Name		(	)	Class	

南 华 中 学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2020

Subject	:	**Mathematics**
Paper	:	**4048/01**
Level	:	**Secondary Four Express**
Date	:	**17 August 2020**
Duration	:	**2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue, correction fluid or correction tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** questions.

1   (a)   Calculate $\sqrt{\dfrac{(74.03)^2 - 2340}{5^3 + 10}}$, writing down the first six digits of your

answer.

Answer .... [1]

(b)   Write your answer to part (a) correct to 3 significant figures.

Answer .... [1]

2   Factorise completely $9a^2 + 6ab + b^2 - 25c^2$.

Answer .... [2]

3   Written as a product of its prime factors, $264 = 2^3 \times 3 \times 11$ and $1980 = 2^2 \times 3^2 \times 5 \times 11$.

The highest common factor and the lowest common multiple of 264, 1980 and A are 12 and 59400 respectively.

Find the largest possible value of A.

Answer $A =$ .... [2]

4   Two bottles of hand sanitisers are geometrically similar.
The height of the smaller bottle and larger bottle are 10 cm and 19 cm respectively.
The price of the smaller and larger bottle are $4.90 and $15.10 respectively.

Explain which bottle gives the better value for money.

Answer .... [3]

5 **(a)** Given that $2 \times 5^x = \dfrac{10}{\sqrt[3]{25}}$, find the value of x.

Answer $x =$ ……...…………… [2]

(b) The radius of the base of a cylinder was increased by 30% and its height was decreased by 30%.

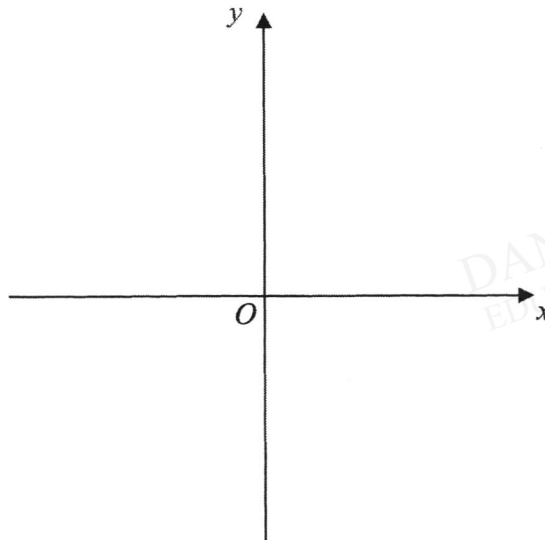Find the percentage change, if any, in its volume.

Answer ……..……………% [3]

6 **(a)** Express $-2x^2 - 6x - 2$, in the form $a(x+h)^2 + k$ where a, h and k are constants.

Answer ……………………...……… [2]

(b) Sketch the graph of $y = -2x^2 - 6x - 2$. Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point on the curve.

Answer [2]

(c) Write down the equation of the line of symmetry of the curve $y = -2x^2 - 6x - 2$

Answer ……………………...……… [1]

7   The graph shows the number of unemployed workers in Country X for the last 10 years.

Unemployed Workers in Country X

State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer ....

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.... [2]

8   The number of students in the parade square is given as 1600, correct to the nearest hundred. Write down values for the smallest and largest possible number of students that could be in the parade square.

Answer   Smallest possible number = ……….…….

Largest possible number = ……….…….   [1]

9   Alex takes 40 hours to harvest a particular plot of rice field.
Ben will take 55 hours to harvest the same plot of rice field.
Alex and Ben worked together for 20 hours before Ben injured himself and could not continue with the harvest.

If Alex continues to harvest the remaining parts of the field himself at the same rate, calculate how long it will take for him to finish harvesting the field.
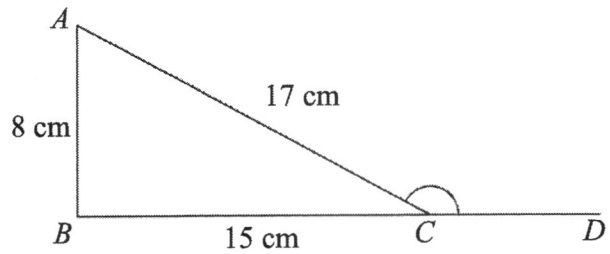
Give you answer in hours and minutes, correct to the nearest minute.

Answer   …………. hours …………. minutes   [3]

10 *ABC* is a triangle with *AB* = 8 cm, *BC* = 15 cm and *AC* = 17 cm.

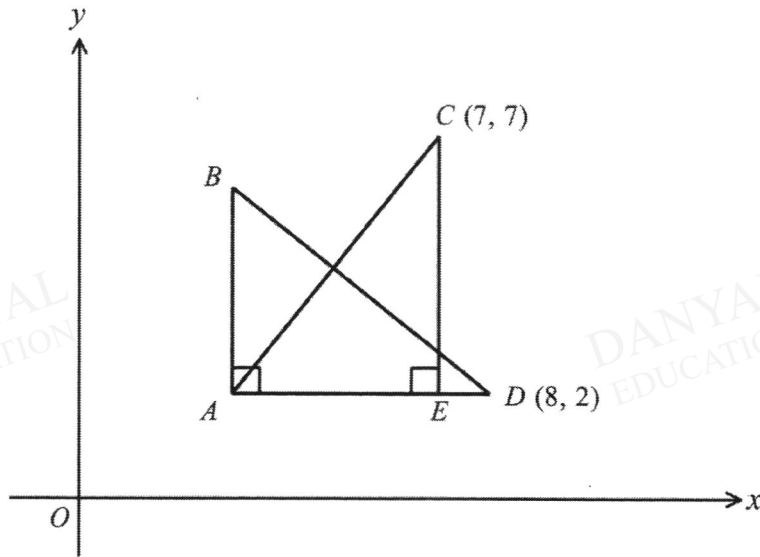**(a)** Explain clearly why triangle *ABC* is a right-angled triangle.

Answer ....

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.... [2]

(b) Without using a calculator, find the value of $\cos \angle ACD$.

Answer .... [1]

11 The diagram shows two congruent right-angled triangles ACE and BDA.
The sides AB and CE are vertical.
The side AD is horizontal and point E lies on it.
Point C has coordinates $(7, 7)$ and D has coordinates $(8, 2)$.

Find the equation of line AC.

Answer .... [3]

12   Solve the inequality $-\dfrac{3x-4}{3} < \dfrac{7x-2}{6} \leq \dfrac{3x+4}{5}$

Answer .... [3]

13   A map of Thailand has a scale of 1 : 500 000.

   (a)   The length of the river Kwai on the map is 76 cm.

   Calculate the actual length, in kilometres, of the river Kwai.

   Answer .... km   [2]

   (b)   The area of Thailand is 513 120 km^2.

   Calculate the area, in square centimetres, of Thailand on the map.
   Leave your answer in standard form, correct to 4 significant figures.

   Answer .... cm^2 [2]

14 A bag contains 30 pieces of tokens, of which x are gold tokens and the remaining are silver tokens.

Two tokens are drawn from the bag one after the other without replacement.

(a) Show that the probability that the second token drawn is a gold token is $\dfrac{x}{30}$.

Answer

[2]

(b) The probability of drawing one gold and one silver token is $\dfrac{25}{87}$.

Find the values of x.

Answer $x =$ .... or $x =$ .... [3]

15 ξ = {all positive integers}
   A = {all prime numbers}
   B = {all even integers}
   C = {perfect squares}

(a)   On the Venn Diagram, shade the region which represents $A \cup B'$.

   Answer   [1]

(b)   Explain why $A \cap B \neq \varnothing$.

   Answer ....

   ....

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   .... [1]

(c)   Explain why $A \cap C = \varnothing$.

   Answer ....

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   .... [2]

16 Indicating the intercepts, if any, sketch the graphs of
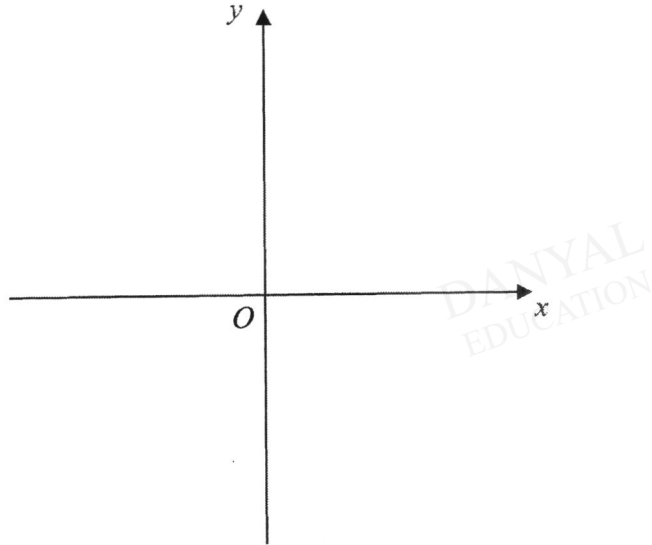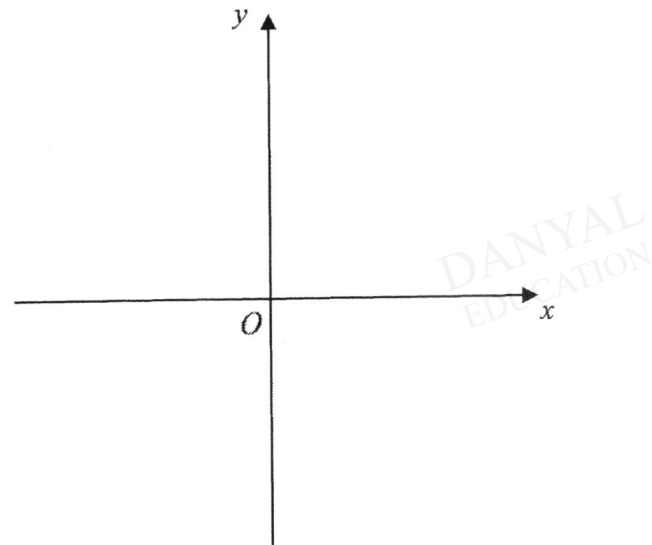
(a) $x^2 y = -3$

Answer [1]

(b) $y = x^3 - 8$

Answer [1]

17   Group A consists of 12 boys who were given milk from Brand A to drink daily.

The stem-and-leaf diagram shows the height gain of the boys after 6 months.

$$
\begin{array}{c|cccc}
2 & 9 \\
3 & 3 & 5 & 6 \\
4 & 1 & 5 & 6 & 9 \\
5 & 4 & 7 & 7 \\
6 & 2 \\
\end{array}
$$

Key: $2 \mid 9$ represents 29 mm

(a)   State the median height gain of the boys.

Answer ………………..……... mm   [1]

(b)   Find the interquartile range of the height gain of the boys.

Answer ………………..……... mm   [2]

(c)   Calculate

   (i)   the mean height gain of the boys,

Answer ………….…………... mm   [1]

   (ii)   the standard deviation of the height gain of the boys.

Answer ………………..……... mm   [1]

(iii) Group B consists of boys who were given milk from Brand B to drink daily.
Their height gains were measured after 6 months and recorded.

The mean height gain was 41 mm.
The standard deviation of the height gain was 17.4 mm.

Compare and comment on the height gain between the boys from Group A and B.

Answer ....

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.... [2]

18 (a) Dr Ng invested $8000 in a savings account with Bank A.
The savings account pays $R\%$ interest per annum, compounded half yearly.
At the end of 2 years, the interest earned was $96.43.

Calculate the value of R.

Answer $R =$ ...…................…... [3]

(b) Bank B has a savings account that offers 0.65% simple interest per annum.

Given that any money invested will be kept in the bank for 2 years, find out if Dr Ng should invest in the savings account from Bank A or Bank B.

Answer ...…..........................…... [2]

19 Container A is in the shape of a cone, where both the base radius and height are r cm.
Container B is in the shape of a hemisphere with radius r cm.

Container A

Container B

Water is leaking from the fully filled container A at a constant rate k cm³/s.
Water is poured into the empty container B at the same constant rate k cm³/s.
It takes 14 seconds to empty container A.

Sketch and label the graphs showing how the depth of water in each container varies with time.

Height of water level (cm)

[3]
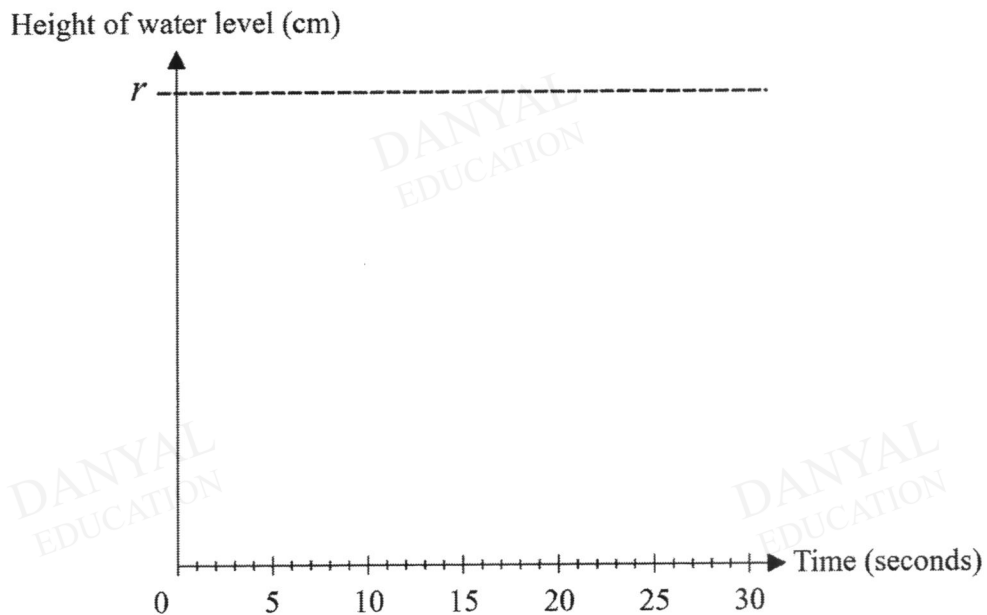
20 (a) The diagram shows an irregular polygon.

Find the value of x.
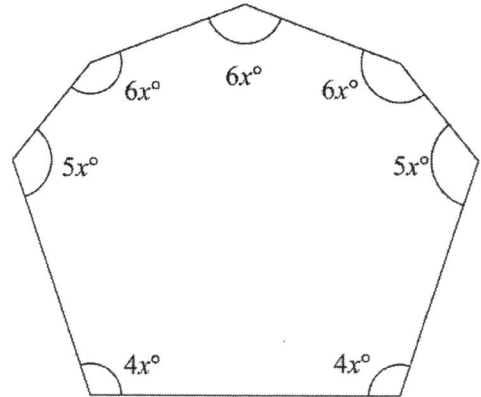
Answer $x =$ .... [2]

(b) The diagram shows a regular pentagon $ABCDE$ with centre O.

Given that the area of the pentagon is 20 m^2, find the length of AB.

Answer   .... m   [3]

21 The diagram below, $BA = BC$ and angle BFC = angle BDA.
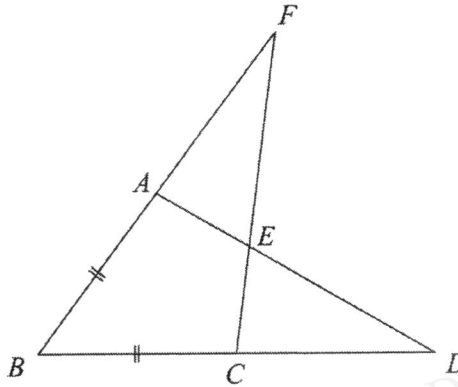
(a) Explain why triangle BCF and triangle BAD are congruent.

Answer ....

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.... [2]

(b) Given further that $AE = CE$, state the name of quadrilateral $ABCE$.

Answer .... [1]

22 The diagram shows the position of a playground P.

A basketball court B is 350 m away from P.

The bearing of B from P is $110°$.

The scale is 1 cm : 50 m.

North

P

(a) Find and label the position of basketball court B. [1]

(b) A community centre C is 450 m away from P and 300 m from B.

Given that the bearing of C from P is between $000°$ to $090°$ find the position of C. [1]

(c) Construct
(i) the perpendicular bisector of PB, [1]

(ii) The angle bisector of angle BCP. [1]

(d) A garden is to be built, nearer to B than to P and nearer to PC than to BC.

Mark clearly a possible position within the region PBC where the garden can be built and label this point G. [1]

23 Consider the following sequence:

$$\frac{3}{4}, \frac{15}{8}, \frac{35}{12}, \frac{63}{16}, \dots$$

(a) Write down the next term in the sequence.

Answer .... [1]

(b) Write down the general form for the nth term of the sequence.

Answer .... [1]

(c) Explain why **none** of the terms are integers.

Answer ....

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.... [1]

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南 华 中 学

NAN HUA HIGH SCHOOL
PRELIMINARY EXAMINATION 2020

Subject : Mathematics

Paper : 4048/02

Level : Secondary Four Express

Date : 19 August 2020

Duration : 2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use

This paper consists of 30 printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions

1.   (a)   Express $\dfrac{4}{2-3x} + \dfrac{4(x+7)}{21x^2 - 5x - 6}$ as a single fraction in its simplest form.

Answer .... [3]

(b)   Simplify $\left(\dfrac{81p^4}{q^8}\right)^{\frac{1}{4}} \div \left(\dfrac{r^3}{3q^2}\right)^{-5}$, leaving your answer in positive index.

Answer .... [3]

(c)   Given that $m = k\sqrt{\dfrac{3n-7}{4}}$, express n in terms of k and m.

Answer .... [2]

2.   Cindy saw a pair of shoes from a Japanese web-site which costs 11080 yen.
      The price excludes the Japanese customs charge of 18% tax for all overseas mail orders
      and freight charge of 3000 yen for product delivered to Singapore.

   (a)   The exchange rate between Japanese Yen and Singapore dollars is
          S$1.3012 = 100 Japanese Yen.
          Calculate the total cost in Singapore dollars, if Cindy decides to buy the pair
          of shoes by mail order.

                                                 Answer $...................................... [2]

   (b)   The same pair of shoes, normally priced locally at S$215.50, exclusive of GST,
          was sold at a discount of 20% during the Black Friday Sale.
          Calculate the selling price of the shoes, taking GST to be 7%, during the sale.

                                                 Answer $...................................... [2]

   (c)   If Cindy purchased the pair of shoes during the Black Friday Sale, express her
          savings as a percentage of the online price.

                                                 Answer ....% [2]

3. A bird flew 36 km at a constant speed of x km/h from A to B (outward journey) when there was no wind. When the bird flew back from B to A (return journey) against the wind direction, its speed was decreased by 2 km/h.

    (a) Write down, in terms of x, the time taken for the bird to fly from A to B.

    Answer ……….……….h [1]

    (b) Write down, in terms of x, the time taken for the bird to fly back from B to A.

    Answer ………….……. h [1]

    (c) Given that the return journey took the bird 75 minutes longer than the outward journey, form an equation in x and show that it simplifies to
    $5x^2 - 10x - 288 = 0$ [2]

(d)   Solve the equation $5x^2 - 10x - 288 = 0$, giving your answers to 2 decimal places.

Answer ……………………………….. [3]

(e)   Find the time taken by the bird on its return journey, giving your answer to the nearest minute.

Answer ……………………………. [2]

4.
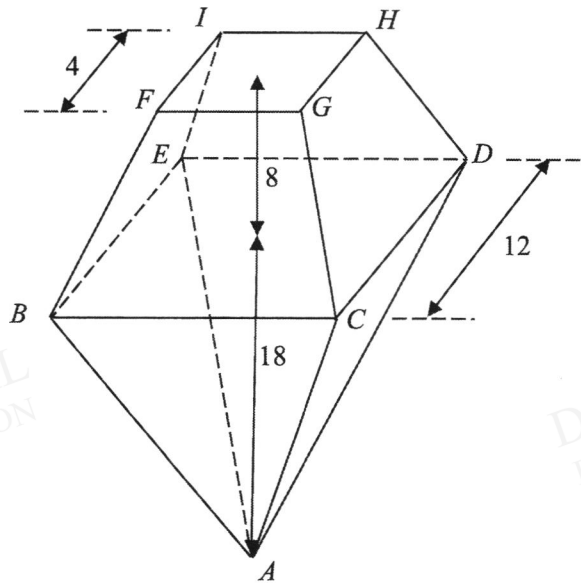
A sculpture consists of an inverted right square pyramid *ABCDE* and a frustum *BCDEIFGH*. The frustum is created by cutting away the top portion of another right square pyramid of height 12 cm to expose a flat surface *FGHI* which is parallel to the base of the pyramid *BCDE*.

BCDE and *FGHI* are squares of side 12 cm and 4 cm respectively.
The vertical height of the inverted pyramid is 18 cm and the vertical height of the frustum is 8 cm. Calculate

(a)   the volume of the sculpture,

Answer .... cm³ [4]

    (b)   the surface area of the sculpture.

Answer .... cm^2 [4]

5.
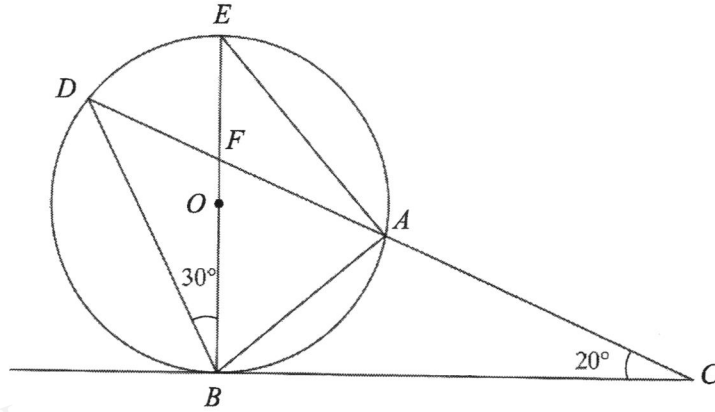
The points A, B, D and E lie on the circumference of a circle, with centre O. EB is a diameter of the circle and BC is tangent to the circle. Angle $EBD = 30°$ and angle $ACB = 20°$. F is the point of intersection of BE and AD.

(a)   Find
        (i)   angle ADB,

                                                        Answer .... ° [2]

        (ii) angle BAC.

                                                        Answer .... ° [2]

(b)   Show that triangles *DFB* and *EFA* are similar.   [2]

If *EF* = 2 cm,

(c)   (i)   calculate the length of *AF*,

Answer .... cm [2]

(ii)   calculate the value of $\dfrac{DF}{BF}$.

Answer .... [2]

6. The speeds of 600 cars passing a checkpoint X were recorded. The cumulative frequency curve below shows the distribution of the speeds.
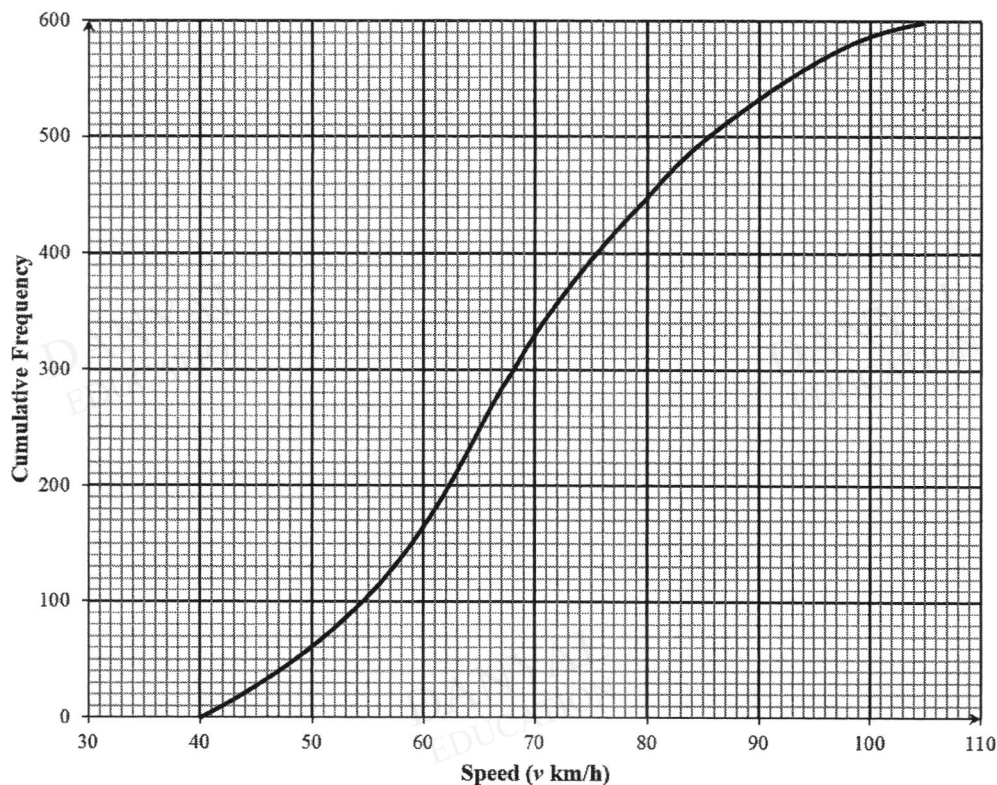
(a) Use the curve to estimate
    (i) the median speed,

Answer .... km/h [1]

    (ii) the 70th percentile speed,

Answer .... km/h [1]

(iii)   the interquartile range.

Answer .... km/h [2]

(b)   Given that 20% of the cars exceeded the speed limit at point *X*, estimate the speed limit at this point.

Answer .... km/h [1]

(c)   Two cars are selected at random. Find the probability that the speeds of these two cars are between 70 km/h and 80 km/h.

Answer .... [2]

(d)   The box-and-whiskers plot below shows the distribution of speeds (km/h) of the same 600 cars that passed through another checkpoint *Y*, further down the same road.
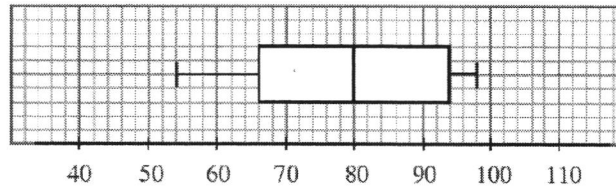
Find
(i)   the median speed,

Answer …………………….. km/h [1]

(ii)   the interquartile range.

Answer …………………….. km/h [1]

(e)   Make **two** comments comparing the speeds of the cars at points *X* and *Y*.

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

……………………………………………………………………………………… [2]

7.   The diagram below shows the speed-time graph of a car.
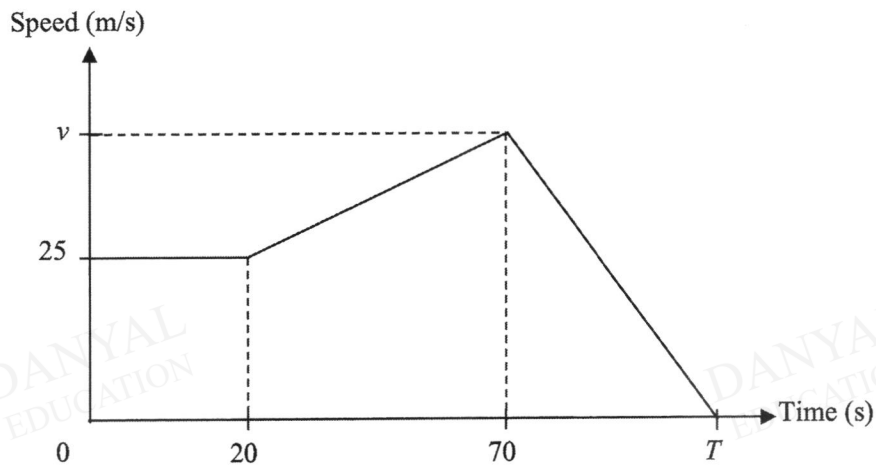
(a)   For how long is the car travelling at an increasing speed?

Answer .... s [1]

(b)   Find the value of *v*, if the total distance covered in the first 70 seconds is 2250 m.

Answer .... [2]

(c)   Find the value of *T* if the uniform retardation of the car is 1.8 m/s^2.

Answer .... [2]

(d)   Use the diagram provided below to draw the distance-time graph of the car for the whole journey.

Answer   [3]

Distance (m)

0   20   70   *T*   Time (s)
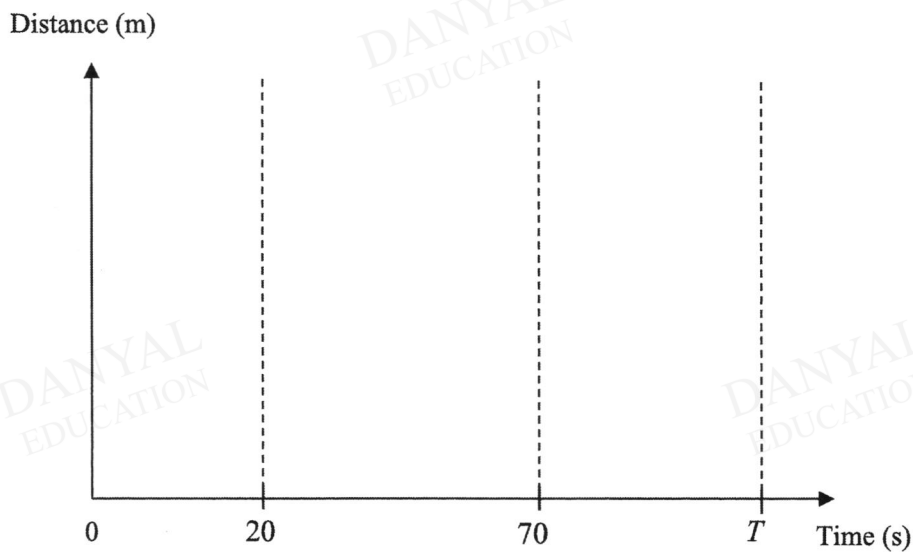
8.   The variables x and y are connected by the equation

$$y = 2x^2 + \frac{12}{x} - 9$$

Some corresponding values of x and y are given in the following table.

x	0.5	0.75	1	1.25	2	2.5	3
y	15.5	8.1	5	3.7	5	p	13

(a)   Find the value of p.

Answer $p = $ .... [1]

(b)   Using a scale of 4 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = 2x^2 + \frac{12}{x} - 9$ for $0.5 \le x \le 3$ [3]

(c)   Use your graph to find the solutions of $\frac{1}{6}x^2 = -\frac{1}{x} + \frac{4}{3}$

Answer   .... [2]

(d)   By drawing a tangent, find the gradient of the curve at the point (2, 5).

Answer   .... [2]

(e) (i) On the same axes, draw the line $y = 12 - \dfrac{3}{2}x$ for $0.5 \leq x \leq 3$. [1]

(ii) Write down the *x*-coordinates of the point where this line intersects the curve.

Answer .... [1]

(f) Use your graph to determine the *x*–coordinate of the point where the gradient of the curve is –4.

Answer .... [2]
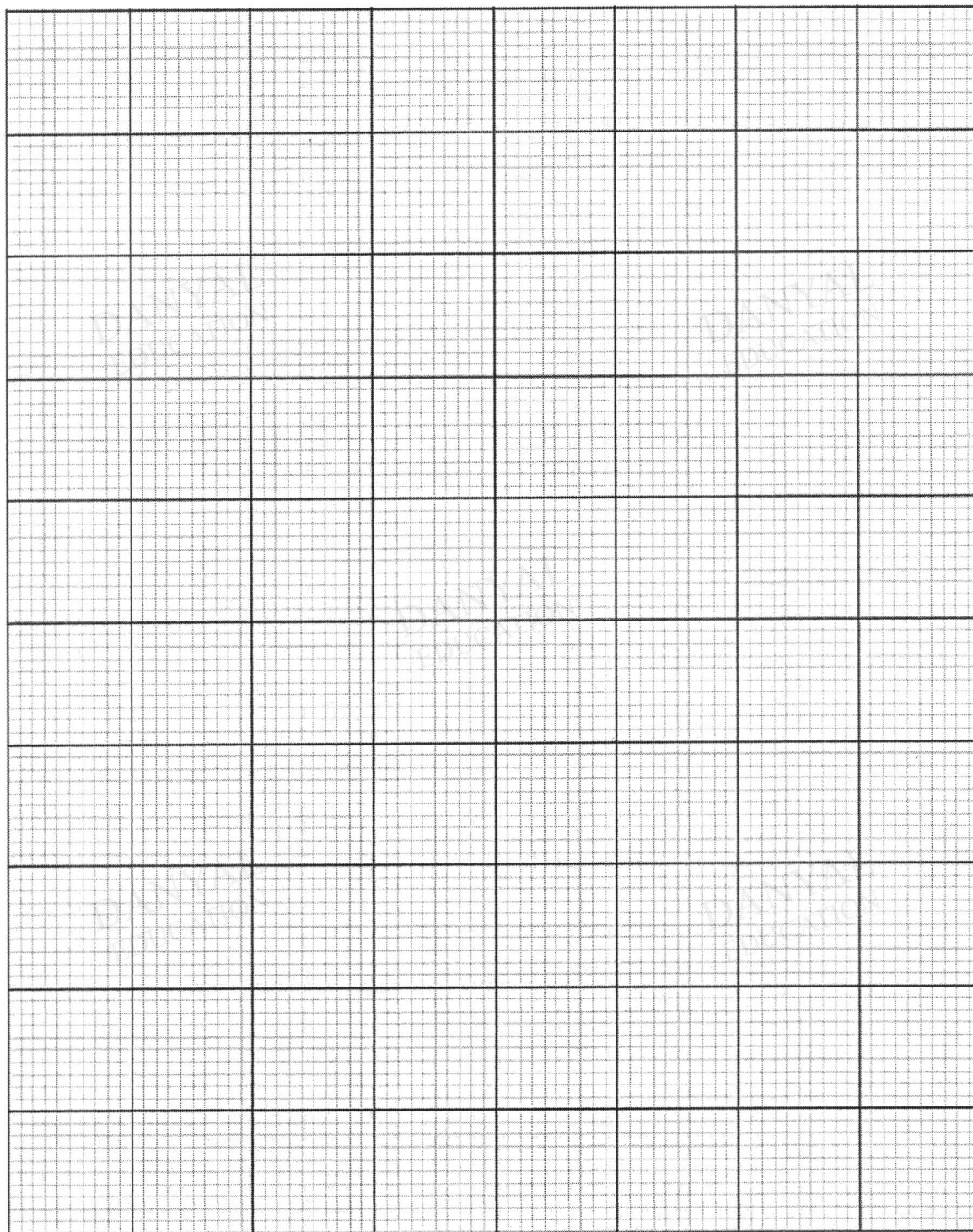
9.
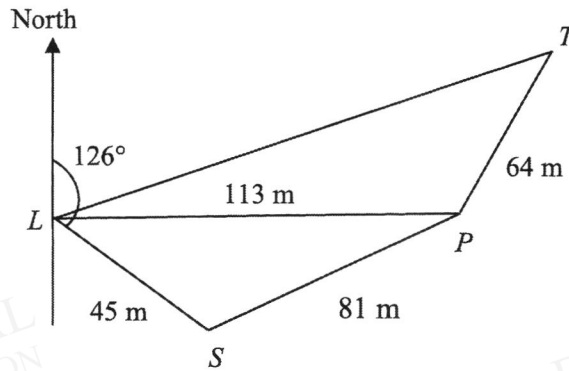
A pirate ship, P, is 113 m due east of the lighthouse L. It is detected by a coast guard S which is 45 m away and at a bearing of 126° from L. The distance between the pirate ship and the coast guard is 81 m.
Find

(a)   the bearing of the pirate ship from the coast guard.

Answer .... ° [3]

An island *T* is located at a bearing of 015° from *P* and the distance *PT* is 64 m.
Find

(b)   the length of *LT*,

Answer .... m [2]

(c)   the area of $\triangle PLT$.

Answer .... m² [2]

(d)   A helicopter is flying at a constant height of 85 m along a straight path above *LT*.
       Find the greatest angle of depression of the pirate ship from the helicopter.

Answer .... ° [3]

10.   The following table shows the number of property transactions of an agent from

      January to March 2019.

Types of Property	Sale Transaction	Rental Transaction
HDB Flats	6	8
Private Condominiums	4	2
Landed House	3	1

The information above is represented by the matrix $\mathbf{P} = \begin{pmatrix} 6 & 8 \\ 4 & 2 \\ 3 & 1 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(a)   Evaluate $\mathbf{W} = \mathbf{PQ}$ and explain the significance of the elements in $\mathbf{W}$.

Answer $\mathbf{W} =$

.... [1]

....

.... [1]

The commission received and the government tax charged under each type of transaction is shown in the table below.

	HDB Flats	Private Condominiums	Landed House
Commission	$3000	$7000	$11000
Government Tax	$800	$1800	$2500

(b)   Represent the above information with matrix **R** and evaluate **S = RW**.
      Explain the significance of the elements in **S**.

Answer **S =**

.............…… [2]

....

....[1]

(c)   Write down another matrix **T** such that their product **TS** gives the amount of money earned by the agent after deducting the government tax from January to March 2019.

Hence, find the amount of money earned by the agent.

Answer **T** = ... [1]

Amount of money earned = \$....................... [2]

11.

New Method in Land Reclamation!

A new method of land reclamation, known as empoldering, will be used at the north-western tip of Pulau Tekong. This method involves building a dike (embankment) around the area to be reclaimed and draining water from it, creating a low-lying tract of land below sea level, called a polder. The polder is buffered from the sea by a dike, and water levels in the polder are controlled by drains and pumps.

The dike will measure 10 km long, up to 15 m wide at its crest and stand about 6 m above sea level.

TRADITIONAL LAND RECLAMATION

Seawall

Infill

Sea

Improved soft clay

Residual soil

How it works

"Infill" such as sand is poured into the reclaimed area above the existing seabed, which is made up of improved soft clay and residual soil.

EMPOLDERING METHOD

Save sand
Save cost

Increased seawall height — Dyke — Pumping station — Reduced infill below sea level

Stormwater collection pond — Drains

Sea

Screen or wall to minimise seawater seepage

How it works

• A dyke is built around the area to be reclaimed and water is drained from it.

• This forms a low-lying tract of land called a polder, which is buffered from the sea by the dyke.

The proposed land to be reclaimed using the empoldering method can be modelled using a trapezoid with uniform cross-sectional area as shown below.

10 km

800 m

4 m

2 m

(a)   Calculate the volume of sand, in cubic metres, needed to reclaim this proposed land.

Answer .... m³ [2]

(b)   This empoldering method reduces the amount of sand used by 40%. What is the estimated volume of sand needed if the traditional method is used?

Answer .... m³ [2]

The following table shows some information about the proposed reclaimed land.

FACTS AND FIGURES

810ha — Total polder land area is about twice the size of Toa Payoh Town.

29km — Total length of drains to be constructed.

21km — Total length of roads to be constructed, about the length of East Coast Parkway.

10km — Total length of dyke to be constructed, about the length of Bukit Timah Expressway.

Source: HOUSING & DEVELOPMENT BOARD
PHOTO: GOOGLE EARTH
STRAITS TIMES GRAPHICS

(c)   Assuming the roads to be constructed on this reclaimed land are straight and of width 12 metres, express the area designated for these roads as a percentage of the polder land area.

$\left(1 \text{ hectare } = 10000 \text{ m}^2\right)$

Answer .... % [2]

(d)   Singapore needs to purchase sand from Country X or Country Y in order to carry out the proposed reclamation work. The following shows the cost details of these two countries.

Country	Cost of Sand (S$ per tonne)	Distance from Singapore by Sea (nautical mile)	Shipment cost (S$ for every 1000 tonne per nautical mile)
Country X	26.37	230	65
Country Y	15.85	591	70

<table>
<tr><td colspan="4" align="center"><u>Useful Information</u>

1 nautical mile = 1.852 km
1 tonne = 1000 kg
Density of sand = 1442 kg/m^3</td></tr>
</table>

Which country should Singapore purchase sand from?
Justify your answer.

....

....

.... [4]

End of Paper

Solutions

南 华 中 学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2020

Subject	:	Mathematics
Paper	:	4048/01
Level	:	Secondary Four Express
Date	:	17 August 2020
Duration	:	2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue, correction fluid or correction tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** questions.

1 **(a)** Calculate $\sqrt{\dfrac{(74.03)^2 - 2340}{5^3 + 10}}$, writing down the first six digits of your answer.

Answer .... $\boxed{4.82312}$ .... [1]

(b) Write your answer to part (a) correct to 3 significant figures.

Answer .... $\boxed{4.82}$ .... [1]

2 Factorise completely $9a^2 + 6ab + b^2 - 25c^2$.

$9a^2 + 6ab + b^2 - 25c^2$
$= (3a + b)^2 - (5c)^2$
$= (3a + b + 5c)(3a + b - 5c)$

Answer $\boxed{(3a+b+5c)(3a+b-5c)}$ [2]

3 Written as a product of its prime factors, $264 = 2^3 \times 3 \times 11$ and $1980 = 2^2 \times 3^2 \times 5 \times 11$.

The highest common factor and the lowest common multiple of 264, 1980 and A are 12 and 59400 respectively.

Find the largest possible value of A.

$\text{HCF} = 2^2 \times 3$
$\text{LCM} = 2^3 \times 3^3 \times 5^2 \times 11$
$C = 2^3 \times 3^3 \times 5^2$
$= 5400$

Answer $A = $ ..... $\boxed{5400}$ .... [2]

4 Two bottles of hand sanitisers are geometrically similar.
The height of the smaller bottle and larger bottle are 10 cm and 19 cm respectively.
The price of the smaller and larger bottle are $4.90 and $15.10 respectively.

Explain which bottle gives the better value for money.

$$\text{Volume of Smaller Bottle} : \text{Volume of Larger Bottle}$$
$$(10)^3 : (19)^3$$
$$1000 : 6859$$

$$\text{Unit price for Smaller Bottle} = \frac{\$4.90}{1000}$$
$$= \$0.0049$$
$$\text{Unit price for Larger Bottle} = \frac{\$15.10}{6859}$$
$$= \$0.0022 \ (4 \ dp)$$

As the unit price for smaller bottle is lower than that of the larger bottle, it will be better value for money to buy the larger bottle.

Answer ...... Larger Bottle ...... [3]

5   (a)   Given that $2 \times 5^x = \dfrac{10}{\sqrt[3]{25}}$, find the value of x.

$$2 \times 5^x = \frac{10}{\sqrt[3]{25}}$$

$$5^x = \frac{5}{\sqrt[3]{25}}$$

$$= 5 \times 5^{-\frac{2}{3}}$$

$$5^x = 5^{\frac{1}{3}}$$

By comparison,

$$x = \frac{1}{3}$$

Answer $x =$ .... $\boxed{\dfrac{1}{3}}$ .... [2]

(b)   The radius of the base of a cylinder was increased by 30% and its height was decreased by 30%.

Find the percentage change, if any, in its volume.

Let the radius of the cylinder be r cm and the height be h cm.

$$\text{Original Volume} = \pi r^2 h$$

$$\text{New Volume} = \pi (1.3r)^2 (0.7h)$$

$$= \frac{1183}{1000} \pi r^2 h$$

$$\text{Percentage change in Volume} = \frac{\dfrac{1183}{1000} \pi r^2 h - \pi r^2 h}{\pi r^2 h} \times 100\%$$

$$= 18.3\%$$

Answer   .... $\boxed{18.3}$ ....% [3]

6 **(a)** Express $-2x^2 - 6x - 2$, in the form $a(x+h)^2 + k$ where a, h and k are constants.

$$-2x^2 - 6x - 2 = -2(x^2 + 3x + 1)$$

$$= -2\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 1\right]$$

$$= -2\left[\left(x + \frac{3}{2}\right)^2 - \frac{5}{4}\right]$$

$$= -2\left(x + \frac{3}{2}\right)^2 + \frac{5}{2}$$

Answer ... $\boxed{-2\left(x + \frac{3}{2}\right)^2 + \frac{5}{2}}$. [2]

(b) Sketch the graph of $y = -2x^2 - 6x - 2$. Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point on the curve.
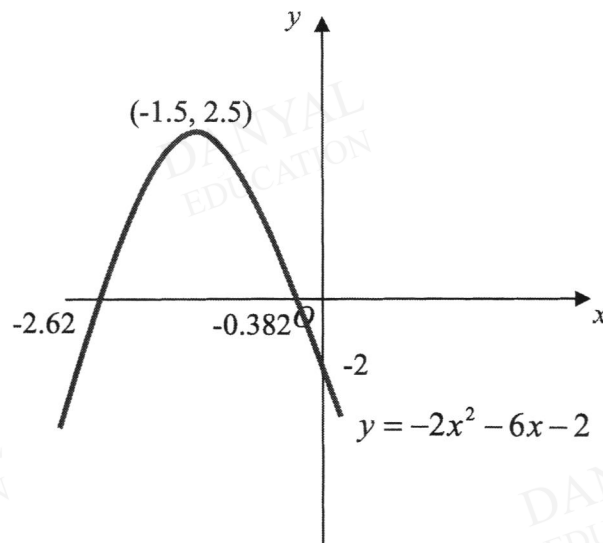
Answer [2]

(c) Write down the equation of the line of symmetry of the curve $y = -2x^2 - 6x - 2$

Answer ..... $\boxed{x = -\dfrac{3}{2} \text{ (o.e)}}$ .... [1]

7   The graph shows the number of unemployed workers in Country X for the last 10 years.

State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer (Suggested Solution 1)
There is no vertical scale and the reader does not know if it starts at zero.
The reader may hence assume that the unemployed workers have halved from 2009 to 2019.

Answer (Suggested Solution 2)
There are missing years in the chart and only 2009, 2014 and 2019 are shown.
The reader may assume there is an overall decrease in unemployment when it may not be true in reality.

Answer (Suggested Solution 3)
The reader may be unsure to compare the height or size of the diagram. From 2009 to 2019, the height in 2009 is double that of 2019 but the area of 2009 is four times of the area in 2019.

8   The number of students in the parade square is given as 1600, correct to the nearest hundred. Write down values for the smallest and largest possible number of students that could be in the parade square.

Answer   Smallest possible number = ... $\boxed{1550}$ ...

Largest possible number = ... $\boxed{1649}$ [1]

9   Alex takes 40 hours to harvest a particular plot of rice field.
Ben will take 55 hours to harvest the same plot of rice field.
Alex and Ben worked together for 20 hours before Ben injured himself and could not continue with the harvest.

If Alex continues to harvest the remaining parts of the field himself at the same rate, calculate how long it will take for him to finish harvesting the field.

Give you answer in hours and minutes, correct to the nearest minute.

Alex will harvest $\dfrac{1}{40}$ of the field in one hour.

Ben will harvest $\dfrac{1}{55}$ of the field in one hour.

$$\text{Fraction of field harvested by both after 20 hours} = \left(\frac{1}{40} + \frac{1}{55}\right) \times 20$$

$$= \frac{19}{22}$$

$$\text{Time taken for Alex to finish the harvest} = \left(1 - \frac{19}{22}\right) \div \frac{1}{40}$$

$$= \frac{60}{11} \text{ hours}$$

$$= 5\frac{5}{11} \text{ hours}$$

$$= 5 \text{ hours } 27 \text{ minutes}$$

Answer   .... $\boxed{5}$ .. hours ... $\boxed{27}$ .. minutes   [3]

10 *ABC* is a triangle with *AB* = 8 cm, *BC* = 15 cm and *AC* = 17 cm.

(a) Explain clearly why triangle *ABC* is a right-angled triangle.
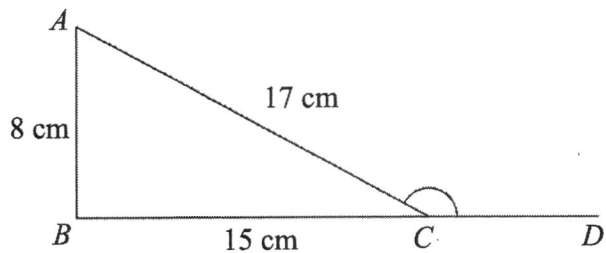
Answer

$$AC^2 = 17^2 = 289$$

$$AB^2 + BC^2 = 8^2 + 15^2 = 289$$

Since $AC^2 = AB^2 + BC^2$,

By Converse of Pythagoras' Theorem,

$$\angle ABC = 90°$$

Hence, *ABC* is a right-angled triangle.

[2]

(b) Without using a calculator, find the value of $\cos \angle ACD$.

$$\cos \angle ACD = -\cos(180° - \angle ACD)$$

$$= -\cos(\angle ACB)$$

$$= -\frac{15}{17}$$

Answer $\boxed{-\dfrac{15}{17}}$ [1]

11   The diagram shows two congruent right-angled triangles *ACE* and *BDA*.
The sides *AB* and *CE* are vertical.
The side *AD* is horizontal and point *E* lies on it.
Point *C* has coordinates (7, 7) and *D* has coordinates (8, 2).
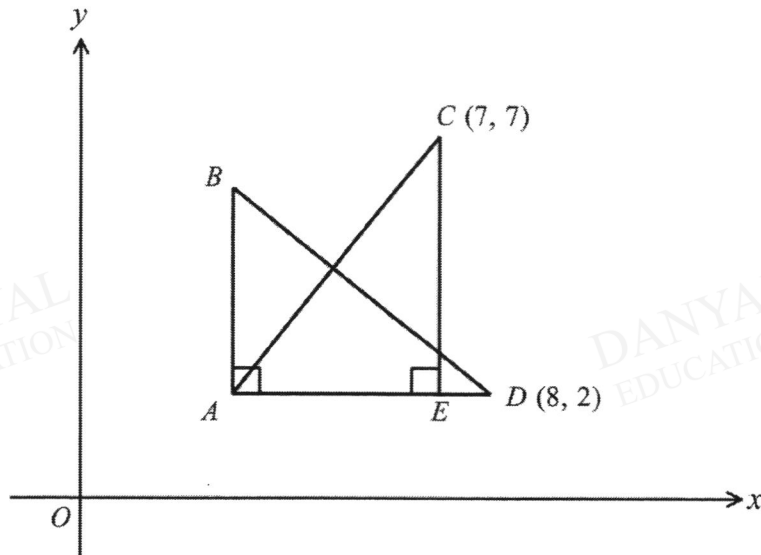
Find the equation of line *AC*.

y-coordinate of A = 2

$$\text{Length of CE} = 7 - 2$$
$$= 5$$

$$\text{x-coordinate of A} = 8 - 5$$
$$= 3$$

$$\text{Gradient of AC} = \frac{7-2}{7-3}$$
$$= \frac{5}{4}$$

Equation of AC:

$$y - 7 = \frac{5}{4}(x - 7)$$
$$y = \frac{5}{4}x - \frac{7}{4}$$

Answer ..... $\boxed{y = \frac{5}{4}x - \frac{7}{4} \text{ (O.E)}}$ [3]

12 Solve the inequality $-\dfrac{3x-4}{3} < \dfrac{7x-2}{6} \leq \dfrac{3x+4}{5}$

$$-\frac{3x-4}{3} < \frac{7x-2}{6}$$
$$-18x+24 < 21x-6$$
$$30 < 39x$$
$$\frac{10}{13} < x$$

and

$$\frac{7x-2}{6} \leq \frac{3x+4}{5}$$
$$35x-10 \leq 18x+24$$
$$17x \leq 34$$
$$x \leq 2$$

$$\frac{10}{13} < x \leq 2$$

Answer ...... $\boxed{\dfrac{10}{13} < x \leq 2}$ .... [3]

13 A map of Thailand has a scale of 1 : 500 000.

(a) The length of the river Kwai on the map is 76 cm.

Calculate the actual length, in kilometres, of the river Kwai.

$$\text{Actual length of River Kwai} = 500000 \times 76 \text{ cm}$$
$$= 38000000 \text{ cm}$$
$$= 380 \text{ km}$$

Answer ..... $\boxed{380}$ ...... km   [2]

(b) The area of Thailand is 513 120 km².

Calculate the area, in square centimetres, of Thailand on the map.
Leave your answer in standard form, correct to 4 significant figures.

$$1 \text{ cm} : 500000 \text{ cm}$$
$$1 \text{ cm} : 5 \text{ km}$$
$$1 \text{ cm}^2 : 25 \text{ km}^2$$
$$\text{Area of Thailand on map} = \frac{513120}{25}$$
$$= 20524.8$$
$$= 2.052 \times 10^4 \text{ cm}^2$$

Answer .. $\boxed{2.052 \times 10^4}$ cm²   [2]

14 A bag contains 30 pieces of tokens of which x are gold tokens and the remaining are silver tokens.

Two tokens were drawn from the bag one after the other without replacement.

(a) Show that the probability that the second token drawn is a gold token is $\dfrac{x}{30}$.

Answer

$$\begin{aligned}
\text{P(Second token is gold)} &= \left(\frac{x}{30}\right)\left(\frac{x-1}{29}\right) + \left(\frac{30-x}{30}\right)\left(\frac{x}{29}\right) \\
&= \frac{x(x-1)+x(30-x)}{30(29)} \\
&= \frac{29x}{30(29)} \\
&= \frac{x}{30}
\end{aligned}$$

[2]

(b) The probability of drawing one gold and one silver token is $\dfrac{25}{87}$.

Find the values of x.

$$\begin{aligned}
\left(\frac{x}{30}\right)\left(\frac{x-1}{29}\right) + \left(\frac{30-x}{30}\right)\left(\frac{x}{29}\right) &= \frac{25}{87} \\
60x - 2x^2 &= 250 \\
2x^2 - 60x + 250 &= 0 \\
x^2 - 30x + 125 &= 0 \\
(x-5)(x-25) &= 0 \\
x = 5 \text{ or } x &= 25
\end{aligned}$$

Answer $x = \boxed{5}$ or $x = \boxed{25}$ [3]

15 ξ = {all positive integers}
  A = {all prime numbers}
  B = {all even integers}
  C = {perfect squares}

(a)   On the Venn Diagram, shade the region which represents $A \cup B'$.
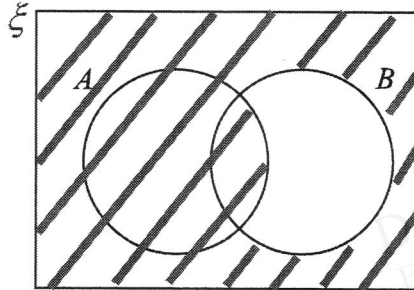
Answer   [1]

(b)   Explain why $A \cap B \neq \varnothing$.

Answer   2 is both a prime number and an even number.

                                                                                          [1]

(c)   Explain why $A \cap C = \varnothing$.

Answer   1 is a perfect square but not a prime number.

All other perfect squares have more than two factors and are hence not prime numbers.

                                                                                          [2]

16 Indicating the intercepts, if any, sketch the graphs of
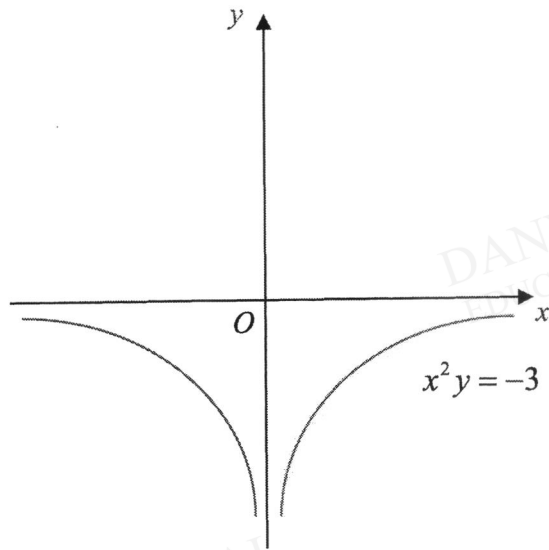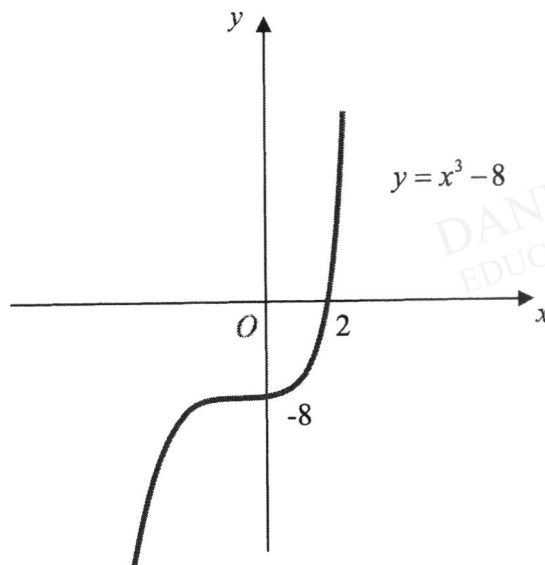
(a) $x^2 y = -3$

Answer   [1]

(b) $y = x^3 - 8$

Answer   [1]

17 Group A consists of 12 boys who were given milk from Brand A to drink daily.

The stem-and-leaf diagram shows the height gain of the boys after 6 months.

```
2 | 9
3 | 3  5  6
4 | 1  5  6  9
5 | 4  7  7
6 | 2
```

Key: 2 | 9   represents 29 mm

(a) State the median height gain of the boys.

Answer .... 45.5 .... mm   [1]

(b) Find the interquartile range of the height gain of the boys.

$55.5 - 35.5 = 20$

Answer .... 20 .... mm   [2]

(c) Calculate

(i) the mean height gain of the boys,

Answer .... 45.3 ... mm   [1]

(ii) the standard deviation of the height gain of the boys.

Answer .... 10.3 ... mm   [1]

(iii) Group B consists of boys who were given milk from Brand B to drink daily. Their height gains were measured after 6 months and recorded.

The mean height gain was 41 mm.
The standard deviation of the height gain was 17.4 mm.

Compare and comment on the height gain between the boys from Group A and B.

Answer ....

In general, boys from group A had a greater height gain than from group B as the mean height gain of group A was larger than group B.

The height gain of boys from group B had a wider/larger spread than that of group A as group B had a larger standard deviation of height gain than group A.

[2]

18 (a)   Dr Ng invested $8000 in a savings account with Bank A.
The savings account pays $R\%$ interest per annum, compounded half yearly.
At the end of 2 years, the interest earned was $96.43.

Calculate the value of R.

$$8000\left(1+\frac{R/2}{100}\right)^4 = 8000 + 96.43$$

$$\left(1+\frac{R/2}{100}\right)^4 = \frac{8096.43}{8000}$$

$$1+\frac{R/2}{100} = \sqrt[4]{\frac{8096.43}{8000}}$$

$$R = 0.600 \text{ (3 sf)}$$

Answer $R =$ .. | 0.600 or 0.6 | [3]

(b)   Bank B has a savings account that offers 0.65% simple interest per annum.

Given that any money invested will be kept in the bank for 2 years, find out if Dr Ng should invest in the savings account from Bank A or Bank B.

Interest from bank B $= 2 \times \dfrac{0.65}{100} \times 8000$

$= \$104$

Answer   ...... | Bank B | .... [2]

19 Container A is in the shape of a cone, where both the base radius and height are r cm.
Container B is in the shape of a hemisphere with radius r cm.

Container *A*   Container *B*
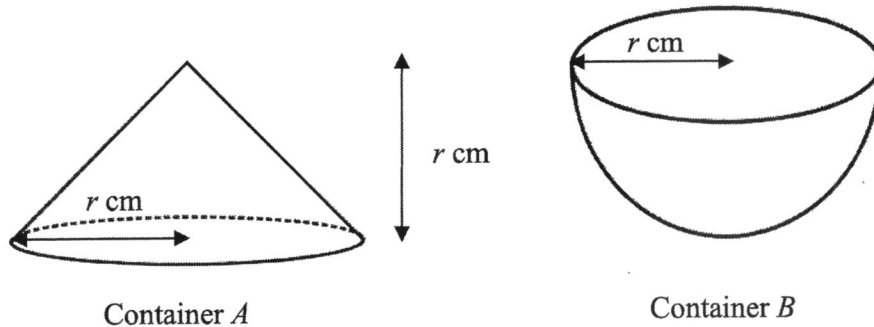
Water is leaking from the fully filled container A at a constant rate k cm^3/s .
Water is poured into the empty container B at the same constant rate k cm^3/s .
It takes 14 seconds to empty container A.

Sketch and label the graphs showing how the depth of water in each container varies with time.

[3]
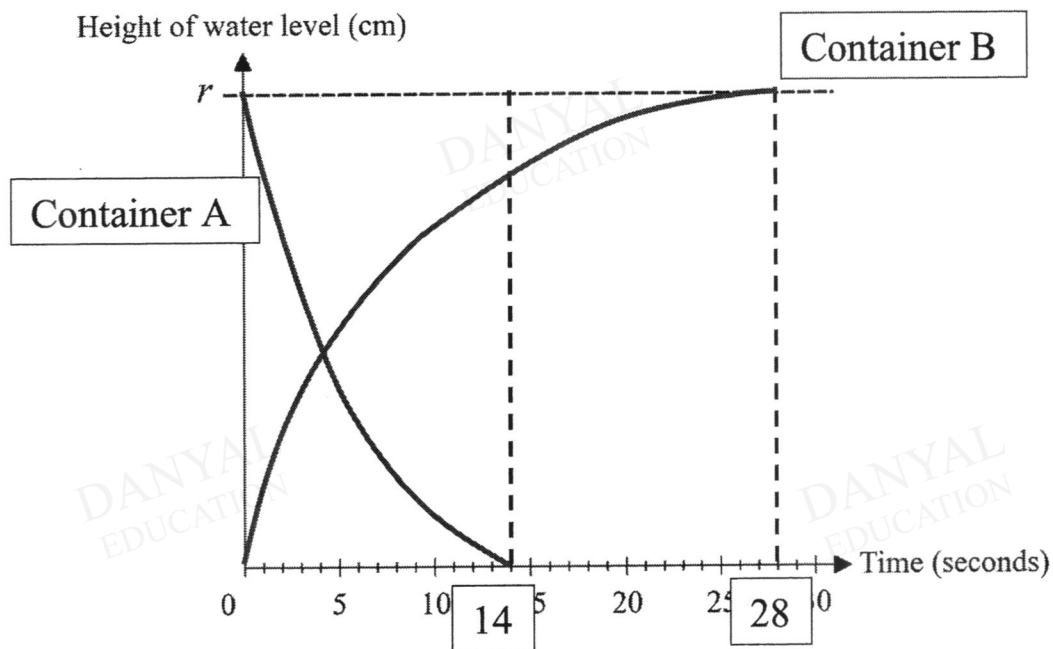
20 (a) The diagram shows an irregular polygon.

Find the value of x.

Sum of interior angles $= (7-2) \times 180°$

$$= 900°$$
$$36x = 900$$
$$x = 25$$
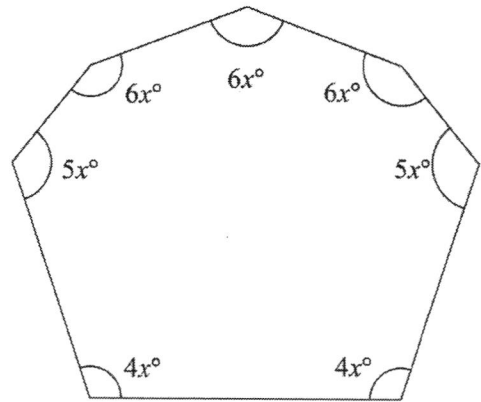
Answer $x =$ .... $\boxed{25}$ .... [2]

(b) The diagram shows a regular pentagon $ABCDE$ with centre O.

Given that the area of the pentagon is 20 m^2, find the length of AB.

Let the length of one side be x cm.

Let $OB = y$ m

$$\angle AOB = 360° \div 5$$
$$= 72°$$
Area of pentagon $= \left[\dfrac{1}{2}(y)(y)\sin 72°\right] \times 5$
$$= 20$$
$$y = 2.9003 \ (5 \text{ sf})$$
$$\angle BAO = \dfrac{(5-2) \times 180°}{5} \times \dfrac{1}{2}$$
$$= 54°$$
$$\dfrac{2.9003}{\sin 54°} = \dfrac{x}{\sin 72°}$$
$$x = 3.41$$
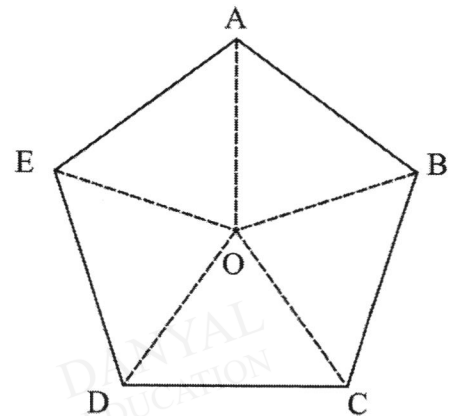
Alternative Solution:

$$\cos 54° = \dfrac{AB\big/2}{2.9003}$$
$$AB = 3.41$$

Answer .... $\boxed{3.41}$ .... m [3]

21 The diagram below, $BA = BC$ and angle BFC = angle BDA.
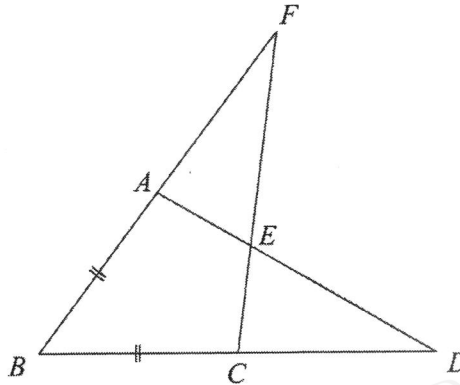
(a) Explain why triangle BCF and triangle BAD are congruent.

Answer ....

$\angle BFC = \angle BDA$ (given)

$\angle FBC = \angle DBA$ (common angle)

$BC = BA$ (given)

$\triangle BCF$ is congruent to $\triangle BAD$ (AAS test)

....

....

.... [2]

(b) Given further that $AE = CE$, state the name of quadrilateral $ABCE$.

Answer .... Kite .... [1]

22   The diagram shows the position of a playground P.

A basketball court B is 350 m away from P.

The bearing of B from P is $110°$.

The scale is 1 cm : 50 m.

North

↑

P •

(a)   Find and label the position of basketball court B.   [1]

(b)   A community centre C is 450 m away from P and 300 m from B.

Given that the bearing of C from P is between $000°$ to $090°$ find the position of
C.   [1]

(c)   Construct
       (i)   the perpendicular bisector of PB,   [1]

       (ii)   The angle bisector of angle BCP.   [1]

(d)   A garden is to be built, nearer to B than to P and nearer to PC than to BC.

Mark clearly a possible position within the region PBC where the garden can be
built and label this point G.   [1]

23 Consider the following sequence:

$$\frac{3}{4}, \frac{15}{8}, \frac{35}{12}, \frac{63}{16}, \ldots$$

(a) Write down the next term in the sequence.

Answer .... $\boxed{\dfrac{99}{20}}$ .... [1]

(b) Write down the general form for the nth term of the sequence.

$$\frac{(2n-1)(2n+1)}{4n} = \frac{4n^2-1}{4n}$$

Answer .... $\boxed{\dfrac{4n^2-1}{4n}}$ .... [1]

(c) Explain why **none** of the terms are integers.

Answer ....
$$\frac{4n^2-1}{4n} = n - \frac{1}{4n}$$

Since n is a positive integer, $\dfrac{1}{4n}$ will not be a positive integer.

Hence, $n - \dfrac{1}{4n}$ will not be an integer.

.... [1]

----- END OF PAPER-----

Answers

1(a)	$-\dfrac{8}{7x+3}$	(b)	$\dfrac{pr^{15}}{81q^{12}}$	(c)	$n=\dfrac{4m^2+7k^2}{3k^2}$
2(a)	\$209.16	(b)	\$184.47	(c)	11.8%
3(a)	$\dfrac{36}{x}$	(b)	$\dfrac{36}{x-2}$	(d)	8.66 , -6.66
(e)	325 mins				
4(a)	1420	(b)	758		
5(a)(i)	40°	(ii)	120°	(b)	AA Similarity
(c)(i)	2.57	(ii)	0.778		
6(a)(i)	68	(ii)	77	(iii)	21
(b)	83	(c)	$\dfrac{119}{2995}$		
(d)(i)	80	(ii)	28		
7(a)	50	(b)	45	(c)	95
8(a)	8.3	(c)	0.8 , 2.35 ($\pm$ 0.05)	(d)	5 ($\pm$ 0.1)
(e)(ii)	0.65 , 2.50	(f)	1.15 ($\pm$ 0.05)		
9(a)	070.9° or 071.0°	(b)	144	(c)	3490
(d)	60.2°				

10(a)	$\begin{pmatrix} 14 \\ 6 \\ 4 \end{pmatrix}$ Represent the total no. of transactions for each type of property.	(b)	$R=\begin{pmatrix} 3000 & 7000 & 11000 \\ 800 & 1800 & 2500 \end{pmatrix}$	$S=\begin{pmatrix} 128000 \\ 32000 \end{pmatrix}$ Represent total amount of commission earned and total govt tax paid
(c)	$T=(1 \;\; -1)$		$TS=(96000)$	\$96000

10(a)	24 000 000	(b)	40 000 000
(c)	3.11%	(d)	Should purchase sand from Country X

南 华 中 学

NAN HUA HIGH SCHOOL
PRELIMINARY EXAMINATION 2020

Subject	:	**Mathematics**
Paper	:	**4048/02**
Level	:	**Secondary Four Express**
Date	:	**19 August 2020**
Duration	:	**2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

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If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper consists of 29 printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions

1. (a) Express $\dfrac{4}{2-3x} + \dfrac{4(x+7)}{21x^2 - 5x - 6}$ as a single fraction in its simplest form.

$$\dfrac{4}{2-3x} + \dfrac{4(x+7)}{(3x-2)(7x+3)}$$

$$= \dfrac{4(7x+3) - 4(x+7)}{(2-3x)(7x+3)}$$

$$= \dfrac{28x + 12 - 4x - 28}{(2-3x)(7x+3)}$$

$$= \dfrac{24x - 16}{(2-3x)(7x+3)} \quad \text{or} \quad \dfrac{16 - 24x}{(3x-2)(7x+3)}$$

$$= -\dfrac{8}{(7x+3)}$$

Answer .... [3]

(b) Simplify $\left(\dfrac{81p^4}{q^8}\right)^{\frac{1}{4}} \div \left(\dfrac{r^3}{3q^2}\right)^{-5}$, leaving your answer in positive index.

$$\dfrac{3p}{q^2} \times \left(\dfrac{r^3}{3q^2}\right)^5$$

$$= 3pq^{-2} \times r^{15} \times \dfrac{q^{-10}}{3^5}$$

$$= \dfrac{1}{81} pq^{-12} r^{15}$$

$$= \dfrac{pr^{15}}{81q^{12}}$$

Answer .... [3]

(c)   Given that $m = k\sqrt{\dfrac{3n-7}{4}}$, express n in terms of k and m.

$$\frac{m}{k} = \sqrt{\frac{3n-7}{4}}$$

$$\frac{m^2}{k^2} = \frac{3n-7}{4}$$

$$\frac{4m^2}{k^2} + 7 = 3n$$

$$n = \frac{4m^2}{3k^2} + \frac{7}{3} \quad \text{or} \quad n = \frac{4m^2 + 7k^2}{3k^2}$$

Answer .... [2]

2. Cindy saw a pair of shoes from a Japanese web-site which costs 11080 yen.
    The price excludes the Japanese customs charge of 18% tax for all overseas mail orders
    and freight charge of 3000 yen for product delivered to Singapore.

    (a) The exchange rate between Japanese Yen and Singapore dollars is
         S$1.3012 = 100 Japanese Yen.
         Calculate the total cost in Singapore dollars, if Cindy decides to buy the pair
         of shoes by mail order.

    $$\text{Total cost in yen} = 11080 \times 1.18 + 3000$$
    $$= 16074.4$$
    $$\text{Total cost in S\$} = \frac{16074.4}{100} \times 1.3012$$
    $$= 209.160$$
    $$= 209.16$$

    Answer $.................................. [2]

    (b) The same pair of shoes, normally priced locally at S$215.50, exclusive of GST,
         was sold at a discount of 20% during the Black Friday Sale.
         Calculate the selling price of the shoes, taking GST to be 7%, during the sale.

    $$\text{Selling price} = 215.50 \times 0.8 \times 1.07$$
    $$= S\$184.468$$
    $$= S\$184.47$$

    Answer $.................................. [2]

    (c) If Cindy purchased the pair of shoes during the Black Friday Sale, express her
         savings as a percentage of the online price.

    $$\text{Percentage saved} = \frac{209.160 - 184.468}{209.160} \times 100\%$$
    $$= 11.805$$
    $$= 11.8\% \ (3 \text{ s.f.})$$

    Answer ....% [2]

3.   A bird flew 36 km at a constant speed of x km/h from A to B (outward journey) when there was no wind. When the bird flew back from B to A (return journey) against the wind direction, its speed was decreased by 2 km/h.

(a)   Write down, in terms of x, the time taken for the bird to fly from A to B.

Time taken $= \dfrac{36}{x}$ h

Answer ....h [1]

(b)   Write down, in terms of x, the time taken for the bird to fly back from B to A.

Time taken $= \dfrac{36}{x-2}$ h

Answer .... h [1]

(c)   Given that the return journey took the bird 75 minutes longer than the outward journey, form an equation in x and show that it simplifies to
$5x^2 - 10x - 288 = 0$ [2]

$$\frac{36}{x-2} - \frac{36}{x} = \frac{75}{60}$$

$$\frac{36x - 36(x-2)}{x(x-2)} = \frac{5}{4}$$

$$\frac{72}{x(x-2)} = \frac{5}{4}$$

$$5x^2 - 10x = 288$$

$$5x^2 - 10x - 288 = 0$$

(d) Solve the equation $5x^2 - 10x - 288 = 0$, giving your answers to 2 decimal places.

$$5x^2 - 10x - 288 = 0$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(5)(-288)}}{2(5)}$$

$$= \frac{10 \pm \sqrt{5860}}{10}$$

$$= 8.655 \text{ or } -6.655$$

$$= 8.66 \text{ or } -6.66 \text{ (2 dp)}$$

Answer ……………………………………….. [3]

(e) Find the time taken by the bird on its return journey, giving your answer to the nearest minute.

Since speed > 0, $x = 8.655$

Time taken $= \dfrac{36}{8.655 - 2}$

$= 5.409 \text{ h}$

$= 324.56 \text{ mins}$

$= 325 \text{ mins (nearest min)}$

Answer ……………………………… [2]
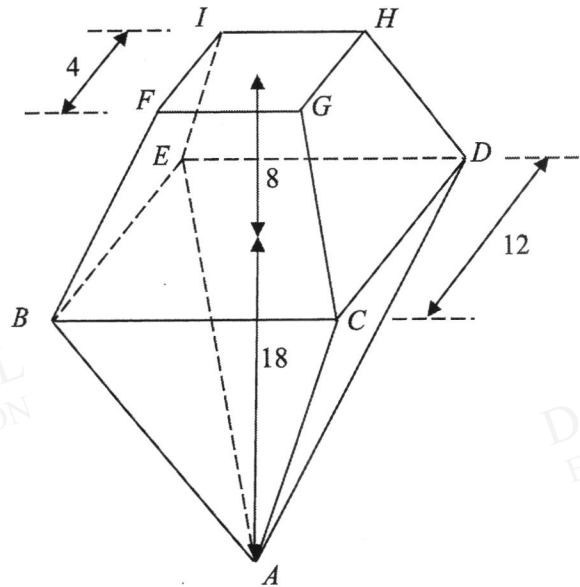
4.

A sculpture consists of an inverted right square pyramid *ABCDE* and a frustum *BCDEIFGH*. The frustum is created by cutting away the top portion of another right square pyramid of height 12 cm to expose a flat surface *FGHI* which is parallel to the base of the pyramid *BCDE*.

BCDE and *FGHI* are squares of side 12 cm and 4 cm respectively.
The vertical height of the inverted pyramid is 18 cm and the vertical height of the frustum is 8 cm. Calculate

(a)   the volume of the sculpture,

Volume of inverted pyramid $= \dfrac{1}{3}(12)^2(18) = 864 \text{ cm}^3$

Volume of frustum $= \dfrac{1}{3} \times 12^2 \times 12 - \dfrac{1}{3} \times 4^2 \times 4$

$= 576 - \dfrac{64}{3}$

$= 554\dfrac{2}{3} \text{ cm}^2$

Volume of sculpture $= 864 + 554\dfrac{2}{3} = 1418\dfrac{2}{3} \text{ cm}^3$

Answer .... cm³ [4]

(b)   the surface area of the sculpture.

Let the mid-point of *BC* be *X*.

For triangle *ABC*,

$$AX^2 = 6^2 + 18^2 = 360$$

$$AX = \sqrt{360}$$

Surface area of inverted pyramid $= 4 \times \dfrac{1}{2}(12)\sqrt{360}$

$$= 24\sqrt{360}$$

$$= 455.36798 \text{ cm}^2$$

Let the height of trapezium *BCGF* be *h* cm.

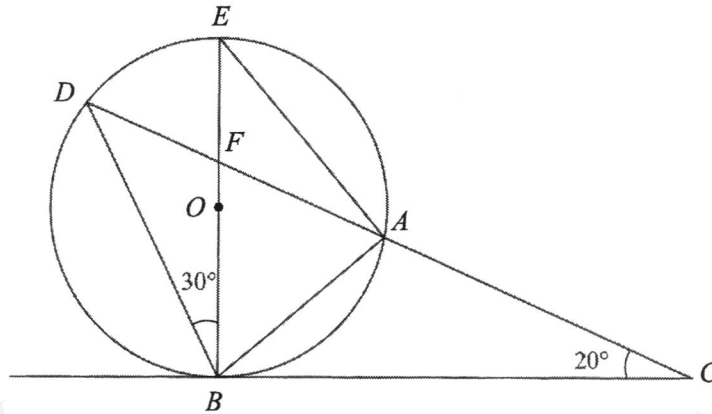$$h = \sqrt{8^2 + 4^2}$$

$$= 8.94427$$

Surface area of frustum $= 4 \times \dfrac{1}{2}(4+12) \times 8.94427 + 4^2$

$$= 302.21664 \text{ cm}^2$$

Total surface area $= 455.36798 + 302.21664$

$$= 757.5846$$

$$= 758 \text{ cm}^2 \quad (3 \text{ s.f.})$$

Answer .... cm² [4]

5.

The points A, B, D and E lie on the circumference of a circle, with centre O. EB is a diameter of the circle and BC is tangent to the circle. Angle $EBD = 30°$ and angle $ACB = 20°$. F is the point of intersection of BE and AD.

(a)   Find
    (i) angle ADB,

$\angle CBE = 90°$ (tangent $\perp$ radius)
$\angle ADB = 180° - 20° - 90° - 30°$ (angle sum of triangle)
        $= 40°$

Answer .... ° [2]

    (ii) angle BAC.

$\angle EAD = 30°$ (angle in same segment)
Since $\angle EAB = 90°$ (right-angle in semi-circle)
$\therefore\ \angle DAB = 90° - 30°$
        $= 60°$
$\therefore\ \angle BAC = 180° - 60°$ (adjacent angles in a straight line)
        $=120°$

Answer .... ° [2]

(b)   Show that triangles *DFB* and *EFA* are similar.   [2]

$\angle DFB = \angle EFA$ (vertically opposite angles)

$\angle DBF = \angle EAF = 30°$ (angles in same segment)

$\therefore$ $\triangle DFB$ is similar to $\triangle EFA$ (AA Similarity)

If $EF = 2$ cm,

(c)   (i)   calculate the length of *AF*,

$$\frac{AF}{\sin 40°} = \frac{2}{\sin 30°}$$

$$AF = \frac{2 \times \sin 40°}{\sin 30°}$$

$$= 2.57115$$

$$= 2.57 \text{ cm} (3 \text{ s.f.})$$

Answer .... cm [2]

(ii)   calculate the value of $\dfrac{DF}{BF}$.

$$\frac{DF}{BF} = \frac{EF}{AF}$$

$$= \frac{2}{\dfrac{2\sin 40°}{\sin 30°}}$$

$$= \frac{\sin 30°}{\sin 40°}$$

$$= 0.778 (3 \text{ s.f.})$$

Answer .... [2]

6.   The speeds of 600 cars passing a checkpoint X were recorded. The cumulative frequency curve below shows the distribution of the speeds.
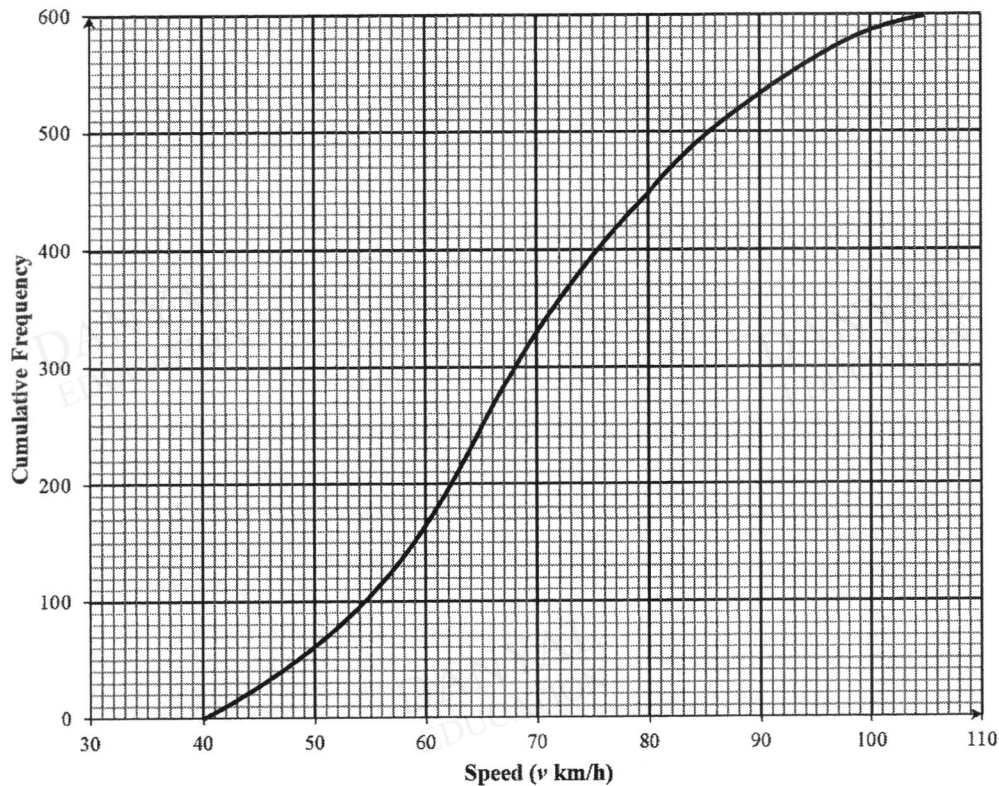
(a)   Use the curve to estimate
       (i)   the median speed,

              68 km/h

                                              Answer .... km/h [1]

       (ii)   the 70th percentile speed,

              77 km/h

                                              Answer .... km/h [1]

(iii)   the interquartile range.

$$\text{Interquartile range} = 80 - 59$$

$$= 21 \text{ km/h}$$

Answer .... km/h [2]

(b)   Given that 20% of the cars exceeded the speed limit at point X, estimate the speed limit at this point.

No. of cars which do not exceed the speed limit $= 0.80 \times 600 = 480$

∴ Speed limit $= 83 \text{ km/h}$

Answer .... km/h [1]

(c)   Two cars are selected at random. Find the probability that the speeds of these two cars are between 70 km/h and 80 km/h.

No. of cars with speed between 70 km/h and 80 km/h $= 450 - 330$

$$= 120$$

Probability $= \dfrac{120}{600} \times \dfrac{119}{599} = \dfrac{119}{2995}$

Answer .... [2]

(d)   The box-and-whiskers plot below shows the distribution of speeds (km/h) of the same 600 cars that passed through another checkpoint *Y*, further down the same road.
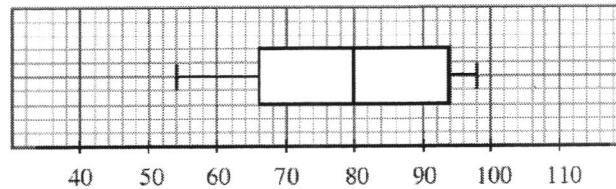
Find

(i)   the median speed,

80 km/h

Answer ………………….. km/h [1]

(ii)   the interquartile range.

$94 - 66 = 28$ km/h

Answer ………………….. km/h [1]

(e)   Make **two** comments comparing the speeds of the cars at points *X* and *Y*.

The median speed of the cars at point *Y* is higher than that at point *X*
……………………………………………………………………………………………
which means the speed of the cars at point *Y* are generally faster.
……………………………………………………………………………………………
The interquartile range of speeds at point *Y* is also higher than that at
……………………………………………………………………………………………
point *X* indicating that there is a wider spread of speed of cars at
……………………………………………………………………………………………
point *Y*
……………………………………………………………………………………………
……………………………………………………………………………………… [2]

7.   The diagram below shows the speed-time graph of a car.
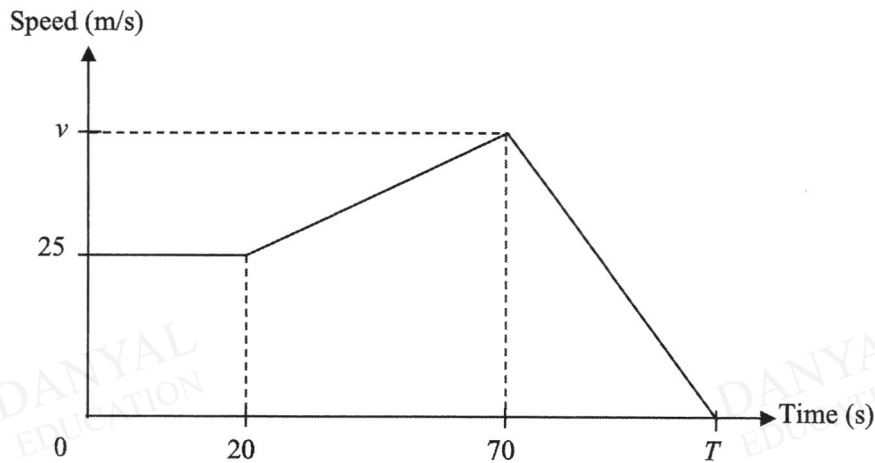
(a)   For how long is the car travelling at an increasing speed?

$$70 - 20 = 50$$

Answer .... s [1]

(b)   Find the value of v, if the total distance covered in the first 70 seconds is 2250 m.

Total distance travelled $= 2250$ m

$$[25 \times 20] + \left[\frac{1}{2} \times (v + 25) \times 50 \right] = 2250$$

$$v = 45$$

Answer .... [2]

(c)   Find the value of T if the uniform retardation of the car is 1.8 m/s^2.

Acceleration $= -1.8$ m/s^2

$$\frac{0-45}{T-70} = -1.8$$

$$T = \frac{-45}{-1.8} + 70$$
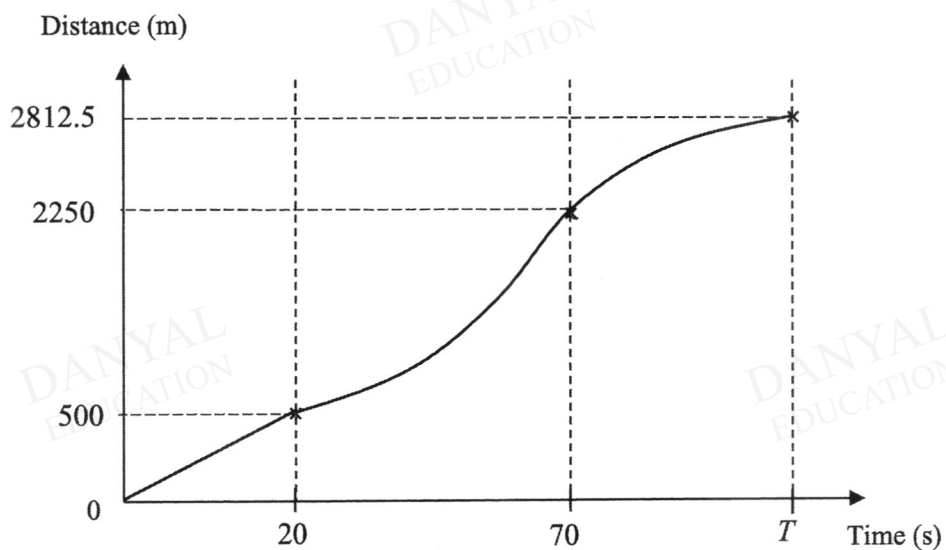
$$= 95$$

Answer .... [2]

(d)   Use the diagram provided below to draw the distance-time graph of the car for the whole journey.

Answer   [3]

8.   The variables x and y are connected by the equation

$$y = 2x^2 + \frac{12}{x} - 9$$

Some corresponding values of x and y are given in the following table.

x	0.5	0.75	1	1.25	2	2.5	3
y	15.5	8.1	5	3.7	5	p	13

(a)   Find the value of p.

$$p = 8.3$$

Answer $p = $ .... [1]

(b)   Using a scale of 4 cm to represent 1 unit on the x-axis and 1 cm to represent

1 unit on the y-axis, draw the graph of $y = 2x^2 + \frac{12}{x} - 9$ for $0.5 \le x \le 3$ [3]

(c)   Use your graph to find the solutions of $\frac{1}{6}x^2 = -\frac{1}{x} + \frac{4}{3}$

$$2x^2 = -\frac{12}{x} + 16$$

$$2x^2 + \frac{12}{x} = 16$$

$$2x^2 + \frac{12}{x} - 9 = 16 - 9$$

$$y = 7$$

$$x = 0.8 \, , \, 2.35 \, (\pm 0.05)$$

Answer .... [2]

(d)   By drawing a tangent, find the gradient of the curve at the point (2, 5).

$$\text{gradient} = \frac{10 - 2}{3 - 1.4}$$

$$= 5 \, (\pm 0.1)$$

Answer .... [2]

(e)   (i)   On the same axes, draw the line $y = 12 - \dfrac{3}{2}x$ for $0.5 \le x \le 3$.   [1]

When $x = 0$, $y = 12$

When $x = 2$, $y = 9$

(ii)   Write down the x-coordinates of the point where this line intersects the curve.
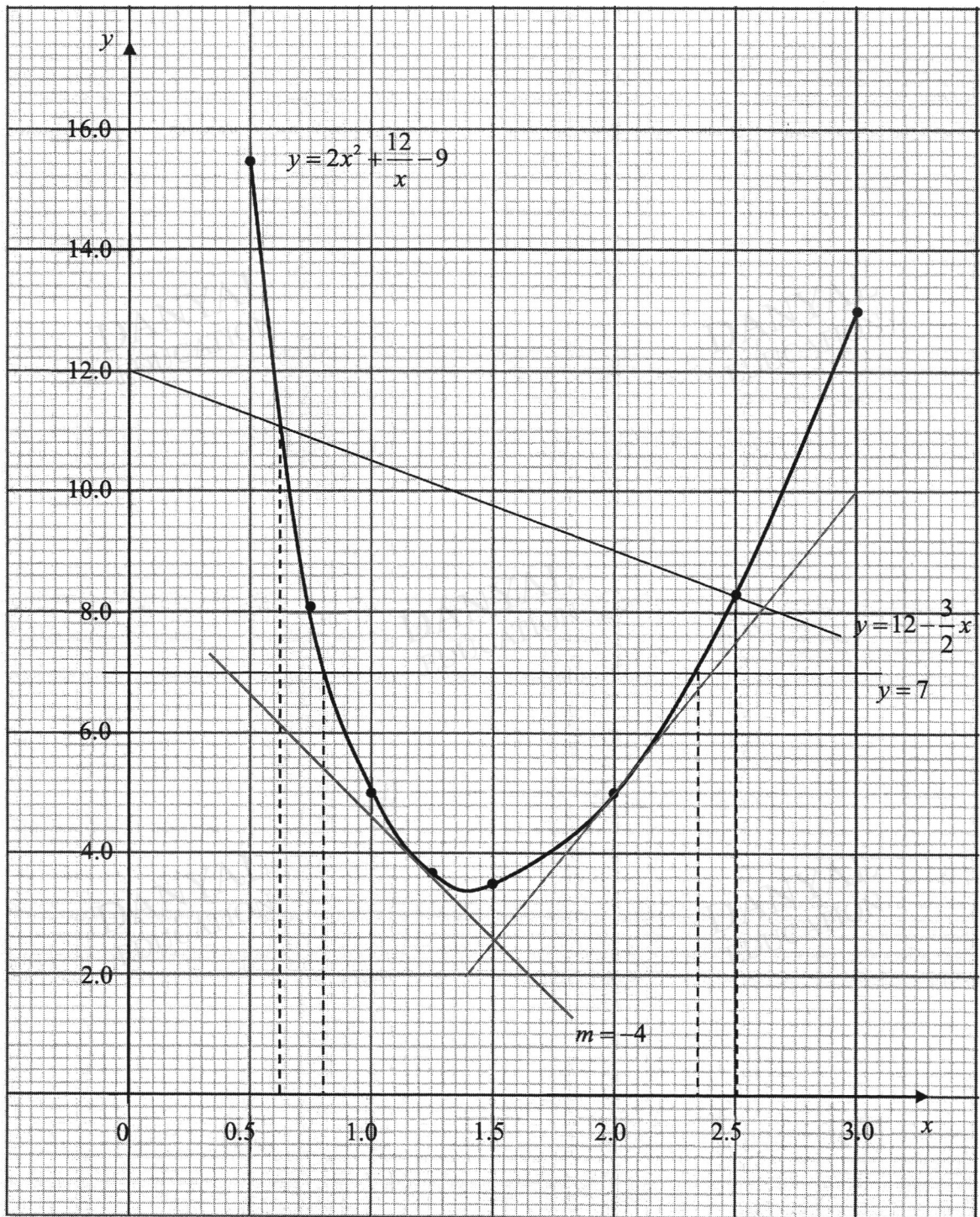
$x = 0.65$ (accept 0.60 - 0.70)
$x = 2.50$ (accept 2.45 - 2.55)

Answer .... [1]

(f)   Use your graph to determine the x–coordinate of the point where the gradient of the curve is –4.

$x = 1.15$ (± 0.05)

Answer .... [2]
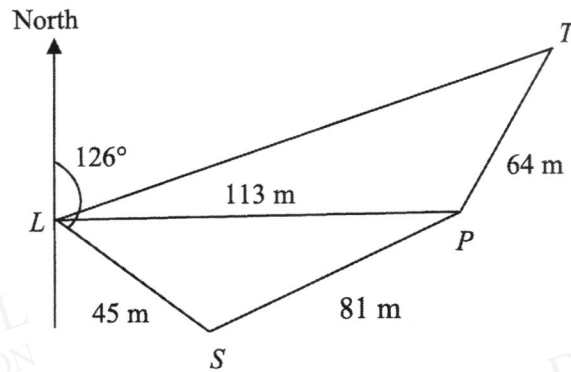
$$y = 2x^2 + \frac{12}{x} - 9$$

$$y = 12 - \frac{3}{2}x$$

$$y = 7$$

$$m = -4$$

9.

A pirate ship, *P*, is 113 m due east of the lighthouse *L*. It is detected by a coast guard *S* which is 45 m away and at a bearing of 126° from *L*. The distance between the pirate ship and the coast guard is 81 m.
Find

(a)   the bearing of the pirate ship from the coast guard.

$$\frac{\sin \angle PSL}{113} = \frac{\sin 36°}{81}$$

$$\sin \angle PSL = \frac{113 \sin 36°}{81}$$

$\angle PSL = 124.92°$ (since $\angle PSL$ is obtuse)

∴ Bearing of ship from coast guard
   $= 124.92° - (180° - 126°)$
   $= 070.9°$ (1 dp)

Alternatively

$$\cos \angle PSL = \frac{45^2 + 81^2 - 113^2}{2(45)(81)}$$

$$= -\frac{4183}{7290}$$

$\angle PSL = 125.02°$

∴ Bearing of ship from coast guard
   $= 125.02° - (180° - 126°)$
   $= 071.0°$ (1 dp)

Answer .... ° [3]

An island T is located at a bearing of 015° from P and the distance PT is 64 m.
Find

(b)   the length of LT,

$$LT = \sqrt{64^2 + 113^2 - 2(64)(113)\cos 105°}$$
$$= \sqrt{20608.55867}$$
$$= 143.5568$$
$$= 144 \text{ m } (3 \text{ s.f.})$$

Answer .... m [2]

(c)   the area of $\triangle PLT$.

$$\text{Area of } \triangle PLT = \frac{1}{2}(64)(113)\sin 105°$$
$$= 3492.7878$$
$$= 3490 \text{ m}^2 \ (3 \text{ s.f.})$$

Answer .... m² [2]

(d)   A helicopter is flying at a constant height of 85 m along a straight path above *LT*.
Find the greatest angle of depression of the pirate ship from the helicopter.

Let the shortest distance of *P* to *TL* be *x* m.

$$3492.7878 = \frac{1}{2} \times 143.5568 \times x$$

$$x = 48.6607$$

Let the angle of depression be θ.

$$\tan\theta = \frac{85}{48.6607}$$

$$\theta = 60.21°$$

Greatest angle of depression is 60.2°

Answer .... ° [3]

10.   The following table shows the number of property transactions of an agent from

January to March 2019.

Types of Property	Sale Transaction	Rental Transaction
HDB Flats	6	8
Private Condominiums	4	2
Landed House	3	1

The information above is represented by the matrix $\mathbf{P} = \begin{pmatrix} 6 & 8 \\ 4 & 2 \\ 3 & 1 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(a)   Evaluate $\mathbf{W} = \mathbf{PQ}$ and explain the significance of the elements in $\mathbf{W}$.

$$\mathbf{W} = \begin{pmatrix} 6 & 8 \\ 4 & 2 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 14 \\ 6 \\ 4 \end{pmatrix}$$

Answer $\mathbf{W} =$

.... [1]

The elements in $\mathbf{W}$ represent the total number of transactions for each

type of property.

.... [1]

The commission received and the government tax charged under each type of transaction is shown in the table below.

	HDB Flats	Private Condominiums	Landed House
Commission	$3000	$7000	$11000
Government Tax	$800	$1800	$2500

(b)   Represent the above information with matrix **R** and evaluate **S** = **RW**.
Explain the significance of the elements in **S**.

$$\mathbf{R} = \begin{pmatrix} 3000 & 7000 & 11000 \\ 800 & 1800 & 2500 \end{pmatrix}$$

$$\mathbf{S} = \begin{pmatrix} 3000 & 7000 & 11000 \\ 800 & 1800 & 2500 \end{pmatrix} \begin{pmatrix} 14 \\ 6 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 128000 \\ 32000 \end{pmatrix}$$

Answer **S** =

.... [2]

The elements in **S** represent the total amount of commission earned and
....

the total government tax paid for each type of transaction.
....[1]

(c)   Write down another matrix **T** such that their product **TS** gives the amount of money earned by the agent after deducting the government tax from January to March 2019.

Hence, find the amount of money earned by the agent.

$$\mathbf{T} = \begin{pmatrix} 1 & -1 \end{pmatrix}$$

$$\mathbf{TS} = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 128000 \\ 32000 \end{pmatrix}$$

$$= \begin{pmatrix} 128000 - 32000 \end{pmatrix}$$

$$= \begin{pmatrix} 96000 \end{pmatrix}$$

Total amount earned = $96000

Answer **T** = ... [1]

Amount of money earned = $........................ [2]

11.

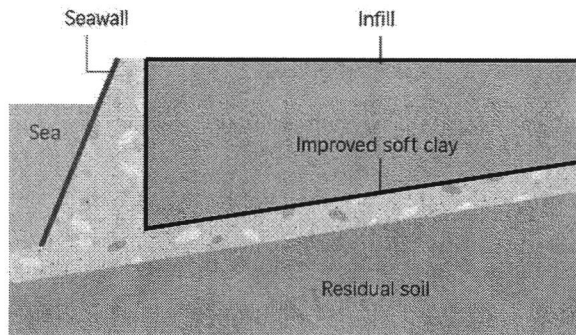# New Method in Land Reclamation!

A new method of land reclamation, known as empoldering, will be used at the north-western tip of Pulau Tekong. This method involves building a dike (embankment) around the area to be reclaimed and draining water from it, creating a low-lying tract of land below sea level, called a polder. The polder is buffered from the sea by a dike, and water levels in the polder are controlled by drains and pumps.

The dike will measure 10 km long, up to 15 m wide at its crest and stand about 6 m above sea level.
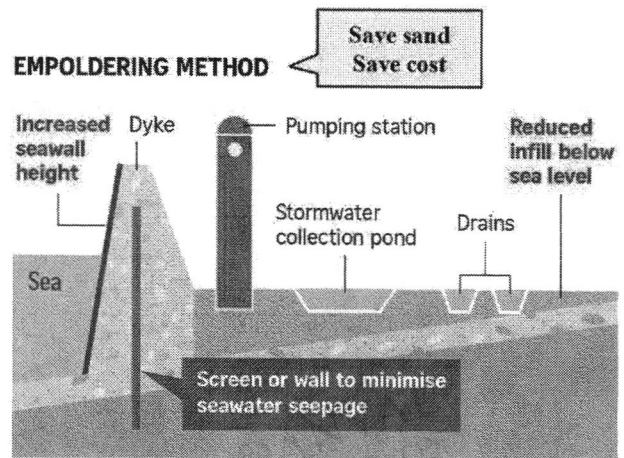
TRADITIONAL LAND RECLAMATION

Seawall

Infill

Sea

Improved soft clay

Residual soil

How it works

"Infill" such as sand is poured into the reclaimed area above the existing seabed, which is made up of improved soft clay and residual soil.
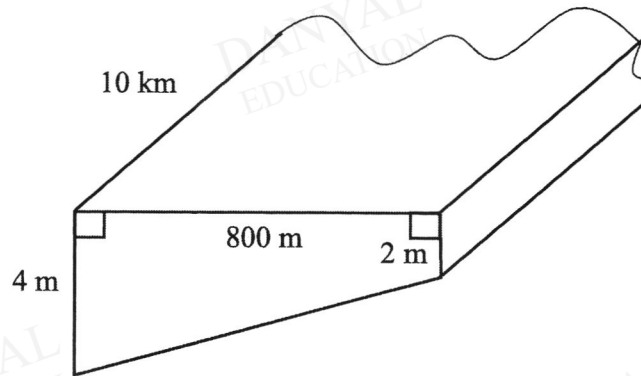
EMPOLDERING METHOD ← Save sand / Save cost

Increased seawall height

Dyke

Pumping station

Reduced infill below sea level

Stormwater collection pond

Drains

Sea

Screen or wall to minimise seawater seepage

How it works

• A dyke is built around the area to be reclaimed and water is drained from it.

• This forms a low-lying tract of land called a polder, which is buffered from the sea by the dyke.

The proposed land to be reclaimed using the empoldering method can be modelled using a trapezoid with uniform cross-sectional area as shown below.

10 km

800 m

4 m

2 m

(a)   Calculate the volume of sand, in cubic metres, needed to reclaim this proposed area.

Volume of sand $= \dfrac{1}{2}(2+4) \times 800 \times 10000$

$= 24\,000\,000$ m^3

Answer .... m^3 [2]

(b) This empoldering method reduces the amount of sand used by 40%. What is the estimated volume of sand needed if the traditional method is used?

$$\text{Volume of sand} = \frac{100}{60} \times 24\,000\,000$$

$$= 40\,000\,000 \text{ m}^3$$

Answer .... m³ [2]

The following table shows some information about the proposed reclaimed land.

FACTS AND FIGURES

810ha Total polder land area is about twice the size of Toa Payoh Town.

29km Total length of drains to be constructed.

21km Total length of roads to be constructed, about the length of East Coast Parkway.

10km Total length of dyke to be constructed, about the length of Bukit Timah Expressway.

Source: HOUSING & DEVELOPMENT BOARD
PHOTO: GOOGLE EARTH
STRAITS TIMES GRAPHICS

(c) Assuming the roads to be constructed on this reclaimed land are straight and of width 12 metres, express the area designated for these roads as a percentage of the polder land area.

$$\left(1 \text{ hectare} = 10000 \text{ m}^2\right)$$

$$\frac{21 \times 1000 \times 12}{810 \times 10000} \times 100\% = 3.11\% \text{ (3 sf)}$$

Answer .... % [2]

Singapore needs to purchase sand from Country X or Country Y in order to carry out the proposed reclamation work. The following shows the cost details of these two countries.

Country	Cost of Sand (S$ per tonne)	Distance from Singapore by Sea (nautical mile)	Shipment cost (S$ for every 1000 tonne per nautical mile)
Country X	26.37	230	65
Country Y	15.85	591	70

Useful Information

1 nautical mile = 1.852 km
1 tonne = 1000 kg
Density of sand = 1442 kg/m^3

(d)   Which country should Singapore purchase sand from?
Justify your answer.

Cost of sand plus shipment from Country X
$$= \frac{24000000 \times 1442}{1000} \times \left(26.37 + \frac{65 \times 230}{1000} \right) = \$1.43 \text{ billion}$$

Cost of sand plus shipment from Country Y
$$= \frac{24000000 \times 1442}{1000} \times \left(15.85 + \frac{70 \times 591}{1000} \right) = \$1.98 \text{ billion}$$

Since the cost of sand plus shipment from Country X is lower as compared to

Country Y, Singapore should purchase sand from Country X.

.... [4]

End of Paper