YUSOF ISHAK SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2020

CANDIDATE NAME	
CLASS	
INDEX NUMBER	

MATHEMATICS 4048/01
4 Express/5 Normal Academic

27 August 2020

Candidates answer on the Question Paper. **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces provided on the work you hand in.
Write in dark blue or black ink on both sides of the paper.
Do not use staples, paper clips, highlighters, glue ' correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

Mathematical formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved Surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

[3]

1 Write the following numbers in order of size, starting with the **largest**.

$$-\frac{22}{7},\ -\pi,\ -3.142,\ -3.\dot{3}$$

Answer: ....., ..., ..., ... [1]
largest *smallest*

---

2 Bob's test marks for six of his tests are listed below.

24	a	22	33	b	26

The median mark is 26.
The mean mark is 27.

Given that $a < b$, find the values of a and b.

Answer: a = ...

b = ... [2]

---

3 n is a positive integer.

Show that $(2n+1)^2-(2n-3)^2$ is a multiple of 8 for all integer values of n.

Answer

[2]

---

[4]

4 Simplify $\left(\frac{27x^9}{y^6}\right)^{-\frac{2}{3}}$.

Answer: ..... [2]

---

5 One solution of the equation $5x^2 - px + 40 = 0$ is $x = 2$.

Find

(a) the value of p,

Answer: *(a)* p = ... [1]

(b) the second possible value of x.

Answer: *(b)* x = ... [1]

---

6 When it is 1100 in Singapore, the time in Dubai is 0700.
A flight from Singapore departs at 1510 and arrives at Dubai at 1825. Given that the distance from Singapore to Dubai is 5840 km, find the average speed of the plane in kilometres per hour.

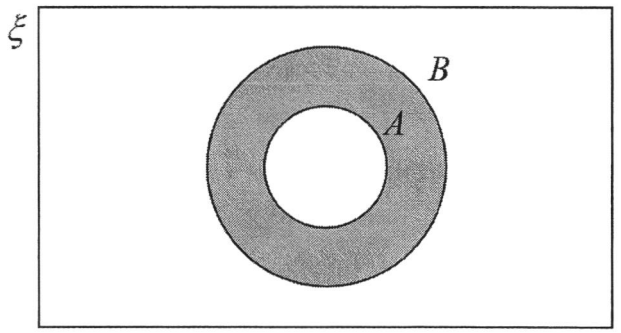
Answer: ..... km/h [2]

---

[5]

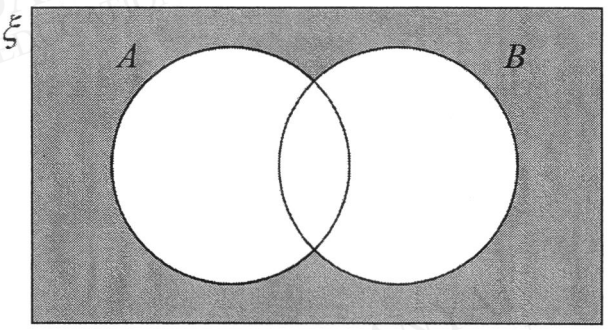
7 Write down the sets represented by the following shaded regions.

(a)



Answer: (a) ..... [1]

(b)



Answer: (b) ..... [1]

[6]

8

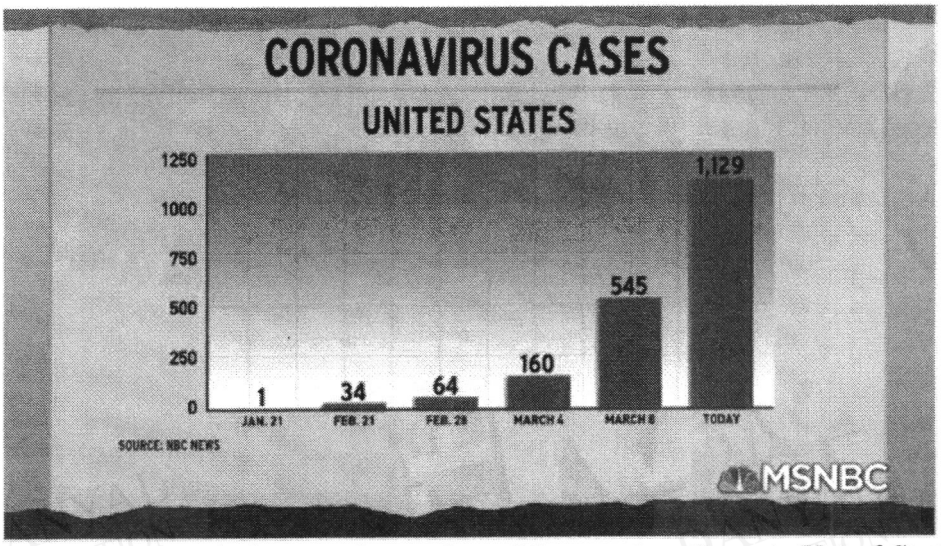


Figure 1: Bar graph showing the number of Coronavirus Cases in United States (Source: NBC NEWS)

(a) State one misleading feature about the presentation of the data in Figure 1.

.....

..... [1]

(b)

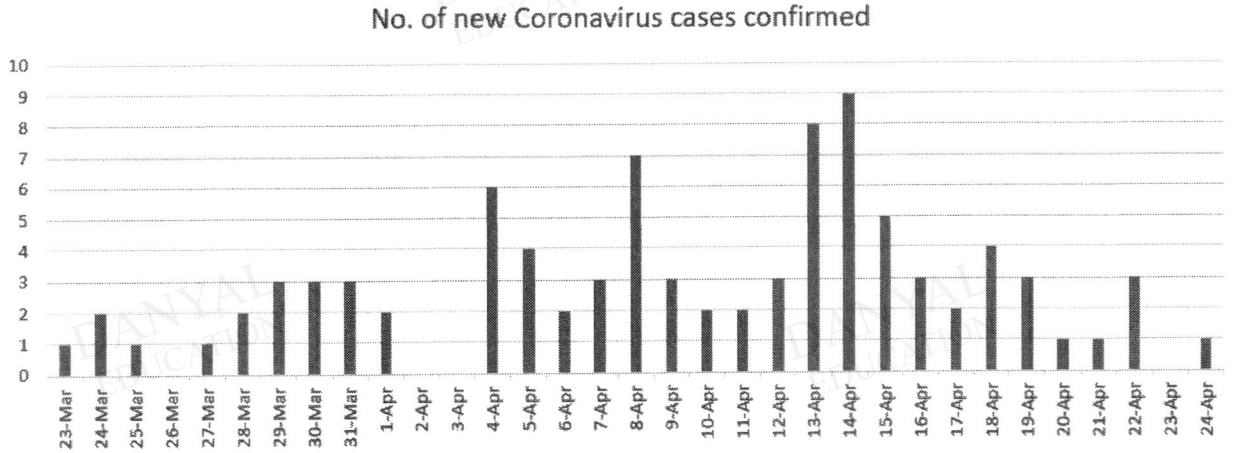


Figure 2: Bar graph showing the number of new coronavirus cases in Singapore everyday (Source: Channel News Asia)

After studying the trend in Figure 2, Sam claims that the total number of Coronavirus cases in Singapore will decrease in April.

Do you agree with Sam? Explain your answer.

.....

.....

..... [1]

[7]

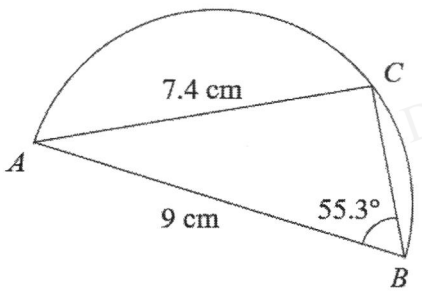
9 To purchase a washing machine, Jamie had to pay a deposit of 15% of the cash price.

The hire-purchase price of the washing machine is \$2106 which comprises of the deposit plus 12 equal monthly payments of \$153.

Find the cash price of the washing machine.

Answer: \$ ..... [3]

10



In the diagram, A, B and C are points on a circle.

$AB = 9$ cm, $AC = 7.4$ cm and angle $ABC = 55.3°$.

Explain why AB is not the diameter of the circle.

Answer: .....

.....

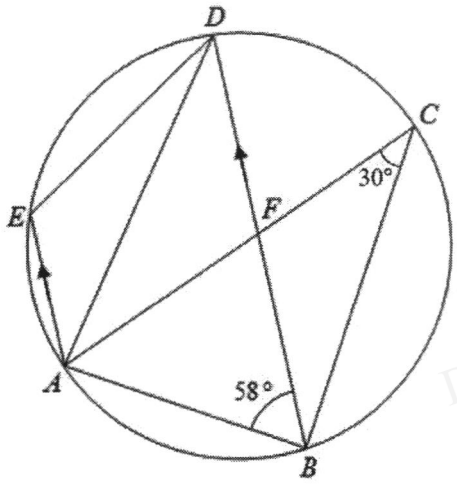
.....

..... [3]

---

[8]

11 In the diagram, A, B, C, D and E are points on a circle.
AC is a diameter of the circle and AE is parallel to BD.
F is the point of intersection of AC and BD.



Given that angle $ABD = 58°$ and angle $ACB = 30°$, find

(a) angle DBC,

Answer: (a) .....° [1]

(b) angle AED,

Answer: (b) .....° [1]

(c) angle ADE.

Answer: (c) .....° [1]

[9]

12 **(a)** 5 men take 9 days to build a house. How many days would 3 men take to build the house?

Answer: *(a)* .....days [1]

(b) The volume of water, V m^3, flowing through a cylindrical pipe is directly proportional to the square of its cross-sectional radius, R m. If the radius of the pipe is increased by 150%, find the percentage increase of the volume.

Answer: *(b)* .....% [2]

[10]

13 The test results of 40 students from Class *A* and 40 students from Class *B* were recorded. The results are shown in the stem-and-leaf diagram.

Class *A*		Class *B*
9 8 8 7 5 3	0	3 4 4 5
8 8 7 7 6 5 4 4 1 0	1	3 3 5 6 8 8 8 8 9
7 7 6 6 5 3 1 1	2	2 2 4 4 5 6 6 7 8 8
9 9 7 4 4 3 1 0 0	3	0 0 1 1 2 2 3 5 5 9
9 9 8 7 7 6 5	4	1 2 2 4 4 5 9
Key (Class *A*) 1 | 2 means 21		Key (Class *B*) 1 | 3 means 13

(a) Write down the median result of Class *B*.

Answer: (a) ..... [1]

(b) Make one comment comparing the test results of Class *A* and Class *B*.

Answer: (b)

.....

.....

.....

..... [2]

14 **(a)** Express 1176 as the product of its prime factors.

Answer: (a) ..... [1]

(b) Two integers, written as product of their prime factors, are

$$2^p \times 3 \times 7^q \text{ and}$$

$$2^2 \times 3^r \times 7^2.$$

The highest common factor of these two integers is 12 and the lowest common multiple of these two integers is 1176. Find the values of p, q and r.

Answer: (b) $p =$ .....

$q =$ .....

$r =$ ..... [3]

---

15 Factorise completely

(a) $27a^2b - 48b^3$,

Answer: (a) ..... [2]

(b) $28dy - 4y - 7d + 1$.

Answer: (b) ..... [2]

---

[12]

16 Rearrange the formula $t = 2\pi\sqrt{\dfrac{h+g}{h}}$ to make h the subject.

Answer: ..... [4]

---

17 The points $(-1,\ 4)$, $(2,\ -5)$ and $(0,\ -3)$ lie on the curve given by the equation $y = ax^2 + bx + c$. Use an algebraic method to find the values of a, b and c.

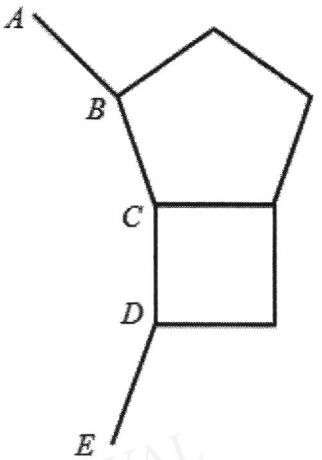
Answer: a = ...

b = ...

c = ... [4]

---

18 The diagram shows an incomplete n-sided regular polygon $ABCDE$, a square and a pentagon. The polygons fit together at C. Find the value of n.



Answer: n = ... [4]

---

19 The ratio of the surface areas of two geometrically similar pyramids is 64 : 121.

(a) Find the ratio of the volume of the smaller pyramid to the volume of the larger pyramid.

Answer: *(a)* ..... : ..... [2]

(b) Given that the volume of the smaller pyramid is 921.6 cm^3, find the volume of the larger pyramid.

Answer: *(b)* .....cm^3 [2]

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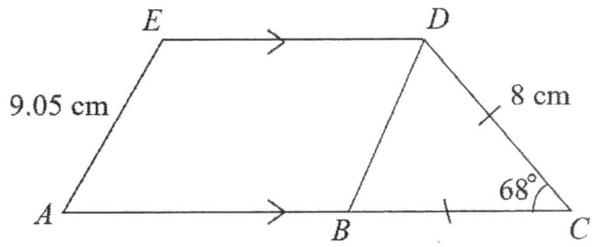
20 $ACDE$ is a trapezium. B is a point on AC such that $BC = DC = 8$ cm, angle $BCD = 68°$ and $AE = 9.05$ cm.



(a) Show that the height of trapezium $ACDE$ is 7.417 cm, correct to 3 decimal places.

Answer

[2]

(b) Explain why AE is not parallel to BD.

Answer: (b) .....

.....

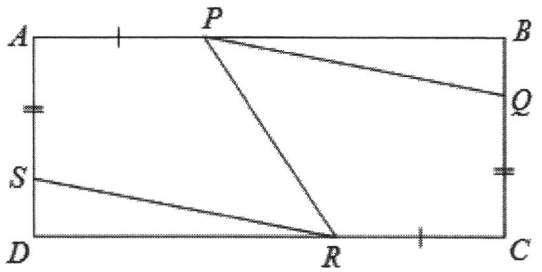
..... [2]

(c) Given that the area of $ACDE$ is 111 cm^2, find the area of $ABDE$.

Answer: (c) .....cm^2 [2]

[15]

21 $ABCD$ is a rectangle. Points P, Q, R and S lie on AB, BC, CD and DA respectively such that $AP = CR$ and $QC = SA$.



(a) Giving reasons clearly,

(i) show that $PB = RD$,

Answer (a)(i)

[1]

(ii) show that triangle PBQ is congruent to triangle RDS.

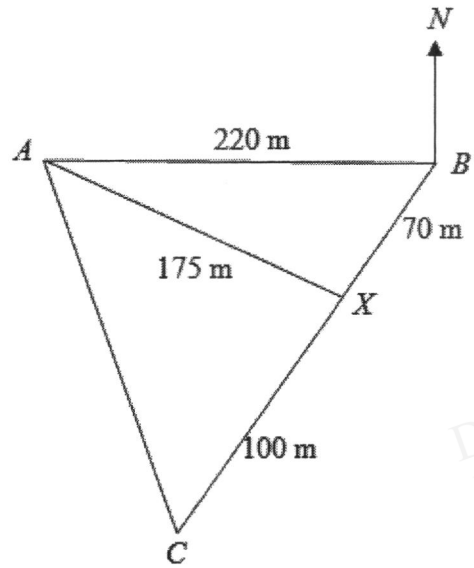
Answer (a)(ii)

[2]

(b) Given that angle $BPQ = 20°$ and angle $APR = 105°$, find angle PRS.

Answer: (b) $\angle PRS =$ ..... ° [1]

22 A, B and C are three points on a horizontal field. A is 220 m due west of B.
X is a point on BC such that $BX = 70$ m, $CX = 100$ m and $AX = 175$ m.



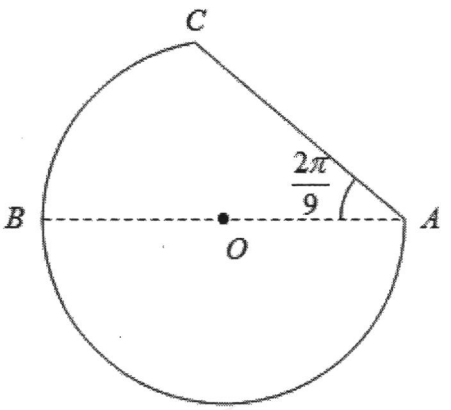
(a) Calculate the bearing of C from B.

Answer: (a) .....° [3]

(b) Calculate the shortest distance from X to AB.

Answer: (b) ..... m [2]

23 The diagram shows a major segment of circle ABC with centre O.



(a) Given that $AB = 24$ cm and angle OAC is $\frac{2\pi}{9}$ radian, show that the length of the arc $BC = 16.8$ cm, correct to 3 significant figures.

Answer

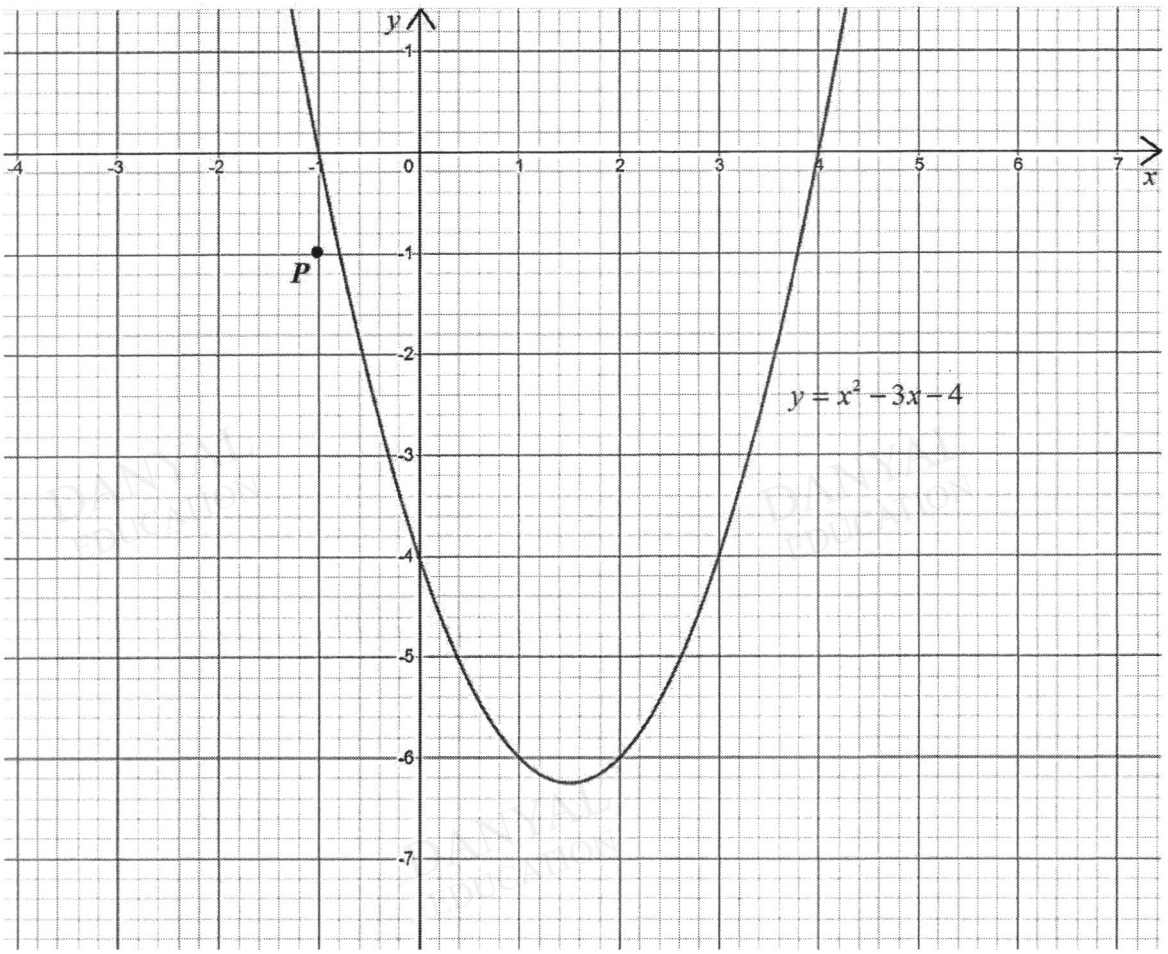
[2]

(b) Hence, find the perimeter of the major segment ABC.

Answer: (b) ..... cm [3]

[18]

24 The graph of $y = x^2 - 3x - 4$ is drawn on the grid.



(a) Write down the equation of the line of symmetry of the curve.

Answer: (a) ..... [1]

(b) Use the graph to solve the equation $x^2 - 3x - 1 = 0$.

Answer: (b) x = ... or ..... [2]

[Continued

(c) The point P has coordinates $(-1,-1)$.

A tangent to the curve can be drawn so that the tangent passes through P.

(i) Draw this tangent on the grid above. [1]

(ii) Find the equation of this tangent.

Answer: (c)(ii) ..... [2]

End of Paper

YUSOF ISHAK SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2020

CANDIDATE NAME	
CLASS	
INDEX NUMBER	

MATHEMATICS 4048 / 02
4 Express / 5 Normal Academic
Paper 2

31 August 2020

Candidates answer on the Question Paper. **2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use
100

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Write as a single fraction in its simplest form

(i) $\frac{27p}{q^3} \div \left(\frac{-6p}{q}\right)^3$,

Answer ..... [2]

(ii) $\frac{1}{1-3y} - \frac{2}{y+2}$.

Answer ..... [2]

(b) Simplify $\frac{9-25x^2}{5x^2-12x-9}$.

Answer ..... [3]

(c) Solve the equation $\frac{5}{x+1} = 2x - 7$.

Answer $x =$ ..... or ..... [3]

---

2 **(a)** In 2019, Bob earned a total of \$45 600.
The percentage increase in his income from 2018 to 2019 is 3.5%.
Calculate his monthly income in 2018, to the nearest dollar.

Answer \$ ..... [2]

(b) A bank offers a savings account with a compound interest rate of 2.2% per annum.
Bob invests \$7000 in his account. Calculate the total amount of interest he earns after 5 years. Give your answer correct to the nearest cent.

Answer \$ ..... [2]

(c) The exchange rate between Singapore dollars and Macau Pataca (MOP$) is $1 = MOP$5.55.
The exchange rate between Hong Kong dollars (HK$) and Singapore dollars is HK$1 = $0.19.

Bob is planning a trip to Hong Kong and Macau.
He finds these hotel prices on a website.

Hong Kong Hotel	HK$825
Macau Hotel	MOP$825

(i) By comparing the exchange rates, explain which hotel costs lower per night.
Show your workings clearly.

Answer

The ..... hotel costs lower per night. [2]

(ii) Bob books 4 nights in the Hong Kong hotel and 2 nights in the Macau hotel.
He pays using his credit card. The credit card company converts the prices to Singapore dollars and charges a fee of k% for the currency conversion.

Given that the total amount Bob pays for the two hotels, including the credit card fee, is $940, find the value of k.

Answer ..... [4]

3 PQR is a triangle. The coordinates of P and Q are (0, 5) and (–2, 1) respectively. The equation of the line PR is $5y - 2x = 25$.

(a) Given the coordinates of R are $(k, k + 2)$, show that $k = 5$.

Answer

[1]

(b) Find the equation of PQ.

Answer ..... [1]

(c) Find the length of diagonal QR.

Answer ..... units [2]

(d) Calculate angle PQR.

Answer ..... [3]

(e) Calculate the area of triangle PQR.

Answer ..... units^2 [3]

4 The first four terms in a sequence of numbers are given below.

$$T_1 = 2^2 + 5 = 9$$
$$T_2 = 4^2 + 3 = 19$$
$$T_3 = 6^2 + 1 = 37$$
$$T_4 = 8^2 - 1 = 63$$

(a) Find T_5.

Answer ..... [1]

(b) Explain why the value of T_n must be odd for all values of n.

.....

..... [1]

(c) Show that the nth term of the sequence, T_n, is given by $4n^2 - 2n + 7$.

Answer

[3]

(d) T_k and T_{k+1} are consecutive terms in the sequence.
Find and simplify an expression, in terms of k, for $T_{k+1} - T_k$.

Answer ..... [3]

(e) Explain why two consecutive terms of the sequence cannot have a difference of 6.

.....

..... [1]

5 An enrichment centre offers music and dance lessons at basic (B) and advanced (A) levels. Each student has a 8-week block of lessons, with one lesson per week.

The matrix **L** shows the number of students attending the lessons each week.

$$\mathbf{L} = \begin{matrix} \\ \text{Music} \\ \text{Dance} \end{matrix} \begin{matrix} \text{B} & \text{A} \\ \begin{pmatrix} 68 & 76 \\ 43 & 38 \end{pmatrix} \end{matrix}$$

(a) Evaluate the matrix $\mathbf{P} = 8\mathbf{L}$.

Answer ..... [1]

(b) The fee for each basic lesson is \$45 and the fee for each advanced lesson is \$65. Represent these fees in a 2×1 column matrix **F**.

Answer ..... [1]

(c) Evaluate the matrix $\mathbf{Q} = \mathbf{PF}$.

Answer ..... [2]

(d) State what the elements of **Q** represent.

.....

..... [1]

(e) The enrichment centre increased the fees by 10%. As a result, the number of students attending basic level and advanced level lessons reduced by 12.5% and 6.25% respectively.

Using appropriate matrix multiplication, determine if the enrichment centre has made a profit or a loss in a 8-week block of lessons after the increase in fees.

Answer

The enrichment centre made a ..... in a 8-week block of lessons after the increase in fees. [3]

---

6 Yan and Zac run a small business that sells hand-painted ornaments.

(a) Yan takes x minutes to paint one ornament.
Write an expression, in terms of x, for the number of ornaments she paints in one hour.

Answer ..... [1]

(b) Zac takes 6 minutes less than Yan to paint one ornament.
Write an expression, in terms of x, for the number of ornaments he paints in one hour.

Answer ..... [1]

(c) One day, Yan and Zac each work for 6 hours.
Altogether, they paint a total of 27 ornaments.
Write down an equation, in terms of x, to represent this information, and show that it reduces to

$$3x^2 - 98x + 240 = 0.$$

Answer

[3]

(d) Solve the equation $3x^2 - 98x + 240 = 0$.

Answer $x =$ ..... or ..... [3]

(e) **(i)** Explain why one of the solutions in part **(d)** must be rejected.

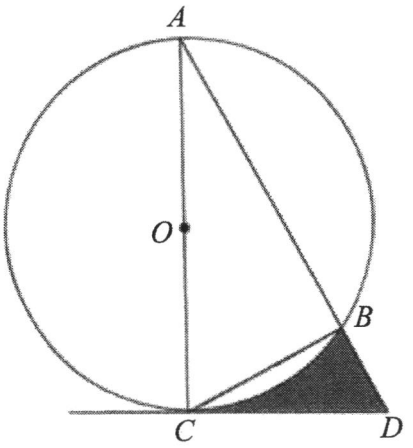
.....

..... [1]

(ii) Hence, find the number of ornaments Zac paints in one hour.

Answer ..... [2]

7



The diagram shows a circle that passes through the points A, B and C, with its centre at O. AC is the diameter of the circle. CD is a tangent that meets AB produced at D.

(a) Show that triangles ACD and CBD are similar.
Give a reason for each statement you make.

.....

.....

.....

..... [3]

(b) The ratio of the area of triangle CBD : area of triangle ACD is 1 : 4.
Show that angle CAD is 30°.

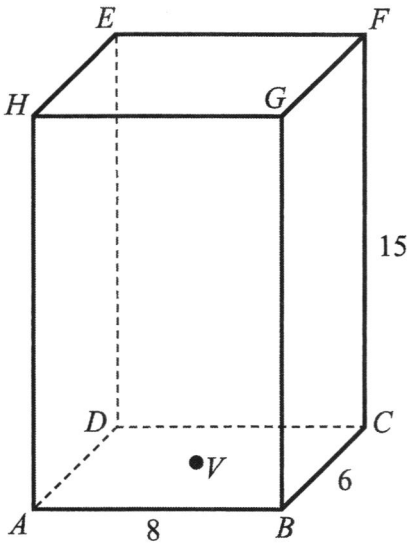
Answer

[2]

(c) Given that the radius of the circle is 5 cm, calculate the shaded area.

Answer ..... cm^2 [4]

8



In the diagram, *ABCDEFGH* is a cuboid with dimensions 8 cm by 6 cm by 15 cm. *V* is the centre of the rectangular base.

(a) Show that $EV = 15.8$ cm, correct to 3 significant figures.

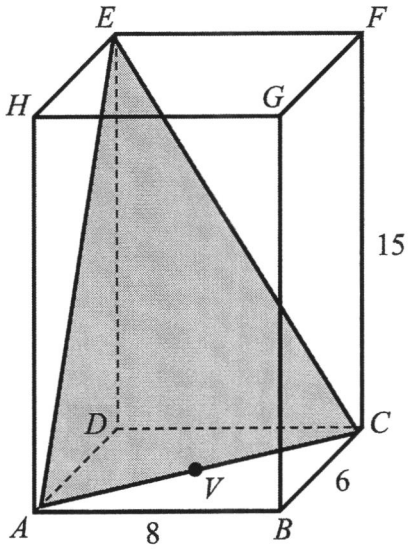
Answer

[2]

(b) Calculate angle *ACE*.

Answer ..... [4]

(c) A pyramid *EDAC* is cut out from the cuboid.



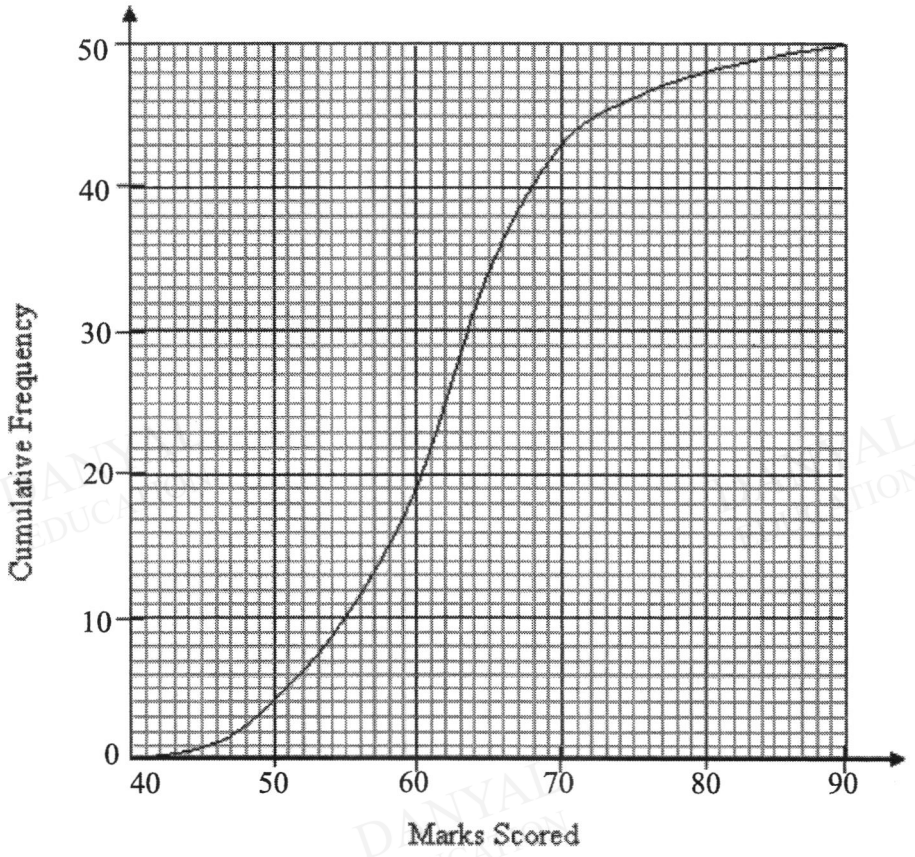
(i) Find the total surface area of the pyramid.

Answer ..... cm^2 [3]

(ii) Another pyramid is to be made with volume half of pyramid *EDAC*.
Given that the two pyramids are geometrically similar, find the vertical height of the smaller pyramid.

Answer ..... cm [2]

9 **(a)** 50 students from School *P* took part in a Mathematics competition.
The cumulative frequency curve below shows the distribution of the marks they scored.



(i) Complete the grouped frequency table of the marks scored by the students. [1]

Marks (x)	$40 \leq x < 50$	$50 \leq x < 60$	$60 \leq x < 70$	$70 \leq x < 80$	$80 \leq x < 90$
Frequency	4	15			

(ii) Calculate an estimate of the mean mark.

Answer ..... [1]

(iii) Calculate an estimate of the standard deviation.

Answer ..... [1]

(iv) The minimum mark for a student to be awarded a certificate of merit is 65.
Find the number of students who were awarded a certificate of merit.

Answer ..... [1]

(v) Another group of 50 students from school Q took part in the same competition and had the same interquartile range of marks as School P. However, half of the students from School Q scored at least 68 marks.

Describe how the cumulative frequency curve for School Q may differ from the curve for School P.

.....

..... [1]

(b) This table shows information about a group of students.

	Wears spectacles	Does not wear spectacles
Lower Secondary	3	9
Upper Secondary	7	5

(i) One of the students is selected at random.
Find, as a fraction in its lowest terms, the probability that the student
(a) is in upper secondary,

Answer ..... [1]

(b) does not wear spectacles.

Answer ..... [1]

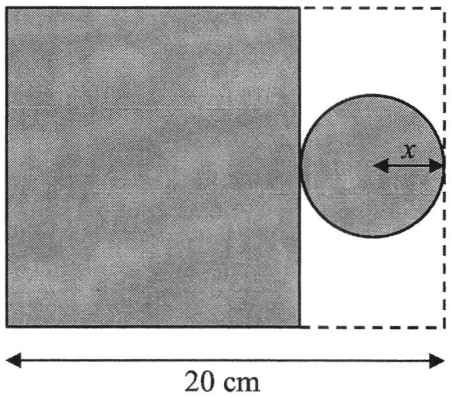
(ii) Two of the students are selected at random.
Find, as a fraction in its lowest terms, the probability that
(a) both students are in lower secondary,

Answer ..... [2]

(b) at least one of them wears spectacles.

Answer ..... [2]

10 Kate is designing open cylindrical gift boxes with base radius x cm.
She uses rectangular pieces of cardboard of length 20 cm and a variable width.



She cuts the net of the gift boxes out from each piece of cardboard.
By changing the base radius of the cylinders, she can change the volume of the gift boxes.

(a) Work out the volume of a gift box of base radius 2 cm.

Answer ..... cm^3 [2]

(b) Show, in terms of x, the volume of the gift boxes she makes is given by $2\pi x^2(10-x)$.

Answer

[1]

(c) Explain why $0 < x < 10$.

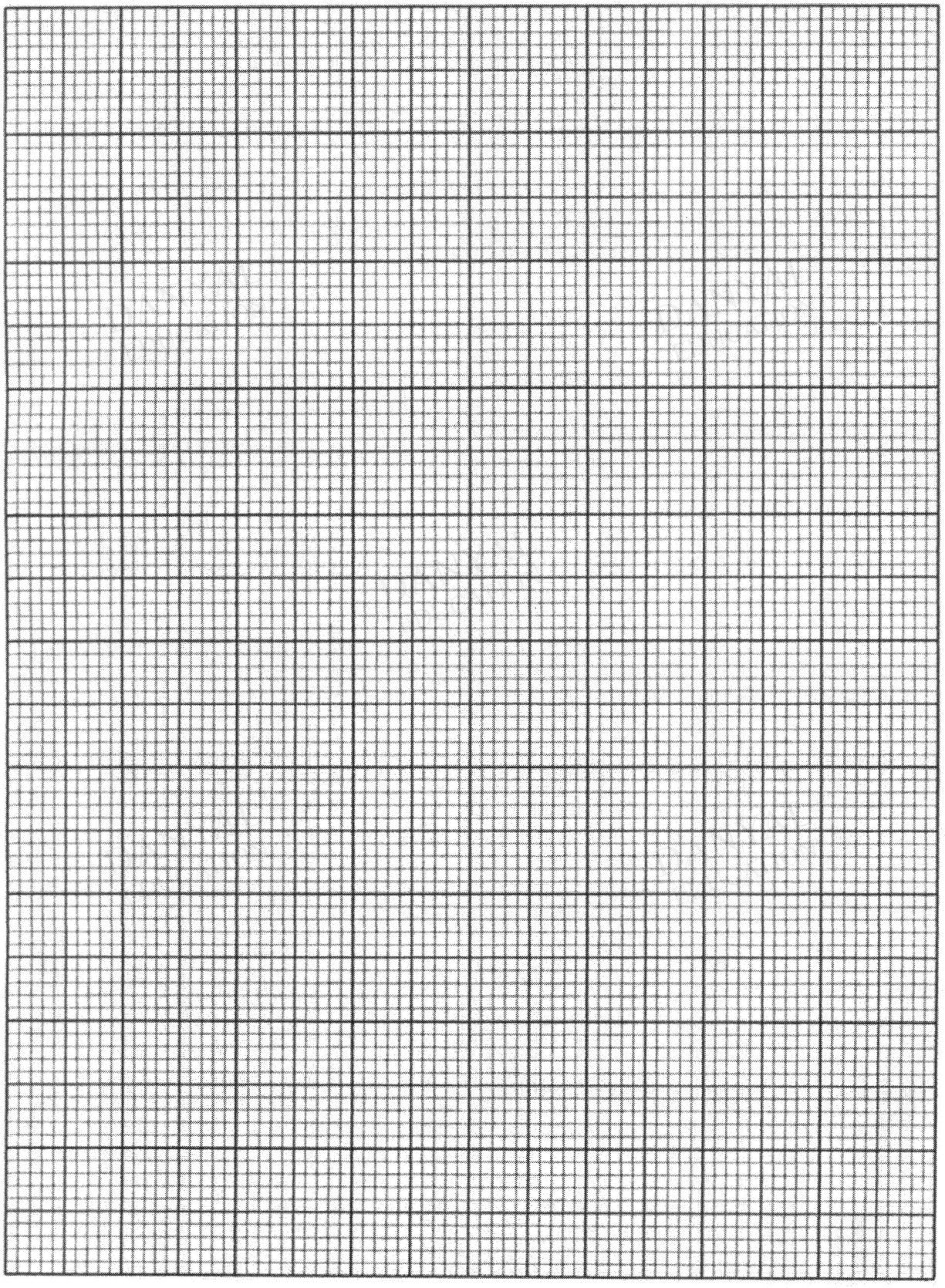
.....

..... [1]

(d) Kate paints the external curved surface area of the gift boxes with acrylic paint.
With one bottle of acrylic paint, Kate can paint up to 0.6 m^2 of cardboard.
At the same time, she wants the volume of the each gift box to be at least 600 cm^3.

By drawing a suitable graph, work out the maximum number of gift boxes she can paint with one bottle of acrylic paint.

Use the grid on page 17 and answer space on page 18. [7]



Answer for Question 10(d)

End of Paper

2020 4E5N Prelims Mathematics Paper 1 Marking Scheme

Setter: Miss Ang Yue Hua

Qns	Worked solution	Marking Scheme
1	$-\pi = -3.141592$ $-\frac{22}{7} = -3.142857$ $-3.\dot{3} = -3.33333...$ $-\pi$, -3.142, $-\frac{22}{7}$, $-3.\dot{3}$ *largest* *smallest*	B1
2	Arrange the 4 known numbers first. 22 24 26 33 Since the median mark is 26, the average of the 3rd and 4th number is 26. So one of the unknown numbers must be 26. Since the mean mark is 27, the total of the 6 numbers is $27 \times 6 = 162$. Hence, the last number is $162 - 22 - 24 - 26 - 26 - 33 = 31$ Since $a < b$, $a = 26$ and $b = 31$	B1 – for finding value of a. B1 – for finding value of b.
3	<u>Method 1: Using special product</u> $(2n+1)^2 - (2n-3)^2$ $= [(2n+1)+(2n-3)][(2n+1)-(2n-3)]$ $= [4n-2][4]$ $= 16n - 8$ $= 8(2n-1)$ <u>Method 2: Directly expanding</u> $(2n+1)^2 - (2n-3)^2$ $= (4n^2 + 4n + 1) - (4n^2 - 12n + 9)$ $= 4n^2 + 4n + 1 - 4n^2 + 12n - 9$ $= 16n - 8$ $= 8(2n-1)$ Hence, for all integer values of n, $(2n+1)^2 - (2n-3)^2$ is a multiple of 8	M1 A1 M1 A1

4	$\left(\frac{27x^9}{y^6}\right)^{-\frac{2}{3}}$ $=\left(\frac{y^6}{27x^9}\right)^{\frac{2}{3}}$ $=\frac{(y^6)^{\frac{2}{3}}}{27^{\frac{2}{3}}(x^9)^{\frac{2}{3}}}$ $=\frac{y^4}{9x^6}$	M1 A1
5(a)	Substitute $x=2$ into $5x^2-px+40=0$ $5(2)^2-p(2)+40=0$ $2p=60$ $p=30$	B1
5(b)	$5x^2-30x+40=0$ $x^2-6x+8=0$ $(x-2)(x-4)=0$ $x-2=0$ or $x-4=0$ $x=2$ or $x=4$ The second possible value of x is 4.	B1
6	Singapore is 4 hours ahead of Dubai. When the flight arrives in Dubai, the time in Singapore is 2225. 1510 → 2225 This means the flight time is 7hr 15min. Average speed $=\frac{5840km}{7.25h}$ $=806km/h$ (to 3 s.f.)	M1 – for finding flight time A1
7(a)	$A'\cap B$	B1
7(b)	$(A\cup B)'$ or $A'\cap B'$	B1
8(a)	The horizontal axis showing the dates have unequal intervals. For example, it is 1 month apart between the first bar and the second bar. But it is 7 days apart between the second bar and the third bar. OR The last bar states "Today" and it is unclear which day that is. OR any other reasonable answers.	B1

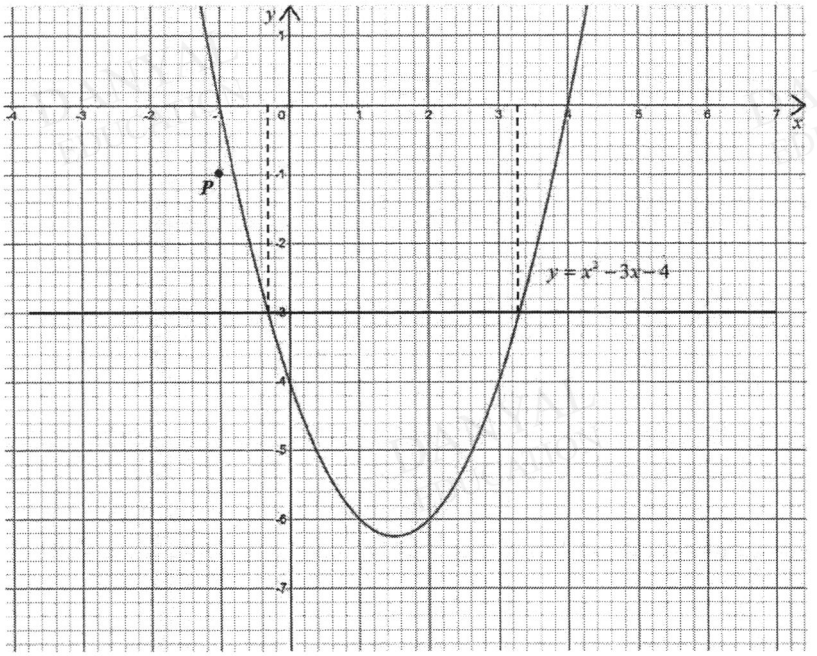
8(b)	No, I do not agree with Sam. The data presents the number of new coronavirus cases confirmed on a daily basis. It does not report the total number of coronavirus cases in Singapore. OR As we do not have the data for the whole of March, there is not enough information to predict the total number of cases in April. OR any other reasonable answers.	B1
9	Deposit $= 2106 - (12 \times 153) = \$270$ 15% → \$270 1% → \$18 100% → \$1800 The cash price of the washing machine is \$1800.	M1 M1 A1
10	By Sine rule, $\frac{\sin \angle ACB}{9} = \frac{\sin 55.3°}{7.4}$ $\sin \angle ACB = \frac{9 \sin 55.3°}{7.4}$ $\angle ACB = 89.2°$ Since $\angle ACB = 89.2° \neq 90°$, angle ACB is not an angle in a semicircle. AB is not the diameter of the circle.	M1 M1 A1
11(a)	$\angle DBC = 90° - 58° = 32°$ ($\angle s$ in a semicircle)	B1
11(b)	$\angle AED = 180° - 58° = 122°$ ($\angle s$ in opp. segment)	B1
11(c)	$\angle ADF = 30°$ ($\angle s$ in the same segment) $\angle ADE = 180° - 30° - 122° = 28°$ (int $\angle s$, $AE//DF$)	 B1
12(a)	5 men → 9 days 1 man → 45 days 3 men → 15 days	 B1
12(b)	$V = kR^2$, where k is a constant Let $V = V_1$ when $R = R_1$, then $k = \frac{V_1}{R_1^2}$ New $R = 250\%$ of R_1 $= 2.5 R_1$ $V_2 = \frac{V_1}{R_1^2}(2.5R_1)^2$ $V_2 = 6.25V_1$ Percentage increase $= \frac{V_2 - V_1}{V_1} \times 100\%$	 M1

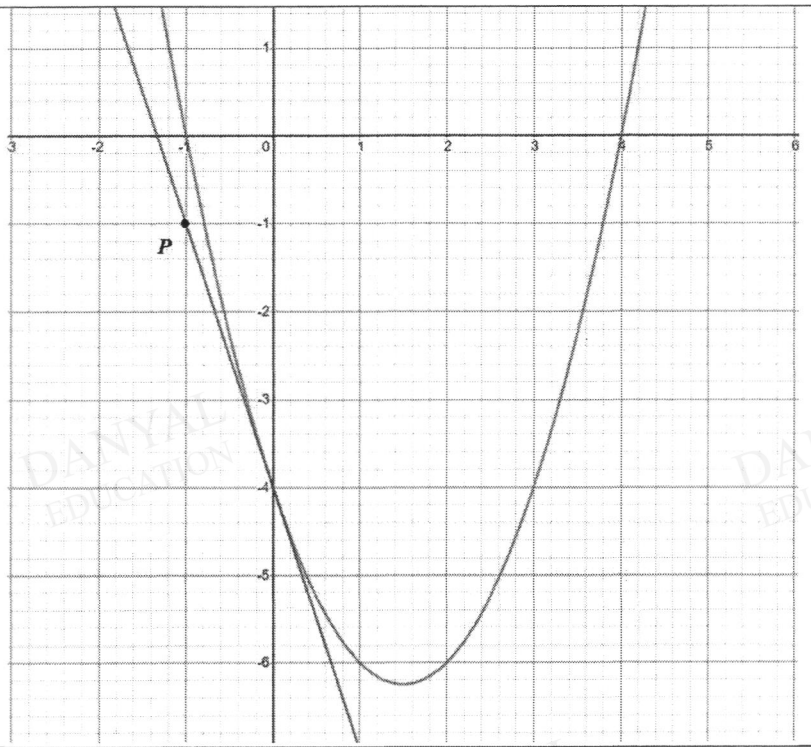
	$= \frac{6.25V_1 - V_1}{V_1} \times 100\%$ $= 525\%$	A1
13(a)	Median is the average value of 20th and 21st student. Median $= \frac{26+27}{2} = 26.5$	B1
13(b)	Any of the comments 1. The mean of Class *A* is 25.6 which is lower than the mean of class *B*, 29.275. On average, Class B's performance in the test is better than Class A's. 2. The median of Class *A* is 25.5 which is lower than the median of class *B*. On average, Class B's performance in the test is better than Class A's.	B1 – for comparing mean/median B1 – for making the correct conclusion on class' performance
14(a)	$1176 = 2^3 \times 3 \times 7^2$	B1
14(b)	$2^p \times 3 \times 7^q$ $2^2 \times 3^r \times 7^2$ $12 = 2^2 \times 3$ $1176 = 2^3 \times 3 \times 7^2$ By comparison, $p = 3$, $r = 1$ and $q = 0$	 B1 for each correct value.
15(a)	$27a^2b - 48b^3$ $= 3b(9a^2 - 16b^2)$ $= 3b(3a + 4b)(3a - 4b)$	 M1 A1
15(b)	$28dy - 4y - 7d + 1$ $= 4y(7d - 1) - (7d - 1)$ $= (7d - 1)(4y - 1)$	 M1 A1

16	$t=2\pi\sqrt{\frac{h+g}{h}}$ $\sqrt{\frac{h+g}{h}}=\frac{t}{2\pi}$ $\frac{h+g}{h}=\frac{t^2}{4\pi^2}$ $4\pi^2(h+g)=ht^2$ $4\pi^2h+4\pi^2g=ht^2$ $4\pi^2h-ht^2=-4\pi^2g$ $h(4\pi^2-t^2)=-4\pi^2g$ $h=\frac{-4\pi^2g}{4\pi^2-t^2}$	M1 – square both sides M1 – simplified fraction M1 A1
17	$c=-3$ So, $y=ax^2+bx-3$ Using $(-1, 4)$, we have $4=a(-1)^2+b(-1)-3$ $a=7+b$ ----------(1) Using $(2, -5)$, we have $-5=a(2)^2+b(2)-3$ $4a+2b=-2$ $2a+b=-1$ ----------(2) Substitute (1) into (2), $2(7+b)+b=-1$ $14+3b=-1$ $3b=-15$ $b=-5$ $a=7+(-5)=2$ $\therefore a=2, b=-5, c=-3$	B1 M1 – Substituting points to find both equations (1) and (2) M1 – any method to solve simultaneous equations A1
18	<u>Method 1: Using ext. $\angle$</u> One int. $\angle$ of pentagon $=\frac{180(5-2)}{5}=108°$ One int. $\angle$ of polygon $=360°-90°-108°=162°$ One ext. $\angle$ of polygon $=180°-162°=18°$ $\therefore n=\frac{360}{18}=20$ <u>Method 2: Using int. $\angle$</u>	M1 M1 M1 A1

	One int. $\angle$ of pentagon $= \dfrac{180(5-2)}{5} = 108°$ One int. $\angle$ of polygon $= 360° - 90° - 108° = 162°$ $162n = 180(n-2)$ $162n = 180n - 360$ $18n = 360$ $n = 20$	M1 M1 M1 A1
19(a)	$\dfrac{A_1}{A_2} = \dfrac{64}{121}$ $\dfrac{l_1}{l_2} = \sqrt{\dfrac{64}{121}} = \dfrac{8}{11}$ $\dfrac{V_1}{V_2} = \left(\dfrac{8}{11}\right)^3 = \dfrac{512}{1331}$ Therefore, the ratio of their volumes is 512 : 1331	M1 A1
19(b)	$\dfrac{912.6}{V_2} = \dfrac{512}{1331}$ $V_2 = 2372.4$ $V_2 = 2370\,\text{cm}^3$ (to 3 s.f.)	M1 A1
20(a)	$\sin 68° = \dfrac{h}{8}$ $h = 8\sin 68°$ $h = 7.417$ cm (SHOWN)	M1 A1
20(b)	$\sin \angle EAB = \dfrac{7.417}{9.05}$ $\angle EAB = 55.041°$ $\angle DBC = \dfrac{180° - 68°}{2} = 56°$ $\angle ABD = 180° - 56° = 124°$ $\angle EAB + \angle ABD = 55.041° + 124° = 179.041°$ Since $\angle EAB + \angle ABD \neq 180°$, the property of interior angles not fulfilled/satisfied. AE is not parallel to BD.	M1 A1
20(c)	Area of triangle $BCD = \dfrac{1}{2}(8)(8)(\sin 68°)$ $= 29.66988\,\text{cm}^2$ Area of $ABDE = 111 - 29.66988 = 81.3\text{cm}^2$ (to 3 s.f.)	M1 A1
21(a)(i)	$AB = CD$ (opp sides of rectangle) $AP = CR$ (given) $PB = AB - AP$	B1 – statements shown

	$= CD - CR$ $= RD$	
21(a) (ii)	$PB = RD$ (from (ai)) $BQ = DS$ (opp sides of rectangle and $AS = CQ$ (given)) $\angle PBQ = \angle RDS = 90°$ (angles in rectangle) $\therefore \Delta PBQ = \Delta RDS$ *(SAS)*	B1 – for correct statements & SAS stated B1 – for correct reasons
21(b)	$\angle DRS = \angle BPQ = 20°$ $\angle PRD = 180° - 105° = 75°$ (int $\angle$s) $\angle PRS = 75° - 20° = 55°$	B1
22(a)	$\cos B = \dfrac{220^2 + 70^2 - 175^2}{2(220)(170)}$ $B = 42.591°$ Bearing of C from B $= 360° - 42.591° - 90°$ $= 227.409°$ $= 227.4°$ (to 1 d.p.)	M1 M1 A1
22(b)	$\sin 42.591° = \dfrac{d}{70}$ $d = 70 \sin 42.591°$ $= 47.4$ m (to 3.s.f.)	M1 A1
23(a)	$\angle BOC = \dfrac{2\pi}{9} + \dfrac{2\pi}{9} = \dfrac{4\pi}{9} rad$ (sum of ext. $\angle$ of triangle) Arc length $BC = 12\left(\dfrac{4\pi}{9}\right)$ $= \dfrac{16\pi}{3}$ $= 16.755$ $= 16.8$ cm (to 3.s.f.) [Shown]	M1 M1 A1
23(b)	Arc length $AB = 12\pi$ $\cos \dfrac{2\pi}{9} = \dfrac{\frac{1}{2}AC}{12}$ (angle from centre bisects chord) $AC = 24 \cos \dfrac{2\pi}{9}$ $AC = 18.385$ Perimeter of major segment ABC = Arc length BC + Arc length AB + AC $= \dfrac{16\pi}{3} + 12\pi + 18.385$ $= 72.839$ cm $= 72.8$ cm (to 3 s.f.)	 M1 M1 A1 (Accept 72.9cm for students who

		used 16.8 instead of $\frac{16\pi}{3}$)
24(a)	Equation of the line of symmetry is $x = 1.5$	B1
24(b)	$x^2 - 3x - 1 = 0$ $x^2 - 3x - 1 - 3 = -3$ $x^2 - 3x - 4 = -3$ Draw $y = -3$  When $y = -3$, $x = -0.3$ or $x = 3.3$.	M1 A1

24(c)(i)		B1 Tangent must pass through one point on the curve.
24(c)(ii)	Gradient $= \frac{-1-(-4)}{-1-0} = -3$ Equation of tangent is $y = -3x - 4$	M1 – for finding gradient of tangent A1

Yusof Ishak Secondary School
2020 4E5N Prelim Math Paper 2 – Marking Scheme

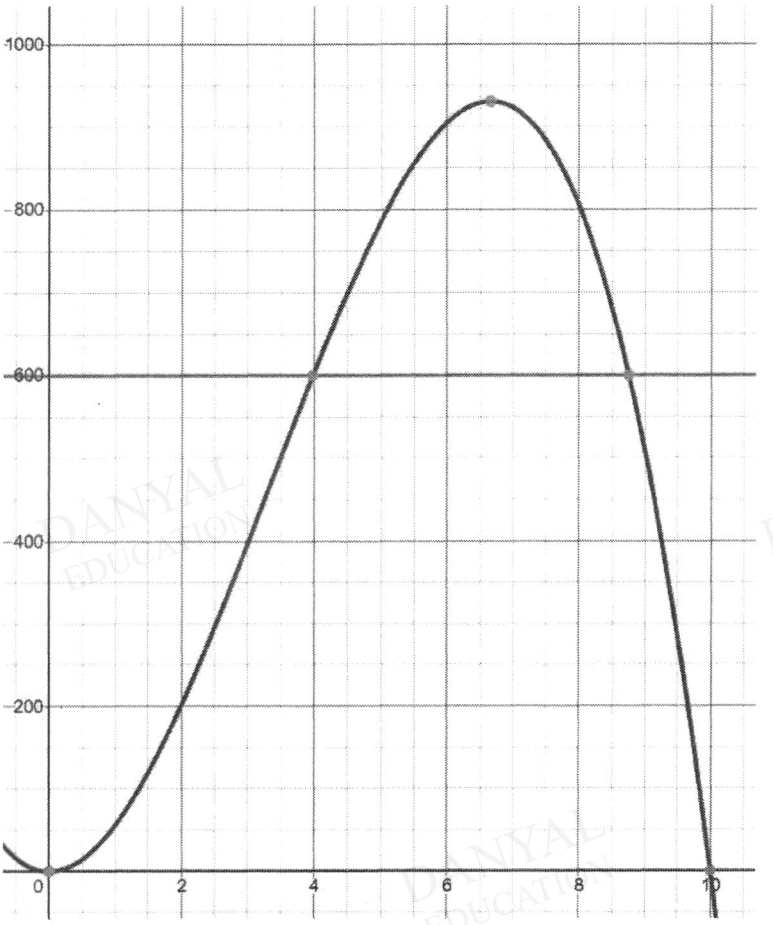
1(a)(i)	$\frac{27p}{q^3} \div \left(\frac{-6p}{q}\right)^3$ $= \frac{27p}{q^3} \times \left(\frac{q^3}{-216p^3}\right)$ $= -\frac{1}{8p^2}$	**M1** **A1**
1(a)(ii)	$\frac{1}{1-3y} - \frac{2}{y+2}$ $= \frac{y+2-2(1-3y)}{(1-3y)(y+2)}$ $= \frac{7y}{(1-3y)(y+2)}$	**M1** **A1**
1(b)	$\frac{9-25x^2}{5x^2-12x-9}$ $= \frac{(3+5x)(3-5x)}{(x-3)(5x+3)}$ $= \frac{3-5x}{x-3}$	**M2** **A1**
1(c)	$\frac{5}{x+1} = 2x-7$ $5 = (2x-7)(x+1)$ $2x^2-5x-12=0$ $(x-4)(2x+3)=0$ $x=4$ or $x=-1.5$	**M1** **M1** **A1**
2(a)	Monthly income in 2018 $= \left(\frac{45600}{103.5} \times 100\right) \div 12$ $= 3671.497585$ $=$ \$3671 (nearest dollar)	**M1** – total income in 2018 **A1**
2(b)	Amount of interest $= 7000\left(1+\frac{2.2}{100}\right)^5 - 7000$ $=$ \$804.63	**M1** – total amount in 5 yrs **A1**
2(c)(i)	HK\$1 = \$0.19 $\$1 = \text{HK}\$\frac{1}{0.19}$ $= \text{HK}\$5.2632$ Since \$1 can only get HK\$5.26, by comparison with \$1 = MOP\$5.55, HK\$ is stronger than MOP\$. Thus, **Macau** hotel costs lower per night.	**M1** (or compare MOP\$1 = \$0.18) **A1**

2(c)(ii)	Total cost without card fee $=(4\times825\times0.19)+\left(2\times825\times\frac{1}{5.55}\right)$ $=\$924.297$ $k=\frac{940-924.297}{924.297}\times100$ $=1.70$ (3 s.f.)	**M2** – Total cost in SGD **M1** **A1**
3(a)	Sub $(k, k+2)$, $5(k+2)-2k=25$ $3k=15$ $k=5$	**B1**
3(b)	Gradient of $PQ=\frac{5-1}{0-(-2)}$ $=2$ Since line passes through (0, 5), y-intercept is 5. Equation of PQ: $y=2x+5$	**B1**
3(c)	$QR=\sqrt{(5+2)^2+(7-1)^2}$ $=\sqrt{85}$ $=9.22$ units (3 s.f.)	**M1** **A1**
3(d)	$\angle PQR=\tan^{-1}\left(\frac{4}{2}\right)-\tan^{-1}\left(\frac{6}{7}\right)$ $=22.834°$ $=22.8°$ (1 d.p.)	**M2** (Alternative: find PQ and PR, then apply Cosine Rule) **A1**
3(e)	$PQ=\sqrt{4^2+2^2}$ $=\sqrt{20}$ Area of $\Delta PQR=\frac{1}{2}(\sqrt{20})(\sqrt{85})\sin22.834$ $=8$ units^2 Alternative: Use "Shoe-lace" method from Add Math	**M1** **M1** **A1**
4(a)	$T_5=10^2-3=97$	**B1** - Award once 97 is seen.
4(b)	The square number of an even number is always even. Hence, adding/subtracting an odd number with/from an even number will always result in an odd number.	**B1** - Award when the idea of difference between odd and even number is seen.
4(c)	$T_n=(2n)^2+[5-2(n-1)]$ $=4n^2+5-2n+2$ $=4n^2-2n+7$ (shown)	**M2** **A1**
4(d)	$T_{k+1}-T_k=[4(k+1)^2-2(k+1)+7]-(4k^2-2k+7)$ $=4(k^2+2k+1)-2k-2+7-4k^2+2k-7$ $=8k+2$	**M1** **M1** **A1**
4(e)	Since $k\geq1$, then $8k+2\geq10$ Hence the difference cannot be 6.	**B1**

5(a)	$P = 8\begin{pmatrix} 68 & 76 \\ 43 & 38 \end{pmatrix}$ $= \begin{pmatrix} 544 & 608 \\ 344 & 304 \end{pmatrix}$	**B1**
5(b)	$F = \begin{pmatrix} 45 \\ 65 \end{pmatrix}$	**B1**
5(c)	$Q = \begin{pmatrix} 544 & 608 \\ 344 & 304 \end{pmatrix}\begin{pmatrix} 45 \\ 65 \end{pmatrix}$ $= \begin{pmatrix} 544(45)+608(65) \\ 344(45)+304(65) \end{pmatrix}$ $= \begin{pmatrix} 64000 \\ 35240 \end{pmatrix}$	**M1** **A1 (or B2)**
5(d)	The elements of Q represent the total amount the centre earns for music and dances lessons respectively over 8 weeks.	**B1**
5(e)	$\begin{pmatrix} 0.875\times544 & 0.9375\times608 \\ 0.875\times344 & 0.9375\times304 \end{pmatrix}\begin{pmatrix} 1.1\times45 \\ 1.1\times65 \end{pmatrix}$ $= \begin{pmatrix} 476(49.5)+570(71.5) \\ 301(49.5)+285(71.5) \end{pmatrix}$ $= \begin{pmatrix} 64317 \\ 35277 \end{pmatrix}$ By comparing the elements in this matrix with that in matrix Q, the enrichment centre made a **profit**.	**M1** **M1** **A1**
6(a)	$\frac{60}{x}$	**B1**
6(b)	$\frac{60}{x-6}$	**B1**
6(c)	$6\left(\frac{60}{x}+\frac{60}{x-6}\right)=27$ $6[60(x-6)+60x]=27x(x-6)$ $720x-2160=27x^2-162x$ $27x^2-882x+2160=0$ $3x^2-98x+240=0$ (shown)	**M1** – forming correct relationship **M1** – multiplying through with denominator **A1**
6(d)	$x=\frac{-(-98)\pm\sqrt{(-98)^2-4(3)(240)}}{2(3)}$ $=\frac{98\pm\sqrt{6724}}{6}$ $=30 \text{ or } 2\frac{2}{3}$	**M1** (or factorisation method) **A2** ($2\frac{2}{3}$ must be exact)
6(e)(i)	Since Zac takes $(x-6)$ minutes to paint one ornament, $x > 6$. Thus, $x \neq 2\frac{2}{3}$.	**B1**
6(e)(ii)	Number of ornaments $=\frac{60}{30-6}=2.5$ (exact)	**M1A1**

7(a)	$\angle ADC = \angle CDB$ (common angle) $\angle ACD = 90°$ (Radius $\perp$ to tangent) Since $\angle ABC = 90°$ ($\angle$ in a semicircle), thus $\angle CBD = 90°$ ($\angle$s on a str. line) $\therefore \angle ACD = \angle CBD = 90°$ $\therefore$ ΔACD and ΔCBD are similar. (AA property)	**M1** **M1** **A1**
7(b)	$\dfrac{\text{Area of } \Delta CBD}{\text{Area of } \Delta ACD} = \dfrac{1}{4}$ $\dfrac{CB}{AC} = \sqrt{\dfrac{1}{4}}$ $= \dfrac{1}{2}$ $\sin \angle CAB = \dfrac{1}{2}$ $\angle CAD = \angle CAB$ $= \sin^{-1} \dfrac{1}{2}$ $= 30°$	 **B1** **B1**
7(c)	$\angle COB = 2\angle CAB$ ($\angle$ at centre = 2$\angle$s at circumference) $= 60°$ $\tan \angle CAD = \dfrac{CD}{AC}$ $CD = 10 \tan 30°$ Shaded area $= \dfrac{1}{2}(10)\left(10 \tan 30°\right) - \dfrac{60°}{360°} \times \pi(5)^2 - \dfrac{1}{2}(5)^2 \sin(180° - 60°)$ $= 4.95 \text{ cm}^2$ (3 s.f.)	 **M1** **M2** **A1**
8(a)	$DV = \dfrac{1}{2}\sqrt{6^2 + 8^2}$ $= 5 \text{ cm}$ $EV = \sqrt{15^2 + 5^2}$ $= \sqrt{250}$ $= 15.811$ $= 15.8 \text{ cm}$ (3 s.f.)	 **M1** **A1**
8(b)	$EC = \sqrt{15^2 + 8^2}$ $= 17$ $\cos \angle ACE = \cos \angle VCE$ $= \dfrac{17^2 + 5^2 - 250}{2(17)(5)}$ $\angle ACE = \cos^{-1} \dfrac{32}{85}$ $= 67.885$ $= 67.9$ (1 d.p.)	**M1** **M1** **M1** **A1** (Accept 67.8 if EV = 15.8 is used)

8(c)(i)	Total surface area of pyramid $=\frac{1}{2}(15)(6)+\frac{1}{2}(15)(8)+\frac{1}{2}(8)(6)+\frac{1}{2}(17)(10)\sin 67.885^\circ$ $=208\ \text{cm}^2$ (3 s.f.)	**M2** (or M1 for at least 2 triangles found) **A1**
8(c)(ii)	$\frac{V_{small}}{V_{large}}=\frac{1}{2}$ $\frac{h_{small}}{h_{large}}=\sqrt[3]{\frac{1}{2}}$ $h_{small}=\sqrt[3]{\frac{1}{2}}\times 15$ $=11.9$ cm (3 s.f.)	**M1** **A1**
9(a)(i)	<table><tr><td>Marks (x)</td><td>$60\le x<70$</td><td>$70\le x<80$</td><td>$80\le x<90$</td></tr><tr><td>Frequency</td><td>24</td><td>5</td><td>2</td></tr></table>	**B1**
9(a)(ii)	Mean mark = 62.2	**B1**
9(a)(iii)	Standard deviation = 8.95 (3 s.f.)	**B1**
9(a)(iv)	Number of students = 50 – 34 = 16	**B1**
9(a)(v)	The curve for School Q will be on **right** of the curve for School P, with an identical slope.	**B1**
9(b)(ia)	Total number of students = 24 P(upper sec) $=\frac{7+5}{24}=\frac{1}{2}$	**B1**
9(b)(ib)	P(does not wear spectacles) $=\frac{9+5}{24}=\frac{7}{12}$	**B1**
9(b)(iia)	P(both are lower sec) $=\frac{12}{24}\times\frac{11}{23}$ $=\frac{11}{46}$	**M1** **A1**
9(b)(iib)	P(at least one wears spectacles) = 1 – P(both do not wear spectacles) $=1-\left(\frac{14}{24}\times\frac{13}{23}\right)$ $=\frac{185}{276}$	**M1** **A1**
10(a)	Volume of gift box $=\pi(2)^2(20-4)$ $=201\ \text{cm}^3$ (3 s.f.)	**M1** – for height of cylinder = 16 **A1**
10(b)	Volume of gift box $=\pi(x)^2(20-2x)$ $=2\pi x^2(10-x)$	**B1**
10(c)	Because volume of gift box > 0, thus $x>0$ and $10-x>0$. Thus, $0<x<10$.	**B1**

10(d)	Plot $V = 2\pi x^2(10-x)$, for $0 < x < 10$.	**P2C1**
		
	When $V = 600$ cm^3, $x = 4$ or 8.7.	**M1** (2 solutions for x)
	To obtain the maximum number of gift boxes, area to be painted should be as small as possible.	
	Curved surface area $= 2\pi(20x - 2x^2)$ Sub $x = 8.7$, curved surface area = 142.13 cm^2	**M1**
	Number of gift boxes $= \dfrac{0.6 \times 100 \times 100}{142.13}$ $= 42.215$	**M1**
	Hence, the maximum number she can paint is **42**.	**A1**