BEATTY SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2018

SUBJECT : Mathematics | **LEVEL : Secondary 3 Express**

PAPER : 4048 / 01 | **DURATION : 1 hour 30 minutes**

SETTER : Ms Joanna Chong | **DATE : 5 October 2018**

CLASS :	NAME :	REG NO :

.....

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **60**.

For Examiner's Use
60

This paper consists of 13 printed pages (including this cover page)

[Turn over

Mathematical formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length = $r\,\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **ALL** questions.

1 Evaluate $\dfrac{\sqrt[3]{3.16\times10^{-6}}}{1.8(2.5\times10^{8})}$ and give your answer in standard form.

Answer ..... [1]

---

2 Simplify $\dfrac{3x}{x^2-4}-\dfrac{2}{2-x}$.

Answer .....[3]

---

3 Given that the three points (3, 4), (2, k) and (7, 5) are collinear. Find the value of k.

Answer k =.....[2]

---

4 Andrew invested some money in a savings account for 5 years.
The rate of compound interest was fixed at 0.3% per annum.
At the end of 5 years, there was $ 875 000 in his account.

How much interest did Andrew earn at the end of 5 years?

Answer $..... [3]

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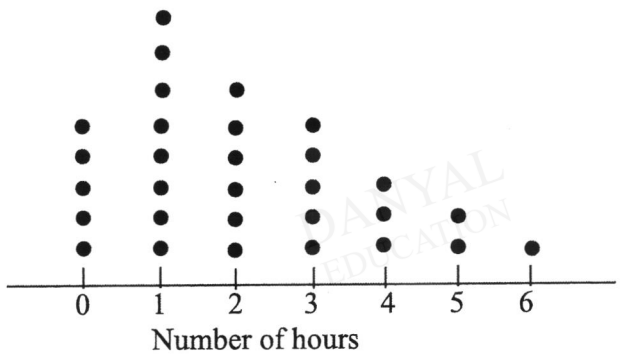
5 The dot diagram shows the number of hours a group teenagers spend on the handphone in a day.



(a) For the above distribution, find
(i) the mode,
(ii) the mean,
(iii) the median.

Answer *(a)(i)* .....hour(s) [1]

(ii) .....hour(s) [1]

(iii) ..... hour(s) [1]

(b) If a teenager is picked at random from the group, what is the probability that this teenager spends at least 4 hours on the handphone in a day?

Answer *(b)* ..... [1]

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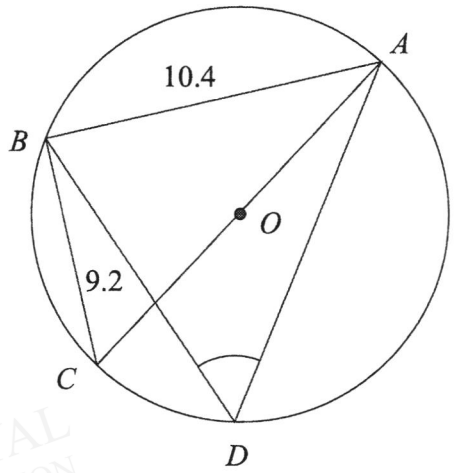
6 In the diagram, O is the centre of the circle $ABCD$. $AB = 10.4$ cm and $BC = 9.2$ cm. Find angle ADB, stating your reasons clearly.



Answer angle ADB =..... [3]

---

7 Factorise completely

(a) $75d^2 - 3$,

(b) $4p^2 - 4pr + r^2 - 2pq + rq$.

Answer *(a)* ..... [2]

(b) ..... [2]

---

8 **(a)** Simplify $\left(\frac{a^3}{3b}\right)^{-2} \div a^2b^5 \times \left(\frac{125a^4b^2}{5}\right)^0$, leaving your answer in positive indices.

(b) Given that $\sqrt{\frac{k^2}{4}} = 8^{-\frac{1}{3}}$, find the value of k.

Answer *(a)* ..... [3]

(b) k =..... [2]

---

9 **(a)** Solve the inequality $\frac{2}{3}x - 1 < x \leq 2(8 - x)$.

(b) Write down largest rational number which satisfies $\frac{2}{3}x - 1 < x \leq 2(8 - x)$.

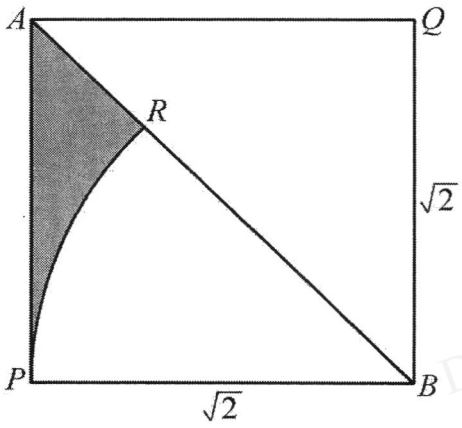
Answer *(a)* ..... [3]

(b) ..... [1]

---

10 In the diagram, $AQBP$ is a square of side $\sqrt{2}$ cm. PR is an arc of a circle with centre at B and radius $\sqrt{2}$ cm.

Find the area of the shaded region APR, leaving your answer in terms of π.

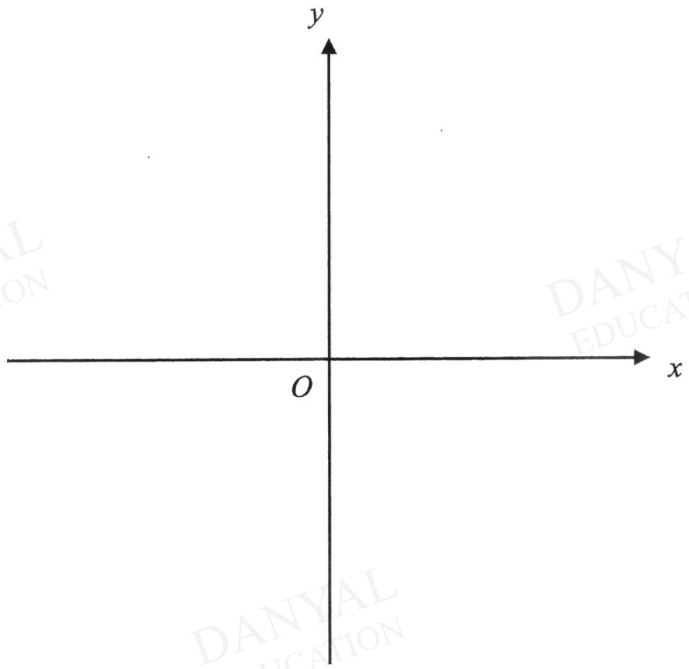


Answer .....cm^2 [3]

11 **(a)** Express x^2+4x+7 in the form $(x+h)^2+k$, where h and k are constants.

(b) Hence, sketch the graph of $y=x^2+4x+7$ on the axes below, indicating its turning point and intercept(s) clearly. [3]

Answer (b)



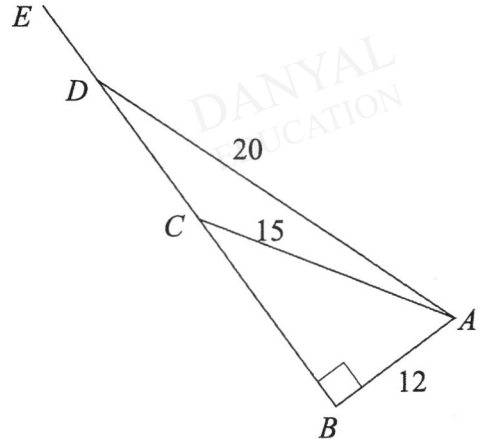
Answer *(a)* ..... [1]

---

12 In the diagram, $\angle ABC = 90°$, $AB = 12$ cm, $AC = 15$ cm, $AD = 20$ cm and $BCDE$ is a straight line. Calculate, giving each answer as a fraction,

(a) $\sin \angle ADB$,

(b) $\cos \angle ACD$.



Answer *(a)* .....[1]

(b)..... [2]

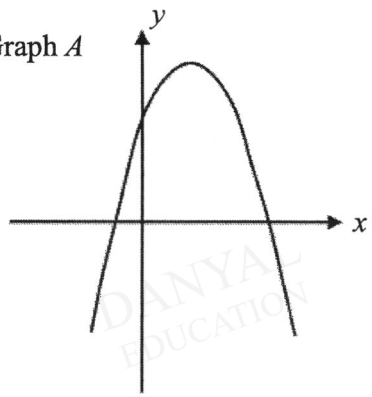
13 State the graph that corresponds to each of following equation.

(a) $y = 3x - 7$,

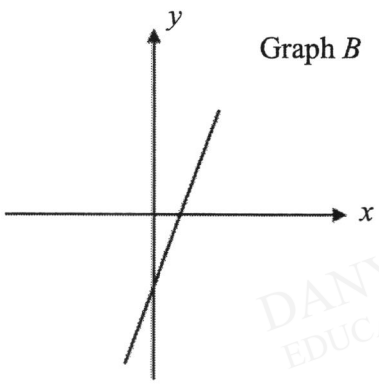
(b) $y = \frac{5}{x}$,

(c) $y = 2 - x^2$.

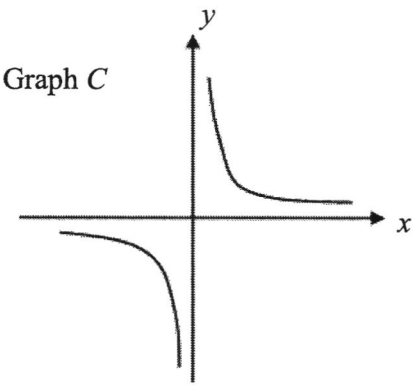
Graph *A*



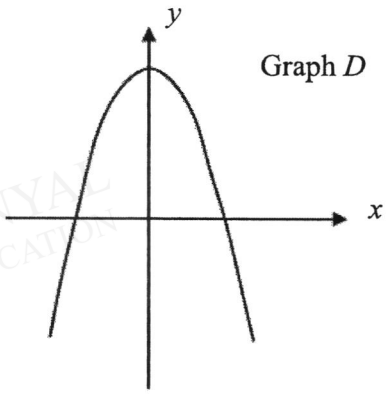
Graph *B*



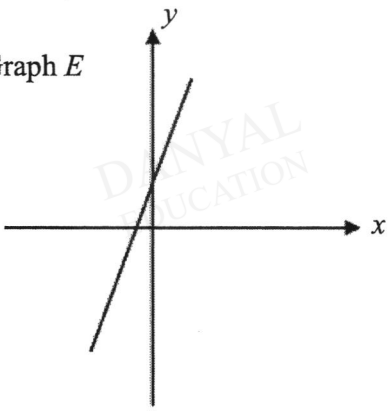
Graph *C*



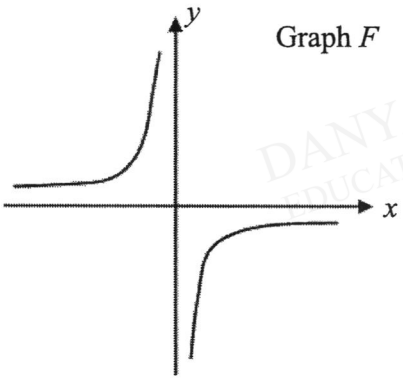
Graph *D*



Graph *E*



Graph *F*



Answer *(a)* .....[1]

(b) .....[1]

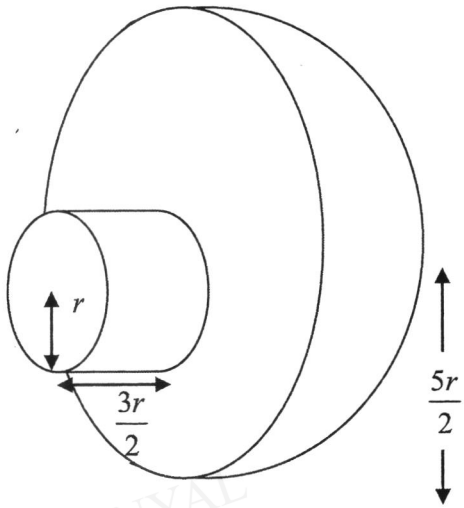
(c) .....[1]

14 The solid shown is made from a cylinder and a hemisphere.

The cylinder has a radius of r and height $\frac{3r}{2}$.

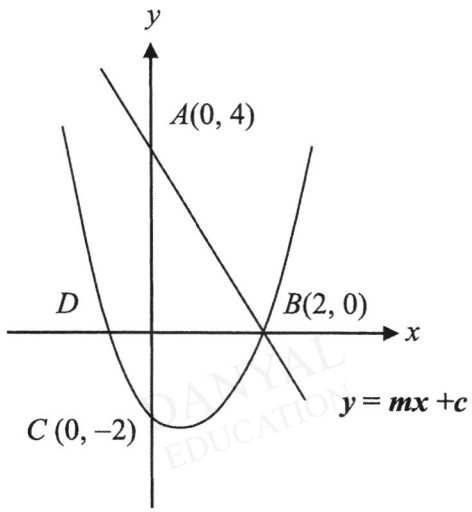
The hemisphere has a radius of $\frac{5r}{2}$.

Find an expression, in terms of π and r, for the total surface area of the solid.



Answer .....cm^2 [3]

15 The diagram below shows a straight line $y = mx + c$ intersecting the y-axis at $A\ (0, 4)$ and x-axis at $B\ (2,0)$.
The curve $y = (x-2)(x+k)$ cuts the x-axis at B and D and the y-axis at $C\ (0, -2)$.



(a) Find

(i) the value of m and of c.

(ii) the length of AB.

(iii) the value of k.

Answer *(a)(i)* m =.....c = ... [2]

(a)(ii) AB =.....[1]

(a)(iii) k =.....[1]

(b) A second curve with the equation $y = ax^2 + bx + d$ cuts the x-axis at points B and D.
Determine whether the value of $b^2 - 4ad$ is positive, negative or zero.
Explain your answer.

Answer $b^2 - 4ad$ is .....

..... [1]

16 Diagram I shows an open metal pail which is in the shape of a frustum. The diameters of the two circular ends are 45 cm and 15 cm. The height of the pail is 40 cm.

The curved surface of the frustum can be found using the model in Diagram II.

(a) By using appropriate triangles in Diagram II, find the value of h.

(b) Hence, find the total area of metallic sheet used to make the pail.

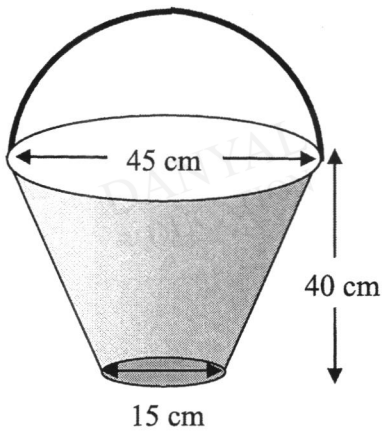


Diagram I

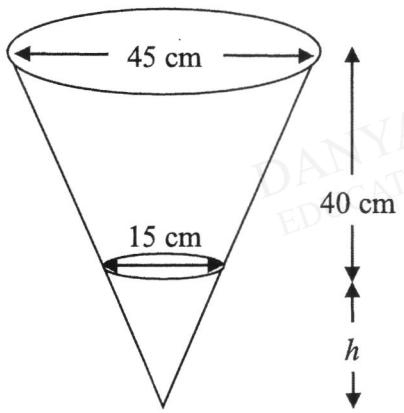


Diagram II

Answer *(a)* h =.....cm [2]

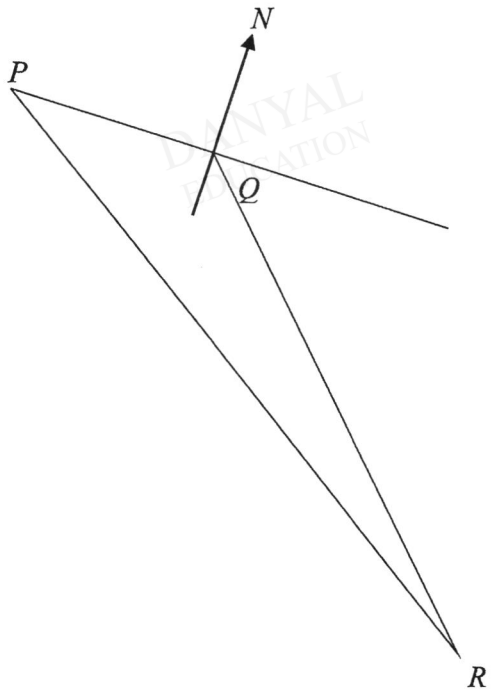
(b) .....cm^2 [4]

17 In the diagram, P, Q and R are three ports. P is 900 m due west of Q. At 07 40, a ship, Pioneer, left P and travelled to R on a bearing of 130°. At the same time, another ship, Queen, left Q and travelled at an average speed of 15 km/h to R. It arrived at 08 10 on the same day.

Calculate

(a) the distance QR, in metres,

(b) the bearing of Q from R.



Answer *(a)* QR =.....m [1]

(b).....[3]

BEATTY SECONDARY SCHOOL
END OF YEAR EXAMINATION 2018

SUBJECT : Mathematics **LEVEL : Sec 3 Express**

PAPER : 4048/2 **DURATION : 2 hours**

SETTER : Mrs Samsol **DATE : 10 October 2018**

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is **80**.

You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** Simplify $\dfrac{4xy-8x^2}{y^2-4x^2}$. [2]

(b) It is given that $a=\dfrac{3b+c}{b-5c}$.

(i) Find a when $b=3$ and $c=-1$. [1]

(ii) Express b in terms of a and c. [2]

(c) Solve the equation $\dfrac{6}{2x+5}=x-3$. [3]

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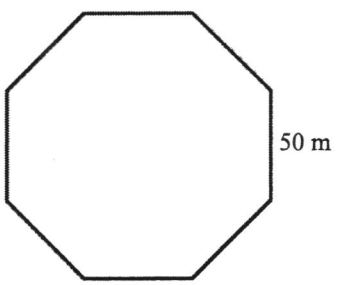
2 **(a)** The force, F newtons, between two magnets is inversely proportional to the square of the distance, d centimetres, between them.
When the force is 25 newtons, the distance is 3 centimetres.

(i) Find the equation connecting F and d. [2]

(ii) The force between the magnets is 10 newtons.
Find the distance between the magnets. [2]

(iii) For two pairs of identical magnets, the ratio of the distances between them is 2 : 3.
Find the ratio of the forces between the two pairs of magnets. [2]

(b) The floor of a recreation hall is in the shape of a regular octagon with sides of length 50 m.



Find the floor area of this hall. [5]

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3 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = \frac{x}{2}(2 + 3x - x^2) .$$

Some corresponding values of x and y are given in the table below.

x	−2	−1	−0.4	0	1	2	2.3	3	4
y	8	1	p	0	2	4	4.2	3	− 4

(a) Find the exact value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-2 \leq x \leq 4$.
Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis of $-6 \leq y \leq 10$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

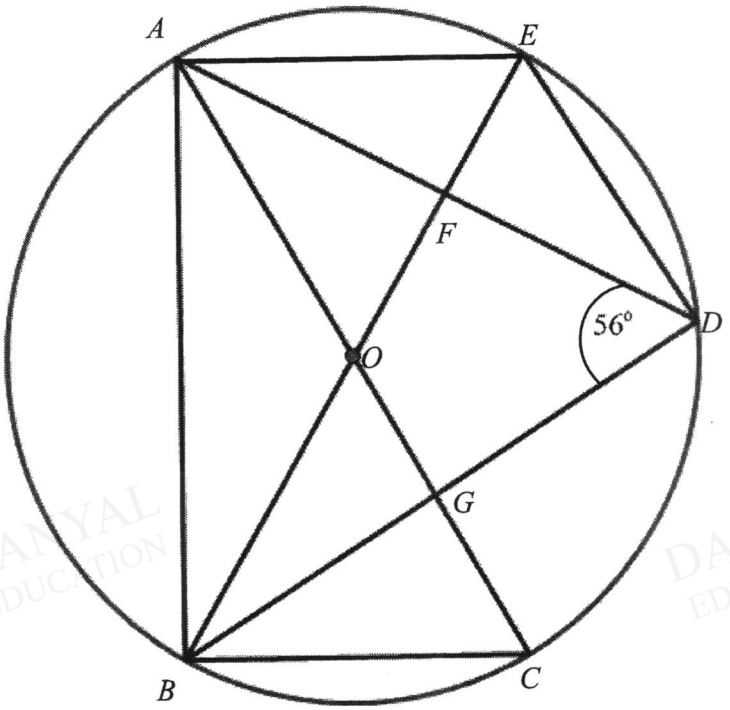
(c) Use your graph to find the solutions of the equation $\frac{x}{2}(2 + 3x - x^2) = 4$. [2]

(d) By drawing a tangent, find the gradient of the curve at the point $(-1, 1)$. [2]

(e) (i) On the same axes, draw the line with gradient $\frac{1}{3}$ that passes through the point with coordinates $(3, 3)$. [1]

(ii) Write down the x-coordinates of the points where this line intersects the curve. [2]

4



The diagram shows a circle $ABCDE$, centre O.
AD and BE intersect at the point F while AC and BD intersect at the point G.
AC and BE pass through O and BE bisects angle ABD.
Angle $ADB = 56°$

Find, giving reasons for each answer,

(i) angle ACB, [1]

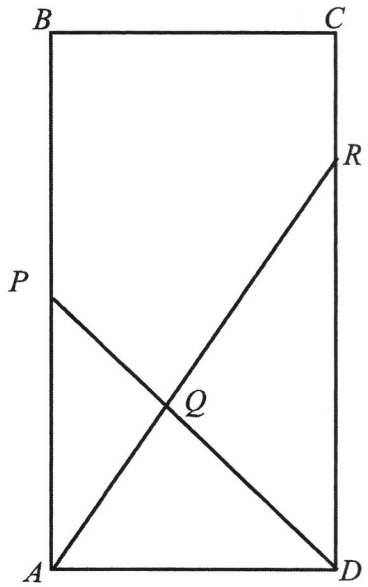
(ii) angle AOB, [1]

(iii) angle ADE, [1]

(iv) angle AED, [2]

(v) angle BGC. [1]

5 $ABCD$ is a rectangle. P is the mid-point of AB and R is a point on CD. AR and DP intersect at the point Q.



(a) Show that triangles AQP and RQD are similar. [2]

(b) Given that $DR = 3RC$, find

(i) $\dfrac{AP}{DR}$, [1]

(ii) $\dfrac{\text{Area of } \Delta AQP}{\text{Area of } \Delta RQD}$, [1]

(iii) $\dfrac{\text{Area of triangle } ARD}{\text{Area of rectangle } ABCD}$. [1]

(c) Given that the area of triangle $AQP = 25.5$ cm^2, find the total area of triangles AQP, AQD and DQR. [2]

6 A water storage tank has a capacity of 8000 litres.

(a) A large pump can empty water from the tank at a rate of x litres per minute.

Write down an expression, in terms of x, for the number of minutes the pump would take to empty a full tank. [1]

(b) A small pump can empty water from the tank at a rate which is 50 litres per minute less than the large pump.

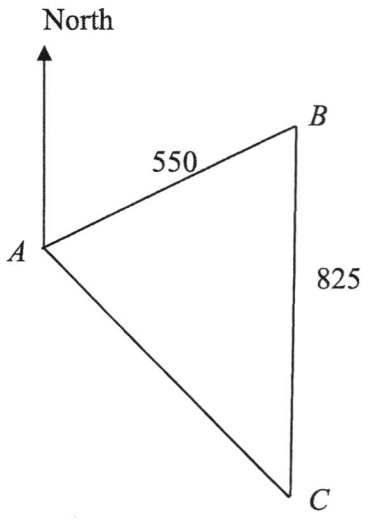
Write down an expression, in terms of x, for the number of minutes the small pump would take to empty a full tank. [1]

(c) It takes 35 minutes longer to empty the tank using the small pump than it does to empty it using the large pump.
Write down an equation in x to represent this information, and show that it reduces to $7x^2 - 350x - 80\,000 = 0$. [3]

(d) Solve the equation $7x^2 - 350x - 80\,000 = 0$, giving your solutions correct to 2 decimal places. [3]

(e) Find the time taken to empty a full tank of water using the small pump.
Give your answer in minutes and seconds, correct to the nearest second. [2]

7 In the diagram, A is a point at sea level at the foot of a vertical cliff.
Two buoys, B and C, are on the surface of the sea and C is due south of B.
$AB = 550$ m and $BC = 825$ m.
The bearing of B from A is $065°$.



(a) Calculate

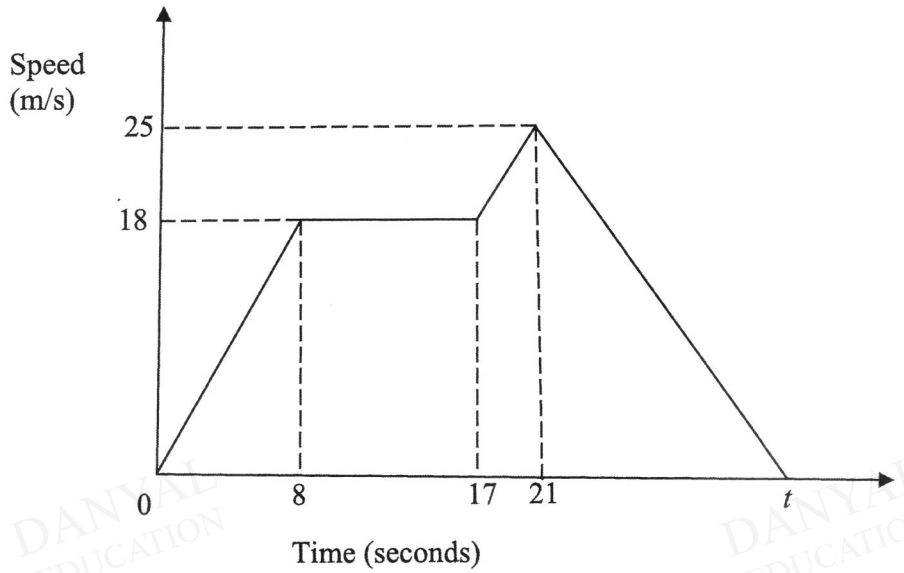
(i) AC, [2]

(ii) the bearing of A from C, [3]

(iii) the shortest distance from B to AC. [2]

(b) P is a point at the top of the cliff vertically above A.
The angle of depression of B from P is $9°$.
Calculate the angle of elevation of P from C. [3]

8 The diagram shows the speed-time graph for a car journey.

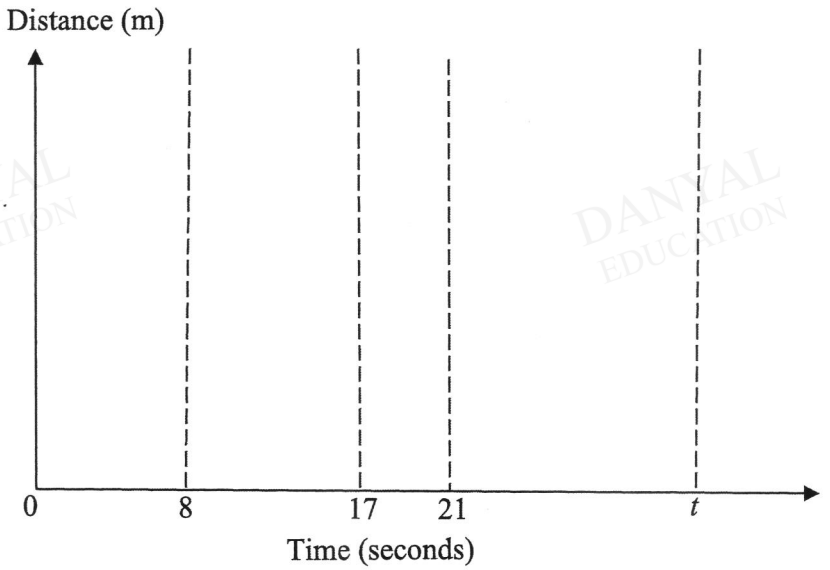


(a) The maximum speed of the car was 25 m/s.
Change 25 m/s into km/h. [1]

(b) Calculate the acceleration of the car after 6 seconds.
[1]

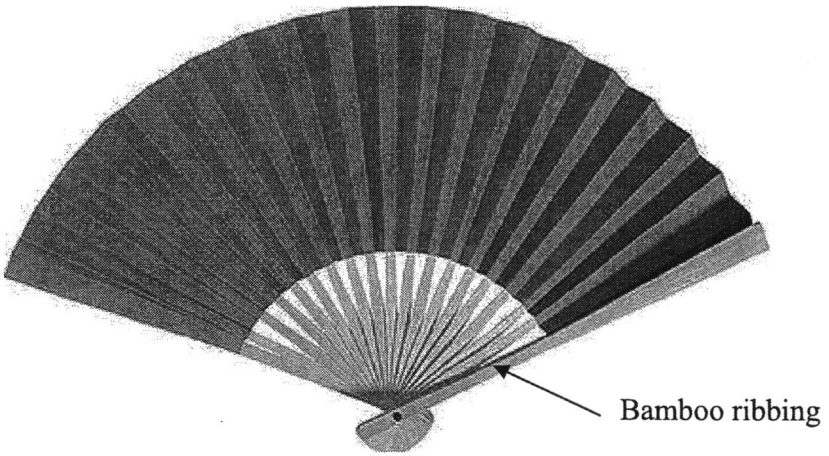
(c) The total distance travelled for the whole journey was 420 m.
Find the value of t. [2]

(d) **Copy** and complete the distance-time graph for the whole journey.



[4]

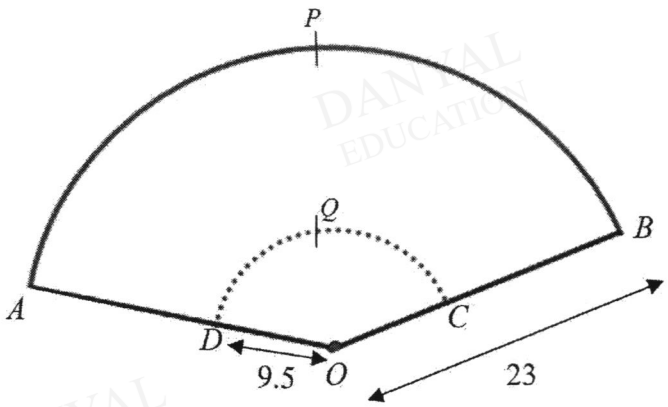
9



http://www.fantastica-uk.co.uk

The handheld fan above is made of paper with bamboo ribbing.

In this question, the fan is modelled as sectors of circles. *APB* and *DQC* are arcs of circles centre *O* with radii 23 cm and 9.5 cm respectively.
The perimeter of *APBCQD* is 64 cm.



(a) Calculate angle *DOC* in radians. [2]

(b) Calculate the area of paper required to cover the region *APBCQD*. [3]

Amanda intends to make 100 identical paper fans to sell at a charity event.
The material needed to make a paper fan are

- 4 sheets of A4 coloured papers,
- 14 bamboo sticks,
- 1 metal craft ring,
- a bottle of craft glue (for making 12 fans).

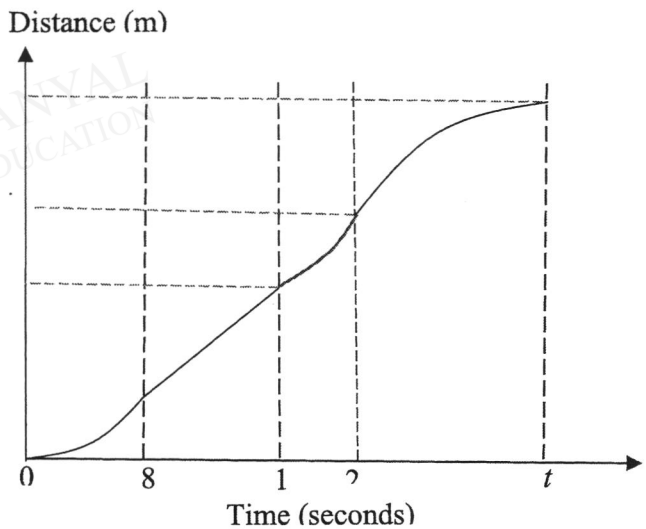
She finds the following information on the internet.

	Material	Unit cost (subject to 7% GST)
1	A pack of 50 sheets of A4 coloured papers	$7.10
2	A pack of 100 ice-cream sticks	$5.35
3	A pack of 10 metal craft rings	$2.40
4	A pack of 48 bamboo sticks	$11.40
5	A bottle of craft glue	$5.00
* GST stands for Goods and Service Tax		

(c) Suggest a reasonable selling price for each fan in order for Amanda to make more than 150% profit of the cost of the material.
Justify your decision and show your workings clearly. [4]

End of Paper

Answer key

1(a) $\frac{4x}{y+2x}$ 1(b)(i) 1 1(b)(ii) $b=\frac{c+5ac}{a-3}$ 1(c) $x=3\frac{1}{2}$ or $x=-3$
2(a)(i) $F=\frac{225}{d^2}$ 2(a)(ii) $d=4.74$ cm 2(a)(iii) 9 : 4 2(b) 12 100 m^2
3(a) $p=-0.128$
3(c) $x=-1.55$ (± 0.05) , $x=2$ or $x=2.55$ (± 0.05)
3(d) Gradient = -3.5 (± 0.8)
3(e)(ii) $x=-1.2$, $x=1.2$ or $x=3$ (± 0.1)
4(i) 56 4(ii) 112 4(iii) 34 4(iv) 112 4(v) 102
5(a) Use of AA property 5(b)(i) $\frac{2}{3}$ 5(b)(ii) $\frac{4}{9}$ 5(b)(iii) $\frac{3}{8}$ 5(c) 121.125 cm^2
6(a) $\frac{8000}{x}$ 6(b) $\frac{8000}{x-50}$ 6(d) $x=134.79$ or $x=-84.79$ 6(e) 94 min 21 seconds
7(a)(i) 774 m 7(a)(ii) 319.9 7(a)(iii) 531 m 7(b) 6.4
8(a) 90 km/h 8(b) 2.25 m/s^2 8(c) 29
8(d) 
9(a) $\frac{74}{65}$ radians [accept 1.14 rad]
9(b) $249\frac{3}{4}$ cm^2 [accept 250 cm^2]
9(c) Accept \$12.52 or more

BEATTY SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2018

MARK SCHEME

SUBJECT : Mathematics | **LEVEL : Secondary 3 Express**

PAPER : 4048 / 01 | **DURATION : 1 hour 30 minutes**

SETTER : Ms Joanna Chong | **DATE : 5 October 2018**

CLASS :	NAME :	REG NO :

.....

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **60**.

For Examiner's Use
60

This paper consists of printed pages (including this cover page)

[Turn over

Mathematical formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length $= r\,\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **ALL** questions.

1 Evaluate $\dfrac{\sqrt[3]{3.16\times10^{-6}}}{1.8(2.5\times10^{8})}$ and give your answer in standard form.

3.26×10^{-11} *B*1

Answer ..... [1]

---

2 Simplify $\dfrac{3x}{x^2-4}-\dfrac{2}{2-x}$

$$\frac{3x}{(x-2)(x+2)}-\frac{2}{2-x} \quad M1(\textit{factorisation})$$

$$=\frac{3x}{(x-2)(x+2)}+\frac{2}{(x-2)} \quad M1(\textit{sign})$$

$$=\frac{3x+2(x+2)}{(x-2)(x+2)}$$

$$=\frac{5x+4}{(x-2)(x+2)} \quad A1$$

Answer .....[3]

---

3 Given that the three points (3, 4), (2, k) and (7, 5) are collinear. Find the value of k.

$$\frac{5-k}{7-2}=\frac{5-4}{7-3} \quad M1$$

$$\frac{5-k}{5}=\frac{1}{4}$$

$$k=\frac{15}{4}=3\frac{3}{4} \quad A1$$

Answer ..... [2]

4 Andrew invested some money in a savings account for 5 years.
The rate of compound interest was fixed at 0.3% per annum.
At the end of 5 years, there was $ 875 000 in his account.

How much interest did Andrew earned at the end of 5 years?

$875000 = P\left(1+\frac{0.3}{100}\right)^5$ *M*1

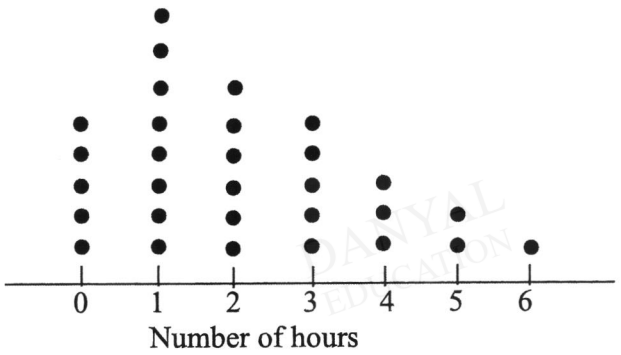
$I = 875\,000 - their\ \frac{875000}{(1+\frac{0.3}{100})^5}\ or\ 861992.30$ *M*1

$= \$13007.70$ *A*1

Answer ..... [3]

---

5 The dot diagram shows the number of hours a group teenagers spend on the handphone in a day.



(a) For the above distribution, find

(i) the mode,

(ii) the mean,

(iii) the median.

(i) 1 *B*1

(ii) mean $= \frac{63}{20}$
$= 2.1$ *B*1

(iii) 2 *B*1

Answer *(a)(i)* .....hour(s) [1]

(ii) .....hour(s) [1]

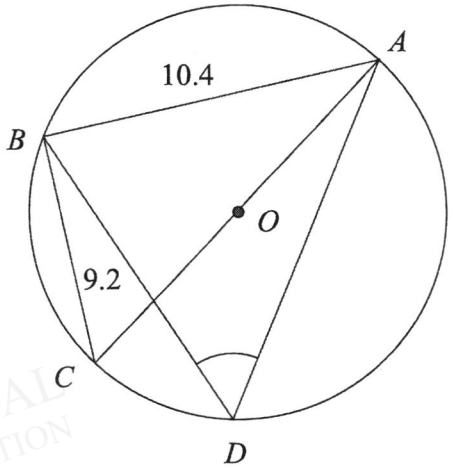
(iii) ..... hour(s) [1]

(b) If a teenager is picked from the group, what is the probability that this teenager spends at least 4 hours on the handphone in a day?

$\frac{6}{30} = \frac{1}{5}$ *B*1

Answer *(b)* ..... [1]

6 In the diagram, O is the centre of the circle $ABCD$. $AB = 10.4$ cm and $BC = 9.2$ cm.
Find angle ADB, stating your reasons clearly.



Since AC passes through O, $\angle ABC = 90^o$ (Angle in a semi-circle) $B1$

$$\tan \angle ACB = \frac{10.4}{9.2}$$

$\angle ACB = 48.503^o$ $M1$

Hence, $\angle ADB = 48.5^o$ (Angles in the same segment) $B1$

Answer angle ADB =..... [3]

7 Factorise completely

(a) $75d^2 - 3$

(b) $4p^2 - 4pr + r^2 - 2pq + rq$

(a)

$3(25d^2 - 1)$ $M1$

$= 3(5d + 1)(5d - 1)$ $A1$

(b)

$4p^2 - 4pr + r^2 - 2pq + rq$

$= (2p - r)^2 - q(2p - r)$ $M1$

$= (2p - r)(2p - r - q)$ $A1$

Or $(r - 2p)(r + q - 2p)$

Answer *(a)* ..... [2]

(b) ..... [2]

8 **(a)** Simplify $\left(\frac{a^3}{3b}\right)^{-2} \div a^2b^5 \times \left(\frac{125a^4b^2}{5}\right)^0$, leaving your answer in positive indices.

(b) Given that $\sqrt{\frac{k^2}{4}} = 8^{-\frac{1}{3}}$, find the positive value of k.

(a)

$$\left(\frac{a^3}{3b}\right)^{-2} \div a^2b^5 \times \left(\frac{125a^4b^2}{5}\right)^0$$

$$= \left(\frac{3b}{a^3}\right)^2 \times \frac{1}{a^2b^5} \quad M1(\textit{for dealing with power zero})$$

$$= \frac{9b^2}{a^6} \times \frac{1}{a^2b^5} \quad M1(\textit{for}\ \frac{9b^2}{a^6})$$

$$= \frac{9b^{-3}}{a^8}$$

$$= \frac{9}{a^8b^3} \quad A1$$

$$\sqrt{\frac{k^2}{4}} = 8^{-\frac{1}{3}}$$

$$\frac{k}{2} = 8^{-\frac{1}{3}} \quad M1$$

$$k = 2 \times 2^{-1}$$

$$k = 2^0$$

$$k = 1 \qquad A1$$

Or $\frac{k^2}{4} = 8^{-\frac{2}{3}}$ *M1*

$$\frac{k^2}{4} = \frac{1}{4}$$

$$k^2 = 1$$

$k = 1$ or $k = -1$(rejected)

Answer *(a)* ..... [3]

(b) ..... [2]

---

9 **(a)** Solve the inequality $\frac{2}{3}x - 1 < x \leq 2(8 - x)$.

(b) Write down the largest rational number which satisfies $\frac{2}{3}x - 1 < x \leq 2(8 - x)$.

(a)

$$\frac{2}{3}x - 1 < x \leq 2(8 - x)$$

$$\frac{2}{3}x - x < 1 \qquad x \leq 16 - 2x$$

$$-\frac{1}{3}x < 1 \qquad x \leq 5\frac{1}{3} \quad M1$$

$$x > -3 \quad M1$$

$$\textit{Hence}\ -3 < x \leq 5\frac{1}{3} \quad A1$$

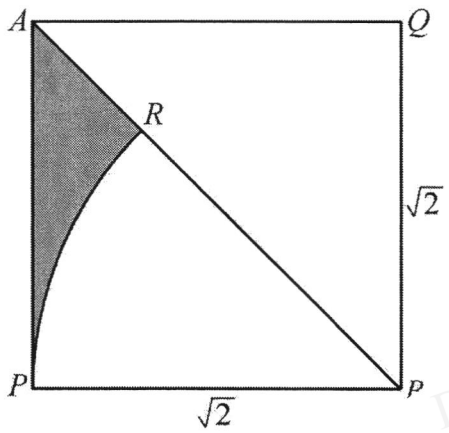
(b) $5\frac{1}{3}$ *B1*

Answer *(a)* ..... [3]

(b) ..... [1]

10 In the diagram, $AQBP$ is a square of side $\sqrt{2}$ cm. PR is an arc of a circle with centre at B and radius $\sqrt{2}$ cm.

Find the area of the shaded region APR, leaving your answer in terms of π.



Area of square

$= \left(\sqrt{2}\right)^2 = 2$ *M1*

Area of quadrant

$= \frac{1}{4}(\pi)\left(\sqrt{2}\right)^2$

$= \frac{\pi}{2}$ *M1*

Area of shaded region

$= \frac{1}{2} \times \left(2 - \frac{\pi}{2}\right)$

$= \left(1 - \frac{\pi}{4}\right) cm^2$ *B1*

Area of $\Delta ABP = \frac{1}{2}(\sqrt{2})^2$

$= 1 \text{ cm}^2$ M1

$\angle ABP = 45 = \frac{\pi}{4}$

Area of sector $= \frac{1}{2}(\sqrt{2})^2\left(\frac{\pi}{4}\right)$

$= \frac{\pi}{4}$

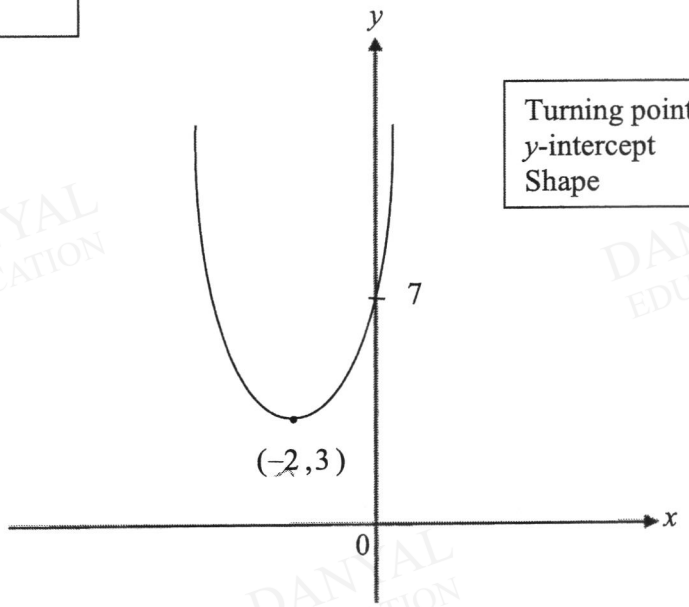
Shaded area $= \left(1 - \frac{\pi}{4}\right) \text{cm}^2$

Answer .....cm^2 [3]

11 **(a)** Express x^2+4x+7 in the form $(x+h)^2+k$, where h and k are constants.

(b) Hence, sketch the graph of $y=x^2+4x+7$ on the axes below, indicating its turning point and intercept(s) clearly. [3]

(a)
$$x^2+4x+7$$
$$=(x+2)^2-2^2+7$$
$$=(x+2)^2+3 \quad B1$$



Turning point	B1
y-intercept	B1
Shape	B1

Answer (a) ..... [1]

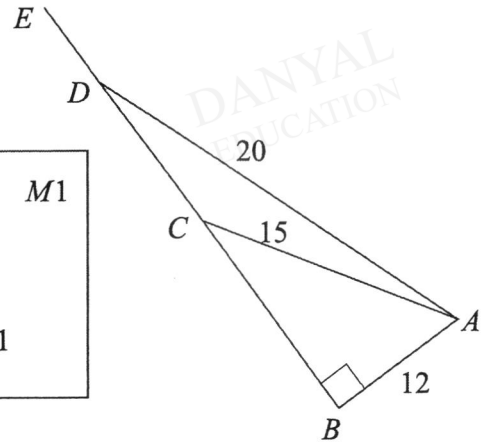
12 In the diagram, $\angle ABC = 90°$, $AB = 12$ cm, $AC = 15$ cm, $AD = 20$ cm and $BCDE$ is a straight line. Calculate, giving each answer as a fraction,

(a) $\sin \angle ADE$,

(b) $\cos \angle ACD$.

(a)
$$\sin \angle ADB = \frac{12}{20}$$
$$= \frac{3}{5} \quad B1$$

(b)
$$BC = \sqrt{15^2-12^2} = 9 \quad M1$$
$$\cos \angle ACB = \frac{9}{15}$$
$$\cos \angle ACD = -\frac{3}{5} \quad A1$$



Answer (a) .....[1]

(b)..... [2]

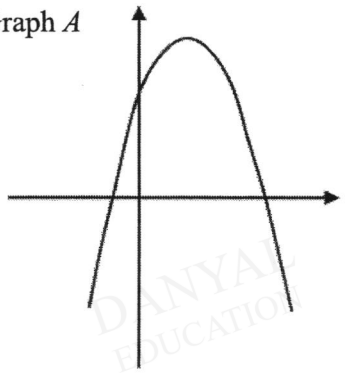
13 State the graph that correspond to each of following equation.

(i) $y = 3x - 7$

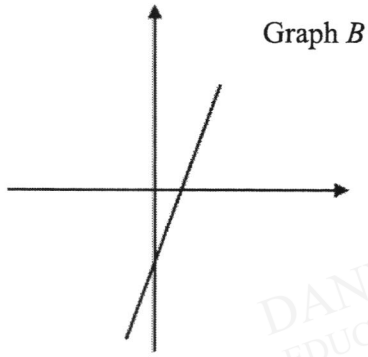
(ii) $y = \frac{5}{x}$

(iii) $y = 2 - x^2$

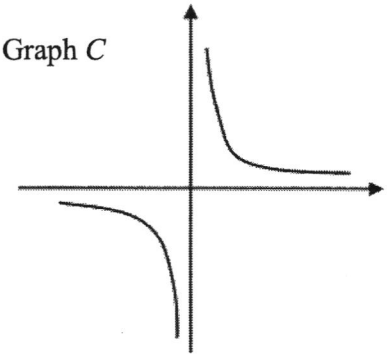
Graph *A*



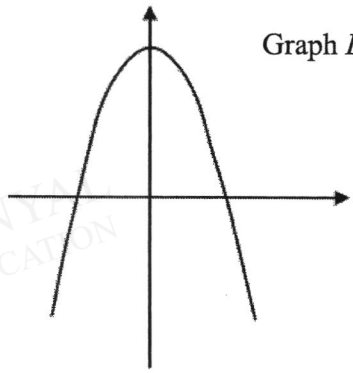
Graph *B*



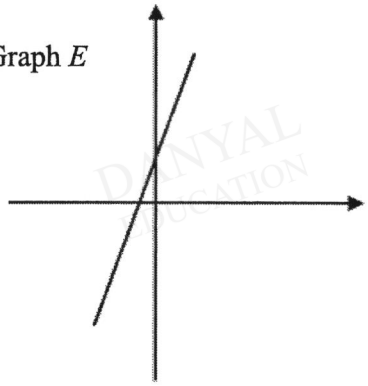
Graph *C*



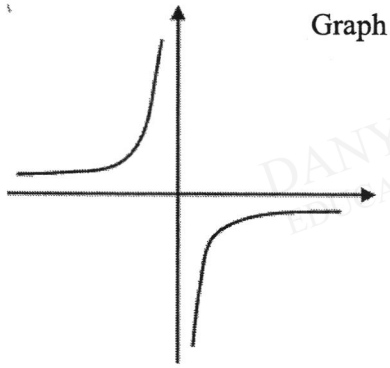
Graph *D*



Graph *E*



Graph *F*



Answer *(i)* .....B.....[1]

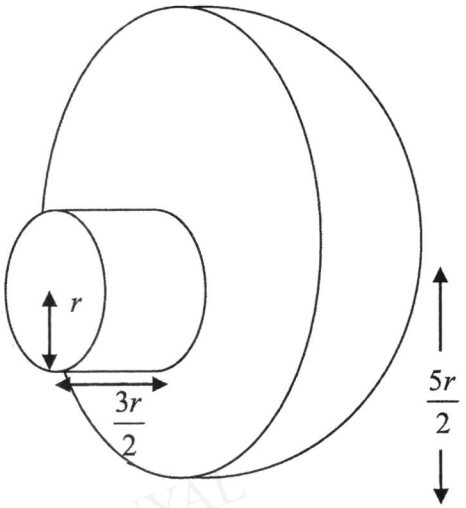
(ii) .....C.....[1]

(iii) .....D.....[1]

14 The solid shown is made from a cylinder and a hemisphere.

The cylinder has a radius of r and height $\frac{3r}{2}$.

The hemisphere has a radius of $\frac{5r}{2}$.



Find an expression, in terms of π and r,
For the total surface area of the solid.

$$curved\ SA\ of\ hemisphere = 2\pi\left(\frac{5r}{2}\right)^2 = \frac{25}{2}\pi r^2 \quad M1$$

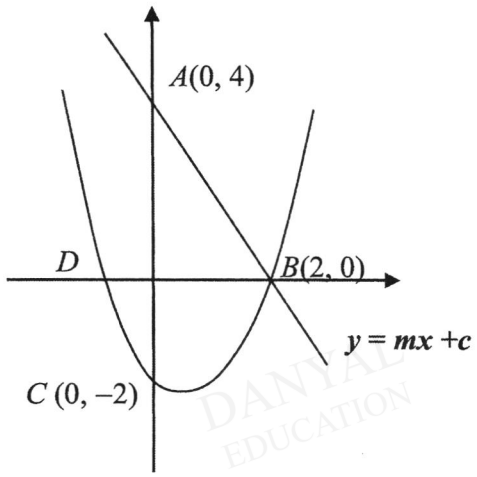
$$curved\ SA\ of\ cylinder = 2\pi r\left(\frac{3r}{2}\right) = 3\pi r^2 \quad M1$$

$$Total\ SA = \frac{25}{2}\pi r^2 + 3\pi r^2 + \pi\left(\frac{5r}{2}\right)^2$$

$$= \frac{87}{4}\pi r^2 = 21\frac{3}{4}\pi r^2\ cm^2 \quad A1$$

Answer .....cm^2 [3]

15 The diagram below shows a straight line $y = mx + c$ intersecting the y-axis at A (0, 4) and x-axis at B (2,0) and D.
The curve $y = (x-2)(x+k)$ cuts the x-axis at B and the y-axis at C (0, – 2).



(a) Find

(i) the value of m and of c.

(ii) the length of AB.

(iii) the value of k.

(a)(i) $m = -2$ *B*1
$c = 4$ *B*1

(ii) $AB = \sqrt{2^2 + 4^2} = \sqrt{20}$
$= 4.4721...$
$= 4.47$ *B*1

(iii) Sub ((0,–2)
$-2k = -2$
$k = 1$ *B*1

Answer *(a)(i)* $m =$.....$c =$ ..... [2]

(a)(ii) .....[1]

(a)(iii) .....[1]

(b) A second curve with the equation $y = ax^2 + bx + d$ cuts the x-axis at points B and D.
Determine whether the value of $b^2 - 4ad$ is positive, negative or zero.
Explain your answer.

Answer $b^2 - 4ad$ is **positive** because **the curve cuts the x-axis at two intersection points.** *B*1

16 Diagram I shows an open metal pail which is the shape of a frustum. The diameter of the two circular ends are 45 cm and 15 cm. The height of the pail is 40 cm.

The curved surface of the frustum can be found using the model in Diagram II.

(a) By using appropriate similar triangles in Diagram II, find the value of h.

(b) Hence, find the total area of metallic sheet used to make the pail.

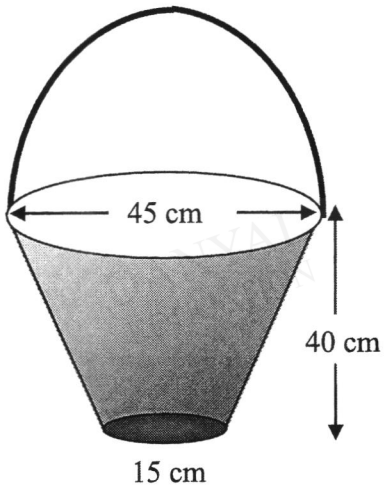


Diagram 1

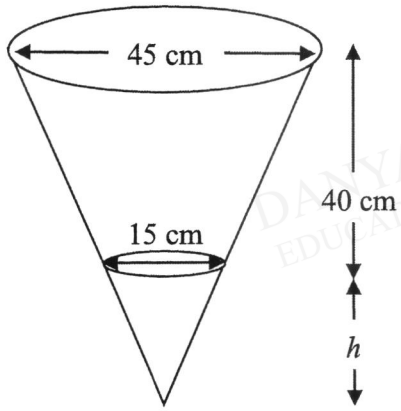


Diagram II

(a)

$$\frac{h}{h+40}=\frac{15}{45} \quad M1$$

$3h = h + 40$

$2h = 40$

$h = 20 \quad A1$

(b)

$\text{Slant height of cone} = \sqrt{60^2 + 22.5^2} = \sqrt{4106.25} \quad M1$

$\text{slant height of smaller cone} = \sqrt{20^2 + 7.5^2} = \sqrt{456.25}$

$\text{Curved SA of entire cone} = \pi(22.5)\sqrt{4106.25} \quad M1$

$\text{Curved SA of smallercone} = \pi(7.5)\sqrt{456.25}$

$\text{area of metallic cone} = \pi(22.5)\sqrt{4106.25}691 - \pi(7.5)\sqrt{456.25} + \pi(7.5)^2 \quad M1$

$= 4202.981497$

$= 4200 \text{ cm}^2 \text{ (3 sf) } A1$

Or

$\text{Slant height of cone} = \sqrt{60^2 + 22.5^2} = \sqrt{4106.25} \quad M1$

$\text{Curved SA of entire cone} = \pi(22.5)\sqrt{4106.25} \quad M1$

$\text{Curved SA of pail} = \frac{8}{9} \times \pi(22.5)\sqrt{4106.25} = 4026.26691$

$\text{Total area of metal sheet} = 4026.26691... + \pi(7.5)^2 \quad M1$

$= 4202.981497$

$= 4200\ cm^2 \ (3sf)\ A1$

17 In the diagram, P, Q and R are three ports. P is 900 m due west of Q. At 07 40, a ship, Pioneer, left P and travelled to R on a bearing of 130°. At the same time, another ship, Queen, left Q and travelled at an average speed of 15 km/h to R. It arrived at 08 10 on the same day.

Calculate

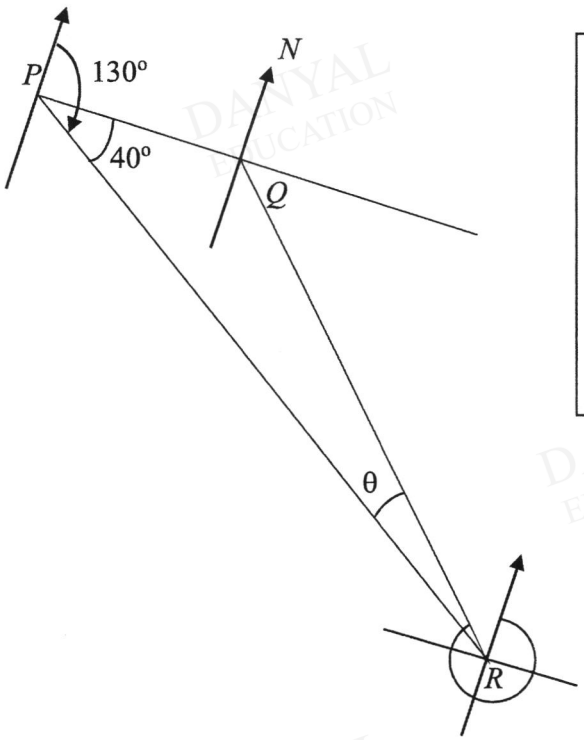
(a) the distance QR, in metres, [1]

(b) the bearing of Q from R. [3]

(a)

$$QR = \frac{1}{2} \times 15 \times 1000$$
$$= 7500 \text{ m} \quad B1$$

(b)



$$\frac{\sin\theta}{900} = \frac{\sin 40^o}{7500} \quad M1$$

$$\sin\theta = \frac{900\sin 40^o}{7500}$$

$$\theta = 4.42387...^o$$

$$\textit{Bearing of } Q \textit{ from } R = 270^o + 40^o + 4.42387...^o \quad M1$$
$$= 314.42387...^o = 314.4^o \quad A1$$

Answer *(a)*.....m [1]

(b).....[4]

Beatty Secondary School

End-of-Year Examination 2018

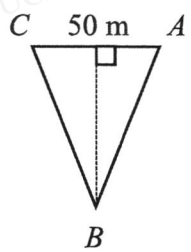
Secondary 3 Express

Mathematics Paper 2 (4048/02)

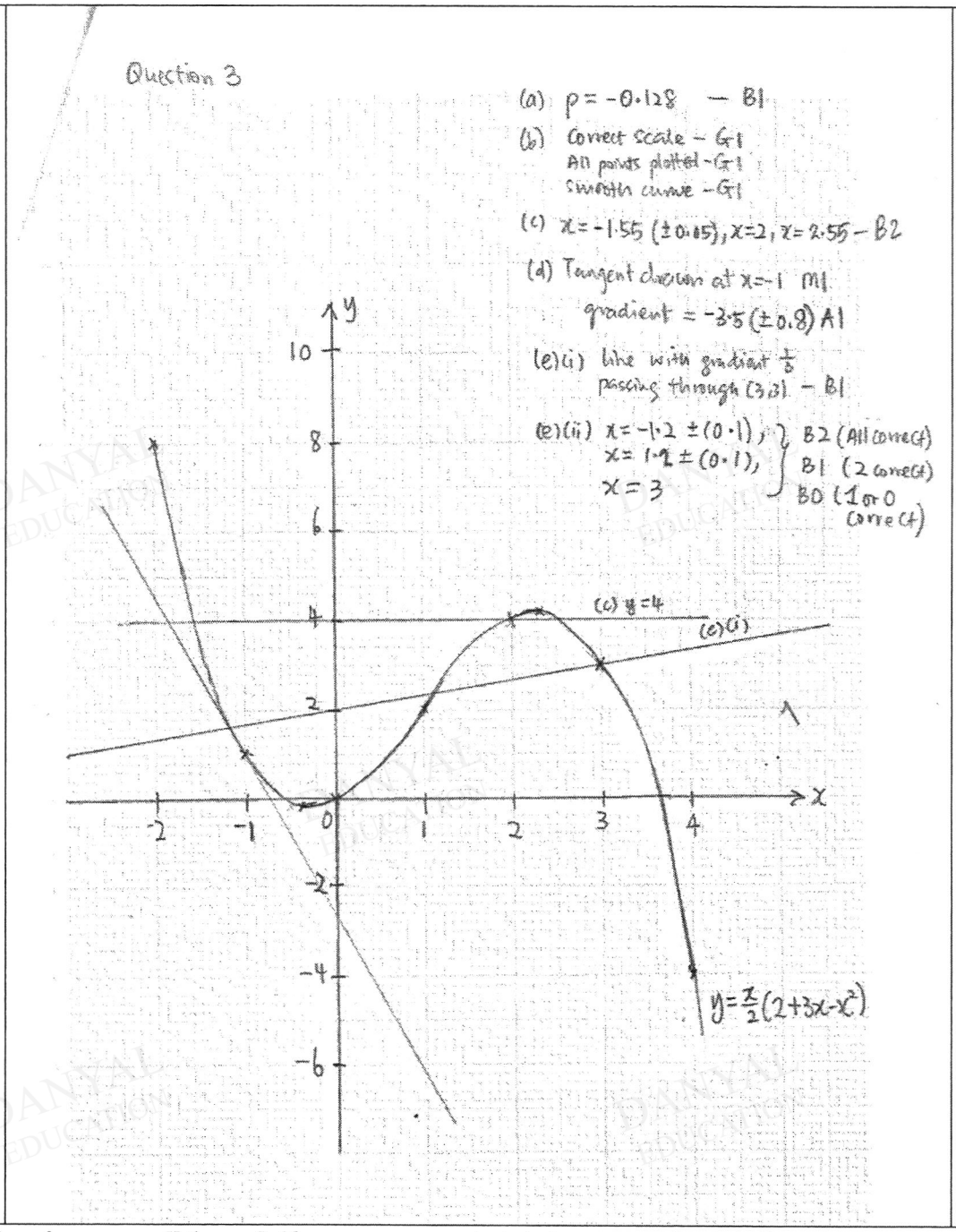
Setter : Mrs Samsol

Mark Scheme

1	**(a)**	$\frac{4xy-8x^2}{y^2-4x^2}$ $=\frac{4x(y-2x)}{(y+2x)(y-2x)}$ ---------- M1 [factorization] $=\frac{4x}{y+2x}$ ---------- A1	**[2]**
1	**(b)(i)**	$a=\frac{3b+c}{b-5c}$ $a=\frac{3(3)+(-1)}{3-5(-1)}$ $a=1$ ---------- B1	**[1]**
1	**(b)(ii)**	$a=\frac{3b+c}{b-5c}$ $ab-5ac=3b+c$ $ab-3b=c+5ac$ ---------- M1 [grouping of like terms] $b(a-3)=c+5ac$ $b=\frac{c+5ac}{a-3}$ ---------- A1	**[2]**
1	**(c)**	$\frac{6}{2x+5}=x-3$ $(2x+5)(x-3)=6$ $2x^2-x-15=6$ ---------- M1 [expansion] $2x^2-x-21=0$ $(2x-7)(x+3)=0$ ---------- M1 [factorization] $x=3\frac{1}{2}$ or $x=-3$ ---------- A1	**[3]**

2	**(a)(i)**	$F = \frac{k}{d^2}$, where k is a constant $25 = \frac{k}{3^2}$ ---------- M1 [calculation of constant] $k = 25 \times 9$ $k = 225$ Equation is $F = \frac{225}{d^2}$ ---------- A1	**[2]**
2	**(a)(ii)**	$10 = \frac{225}{d^2}$ $d^2 = 22.5$ ---------- M1√ $d = \sqrt{22.5}$ **[Accept** $d = \pm\sqrt{22.5}$ **]** $d = 4.74$ cm (3 sf) ---------- A1 [A0 if negative value is not rejected]	**[2]**
2	**(a)(iii)**	Given that $\frac{d_1}{d_2} = \frac{2}{3}$ $\frac{F_1}{F_2} = \left(\frac{k}{d_1^2}\right) \div \left(\frac{k}{d_2^2}\right)$ $\frac{F_1}{F_2} = \left(\frac{d_2}{d_1}\right)^2$ ---------- M1[calculation of ratio between force & Distance] $\frac{F_1}{F_2} = \left(\frac{3}{2}\right)^2$ $\frac{F_1}{F_2} = \frac{9}{4}$ or 9 : 4 ---------- A1	**[2]**
2	**(b)**	Divide octagon into 8 identical isosceles triangles, ABC.  angle $ABC = \frac{360^\circ}{8}$ ---------- M1 $= 45^\circ$	

		$\sin\left(\frac{45^\circ}{2}\right)=\frac{25}{AB}$ ---------- M1 $AB=\frac{25}{\sin\left(\frac{45^\circ}{2}\right)}$ $AB = 65.328$ ---------- A1 area of $\Delta ABC=\frac{1}{2}(65.328)^2\sin 45^\circ$ ---------- M1 $= 1508.87$ Floor area of hall = 8 x 1508.87 =12070.96 = 12 100 m^2 (3 sf) ---------- A1 Accept other alternatives	**[5]**
3	**(a)**	$p = -0.128$ ---------- B1	**[1]**
3	**(b)**	Correct scale ---------- G1 All points plotted correctly ---------- G1 Smooth curve drawn ---------- G1	**[3]**
3	**(c)**	$x = -1.55$ (± 0.05) , $x = 2$ or $x = 2.55$ (± 0.05) ---------- B2 B1 for any 2 solutions. B1 for only 1 solution	**[2]**
3	**(d)**	Tangent drawn at (–1,1) ---------- M1 Gradient = –3.5 (± 0.8) ---------- G1	**[2]**
3	**(e)(i)**	Straight line drawn ---------- B1	**[1]**
3	**(e)(ii)**	$x = -1.2$, $x = 1.2$ or $x = 3$ (± 0.1) ---------- B2 (All correct) B1 (2 correct) B0 (1 or 0 correct)	**[2]**

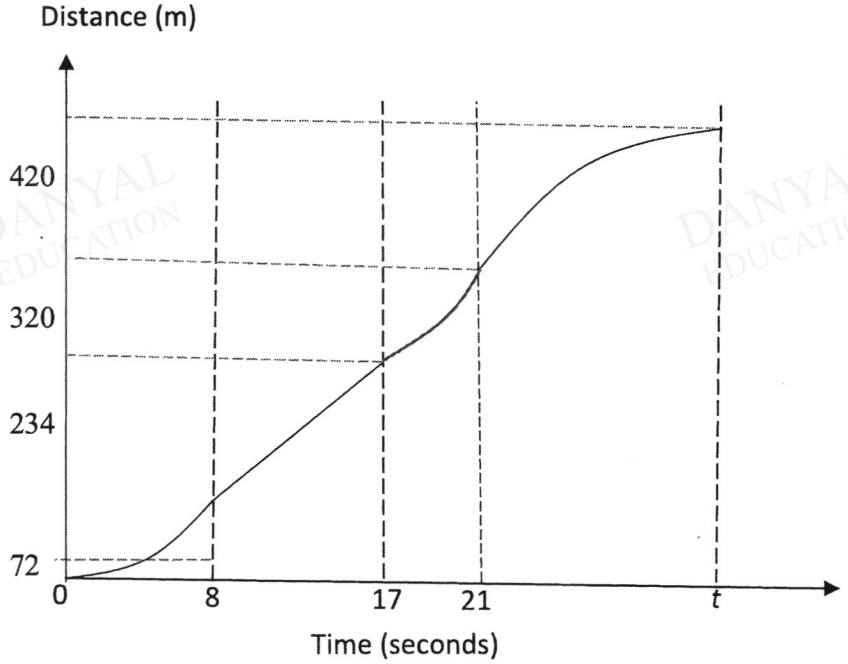


	For Q4 , minus overall 1 mark if reasons are not stated or wrongly stated		
4	**(i)**	$\angle ACB = 56^\circ$ [angles in same segment] ------- B1	**[1]**
4	**(ii)**	$\angle AOB = 2 \times 56^\circ$ $= 112^\circ$ [angle at centre = 2 angle at circumference] -------- B1	**[1]**
4	**(iii)**	$\angle ADE = 90^\circ - 56^\circ$ [angle in semicircle] $= 34^\circ$ ---------- B1	**[1]**

4	(iv)	$\angle ABO = \angle ADE = 34^\circ$ [angle in same segment] $\angle ABD = 2 \times 34^\circ$ [BE bisects angle ABD] ---------- M1 $= 68^\circ$ $\angle AED = 180^\circ - 68^\circ$ [angles in opposite segment] $= 112^\circ$ ---------- A1	[2]
4	(v)	$\angle BGC = 3 \times 34^\circ$ [1 exterior angle = 2 interior opposite angles] $\angle BGC = 102$ ---------- B1	[1]
5	(a)	In triangles AQP and RQD , $\angle PQA = \angle DQR$ (vertically opposite angles) $\angle APQ = \angle RDQ$ (PA parallel to RD, alternate angles) } any 2 $\angle PAQ = \angle DRQ$ (PA parallel to RD, alternate angles) ----- B1 Therefore triangles AQP and RQD are similar. (AA property) ----- B1	[2]
5	(b)(i)	$\frac{AP}{DR} = \frac{2}{3}$ ---------- B1	[1]
5	(b)(ii)	$\frac{\text{Area of } \Delta AQP}{\text{Area of } \Delta RQD} = \left(\frac{2}{3}\right)^2$ $= \frac{4}{9}$ ---------- B1	[1]
5	(b)(iii)	$\frac{\text{Area of triangle } ARD}{\text{Area of rectangle } ABCD} = \frac{\frac{1}{2}(DR)(AD)}{(DC)(AD)}$ $= \frac{1}{2}\left(\frac{DR}{DC}\right)$ $= \frac{1}{2}\left(\frac{3}{4}\right)$ $= \frac{3}{8}$ ---------- B1	[1]

5	**(c)**	Given triangle $AQP = 25.5$ cm^2. Area of triangle $AQD = \frac{3}{2}(25.5)$ $= 38.25$ cm^2 } M1 for knowing ratios of triangles Area of triangle $DQR = \frac{9}{4}(25.5)$ $= 57.375$ cm^2 total area of triangles AQP, AQD and DQR $= 25.5 + 38.25 + 57.375$ $= 121.125$ cm^2 ---------- A1 **OR** $= \Delta APQ : \Delta AQD : \Delta DQR$ ---------- M1 $= 4 : 6 : 9$ Total area $= \frac{25.5}{4} \times 19$ [accept all other alternatives] $= 121\frac{1}{8}$ or 121.125 cm^2 ---------- A1	**[2]**
6	**(a)**	Time taken $= \frac{8000}{x}$ minutes ---------- B1	**[1]**
6	**(b)**	Time taken for small pump $= \frac{8000}{x-50}$ minutes ---------- B1	**[1]**
6	**(c)**	$\frac{8000}{x-50} - \frac{8000}{x} = 35$ ---------- M1 $\frac{8000x - 8000(x-50)}{x(x-50)} = 35$ $400000 = 35x(x-50)$ ---------- M1√ $35x^2 - 1750x - 400000 = 0$ Divide by 5 $7x^2 - 350x - 80000 = 0$ ---------- A1	**[3]**
6	**(d)**	$x = \frac{-(-350) \pm \sqrt{(-350)^2 - 4(7)(-80000)}}{2(7)}$ ---------- M1 $x = 134.7887$ or $x = -84.78875$ $x = 134.79$ or $x = -84.79$ (correct to 2 d p) ------- A2	**[3]**
6	**(e)**	Time taken by small pump $= \frac{8000}{134.788 - 50}$ $= 94.35297$ minutes ---------- B1 $=$ 94 minutes 21 seconds ---------- B1	**[2]**

7	(a)(i)	$AC = \sqrt{550^2 + 825^2 - 2(550)(825)\cos 65^\circ}$ ---------- M1 $AC = 774.33$ $AC = 774$ m (3 sf) ---------- A1	[2]
7	(a)(ii)	$\frac{\sin \angle ACB}{550} = \frac{\sin 65}{774.33}$ $\angle ACB = \sin^{-1}\left(\frac{550 \sin 65}{774.33}\right)$ ---------- M1 $\angle ACB = 40.071$ Bearing of A from $C = 360 - 40.071$ ---------- M1 $= 319.929$ $= 319.9$ (to 1 dp) ---------- A1	[3]
7	(a)(iii)	$\sin 40.071 = \frac{d}{825}$ $d = 825 \sin 40.071$ ---------- M1√ $d = 531.08$ shortest distance = 531 m (to 3 sf) ---------- A1 Accept other alternatives $\frac{1}{2} \times 774.33 \times d = \frac{1}{2}(550)(825)\sin 65$ ----- M1 $d = \frac{205618.5792 \times 2}{774.337}$ $d = 531.08$ $d = 531$ m ----------A1	[2]
7	(b)	$\tan 9 = \frac{h}{550}$ $h = 550 \tan 9$ ---------- M1 $\tan \theta = \frac{550 \tan 9}{774.33}$ ---------- M1√ $\theta = \tan^{-1}\left(\frac{550 \tan 9^\circ}{774.33}\right)$ $\theta = 6.418^\circ$ Angle of elevation of P from C = 6.4° (to 1 dp) ---------- A1	[3]

8	**(a)**	25 m/s = 25 x 3.6 = 90 km/h ---------- B1	**[1]**
8	**(b)**	Acceleration $= \frac{18}{8}$ $= 2\frac{1}{4}$ m/s^2 ---------- B1	**[1]**
8	**(c)**	Distance travelled for 21 seconds : $\frac{1}{2}(8)(18) = 72$ $9 \times 18 = 162$ $\frac{1}{2}(18+25)(4) = 86$ ------ M1 [area under curve] Total distance = 72 +162 +86 = 320 m Or Distance for 21 seconds $= \frac{1}{2}(17+9)(18) + \frac{1}{2}(18+25)(4)$ ---------- M1 [area under curve] = 320 m $\frac{1}{2}(t-21)(25) = 420-320$ $t-21=8$ $t=29$ ---------- A1	**[2]**
8	**(d)**	 B4 - I mark for each part of the graph	**[4]**

		Give B3 if distances are not indicated on vertical axis.	
9	(a)	$23\theta + 9.5\theta + 2(23-9.5) = 64$ ---- M1 [sum of 2 arc = 37] $32.5\theta = 37$ $\theta = \frac{74}{65}$ Angle $DOC = \frac{74}{65}$ radians [accept 1.14 rad] ---------- A1	[2]
9	(b)	Area $= \frac{1}{2}(23)^2\left(\frac{74}{65}\right) - \frac{1}{2}(9.5)^2\left(\frac{74}{65}\right)$ ---------- M2 √ $= 249\frac{3}{4}$ cm^2 ---------- A1 Or Area $= \frac{1}{2}(23)^2(1.1384) - \frac{1}{2}(9.5)^2(1.1384)$ ---------- M2 √ $= 250$ cm^2 (to 3 sf) ---------- A1	[3]
9	(c)		[4]

Material	Amount required	cost
A4 paper	$\frac{100\times4}{50} = 8$ packs	$8 \times 7.10 = 56.80$
Metal ring	$\frac{100}{10} = 10$ packs	$10 \times 2.40 = 24.00$
Bamboo sticks	$\frac{100\times14}{48} = 29.16$	$30 \times 11.40 = 342.00$
Glue	$\frac{100}{12} = 8.33$	$9 \times 5 = 45.00$
	P1 – calculation of amount of materials	Q1 – calculation of cost

Average cost for 1 fan $= \frac{56.80 + 24.00 + 342.00 + 45.00}{100}$

$= \frac{467.80}{100}$

$= \$4.678$

price of 1 fan with GST $= 2.5 \times 4.678 \times 1.07$ -- R1 [for using 2.5]

$= \$ 12.51365$

Accept suggested selling price of \$ 12.52 or more. ----------S1

Accept alternative workings

Material	Amount required	Cost (including 7% GST)
A4 paper	$\frac{100\times4}{50}=8$ packs	$8\times7.10\times1.07=60.78$
Metal ring	$\frac{100}{10}=10$ packs	$10\times2.40\times1.07=25.68$
Bamboo sticks	$\frac{100\times14}{48}=29.16$	$30\times11.40\times1.07=365.94$
Glue	$\frac{100}{12}=8.33$	$9\times5\times1.07=48.15$
	P1 – calculation of amount of materials	Q1 – calculation of cost

Total cost with GST $= \frac{500.55}{100}$

$= \$5.0055$

Minimum selling price of 1 fan $= 2.5 \times 5.0055$ ---------- R1

$= \$ 12.51375$

Accept suggested selling price of 1 fan is \$12.52 or more -------- S1