]

[The volume of a cone is $\frac{1}{3}\pi r^2 h$.]



8 **Answer the whole of this question on a sheet of graph paper.**

A pebble was thrown from the top of a cliff next to the sea.

The height, h metres, of the pebble above sea level t seconds after it is released can be modelled by the equation $h = 3(8 + 5t - t^2)$.

Some corresponding values of t and h are given in the table below.

t	0	1	2	3	4	5	6	7
h	24	36	42	42	36	24	6	p

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \leq t \leq 7$.

Using a scale of 2 cm to represent 10 metres, draw a vertical h-axis for $-20 \leq h \leq 50$.

On your axes, plot the points given and join them with a smooth curve. [3]

(c) Use your graph to estimate the

(i) maximum height of the pebble above sea level, [1]

(ii) length of time that the pebble was more than 32 m above sea level. [2]

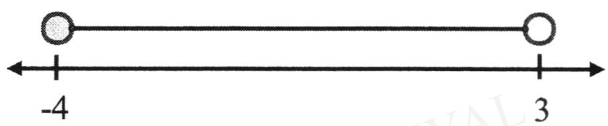
(iii) time taken for the pebble to hit the water. [1]

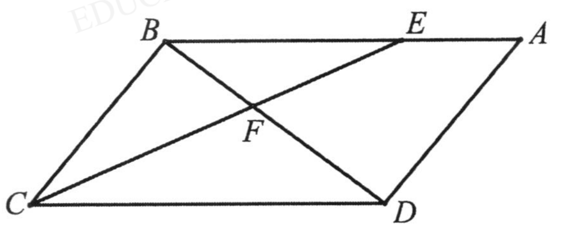
(d) By drawing a tangent, find the gradient of the curve at (4, 36).

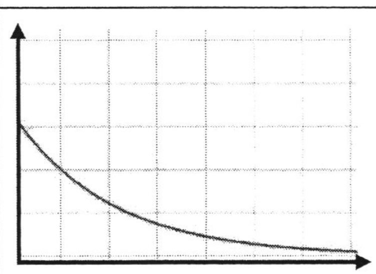
State the units of your answer. [3]

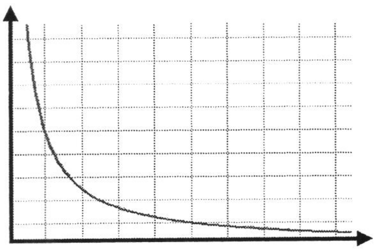
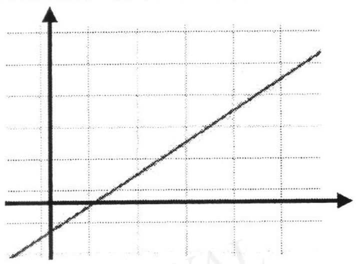
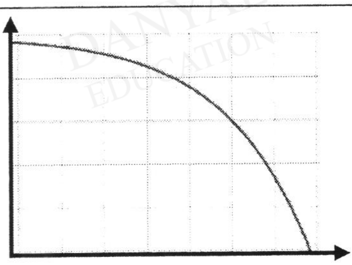
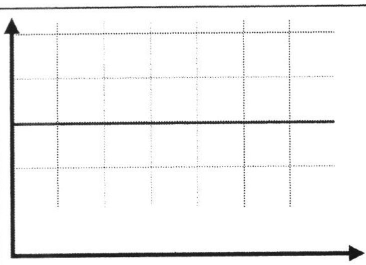
End of Paper

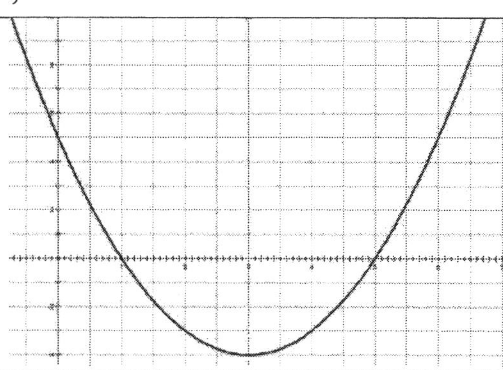
"You may be disappointed if you fail, but you are doomed if you don't try."
Beverly Sills

Qn	Solution	Marks
1a	$1485 = 3^3 \times 5 \times 11$	B1
1b	$k = 3 \times 5 \times 11$ $= 165$	B1
1c	$1485X = LCM \times HCF$ $X = \dfrac{5940 \times 45}{1485} = 180$	B1
2a	$x - 1 < 5 - x$ $2x < 6$ $x < 3$ $5 - x \leq 2x + 17$ $-12 \leq 3x$ $-4 \leq x$ $-4 \leq x < 3$	B2
2b		B1
3	Interior angle of equilateral triangle = 60° Interior angle of nonagon = $\dfrac{(9-2) \times 180}{9} = 140°$ Interior angle of unknown polygon = 160° Exterior angle of unknown polygon = 20° Number of sides = $\dfrac{360}{20} = 18$ sides.	M1 (Nonagon) M1 (Int. of Unknown) A1
4a	Let d be the distance that Billy travels. $3.90 + \dfrac{d}{0.4} \times 0.3 \leq 14$	B1
4b	$3.90 + \dfrac{3d}{4} \leq 14$ $15.6 + 3d \leq 56$ $3d \leq 40.4$ $d \leq 13\dfrac{7}{15} \approx 13.466$ Maximum distance = 13.2 km	M1 A1

5a	1cm : 200000cm 1cm : 2000m 1cm : 2km 1.6cm : 3.2km	B1
5b	1cm : 2km $1cm^2 : 4km^2$ $0.25cm^2 : 1km^2$ $0.2525cm^2 : 1.01km^2$	M1 (Square) A1
6a	$3.2\times10^{14}+7.9\times10^{13}$ $=3.2\times10^{14}+0.79\times10^{14}$ $=3.99\times10^{14}$	B1
6b	$1.2\times10^{-17}-1.9\times10^{-18}$ $=1.2\times10^{-17}-0.19\times10^{-17}$ $=1.01\times10^{-17}$	B1
6c	$(3\times10^{20})^3$ $=27\times10^{60}$ $=2.7\times10^{61}$	B1
7a	$5,11,17,23,29,35,41,47$ Ans: 47	B1
7b	$T_n=6n-1$	B1
7c	$T_n=591$ $6n-1=591$ $6n=592$ $n=98\frac{2}{3}$ No. Since n has to be a positive integer, 591 is not a term in the sequence.	DM1
8a	 $\angle FEB=\angle FCD$ (Alt Angles, BE//CD) $\angle EFB=\angle CFD$ (Vertically Opposite Angles) $\therefore \Delta BFE$ is similar to ΔDFC (AA)	B2 (Reasons)
8bi	$\frac{Area\ of\ \Delta BFE}{Area\ of\ \Delta DFC}$ $=\left(\frac{BE}{CD}\right)^2=\frac{4}{9}$	B1

8bii	$\frac{Area\ of\ \Delta BFC}{Area\ of\ \Delta DFC}$ $=\frac{\frac{1}{2}\times BF\times h}{\frac{1}{2}\times DF\times h}=\frac{BF}{DF}$ (Triangles of common height) $=\frac{2}{3}$	B1
9a	$T\propto\sqrt{L}$ $T=k\sqrt{L}$ $1.2=k\sqrt{0.36}$ $k=\frac{1.2}{\sqrt{0.36}}=2$ $T=2\sqrt{L}$	M1 (Form Equation) A1
9b	L increased by 300% to $4L$ $T_f=2\sqrt{4L}=4\sqrt{L}$ Percentage change $=\frac{4\sqrt{L}-2\sqrt{L}}{2\sqrt{L}}\times100\%=100\%$	M1 (New L) M1 (New T) A1
10a	250 billion $=250\times10^9$ $=2.50\times10^{11}$	B1
10b	$3\times10^8 m/s\times60s\times60\min\times24h\times365.25days\times12years$ $=1.13607\times10^{17}m$ $\approx1.1\times10^{17}m$	M1 (Form expression) A1
10c	Time taken $=\frac{1.13607\times10^{17}m}{265000km/h}$ $=\frac{1.13607\times10^{17}m}{2.65\times10^8 m/h}$ $=428705660h$ $=17862730days$ $=48905.5years$ $\approx49000years$ (2s.f.)	M1 A1
11A	 Equation **A**	B1

11B	 Equation **B**	B1
11C	 Equation **C**	B1
11D	 Equation **D**	B1
11E	 Equation **E**	B1
12a	$\lvert AB\rvert=\sqrt{(5-(-1))^2+(7-4)^2}$ $=\sqrt{6^2+3^2}$ $=3\sqrt{5}units$ $=6.7082$ $\approx 6.71units$	M1 A1
12b	$\lvert BC\rvert=\sqrt{(5-3)^2+(7-(-4))^2}$ $=\sqrt{2^2+11^2}=5\sqrt{5}units$ $\lvert AC\rvert=\sqrt{(-1-3)^2+(4-(-4))^2}$ $=\sqrt{4^2+8^2}=4\sqrt{5}units$ $\because \lvert AC\rvert^2+\lvert AB\rvert^2=\left(3\sqrt{5}\right)^2+\left(4\sqrt{5}\right)^2=\left(5\sqrt{5}\right)^2=\lvert BC\rvert^2$ $\therefore \Delta ABC$ is a right-angled triangle (Converse of Pythagoras Thm)	M1 (Find BC) M1 (Find AC) A1 (Reason)
12c	Angle *BAC* is the right angle.	B1

13a	When $y = 1$ $2y+3x-4=0$ $2(1)+3x-4=0$ $3x=2$ $x=\frac{2}{3}$ $Q\left(\frac{2}{3},1\right)$ $m_{PQ}=\frac{7-1}{-6-\frac{2}{3}}$ $=-\frac{9}{10}$	M1 (Find x) M1 (Gradient Formula) A1
13b	$m_{PR}=\frac{7-1}{-6-(-1)}$ $=-\frac{6}{5}$ Subst (-6, 7) and m = -1.2 in $y = mx + c$ $7=-\frac{6}{5}(-6)+c$ $c=-\frac{1}{5}$ $y=-\frac{6}{5}x-\frac{1}{5}$	M1 (Find m) M1 (Find c) A1
13c	Area $=\frac{1}{2}\times(7-1)\times\left(1+\frac{2}{3}\right)=5units^2$	B1
14a	x^2-6x+5 $=x^2-6x+9-4$ $=(x-3)^2-4$	B2
14b	$x=3$	B1
14c	$(x-3)^2=4$ $x-3=\pm 2$ $x=3\pm 2=1,5$	B2 (Must use part (a))
14d		B2 - labels of critical points (Turning point, y-intercept, x-intercept) B1 - Shape/Smoothness

Solutions to MYE 2018 Paper 2

1 Smallest value of $xy = -8$

Largest value of $(2x + y)(2x - y) = 64$

2(a) $\dfrac{2p(1-2p)}{3}$

2(b) $x = \pm\sqrt{\dfrac{b^2 y}{a} + 8}$

2(c) $x : y = 17 : 7$

3(a) $(a - 4b)(a - 4b - 2)$

3(b) $x = \dfrac{3}{8}$ or $x = 1$

$a = \dfrac{3}{4}$ or $a = 2$

4(a) Proof

4(b) $XP = PQ$ (PQX is equilateral Δ)

$= SP$ ($PQRS$ is a square)

In ΔXPY and ΔSPY,

$XP = SP$ (proven above)

$\angle XPY = \angle SPY = 75°$ (proven in part **a**)

$PY = PY$ (common side)

ΔXPY is congruent to ΔSPY. (SAS)

4(c) $\angle PXY = \dfrac{60°}{2}$ (line XY bisects $\angle PXQ$)

$= 30°$

$\angle PSY = \angle PXY$ ($\Delta XPY \equiv \Delta SPY$)

$= 30°$

$\angle PYS = 180° - 75° - 30°$ ($\angle$ sum of Δ)

$= 75°$

$\angle PYS = \angle SPY = 75°, \therefore$ ΔSPY is an isosceles triangle. (proven)

5(a) $\dfrac{a^3}{3b^{\frac{3}{2}}}$

5(b) $7^{2b-3a} = 2\dfrac{10}{27}$

5(ci) $x = -8$

5(cii) $x = 1$

6(i) Time taken $= \dfrac{400}{v+3+v} = \left(\dfrac{400}{2v+3}\right)$ seconds

6(ii) Time taken $= \dfrac{400}{v-4+v} = \dfrac{400}{2v-4}$

$= \left(\dfrac{200}{v-2}\right)$ seconds

6(iii) $\left(\dfrac{200}{v-2}\right) - \left(\dfrac{400}{2v+3}\right) = 24$

… $6v^2 - 3v - 193 = 0$ (shown)

6(iv) $v = -5.43$ (3 sig. fig.) or 5.93 (3 sig. fig.)

6(v) Time taken by Joshua = 44.8 seconds (3 sig. fig.)

7 Volume of each popcorn cup = 3440 cm^3 (3 sig. fig.)

8(a) $p = -18$

8(ci) maximum height of pebble ≈ 43 m [Accept 42.5 m to 43 m]

8(cii) Length of time ≈ 3.8 s [Accept 3.6 s to 4 s]

8(ciii) Time taken ≈ 6.3 s [Accept 6.2 s to 6.4 s]

8(d) Gradient ≈ −9 m/s [Accept −9.9 m/s to −8.1 m/s]

E Maths MYE 2018 Paper 2

Q1.

$-7 < 3x - 1 \leq x + 7$

$-7 < 3x - 1$ and $3x - 1 \leq x + 7$

$-6 < 3x$ $\quad$ $2x \leq 8$

$x > -2$ $\quad$ $x \leq 4$

$\therefore -2 < x \leq 4$ <M1>

$-14 \leq 7y \leq 49$

$-2 \leq y \leq 7$ <M1>

Smallest value of $xy = (4)(-2)$

$= \underline{-8}$ # <B1>

largest value of $(2x+y)(2x-y)$

= largest value of $(4x^2 - y^2)$

$= 4(4)^2 - 0^2$

$= \underline{64}$ # <B1>

Q2(a)

$$\frac{2p-8p^3}{6p+3} = \frac{2p(1-4p^2)}{3(2p+1)}$$ <M1>

$$= \frac{2p\cancel{(1+2p)}(1-2p)}{3\cancel{(2p+1)}}$$ <M1>

$$= \underline{\frac{2p(1-2p)}{3}}$$ # <A1>

(b)

$$b = \sqrt{\frac{a(x^2-8)}{y}}$$

$$b^2 = \frac{a(x^2-8)}{y}$$ <M1>

$$\frac{b^2y}{a} = x^2 - 8$$

$$x^2 = \frac{b^2y}{a} + 8$$

$$\underline{x = \pm\sqrt{\frac{b^2y}{a} + 8}}$$ # <A1> → must have ±!

(c)

$$\frac{x+3y}{5x-4y} = \frac{2}{3}$$ ✓

$$3x + 9y = 10x - 8y$$

$$17y = 7x$$

<M1>

$$\frac{17}{7} = \frac{x}{y}$$

$$\Rightarrow \underline{x : y = 17 : 7}$$ # <A1>

Q3 (a) $a^2 + 16b^2 - 8ab - 2a + 8b$

$= (a-4b)^2 - 2(a-4b)$ <M1>

$= \underline{(a-4b)(a-4b-2)}$ # <A1>

(b) $\frac{1}{4x^2-4x+1} + \frac{3}{2x-1} = 4$

$\frac{1}{(2x-1)^2} + \frac{3}{2x-1} = 4$ <M1> — Factorising $4x^2-4x+1$

$1 + 3(2x-1) = 4(2x-1)^2$

$1 + 6x - 3 = 4(4x^2 - 4x + 1)$

$6x - 2 = 16x^2 - 16x + 4$

$16x^2 - 22x + 6 = 0$

$8x^2 - 11x + 3 = 0$ <M1>

$(8x-3)(x-1) = 0$

$\underline{x = \frac{3}{8} \quad \text{or} \quad x = 1}$ # <A1>

$\frac{1}{(a-1)^2} + \frac{3}{a-1} = 4$

By comparing: $a = 2x$

$\therefore a = 2\left(\frac{3}{8}\right)$ or $a = 2$ } <M1>

$a = \frac{3}{4}$

$\therefore a = \frac{3}{4}$ or $a = 2$ # <A1>

Q4(a) $X\hat{P}Q = 60^\circ$ ($\because$ PQX is equilateral $\triangle$)

$\therefore \angle XPY = 60^\circ + 15^\circ$

$= 75^\circ$ <A1>

$S\hat{P}Y = 90^\circ - 15^\circ$ ($\because$ PQRS is a square)

$= 75^\circ$ <A1>

$\therefore X\hat{P}Y = S\hat{P}Y = 75^\circ$ (proven) #

(b) $XP = PQ$ ($\because$ PQX is equilateral $\triangle$)

$= SP$ ($\because$ PQRS is a square)

$\therefore XP = SP$ <M1>

In $\triangle XPY$ and $\triangle SPY$,

$XP = SP$ (proven above)

$X\hat{P}Y = S\hat{P}Y = 75^\circ$ (proven in part (a))

$PY = PY$ (common side)

Correct test with correct order & reasons! — <M1>

$\therefore \triangle XPY \equiv \triangle SPY$ (SAS) # <A1> → must state test SAS!

(c) $P\hat{X}Y = \frac{60^\circ}{2}$ (line XY bisects $\angle PXQ$)

$= 30^\circ$ <M1>

$P\hat{S}Y = P\hat{X}Y$ ($\because \triangle XPY \equiv \triangle SPY$)

$= 30^\circ$ <M1>

$P\hat{Y}S = 180^\circ - 75^\circ - 30^\circ$ ($\angle$ sum of $\triangle$)

$= 75^\circ$ <A1>

Q5(a) $\sqrt[3]{27a^9b^{-3}} \times \frac{1}{9}(a^{-\frac{1}{2}}b^{\frac{1}{4}})^{-2} \div (ab^0)$

$= 3a^3b^{-1} \times \frac{1}{9}(ab^{-\frac{1}{2}}) \div a$ <M2> — $3a^3b^{-1}$; $\frac{1}{9}(ab^{-\frac{1}{2}}) \div a$

$= \frac{1}{3}a^3b^{-\frac{3}{2}}$

$= \frac{a^3}{3b^{\frac{3}{2}}}$ # <A1>

(b) Given $7^a = 3$, $7^b = 8$

$7^{2b-3a} = \frac{7^{2b}}{7^{3a}}$

$= \frac{(7^b)^2}{(7^a)^3}$ } <M1>

$= \frac{8^2}{3^3}$

$= 2\frac{10}{27}$ # <A1>

(c)(i) $9^{x+5} = \frac{1}{729}$

$9^{x+5} = 9^{-3}$ <M1>

Comparing: $x + 5 = -3$

$x = -8$ # <A1>

(ii) $4^x \times 3^{2x} = 36$

$4^x \times 9^x = 36$

$36^x = 36$ } <M1>

$\therefore x = 1$ # <A1>

OR $4^x \times 3^{2x} = 36$

$2^{2x} \times 3^{2x} = 6^2$

$6^{2x} = 6^2$ } <M1>

$\therefore 2x = 2$

$x = 1$ // <A1>

Q6 (i) Time passed $= \dfrac{400}{V+3+V}$

$= \left(\dfrac{400}{2V+3}\right)$ seconds (shown) # <A1>

(ii) Time passed $= \dfrac{400}{V-4+V}$

$= \dfrac{400}{2V-4}$

$= \left(\dfrac{200}{V-2}\right)$ seconds # <B1> need to simplify!

(iii) $\dfrac{200}{V-2} - \dfrac{400}{2V+3} = 24$ <M1>

$200(2V+3) - 400(V-2) = 24(V-2)(2V+3)$

$\cancel{400V} + 600 - \cancel{400V} + 800 = 24(2V^2 - V - 6)$ <M1>

$1400 = 24(2V^2 - V - 6)$

$175 = 3(2V^2 - V - 6)$

$175 = 6V^2 - 3V - 18$

$6V^2 - 3V - 193 = 0$ (shown) # <A1>

(iv) $V = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(6)(-193)}}{2(6)}$ <M1>

$V = \dfrac{3 \pm \sqrt{4641}}{12}$

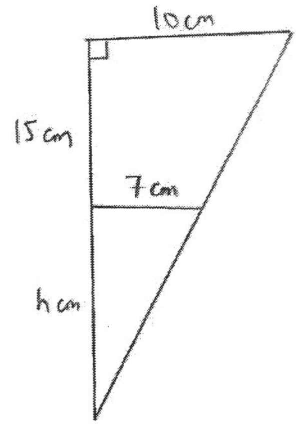
$V = 5.927073777$ or -5.427073777

$V \approx 5.93$ (3 sig. fig.) or -5.43 (3 sig. fig.)

$\therefore V \approx -5.43$ or 5.93 # <A1, A1>

(v) Time taken by Joshua $= \dfrac{400}{5.927073777 + 3}$ $(\because V > 0)$ <M1>

Q7. Using similar triangles,



$$\frac{h}{h+15} = \frac{7}{10} \quad \langle M1 \rangle$$

$$10h = 7h + 105$$

$$3h = 105$$

$$h = 35 \quad \langle M1 \rangle$$

Volume of each popcorn cup

$$= \frac{1}{3}\pi(10)^2(35+15) - \frac{1}{3}\pi(7)^2(35) \quad \langle M1 \rangle$$

$$= 3440.043956$$

$$\approx \underline{3440 \text{ cm}^3 \text{ (3 sig. fig.)}} \# \quad \langle A1 \rangle$$

Q8. $h = 3(8 + 5t - t^2)$

(a) When $t = 7$, $p = 3(8 + 5(7) - 7^2)$

$p = -18$ # <B1>

(c)(i) From graph, max. height of pebble ≈ 43 m # <A1> [Accept 42.5 m — 43 m]

(ii) Length of time ≈ 4.4 − 0.6 <M1>

≈ 3.8 s # <A1> [Accept 3.6 s → 4 s]

(iii) time taken ≈ 6.3 s # <A1> [Accept 6.2 s → 6.4 s]

(d) (3, 45) and (6, 18) lies on tangent drawn.

gradient of curve ≈ $\frac{45 - 18}{3 - 6}$ <M1> → correct tangent drawn.

≈ −9 m/s # <A1>, <A1> → correct units.

[Accept −10 m/s → −8 m/s]