# Queenstown Secondary School 



## Preliminary Examination 2020 Secondary Four Express / Five Normal (Academic) Mathematics Paper 1 4048/01

25 August 2020
Tuesday
Setter: Mr Norman Lim

Time: 0800 - 1000
Duration: 2 hours

Candidates answer on the Question Paper.

## READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place. For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.

The number of marks is given in brackets [ ] at the end of each question or part question.


## Mathematical Formulae

Compound interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

## Mensuration

Curved surface area of a cone $=\pi r l$
Surface area of a sphere $=4 \pi r^{2}$

> Volume of a cone $=\frac{1}{3} \pi r^{2} h$
> Volume of a sphere $=\frac{4}{3} \pi r^{3}$

Area of a triangle $A B C=\frac{1}{2} a b \sin C$

Arc length $=r \theta$, where $\theta$ is in radians
Sector area $=\frac{1}{2} r^{2} \theta$, where $\theta$ is in radians

Trigonometry

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{gathered}
$$

## Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

Answer all the questions.
1 Write the following numbers in order of size, starting with the smallest.
0.707
0.707
$\frac{7}{10}$
0.707

Answer
[1]

2 (a) The chart shows the annual sales of bicycles of four companies in 2019.


Explain how the chart above may be misleading.
$\qquad$
$\qquad$
(b) The number of bicycles sold by BB company in 2019 was 5000 , correct to the nearest thousand.

Write down the maximum number of bicycles BB company could have sold in 2019.

3 Simplify $\left(x^{9}\right)^{\frac{2}{3}} \times\left(y^{3}\right)^{0}$.

## Answer

4 (a) Express 504 as a product of its prime factors.

## Answer

(b) $m$ and $n$ are both prime numbers.

Find the values of $m$ and $n$ so that $504 \times \frac{m}{n}$ is a perfect square.

$$
\begin{aligned}
& \text { Answer } \quad m= \\
& n=.
\end{aligned}
$$

5 Karen invested $\$ 5000$ in a savings fund for 5 years.
The fund increased by $3 \%$ every year.
Find the total amount of interest Karen will earn at the end of 5 years.

> Answer \$.

6 Solve the equation $\frac{5}{3 x-1}=\frac{7}{2 x-3}$.

$$
\text { Answer } \quad x=
$$

7 A box contains 5 green pens and 7 blue pens.
Two pens are taken from the box at random with replacement.
Find, as a fraction in its simplest form, the probability that the two pens are the same colour.

8 Solve the inequalities $7 \leq \frac{2 x+5}{3}<15$.

## Answer

$9 \quad F$ is inversely proportional to $d^{2}$.
When $F=4, d=3$.
Find $d$ when $F=9$.

Answer $\quad d=$

10 (a) Express $x^{2}-6 x-8$ in the form $(x-p)^{2}-q$.

> Answer
(b) Hence, solve the equation $x^{2}-6 x-8=0$, giving your answers correct to two decimal places.

$$
11 \begin{aligned}
\xi & =\text { \{integers } x: 10 \leq x<20\} \\
P & =\text { prime numbers }\} \\
Q & =\text { \{multiples of } 3\}
\end{aligned}
$$

(a) List the elements in
(i) $P$,

> Answer
(ii) $P \cap Q$.

Answer
[1]
(b) Write down the set represented by the shaded region.


12 (a) Sketch the graph of $y=-(x-6)(x+4)$ on the axes below. Indicate clearly the coordinates of the points where the graph crosses the $x$ - and $y$-axes.

Answer

(b) Write down the equation of the line of symmetry of the graph.


The diagram shows the positions of three pavilions, $P, Q$ and $R$ in a park.
Pavilion $R$ is 600 m from $Q$.
Angle $P R Q=94^{\circ}$ and angle $R P Q=39^{\circ}$.
The bearing of $R$ from $Q$ is $300^{\circ}$.
Find
(a) the bearing of $Q$ from $R$,
(b) the distance between $P$ and $R$.
$14 P Q R S$ represents the area of a school garden.
Answer


On the diagram,
(a) construct the bisector of angle $P Q R$,
(b) construct the perpendicular bisector of $P Q$,
(c) A shelter is to be built in the garden.

The shelter is to be located midway between points $P$ and $Q$ and is nearer to lines $R Q$ than $P Q$.

Mark clearly a possible location for the shelter. Label the location as $\boldsymbol{T}$.


In the diagram, $C A D$ and $C B E$ are straight lines.
$C A$ is the diameter of the circle $A B C$.
Explain why $A E$ is the diameter of the circle $A D E B$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$


In the diagram, $A, B$ and $C$ are points on a circle, centre $O$. Angle $A C O=x^{\circ}$ and angle $A B C=3 x^{\circ}$. Find $x$.

17 Factorise completely
(a) $9 x^{3}-4 x y^{2}$,

## Answer

(b) $4 c p+10 d q-8 d p-5 c q$.

## Answer

18 The mean height of the group of students is 169.5 cm .
The mean height of the girls is 162 cm .
The ratio mean height of the girls : mean height of the boys is $27: 29$.
There are 2 more boys than girls.
Find the number of girls.


The diagram shows the speed-time graph of two cars, A and B.
Car A, starting from rest, accelerates uniformly for 2 seconds until it reaches a speed of $11 \mathrm{~m} / \mathrm{s}$. It then continues to travel at constant speed.

Car B starts from the same point as Car A, but 2 seconds later.
(a) Find
(i) the acceleration of Car A when $t=1$,

> Answer $\mathrm{m} / \mathrm{s}^{2}$
(ii) the distance travelled by Car A for the first 2 seconds.

$$
\text { Answer ............................... } \mathrm{m}
$$

(b) Car B accelerates uniformly from rest.

It overtakes Car A when $t=9$ seconds.
Find $v$, the speed of Car B when it overtakes Car A.

## 15

20 An aeroplane leaves San Francisco at 0210 local time on Saturday.
The flight distance between Singapore and San Francisco is 13563 km .
The average speed of the plane is $775 \mathrm{~km} / \mathrm{h}$.
Singapore is ahead of San Francisco by 15 hours.
Find the local time and the day on which the aeroplane arrives in Singapore.
Show your working.


44-litre backpack


18-litre backpack

The diagram shows two geometrically similar backpacks.
The larger backpack has a capacity of 44 litres and the smaller backpack has a capacity of 18 litres.
(a) The height of the 18 -litre backpack is 40 cm .

Calculate the height of the 44-litre backpack, giving your answer to the nearest centimetre.
(b) The ratio surface area of larger backpack : surface area of smaller backpack can be written in the form $k: 1$.

Find the value of $k$, giving your answer to 2 decimal places.

22 A sequence is given by:

$$
\begin{gathered}
y_{1}=1 \\
y_{n+1}=y_{n}\left(k+y_{n}\right)
\end{gathered}
$$

where $k$ is a constant and $k \neq 0$.
(a) Find $y_{2}$ in terms of $k$.

## Answer

(b) Show that $y_{3}=2 k^{2}+3 k+1$.

Answer
(c) Given that $y_{3}=1$, find the value of $k$.

23 (a) The stem-and-leaf diagram shows the number of working hours of 15 employees in a week for company A .

Company A

|  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | Company A |  |  |  |  |  |  |  |
| 4 | 5 | 6 | 8 |  |  |  |  |  |
| 5 | 1 | 3 | 4 | 5 | 5 | 5 | 9 |  |

Key: $\quad 3 \mid 1 \quad$ means 31 hours
(i) Find the median working hours for company A.

Answer ........................ hours
(ii) Find the range of working hours for company A.

Answer ........................ hours
(iii) Calculate the percentage of workers who worked less than 40 hours.

> Answer ............................. \%
(b) The number of working hours in a week for company B is represented by the box and whisker plot below.


Make two comments on the working hours in a week for companies A and B .
1.
2.
$\qquad$
$\qquad$
$\qquad$

24 A satay shop offers two types of combo sets.
Combo Set X includes 20 sticks of chicken (C), 10 sticks of mutton (M) and 10 sticks of beef (B).

Combo Set Y includes 25 sticks of chicken, 15 sticks of mutton and no beef.
(a) Represent this information in a $2 \times 3$ matrix, $\mathbf{P}$.

$$
\text { Answer } \quad \mathbf{P}=\left(\begin{array}{ccc}
\mathrm{C} & \mathrm{M} & \mathrm{~B}  \tag{1}\\
& &
\end{array}\right) \mathrm{Y}
$$

(b) The cost of a stick of satay is 90 cents for chicken, $\$ 1.20$ for mutton and $\$ x$ for beef.

Represent this information in dollars, in a $3 \times 1$ matrix, $\mathbf{T}$.

$$
\begin{equation*}
\text { Answer } \quad \mathbf{T}=\quad( \tag{1}
\end{equation*}
$$

(c) Find, in terms of $x$, the matrix $\mathbf{Q}=\mathbf{P T}$.

$$
\text { Answer } \quad \mathbf{Q}=
$$

(d) Explain what each element in matrix $\mathbf{Q}$ represents.
$\qquad$
$\qquad$
(e) The total cost is the same for both Set X and Y . Calculate $x$.

$$
\text { Answer } \quad x=
$$

25


Rectangle $A B C D$ represents the floor plan of Zoe's dance studio. It is drawn to a scale of 1 cm represents $n$ meters.

The actual area of the floor is $128 \mathrm{~m}^{2}$.
(a) Using the plan, find the value of $n$.

$$
\text { Answer } n=
$$

(b) Zoe decides to varnish the floor in her dance studio. The varnish she needs is sold in tins of two sizes.

| 500 ml |
| :--- |
| $\$ 13.00$ |$\quad$| 1 litre |
| :--- |
| $\$ 25.00$ |

1 litre of varnish covers $12 \mathrm{~m}^{2}$.
Calculate the least amount of money that Zoe needs to pay for the varnish.
$\qquad$

# Queenstown Secondary School 



## Preliminary Examination 2020 Secondary Four Express / Five Normal (Academic) Mathematics Paper 2 <br> 4048/02

## 27 August 2020

Thursday
Setter: Mr Tay Yee Kai

Time: 0800-1030
Duration: 2 hours 30 minutes

Candidates answer on the Question Paper.

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\end{aligned}
$$

1 (a) The compression constant of combustion engine is given by $C=1+\frac{\pi b^{2} s}{4 V}$.
(i) Evaluate $C$ when $b=2, s=2.5$ and $V=3$.

$$
\text { Answer } C=
$$

(ii) Express $b$ in terms of $C, V$ and $s$.

$$
\begin{equation*}
\text { Answer } b= \tag{2}
\end{equation*}
$$

(b) Express as a single fraction in its simplest form
(i) $\frac{8 a^{2} b}{3} \div \frac{2 a}{b}$,

> Answer
[1]
(ii) $\frac{5}{3 x-1}+\frac{4}{x-2}$.
(c) Given that $25^{3 x-11}=125^{x}$, find the value of $x$.
$\qquad$
(d) Solve these simultaneous equations.

$$
\begin{aligned}
& 4 x+3 y=3 \\
& 6 x-2 y=11
\end{aligned}
$$

$$
\begin{aligned}
\text { Answer } & x=. \\
y & =.
\end{aligned}
$$



The diagram shows a regular hexagon, $A B C D E F$, and five of the sides, $P Q, Q D$, $D E, E R$ and $R S$, of a second regular polygon.
Angle $Q D C=90^{\circ}$ and $A B=2 \mathrm{~cm}$.
(a) Find angle $E D C$.

## Answer

 .(b) Find the number of sides in the second regular polygon.
(c) Calculate the area of regular hexagon $A B C D E F$.
$\qquad$ $\mathrm{cm}^{2}$


The diagram shows a cuboid $A B C D E F G H$ with a volume of $4800 \mathrm{~cm}^{3}$. $A B=30 \mathrm{~cm}$ and $C H=8 \mathrm{~cm}$.
(a) Show that $A C=36.1 \mathrm{~cm}$, correct to 1 decimal place.

Answer
(b) Find angle of elevation of $H$ from $A$.
(c) Find angle $A E C$.

4 (a) The table below summarises the times taken by 500 participants in a 5 km run in 2019.

| Time <br> $(t$ minutes $)$ | Frequency |
| :---: | :---: |
| $30<t \leq 40$ | 35 |
| $40<t \leq 50$ | 103 |
| $50<t \leq 60$ | 251 |
| $60<t \leq 70$ | 89 |
| $70<t \leq 80$ | 22 |

(i) Calculate the estimate of the mean time.

> Answer
(ii) Calculate the estimate of the standard deviation.

Answer
(iii) Explain why the mean and standard deviation are estimates.
$\qquad$
$\qquad$
The table below shows the mean time and standard deviation of 500 participants in a 5 km run in 2020.

| Mean time | 51.6 |
| :---: | :--- |
| Standard deviation | 9.51 |

(iv) The organiser for the 5 km run states that a better running record has been attained in 2020.

Comment on whether the data above supports this claim.
$\qquad$
$\qquad$
(b)


The cumulative frequency graph represents the heights of all Primary 4 pupils in Queens Primary School.
(i) How many Primary 4 pupils are there in the school?

Answer
(ii) Find the median height.

Answer
cm
(iii) Find the interquartile range of the heights.

Answer
cm
(iv) Only $4 \%$ of the Primary 4 pupils could get into the school basketball team based on their height.
What is the minimum height requirement to join school basketball team?

5 (a) A football team takes part in two matches to qualify for the next round. The probability that the team wins is 0.6 .
The probability that the team draws is 0.3 .
(i) Complete the tree diagram below.

(ii) To qualify for the next round, the team needs at least a win and a draw. Find the probability that the team will qualify for the next round.
(b) A bag contains 2 red buttons and $x$ yellow buttons.

Two buttons are chosen at random, one after another, without replacement.
The probability that both buttons chosen are red is $\frac{1}{10}$.
Find $x$.

6 Derrick and Elle went on a cycling expedition using different routes. Derrick travelled by route A, which is 115 km long, at an average speed of $x \mathrm{~km} / \mathrm{h}$. Meanwhile Elle travelled by route $B$, which is 5 km shorter, but has more challenging terrain. She covered the distance at an average speed that was slower than Derrick by $3 \mathrm{~km} / \mathrm{h}$.
(a) Write down an expression, in terms of $x$, for the time taken by Derrick.

> Answer ..................... hours
(b) Write down an expression, in terms of $x$, for the time taken by Elle.

> Answer ...................... hours
(c) Given that Derrick waited 40 minutes before Elle arrived at the same destination, form an equation and show that it reduces to

$$
2 x^{2}+9 x-1035=0
$$

Answer
(d) Solve the equation $2 x^{2}+9 x-1035=0$, giving both answers correct to 2 decimal places.
$\qquad$
(e) Explain why one of the solutions in part (d) must be rejected.
$\qquad$
(f) Hence, calculate the total time taken by the two cyclists.

7 (a) In 2019, Mei Ling earned an annual income of \$54000. In 2020, due to economic recession, she will have a pay reduction of $\$ 300$ per month from August to December. Calculate the percentage decrease in her annual income from 2019 to 2020.
$\qquad$
\%
[2]
(b) In 2010, the number of mobile phones sold in Singapore was 6.95 million units.
Calculate the mean number of mobile phones sold in Singapore each month.
Give your answer in standard form.
(c) The price of a smart watch is $\$ 1250$ in Singapore.

For the same smart watch, it is sold at $¥ 89999$ from a shopping site in Japan.
The exchange rate between Singapore dollars (\$) and Japanese Yen ( $¥$ ) is $¥ 100=\$ 1.30$.

Calculate how much cheaper the smart watch is in Japan than in Singapore.
(d) Sharon buys a car on hire purchase which includes a deposit and 5 years of monthly payments.
She pays a deposit of one quarter of the cash price.
She then makes 5 years payment of $\$ 1250$ per month.
With hire purchase, she realises that she pays $5 \%$ more than the cash price.
Find the hire purchase price of the car.


The diagram shows the points $A, B$ and $Q$. $A$ lies on the $y$-axis and $B$ is $(3,1)$.
(a) The gradient of $A B$ is $-\frac{4}{3}$.

Find the equation of the line $A B$.

> Answer
(b) $A O C B$ is a rhombus.

Find the coordinates of $C$.
(c) The equation of line $l$ is $a y+6 x=10$. Line $l$ does not intersect the line $O B$. Find the value of $a$.

$$
\text { Answer } a=
$$

(d) $Q$ is a point on $O B$ so that $A Q$ is perpendicular to $O B$.

Show that triangle $O A Q$ is congruent to triangle $B A Q$.
Give a reason for each statement you make.

Answer

9 The variables $x$ and $y$ are connected by the equation $y=\frac{4}{x}+x-5.2$.
Some corresponding values of $x$ and $y$ are given in the table below.

| $x$ | 0.7 | 1 | 1.5 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 1.2 | $p$ | -1.0 | -1.2 | -0.9 | -0.2 | 0.6 | $q$ | 2.4 |

(a) Find the values of $p$ and $q$.

$$
\text { Answer } \begin{align*}
& p= \\
q & = \tag{2}
\end{align*}
$$

(b) Draw the graph of $y=\frac{4}{x}+x-5.2$ for $0.7 \leq x \leq 7$ on the grid next page.
(c) Use your graph, estimate the solutions of the equation $\frac{4}{x}+x=5.2$.

$$
\begin{equation*}
\text { Answer } x=\ldots \ldots \ldots \text { or } \ldots \ldots \ldots . \tag{2}
\end{equation*}
$$

(d) By adding a suitable straight line on your graph, estimate the solutions of the equation $\frac{4}{x}+x=6.2-\frac{1}{2} x$.
or $\qquad$
(e) On your graph of $y=\frac{4}{x}+x-5.2$ for $0.7 \leq x \leq 7$, the gradient of the tangent is negative for $x<b$. State the largest value of $b$.

$$
\text { Answer } b=
$$



10 A windscreen wiper helps clear away water or snow from the windscreen so that drivers can see the road clearly.

There are two types of wipers, namely the single arm wiper system or the tandem wiper system.


In both systems, wipers rotate about a hinge.
Areas covered by the two systems are different.
(a) In a single arm wiper system, the wiper blade length is 60 cm .

The length from the tip of the wiper to the hinge is 80 cm .
The angle subtended at the hinge during one full swipe is $150^{\circ}$.


Single arm wiper system
Show that the perimeter of the region (shaded portion) covered by wiper blade is 382 cm , correct to the nearest cm .

Answer
(b) In a tandem wiper system, wiper blades $P$ and $Q$ of lengths 60 cm each clean the windscreen simultaneously. The tip of each blade is 80 cm from the hinge.
The angles subtended at the hinge during one full swipe by $P$ and $Q$ are $120^{\circ}$ and $\theta$ respectively.
The total area covered by both wipers is $9980 \mathrm{~cm}^{2}$.
The intersecting area (shaded portion) of the two wipers is $800 \mathrm{~cm}^{2}$.


Tandem wiper system
Calculate the area covered by wiper $Q$.
$\mathrm{cm}^{2}$
(c) Both wiper blade and arm have to be replaced periodically due to wear and tear. Mr Smith is currently driving a large family car which uses the tandem wiper system ( 2 wiper arms and 2 wiper blades).

Wiper blades have to be replaced every 6 months.
The combined length of wiper and arm should cover about $80 \%$ of the height of the windscreen.

Mr Smith uses the following information to work out the replacement cost.

| Dimension of Windscreen |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Car Type | Mini | Small family | Large family | SUV |
| Height | 60 cm | 75 cm | 85 cm | 100 cm |


| Wiper Blade | - | Wiper Arm |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Length of blade and arm combined (inches) | Cost of wiper blade | Material of wiper arm | Cost of wiper arm | Durablity |
| 22 | \$24 | Plastic | \$12 | 6 months |
| 25 | \$28 | Stainless Steel | \$16 | 1 year |
| 27 | \$32 | Aluminium | \$30 | 2 years |
| 30 | \$34 |  |  |  |
| 33 | \$36 |  |  |  |
| 35 | \$38 |  |  |  |
| 37 | \$40 |  |  |  |
| 39 | \$42 |  |  |  |
| 42 | \$45 | $1 \mathrm{inch}=2.54$ ce | imetres |  |

Mr Smith has the loyalty card which entitles him a 5\% discount for wiper arm only.

Please advise Mr Smith what to purchase and why?
Assuming that both wiper blades have the same length.

Continuation of working space for Question 10(c).

| QN | Solutions and Answers | Mark |
| :---: | :---: | :---: |
| 1 | $\frac{7}{10}, 0.707,0.7 \dot{0}, 0 . \dot{7} 0 \dot{7}$ | B1 |
| 2a | No because the width of the bar presenting $P P$ company is wider than other bar (or twice the width as others), giving a false impression that the $P P$ annual sales company is not just 4 times as many as the annual sales of $B B$ company (or anything similar) | B1 |
| 2b | 5499 | B1 |
| 3 | $\begin{aligned} & \left(x^{9}\right)^{\frac{2}{3}} \times\left(y^{3}\right)^{0} \\ & =x^{6} \times 1 \\ & =x^{6} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 4a | $504=2^{3} \times 3^{2} \times 7$ | B1 |
| 4b | $\begin{aligned} & m=7 \\ & n=2 \end{aligned}$ <br> or vice versa | B1 |
| 5 | $\begin{aligned} & \text { Total amount } \\ & =5000\left(1+\frac{3}{100}\right)^{5} \\ & =5796.37 \\ & \text { Interest }=\$ 796.37 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 6 | $\begin{aligned} & \frac{5}{(3 x-1)}=\frac{7}{(2 x-3)} \\ & 5(2 x-3)=7(3 x-1) \\ & 10 x-15=21 x-7 \\ & 11 x=-8 \\ & x=-\frac{8}{11} \end{aligned}$ | M1 - cross multiply <br> A1 |
| 7 | $\begin{aligned} \mathrm{P}(\mathrm{GG} \text { or } \mathrm{BB}) & =\frac{5}{12} \times \frac{5}{12}+\frac{7}{12} \times \frac{7}{12} \\ & =\frac{37}{72} \end{aligned}$ | M1 A1 |
| 8 | $\begin{aligned} & 7 \leq \frac{2 x+5}{3} \\ & 21 \leq 2 x+5 \\ & 16 \leq 2 x \\ & 8 \leq x \\ & \frac{2 x+5}{3}<15 \\ & 2 x+5<45 \\ & 2 x<40 \\ & x<20 \end{aligned}$ <br> Ans: $8 \leq x<20$ | M1 - <br> 2 inequalities <br> A1 |


| 9 | $\begin{aligned} & F=\frac{k}{d^{2}} \\ & 4=\frac{k}{3^{2}} \\ & k=36 \\ & 9=\frac{36}{d^{2}} \\ & d=2 \text { or }-2 \end{aligned}$ | $\mathrm{B} 1-$ value of $k$ A1 |
| :---: | :---: | :---: |
| 10a | $x^{2}-6 x-8=(x-3)^{2}-17$ | B1 |
| 10b | $\begin{aligned} & (x-3)^{2}-17=0 \\ & (x-3)^{2}=17 \\ & x-3= \pm \sqrt{17} \\ & x=-1.12 \text { or } 7.12 \end{aligned}$ | M1 - no marks awarded if wrong method is used. A1 |
| 11ai | $\{11,13,17,19\}$ | B1 |
| 11aii | $\varnothing$ | B1 |
| 11b | $A \cup B^{\prime}$ | B1 |
| 12a |  | Shape - B1 Intercepts-B1 |
| 12b | $x=1$ | B1 |
| 13a | $120^{\circ}$ | B1 |
| 13b | $\begin{aligned} & \frac{P R}{\sin 47}=\frac{600}{\sin 39} \\ & P R=697 \mathrm{~m} \text { (to } 3 \mathrm{sf} \text { ) } \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { M1 } \\ \text { B1 } \end{array}$ |
| 14 | B 1 for proper construction of angle bisector. <br> B1 for proper construction of perpendicular bisector. <br> B1 for any correct location |  |
| 15 | $\begin{aligned} & \angle A B C=90^{\circ}(\angle \text { in a semi-circle }) \\ & \angle A B E=90^{\circ}(\angle s \text { on straight line }) \\ & \therefore A E \text { is diameter }(\angle \text { in a semi-circle }) \end{aligned}$ | $\begin{aligned} & \hline \text { B1 } \\ & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 16 | $\begin{aligned} & \angle A O C=6 x(\angle \text { at centre }=2 \angle \text { at circumference }) \\ & 6 x+x+x=180 \\ & 8 x=180 \\ & x=22.5 \end{aligned}$ | M1 <br> M1 <br> A1 |


| 17a | $\begin{aligned} & 9 x^{3}-4 x y^{2} \\ & =x\left(9 x^{2}-4 y^{2}\right) \\ & =x(3 x-2 y)(3 x+2 y) \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| :---: | :---: | :---: |
| 17b | $\begin{aligned} & 4 c p+10 d q-8 d p-5 c q \\ & =4 c p-5 c q+10 d q-8 d p \\ & =c(4 p-5 q)-2 d(4 p-5 q) \\ & =(4 p-5 q)(c-2 d) \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 18 | Let number of girls be $x$ <br> Mean height of boy $=174$ <br> Total height of girls $=162 x$ <br> Number of boys $=x+2$ <br> Total height $=169.5(x+x+2)$ $\begin{aligned} & 162 x+174(x+2)=169.5(x+x+2) \\ & 162 x+174 x+348=339 x+339 \\ & 3 x=9 \\ & x=3 \end{aligned}$ <br> Number of girls $=3$ |  |
| 19ai | $5.5 \mathrm{~m} / \mathrm{s}^{2}$ | B1 |
| 19aii | 11 m | B1 |
| 19b | $\begin{aligned} & \hline 3.5 v=11+77 \\ & v=25.1 \mathrm{~m} / \mathrm{s} \\ & \hline \end{aligned}$ | M1 |
| 20 | $\begin{aligned} & \begin{array}{l} \text { Flight duration }=\frac{13563}{775} \\ =17.5 \text { hour } \\ \text { Arrival time (San Francisco time) } \\ =0210+1730 \\ =1940 \text { Saturday } \\ \\ \text { Arrival time (Singapore time) } \\ =1940 \text { Saturday + } 15 \\ =1040 \text { Sunday } \end{array} \\ & \hline \end{aligned}$ | B1 <br> B1 <br> B2 |
| 21a | $\begin{aligned} & \frac{44}{18}=\left(\frac{x}{40}\right)^{3} \\ & x=54 \mathrm{~cm} \end{aligned}$ | M1 A1 |
| 21b | $\left(\frac{53.883}{40}\right)^{2}=\frac{k}{1}$ | $\begin{aligned} & \mathrm{M} 1 \\ & \mathrm{Al} \end{aligned}$ |
| 22a | $\begin{aligned} & y_{2}=y_{1}\left(k+y_{1}\right) \\ & y_{2}=1(k+1) \\ & y_{2}=k+1 \end{aligned}$ | B1 |
| 22b | $\begin{aligned} & y_{3}=y_{2}\left(k+y_{2}\right) \\ & y_{3}=(k+1)(k+k+1) \\ & y_{3}=(k+1)(2 k+1) \\ & y_{3}=2 k^{2}+3 k+1 \quad \text { (shown) } \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |


| 22c | $\begin{aligned} & 1=2 k^{2}+3 k+1 \\ & 2 k^{2}+3 k=0 \\ & k(2 k+3)=0 \\ & k=-\frac{3}{2}(\text { reject } k=0) \end{aligned}$ | $\begin{array}{\|l} \text { M1 } \\ \text { A1 } \end{array}$ |
| :---: | :---: | :---: |
| 23ai | 44 hours | B1 |
| 23aii | 26 hours | B1 |
| 23aiii | 26.7\% | B1 |
| 23b | Working hour in company B is longer than company A as it has a higher median. <br> Company A has a wider spread of working hours since it has a bigger range. | B1 B1 |
| 24a | $\left(\begin{array}{ccc}20 & 10 & 10 \\ 25 & 15 & 0\end{array}\right)$ | B1 |
| 24b | $\left(\begin{array}{c} 0.9 \\ 1.2 \\ x \end{array}\right)$ | B1 |
| 24c | $\begin{aligned} & \left(\begin{array}{lll} 20 & 10 & 10 \\ 25 & 15 & 0 \end{array}\right)\left(\begin{array}{c} 0.9 \\ 1.2 \\ x \end{array}\right) \\ & =\binom{30+10 x}{40.5} \end{aligned}$ | B2 |
| 24d | Cost of Set X \& Y respectively | B1 |
| 24 e | $\begin{aligned} & 30+10 x=40.5 \\ & x=1.05 \end{aligned}$ | 1 |
| 25a | Area of rectangle $A B C D=4 \times 8=32 \mathrm{~cm}^{2}$ $\begin{aligned} & 32 \mathrm{~cm}^{2}: 128 \mathrm{~m}^{2} \\ & 1 \mathrm{~cm}^{2}: 4 \mathrm{~m}^{2} \\ & 1 \mathrm{~cm}: 2 \mathrm{~m} \end{aligned}$ $n=2$ | $\begin{array}{\|l} \text { M1 } \\ \text { M1 } \\ \text { A1 } \\ \hline \end{array}$ |
| 25b | $\begin{aligned} & \frac{128}{12}=10.7 \text { litres } \\ & 500 \mathrm{ml} \\ & 22 \text { tins are needed }>\$ 286 \\ & 1 \text { litre } \\ & 11 \text { tins are needed }>\$ 275 \\ & \text { Least amt }=\$ 275 \end{aligned}$ | M1 <br> M1 <br> A1 |

QTSS 2020 4EXP MATH PRELIM P2 MARKING SCHEME

| QN | Solutions and Answers | Mark |
| :---: | :---: | :---: |
| 1ai | 3.62 | B1 |
| 1aii | $\begin{aligned} & C-1=\frac{\pi b^{2} s}{4 V} \\ & b^{2}=\frac{4 V(C-1)}{\pi s} \\ & b= \pm \sqrt{\frac{4 V(C-1)}{\pi s}} \end{aligned}$ | M1 <br> A1 |
| 1bi | $\frac{4 a b^{2}}{3}$ | B1 |
| 1 bii | $\begin{aligned} & \frac{5(x-2)+4(3 x+1)}{(3 x+1)(x-2)} \\ & =\frac{5 x-10+12 x+4}{(3 x+1)(x-2)} \\ & =\frac{17 x-6}{(3 x+1)(x-2)} \end{aligned}$ | M1 <br> A1 |
| 1c | $\begin{aligned} & \left(5^{2}\right)^{3 x-11}=\left(5^{3}\right)^{x} \\ & 5^{6 x-22}=5^{3 x} \end{aligned}$ <br> Hence, $6 x-22=3 x$ $\begin{equation*} x=\frac{22}{3} \tag{3} \end{equation*}$ | $\begin{aligned} & \text { M1 } \\ & \\ & \text { A1 } \end{aligned}$ |
| 1d | (1) $\times 2: 8 x+6 y=6$. <br> (2) $\times 3: 18 x-6 y=33$. <br> (3) $+(4): 26 x=39$ $x=\frac{3}{2}$ <br> Substitute $x=\frac{3}{2}$ into (1): $4\left(\frac{3}{2}\right)+3 y=3$ $y=-1$ | M1 Accept substitution method A2 |
| 2a | $120^{\circ}$ | B1 |
| 2b | $\begin{aligned} & \angle E D Q=360-120-90=150^{\circ} \\ & \text { exterior angle }=30^{\circ} \\ & n=\frac{360}{30} \\ & =12 \end{aligned}$ | M1 <br> M1 <br> A1 |


| 2c | Let $X$ be the foot of perpendicular from $F$ to $E A$. $\begin{aligned} & F X=2 \sin 30^{\circ}=1 \mathrm{~cm} \\ & E X=2 \cos 30^{\circ} \approx 1.732 \mathrm{~cm} \end{aligned}$ <br> Area of trapezium $A B C F=\frac{1}{2}(2+4)(1.732)$ $\approx 5.196 \mathrm{~cm}^{2}$ <br> Area of hexagon $=5.196 \times 2$ <br> $\square 10.4 \mathrm{~cm}^{2}$ | $\begin{aligned} & \text { M1 } \\ & \text { M1 } \\ & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| :---: | :---: | :---: |
| 3a | $\begin{aligned} & B C=\frac{4800}{8 \times 30}=20 \mathrm{~cm} \\ & A C=\sqrt{30^{2}+20^{2}} \\ & \approx 36.0555 \\ & \approx 36.1 \mathrm{~cm}(1 \text { d.p. }) \text { (shown) } \end{aligned}$ | M1 <br> M1 <br> Answer given |
| 3b | $\begin{aligned} & \tan \angle H A C=\frac{8}{36.0555} \\ & \angle H A C=\tan ^{-1}\left(\frac{8}{36.0555}\right) \\ & \approx 12.5^{\circ} \end{aligned}$ | M1 <br> A1 |
| 3c | $\begin{aligned} & A E=\sqrt{20^{2}+8^{2}} \approx 21.54 \mathrm{~cm} \\ & E C=\sqrt{30^{2}+8^{2}} \approx 31.05 \mathrm{~cm} \end{aligned}$ <br> Using cosine rule, $\begin{aligned} & \cos \angle A E C=\frac{21.54^{2}+31.05^{2}-36.0555^{2}}{2(21.54)(31.05)} \\ & \quad=0.09574743147 \\ & \angle A E C=\cos ^{-1}(0.09574743147) \\ & \approx 84.5^{\circ} \end{aligned}$ | M1 <br> M1 <br> M1 <br> A1 |
| 4ai | 54.2 | B1 |
| 4aii | 9.13 | B1 |
| 4aiii | For grouped data, we use the mid-poiny of each class interval which are no exact data values. | B1 accept any reasonable explanation |
| 4aiv | The organiser is right in his claim because the mean time taken in 2020 is lower than the mean time taken in 2019. | B1 |
| 4bi | 200 | B1 |
| 4bii | 112 | B1 |
| 4biii | $\begin{aligned} & 119-104 \\ & =15 \end{aligned}$ | M1--Upper quartile or Lower quartile correct A1 |
| 4aiv | $4 \% \times 200=8$ <br> Minimum height $=129 \mathrm{~cm}$ | $\begin{array}{\|l\|} \hline \text { M1 } \\ \text { A1 } \\ \hline \end{array}$ |


| 5ai |  | All correct --- 2 marks <br> 1-2 worng --- 1 mark <br> $>2$ wrong --- 0 mark |
| :---: | :---: | :---: |
| 5aii | $\begin{aligned} & \mathrm{P}(\mathrm{WW} \text { or WD or DW }) \\ & =(0.6)(0.6)+(0.6)(0.3)+(0.3)(0.6) \\ & =0.72 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 5b | $\begin{aligned} & P(R)=\frac{2}{2+x} \\ & P(R R)=\left(\frac{2}{2+x}\right)\left(\frac{1}{1+x}\right) \\ & \left(\frac{2}{2+x}\right)\left(\frac{1}{1+x}\right)=\frac{1}{10} \quad[\times 10(2+x)(1+x)] \\ & 20=(2+x)(1+x) \\ & 20=2+2 x+x+x^{2} \\ & x^{2}+3 x-18=0 \\ & (x+6)(x-3)=0 \\ & x=-6 \text { (rej) or } 3 \end{aligned}$ | M1 <br> M1 <br> M1 <br> A1 |
| 6a | $\frac{115}{x}$ | B1 |
| 6 b | $\frac{110}{x-3}$ | B1 |
| 6c | $\begin{aligned} & \frac{110}{x-3}-\frac{115}{x}=\frac{40}{60} \\ & \frac{110}{x-3}-\frac{115}{x}=\frac{2}{3} \quad[\text { multiply by } 3 x(x-3)] \\ & 330 x-345(x-3)=2 x(x-3) \\ & 330 x-345 x+1035=2 x^{2}-6 x \\ & 2 x^{2}+9 x-1035=0 \text { (shown) } \end{aligned}$ | M1 <br> M1 <br> M1 <br> Answer given |
| 6d | $\begin{aligned} & x=\frac{-(9) \pm \sqrt{(9)^{2}-4(2)(-1035)}}{2(2)} \\ & \approx 20.61 \text { or }-25.11 \text { (2d.p.) } \end{aligned}$ | M1 <br> A1 A1 |
| 6 e | $x=-25.11$ must be rejected because average speed cannot be negative. | B1 |



| 8b | Let coordinates of $C$ be $(a, b)$ $\begin{aligned} & a=3 \\ & 1-b=5 \\ & b=-4 \end{aligned}$ <br> Hence $C$ <br> $(3,-4)$ | M1 $\mathrm{A} 1 \quad \text { or } \mathrm{B} 2$ |
| :---: | :---: | :---: |
| 8c | $\begin{aligned} & a y+6 x=10 \\ & y=-\frac{6}{a} x+\frac{10}{a} \end{aligned}$ <br> Gradient of line $O B=\frac{1}{3}$ <br> Line $l$ is parallel to line $O B$ <br> Hence $-\frac{6}{a}=\frac{1}{3}$ $a=-18$ | M1 (either $-\frac{6}{a}$ or $\frac{1}{3}$ seen) A1 |
| 8d | $\text { Length of } \begin{aligned} A B & =\sqrt{(3-0)^{2}+(1-5)^{2}} \\ & =5 \text { units } \end{aligned}$ <br> Hence $A O=A B=5$ units ( H ) <br> $A Q=A Q$ (common side) (S) <br> $\angle A Q O=\angle A Q B=90^{\circ}(O Q B$ is a straight line $)(\mathrm{R})$ <br> Hence $\triangle O A Q$ is congruent to $\triangle B A Q$ (RHS) (shown) | $\begin{array}{\|l} \text { M1 } \\ \text { M1 } \\ \text { M1 } \\ \text { M1 } \\ \text { A1 } \end{array}$ |
| 9 a | $\begin{aligned} & p=-0.2 \\ & q=1.5 \end{aligned}$ | $\begin{aligned} & \mathrm{B} 1 \\ & \text { B1 } \end{aligned}$ |
| 9 b |  | Points plotted correctly M1 <br> Graph passing through all points M1 <br> Smooth graph M1 |


|  |  |  |
| :---: | :---: | :---: |
| 9c | $x=0.95$ or 4.25 | B1 B1 <br> Accuracy $\pm 0.05$ |
| 9d | $\begin{aligned} & \frac{4}{x}+x=6.2-\frac{1}{2} x \\ & \frac{4}{x}+x-5.2=6.2-\frac{1}{2} x-5.2 \\ & \frac{4}{x}+x-5.2=-\frac{1}{2} x+1 \end{aligned}$ <br> Draw $y=-\frac{1}{2} x+1$ <br> From graph, $x=0.8$ or 3.35 | M1 <br> M1 <br> A1 <br> Accuracy $\pm 0.1$ |
| 9 e | $b=2$ | B1 |
| 10a | $\begin{aligned} & \text { Perimeter covered }= \\ & \frac{150}{360} \times 2 \pi(80)+\frac{150}{360} \times 2 \pi(20)+60+60 \\ & =381.7993878 \\ & \approx 382 \mathrm{~cm} \text { (shown) } \end{aligned}$ | M1-Either larger arc length or smaller arc length M1- correct addition of various lengths Answer given |
| 10b | Area covered by wiper $P$ $\begin{aligned} & =\frac{120}{360} \times \pi(80)^{2}-\frac{120}{360} \times \pi(20)^{2} \\ & =6283.185307 \mathrm{~cm}^{2} \end{aligned} \begin{aligned} & 6283.185307+\operatorname{Area}(\mathrm{Q})-800=9980 \\ & \text { Area }(\mathrm{Q})=9980-6283.185307+800 \\ &=4496.814693 \\ & \approx 4500 \mathrm{~cm}^{2} \end{aligned}$ | M1 <br> M1 <br> A1 |
| 10c | Height of windscreen $=\frac{1}{2.54} \times 85 \approx 33.46$ inches <br> $80 \% \times 33.46 \approx 26.8$ inches <br> Recommended combined length he should purchase is 27 inches. <br> Cost of blades + plastic arms $=\$ 32 \times 2+\$ 12 \times 2 \times 0.95=\$ 86.80$ (for 6 months) <br> Cost of blades + steel arms $\begin{aligned} =\$ 32 \times 4+\$ 16 \times 2 \times 0.95 & =\$ 158.40 \text { (for } 1 \text { year) } \\ & =\$ 79.20 \text { (for } 6 \text { months) } \end{aligned}$ <br> Cost of blades + aluminium arms $\begin{aligned} =\$ 32 \times 8+\$ 30 \times 2 \times 0.95 & =\$ 313(\text { for } 2 \text { years }) \\ & =\$ 78.25(\text { for } 6 \text { months }) \end{aligned}$ | M1 <br> M1 <br> M1 <br> M1 |


|  | Hence he should purchase 27 inch blades and <br> aluminium arms | A1 |
| :--- | :--- | :--- |

