HUA YI SECONDARY SCHOOL

4E5N

Preliminary Examination

4E5N

MATHEMATICS

4048/1

Paper 1

2 September 2020

2 hours

Candidates to answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index Number in the spaces provided at the top of this page.

Write your answers on the answer paper provided.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.

Answer **all** questions. If working is needed for any question, it must be shown in the space below the question. Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give your answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is **80**.

For Examiner's Use
80

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1 + \frac{r}{100})^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1   (a)   Express 480 as the product of its prime factors, giving your answer in index notation.

Answer .... [1]

(b)   The number $480k$ is a perfect square. Find the smallest positive integer value of k.

Answer .... [1]

(c) x is a number between 110 and 190.
The highest common factor of x and 480 is 15.
Find the smallest possible value of x.

Answer ....[2]

2   The diagram shows the speed-time graph for a car's journey.
The shaded area represents the distance travelled. The distance travelled to its destination is 1140 m.
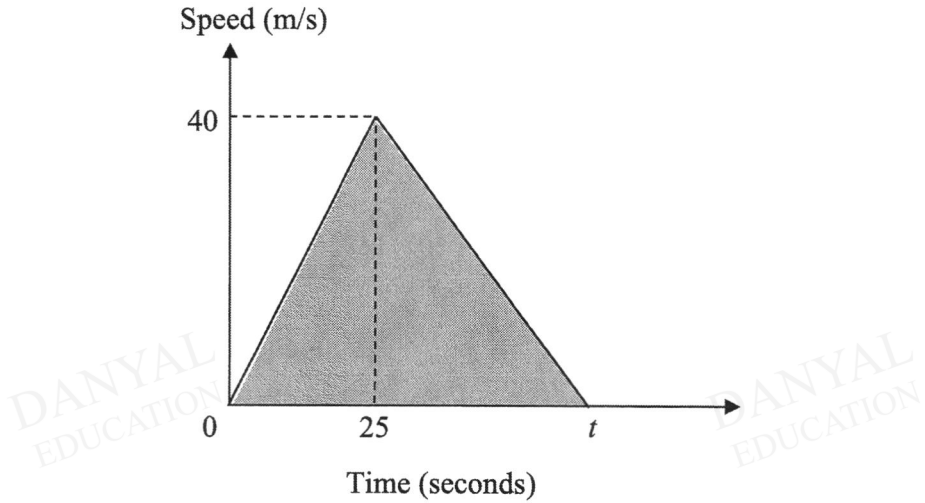
(a)   Calculate the value of t.

Answer ....s [1]

(b)   Calculate the acceleration of the car for the second part of the journey (i.e. after the first 25 seconds).

Answer ....m/s^2 [1]

(c)   **(i)**   A second car starts off from the same point as the first car and travels at a constant speed of 60 km/h. Express 60 km/h in metres per second.

Answer .... m/s [1]

(ii)   Ben claims that the second car will arrive at the destination earlier. Explain whether you agree with him.

....

....

.... [2]

3 **(a)** Simplify $\left(\dfrac{x^9}{y^6}\right)^{-\frac{1}{3}}$, leaving your answer in positive index form.

Answer .... [2]

(b) $5^b = 5^{15} + 5^{15} + 5^{15} + 5^{15} + 5^{15}$

Find the value of b.

Answer $b =$ .... [1]

___

4 The volume of a newly-designed rocket fuel tank is 405 m³. A smaller model of this rocket fuel tank is built to a scale of 1 : 30.

(a) Find the volume, in m³, of the model. Give your answer in standard form.

Answer .... [3]

(b) The older version of the rocket fuel tank is geometrically similar to this newly-designed fuel tank. The surface area of the newly-designed rocket fuel tank is 54 m^2.

The height of the older version of the fuel tank is 3.2 m.

The height of the newly-designed fuel tank is 2.4 m.

Calculate the surface area of the older version of the rocket fuel tank.

Answer .... m^2 [2]

---

5 A company sells the same brand and type of paint in tins of three sizes.

Volume of Paint in One Can (V)	250 *ml*	600 *ml*	1 *l*
Cost of One Can of Paint (C)	$3.50	$8.40	$13.50

Show that the cost of the paint is **not** directly proportional to the volume of paint.

Answer

[2]

6 Factorise each of the following completely.

(a) $8a^3b^2 - 50ab^4$

Answer .... [2]

(b) $6c^2d - 3cd + 1 - 2c$

Answer .... [2]

---

7 Write as a single fraction in its simplest form $\dfrac{5}{2-3y} - \dfrac{3}{y-1}$.

Answer .... [2]

8   Rearrange the formula $s = \sqrt[3]{pq - r^2p}$ to make q the subject.

Answer $q =$ .... [3]

9   The graph of $y = -2(x^2 + 3x - 10)$ is drawn below. It cuts the axes at points A, B and C.
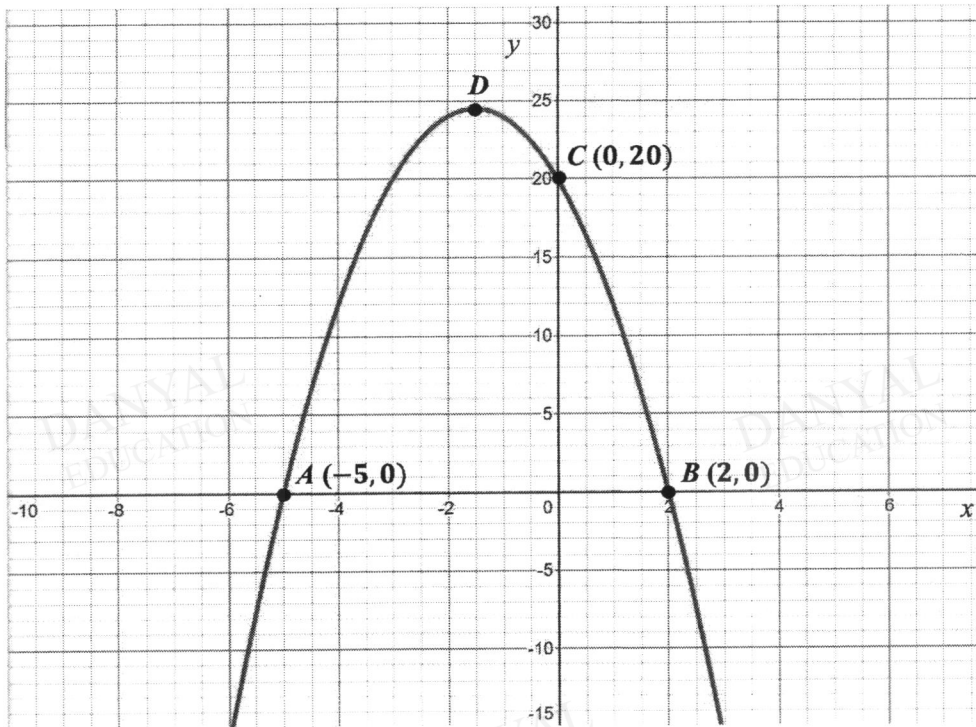
(a) Find the coordinates of the maximum point D.

Answer .... [1]

(b) Write down the equation of the line of symmetry of the graph.

Answer .... [1]

(c) Use the graph to solve the equation $x^2 + 3x - 10 = 4$.

Answer $x =$ .... or ....[2]

(d) Explain why the equation $-2(x^2 + 3x - 10) = k$ does not have solutions for some values of k.
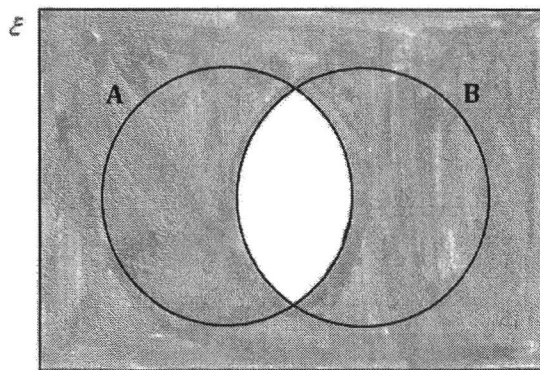
....

....

.... [1]

10 **(a)** Solve the inequalities $5 \le 3y + 7 < 17$.

Answer .... [2]

(b) **Hence**, find the smallest integer that satisfies the above inequalities.

Answer .... [1]

---

11 **(a)** Write down the set represented by the shaded region.

Answer .... [1]

(b) Quadrilaterals can be classified into various categories depending on their properties. Some sets have been listed below.

ξ = {all quadrilaterals}
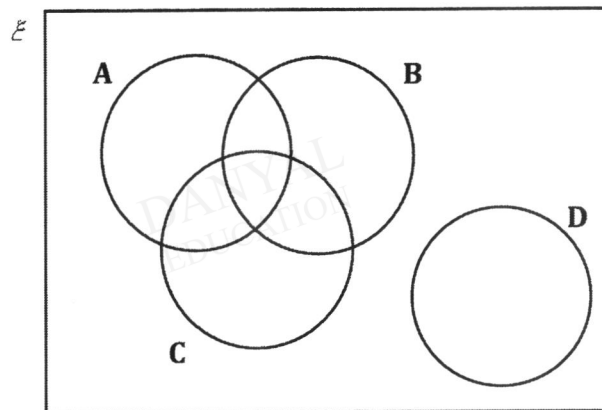
A = {quadrilaterals with four equal lengths}

B = {quadrilaterals with four 90° interior angles}

C = {quadrilaterals with two pairs of parallel sides}

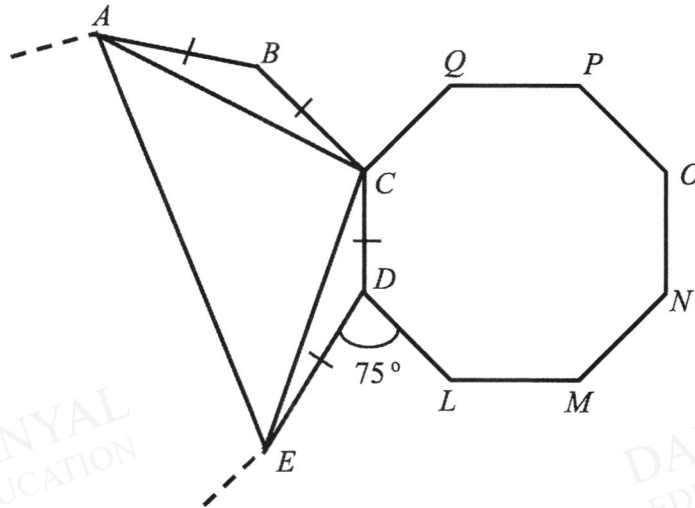D = {quadrilaterals with only one pair of parallel sides}

On the Venn diagram below, state where these six quadrilaterals should be placed: Parallelogram (**P**), Rectangle (**R**), Rhombus (**H**), Square (**S**), Trapezium (**T**), Kite (**K**).

Answer

[2]

12   The diagram shows regular octagon *CDLMNOPQ* and part of a regular n-sided polygon, *ABCDE... .*

(a) Find

   (i)   angle *CDL*,

   Answer   ⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯° [1]

   (ii)   the number of sides in the regular polygon *ABCDE*,

   Answer   ⋯⋯⋯⋯⋯⋯⋯⋯sides [2]

   (iii)   angle *BAC*,

   Answer   ⋯⋯⋯⋯⋯⋯⋯⋯° [1]

(iv) angle *CAE*.

Answer ....° [2]

(b) Explain why triangles *ABC* and *CDE* are congruent.

....

....

.... [2]

(c) Is it possible to have a circle with diameter *AE* and point C on its circumference? Explain.

....

....

.... [1]

13   *PQRS* is a parallelogram.
      PT bisects angle *SPQ*.
      PQ = 7 cm, *PS* = 5 cm and angle *SPT* = 42 °.
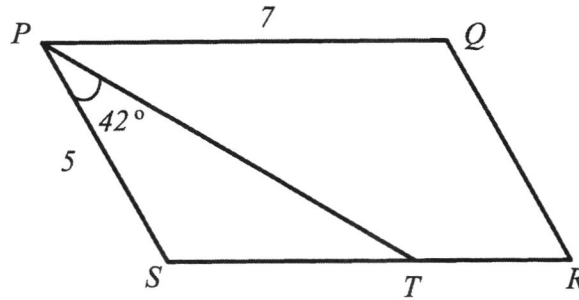
(a) Prove that *ST* = 5 cm.

   Answer

                                                                                    [2]

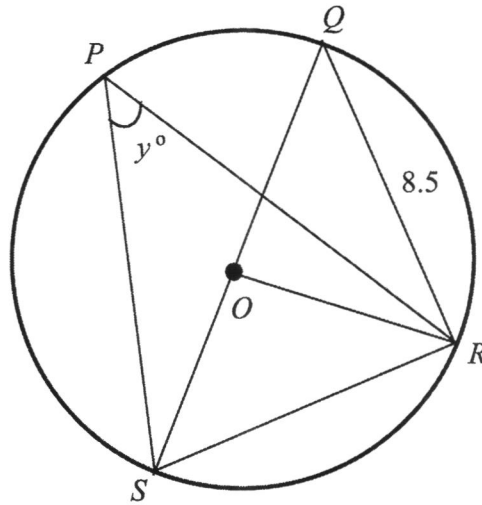(b) Calculate the area of triangle *PST*.

                                            Answer ....cm^2 [1]

(c) Calculate the area of trapezium *PQRT*.

                                            Answer ....cm^2 [3]

14

In the diagram, P, Q, R, S are points on a circle, centre O and radius 7.2 cm.

$QR = 8.5$ cm and angle $SPR = y°$.

Find, giving reasons for each answer,

(a) y,

Answer $y = $ _____ [3]

(b) angle SOR,

Answer   _____ ° [1]

(c) major arc length SR.

Answer   _____ cm [2]

15   *AB* is a diameter of the large circle, centre *O* and radius 10 cm. Angle *OAC* = 55°.
      CD is a diameter of the small circle, centre *O*.
      Triangle *OCA* is congruent to triangle *ODB*.
      AC and *BD* are tangents to the small circle.
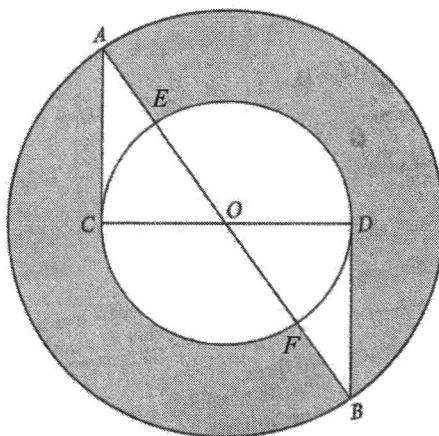
(a) Calculate the area of triangle *OCA*.

Answer ....cm^2 [3]

(b) Calculate the shaded area.

Answer ....cm^2 [4]

16
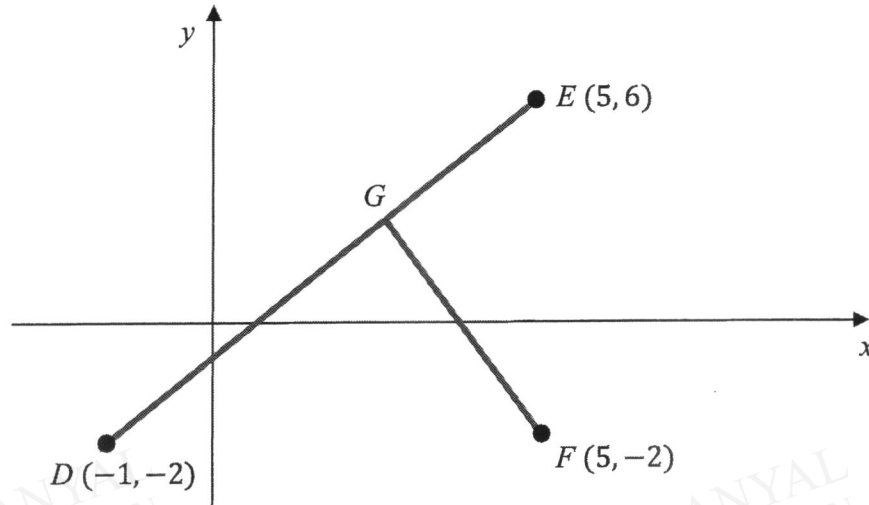
The diagram shows the points $D\ (-1, -2)$, $E\ (5, 6)$ and $F(5, -2)$.

G is a point on DE so that FG is perpendicular to DE.

The product of (gradient of FG) × (gradient of DE) $= -1$.

(a) **(i)** Find the gradient of *DE*.

Answer .... [1]

(ii) Use this information to find the equation of the line *FG*.

Answer .... [2]

(b) **(i)** Find the area of triangle *DEF*.

Answer ....units^2 [2]

(ii) **Hence**, find the perpendicular distance from F to DE.

Answer ....units^2 [2]

---

17 **(a)** The 4 x 10m shuttle run timings, in seconds, of the 24 girls and 15 boys from Class 3G of Hua Yi Secondary School were recorded in the stem-and-leaf diagram below.

Girls		Boys
8 7	9	4 5
9 8 6 5 4	10	0 2 6 6 8
8 8 7 4 3 3 2	11	0 1 3 3 4 5 8
9 8 6 5 5 2 0	12	y
5 1 0	13	
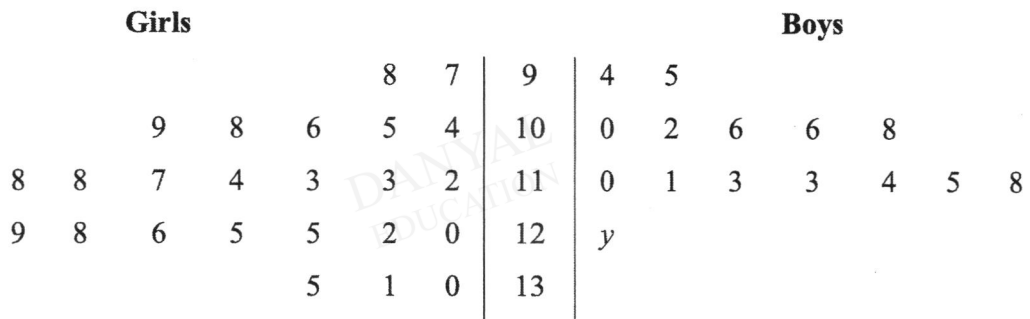
Key: 7 | 9 | 4 means a timing of 9.7*s* for girls and a timing of 9.4*s* for boys

(i) Find the median timing for girls.

Answer ....*s* [1]

(ii) The range of the timings for the boys is 2.8 *s*. Find y.

Answer ....*s* [1]

(iii) The PE teacher selects a girl from Class 3G to be the PE class representative. Find, as a fraction in its simplest form, the probability that the PE class representative runs slower than 11.7 *s*.

Answer .... [1]

(iv) Two students from Class 3G volunteer to participate in the school's Sports Day. Find, as a fraction in its simplest form, the probability that these two students have an individual shuttle run timing of 10.5 *s* or below.

Answer .... [1]

(b)

Shuttle Run Timings of Boys from Class 3B	
Median	10.7 *s*
Range	2.4 *s*

The PE teacher commented that the boys from Class 3G generally performed better at the shuttle run than the boys from Class 3B. Do you agree? Explain your answer.

....

....

.... [2]

~ END OF PAPER ~

HUA YI SECONDARY SCHOOL

4E5N

Preliminary Examination

4E5N

MATHEMATICS

4048/02

Paper 2

27 August 2020

2 hours 30 minutes

Candidates to answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index Number in the spaces provided at the top of this page.

Write your answers on the answer paper provided.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.

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If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give your answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is **100**.

For Examiner's Use
100

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1+\frac{r}{100})^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1   **(a)**   Simplify $24a^2b^3 \div 15a^3b^{-2}$, leaving your answer in positive index form.

Answer .... [2]

(b)   Simplify $\dfrac{8w^2 - 2}{6w^2 - 7w - 5}$.

Answer .... [3]

(c)   The total surface area of a Rubik's cube is given by $294x^4 \, \text{cm}^2$.

Find an expression for the volume of the Rubik's cube in the form px^n, where p and n are integers.

Answer .... cm^3 [3]

(d) In a recent study, it was reported that usually about 8% of the world's emperor penguin population breeds at Halley Bay.

The estimated number of emperor penguin pairs that bred in that area increased from 11 000 pairs in 2015 to 14 610 pairs in 2018.

The number of breeding pairs are expected to increase by r % every year.

Find the value of r.

Answer $r =$ _____ [3]

2 **(a)** Elvin's car has the following specifications:

- Fuel capacity: 45 litres
- Estimated fuel consumption: 31.6 miles per gallon (mpg)

It is given that 1 mpg = 0.354 km per litre.

He drove the car for 280 kilometres with a full tank of fuel and was left with 15 litres of petrol.

Showing your working clearly, determine if the actual fuel consumption of the car is more or less than the estimated fuel consumption.

Answer

....

....

[3]

(b) The force, F, between two particles is inversely proportional to the square of the distance, D, between them.

The distance between two particles is increased by 40%.

Calculate the percentage reduction in the force between the particles, leaving your answer correct to 1 decimal place.

Answer .... % [3]

(c) The diagram shows a vessel made up of a conical base and an open cylindrical top.

The vessel has a depth of 36 cm.

The depth of the conical base is $\dfrac{1}{4}$ of the entire depth of the vessel.

It took 10 seconds to fill up the vessel fully.
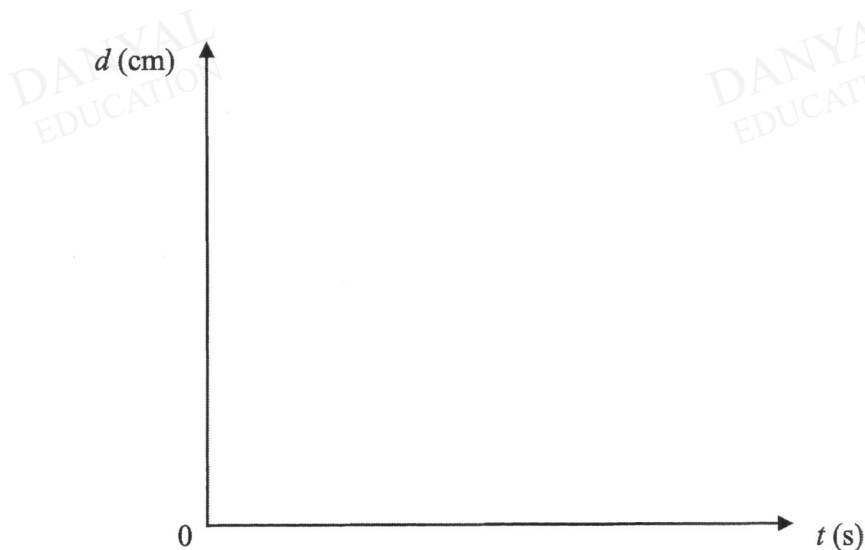
After t seconds, the depth of the liquid is d cm.

(i) Find the depth of the conical base.

36 cm

Answer .... cm [1]

(ii) On the axes provided below, sketch a graph to show the change in the depth of liquid in the vessel within 10 seconds.

Answer

d (cm)

0   t (s)

[2]

3   A fitness centre offers daily zumba classes for morning and afternoon classes.

On every weekday morning, 15 males and 10 females will attend the zumba class, while 20 males and 15 females attend the afternoon zumba class.

This information may be represented by the matrix **D** below.

$$\begin{array}{cc} \text{males} & \text{females} \end{array}$$
$$\mathbf{D} = \begin{pmatrix} 15 & 10 \\ 20 & 15 \end{pmatrix} \begin{array}{l} \text{morning} \\ \text{afternoon} \end{array}$$

On each weekend, 30 males and 20 females will attend the morning class, while 25 males and 25 females will attend the afternoon class.

(a)   Represent the number of people attending the fitness class on each weekend by a 2×2 matrix **E**.

Answer   **E** = ... [1]

(b)   Evaluate the matrix **S** = 5**D** + 2**E**.

Answer   **S** = ... [1]

(c)   The fitness centre charges each person $10 for a morning session and $15 for an afternoon session. Represent the charges by a row matrix **P**.

Answer   **P** = ... [1]

(d) Evaluate the matrix **T = PS**.

Answer   **T** = _____ [1]

(e) State what the elements of **T** represent.

Answer   .... :

.... [1]

4 Hazelle recorded the mass of her newborn nephew, Kinston, every 7 days.

The table below shows some of the values she recorded.

Age of Kinston (t days)	0	7	14	21	28	35	42	49	56
Mass (m kg)	3.50	3.25	3.15	3.15	3.40	3.70	4.10		4.90

(a) *Answer this part of the question on the grid on page 11.*

On the axes provided, draw a horizontal t-axis for $0 \le t \le 60$ and a vertical m-axis for $2 \le m \le 5$.

Plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to estimate

(i) the mass of Kinston after 49 days,

Answer .... kg [1]

(ii) when Kinston's mass was least,

Answer after .... days [1]

(iii) when Kinston regained his birth mass.

Answer after .... days [1]

(c) (i) By drawing a tangent, find the gradient of the curve at (7, 3.25).

Answer .... [2]

(ii) What does this gradient represent?

Answer ....

.... [1]

(d) Hazelle wishes to estimate what the mass of Kinston will be after 260 days from the day he was born.

She proposes to extend the graph line up to $t = 260$.

Justify why it is not possible to estimate the mass of Kinston in this way.

Answer ....: [1]
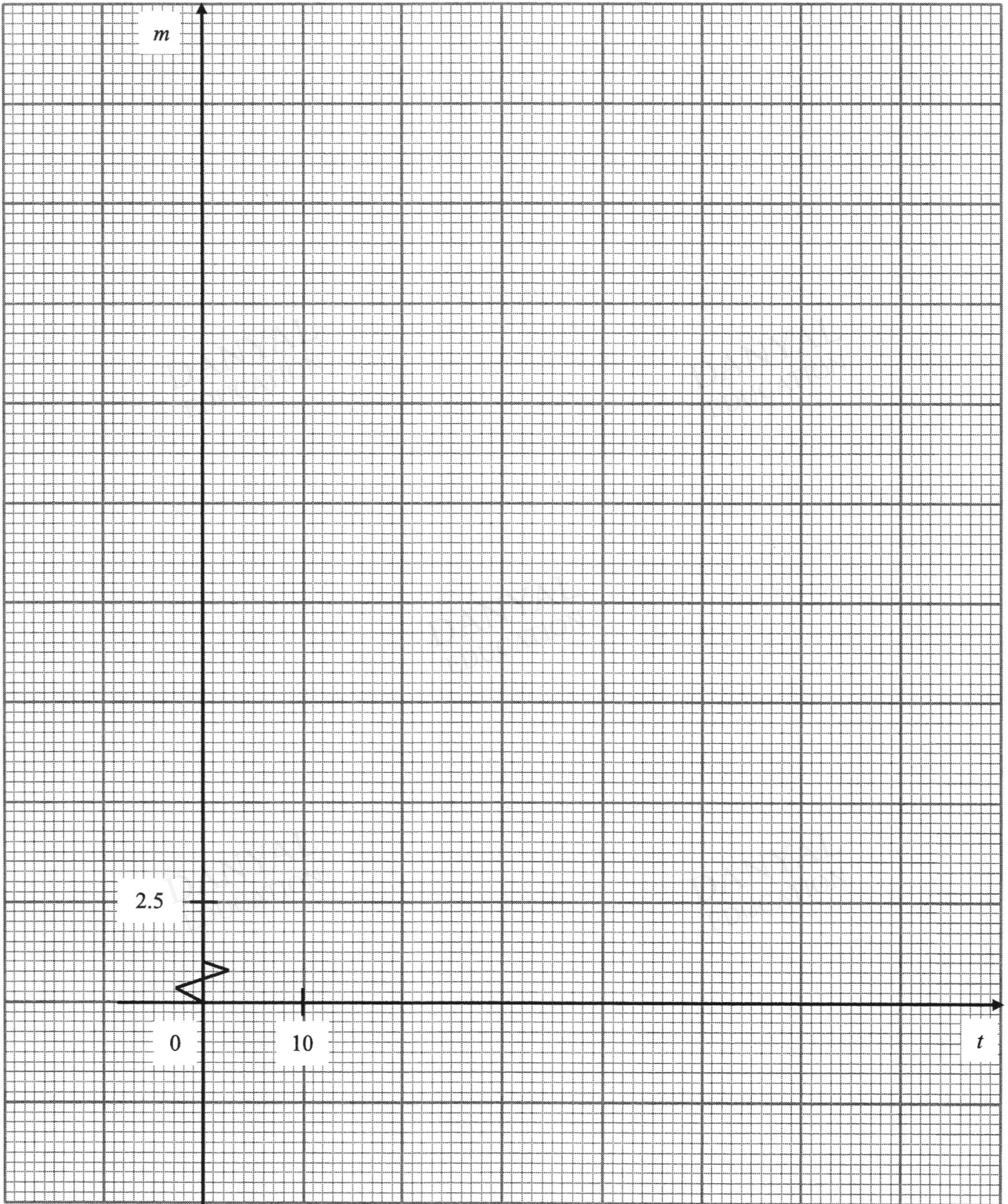
5 The first four terms in a sequence of numbers are given below.

$$T_1 = 1^2 + 2 \times 1 = 3$$
$$T_2 = 2^2 + 3 \times 3 = 13$$
$$T_3 = 3^2 + 4 \times 5 = 29$$
$$T_4 = 4^2 + 5 \times 7 = 51$$

(a) Write down the 5th line of the sequence.

Answer .... [1]

(b) Prove that the nth term of the sequence, T_n, is given by $3n^2 + n - 1$.

Answer

[3]

(c) T_{m-1} and T_m are consecutive terms in the above sequence.

Find and simplify an expression, in terms of m,

    (i) for $T_m - T_{m-1}$ and

Answer .... [3]

    (ii) explain with suitable working why the difference between two consecutive terms of the sequence will always be even.

Answer ....As above............ [2]

6 A room has an enclosed space of 120 m^3.

(a) An air-conditioning (AC) unit releases cool air at a rate of x m^3 per hour.

Write down an expression, in terms of x, for the number of hours which the AC unit will take to cool the room to a particular temperature.

Answer ....hours [1]

(b) When the AC unit is faulty, it will release cool air at a rate of $(x - 450)$ m^3 per hour. Write down an expression, in terms of x, for the number of hours which the faulty AC unit will take to cool the room to the same temperature.

Answer ....hours [1]

(c) It takes 30 minutes longer to cool the room when the AC unit is faulty.

Write down an equation in x to represent this information and show that it reduces to $x^2 - 450x - 108000 = 0$.

Answer ....As above......... [3]

(d) Solve the equation $x^2 - 450x - 108000 = 0$, leaving your answers correct to 2 decimal places.

Answer $x =$ _____ [3]

(e) Hence, find the time it takes to cool the room to the same temperature when the AC unit is faulty.

Give your answer correct to the nearest minute.

Answer   _____ minutes [2]

7   In the diagram P, Q and R represent three points on level ground.

P is 15 m from Q on a bearing of $220°$.

R is 8 m from Q on a bearing of $103°$.
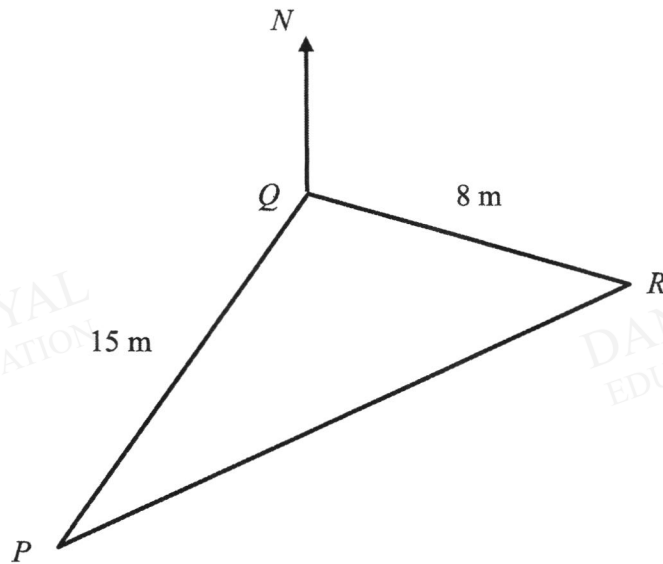
(a)   Calculate the distance of P from R.

Answer .... m [3]

(b)   Show that $\angle PRQ = 42.1°$, correct to 1 decimal place.

Answer .... ° [2]

(c)   Calculate the bearing of P from R.

Answer ....° [2]

(d)   A tower of height 10 m was built at Q.

Joyce leaves R and walks towards P.

(i)   Calculate the distance from R, where she is equidistant from points Q and R.

Answer .... m [2]

(ii)   Calculate the **greatest** angle of elevation of Joyce to the top of the tower as she walks from R to P.

Answer ....° [3]

8 The diagram below shows the logo of Pac-Man.

It was a popular electronic game in the 1980s.
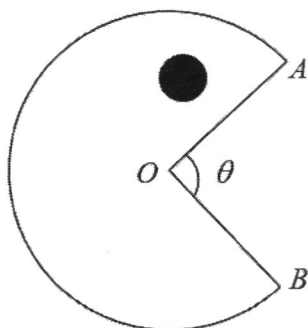
In the diagram, OA and OB are radii to the arc and $OA = OB = 5$ cm.

The eye of Pac-Man, as represented by the shaded circle, has a diameter of 1 cm.

During the game, the angle θ varies between 0 radians and $\dfrac{\pi}{2}$ radians.

(a) Leaving your answer in terms of π and θ,

   (i) write down the reflex angle AOB,

   Answer .... radians [1]

   (ii) find the area of major sector AOB.

   Answer .... cm^2 [1]

(b) If the length of major arc AB is 24 cm, calculate angle AOB.

Answer .... radians [2]

(c) Is it possible for the **maximum** length of the major arc AB to be 34 cm?

Justify your answer with suitable workings and/or mathematical reasons.

Answer ....

.... [2]

(d) The developer of Pac-Man intends to create a limited edition token of the game logo to mark the 40th anniversary of the game.

To enhance the attractiveness of the token, the designer proposed for the eye on the Pac-Man logo to be hollow and the material for the token to be acrylic.

The token will also have a thickness of 4 mm.

Taking $\theta = \dfrac{\pi}{2}$ and considering that the manufacturer charges $0.46 per cubic centimetre of acrylic, calculate the **greatest** number of tokens that can be made with a budget of $500.

Answer .... tokens [5]

9 **(a)** The Marina Costal Expressway (MCE) is Singapore's first undersea road.

The timings of 160 vehicles passing through a certain stretch of MCE on a Sunday morning were recorded.

The cumulative frequency curve below shows the distribution of the timings in seconds.
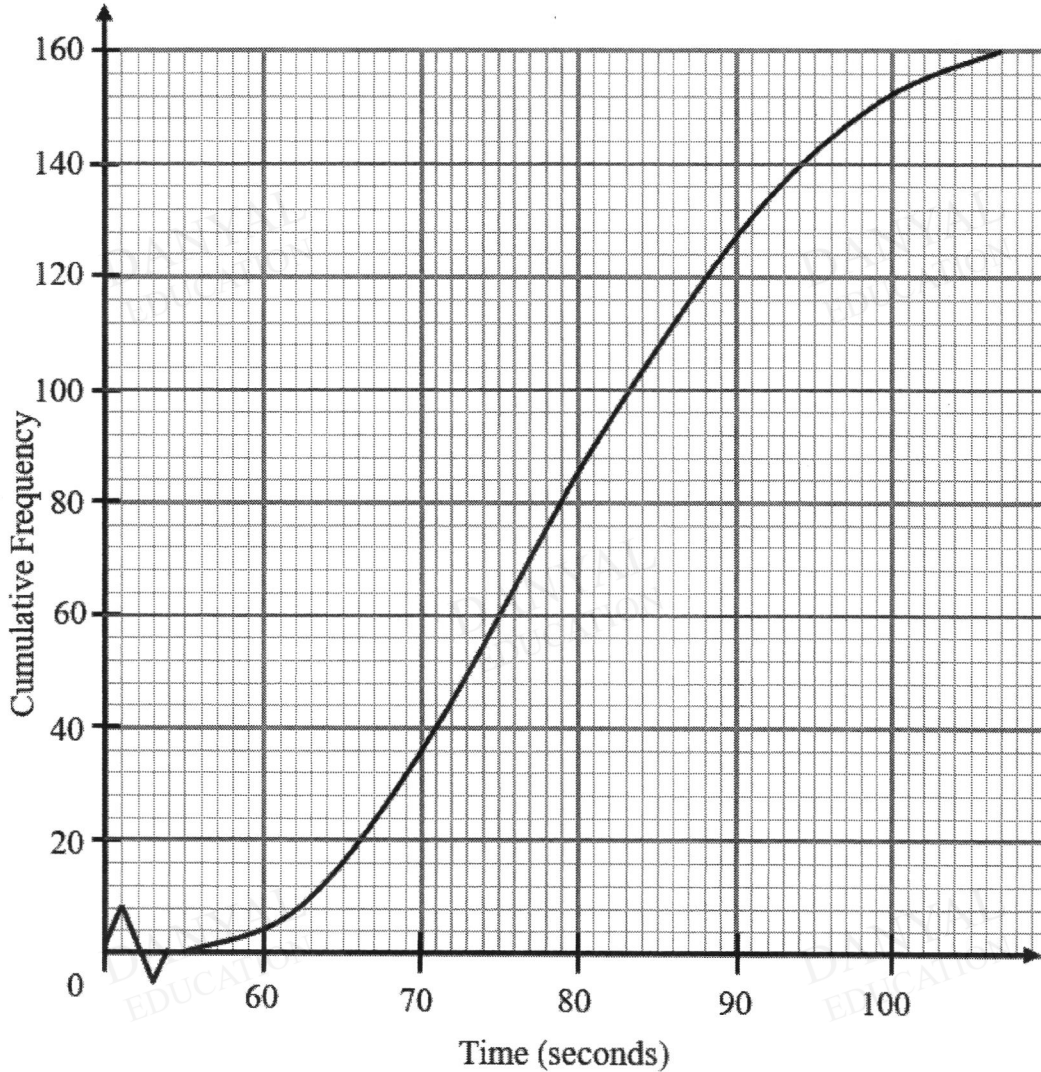
(i) Use the graph to estimate

(a) the median time,

Answer .... seconds [1]

(b) the interquartile range of the timings.

Answer .... seconds [2]

(c)   the value of *x*, if 30% of the vehicles took more than *x* seconds to pass through a certain stretch of MCE.

Answer   *x* = ... [2]

(ii)   The timings of another 160 vehicles passing through the same stretch of MCE on a Monday morning were also recorded.
The box-and-whisker plot shows the distribution of the timings in seconds.

Time (seconds)
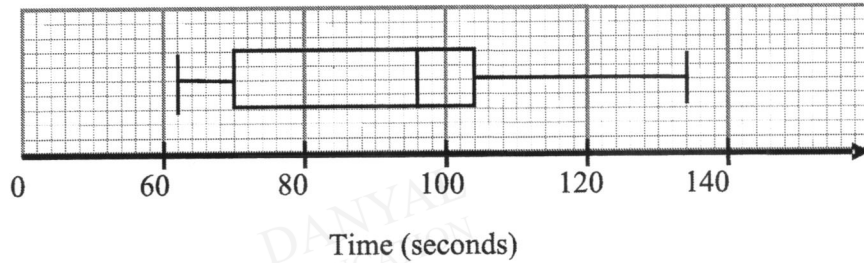
Make **two** comments to compare the timings of the vehicles on the Sunday morning and the Monday morning.

Answer

1. ....

....

....

2. ....

....

....

[2]

9 **(b)** A jar contains 10 identical balls of which 4 are pink and 6 are yellow.

The first person to take a yellow ball wins and the game ends.

To start the game, Emily took the first draw and Theo took the second draw.

The game continued with them taking turns, to draw a ball from the jar **without** replacement.

(i) Find the probability that Theo wins on his first draw.

Answer .... [2]

(ii) Find the probability that **no** fourth draw is required.

Answer .... [2]

(iii) If the game ends at the kth draw, state the largest value of k.

Answer .... [1]

10   Veron owns a local vintage shop.

She imports vintage furniture from overseas and sells them locally.

A week ago, she came across a vintage armchair from an overseas shop and intends to import it from USA.

Below are some information that Veron has gathered.

Product information for vintage armchair

Retail Price	US$ 380
Item Weight	34 pounds
NOTE: The product and parcel dimensions stated **below** are approximated.	
Product Dimensions	32 x 30 x 25 inches
Parcel Length	36 inches
Parcel Width	36 inches
Parcel Height	30 inches
Parcel Weight	40 pounds

Other useful information:

- 1 inch = 2.54 centimetres

- 1 pound = 0.454 kg

- USD 1 = SGD 1.38

What is Volumetric Weight & Actual Weight?

Volumetric Weight (kg) = $\dfrac{L \times W \times H}{5000}$

L = Length of parcel in centimetres

W = Width of parcel in centimetres

H = Height of parcel in centimetres

Actual Weight (kg) = Parcel Weight (kg)

Source: www.vpost.com.sg

USA to Singapore Shipping Modes

Economy	Standard	Sea Freight
9 – 12 ^working days	4 – 6 ^working days	35 – 60 ^working days
Actual Weight	Actual Weight or Volumetric Weight*	Bulky Items* >10kg Weight

^Working days do not include weekends, both in Singapore or product source country's and public holidays.

*For Standard Air Freight and Sea Freight, the chargeable weight of a package is calculated based on **Actual Weight** or **Volumetric Weight**, <u>whichever is higher</u>.

USA Shipping Rates

Country	Shipping Method	Base Charge	Weight Charge (Per 0.1 kg)
USA	Economy — Air Freight	S$13.40	S$0.91
	Standard * — Air Freight	S$13.40	S$0.91
	Sea Freight *	S$39.90	S$0.39

NOTE:

Air Freight and Sea Freight shipping modes are available when the chargeable weight is at least 0.5 kg and 10 kg respectively. Varied charges apply with an incremental weight break of 0.1 kg thereafter.

(a)   Using the relevant information on page 22, calculate

    (i)   the **Actual Weight** in kilograms and,

Answer .... kg [1]

    (ii)   the **Volumetric Weight**, in kilograms, of a vintage armchair.

Answer .... kg [2]

Veron needs to decide on the shipping method to import the vintage armchair from USA.

(b) Justify with calculations, a most suitable shipping method for the imported vintage armchair.

Answer

....

....

[6]

(c)   Suggest a sensible selling price in SGD for the imported vintage armchair.

Explain your answer.

Answer   SGD ....

Answer

....

....

[2]

~ END OF PAPER ~
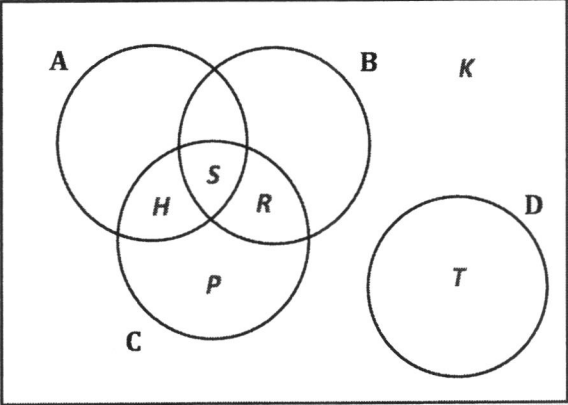
1.	(a)	$480 = 2^5 \times 3 \times 5$ --- B1
	(b)	$k = 2 \times 3 \times 5 = 30$ --- B1
	(c)	$HCF = 15 = 3 \times 5$ --- M1 Smallest possible value of $x = 3^3 \times 5 = 135$ --- A1

2.	(a)	$t = 57s$ --- B1
	(b)	Acceleration $= -1.25 \, m/s^2$ --- B1
	(ci)	$16\frac{2}{3}$ m/s --- B1
	(cii)	Time taken $= 1140 \div 16\frac{2}{3} = 68.4s$ [M1] I disagree with Ben, because the second car took a longer time [A1] of 68.4s, compared to the 57s that the first car took for the 1140 m travelled.

3.	(a)	$(\frac{x^9}{y^6})^{-\frac{1}{3}}$ $= (\frac{y^6}{x^9})^{\frac{1}{3}}$ --- M1 $= (\frac{y^2}{x^3})$ --- A1
	(b)	$b = 16$ --- B1

4.	(a)	$(0.01)^3 \, m^3 = (0.3)^3 \, m^3$ $(1 \times 10^{-6}) \, m^3 = 0.027 \, m^3$ --- M1 for scale (volume) $\frac{(1 \times 10^{-6})}{0.027} m^3 = 1 \, m^3$ $405 \, m^3 = 405 \times \frac{(1 \times 10^{-6})}{0.027} m^3$ --- M1 $\qquad = 15000 \times 10^{-6} \, m^3$ $\qquad = 1.5 \times 10^{-2} \, m^3$ --- A1
	(b)	$(\frac{4}{3})^2 = (\frac{SA1}{54})$ --- M1 $(\frac{4}{3})^2 = (\frac{16}{9}) = (\frac{SA1}{54})$ $SA1 = \frac{16 \times 54}{9} = 96 \, m^3$ --- A1

5.		**Method 1:** For the cost to be directly proportional to the volume of paint, $C = kV$, where k is a constant. • When C = \$3.50 and V = 250 ml, $k = \frac{3.50}{250} = 0.014$. • When C = \$8.40 and V = 600 ml, $k = \frac{8.40}{600} = 0.014$. • When C = \$13.50 and V = 1000 ml, $k = \frac{13.50}{1000} = 0.0135$. --- M1 Since **the values of k are not the same ($k = 0.014 \neq 0.0135$)**, the cost (C) of the paint is not directly proportional to the volume (V) of paint. --- A1

		Method 2: $4 \times 250\ ml = 1l$ But $\$3.50 \times 4 = \$14 \neq \$13.50$ --- B2
6.	**(a)**	$8a^3b^2 - 50ab^4$ $= 2ab^2(4a^2 - 25b^2)$ --- M1 $= 2ab^2(2a - 5b)(2a + 5b)$ --- A1
	(b)	$6c^2d - 3cd + 1 - 2c$ $= 3cd\,(2c - 1) + (1 - 2c)$ --- M1 *or* $= 3cd\,(2c - 1) - (2c - 1)$ --- M1 $= (3cd - 1)(2c - 1)$ --- A1
7.		$\dfrac{5}{2 - 3y} - \dfrac{3}{y - 1}$ $= \dfrac{5(y-1) - 3(2-3y)}{(2-3y)(y-1)}$ --- M1 *or* $= \dfrac{5y - 5 - 6 + 9y}{(2-3y)(y-1)}$ --- M1 $= \dfrac{14y - 11}{(2-3y)(y-1)}$ --- A1
8.		$s = \sqrt[3]{pq - r^2 p}$ $s^3 = pq - r^2 p$ --- M1 for cube-ing both sides $s^3 = p(q - r^2)$ --- M1 for extracting common factor *or* $\dfrac{s^3}{p} = q - r^2$ --- M1 $\dfrac{s^3}{p} + r^2 = q$ --- A1
9.	**(a)**	$D(-1.5, 24.5)$ --- B1
	(b)	$x = -1.5$ --- B1
	(c)	$x^2 + 3x - 10 = 4$ Draw the horizontal line y $= -8$ on the graph. --- M1 $x = -5.5$ or $x = 2.5$ --- A1 / B2 (*Accepted range*: $x \pm 0.25$) (Actual answers: $x = -5.24$ or $x = 2.24$)
	(d)	For $-2(x^2 + 3x - 10)$ to have no solutions, **the horizontal line y $= k$ does not have any intersection points with the curve**, and **lies entirely <u>above</u> the curve**. --- B1
10.	**(a)**	$y \geq -\dfrac{2}{3}$ and $y < 3\dfrac{1}{3}$ --- M1 $-\dfrac{2}{3} \leq y < 3\dfrac{1}{3}$ --- A1
	(b)	Smallest integer $= 0$ --- B1

11.	**(a)**	$A' \cup B'$ or $(A \cap B)'$ --- B1
	(b)	--- B2 for 6 correct answers, --- B1 for 3 correct answers

12.	**(ai)**	Angle $CDL = \frac{(8-2) \times 180°}{8} = 135°$ --- B1
	(aii)	Angle CDE (1 interior angle of regular polygon) $= 360° - 135° - 75° = 150°$ --- M1 *or* $\frac{(n-2) \times 180°}{n} = 150°$ $180°n - 360° = 150°n$ --- M1 $180°n - 150°n = 360°$ $30°n = 360°$ $n = \frac{360°}{30°} = 12$ --- A1
	(aiii)	Angle $ABC = 150°$ Angle $BAC = \frac{180° - 150°}{2} = 15°$ (base angles of isos. triangle) --- B1
	(aiv)	Angle ACE $= 150° - 15° - 15° = 120°$ --- M1 Angle CAE $= \frac{180° - 120°}{2} = 30°$ (base angles of isos. triangle) --- A1
	(b)	Angle ABC = Angle CDE (interior angles of the regular polygon $ABCDE$) --- B1 $AB = CD$ (sides of the regular polygon) $BC = DE$ (sides of the regular polygon) ---- B1 Therefore, triangles ABC and CDE are congruent (**SAS**).
	(c)	No, because Angle $ACE = 120°$. For AE to be the diameter of the circle touching point C, Angle ACE must equal 90° (angles in semi-circle). --- B1

13.	**(a)**	Angle $SPQ = 42° \times 2 = 84°$ *(PT bisected angle SPQ)* Angle $PST = 180° - 84° = 96°$ *(interior angles)* --- M1 *or* Angle $STP = 180° - 42° - 96° = 42°$ *(sum of angles in a triangle)* --- M1 Since Angle SPT = Angle $STP = 42°$, triangle SPT is an isosceles triangle, and $PS = ST = 5$ cm. --- A1

	(b)	Area of triangle PST $= \frac{1}{2} \times 5 \times 5 \times \sin 96°$ $= 12.432$ $= 12.4 \text{ cm}^2$ (3 sf) --- B1
	(c)	**Method 1:** Let h be the perpendicular distance from P to RS produced. $\sin 84° = \frac{h}{5}$ $h = 5 \sin 84° = 4.9726 \text{ cm}$ --- M1 Area of parallelogram $PQRS$ $= PQ \times h$ $= 7 \times 4.9726$ $= 34.8082 \text{ cm}^2$ --- M1 Area of trapezium $PQRT$ $=$ Area of parallelogram $PQRS$ - Area of triangle PST $= 34.8082 - 12.432$ $= 22.3762$ $= 22.4 \text{ cm}^2$ --- A1 **Method 2:** Since triangle SPT is an isosceles triangle, and $PS = ST = 5$ cm. $\Rightarrow$ Therefore, $TR = 7 - 5 = 2$ cm Let h be the perpendicular distance from P to RS produced. $\sin 84° = \frac{h}{5}$ $h = 5 \sin 84° = 4.9726 \text{ cm}$ --- M1 Area of trapezium $PQRT$ $= \frac{1}{2} \times (sum\ of\ parallel\ sides) \times height$ $= \frac{1}{2} \times (7 + 2) \times 4.9726$ --- M1 $= 22.3767$ $= 22.4 \text{ cm}^2$ --- A1 **Method 3:** Area of triangle PRS $= \frac{1}{2} \times 5 \times 7 \times \sin 96°$ $= 17.404 = 17.4 \text{ cm}^2$ (3 sf) --- M1 $=$ Area of triangle PQR Area of triangle PRT $=$ Area of triangle PRS – Area of Triangle PST $= 17.404 - 12.432$ $= 4.972 = 4.97 \text{ cm}^2$ (3 sf) --- M1

		Area of trapezium $PQRT$ = Area of triangle PQR + Area of Triangle PRT = 17.404 + 4.972 = 22.376 = 22.4 cm^2 (3 sf) --- A1
14.	**(a)**	Angle $QRS = 90°$ *(Angle in semi-circle)* --- M1 $\cos SQR = \dfrac{8.5}{7.2 \times 2} = \dfrac{85}{144}$ Angle $SQR = \cos^{-1}\left(\dfrac{85}{144}\right) = 53.82° = 53.8°$ (1 dp) --- M1 $y° = 53.8°$ (1 dp) *(angles in the same segment)* --- A1
	(b)	Angle $SOR = 2 \times$ Angle $SQR = 2 \times$ Angle $SPR = 2 \times$ Angle $y°$ $\qquad = 2 \times 53.82°$ $\qquad = 107.64° = 107.6°$ (1 dp) *(Angle at centre = 2 Angles at circumference)* --- B1
	(c)	Major arc length SR $= \dfrac{360° - 107.64°}{360°} \times 2\pi(7.2)$ --- M1 *or* $= \dfrac{252.36°}{360°} \times 14.4\pi$ --- M1 $= 31.712$ $= 31.7$ cm (to 3 sf) --- A1
15.	**(a)**	**Method 1:** $\cos 55° = \dfrac{AC}{10}$ $AC = 10\cos 55° = 5.7358$ cm $= 5.74$ cm --- M1 Area of triangle OCA $= \dfrac{1}{2} \times 5.7358 \times 10 \times \sin 55°$ --- M1 $= 23.492$ $= 23.5$ cm^2 (3 sf) --- A1 **Method 2:** $\sin 55° = \dfrac{OC}{10}$ $OC = 10\sin 55° = 8.1915$ cm $= 8.19$ cm --- M1 *or* $\cos 55° = \dfrac{AC}{10}$ $AC = 10\cos 55° = 5.7358$ cm $= 5.74$ cm --- M1 Area of triangle OCA $= \dfrac{1}{2} \times 8.1915 \times 5.7358$ --- M1 $= 23.492$ $= 23.5$ cm^2 (3 sf) --- A1

15.	(b)	**Method 1:**
		Area of outer ring $= \pi(10)^2 - \pi(8.1915)^2$ $= 103.36 \text{ cm}^2$ --- M1
		Angle $OCA = 90°$ *(radius perpendicular to tangent)* Angle $AOC = 180° - 90° - 55° = 35°$ *(sum of angles in a triangle)*
		Area of sector OCE $= \dfrac{35°}{360°} \times \pi(8.1915)^2$ $= 20.495 \text{ cm}^2$ --- M1
		Area of ACE $= 23.492 - 20.495$ $= 2.997 \text{ cm}^2$ --- M1 *or*
		Therefore, shaded area $= 103.36 - 2(2.997)$ --- M1 $= 103.36 - 5.994$ $= 97.366$ $= 97.4 \text{ cm}^2$ (3sf) --- A1
		Method 2:
		Area of unshaded parts $= 2(\text{Area of triangle } AOC) + (\text{Area of sectors } ODE \text{ and } OCF)$ $= 2(23.492) + \dfrac{360° - 35° - 35°}{360°} \times \pi(8.1915)^2$ --- M1 for area of sectors $= 46.984 + 169.81$ $= 216.794 \text{ cm}^2$ --- M1
		Area of large circle $= \pi(10)^2$
		Therefore, shaded area $= \pi(10)^2 - 216.794$ --- M1 $= 97.365$ $= 97.4 \text{ cm}^2$ (3sf) --- A1
16.	(ai)	$DE = \dfrac{6-(-2)}{5-(-1)} = \dfrac{8}{6} = \dfrac{4}{3} = 1\dfrac{1}{3}$ --- B1
	(aii)	$\dfrac{4}{3} \times Gradient \text{ of } FG = -1$ $Gradient \text{ of } FG = -1 \div \dfrac{4}{3} = -\dfrac{3}{4}$ --- M1 At $F(5, -2)$, $y = -\dfrac{3}{4}x + c$ $(-2) = -\dfrac{3}{4}(5) + c \Rightarrow c = 1\dfrac{3}{4}$ Therefore, the equation of the line FG is $y = -\dfrac{3}{4}x + 1\dfrac{3}{4}$. --- A1

6

	(bi)	Area of triangle DEF $= \frac{1}{2} \times 8 \times 6$ --- M1 $= 24$ units^2 --- A1
	(bii)	$DE = \sqrt{(6-(-2))^2 + (5-(-1))^2} = \sqrt{(8)^2 + (6)^2} = \sqrt{100} = 10$ units --- M1 Area of triangle DEF $= \frac{1}{2} \times DE \times FG = \frac{1}{2} \times 10 \times FG = 24$ units^2 $FG = \frac{24}{5}$ $= 4.8$ units --- A1
17.	(ai)	Median timing for girls $= 11.75 \ s$ --- B1 *(Median: 12.5th position)*
	(aii)	$9.4 + 2.8 = 12.2 \ s$ $y = 2$ --- B1
	(aiii)	Probability (girl runs slower than $11.7 \ s$) $= \frac{12}{24} = \frac{1}{2}$ --- B1
	(aiv)	Probability (two students have a timing of $10.5 \ s$ and under) $= \frac{8}{39} \times \frac{7}{38} = \frac{28}{741}$ --- B1
	(b)	No, I do not agree. The 3G boys had a **higher median [B1] of 11.0 s** *(Median: 8th position)*, compared to the median of the 3B boys of 10.7 s, which means that the 3G boys had **slower shuttle run timings** and hence, worse performance. --- B2

HUA YI SECONDARY SCHOOL

4E5N

Preliminary Examination

4E5N

MATHEMATICS

4048/02

Paper 2

27 August 2020

2 hours 30 minutes

Candidates to answer on the Question Paper.

MARKING SCHEME

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is **100**.

For Examiner's Use
100

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1 + \frac{r}{100})^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Simplify $24a^2b^3 \div 15a^3b^{-2}$, leaving your answer in positive index form.

$$24a^2b^3 \div 15a^3b^{-2}$$
$$= \frac{8}{5}a^{2-3}b^{3+2} \quad --M1 \text{ (Apply BOTH indices laws correctly)}$$
$$= \frac{8b^5}{5a} \quad --A1$$

Answer .... [2]

(b) Simplify $\dfrac{8w^2-2}{6w^2-7w-5}$.

$$\frac{8w^2-2}{6w^2-7w-5} = \frac{2(4w^2-1)}{6w^2-7w-5} \quad --M1 \text{ (Numerator: cross-factorise OR by HCF)}$$
$$= \frac{2(2w+1)(2w-1)}{(3w-5)(2w+1)} \quad --M1 \text{ (Base: cross-factorise)}$$
$$= \frac{2(2w-1)}{3w-5} \quad OR \quad \frac{4w-2}{3w-5} \quad --A1$$

Answer .... [3]

(c) The total surface area of a Rubik's cube is given by $294x^4 \text{ cm}^2$.

Find an expression for the volume of the Rubik's cube in the form px^n, where p and n are integers.

Area of 1 square face $= \dfrac{294x^4}{6}$
$$= 49x^4 \text{ cm}^2 \quad --M1$$

Length of cube $= \sqrt{49x^4}$
$$= 7x^2 \text{ cm} --M1 \text{ (express length in terms of } x)$$

Volume of cube $= y^3$
$$= (7x^2)^3$$
$$= 343x^6 \text{ cm}^3 \quad --A1$$

Answer .... cm^3 [3]

(d) In a recent study, it was reported that usually about 8% of the world's emperor penguin population breeds at Halley Bay.

The estimated number of emperor penguin pairs that bred in that area increased from 11 000 pairs in 2015 to 14 610 pairs in 2018.

The number of breeding pairs are expected to increase by r % every year.

Find the value of r.

By Compound Interest formula:

$$\text{Total amount} = P(1+\frac{r}{100})^{n}$$

$$14610 = 11000\left(1+\frac{r}{100}\right)^{3} \quad --M1$$

$$\sqrt[3]{\frac{14610}{11000}} = 1+\frac{r}{100} \quad --M1 \text{ (takes cube root)}$$

$$r = 9.92 \text{ (3 s.f.)} \quad --A1$$

Answer $r = $ .... [3]

2 (a) Elvin's car has the following specifications:

- Fuel capacity: 45 litres

- Estimated fuel consumption: 31.6 miles per gallon (mpg)

It is given that 1 mpg = 0.354 km per litre.

He drove the car for 280 kilometres with a full tank of fuel and was left with 15 litres of petrol.

Showing your working clearly, determine if the actual fuel consumption of the car is more or less than the estimated fuel consumption.

➤ Fuel Capacity = Full Tank = 45 litres

Actual Consumption

Amount of fuel used = 30 litres

$$30l \rightarrow 280km$$
$$1l \rightarrow 280 \div 30$$
$$= 9.33333km --M1 \text{ (convert to } km/l)$$

$$0.354 \, km/l \rightarrow 1 \, mpg$$
$$9.33333 \, km/l \rightarrow 9.33333 \div 0.354$$
$$= 26.36534 \, mpg --M1 \text{ (convert to } mpg)$$

➤ From above, we see that the actual fuel consumption is **more than** the estimated, since it covers a shorter distance for each gallon of fuel.   --- A1

Alternative possible methods:

- Calculations and comparisons done based on amount of petrol used.

- Calculations and comparisons done based on amount of petrol left.

Answer

....

....

[3]

(b) The force, F, between two particles is inversely proportional to the square of the distance, D, between them.

The distance between two particles is increased by 40%.

Calculate the percentage reduction in the force between the particles, leaving your answer correct to 1 decimal place.

$F = \dfrac{k}{D^2}$, where k is a constant

When distance increased BY 40%, new distance, $d_2 = 140\% \Rightarrow d_2 = 1.4D$

$\therefore F_2 = \dfrac{k}{(1.4D)^2}$ $\quad$ --M1

Decrease in force $= F - F_2$

$\qquad\qquad = \dfrac{k}{D^2} - \dfrac{k}{\frac{49}{25}D^2}$ $\quad$ --M1

$\qquad\qquad = \dfrac{24}{49}\left(\dfrac{k}{D^2}\right)$

Percetange reduction in force $= \dfrac{\dfrac{24}{49}\left(\dfrac{k}{D^2}\right)}{\left(\dfrac{k}{D^2}\right)} \times 100\%$

$\qquad\qquad = 48\dfrac{48}{49}\%$

$\qquad\qquad \approx 49.0\%$ (to 1 d.p.) $\quad$ --A1

Answer .... [3]

(c) The diagram shows a vessel made up of a conical base and an open cylindrical top.

The vessel has a depth of 36 cm.

The depth of the conical base is $\frac{1}{4}$ of the entire depth of the vessel.

It took 10 seconds to fill up the vessel fully.

After t seconds, the depth of the liquid is d cm.
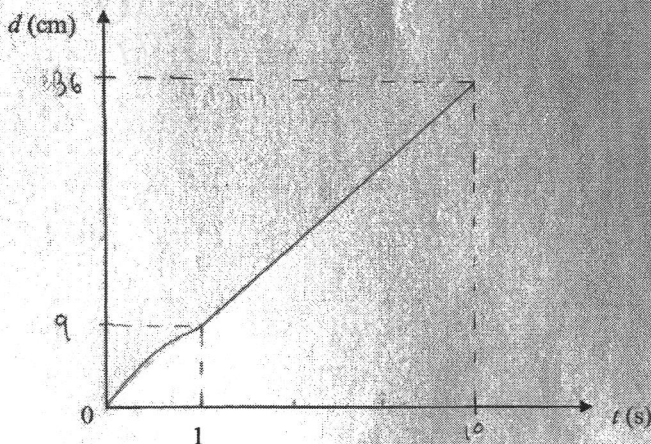
(i) Find the depth of the conical base.

36 cm
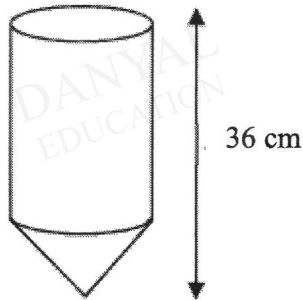
$$\frac{36}{4} = 9cm \quad --B1$$

Answer .... cm [1]

(ii) On the axes provided below, sketch a graph to show the change in the depth of liquid in the vessel within 10 seconds.

Answer

➢ Graph for cone: Increasing curve with decreasing gradient **OR** draws str. line for the 2ⁿᵈ section --- *B1*

To obtain **full 2 marks**:
➢ Must draw correct curve and str. line
➢ Indicates correct values of (10, 36) , (1, 9) [optional] & has correct proportion on axes --- *B1*

[2]

3   A fitness centre offers daily zumba classes for morning and afternoon classes.

On every weekday morning, 15 males and 10 females will attend the zumba class, while 20 males and 15 females attend the afternoon zumba class.

This information may be represented by the matrix **D** below.

$$\mathbf{D} = \begin{array}{cc} \text{males} & \text{females} \\ \begin{pmatrix} 15 & 10 \\ 20 & 15 \end{pmatrix} & \begin{array}{l} \text{morning} \\ \text{afternoon} \end{array} \end{array}$$

On each weekend, there will be 30 males and 20 females attend the morning class, while 25 males and 25 females will attend the afternoon class.

(a) Represent the number of people attending the fitness class on each weekend by a 2×2 matrix **E**.

$$E = \begin{pmatrix} 30 & 20 \\ 25 & 25 \end{pmatrix} \quad --B1$$

Answer   **E** = ... [1]

(b) Evaluate the matrix **S** = 5**D** + 2**E**.

$$S = 5\begin{pmatrix} 15 & 10 \\ 20 & 15 \end{pmatrix} + 2\begin{pmatrix} 30 & 20 \\ 25 & 25 \end{pmatrix}$$

$$= \begin{pmatrix} 135 & 90 \\ 150 & 125 \end{pmatrix} \quad --B1$$

Answer   **S** = ... [1]

(c) The fitness centre charges each person $10 for a morning session and $15 for an afternoon session. Represent the charges by a row matrix **P**.

$$P = \begin{pmatrix} 10 & 15 \end{pmatrix} \quad --B1$$

Answer   **P** = ... [1]

(d) Evaluate the matrix **T = PS**.

$$T = \begin{pmatrix} 10 & 15 \end{pmatrix} \begin{pmatrix} 135 & 90 \\ 150 & 125 \end{pmatrix}$$
$$= \begin{pmatrix} 1350+2250 & 900+1875 \end{pmatrix}$$
$$= \begin{pmatrix} 3600 & 2775 \end{pmatrix} \quad --B1$$

Answer   **T =** .... [1]

(e) State what the elements of **T** represent.

Answer   The elements of **T** represent the total charges of Zumba classes for all male and female participants respectively, over a 1-week period.   ---- B1 .... [1]

4   Hazelle recorded the mass of her newborn nephew, Kinston, every 7 days.

The table below shows some of the values she recorded.

Age of Kinston (t days)	0	7	14	21	28	35	42	49	56
Mass (m kg)	3.50	3.25	3.15	3.15	3.40	3.70	4.10		4.90

(a)   *Answer this part of the question on the grid on page 12.*

On the axes provided, draw a horizontal t-axis for $0 \le t \le 60$ and a vertical m-axis for $2 \le m \le 5$.

Plot the points given in the table and join them with a smooth curve.   [3]

(b)   Use your graph to estimate

   (i)   the mass of Kinston after 49 days,

   From graph, at 49th day, the mass is 4.5 kg. --- *A1*

   ➢   Acceptable range: 4.45 to 4.55 kg   (allowance of ± 0.05 kg)

                                     Answer .... kg [1]

   (ii)   when Kinston's mass was least,

   At minimum point, it was after 17 days that Kinston's mass was least. --- *A1*

   ➢   Acceptable range: 16 to 18 days   (allowance of ± 1 day)

                                     Answer   after .... days [1]

   (iii)   when Kinston regained his birth mass.

   From graph, it was after 31 days that Kinston regained his initial mass. --- *A1*

   ➢   Acceptable range: 30 to 32 days   (allowance of ± 1 day)

                                     Answer   after .... days [1]

(c)   (i)   By drawing a tangent, find the gradient of the curve at (7, 3.25).

   ➢   Draw acceptable tangent at (7, 3.25) --- *A1*

   ➢   Points used: (0, 3.4) & (50, 2.2) , Gradient $= -0.024$ $--A1$

   ➢   Acceptable range: -0.0204 to -0.0276 (15% allowance from above gradient)

                                     Answer .... [2]

(ii) What does this gradient represent?

➤ It represents the <u>rate of change</u> in Kinston's <u>mass</u> after 7 days. **OR**

➤ It represents the <u>rate of loss of mass</u> in Kinston after 7 days. **OR**

➤ It represents <u>how fast Kinston decreased in mass</u> after 7 days. --- *B1*

Answer ....:

....: [1]

(d) Hazelle wishes to estimate what the mass of Kinston will be after 260 days from the day he was born.

She proposes to extend the graph line up to $t = 260$.

Justify why it is not possible to estimate the mass of Kinston in this way.

➤ The mass of Kinston <u>may not always increase or decrease by the same amount</u>. **OR**

➤ The mass of Kinston <u>may not change at a constant rate</u>. **OR**

➤ <u>Day 365 is too far</u> from Day 70 for extrapolation. --- *B1*

Answer ....: [1]

4(a) Graph as plotted below:

  ➢   Mark allocation: Scale --- *B1*, Smooth curve --- *B1*, Accurate plot of points --- *B1*
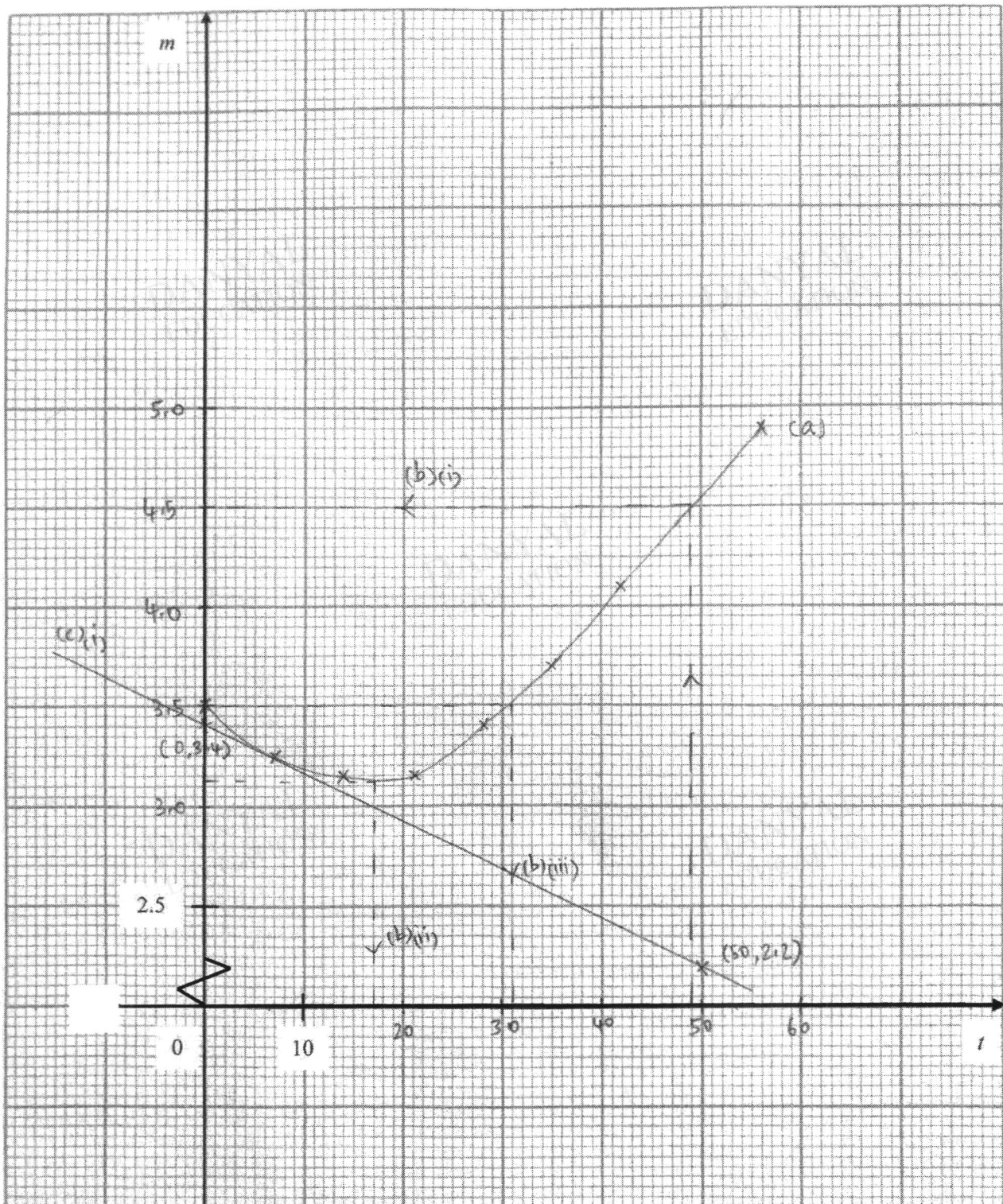
5 The first four terms in a sequence of numbers are given below.

$$T_1 = 1^2 + 2 \times 1 = 3$$
$$T_2 = 2^2 + 3 \times 3 = 13$$
$$T_3 = 3^2 + 4 \times 5 = 29$$
$$T_4 = 4^2 + 5 \times 7 = 51$$

(a) Write down the 5th line of the sequence.

$$T_5 = 5^2 + 6 \times 9 = 79 \quad --B1$$

Answer .... [1]

(b) Prove that the nth term of the sequence, T_n, is given by $3n^2 + n - 1$.

$$T_n = n^2 + (n+1)(2n-1) \quad --M1 \text{ (1st 2 parts)}, \quad --M1 \text{ (linear seq)}$$
$$= n^2 + 2n^2 - n + 2n - 1$$
$$= 3n^2 + n - 1 \text{ (proven)} \quad --A1$$

Answer .... As above .... [3]

(c) T_{m-1} and T_m are consecutive terms in the above sequence.

Find and simplify an expression, in terms of m,

(i) for $T_m - T_{m-1}$ and

$$T_m - T_{m-1} = 3m^2 + m - 1 - \left[3(m-1)^2 + (m-1) - 1 \right] \quad --M1 \text{ (Able to write } T_{m-1})$$
$$= 3m^2 + m - 1 - \left[3(m^2 - 2m + 1) + m - 2 \right] \quad --M1 \text{ (expand special product)}$$
$$= 3m^2 + m - 1 - 3m^2 + 6m - 3 - m + 2$$
$$= 6m - 2 \quad --A1$$

Answer .... [3]

(ii) explain with suitable working why the difference between two consecutive terms of the sequence will always be even.

From (c)(i), we observe that the difference of 2 consecutive terms is

$$6m - 2 = 2(3m - 1) \quad --M1 \text{ (Factorise HCF 2)}$$

Since the expression has a factor of 2, it means that the expression will result will be divisible by 2, hence resulting in an even sequence. --- *A1*

Answer .... As above .... [2]

6 A room has an enclosed space of 120 m^3.

(a) An air-conditioning (AC) unit releases cool air at a rate of x m^3 per hour.

Write down an expression, in terms of x, for the number of hours which the AC unit will take to cool the room to a particular temperature.

No. of hours needed for AC unit to cool the room $= \dfrac{120}{x}$ hours ----- B1

Answer ....hours [1]

(b) When the AC unit is faulty, it will release cool air at a rate of $(x - 450)$ m^3 per hour. Write down an expression, in terms of x, for the number of hours which the faulty AC unit will take to cool the room to the same temperature.

No. of hours needed for faulty AC unit to cool the room $= \dfrac{120}{x-450}$ hours ----- B1

Answer ....hours [1]

(c) It takes 30 minutes longer to cool the room when the AC unit is faulty.

Write down an equation in x to represent this information and show that it reduces to $x^2 - 450x - 108000 = 0$.

Faulty AC timing $=$ Normal AC timing $+ 0.5$ hours

$\dfrac{120}{x-450} = \dfrac{120}{x} + \dfrac{1}{2}$ $---M1$ (forms equation)

$\dfrac{120}{x-450} = \dfrac{240+x}{2x}$

$240x = (240+x)(x-450)$ $---M1$ (removes denominator)

$240x = 240x - 108000 + x^2 - 450x$

$x^2 - 450x - 108000 = 0$ (*shown*) $---A1$

Answer ....As above........ [3]

(d) Solve the equation $x^2 - 450x - 108000 = 0$, leaving your answers correct to 2 decimal places.

$$x^2 - 450x - 108000 = 0$$

$$x = \frac{450 \pm \sqrt{(-450)^2 - 4(1)(108000)}}{2(1)} \quad ---M1$$

$$x = -173.28 \quad or \quad x = 623.27754 \approx 623.28 \quad (2 \text{ d.p.}) \quad ---A2$$

Answer $x =$ .... [3]

(e) Hence, find the time it takes to cool the room to the same temperature when the AC unit is faulty.

Give your answer correct to the nearest minute.

➤ Since x is a volume quantity, it cannot be negative and so $x = -173.28$ is rejected.

Time taken to cool the room when to the same temperature when the AC unit is faulty

$$= \frac{120}{623.27754 - 450} \quad ---M1$$

$$= 0.69253 \text{ hours}$$

$$= 42 \text{ min (to nearest min)} \quad ---A1$$

Answer .... minutes [2]

7   In the diagram P, Q and R represent three points on level ground.

P is 15 m from Q on a bearing of $220°$.

R is 8 m from Q on a bearing of $103°$.
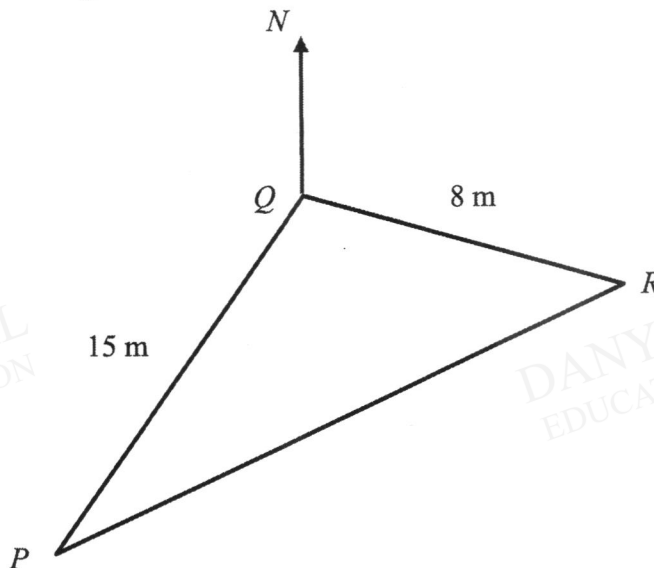
(a)   Calculate the distance of P from R.

By Cosine Rule:

$PR^2 = 15^2 + 8^2 - 2(15)(8)\cos(220° - 103°)$ $--M2$ (angle, correct substitution)

$PR = 19.948\,878$

$\approx 19.9\text{m}$ $--A1$

Answer   .... m [3]

(b)   Show that $\angle PRQ = 42.1°$, correct to 1 decimal place.

By Sine Rule:

$\dfrac{\sin \angle PRQ}{15} = \dfrac{\sin 117°}{19.948\,878}$ $--M1$

$\angle PRQ = 42.06455°$

$\approx 42.1°$ $--A1$ (shown)

Answer   .... ° [2]

(c) Calculate the bearing of *P* from *R*.

Bearing of *P* from *R*

$= 360° - 042.1° - (180° - 103°)$ (can use $\angle PRQ$ from part b)   M1

$= 240.9° --B1$

Answer ....° [2]

(d) A tower of height 10 m was built at *Q*.

Joyce leaves *R* and walks towards *P*.

(i) Calculate the distance from *R*, where she is equidistant from points *Q* and *R*.

➤ Let the point where Joyce **stops on *RP* be *Y*.**

➤ To be equidistant from line *QR*, *Y* is the <u>intersection</u> of the perpendicular bisector of line *QR* with *RP*.

$\cos 42.1° = \dfrac{4}{YR}$ $--M1$

$YR \approx 5.39\text{m}$ $--A1$

Midpoint of *QR* | 4 m | *R* | From (b) | *Y*
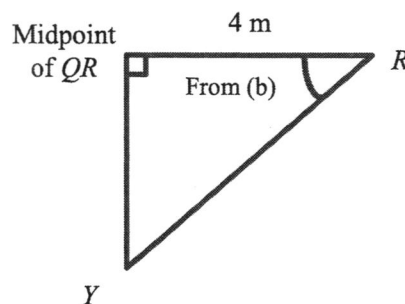
NOTE: Final answer is <u>still 5.39 m</u> if student uses more d.p. for angle PRQ.

Answer .... m [2]

(ii) Calculate the **greatest** angle of elevation of Joyce to the top of the tower as she walks from *R* to *P*.

Let <u>perpendicular distance</u> from *Q* to *RP* be *QA* and the <u>top</u> of the tower be *T*.

In triangle *QAR*, $\sin 42.1° = \dfrac{QA}{8}$ $--M1$

$QA \approx 5.36341m$

In triangle *TQA*, $\tan \angle TAQ = \dfrac{10}{5.36341}$ $--M1$

Greatest angle of elevation, $\angle TAQ = 61.8°$ $--A1$

NOTE: Final answer is <u>still 61.8 °</u> if student uses more d.p. for angle PRQ.

Answer ....° [3]

8   The diagram below shows the logo of Pac-Man.

It was a popular electronic game in the 1980s.
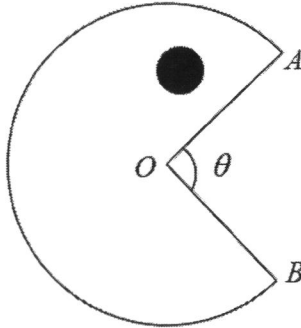
In the diagram, OA and OB are radii to the arc and $OA = OB = 5$ cm.

The eye of Pac-Man, as represented by the shaded circle, has a diameter of 1 cm.

During the game, the angle θ varies between 0 radians and $\dfrac{\pi}{2}$ radians.

(a)   Leaving your answer in terms of π and θ,

    (i)   write down the reflex angle AOB,

$$\left(2\pi - \theta\right) \text{ radians ---- B1}$$

Answer   .... radians [1]

    (ii)   find the area of major sector AOB.

$$\text{Area of major sector } AOB = \frac{1}{2}(5)^2 \left(2\pi - \theta\right)$$
$$= 12.5\left(2\pi - \theta\right) \text{ radians } --B1$$

Answer   .... cm^2 [1]

(b)   If the length of major arc AB is 24 cm, calculate angle AOB.

$$\text{Major arc } AB = 24$$
$$24 = 5\left(2\pi - \theta\right) \;\; --M1$$
$$\theta = 2\pi - 4.8$$
$$= 1.48319 \approx 1.48 \text{ radians } --A1$$

Answer   .... radians [2]

(c) Is it possible for the **maximum** length of the major arc AB to be 34 cm?

Justify your answer with suitable workings and/or mathematical reasons.

> ➢ For maximum length of major arc AB, it means that $\theta = 0$ radians thus, major arc AB becomes a circle of radius 5 cm.

Circumference of circle $= 2\pi(5)$ $--M1$

$$= 31.4159 \approx 31 \text{ cm (nearest whole number)}$$

From above, it the largest possible value for major arc AB when rounded to nearest whole number can only 31 cm and not 34 cm. Thus, it <u>will not be possible</u>. ---- $A1$

Answer ....:

....[2]

(d) The developer of Pac-Man intends to create a limited edition token of the game logo to mark the 40th anniversary of the game.

To enhance the attractiveness of the token, the designer proposed for the eye on the Pac-Man logo to be hollow and the material for the token to be acrylic.

The token will also have a thickness of 4 mm.

Taking $\theta = \dfrac{\pi}{2}$ and considering that the manufacturer charges \$0.46 per cubic centimetre of acrylic, calculate the **greatest** number of tokens that can be made with a budget of \$500.

When $\dfrac{\pi}{2}$ radians $= 90°$, the major sector AOB is $\dfrac{3}{4}$ of a circle.

*Award M2 for *Area of major sector AOB* and *Area of eye*:

Area of major sector $AOB = \dfrac{3}{4}\pi(5)^2 = \dfrac{75}{4}\pi \approx 58.904\,862 \text{ cm}^2$

OR

Area of eye $= \pi(0.5)^2 = \dfrac{\pi}{4} = 0.785\,398 \text{ cm}^2$

Cross-sectional area of manufactured token $= \dfrac{75}{4}\pi - \dfrac{\pi}{4}$

$$= 18.5\pi \approx 58.119\,464 \text{ cm}^2$$

Volume of 1 token $= 18.5\pi \times 0.4$

$$= 7.4\pi cm^3 \approx 23.247\,786 \quad --M1\,(ECF)$$

No. of tokens made with budget $= 500 \div (7.4\pi \times 0.46) \quad --M1\,(ECF)$

$$= 46.755 \approx 46 \text{ (nearest whole number)} \quad --A1$$

Answer .... tokens [5]

9   (a)   The Marina Costal Expressway (MCE) is Singapore's first undersea road.

The timings of 160 vehicles passing through a certain stretch of MCE on a Sunday morning were recorded.

The cumulative frequency curve below shows the distribution of the timings in seconds.
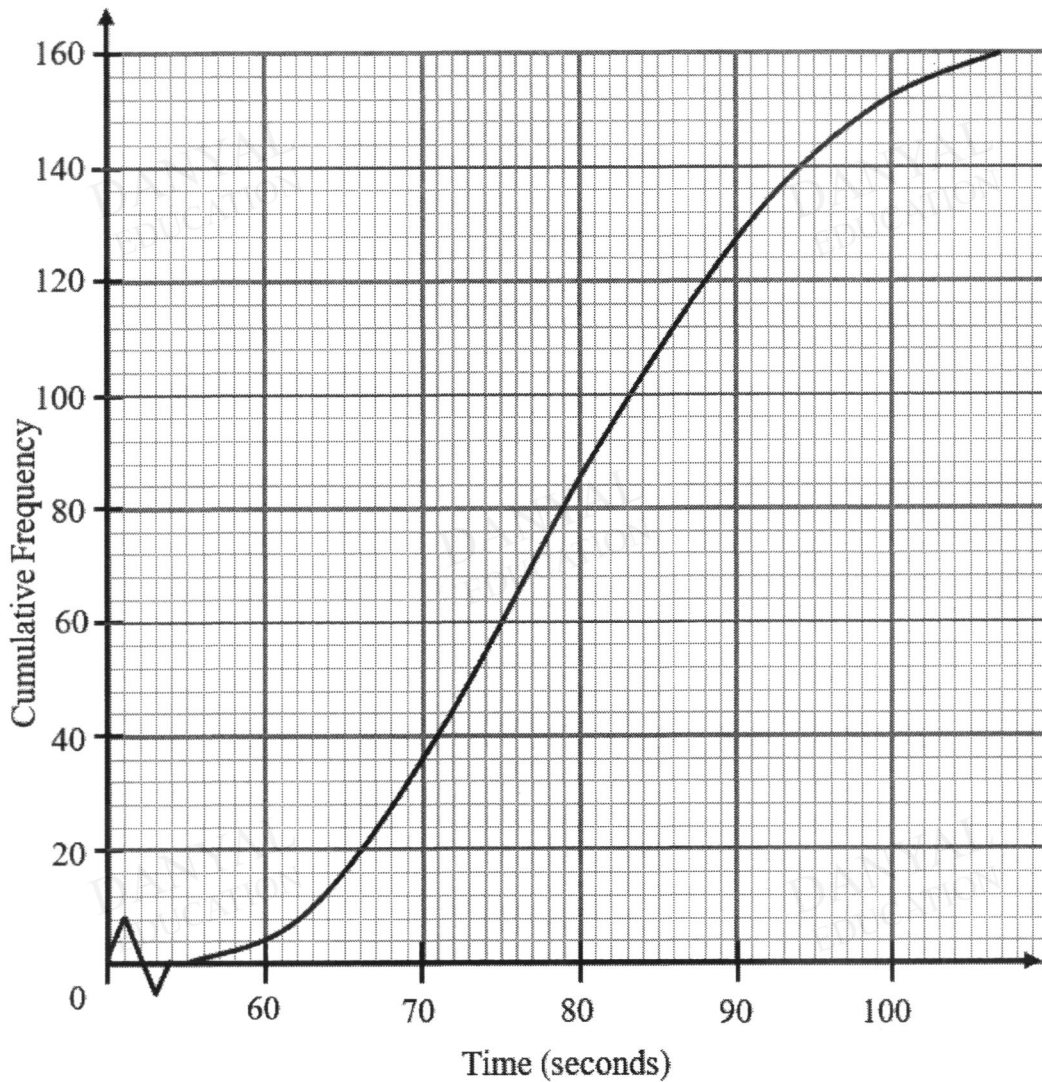
(i)   Use the graph to estimate

(a)   the median time,

Median = 79 s --- B1

Answer   .... seconds [1]

(b)   the interquartile range of the timings.

$Q_3 = 120$th term, $Q_1 = 40$th term

$IQR = 88 - 71$ $--M1$

$\quad = 17s$ $--A1$ OR $B2$

Answer .... seconds [2]

(c)   the value of x, if 30% of the vehicles took more than x seconds to pass through a certain stretch of MCE.

$P_{70} = 112$th term $--M1$

$x = 86$ $--A1$ OR $B2$

Alternative solution:

$\quad P_{30} = 48$th term $= 73$ $--M1$

112th term $= x = 87$ $--A1$ OR $B2$

Answer $x =$ .... [2]

(ii)   The timings of another 160 vehicles passing through the same stretch of MCE on a Monday morning were also recorded.

The box-and-whisker plot shows the distribution of the timings in seconds.

Time (seconds)
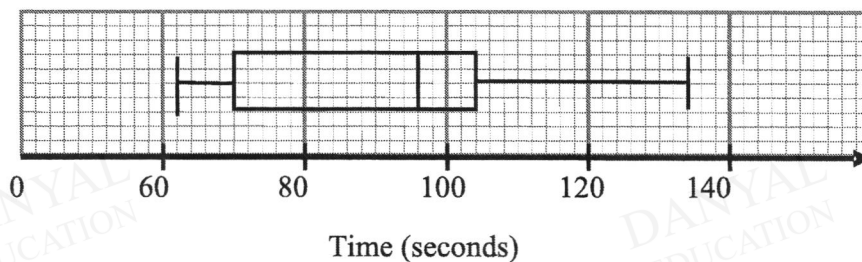
Make **two** comments to compare the timings of the vehicles on the Sunday morning and the Monday morning.

	Sunday Morning	*Monday Morning*
Median	79 seconds	96 seconds
IQR	17 seconds	34 seconds

Answer

1. **Median of Sunday** morning is of a **lower value** compared to that of Monday. This means that the **vehicles generally take a shorter time** to pass through that certain stretch of MCE. ----- B1

2. The **interquartile range of Sunday morning** is of a **lower value** compared to that of Monday. This means that the **vehicles are generally more consistent** in their timings. ----- B1

[2]

9 **(b)** A jar contains 10 identical balls of which 4 are pink and 6 are yellow.

The first person to take a yellow ball wins and the game ends.

To start the game, Emily took the first draw and Theo took the second draw.

The game continued with them taking turns, to draw a ball from the jar **without** replacement.

(i) Find the probability that Theo wins on his first draw.

P (Theo wins on his 1st draw) = P (pink, yellow)

$$= \frac{4}{10} \times \frac{6}{9} \quad --M1$$

$$= \frac{4}{15} \quad --A1 \qquad OR \qquad B2$$

Answer .... [2]

(ii) Find the probability that **no** fourth draw is required.

P (no 4th draw needed)

= P (yellow) + P (pink, yellow) + P (pink, pink, yellow)

$$= \frac{6}{10} + \left(\frac{4}{10} \times \frac{6}{9} \right) + \left(\frac{4}{10} \times \frac{3}{9} \times \frac{6}{8} \right) \quad --M1$$

$$= \frac{29}{30} \quad --A1$$

Answer .... [2]

(iii) If the game ends at the kth draw, state the largest value of k.

There are only 4 pink balls, so the 1ˢᵗ chance to pick a yellow ball, if all previous draws fail is on the 5ᵗʰ draw. So, $\underline{k = 5}$. ---- B1

Answer .... [1]

10   Veron owns a local vintage shop. She imports vintage furniture from overseas and sells them locally.

A week ago, she came across a vintage armchair from an overseas shop and intends to import it from USA.

Below are some information that Veron has gathered.

Product information for vintage armchair

Retail Price	US$ 380
Item Weight	34 pounds

NOTE:

The product and parcel dimensions stated **below** are approximated.

Product Dimensions	32 x 30 x 25 inches
Parcel Length	36 inches
Parcel Width	36 inches
Parcel Height	30 inches
Parcel Weight	40 pounds

Other useful information:

- 1 inch = 2.54 centimetres
- 1 pound = 0.454 kg
- USD 1 = SGD 1.38

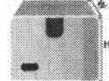## What is Volumetric Weight & Actual Weight?

Volumetric Weight (kg) = $\dfrac{L \times W \times H}{5000}$

L = Length of parcel in centimetres

W = Width of parcel in centimetres

H = Height of parcel in centimetres

Actual Weight (kg) = Parcel Weight (kg)

Source: www.vpost.com.sg

USA to Singapore Shipping Modes

Economy	Standard	Sea Freight
9 – 12 ^ working days	4 – 6 ^ working days	35 – 60 ^ working days
Actual Weight	Actual Weight or Volumetric Weight*	Bulky Items* >10kg Weight

^Working days do not include weekends, both in Singapore or product source country's and public holidays.

*For Standard Air Freight and Sea Freight, the chargeable weight of a package is calculated based on Actual Weight or Volumetric Weight, _whichever is higher_.

USA Shipping Rates

Country	Shipping Method	Base Charge	Weight Charge (Per 0.1 kg)
USA	Economy Air Freight	S$13.40	S$0.91

	Standard * Air Freight	S$13.40	S$0.91
	Sea Freight *	S$39.90	S$0.39

NOTE:

Air Freight and Sea Freight shipping modes are available when the chargeable weight is at least 0.5 kg and 10 kg respectively. Varied charges apply with an incremental weight break of 0.1 kg thereafter.

(a) Using the relevant information on page 23, calculate

(i) the **Actual Weight** in kilograms and,

Actual weight $= 40 \times 0.454$
$= 18.16 \text{ kg} \quad --B1$

Answer .... kg [1]

(ii) the **Volumetric Weight**, in kilograms, of a vintage armchair.

Volumetric weight $= \dfrac{(36 \times 2.54) \times (36 \times 2.54) \times (30 \times 2.54)}{5000} \quad --M1$
$= 127.42581 \text{ kg}$
$= 127 \text{ kg} \quad --A1$

Answer .... kg [2]

Veron needs to decide on the shipping method to import the vintage armchair from USA.

(b) Justify with calculations, a most suitable shipping method for the imported vintage armchair.

NOTE:

- *Weight charge of 0.1 kg → <u>Use 1 d.p</u> for the actual/ volumetric weights*
- *All marks in this part of the qn can <u>apply ECF</u>.*

<u>Shipping Mode: **Economy** (uses **Actual Weight**)</u>

Amount to pay = Base Charge + Incremental Weight Charge

$$= 13.40 + \left(\frac{18.2}{0.1} \times 0.91 \right) \quad --E1$$

$$\approx \$179.02 \ (2 \ d.p.) \quad --EA1$$

<u>Shipping Mode: **Standard** (uses **Volumetric Weight** since it's higher than actual weight)</u>

Amount to pay = Base Charge + Incremental Weight Charge

$$= 13.40 + \left(\frac{127.4}{0.1} \times 0.91 \right) \quad --S1$$

$$\approx \$1172.74 \ (2 \ d.p.)$$

<u>Shipping Mode: **Sea Freight** (uses **Volumetric Weight** since it's higher than actual weight)</u>

Amount to pay = Base Charge + Incremental Weight Charge

$$= 39.90 + \left(\frac{127.4}{0.1} \times 0.39 \right) \quad --M1 \text{ (uses volumetric weight)}$$

$$\approx \$536.76 \ (2 \ d.p.) \quad --MA1$$

Answer Veron should import the armchair by the **Economy class** of air freight because

it is the **cheapest among** all 3 shipping modes. ---- *A1 (ecf)*

[6]

(c) Suggest a sensible selling price in SGD for the imported vintage armchair.

Explain your answer.

NOTE:

- *All marks in this part of the qn can <u>apply ECF</u>.*

Total cost of importing armchair = Shipping Fees + Cost price of armchair

$$= 179.02 + (380 \times 1.38)$$
$$= \$703.42 \quad --M1\,(ecf)$$

➢ To ensure that she makes a profit, Veron can sell at any price **above \$703** (rounded to nearest dollar), depending on how much profit she wants to make.

➢ Suggested selling price: <u>\$900</u> ---- A1 (*ecf*)

Answer SGD ....:

Answer

....

....

[2]

~ END OF PAPER ~