| Class | Index Number | Candidate Name |
| :--- | :--- | :--- |

## ANG MO KIO SECONDARY SCHOOL <br> FINAL EXAMINATION 2018 SECONDARY THREE EXPRESS

## MATHEMATICS

4048/01
Paper 1

Candidates answer on the Question Paper.

## READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is $\mathbf{8 0}$.


## Mathematical Formulae

## Compound interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

## Mensuration

Curve surface area of a cone $=\pi r l$
Surface area of a sphere $=4 \pi r^{2}$

$$
\text { Volume of a cone }=\frac{1}{3} \pi r^{2} h
$$

$$
\text { Volume of a sphere }=\frac{4}{3} \pi r^{3}
$$

$$
\text { Area of triangle } A B C=\frac{1}{2} a b \sin C
$$

Arc length $=r \theta$, where $\theta$ is in radians

$$
\text { Sector Area }=\frac{1}{2} r^{2} \theta, \text { where } \theta \text { is in radians }
$$

Trigonometry

$$
\begin{aligned}
& \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{aligned}
$$

Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

1 (a) Simplify $5 x-2(3 x-7)$.

## Answer

(b) Factorise completely $6 x^{2}+20 x-16$.

## Answer

2 The diameter of a spherical organism is 807 micrometres. Giving your answers in standard form,
(a) express 807 micrometres in metres,
(b) find the surface area, in square metres, of the spherical organism. [ 1 micrometre $=10^{-6}$ metres]

3 The first four terms of a sequence are $7,12,17$ and 22.
(a) Write down the $6^{\text {th }}$ term of the sequence.

Answer
[1]
(b) Find an expression, in terms of $n$, for the $n^{\text {th }}$ term of the sequence.

Answer
(c) Explain why 200 is not a term of this sequence.

Answer $\qquad$

4 The volume of air, $V \mathrm{~cm}^{3}$, inside an air pump is inversely proportional to the cube root of the air pressure, $P \mathrm{~Pa}$. When $15 \mathrm{~cm}^{3}$ of air is pumped, the air pressure reaches 2744 Pa . Find
(a) an equation connecting $V$ and $P$,

> Answer
(b) the air pressure when $21 \mathrm{~cm}^{3}$ of air is pumped.

Answer
Pa

5 Solve
(a) $\frac{p}{6}-\frac{3(2-p)}{4}=1$,
(b) $\frac{1}{x}+\frac{3}{x-1}+\frac{2}{x+1}=0$.

6 (a) Express $x^{2}-6 x+7$ in the form $(x-p)^{2}-q$.

Answer
(b) Hence write down the minimum value of $x^{2}-6 x+7$.

Answer
(c) Write down the equation of the line of symmetry of the graph of $y=x^{2}-6 x+7$.

Answer

7 Linda is offered a choice of the following rates of pay per week.

| Rate $A$ | Rate $B$ |
| :--- | :--- |
| $\$ 15$ per hour up to 40 hours | $\$ 18.50$ per hour up to 30 hours |
| $\$ 12$ per hour for the remaining hours | $\$ 5$ per $\frac{1}{2}$ hour for the remaining hours |

If Linda works 55 hours a week, which pay rate would be a better choice?
Show your working clearly in the space provided.

Answer Linda should choose Rate ............. because
$\qquad$

8 The diagram shows triangle $P Q R$ with coordinates $P(-8,0), Q(10,0)$ and $R(-5,9)$. The line $Q R$ cuts the $y$-axis at the point $S$.


Find
(a) the length of $Q R$,

## Answer

(b) the equation of the line $Q R$,

## Answer

(c) the coordinates of $S$,
Answer (......................... ,
(d) the area of quadrilateral $O P R S$.

9 (a) Simplify
(i) $\frac{1}{5} x^{3} \times(-3 x y)^{2}$,
Answer
(ii) $a^{\frac{5}{3}} \div \sqrt[3]{a^{2}}$.

Answer
(b) Solve $7^{2 x-1} \times 49^{x}=1$.

Answer $x=$

10 Sketch the graph of $y=-(x+5)(x-3)$ on the axes below.
Indicate clearly the values where the graph crosses the $x$ - and $y$-axes.


11 Written as the product of its prime factors, $1350=2 \times 3^{3} \times 5^{2}$.
(a) Express 540 as the product of its prime factors.

> Answer
(b) Find the smallest positive integer $m$ such that $1350 m$ is a perfect cube.

## Answer

(c) Write down the greatest integer that will divide both 1350 and 540 exactly.

12 An open field has an area of $27 \mathrm{~km}^{2}$. It is represented by an area of $3 \mathrm{~cm}^{2}$ on map $X$.
(a) Find the scale of $\operatorname{map} X$ in the form $1: n$.

Answer
(b) A road is measured 2.4 cm on map $X$. Find, in centimetres, the length representing this road on map $Y$ if map $Y$ has a scale of $1: 400000$.

> Answer
cm

13 A vehicle travels $2 w \mathrm{~km}$ in 10 minutes. Find its speed in $\mathrm{km} / \mathrm{h}$, leaving your answer in terms of $w$.

14 In the diagram below, $A B=18 \mathrm{~cm}, B C=7.5 \mathrm{~cm}, C D=16.5 \mathrm{~cm}, A D=30 \mathrm{~cm}$ and $B C D$ is a straight line.

(a) Show that $\triangle A B D$ is a right-angled triangle.

Answer
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) Givng your answer as a fraction, find the exact value of (i) $\sin \angle A D B$,

> Answer
(ii) $\cos \angle A C D$.

15 In the diagram, $G H=L K=4 \mathrm{~cm}, H J=20 \mathrm{~cm}, H L=6 \mathrm{~cm}$ and $G L=8 \mathrm{~cm}$.

(a) Show that $\triangle G J K$ is similar to $\triangle G L H$.

Answer $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) Find $J K$.

Answer JK = cm

16 The diagram below shows two open troughs that are geometrically similar. The ratio of the base areas of the two troughs is $9: 4$. Both troughs are filled with sand to the brim. The mass of sand in the smaller trough is 3.2 kg . Find the mass of sand in the larger trough.


17 In the figure below, $A B C D E F$ is a regular hexagon and $H B C$ is a straight line.


Find the value of
(a) $p$,

$$
\begin{equation*}
\text { Answer } p= \tag{1}
\end{equation*}
$$

(b) $q$,

$$
\text { Answer } q=
$$

(c) $r$.

$$
\begin{equation*}
\text { Answer } r= \tag{1}
\end{equation*}
$$

18 The diagram below shows a speed-time graph of a moving particle over a period of 12 seconds.

(a) Find the acceleration of the particle in the first 3 seconds.

$$
\text { Answer ............................................. m/s }{ }^{2}
$$

(b) If the particle moved a total distance of 100 m , calculate its speed $v$, at $t=12$ seconds.

Answer
$\mathrm{m} / \mathrm{s}$
(c) Use the grid below to sketch the distance-time graph for the 12 seconds.

Distance (m)


19 In $\triangle A D C, B E$ is parallel to $C D, \angle B C D=71^{\circ}, \angle D B E=53^{\circ}$, and $\angle B D E=21^{\circ}$.


Calculate
(a) $\angle C D B$,

$$
\begin{equation*}
\text { Answer } \angle C D B= \tag{1}
\end{equation*}
$$

(b) $\angle C B D$,

$$
\begin{equation*}
\text { Answer } \angle C B D=\text {.................................. } \tag{2}
\end{equation*}
$$

(c) reflex $\angle D A B$.

$$
\text { Answer } \angle D A B=
$$

20 The diagram shows a quadrant of a circle $P O R$ with centre $O$ and radius $6 \mathrm{~cm} . Q$ is a point such that $Q R$ is parallel to $P O$ and $\angle P Q R=45^{\circ}$.

(a) Explain why $Q R=12 \mathrm{~cm}$.

Answer $\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) Find the area of the shaded region. Give your answer in the form $a-b \pi$.

$$
\text { Answer } \mathrm{cm}^{2}
$$

# ANG MO KIO SECONDARY SCHOOL FINAL EXAMINATION 2018 SECONDARY THREE EXPRESS 

MATHEMATICS

Paper 2
Setter: Mrs Koh Hui Teng

Thursday

Additional Materials:

04 October 2018

## 2 hours 30 minutes

Answer Paper
Graph Paper

## READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Compound interest
Total amount $=P\left(1+\frac{r}{100}\right)^{n}$

Mensuration
Curved surface area of a cone $=\pi r l$
Surface area of a sphere $=4 \pi r^{2}$
Volume of a cone $=\frac{1}{3} \pi r^{2} h$
Volume of a sphere $=\frac{4}{3} \pi r^{3}$
Area of triangle $A B C=\frac{1}{2} a b \sin C$
Arc length $=r \theta$, where $\theta$ is in radians
Sector area $=\frac{1}{2} r^{2} \theta$, where $\theta$ is in radians

Trigonometry

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{gathered}
$$

## Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

## Answer all the questions.

1 (a) Simplify, leaving your answers as positive indices where necessary,
(i) $\quad\left(a^{\frac{2}{5}}\right)^{10} \div\left(a^{\frac{1}{3}}\right)^{6}$,
(ii) $\left(81 m^{-6}\right)^{\frac{1}{2}}$,
(iii) $\frac{x^{4}-x^{2}}{x^{2}-x}$.
(b) Solve the equations
(i) $x^{-\frac{2}{3}}=\frac{1}{4}$,
(ii) $\quad 243^{x+7}=27^{x-1}$.
(c) (i) Solve the inequality $\frac{3}{4} x-23<5-x \leq x-10$.
(ii) State the smallest prime number which satisfies the above inequality.

2 (a) Mr and Mrs Lee open separate bank accounts.
(i) Mr Lee deposits $\$ 1000$ in his account. This account pays simple interest at the rate of $4 \%$ per year. Calculate the total amount in his account after 3 years.
(ii) Mrs Lee deposits $\$ 1000$ in her account. This account pays compound interest at the rate of $4 \%$ per year. Find the difference of money in both their accounts after 3 years.
(b) The cash price of a new laptop is $\$ 2400$. Andy buys this laptop on hire purchase. He pays a deposit of one third of the cash price followed by 24 monthly instalments of $\$ 72.50$. Calculate the total amount that Andy will pay for the computer.
(c) The exchange rate between Euros ( $€$ ) and Singapore dollars ( $\mathrm{S} \$$ ) was $€ 1=\mathrm{S} \$ 1.56$. Tammy bought a wallet from an online shop for $€ 298$ after $20 \%$ discount. Find the original price of the wallet in Singapore dollars.

3 The Nature Society chartered an air-conditioned bus for $\$ 1500$ to take a group of $x$ members to Malaysia for a trekking trip. It was agreed that each member of the group would pay an equal share of this transport fee.
(a) Write down an expression, in terms of $x$, for the amount of money each member of the group had to pay.

On the day of departure, three members of the group could not make it for the trip. The Nature Society decided that it would contribute $\$ 140$ from its funds and that the balance of the transport fee was to be shared equally by the remaining members.
(b) Write down an expression, in terms of $x$, for the amount which each remaining member had to pay after the three members had withdrawn from the trip.
(c) As a result of the three members withdrawing from the trip, the amount each member had to pay was $\$ 5$ more than the initial amount.
Form an equation in $x$ and show that it reduces to

$$
\begin{equation*}
x^{2}+25 x-900=0 \tag{3}
\end{equation*}
$$

(d) Solve the equation $x^{2}+25 x-900=0$.
(e) Find the transport fee that each member had to pay initially.

4 (a) It is given that $2 c=\sqrt[3]{\frac{e^{2}}{d}}$.
(i) Find $c$ when $d=3$ and $e=9$.
(ii) Express $e$ in terms of $c$ and $d$.
(b) Factorise completely $16 a^{2}-10 a b-8 a+5 b$.
(c) Express as a single fraction in its simplest form $\frac{2}{x-3}-\frac{x+3}{2 x^{2}-5 x-3}$.
(d) Solve the simultaneous equations

$$
\begin{gather*}
3 x-5 y=31 \\
x+3 y=1 \tag{3}
\end{gather*}
$$

5


## Diagram I

Diagram II
Diagram I shows a spinning top made by joining together a cylinder and a cone with base radius 6 cm . The height of the cylinder is 2 cm and the vertical height of the cone is 8 cm . Diagram II shows a vertical cross-section of the cone.
(a) Find the slant height of the cone.
(b) Leaving your answers in terms of $\pi$, calculate
(i) the volume of the top,
(ii) the total surface area of the top.

Each complete spin made by point $P$ along the circumference of the cylinder is taken to be 1 revolution. The top spins at 3 revolutions per second.
(c) Calculate the distance, in cm , moved by the point $P$ in 1 minute.

Answer the whole of this question on a sheet of graph paper.

The table below shows some values of $x$ and the corresponding values of $y$, correct to the nearest whole number, where

$$
y=2 x^{2}+\frac{80}{x}-30 .
$$

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 52 | 18 | 15 | 22 | $p$ | 55 | 108 |

(a) Find the value of $p$.
(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal $x$-axis for $0 \leq x \leq 8$.

Using a scale of 2 cm to represent 10 units, draw a vertical $y$-axis for $0 \leq y \leq 110$.
On your axes, plot the points given in the table and join them with a smooth curve.
(c) Use your graph to find the values of $x$ for which $y=20$.
(d) By drawing a tangent, find the gradient of the curve at $(4,22)$.
(e) Use your graph to find solutions to the equation $2 x^{2}-20 x+\frac{80}{x}-30=0$ in the range

$$
0 \leq x \leq 8
$$



In the diagram, $A B$ is parallel to $F C$ and $G F$ is parallel to $B E$. Given that $B C=C D=D E$ and $G B=6 \mathrm{~cm}$.
(a) Show that $\triangle P C D$ is congruent to $\triangle P F G$.
(b) Name another triangle that is similar to $\triangle P C D$.
(c) Calculate the length of $P C$.
(d) Write down the numerical value of
(i) $\frac{\text { area of } \triangle F C D}{\text { area of } \triangle F C E}$,
(ii) $\frac{\text { area of } \triangle A G F}{\text { area of trapezium } G B E F}$.


The diagram shows a straight path $A C D$ running from $A$ in a direction $060^{\circ}$. A building at $B$ is 70 m due south of $A . A C=95 \mathrm{~m}$ and $B D=180 \mathrm{~m}$.
(a) Calculate
(i) the distance $B C$,
(ii) the bearing of $D$ from $B$.
(b) Given that from $D$, the angle of elevation of the top of the building that stands at $B$ is $24^{\circ}$, calculate the height of the building.
(c) Calculate the area of $\triangle B C D$, giving your answer correct to the nearest $\mathrm{m}^{2}$.


The diagram shows an open fish tank, constructed by removing a portion of the cylinder of radius 10 cm and length 20 cm . The cross-section APC of the tank is the major segment of the circle centred at $O$ and angle $A O C=1.2 \mathrm{rad}$.

Find
(a) the length of the major arc $A P C$,
(b) the area of the major segment $A P C$,
(c) the total volume of the fish tank,
(d) the total external surface area of the fish tank.

10 Container ships are cargo ships that carry all of their load in truck-size intermodal containers. They are a common means of commercial intermodal freight transport.
Below is an example of a container ship :


Name : MSC Beatrice
Length : 366 m
Breadth : 51 m
Capacity : 14000 TEU
Cost of diesel fuel per litre : US\$1.63
Diesel consumption per year without retractable sails : Approximately 3600000 litres
(a) Calculate the cost of diesel consumption in a year.
(b) TEU stands for Twenty-Foot Equivalent Unit which can be used to measure a ship's cargo carrying capacity. The dimensions of one TEU are equal to that of a standard shipping container measuring 2.44 m by 2.17 m by 2.17 m .
Calculate the total volume of the maximum number of containers MSC Beatrice can carry in cubic metres.
(c) Due to high diesel fuel costs, the owners of MSC Beatrice decide to equip giant, retractable sails on their ship. They can be used to maximise wind energy while simultaneously cutting down on fuel consumption.

## Useful Information

- Cost of equipping retractable sails : \$2,800,000
- Estimated to reduce diesel consumption by $20 \%$
(i) Calculate the cost of diesel consumption in a year, after equipping retractable sails.
(ii) The owners expect to recover the cost of equipping retractable sails by the end of 2 years.
Do you think this is possible? Justify your answer by showing your workings clearly.

AMKSS FE 2018 3E EM P1 Marking Scheme
1 mark deducted from whole paper if answers to fractions were not reduced to lowest term and mixed fractions.

1 mark deducted from whole paper for answers not given to 3 significant figures. (Q5a)

| Qn | Answers | Marks |
| :---: | :---: | :---: |
| 1a | $\begin{aligned} & 5 x-2(3 x-7) \\ & =5 x-6 x+14 \\ & =-x+14 \end{aligned}$ | B1 |
| 1b | $\begin{aligned} & 6 x^{2}+20 x-16 \\ & =2\left(3 x^{2}+10 x-8\right) \\ & =2(3 x-2)(x+4) \end{aligned}$ | M1 <br> A1 |
| 2a | $8.07 \times 10^{-4} \mathrm{~m}$ | B1 |
| 2b | $\begin{aligned} & 4 \pi\left(\frac{8.07 \times 10^{-4}}{2}\right)^{2} \\ & =\pi \times 65.1249 \times 10^{-8} \\ & =204.5959074 \times 10^{-8} \\ & =2.05 \times 10^{-6} \mathrm{~m}^{2} \end{aligned}$ | M1 <br> A1 |
| 3a | 32 | B1 |
| 3b | $5 n+2$ | B1 |
| 3c | $\begin{aligned} 200 & =5 n+2 \\ 5 n & =198 \\ n & =39.6 \end{aligned}$ <br> $n$ must be an integer OR 198 is not a multiple of 5 | B1 |
| 4a | $\begin{aligned} V & =\frac{k}{\sqrt[3]{P}} \\ 15 & =\frac{k}{\sqrt[3]{2744}} \\ k & =210 \\ V & =\frac{210}{\sqrt[3]{P}} \end{aligned}$ | M1 <br> A1 |
| 4b | $\begin{aligned} & \sqrt[3]{P}=\frac{210}{21}=10 \\ & P=10^{3}=1000 \mathrm{~Pa} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |


| 5 a | $\begin{aligned} \frac{p}{6}-\frac{3(2-p)}{4} & =1 \\ \frac{2 p-3(6-3 p)}{12} & =1 \\ 11 p-18 & =12 \\ 11 p & =30 \\ p & =2 \frac{8}{11} \end{aligned}$ | M1 <br> A1 |
| :---: | :---: | :---: |
| 5b | $\begin{aligned} & \frac{1}{x}+\frac{3}{x-1}+\frac{2}{x+1}=0 \\ & \frac{3(x+1)+2(x-1)}{x^{2}-1}=-\frac{1}{x} \\ & \frac{5 x+1}{x^{2}-1}=-\frac{1}{x} \\ & 5 x^{2}+x=-x^{2}+1 \\ & 6 x^{2}+x-1=0 \\ & (3 x-1)(2 x+1)=0 \\ & x=\frac{1}{3} \text { or }-\frac{1}{2} \end{aligned}$ | M1 <br> M1 <br> A1 |
| 6a | $\begin{aligned} & x^{2}-6 x+7 \\ & =x^{2}-6 x+\left(\frac{6}{2}\right)^{2}-\left(\frac{6}{2}\right)^{2}+7 \\ & =(x-3)^{2}-2 \end{aligned}$ | M1 <br> A1 <br> or B2 |
| 6b | -2 | B1 |
| 6c | $x=3$ | B1 |
| 7 | $\left.\begin{array}{l} \frac{\text { Rate } A}{(15 \times 40)+(12 \times 15)} \\ =\$ 780 \\ \frac{\text { Rate } B}{(18.5 \times 30)+(10 \times 25)} \\ =\$ 805 \end{array}\right\}$ <br> Linda should choose Rate $\underline{B}$ because she will get $\$ 25$ more. | $\mathrm{A} 1, \mathrm{~A} 1$ |


| 8a | $\begin{aligned} Q R & =\sqrt{(10+5)^{2}+(0-9)^{2}} \\ & =\sqrt{306} \\ & =17.4928556845 \\ & =17.5 \text { units } \end{aligned}$ | M1 A1 |
| :---: | :---: | :---: |
| 8b | $\begin{aligned} & m=\frac{0-9}{10-(-5)} \\ &=\frac{-9}{15}=-\frac{3}{5} \\ & 0=-\frac{3}{5}(10)+c \\ & c=6 \\ & \Rightarrow y=-\frac{3}{5} x+6 \end{aligned}$ | M1 A1 |
| 8c | $S(0,6)$ | B1 |
| 8d | $\begin{aligned} & \frac{1}{2}(3)(9)+\frac{1}{2}(9+6)(5) \\ = & \frac{27}{2}+\frac{75}{2} \\ = & 51 \text { units }^{2} \end{aligned}$ | M1 <br> A1 |
| OR | $\begin{aligned} & \Delta R P Q-\triangle S O Q \\ = & 81-30 \\ = & 51 \text { units }^{2} \end{aligned}$ |  |
| 9 ai | $\begin{aligned} & \frac{1}{5} x^{3} \times(-3 x y)^{2} \\ & =\frac{1}{5} x^{3} \times 9 x^{2} y^{2} \\ & =\frac{9}{5} x^{5} y^{2} \end{aligned}$ | M1 <br> A1 |
| 9aii | $\begin{aligned} & a^{\frac{5}{3}} \div \sqrt[3]{a^{2}} \\ & =a^{\frac{5}{3}} \div a^{\frac{2}{3}} \\ & =a^{\frac{5-2}{3}}=a \end{aligned}$ | M1 for fractional index A1 |
| 9 b | $\begin{aligned} & 7^{2 x-1} \times 49^{x}=1 \\ & 7^{2 x-1} \times\left(7^{2}\right)^{x}=7^{0} \\ & 2 x-1+2 x=0 \\ & 4 x=1 \\ & x=\frac{1}{4} \end{aligned}$ | M1 for zero index <br> M1 <br> A1 |


| 10 |  | B1 <br> for correct shape \& y intercept <br> B1 <br> For correct $x$ intercepts |
| :---: | :---: | :---: |
| 11a | $2^{2} \times 3^{3} \times 5$ | B1 |
| 11b | $m=2^{2} \times 5=20$ | B1 |
| 11c | $\mathrm{HCF}=2 \times 3^{3} \times 5=270$ | B1 |
| 12a | $\begin{aligned} & 3 \mathrm{~cm}^{2}: 27 \mathrm{~km}^{2} \\ & 1 \mathrm{~cm}^{2}: 9 \mathrm{~km}^{2} \\ & 1 \mathrm{~cm}: 3 \mathrm{~km} \\ & 1: 300000 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 12b | $\begin{aligned} & \frac{\operatorname{Map} X}{2.4 \mathrm{~cm}}: 7.2 \mathrm{~km} \\ & \frac{\operatorname{Map} Y}{1: 400000} \\ & 1 \mathrm{~cm}: 4 \mathrm{~km} \\ & \text { Length of road }=7.2 \div 4=1.8 \mathrm{~cm} \end{aligned}$ | M1 <br> A1 |
| 13 | $\begin{aligned} & 2 w \div \frac{1}{6} \\ & =12 w \mathrm{~km} / \mathrm{h} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 14a | $\left.\begin{array}{l} A B^{2}=18^{2}=324 \\ B D^{2}=(7.5+16.5)^{2}=576 \\ A D^{2}=30^{2}=900 \\ A B^{2}+B D^{2}=324+576=900 \\ \text { Since } A B^{2}+B D^{2}=A D^{2} \\ \text { By Pythagoras' Theorem } \\ \triangle A B D \text { is a right-angled } \triangle \end{array}\right]$ | M1 <br> M1 |
| 14bi | $\frac{18}{30}=\frac{3}{5}$ | B1 |


| 14bii | $\begin{aligned} & A C=\sqrt{18^{2}+7.5^{2}}=19.5 \mathrm{~cm} \\ & \cos \angle A C B=\frac{7.5}{19.5}=\frac{5}{13} \\ & \cos \angle A C D=-\frac{5}{13} \end{aligned}$ | M1 <br> A1 |
| :---: | :---: | :---: |
| 15a | $\left.\begin{array}{l} \angle L G H=\angle J G K \quad(\text { common } \angle) \\ \frac{G L}{G J}=\frac{8}{24}=\frac{1}{3} \\ \frac{G H}{G K}=\frac{4}{12}=\frac{1}{3} \end{array}\right\}$ <br> $\triangle G J K$ is similar to $\triangle G L H$ (2 ratios of corr. sides and included $\angle$ equal) or (SAS) | M1 <br> M1 for stating test |
| 15b | $\begin{aligned} \frac{L H}{J K} & =\frac{1}{3} \\ J K & =3 \times 6 \\ & =18 \mathrm{~cm} \end{aligned}$ | $\begin{array}{\|l} \text { M1 } \\ \text { A1 } \end{array}$ |
| 16 | $\begin{aligned} & \frac{l_{1}}{l_{2}}=\sqrt{\frac{9}{4}}=\frac{3}{2} \\ & \frac{V_{1}}{V_{2}}=\frac{27}{8}=\frac{m_{1}}{3.2} \\ & m_{1}=\frac{27 \times 3.2}{8}=10.8 \mathrm{~kg} \end{aligned}$ | M1 <br> M1 <br> A1 |
| 17a | $\frac{360}{6}=60^{\circ}$ | B1 |
| 17 b | $180-60=120^{\circ}$ | B1 |
| 17c | $\frac{60}{2}=30^{\circ}$ | B1 |
| 18a | $\frac{10.5}{4}=2.625 \mathrm{~m} / \mathrm{s}^{2}$ | B1 |
| 18b | $\begin{gathered} \frac{1}{2}(4 \times 10.5)+(6 \times 10.5)+\frac{1}{2}(2)(10.5+v)=100 \\ 21+63+10.5+\mathrm{v}=100 \\ v=5.5 \mathrm{~m} / \mathrm{s} \end{gathered}$ | M1 ( $\Delta$ area), <br> M1 (trapezium) <br> A1 |


| 18c |  | A1 (0-4) <br> A1 (4-10) <br> A1 (10-12) <br> Minus 1M if <br> no distance <br> labelled on <br> axis |
| :---: | :---: | :---: |
| 19a | $\angle C D B=53^{\circ}$ (alt $\angle \mathrm{s}$ ) | B1 |
| 19b | $\begin{aligned} \angle C B D & =180-71-53(\text { interior } \angle \mathrm{s}) \\ & =56^{\circ} \end{aligned}$ | $\begin{aligned} & \mathrm{M} 1 \\ & \mathrm{~A} 1 \end{aligned}$ |
| 19c | $\begin{aligned} \angle D A B & =56-21(\text { ext } \angle \text { of } \Delta) \\ & =35^{\circ} \\ \text { Reflex } \angle D A B & =360-35(\angle \mathrm{~s} \text { at a pt }) \\ & =325^{\circ} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 20a | Let $P X$ be perpendicular to $Q R$. <br> $\triangle P Q X$ is an isosceles triangle $\begin{aligned} & \Rightarrow Q X=P X=6 \mathrm{~cm} \\ & \Rightarrow Q R=12 \mathrm{~cm} \end{aligned}$ | M1 for <br> stating isosceles $\Delta$ |
| 20b | $\begin{aligned} \text { Area of shaded region } & =\frac{1}{2}(6+12)(6)-\frac{1}{4} \pi(6)^{2} \\ & =(54-9 \pi) \mathrm{cm}^{2} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |

## AMKSS 3E MATHEMATICS PAPER 2

## SOLUTIONS

| Question | Solutions | Marks |
| :---: | :---: | :---: |
| 1(a)(i) | $\begin{aligned} & \left(a^{\frac{2}{5}}\right)^{10} \div\left(a^{\frac{1}{3}}\right)^{6} \\ = & a^{4} \div a^{2} \\ = & a^{2} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 1(a)(ii) | $\begin{aligned} & \left(81 m^{-6}\right)^{\frac{1}{2}} \\ & =9 m^{-3} \\ & =\frac{9}{m^{3}} \end{aligned}$ | M1 <br> A1 |
| 1(a)(iii) | $\begin{aligned} & \frac{x^{4}-x^{2}}{x^{2}-x} \\ & =\frac{\left(x^{2}-x\right)\left(x^{2}+x\right)}{x^{2}-x} \\ & =x^{2}+x \end{aligned}$ | M1 <br> A1 |
| 1(b)(i) | $\begin{aligned} & x^{-\frac{2}{3}}=\frac{1}{4} \\ & x^{\frac{2}{3}}=4 \\ & x=4^{\frac{3}{2}} \\ & x=8 \end{aligned}$ | M1 <br> A1 |
| 1(b)(ii) | $\begin{aligned} & 243^{x+7}=27^{x-1} \\ & 3^{5 x+35}=3^{3 x-3} \\ & 5 x+35=3 x-3 \\ & 2 x=-38 \\ & x=-19 \end{aligned}$ | M1 <br> M1 <br> A1 |
| 1(c)(i) | $\begin{array}{ll} \frac{3}{4} x-23<5-x \leq x-10 \\ \frac{3}{4} x-23<5-x & \text { or } \\ 5-x \leq x-10 \\ \frac{7}{4} x<28 & \\ x<16 & \\ 7.5 \leq x<16 & \\ l .5 \leq x \\ 7 . & \end{array}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 1(c)(ii) | 11 | A1 |
| 2(a)(i) | $\begin{aligned} & I=\frac{P R T}{100} \\ & I=\frac{(1000)(4)(3)}{100}=\$ 120 \\ & \text { Total }=\$ 1120 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |


| 2(a)(ii) | $\begin{aligned} & A=P\left(1+\frac{r}{100}\right)^{n} \\ & A=1000\left(1+\frac{4}{100}\right)^{3} \\ & A=\$ 1124.864 \\ & \$ 1124.864-\$ 1120=\$ 4.86 \end{aligned}$ | $\begin{array}{\|l} \text { M1 } \\ \text { M1 } \\ \text { A1 } \end{array}$ |
| :---: | :---: | :---: |
| 2(b) | $\begin{aligned} & 24 \times \$ 72.50=\$ 1740 \\ & \frac{1}{3} \times \$ 2400=\$ 800 \\ & \$ 800+\$ 1740=\$ \$ 2540 \end{aligned}$ | M1 <br> A1 |
| 2(c) | $\begin{aligned} & \frac{298}{80} \times 100=€ 372.50 \\ & 372.50 \times 1.56=S \$ 581.10 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 3(a) | $\frac{1500}{x}$ | B1 |
| 3(b) | $\frac{1360}{x-3}$ | B1 |
| 3(c) | $\begin{aligned} & \frac{1360}{x-3}-\frac{1500}{x}=5 \\ & \frac{1360 x-1500 x+4500}{x(x-3)}=5 \\ & -140 x+4500=5 x^{2}-15 x \\ & 5 x^{2}+125 x-4500=0 \\ & x^{2}+25 x-900=0 \text { (shown) } \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { M1 } \\ & \text { M1 } \end{aligned}$ |
| 3(d) | $\begin{aligned} & x^{2}+25 x-900=0 \\ & x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\ & x=\frac{-25 \pm \sqrt{(25)^{2}-4(1)(-900)}}{2(1)} \\ & x=\frac{-25 \pm \sqrt{4225}}{2} \\ & x=20 \text { or } \quad x=-45 \end{aligned}$ | M1 <br> A1, A1 |
| 3(e) | $\begin{aligned} & \frac{1500}{20} \\ & =\$ 75 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 4(a)(i) | $\begin{aligned} & 2 c=\sqrt[3]{\frac{e^{2}}{d}} \\ & 2 c=\sqrt[3]{\frac{9^{2}}{3}} \\ & 2 c=3 \\ & c=1.5 \end{aligned}$ | B1 |


| 4(a)(ii) | $\begin{aligned} & 2 c=\sqrt[3]{\frac{e^{2}}{d}} \\ & 8 c^{3}=\frac{e^{2}}{d} \\ & 8 c^{3} d=e^{2} \\ & e= \pm \sqrt{8 c^{3} d} \end{aligned}$ | M1 <br> A1 |
| :---: | :---: | :---: |
| 4(b) | $\begin{aligned} & 16 a^{2}-10 a b-8 a+5 b \\ & =2 a(8 a-5 b)-(8 a-5 b) \\ & =(8 a-5 b)(2 a-1) \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 4(c) | $\begin{aligned} & \frac{2}{x-3}-\frac{x+3}{2 x^{2}-5 x-3} \\ & =\frac{2}{x-3}-\frac{x+3}{(2 x+1)(x-3)} \\ & =\frac{2(2 x+1)}{(x-3)(2 x+1)}-\frac{x+3}{(2 x+1)(x-3)} \\ & =\frac{4 x+2-x-3}{(x-3)(2 x+1)} \\ & =\frac{3 x-1}{(x-3)(2 x+1)} \end{aligned}$ | M1 <br> M1 A1 |
| 4(d) | $\begin{align*} & 3 x-5 y=31  \tag{1}\\ & x+3 y=1 \tag{2} \end{align*}$ <br> From (2), $\begin{align*} & x+3 y=1 \\ & x=1-3 y \tag{3} \end{align*}$ <br> Sub (3) into (1) $\begin{aligned} & 3(1-3 y)-5 y=31 \\ & 3-9 y-5 y=31 \\ & -14 y=28 \\ & y=-2 \end{aligned}$ <br> Sub $y=-2$ into (3) $x=1-3(-2)=7$ | Any appropriate method M1 <br> A1 <br> A1 |
| 5(a) | $\begin{aligned} & \begin{array}{l} \sqrt{6^{2}+8^{2}} \\ =10 \mathrm{~cm} \end{array} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 5(b)(i) | $\begin{aligned} & \text { Volume }=\pi(6)^{2}(2)+\frac{1}{3} \pi(6)^{2}(8) \\ & =168 \pi \end{aligned}$ | $\begin{aligned} & \text { M1, M1 } \\ & \text { A1 } \end{aligned}$ |
| 5(b)(ii) | Curved SA of Cylinder $\begin{aligned} & =2 \pi(6)(2) \\ & =24 \pi \end{aligned}$ | M1 |


|  | $\begin{aligned} & \text { Curved SA of Cone } \\ & =\pi(6)(10) \\ & =60 \pi \\ & \text { Total SA } \\ & =24 \pi+60 \pi+\pi(6)^{2} \\ & =120 \pi \end{aligned}$ | M1 A1 |
| :---: | :---: | :---: |
| 5(c) | $\begin{aligned} & 2 \pi(6) \times 3 \times 60 \\ & =6785.840132 \\ & =6790 \mathrm{~cm} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 6(a) | $p=36$ | B1 |
| 6(b) | Correct scale Correct points Smooth curve | $\begin{aligned} & \hline \text { B1 } \\ & \text { B1 } \\ & \text { B1 } \\ & \hline \end{aligned}$ |
| 6(c) | $x=1.8$ or 3.8 <br> (Accept 1.7 to 1.9)  (Accept 3.7 to 3.9) | B1, B1 |
| 6(d) | Draw tangent <br> Gradient $=10.7$ (Accept 10 to 12) | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 6(e) | $\begin{aligned} & \text { Draw } y=20 x \\ & x=1.4 \text { (Accept } 1.3 \text { to } 1.5 \text { ) } \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 7(a) | $\begin{aligned} & \angle G F P=\angle D C P \text { (Alternate angles) } \\ & \angle F G P=\angle C D P \text { (Alternate angles) } \\ & \mathrm{GF}=\mathrm{DC} \text { (sides of parallelogram) } \\ & \triangle P C D \text { is congruent to } \triangle P F G \text { (AAS) } \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 7(b) | $\triangle G B D$ or $\triangle A G F$ or $\triangle A B E$ or $\triangle F C E$ | B1 |
| 7(c) | $\begin{aligned} & \frac{P C}{6}=\frac{1}{2} \\ & P C=3 \mathrm{~cm} \end{aligned}$ <br> Or <br> Since $\triangle P C D$ is congruent to $\triangle P F G$, Therefore $P C=P F=6 \div 2=3 \mathrm{~cm}$ | M1 <br> A1 $\mathrm{M} 1, \mathrm{~A} 1$ |
| 7(d)(i) | $\frac{1}{2}$ | B1 |
| 7(d)(ii) | $\begin{aligned} & \frac{\text { area of } \triangle A G F}{\text { area of } \triangle \mathrm{ABE}}=\frac{1}{9} \\ & \frac{\text { area of } \triangle A G F}{\text { area of trapezium } G B E F}=\frac{1}{8} \end{aligned}$ | B1 |
| 8(a)(i) | $\begin{aligned} & B C^{2}=70^{2}+95^{2}-2(70)(95) \cos 120^{\circ} \\ & B C^{2}=20575 \\ & B C=143.4398829 \\ & B C=143 \mathrm{~m} \end{aligned}$ | M1 <br> M1 <br> A1 |


| 8(a)(ii) | $\begin{aligned} & \frac{180}{\sin 120^{\circ}}=\frac{70}{\sin \angle A D B} \\ & \sin \angle A D B=0.336787657 \\ & \angle A D B=19.68128117^{\circ} \\ & 180^{\circ}-19.68128117^{\circ}-120^{\circ} \\ & =40.31871883 \\ & \approx 040.3^{\circ} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| :---: | :---: | :---: |
| 8(b) | $\begin{aligned} & \tan 24^{\circ}=\frac{h}{180} \\ & h=80.14116336 \approx 80.1 \mathrm{~m} \end{aligned}$ | $\begin{array}{\|l} \hline \mathrm{M} 1 \\ \mathrm{~A} 1 \end{array}$ |
| 8(c) | $\begin{aligned} & \frac{143.4398829}{\sin 120^{\circ}}=\frac{95}{\sin \angle A B C} \\ & \sin \angle A B C=0.5735672095 \\ & \angle A B C=34.99935463^{\circ} \\ & \angle C B D=40.31871883-34.99935463 \\ & =5.319364204^{\circ} \end{aligned}$ $\begin{aligned} & \text { Area }= \\ & \frac{1}{2}(180)(143.4398829) \sin (5.319364204) \\ & =1196.810683 \\ & =1197 \mathrm{~m}^{2} \end{aligned}$ <br> or $\frac{1}{2}(180)(70) \sin 40.3187=4076.34515$ $\frac{1}{2}(70)(95) \sin 120=2879.534468$ $4076.34515-2879.5344$ $=1196.810682$ $\approx 1197 m^{2}$ | M1 <br> M1 <br> A1 <br> M1 <br> M1 <br> A1 |
| 9(a) | $\begin{aligned} & \text { Length of Major arc APC }=(10)(2 \pi-1.2) \\ & =50.83185307 \mathrm{~m} \\ & =50.8 \mathrm{~m} \end{aligned}$ | $\begin{array}{\|l} \hline \text { M1 } \\ \text { A1 } \end{array}$ |
| 9(b) | $\begin{aligned} \text { Area of Sector APCO } & =\frac{1}{2}(10)^{2}(2 \pi-1.2) \\ & =254.1592654 \\ \text { Area of triangle AOC } & =\frac{1}{2}(10)(10) \sin 1.2 \\ & =46.6019543 \end{aligned}$ <br> Total area of segment APC $\begin{aligned} & =254.1592654+46.6019543 \\ & =300.7612197 \end{aligned}$ | M1 <br> M1 <br> A1 |


|  | $=301 \mathrm{~cm}^{2}$ |  |
| :---: | :---: | :---: |
| 9(c) | $\begin{aligned} & \text { Total Volume }=300.7612197 \times 20 \\ & =6015.224=6020 \mathrm{~cm}^{3} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 9(d) | $\begin{aligned} & \hline \text { Curved Surface area }=50.83185307 \times 20 \\ &=1016.637061 \\ & \begin{aligned} & \text { Total surface area }=1016.637061+2 \\ &(300.7612197) \end{aligned} \\ &= 1618.159501 \\ &=1620 \mathrm{~cm}^{2} \end{aligned}$ | M1 <br> M1 <br> A1 |
| 10(a) | $1.63 \times 3600000=$ US $\$ 5868000$ | B1 |
| 10(b) | $\begin{aligned} & 2.44 \times 2.17 \times 2.17=11.489716 \\ & 11.489716 \times 14000=160856.024 \mathrm{~m}^{3} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 10(c)(i) | $\begin{aligned} & \frac{80}{100} \times 5868000 \\ & =\text { US } \$ 4694400 \\ & \hline \end{aligned}$ | M1 <br> A1 |
| 10(c)(ii) | $\begin{aligned} & \text { Savings per year }= \\ & \frac{20}{100} \times 5868000=\text { US } \$ 1173600 \\ & 2800000 \div 1173600 \\ & =2.385821404 \end{aligned}$ <br> Not possible <br> Or <br> US $\$ 1173600 \times 2=$ US $\$ 2345200$ <br> US\$2880000 - US\$2345200 $=\text { US } \$ 454800$ <br> Not possible as they are short of US\$454800. | M1 <br> M1 <br> A1 <br> M1 <br> M1 <br> A1 |

3E Raper 2 Question 6


